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SUMMER SCHOOL ON PARTICLE PHYSICS

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LECTURES ON SUPERSYMMETRY

Lecture I

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Lectures on Supersymmetry

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Lecture 1:

- Limitations of the Standard Model
- The SUSY algebra
- Consequences of the SUSY algebra, SUSY multiplets

The Standard Model (SM) cannot be the ultimate theory

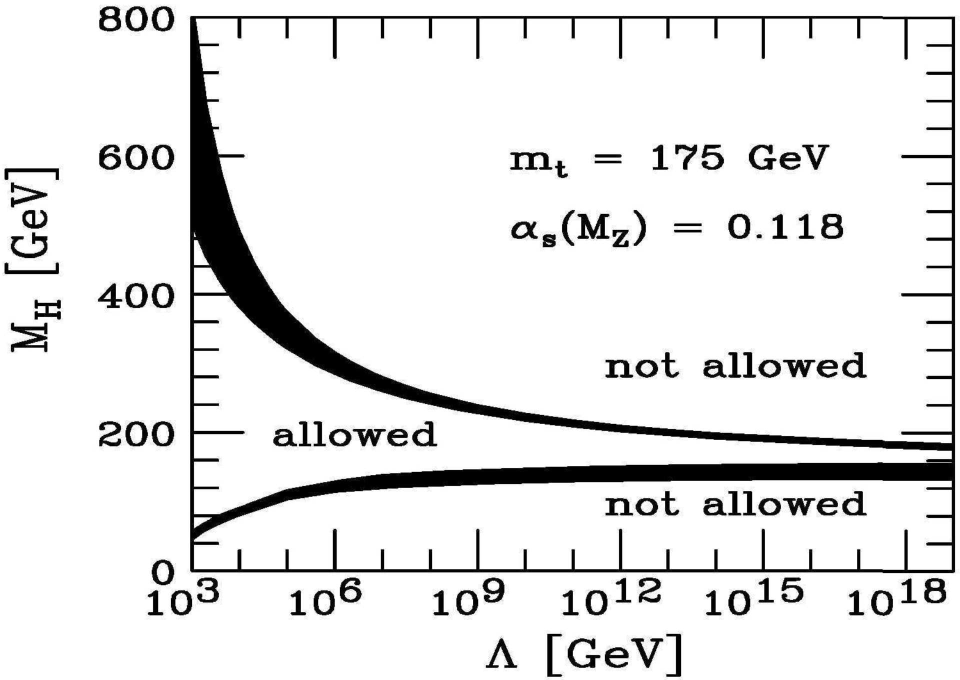
– doesn't contain gravity

– further problems: "hierarchy problem", ...

Up to which energy scale Λ can it be valid?

– $\Lambda < M_{\mathrm{Pl}} \approx 10^{19}$ GeV (inclusion of gravity effects necessary)

– stability of Higgs potential:
[*T. Hambye, K. Riesselmann '97*]



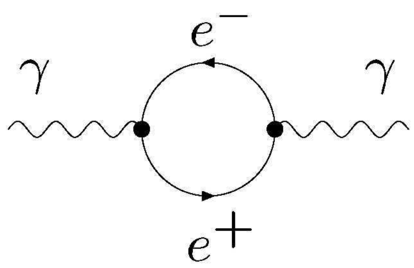
– hierarchy problem: Higgs mass unstable w.r.t. large quantum corrections
$\Rightarrow \delta M_{\mathrm{H}}^2 \sim \Lambda^2$

The hierarchy problem

Consider loop corrections to propagators ⇔ corrections to particle masses

$$\Delta(p^2) \sim \frac{1}{p^2 - m^2 + \Sigma(p^2)}$$

Photon self-energy in QED:

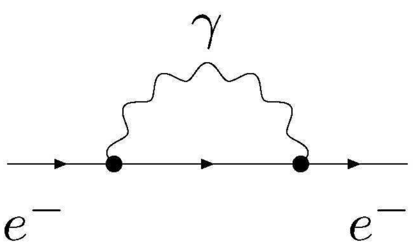


$$\Sigma^{\gamma\gamma}(0) = 0$$

consequence of U(1) gauge invariance of QED ⇒ photon stays massless

$$\Delta^{-1}_{\gamma\gamma}(p^2) \to 0 \text{ for } p^2 \to 0$$

Electron self-energy in QED:



for $\Lambda \to \infty$: $\Sigma^{ee} \sim m_\mathrm{e} \int^\Lambda \frac{dk}{k} \to \ln \Lambda$

$\Rightarrow$ logarithmically divergent correction to electron mass, δm_e

Within QED: divergence can be removed via renormalization
$\Rightarrow k \to \infty$ possible

QED as effective theory, underlying more fundamental theory at scale Λ
$\Rightarrow$ cutoff scale Λ

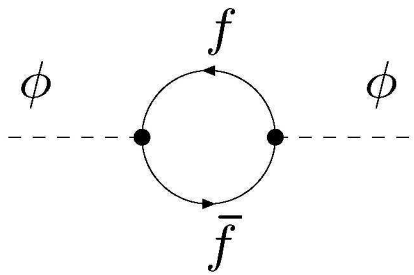
For $\Lambda = M_\mathrm{Pl}$: $\delta m_\mathrm{e} \approx 2 \frac{\alpha}{\pi} m_\mathrm{e} \log(M_\mathrm{Pl}/m_\mathrm{e}) \approx 0.2\, m_\mathrm{e}$

$\Rightarrow$ modest correction, proportional to m_e

reason: chiral symmetry in limit $m_\mathrm{e} \to 0$, $\psi_\mathrm{e} \to e^{i\gamma_5\theta}\psi_\mathrm{e}$

$\Rightarrow$ breaking proportional to $m_\mathrm{e} \Rightarrow$ symmetry “protects” m_e

Contribution of heavy fermions to Higgs self-energy:



$$\Sigma_f^{\phi\phi} \sim -2\, N(f)\, \lambda_f^2 \int d^4k \left(\frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{(k^2 - m_f^2)^2} \right)$$

for $\Lambda \to \infty$:
$$\Sigma_f^{\phi\phi} \sim -2\, N(f)\, \lambda_f^2 \left(\underbrace{\int \frac{d^4k}{k^2}}_{\sim \Lambda^2} + 2m_f^2 \underbrace{\int \frac{dk}{k}}_{\sim \ln \Lambda} \right)$$

$\Rightarrow$ quadratically divergent!

For $\Lambda = M_P$: $\delta M_\phi^2 \sim M_P^2 \Rightarrow \delta M_\phi^2 \approx 10^{30}\, M_\phi^2$ ($M_\phi \lesssim 1$ TeV)

no additional symmetry for $M_\phi = 0$, no protection against large corrections

$\Rightarrow$ in general: scalar masses tend to be near highest mass scale of the theory

$\Rightarrow$ hierarchy problem, extreme fine-tuning necessary to get small M_ϕ

⇒ Hierarchy problem is instability of small Higgs mass to large corrections in a theory with a large mass scale in addition to the weak scale

E.g.: Grand Unified Theory (GUT): $\delta M_\phi^2 \approx \lambda \langle v_{\mathrm{GUT}} \rangle^2$

Even if $\lambda = 0$ at tree level: non-zero coupling regenerated by radiative corrections: $\delta M_\phi^2 \approx \mathcal{O}(\alpha/\pi) \langle v_{\mathrm{GUT}} \rangle^2$ (cf. 'Little Higgs' models)

Would need a symmetry to suppress *many* orders of perturbation theory

Hierarchy problem is not just a problem of the Higgs mass;

problem: why is $M_{\mathrm{W}} \ll M_{\mathrm{GUT}}, M_{\mathrm{Pl}}$, why is $V_{\mathrm{Coulomb}} \gg V_{\mathrm{Newton}}$?

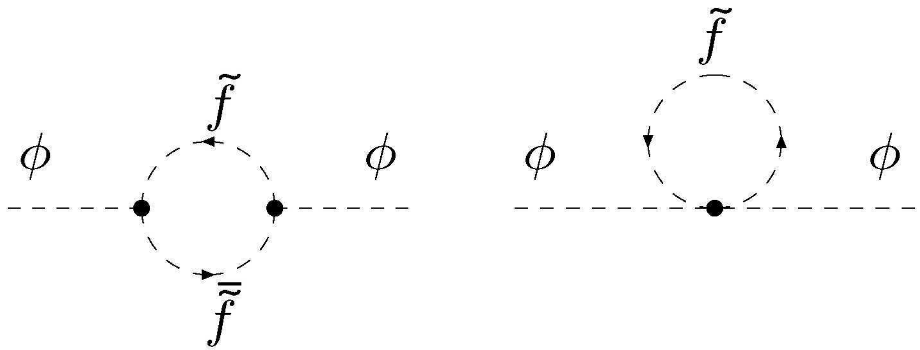
Note however: there is another fine-tuning problem in nature, for which we have no clue so far – cosmological constant

Supersymmetry:

Symmetry between fermions and bosons

$$Q|\text{boson}\rangle = |\text{fermion}\rangle$$
$$Q|\text{fermion}\rangle = |\text{boson}\rangle$$

SUSY: additional contributions from scalar fields:



$$\Sigma_{\tilde{f}}^{\phi\phi} \sim N(\tilde{f})\, \tilde{\lambda}_f \int d^4k \left(\frac{1}{k^2 - m_{\tilde{f}_L}^2} + \frac{1}{k^2 - m_{\tilde{f}_R}^2} \right) + \text{terms without quadratic div.}$$

for $\Lambda \to \infty$: $\Sigma_{\tilde{f}}^{\phi\phi} \sim 2\, N(\tilde{f})\, \tilde{\lambda}_f\, \Lambda^2$

$\Rightarrow$ quadratic divergencies cancel for

$$N(\tilde{f}_L) = N(\tilde{f}_R) \; = \; N(f)$$
$$\tilde{\lambda}_f \; = \; \lambda_f^2$$

complete correction vanishes if furthermore

$$m_{\tilde{f}} = m_f$$

For $m_{\tilde{f}}^2 = m_f^2 + \Delta^2, \quad \tilde{\lambda}_f = \lambda_f^2,$ "soft SUSY breaking"

$$\Rightarrow \; \Sigma_{f+\tilde{f}}^{\phi\phi} \sim N(f)\, \lambda_f^2\, \Delta^2 + \ldots$$

$\Rightarrow$ correction stays acceptably small if mass splitting is of weak scale

$\Rightarrow$ realized if mass scale of SUSY partners

$$M_{\text{SUSY}} \lesssim 1\,\text{TeV}$$

$\Rightarrow$ SUSY at TeV scale provides attractive solution of hierarchy problem

Other recent proposals for solving hierarchy problem in models with extra spatial dimensions:

E.g.: Gravity in large extra dimensions

[*N. Arkani-Hamed, S. Dimopoulos, G. Dvali '98*]

Gravity propagates in δ large extra spatial dimensions

fundamental Planck scale in $4+\delta$-dimensional space, M_*, is of order 1 TeV

$$M_{\mathrm{Pl}}^2 = V_\delta \, M_*^{2+\delta}$$

⇒ large value of M_{Pl} related to relatively large volume V_δ (10^{-3} m–10^{-15} m for $\delta = 2$–6) of extra dimensions

The SUSY algebra

Symmetry: a group of transformations that leave Lagrangian invariant

generators of the group fulfill certain algebra

Noether's theorem: symmetries $\Leftrightarrow$ conservation laws

Unification of fundamental interactions?

electroweak and strong interactions:

described by gauge theories: internal symmetries

γ, Z, $W^{\pm}$: spin 1

gravity:

described by general relativity: invariance under space–time transformations

graviton G: spin 2

Space–time symmetries: Poincaré group

Lorentz transformations:

$M^{\rho\nu} = -M^{\nu\rho}$ (6 generators), translations P^ρ (4 generators)

continous group: Lie-group

particle states characterized by eigenvalues of mass and spin

For later use it is convenient to describe spin $\frac{1}{2}$ particles not in terms of 4-component 'Dirac spinors' but in terms two 2-component 'Weyl spinors'

Spinor representation of the Lorentz group

Form (complex) linear combinations of the generators of rotations and Lorentz boosts such that commutation relations become

$$\left[J_i^\pm, J_j^\pm\right] = i\epsilon_{ijk} J^{\pm,k}; \quad \left[J_i^+, J_j^-\right] = 0$$

$\Rightarrow$ two SU(2) algebras, commute with each other

$\Rightarrow$ Lorentz group is equivalent to a (complexified) version of SU(2) $\times$ SU(2) ($\rightarrow$ SL(2, C))

$\Rightarrow$ Representations of Lorentz group are labelled by two 'spins', j_1, j_2, where $j_1, j_2 = 0, \frac{1}{2}, 1, \ldots$ as usual for SU(2)

Basic representations:

$(\frac{1}{2}, 0)$: LEFT-handed 2-component Weyl spinor, ψ_α

$(0, \frac{1}{2})$: RIGHT-handed 2-component Weyl spinor, $\bar{\psi}^{\dot{\alpha}}$

The two component Weyl spinors ψ_α (left-handed) and $\bar{\psi}^{\dot{\alpha}}$ (right-handed) transform under Lorentz transformations as follows:

$$\begin{aligned} \psi'_\alpha &= M_\alpha{}^\beta \psi_\beta; & \bar{\psi}'_{\dot{\alpha}} &= (M^*)_{\dot{\alpha}}{}^{\dot{\beta}} \bar{\psi}_{\dot{\beta}} \\ \psi'^\alpha &= (M^{-1})_\beta{}^\alpha \psi^\beta; & \bar{\psi}'^{\dot{\alpha}} &= (M^{*-1})_{\dot{\beta}}{}^{\dot{\alpha}} \bar{\psi}^{\dot{\beta}} \end{aligned}$$

where $M = \exp(i\frac{\vec{\sigma}}{2}(\vec{\vartheta} - i\vec{\varphi}))$ and $\vec{\vartheta}$ and $\vec{\varphi}$ are the three rotation angles and boost parameters, respectively

$\Rightarrow$ spinors with undotted indices (first two components of Dirac spinor) transform according to $(\frac{1}{2}, 0)$-representation of Lorentz group,

spinors with dotted indices (last two components of Dirac spinor) transform according to $(0, \frac{1}{2})$-representation

Internal symmetries, e.g. gauge symmetries

QED: $\psi \to e^{iQ\lambda}\psi$, $A_\mu \to A_\mu + \partial_\mu \lambda$, generator: Q, group U(1)

QCD: SU(3), SM: SU(2) $\times$ U(1)

generators T^a of internal symmetries satisfy Lie algebra, $\left[T^a, T^b\right] = i f^{abc} T_c$

‘Trivial’ extension of Poincaré group, T^a commute with Poincaré generators

$[P^\mu, T^a] = 0$, $[M^{\mu\nu}, T^a] = 0$

$\Rightarrow$ direct product

(Poincaré group) $\otimes$ (internal symmetry group)

Particle states characterized by maximal set of commuting observables:

$$|\underbrace{m, s; \vec{p}, s_3}_{\text{space–time}}; \underbrace{Q, I, I_3, Y, \ldots}_{\text{internal}}\rangle$$

quantum numbers

Direct product $\Rightarrow$ no irreducible multiplets containing particles with different mass or different spin

$\Rightarrow$ Look for extension of space–time symmetry with new generators Q_α such that
$[Q_\alpha, M^{\mu\nu}] \neq 0$ or $[Q_\alpha, P^\mu] \neq 0$

But: general "no go theorems" of QFT

Coleman–Mandula theorem '67:

Any Lie-group containing Poincaré group P and internal symmetry group G must be direct product $P \otimes G$

No go theorem can be evaded if instead of Lie-group (generators fulfill commutator relations):
[*Gol'fand, Likhtman '71*] [*Volkov, Akulov '72*] [*Wess, Zumino '73*]

$$[\ldots,\ldots] \to \{\ldots,\ldots\}$$

Anticommutator: $\{A, B\} = AB + BA$

$\Rightarrow$ 'graded Lie algebra', 'superalgebra'

Haag, Lopuszanski, Sohnius theorem '75:

no direct symmetry transformations between fields with different integer spins

$\Rightarrow$ particles with different spin in the same multiplet only possible for SUSY theories, $Q|\text{boson}\rangle = |\text{fermion}\rangle$, $Q|\text{fermion}\rangle = |\text{boson}\rangle$

symmetry generator Q: fermionic operator, needs to have spin 1/2

E.g.:

spin 2	$\rightarrow$	spin $\frac{3}{2}$	$\rightarrow$	spin 1
graviton		gravitino		photon

Q changes spin (behavior under spatial rotations) by $\frac{1}{2}$

$\Rightarrow$ SUSY transformation influences in general both space–time and internal quantum numbers

Simplest case:

only one fermionic generator Q_α (and conjugate $\bar{Q}_{\dot\beta}$, $\alpha = 1, 2$, $\dot\beta = 1, 2$)

$\Rightarrow N = 1$ SUSY algebra:

$$
\begin{aligned}
[Q_\alpha, P_\mu] = \left[\bar{Q}_{\dot\beta}, P_\mu\right] &= 0 \\
[Q_\alpha, M^{\mu\nu}] &= i\,(\sigma^{\mu\nu})_\alpha{}^\beta\, Q_\beta \\
\left\{Q_\alpha, Q_\beta\right\} = \left\{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\right\} &= 0 \\
\{Q_\alpha, \bar{Q}_{\dot\alpha}\} &= 2(\sigma^\mu)_{\alpha\dot\alpha} P_\mu
\end{aligned}
$$

Haag, Lopuszanski, Sohnius theorem $\Rightarrow$ only possibility

$P_0 = H$, $[Q_\alpha, H] = 0$ $\Rightarrow$ conserved charge

$\Rightarrow$ SUSY: symmetry that relates bosons to fermions

unique extension of Poincaré group of symmetries of $D = 4$ relativistic QFT

'generalizes classical notion of a dimension to quantum domain'

'so beautiful it must be true . . . '

Consequences of the SUSY algebra

Global SUSY transformation:

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = \underbrace{2(\sigma^\mu)_{\alpha\dot{\alpha}} P_\mu}_{\text{constant translation in space–time}}$$

If SUSY transformations are made local
⇒ space–time transformation differing from point to point

Invariance under local SUSY transformations:

⇒ invariance under local coordinate change

⇒ general relativity

⇒ local SUSY includes gravity, called "supergravity"

In the following: mostly global SUSY transformations considered
(flat space–time)

Q_α changes spin of particle by $\frac{1}{2}$

$Q_\alpha|\text{boson}\rangle = |\text{fermion}\rangle$, $Q_\alpha|\text{fermion}\rangle = |\text{boson}\rangle$

Consider fermionic state $|f\rangle$ with mass m: bosonic state $|b\rangle = Q_\alpha|f\rangle$

$P^2|f\rangle = m^2|f\rangle$

$\Rightarrow P^2|b\rangle = P^2 Q_\alpha|f\rangle = Q_\alpha P^2|f\rangle = Q_\alpha m^2|f\rangle = m^2|b\rangle$

$\Rightarrow$ for each fermionic state there is a bosonic state with the same mass

$\Rightarrow$ states are paired bosonic $\leftrightarrow$ fermionic

$$\left\{Q_\alpha, \bar{Q}_{\dot{\beta}}\right\} = 2\sigma^\mu_{\ \alpha\dot{\beta}} P_\mu$$

$$\Rightarrow \left\{Q_\alpha, \bar{Q}_{\dot{\beta}}\right\} \bar{\sigma}^{\alpha\dot{\beta}}_\nu = 2 \underbrace{\sigma^\mu_{\ \alpha\dot{\beta}} \bar{\sigma}^{\alpha\dot{\beta}}_\nu}_{2g^\mu_{\ \nu}} P_\mu = 4P_\nu$$

$$\nu = 0 \Rightarrow H = P_0 = \frac{1}{4}\left\{Q_\alpha, \bar{Q}_{\dot{\beta}}\right\} \bar{\sigma}^{\alpha\dot{\beta}}_0 = \frac{1}{4}\left(\left\{Q_1, Q_1^\dagger\right\} + \left\{Q_2, Q_2^\dagger\right\}\right)$$

where $(\sigma^\mu)_{\alpha\dot{\alpha}} \equiv \{1, \sigma^1, \sigma^2, \sigma^3\}_{\alpha\dot{\alpha}}$; $(\sigma_\mu)_{\alpha\dot{\alpha}} = g_{\mu\nu}(\sigma^\nu)_{\alpha\dot{\alpha}}$; $(\bar{\sigma}^\mu)_{\dot{\alpha}\alpha} \equiv (\sigma^\mu)_{\alpha\dot{\alpha}}$

and $\bar{Q}_{\dot{\alpha}} = (Q_\alpha)^\dagger$

$\left\{Q_i, Q_i^\dagger\right\} = Q_i Q_i^\dagger + Q_i^\dagger Q_i$: hermitean operator, eigenvalues ≥ 0

$\Rightarrow$ for any state $|\alpha\rangle$: $\langle\alpha|H|\alpha\rangle \geq 0$

spectrum of H is bounded from below, ≥ 0

$\Rightarrow$ no negative eigenvalues

State with lowest energy: vacuum state $|0\rangle$

if vacuum state is symmetric, i.e. $Q|0\rangle = 0$, $Q^\dagger|0\rangle = 0$ for all Q

$\Rightarrow$ vacuum has zero energy, $\langle 0|H|0\rangle = E_{\text{vac}} = 0$

For spontaneous symmetry breaking: vacuum state is not invariant

$\Rightarrow$ If SUSY is spontaneously broken, i.e. $Q_\alpha|0\rangle \neq 0$,

then $\langle 0|H|0\rangle = E_{\text{vac}} > 0$

$\Rightarrow$ non-vanishing vacuum energy

SUSY multiplets:

Particularly important: massless representations

go to reference frame where $P_\mu = (E, 0, 0, E)$

$\Rightarrow Q_1, Q_{\dot{1}} = 0$ as operators; thus left with $Q_2, Q_{\dot{2}}$ only

define $a = \frac{1}{\sqrt{4E}} Q_2$, $a^\dagger = \frac{1}{\sqrt{4E}} \bar{Q}_{\dot{2}}$

$\Rightarrow$ annihilation and creation operators: $\{a, a^\dagger\} = 1, \{a, a\} = 0, \{a^\dagger, a^\dagger\} = 0$

start with state of lowest helicity λ_0

application of $a^\dagger \Rightarrow$ one additional state with helicity $\lambda_0 + \frac{1}{2}$

$\Rightarrow$ one fermionic + one bosonic state

(N SUSY generators $\Rightarrow 2^{N-1}$ bosonic and 2^{N-1} fermionic states)

$\Rightarrow$ equal number of bosonic and fermionic states in supermultiplet

Most relevant multiplets (possess also CPT conjugate 'mirrors'):

- chiral supermultiplet: $-\frac{1}{2}, 0$

  Weyl fermion (quark, lepton, ...) + complex scalar (squark, slepton)

- vector supermultiplet: $-1, -\frac{1}{2}$

  Gauge boson (massless vector) + Weyl fermion (gaugino)

- graviton supermultiplet: $-2, -\frac{3}{2}$

  graviton + gravitino

Summary of Lecture 1:

- Standard Model cannot be the ultimate theory

  Hierarchy problem, . . .
  $\Rightarrow$ strong motivation for low-energy (TeV scale) SUSY

- SUSY: relates bosons to fermions

  SUSY algebra involves anticommutators: 'graded Lie algebra'

  SUSY is unique extension of Poincaré group of space–time symmetries

  local SUSY includes gravity: "supergravity"

- Exact SUSY $\Leftrightarrow E_{\text{vac}} = 0$
  Spontaneous breaking of global SUSY $\Rightarrow E_{\text{vac}} > 0$

- SUSY multiplets: states are paired bosonic $\leftrightarrow$ fermionic
  chiral, vector, graviton supermultiplet, . . .