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SMR.1508 - 12

*SUMMER SCHOOL ON PARTICLE PHYSICS*

**16 June - 4 July 2003**

**FLAVOUR PHYSICS**

**Part 4**

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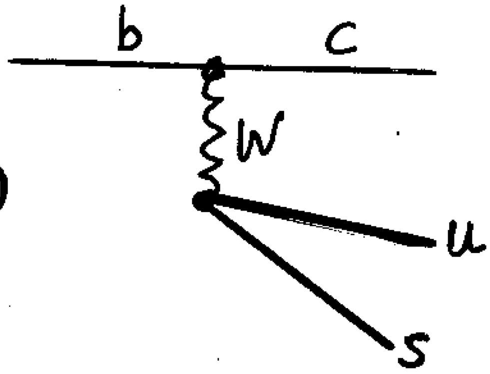
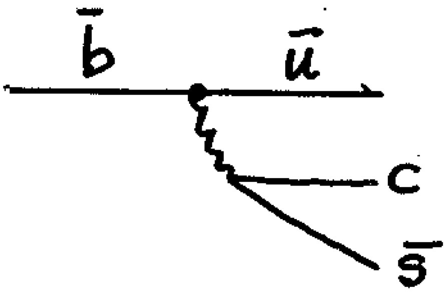
# Idea ($B \to DK$)

① 


$$\left.\begin{array}{l} B^+ \to \bar{D}^0 K^+ \\ B^- \to D^0 K^- \end{array}\right\} (V_{us} V_{cb}^*)\, e^{i\delta}$$

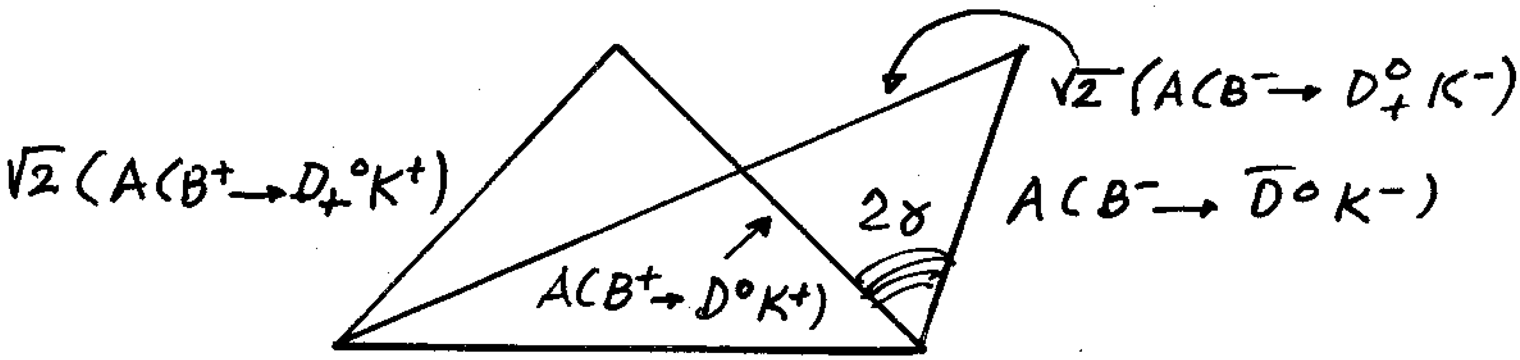
② 


$$B^+ \to D^0 K^+ : \; V_{ub}^* C_{cs} \simeq e^{i\gamma} e^{i\delta'}$$

$$B^- \to \bar{D}^0 K^- \simeq e^{-i\gamma} e^{i\delta'}$$

$$D_\pm^0 = \frac{1}{\sqrt{2}} \left( D^0 + \bar{D}^0 \right)$$

$$\Longrightarrow \quad A(B^+ \to \bar{D}^0 K^+) + A(B^+ \to D^0 K^+) = \sqrt{2}(A \to D_+^0 K^+)$$



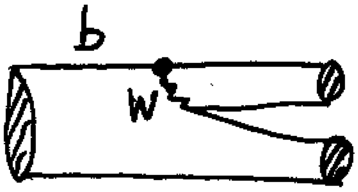

$$A(B^+ \to \bar{D}^0 K) = A(B^- \to D^0 K^-)$$

clean, but hard!

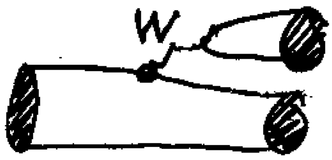
# 6. Heavy quarks; hadrons

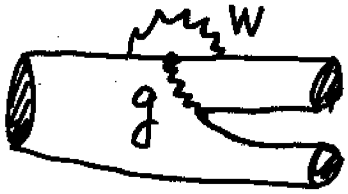
## a) General comments

- Hadronic decays: $B \to K\pi$, $B \to D\pi$, $B \to \pi\pi$ interesting for CP-violation, QCD, etc.



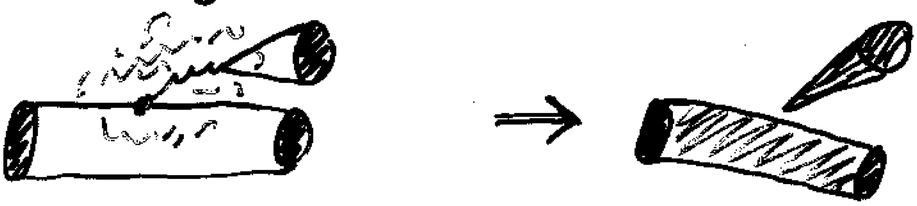

Tree level



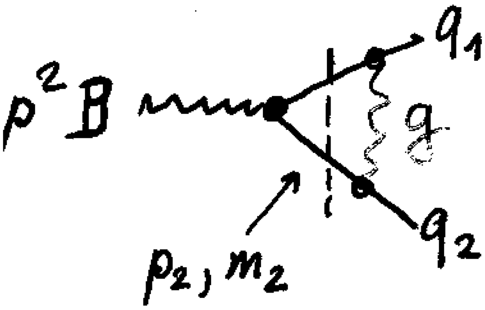




Penguin different decays

- Factorization



- Final state interaction (strong phases)




$H \to q_1 q_2$

diagram gets an imaginary part

$$\operatorname{Im} \mathcal{D} \sim \theta(q^2 - 4m^2)\, \delta(p_1^2 - m_1^2)\, \delta(p_2^2 - m_2^2) \,..$$

intermed. on mass shell

Amplitude $A \sim A e^{i\delta} \leftarrow$ scattering phase

physical picture

$$B \underset{\text{weak}}{\longrightarrow} I \underset{\text{strong}}{\longrightarrow} F \qquad \text{phase} \Longleftrightarrow \text{Intermediate states}$$

$$\text{CPT}: \sum_F \Gamma(B \to F) = \sum_F \Gamma(\bar{B} \to \bar{F})$$

If only one F : no CP-violation

$\Rightarrow$ need two states

need their phases p. C61

$$A(B \to F) = (B \to F_1 \to F) + (B \to F_2 \to F)$$

$$\text{Weak: CKM} \longrightarrow A_1 e^{i\delta_1} \quad + \quad A_2 e^{i\delta_2}$$

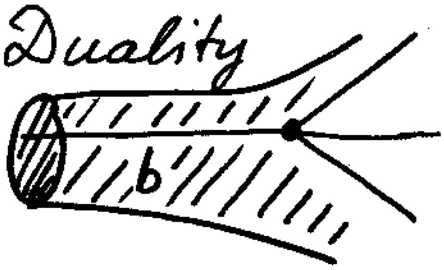
$$A(\bar{B} \to F) = (\bar{B} \to \bar{F}_1 \to \bar{F}) + (\bar{B} \to \bar{F}_2 \to \bar{F})$$

$$A_1^* e^{+i\delta_1} \quad + \quad A_2 e^{+i\delta_2}$$

$$A_{CP} \mathrel{\hat{=}} \Gamma(B \to F) - \Gamma(\bar{B} \to \bar{F})$$

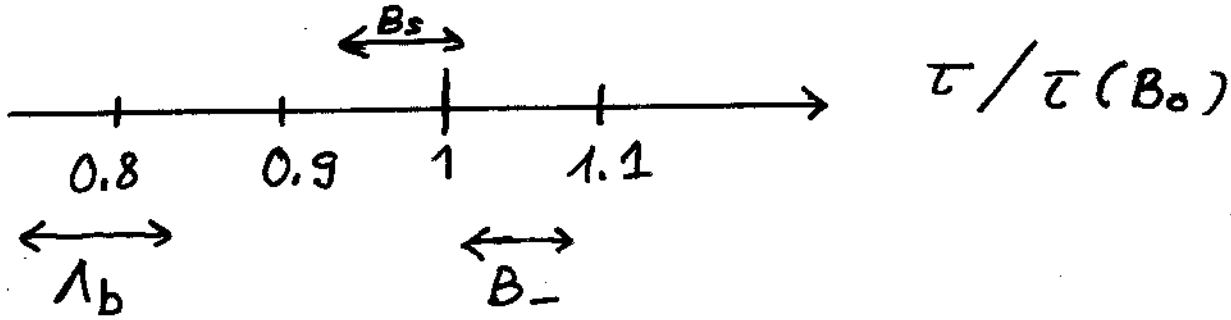
$$\simeq \mathrm{Im}(A_1 A_2^*) \sin(\delta_1 - \delta_2)$$

• Duality




$\bar{b}$ decays "unaffected" by surrounding (heavy, nucleus in atom)

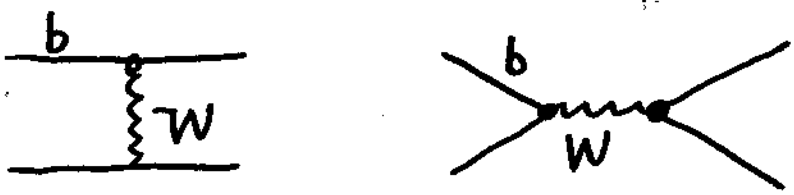
implies equal lifetimes for all Hadrons with a $b$




"Quark hadron duality"

Physical results: $A_0 \left(1 + \frac{A_1}{m} + \frac{A_2}{m^2} + \dots\right)$

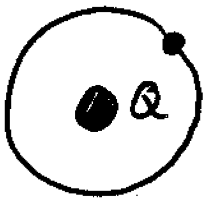
corrections: exchange + annihilation




small

Problem: $\tau(\Lambda_B)$ too small!

# HQET – short exposition




$H_Q$: hadron, containing $Q$ ($b$, $c$) and light quark $q$

$$R_{\text{hadron}} \sim \frac{1}{\Lambda_{QCD}} \leftrightarrow \text{Bohr radius } e, m$$

$$\lambda_Q \sim \frac{1}{m_Q} \ll R_{\text{hadron}}$$

* $q$ moves independently of $Q$
  (atom: e "independent of Nuleus)

  $\Rightarrow$ flavour Symmetry

  $b \leftrightarrow c$

  Compare spectra of B's and C's

* $Q$ at rest: only electric colour field
  Spin of heavy quark decouples

  $\Rightarrow$ Spin symmetry

  $B \leftrightarrow B^*$ $\quad$ $D \leftrightarrow D^*$

  relate spectra of B and $B^*$

  $$m_{B^*} - m_B \sim O(1/m_b)$$

* heavy Quark decay constants

  $$f_B \sim \frac{1}{\sqrt{m_B}} \qquad f_B/f_D = \sqrt{\frac{m_D}{m_B}}$$

Effective Lagrangian
( systematic expansion in $1/m_Q$ )

if $E \ll m_Q \Rightarrow$ no antiparticles
4 components $\rightarrow$ 2 components

$$Q \simeq \psi_{Dirac} = \begin{cases} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \sim 1 & \text{particle} : \psi_p \\ \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix} \sim \frac{E}{m} & \text{antiparticle} : \psi_{\bar{p}} \end{cases}$$

Dirac equation : $(i\not{D} - m_Q)\, Q = 0$

$D_\mu = \partial_\mu - ig A_\mu \qquad |A_\mu| \simeq \Lambda_{QCD}$

$m_Q \gg \Lambda_{QCD}$

$Q$ at rest : $p_\mu = m v_\mu \qquad v_\mu = (1,0,0,0)$

$$\left(i\gamma_0 \frac{\partial}{\partial t} - m\right) Q = ig \not{A} Q \qquad \left(i\frac{\partial}{\partial t} - m\right)\psi_p \hat{=} \Lambda Q$$

$$Q \simeq e^{-im_Q t}\, \tilde{Q} \qquad \left(i\frac{\partial}{\partial t} + m\right)\psi_{\bar{p}} \hat{=} \Lambda Q$$

$$i\frac{\partial}{\partial t} Q \simeq m \tilde{Q} e^{-im_Q t}$$

$$M = m_Q \qquad \left.\begin{aligned} (m - M)\, \tilde{\psi}_p &= \Lambda Q \\ (m + M)\, \tilde{\psi}_{\bar{p}} &= \Lambda Q \end{aligned}\right\} \; \psi_{\bar{p}} \sim \frac{\Lambda}{2m_Q}\psi_p$$

Idea: use EOM for $\tilde{\psi}_{\bar{p}}$ in terms of $\psi_p$ and get effective Lagrangian in terms of $\psi_p$ ( etc ).

Recall: $\psi_{\bar p} \sim \left(\frac{\vec p\,\vec\sigma + \vec A\,\vec\sigma}{2m}\right)\psi_p$

systematic: set $p_\mu = m v_\mu + k_\mu$

$v^2 = 1$ 4-veloc.

$k$ small $\sim \Lambda_{QCD}$

$(\not p - m)Q \hat= (m(\not v - 1) + \not k)Q = 0$

$m \to \infty$ : $m(\not v - 1)Q = 0$

$$Q = \left[\frac{(1+\not v)}{2} + \frac{(1-\not v)}{2}\right]Q = Q_+ + Q_-$$

Dirac-Eqn: $0 \cdot Q_+ = 0 \to Q_+$ leading

$Q_- = 0 \to Q_- \simeq \frac{1}{M}$

$h_v(x) = e^{im_Q v x}\, Q_+$    $v$: label

$H_v(x) = e^{im_Q v x}\, Q_-$

insert in $\mathcal{L} = \bar Q\,((p - eA)^\mu \gamma_\mu - m)Q$

and neglect small terms

$$\mathcal{L}_{HQET} = \bar h_v\, i v \mathcal{D}\, h_v$$

$$\left(\text{use } \left(\frac{1+\not v}{2}\right)\gamma^\mu\left(\frac{1+\not v}{2}\right) = v^\mu\right)$$

$\mathcal{D}_\mu = k_\mu - eA_\mu$ (no $m v_\mu$

Propagator: $\frac{1}{k \cdot v + i\varepsilon}$

Including also next term, find

$$(ivD + 2m_Q)\, H_v(x) = i \not{D}_\perp h_v \qquad \mathcal{D}_\perp^\mu = \mathcal{D}^\mu - v^\mu v\mathcal{D}$$

$$\boxed{\mathcal{L}_{eff} = h_v iv\mathcal{D} h_v + \underbrace{h_v i\not{D}_\perp \frac{1}{ivD + 2m_Q} i\not{D}_\perp h_v}_{\frac{1}{2m_Q}\left(h_v (i\not{D}_\perp)^2 h_v + \frac{g}{2} h_v \sigma^{\alpha\beta} G_{\alpha\beta} h_v + \dots\right.}}$$

use $(\vec{a}\vec{\sigma})(\vec{b}\vec{\sigma}) = \vec{a}\vec{b} + (\vec{a}\times\vec{b})\vec{\sigma}$

Effective Lagrangian for $m_Q \gg \Lambda_{QCD}$

<u>currents:</u>

- $$\bar{c}\gamma_\mu b \to \bar{h}^c_{v'} \gamma_\mu h^b_v + \frac{1}{2m_c} \bar{h}^c_{v'} (-i\overleftarrow{\not{D}}_\perp)\gamma_\mu h^b_v + \frac{1}{2m_b} \bar{h}^c_{v'} \gamma_\mu (i\not{D}_\perp) h^b_v + \dots$$

- $$\bar{q}\gamma_\mu Q_b \to \bar{q}\gamma_\mu h^b_v + \dots$$

Renormalization: replace by

$$C(m_Q, \mu)\, \bar{q}\gamma_\mu h^b_v$$

masses

$$m_H = m_Q + \overbrace{\langle H|\,\bar h_v\,(i\overset{\simeq \vec{D}}{D_\perp})^2 h_v\,|H\rangle}^{\equiv \lambda_1}\frac{1}{2m_Q}$$

$$+ \underbrace{\langle H|\,\bar h_v\,(\vec\sigma\vec B)\,h_v\,|H\rangle \frac{1}{4m_Q}}_{\equiv \lambda_2} + \bar\Lambda$$

- $D^2$ spin independent
- $\vec\sigma\vec B \sim \vec\sigma(\vec E\times\vec v) \sim \dfrac{(\vec\sigma\vec L)}{r^3 m_Q}$

$$\vec E \sim \frac{\vec r}{r^3}$$

$$\vec\sigma\vec L = \frac{1}{2}\left((\vec\sigma+\vec L)^2 - \vec\sigma^2 - \vec L^2\right) = \frac{1}{2}\left(\overset{\text{Spin}}{J}(J+1)+\dots\right)$$

$$m_B = \mathcal{M}_0 - \frac{3\lambda_2}{2m_Q}\ ;\ m_{B^*} = \mathcal{M}_0 + \frac{\lambda_2}{2m_Q}$$

$$\lambda_2 \simeq 0.12\ \text{GeV}^2$$

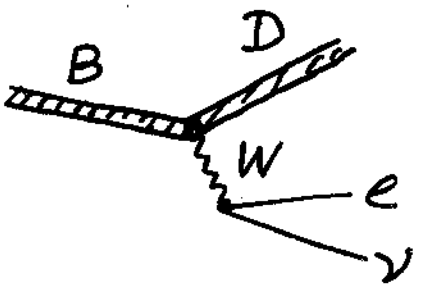
similar for $D$.

In this way construct $\mathcal{L}_{eff}(h_v)$.

$$\lambda_1 \simeq -0.3 \pm 0.2\ \text{GeV}^2$$

# Applications

## Semileptonic decays of B




$\longrightarrow V_{cb}, V_{ub}$

$$\langle D | (\bar{c}\gamma^\mu b) | B \rangle \equiv f_+(q^2)\cdot(p_B+p_D)^\mu + f_-(q^2)(p_B-p_D)^\mu$$

in HQET: $p_B = m_B v \quad p_D = m_D v'$

$$\langle \dots \rangle = \frac{1}{2}\left[(m_B+m_D)f^+ + (m_B-m_D)f^-\right]_1 (v+v')^\mu + \frac{1}{2}\left[(m_B-m_D)f^+ + (m_B+m_D)f^-\right]_2 (v-v')^\mu$$

- $v=v'$ : flavor symmetry: $b \to c$
  "no other effect"
- "Dirac-equation" for $h_v$ mass independent: current conserved, only one form factor $\left((v-v')\langle\ \rangle = 0 \to [\dots]_2 = 0\right)$

$$\langle D(v') \dots B(v) \rangle \equiv \xi(vv')^{*}(v+v')^\mu$$

$\xi(1) = 1$ Isgur/Wise

$$f^\pm = \frac{m_B \pm m_D}{2\, m_B m_D}\,\xi(vv') \qquad \omega \equiv vv'$$

$$q^2 = m_B^2 + m_D^2 - 2 m_B m_D\, vv'$$

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* why only $vv'$?

$$\frac{d\Gamma}{dw}(B \to D \ell \nu) = (\dots)\, \mathcal{F}^2(\omega)$$

$\xi(\omega)$ + corrections

$$\frac{1}{m_Q^2},\ QCD \longrightarrow \quad \mathcal{F}(1) \simeq 0.91 \pm 0.03$$

$$\frac{d\Gamma}{dw}(B \to D^* \ell \nu) = (\dots\dots)\, \mathcal{F}^2(\omega)$$

many calculations done

Total (inclusive) rates

$$\Gamma(H_b \to X) = \frac{G_F^2 m_b^5}{192\pi^2}\left\{ c\left(1 + \frac{\lambda_1 - 3\lambda_2}{2m_b^2}\right) + \dots \right\}$$

$X$ = final state (with some specification)

$X = (X_H + \ell) \to \quad c = |V_{cb}|^2 (1 - 8x^2 + \dots\dots)$

$x = m_c / m_b$

$\to$ find $V_{cb}$

# Latest developments: SCET

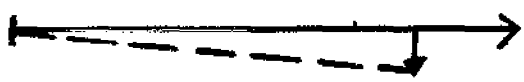
Soft-collinear effective theory

In decays of heavy mesons into light particles: <u>collinear</u> momenta

- $p_\mu \sim (1, 0, 0, 1)$: $p^2 \simeq \Lambda^2_{QCD} \ll m_B^2$

$$n = (1,0,0,1) \qquad \bar{n} = (1,0,0,-1)$$

$$p_\mu = \underbrace{\frac{(\bar{n}p)}{2}\, n_\mu}_{O(m_b)} + \underbrace{\frac{np}{2}\,\bar{n}_\mu}_{O\left(\frac{\Lambda^2}{m_b}\right)} + \underbrace{p_\mu^\perp}_{O(\Lambda)} \qquad \begin{array}{l} n^2 = 0 \\ \bar{n}^2 = 0 \\ n\bar{n} = 2 \end{array}$$



"needs QCD to go off n-directions"

quarks with collinear momenta dominate (similar to particle over antiparticle)
→ collinear particles ($q$ and $g$)

Idea: write an effective theory in terms of these fields (+ soft)

soft: $p_\mu \sim (\Lambda, \Lambda, \Lambda)$ $\quad \Lambda = \Lambda_{QCD}$

recall: $h, H \sim \left(\frac{1 \pm \not{v}}{2}\right) Q$

here $P_c = \frac{\not n \not{\bar n}}{4} q \qquad P_a = \frac{\not{\bar n} \not n}{4} q$

$$P_c^2 = P_a^2 = 1 \qquad P_c \cdot P_a = 1 \qquad P_a + P_c = 1$$

$$\not p \cdot \left( \frac{\not n \not{\bar n}}{4} q \right) \sim \not n \not n \not{\bar n} q = 0$$

$$\not p \left( \frac{\not{\bar n} \not n}{4} q \right) \sim \frac{n \not{\bar n} \not n}{4} q = \not n q \neq 0$$

Same ideas as before (p. 72/73)

$$\mathcal{L} = \bar\xi_c \left( i n D + g n A_c + \not D_\perp \frac{1}{\bar n \mathcal{D}_c} \not D_\perp \right) \xi_c$$

$$\xi_c = P_c q \qquad A_c \text{ collin. gluons}$$

Advantage: soft gluons decouple

Feynman rule

- $\bar\xi_c \, i n D \, \xi_c \Rightarrow n_\mu A^\mu \xrightarrow{\text{L.C.G.}} 0$
  light cone gauge

- General:
  Redefine fields $\xi \to W(nA)\,\xi$

$$Y = P \exp \int^x dy \cdot nA$$

recall: $\bar\psi(\partial - iA)\psi \simeq \bar\psi \partial \left( e^{i\int_0^x dy A} \right) \psi$

$\Rightarrow$ Can show several "decouplings"

$\rightarrow$ factorization

- quite technical
- not yet best formulation

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