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SMR.1508 - 16

SUMMER SCHOOL ON PARTICLE PHYSICS

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THE STANDARD MODEL AND HIGGS PHYSICS

Lecture III

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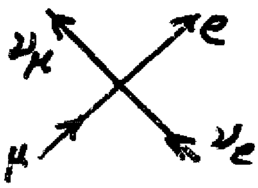
iii) Fermi Constant G_μ & τ_μ (Muon Lifetime)

Muon decay $\mu^- \to e^- \bar{\nu}_e \nu_\mu \; (\gamma, \gamma\gamma, e^+e^- \ldots)$ simple

Lifetime $\tau_\mu = 2.197035(40) \times 10^{-6}$ sec New PSI exp. $\to$ factor 20 improvement

Historical Approach Effective 4 Fermion Int.

$$\mathcal{L} = \frac{G_\mu}{\sqrt{2}} \bar{\nu}_\mu \gamma^\alpha \mu_L \bar{e}_L \gamma_\alpha \nu_e$$

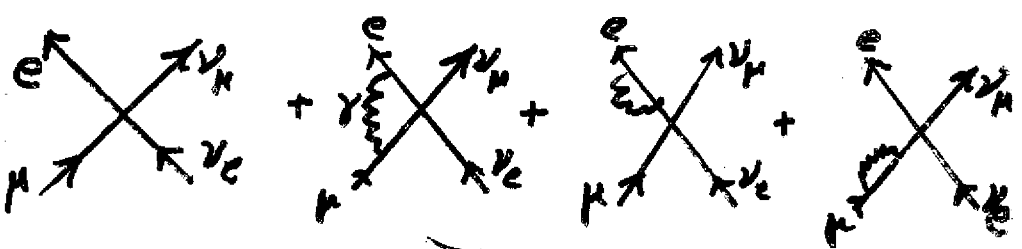


$$\tau_\mu^{-1} = \Gamma_0(\mu \to e \bar{\nu}_e \nu_\mu) = \frac{G_\mu^2 m_\mu^5}{192\pi^3} f\left(\frac{m_e^2}{m_\mu^2}\right)$$

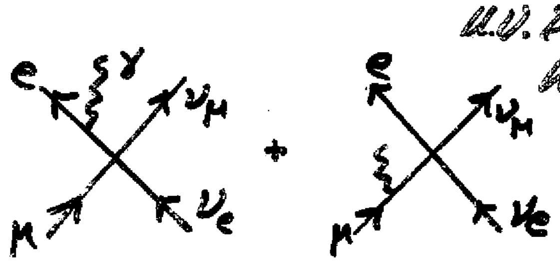
$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$ Phase-Space Factor

QED Radiative Corrections Finite! (to all orders in α!)

Fiertz:



U.V. Divergences cancel
Ward Identity



$\tau_\mu^{-1} = \Gamma(\mu \to e\nu\bar{\nu}) + \Gamma(\mu \to e\nu\bar{\nu}\gamma) \ldots = \Gamma_0 \, (1 + \text{R.C.})$ rad. corr.

$$R.C. = \frac{\alpha}{2\pi}\left(\frac{25}{4}-\pi^2\right) + O(\alpha^2)$$ Kinoshita & Sirlin, Berman

Old Fashioned Renormalization Group Eq. (Gell-Mann - Low)

$$\left(m_e\frac{\partial}{\partial m_e} + \beta(\alpha)\frac{\partial}{\partial\alpha}\right) R.C. = 0$$

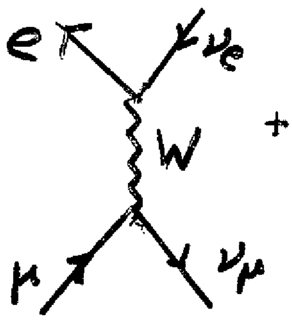
$$\beta(\alpha) = \frac{2}{3}\frac{\alpha^2}{\pi} + \frac{1}{2}\frac{\alpha^3}{\pi^2} + \cdots$$

$$R.C. = \underset{\substack{\uparrow\\ \text{1 loop}}}{\frac{\alpha}{2\pi}\left(\frac{25}{4}-\pi^2\right)}\Bigg(1 + \underset{\substack{\uparrow\\ \text{2 loop}}}{\frac{\alpha}{\pi}\left(\frac{2}{3}\ln\frac{m_\mu}{m_e} - 3.7\right)} + \underset{\substack{\uparrow\\ \text{leading + next to leading}\\ \text{3 loop}\\ C \text{ not known}}}{\left(\frac{\alpha}{\pi}\right)^2\left(\frac{4}{9}\ln^2\frac{m_\mu}{m_e} - 2.0\ln\frac{m_\mu}{m_e} + C\right)}\ldots\Bigg)$$

$$\tau_\mu^{-1} = \Gamma_0(1+R.C.) = 1/2.197035(40)\times 10^{-6}\,\text{sec}$$

$$\boxed{G_\mu = 1.16637(1)\times 10^{-5}\,\text{GeV}^{-2}}$$ Fermi Constant
Most Precise Weak
Interaction Parameter

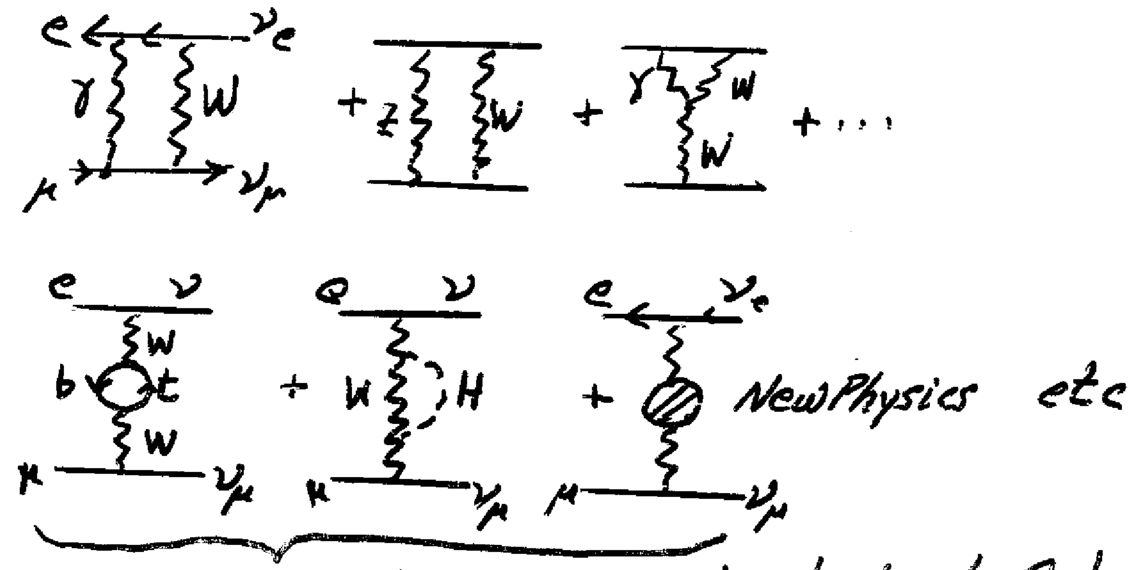
What Happens in the Standard Model?



+ higher orders $$\frac{g_{2_0}^2}{8 m_W^{0\,2}}\Big\{1 + \underbrace{\text{New Rad. Corr}}_{\substack{\text{infinite}\\ \text{+ Finite Part}}}\Big\} = \frac{G_\mu}{\sqrt{2}}$$

What New Radiative Corrections Absorbed Into G_μ?

examples:



Very interesting physics absorbed into G_μ!
(loops, W^*, dynamics...)
<u>How Can it be retrieved</u>?

By comparing G_μ against similar quantities with different quantum loops*!

* Note, recently the full 2 loop SM corrections to G_μ have been computed! An important advancement!

b) <u>m_Z, m_W, $\sin^2\theta_W$, Γ_Z, A_{LR} ...</u> Other Important Meas.

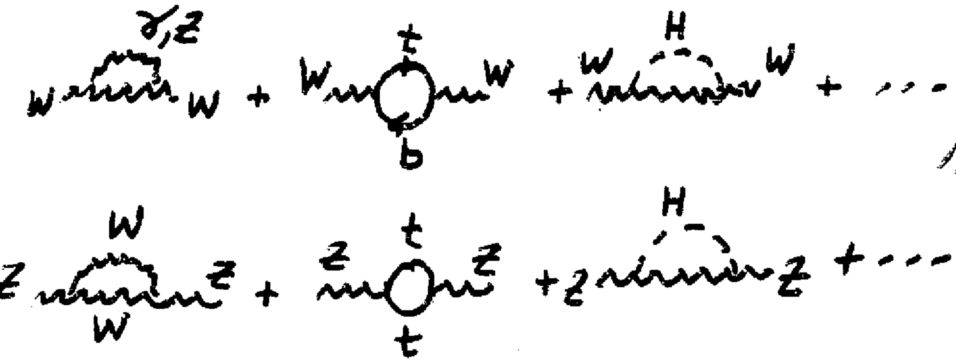
Definitions: Gauge Boson Masses

$m_W^{0\,2} = m_W^2 - \delta m_W^2$, Γ_W (full width) $\qquad \delta m^2$ = infinite counterterm

$m_Z^{0\,2} = m_Z^2 - \delta m_Z^2$, Γ_Z (full width)

Compute full one loop corrections to W + Z propagators:

Include:



Many Diagrams

Pole in propagators has a real + imaginary part

$$m_Z^2 = m_Z^2(\text{real part of pole}) + \Gamma_Z^2$$

$$m_W^2 = m_W^2(\text{real part of pole}) + \Gamma_W^2$$

Note $\Gamma_Z \simeq 2.5\,GeV$ $\quad m_Z \simeq 91\,GeV$ $\qquad \frac{1}{2}\,\Gamma_Z^2/m_Z^2 \simeq 3.8\times10^{-4}$

$3.8\times10^{-4}\,m_Z \simeq 0.034\,GeV$

Experiment: $m_Z = 91.1875(21)\,GeV$ $\quad$ very precise!

$m_W = 80.451(33)\,GeV$ $\quad$ Factor of 15 worse

The weak mixing angle $\sin^2\theta_W^0$

$\tan\theta_W^0 = g_0'/g_{20}$

$$\sin^2\theta_W^R \equiv \sin^2\theta_W^0 + \underbrace{\delta s^2}_{\substack{\text{infinite}\\ \text{counterterm}}}$$

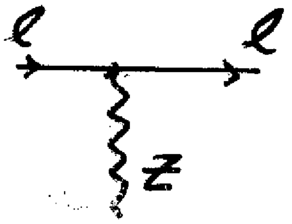
Many Choices For Finite Part

Simplest choice $\overline{MS}$ Modified Minimal Subtraction

$$\delta s^2 = \text{cons} \times \left\{ \frac{1}{n-4} + \frac{\gamma}{2} - \ln\sqrt{4\pi} \right\}$$

$\sin^2\theta_W(\mu)_{\overline{MS}}$ Not Physical, but very practical, simple

Another choice used at LEP + SLAC $\sin^2\theta_W^{\text{eff}}$



$$\frac{+ig_{Z_0}\gamma_\mu}{2\cos\theta_W^0} \left\{ \tfrac{1}{2}(1-4\sin^2\theta_W^0) - \tfrac{1}{2}\gamma_5 \right\}$$

+ quantum loop effects (very complicated)

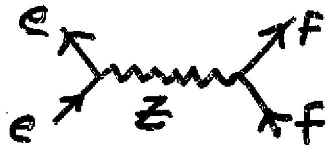
$$\downarrow$$

$$\left\{ \tfrac{1}{2}(1-4\sin^2\theta_W^{\text{eff}}) - \tfrac{1}{2}\gamma_5 \right\}$$

<u>Numerically</u> $\sin^2\theta_W^{\text{eff}} = \sin^2\theta_W(m_Z)_{\overline{MS}} + 0.00028$ (loops)

very close in value

<u>Asymmetries:</u>
at Z pole



$e^+e^- \to f\bar{f}$

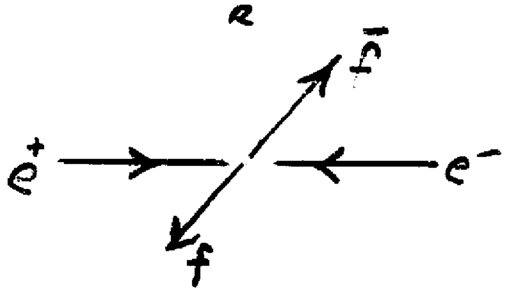
<u>Left-Right Parity Violating:</u> $A_{LR} = \dfrac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$

$e_L^- + e^+ \to$ hadrons vs $e_R^- + e^+ \to$ hadrons

$$A_{LR} = \frac{2(1-4\sin^2\theta_W^{\text{eff}})}{1+(1-4\sin^2\theta_W^{\text{eff}})^2}$$

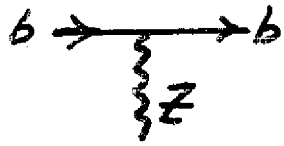
measured very precisely at SLAC

Other Asymmetries: $e^+e^- \to f\bar{f}$ Forward-Backward Asym
$e^-_{L,R}e^+ \to f\bar{f}$ Polarized Forward-Backward



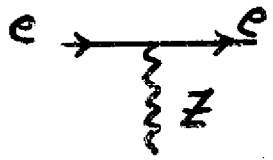
$e^+e^- \to \bar{b}b$ Forward-Backward Asym. Large gives very good determination of $\sin^2\theta_W(m_Z)_{\overline{MS}}$ (or $\sin^2\theta_W^{eff}$)

$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$



$$\frac{ig_2\gamma_\mu}{4\cos\theta_W}\left(1 - \frac{4}{3}\sin^2\theta_W - \gamma_5\right)$$

measured



$$\frac{ig_2\gamma_\mu}{4\cos\theta_W}\left(1 - 4\sin^2\theta_W - \gamma_5\right)$$

$$A_{FB}(e^+e^- \to b\bar{b}) = 3\,\frac{(1-4s^2)(1-\frac{4}{3}s^2)}{[1+(1-4s^2)^2][1+(1-\frac{4}{3}s^2)^2]} \quad + \text{ Rad. Corr.}$$

$$s^2 = \sin^2\theta_W^{eff}$$

$b\bar{b}$ FB: $\longrightarrow$ $\underline{\sin^2\theta_W(m_Z)_{\overline{MS}} = 0.2320 \pm 0.0003}$

somewhat high compared to A_{LR}, $A_{FB}(\mu^+\mu^-)$..

$\downarrow$

$\underline{\sin^2\theta_W(m_Z)_{\overline{MS}} = 0.2308 \pm 0.0002}$

About 3.5σ discrepency

Other Important Z Pole Measurements:

Γ_Z, $\Gamma(Z\to \ell^+\ell^-)$, $\Gamma(Z\to \nu\bar{\nu})$ ...

Many of the precision measurements have reached the $\pm 0.1\%$ level or better

$$\alpha^{-1} = 137.03599877(40)$$

$$G_\mu = 1.16637(1)\times 10^{-5}\,\text{GeV}^{-2}$$

$$m_Z = 91.1875(21)\ \text{GeV}$$

$$m_W = 80.451(33)\ \text{GeV} \qquad \pm 0.04!$$

$$\sin^2\theta_W(m_Z)_{\overline{MS}} = 0.2308 \pm 0.0002 \qquad \pm 0.1\%$$
(leptonic)

$$\Gamma_Z = 2.4952 \pm 0.0023\ \text{GeV} \qquad \pm 0.1\%$$

$$\Gamma(Z\to \ell^+\ell^-) = 83.984 \pm 0.086\ \text{MeV} \qquad \pm 0.1\%$$

$$\Gamma(Z\to \text{invisible}) = 499.0 \pm 1.5\ \text{MeV} \qquad \pm 0.3\%$$

$\vdots$

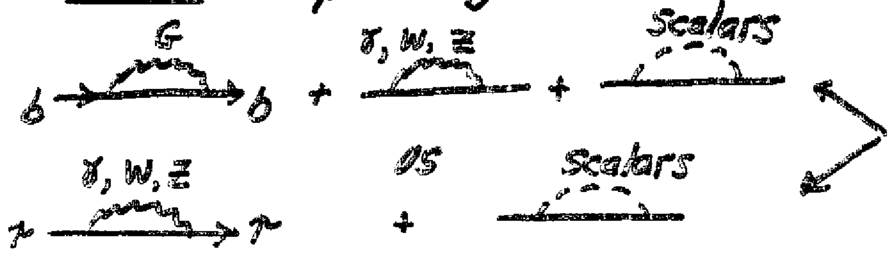
Comparison Provides Powerful Constraints on "NewPhyics"

eg. same $\sin^2\theta_W(m_Z)$ values?

4.) Natural Relations – Loop Probes

Sometimes two parameters are related at the bare level → Radiative Corrections Finite + Calculable

eg some GUTS $m_\tau^0 = m_b^0$



same divergent parts

Predict $m_b \simeq 2.5\, m_\tau$ gluonic effects

Others: All doublets have same g_{2_0} (Gauge Sym)

Rad. Corr test Universality

eg $\Gamma(W \to e\nu)$ vs $\Gamma(W \to \mu\nu)$ vs $\Gamma(W \to \tau\nu)$

or $\Gamma(\tau \to \mu\nu\bar{\nu})$ vs $\Gamma(\tau \to e\nu\bar{\nu})$

Comparisons constrain "New Physics"

More Subtle: due to $SU(2)_V$ Global Sym.

$$m_W^{0\,2} = m_Z^{0\,2} \cos^2\theta_W^0$$

$$\tan\theta_W^0 = g_0'/g_{2_0} \quad + \quad e_0 = g_{2_0} \sin\theta_W^0$$

$$\frac{e_0^2}{g_{Z_0}^2} = 1 - \frac{m_W^{0\,2}}{m_Z^{0\,2}} = \sin^2\theta_W^0 \quad \text{Natural Relation}$$

↖ ↑ ↗

Same Divergent Parts !

Radiative Corrections to ratios are finite + calculable !

Very Powerful Constraints
on m_t, m_H, "New Physics"

<u>Example</u>: Use $G_\mu = \frac{g_{Z_0}^2}{4\sqrt{2}\, m_W^{0\,2}} \Big(1 + \underbrace{\text{Rad. Corr of S.M.}}_{\text{divergent loops}} \Big)$

$$\alpha_0 = \frac{e_0^2}{4\pi} - \frac{2e_0\,\delta e_0}{4\pi} \quad \text{loops } e^+e^-, \mu^+\mu^-, q\bar{q}, W^+ \ldots$$

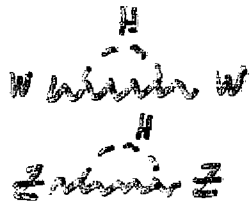
$$\left.\begin{aligned} m_W^{0\,2} &= m_W^2 - \delta m_W^2 \\ m_Z^{0\,2} &= m_Z^2 - \delta m_Z^2 \\ \sin^2\theta_W^0 &= \sin^2\theta_W - \delta s^2 \end{aligned}\right\} \text{All Computed to 1 or 2 loop order}$$

$$G_\mu \big(1 - \Delta r(m_t, m_H, \text{"New Phys."}) \big) = \frac{\pi\alpha}{\sqrt{2}\, m_W^2 \left(1 - \frac{m_W^2}{m_Z^2}\right)}$$

↑
Finite R.C.

$\Delta r \sim O\left(\frac{\alpha}{\pi} \frac{m_t^2}{m_W^2}\right)$

$O\left(\frac{\alpha}{\pi} \ln \frac{m_H}{m_Z}\right)$



Precision Measurements $\rightarrow \Delta r = 0.03206 \pm 0.0022$! 15σ effect!

$$S.M. \rightarrow \frac{\alpha}{\pi s^2}\left\{ \frac{-3}{16}\frac{m_t^2}{m_W^2}\frac{c^2}{s^2} + \frac{11}{48}\ln\frac{m_H^2}{m_Z^2} \right\} + \cdots$$

$\underbrace{\pm 0.0011}_{\text{uncertainty from Hadronic Vacuum Pol}}$

u,d,s... determine using $e^+e^- \rightarrow$ hadrons data

$\alpha = 1/137 \rightarrow \alpha(m_Z) \simeq 1/129$ ~ 7% effect

Main Source of UNC.

Remember e^+e^- vs τ data discrepency

Other Natural Relations:

$$G_\mu\left(1 - \Delta\hat{r}(m_t, m_H)\right) = \frac{2\sqrt{2}\,\pi\alpha}{m_Z^2 \sin^2 2\theta_W(m_Z)_{\overline{MS}}} \qquad \text{without } m_W$$

$$G_\mu\left(1 - \Delta r_{\overline{MS}}\right) = \frac{\pi\alpha}{\sqrt{2}\, m_W^2 \sin^2\theta_W(m_Z)_{\overline{MS}}} \qquad \text{without } m_Z$$

All Δr's Have the same hadronic vac. pol. unc. steming from α

i) Top Quark Mass

Before top discovery, R.C. $\rightarrow m_t < 200\,GeV$
with $m_t \simeq 165\,GeV$ Favored

Top Discovered at Fermilab $m_t \simeq 174.3 \pm 5.1\,GeV$

That is pole mass

$$m_t(\text{pole}) = m_t(m_t)_{\overline{MS}}\left\{1 + \frac{4}{3}\frac{\alpha_{QCD}(m_t)}{\pi} \cdots\right\}$$

$\sim +5\%$ $\rightarrow$ 8GeV difference

Loop corrections very sensitive to m_t

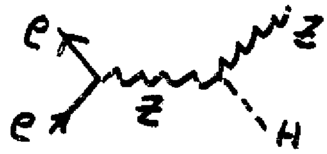
$$\Delta r \simeq 0.007 \frac{m_t^2}{m_W^2}$$

Recent Dϕ Analysis suggests $m_t \rightarrow 180\,GeV$ (?)

Current Fermilab Goal $\Delta m_t \rightarrow \pm 2\,GeV$

Long Term $e^+e^- \rightarrow t\bar{t}$ $\Delta m_t \rightarrow \pm 0.2\,GeV$ (Better ?)

iii) <u>Higgs Mass</u>

LEP II Search $e^+e^- \to ZH$  Not Seen

$\underline{m_H > 114.4\,GeV}$

Loop Corrections $\sim \frac{\alpha}{\pi} \ln \frac{m_H}{m_Z}$ mild dependence

<u>Global Fit to all precision data</u>

$$m_H \simeq 86^{+49}_{-32}\ GeV \qquad \text{(P. Langacker 2002)}$$

$$m_H < 215\ GeV\ (95\%\ CL) \qquad (\text{or } < 185\ GeV)$$

Note small errors because of small central value

<u>χ squared fit not very good</u>

m_W & $\sin^2\theta_W(m_Z)_{\overline{MS}}$ (leptonic) $\to$ m_H very small

$A_{FB}(e^+e^- \to Z \to b\bar{b})$ $\to m_H$ large

What if we only use m_W not various $\sin^2\theta_W(m_Z)_{\overline{MS}}$?

α, G_μ, m_Z + $\underline{m_W = 80.451(33)\,GeV}$ + $\Delta r(m_H)$ + $m_t = 174.3 \pm 5.1\,GeV$

$$m_H = 23^{+49}_{-23}\ GeV$$

$\underline{m_H < 122\,GeV\ (95\%\,CL)}$ squeezed

<u>If τ data is used instead of e^+e^-</u>

$$m_H = 21^{+46}_{-21}\ GeV$$

$$m_H < 115\,GeV \quad 95\%\,CL$$

<u>Problem?</u> Larger $m_t \to 180\,GeV$

Smaller $m_W \to 80.40\,GeV$

or "New Physics" in Loops

Note, $g_\mu - 2$ + m_W together suggest light Higgs and/or "New Physics"

A Sign of light SUSY loops?

Perhaps

But deviations not significant enough

Nice to have more precise: $g_\mu - 2/2$, m_W, $\sin^2\theta_W(m_Z)_{\overline{MS}}$, m_t + $e^+e^- \to$ hadrons data

$$\Delta r(m_H, \text{"NP"}) = 1 - \frac{\pi\alpha}{\sqrt{2}G_\mu m_W^2\left(1-\frac{m_W^2}{m_Z^2}\right)} \quad \sim 0.035$$

$$\Delta\hat{r}(m_H, \text{"NP"}) = 1 - \frac{4\pi\alpha}{\sqrt{2}G_\mu m_Z^2 \sin^2 2\theta_W(m_Z)_{\overline{MS}}} \quad \sim 0.060$$

$$\Delta r_{\overline{MS}} = 1 - \frac{\pi\alpha}{\sqrt{2}G_\mu m_W^2 \sin^2\theta_W(m_Z)_{\overline{MS}}}$$ (very little m_H, m_t dep.)

~ 0.0696

Predict $m_W = 80.385 \pm 0.032 \pm 0.003\ \text{GeV}$

$\times\left(1 - 0.00072 \ln m_H/100\,\text{GeV} - 1\times10^{-4}\ln^2\left(\frac{m_H}{100\,\text{GeV}}\right)\right)$

vs

$m_W^{exp} = 80.451(33)\ \text{GeV}$