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LECTURES ON SUPERSYMMETRY

Lecture II

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Lectures on Supersymmetry

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Lecture 2:

- Construction of SUSY field theories
- $N > 1$ SUSY
- Soft SUSY breaking
- The Minimal Supersymmetric Standard Model (MSSM)

More on spinors:

The components of the spinors are Grassmann variables, i.e. anticommuting c-numbers

Raising and lowering of indices through the totally antysymmetric ϵ-tensor:

$$\epsilon^{\alpha\beta} = \epsilon^{\dot{\alpha}\dot{\beta}} \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \;\; \epsilon_{\alpha\beta} = \epsilon_{\dot{\alpha}\dot{\beta}} \equiv \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\Rightarrow \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta; \quad \psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta; \quad \bar{\psi}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\psi}^{\dot{\beta}}; \quad \bar{\psi}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\psi}_{\dot{\beta}};$$

In particular

$$\epsilon^{\alpha}{}_{\beta} = \epsilon^{\alpha\gamma}\epsilon_{\gamma\beta} = \delta^{\alpha}{}_{\beta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The product of two Grassmann spinors is defined through

$$\theta\zeta \equiv \theta^\alpha\zeta_\alpha = \theta^\alpha\epsilon_{\alpha\beta}\zeta^\beta = -\epsilon_{\alpha\beta}\zeta^\beta\theta^\alpha = \zeta^\beta\epsilon_{\beta\alpha}\theta^\alpha = \zeta^\beta\theta_\beta = \zeta\theta$$

$$\bar{\theta}\bar{\zeta} \equiv \bar{\theta}_{\dot{\alpha}}\bar{\zeta}^{\dot{\alpha}} = -\bar{\zeta}^{\dot{\alpha}}\bar{\theta}_{\dot{\alpha}} = \bar{\zeta}\bar{\theta}$$

The γ-matrices are defined by

$$\gamma^\mu \equiv \begin{pmatrix} 0 & (\sigma^\mu)_{\alpha\dot\alpha} \\ (\bar\sigma^\mu)^{\dot\alpha\alpha} & 0 \end{pmatrix}; \quad \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

have the usual commutation relation, $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$; follows from

$$(\sigma^\mu)_{\alpha\dot\alpha}(\bar\sigma^\nu)^{\dot\alpha\alpha} = \mathrm{Tr}(\sigma^\mu\bar\sigma^\nu) = 2g^{\mu\nu}$$

The Lorentz covariant expressions involving 4-component Dirac spinors can be written in two-component notation using

$$\Phi = \begin{pmatrix} \lambda_\alpha \\ \bar\phi^{\dot\alpha} \end{pmatrix}; \quad \overline{\Psi} = \begin{pmatrix} \chi^\alpha & \bar\psi_{\dot\alpha} \end{pmatrix}$$

$$\begin{aligned}
\overline{\Psi}\Phi &= \chi\lambda + \bar\psi\bar\phi = \chi^\alpha\lambda_\alpha + \bar\psi_{\dot\alpha}\bar\phi^{\dot\alpha} \\
\overline{\Psi}\gamma^5\Phi &= \bar\psi\bar\phi - \chi\lambda = \bar\psi_{\dot\alpha}\bar\phi^{\dot\alpha} - \chi^\alpha\lambda_\alpha \\
\overline{\Psi}\gamma^\mu\Phi &= \chi\sigma^\mu\bar\phi - \lambda\sigma^\mu\bar\psi = \chi^\alpha(\sigma^\mu)_{\alpha\dot\alpha}\bar\phi^{\dot\alpha} - \lambda^\alpha(\sigma^\mu)_{\alpha\dot\alpha}\bar\psi^{\dot\alpha} \\
\overline{\Psi}\gamma^\mu\gamma^5\Phi &= \chi\sigma^\mu\bar\phi + \lambda\sigma^\mu\bar\psi = \chi^\alpha(\sigma^\mu)_{\alpha\dot\alpha}\bar\phi^{\dot\alpha} + \lambda^\alpha(\sigma^\mu)_{\alpha\dot\alpha}\bar\psi^{\dot\alpha} \\
\overline{\Psi}\gamma^\mu\gamma^\nu\Phi &= \chi\sigma^\mu\bar\sigma^\nu\lambda + \bar\psi\bar\sigma^\mu\sigma^\nu\bar\phi = \chi^\alpha(\sigma^\mu)_{\alpha\dot\alpha}(\bar\sigma)^{\dot\alpha\beta}\lambda_\beta + \bar\psi_{\dot\alpha}(\bar\sigma^\mu)^{\dot\alpha\beta}(\sigma^\nu)_{\beta\dot\beta}\bar\phi^{\dot\beta}
\end{aligned}$$

Superspace and superfields

For compact representation of SUSY transformations

⇒ 'fermionic variables' θ^α, $\bar{\theta}^{\dot{\alpha}}$ introduced: anticommuting c-numbers (Grassmann variables)

E.g.: translations, generators P_μ, parameters of transformation x^μ

$N = 1$ SUSY, generators Q_α, $\bar{Q}_{\dot{\alpha}}$, parameters of transformation θ^α, $\bar{\theta}^{\dot{\alpha}}$

⇒ Extension of 4-dim. space–time by coordinates θ^α, $\bar{\theta}^{\dot{\alpha}}$: superspace

Point in superspace: $X = (x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$

Superfield: $\phi(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$

Taylor expansion in Grassmann variable: $\theta^\alpha \theta^\beta \theta^\gamma = 0$

$\Rightarrow$ Taylor expansion terminates after second term, i.e. $\phi(\theta) = a + \theta\psi + \theta\theta\, f$ $(\theta\theta \equiv \theta^\alpha \theta_\alpha)$

Integration: $\int d\theta = 0$, $\int d\theta\, \theta = 1$

$d^2\theta = -\frac{1}{4}\epsilon_{\alpha\beta} d\theta^\alpha d\theta^\beta$

$$\Rightarrow \quad \int d^2\theta\, \phi(\theta) = \int d^2\theta (a + \theta\psi + \theta\theta\, f) = f$$

With Grassmann variables:

SUSY algebra can be written in terms of commutators only

$$\begin{aligned} [\theta Q, \bar\theta\bar Q] &= 2\theta\sigma^\mu\bar\theta P_\mu \\ [\theta Q, \theta Q] = [\bar\theta\bar Q, \bar\theta\bar Q] &= 0 \\ [P^\mu, \theta Q] = [P^\mu, \bar\theta\bar Q] &= 0 \end{aligned}$$

$\Rightarrow$ can be treated like Lie-group with anticommuting parameters

SUSY transformations

Group element of finite SUSY transformation:

$$S(y,\xi,\bar{\xi}) = \exp i\left(\xi Q + \bar{\xi}\bar{Q} - y^\mu P_\mu\right)$$

in analogy to group elements for Lie-groups

ξ, $\bar{\xi}$ are independent of x^μ: global SUSY transformation

Transformation of superfield: $S(y,\xi,\bar{\xi})\phi(x,\theta,\bar{\theta})$

Use Hausdorff's formula ($e^A e^B = \ldots$) and SUSY algebra

$$\Rightarrow \quad S(y,\xi,\bar{\xi})\phi(x,\theta,\bar{\theta}) = \phi(x^\mu + y^\mu - i\xi\sigma^\mu\bar{\theta} + i\theta\sigma^\mu\bar{\xi}, \xi+\theta, \bar{\xi}+\bar{\theta})$$

representations of generators obtained from infinitesimal transformation of superfield

$$\Rightarrow \quad P_\mu = i\partial_\mu, \quad Q_\alpha = -i\,\partial_\alpha + (\sigma^\mu\bar{\theta})_\alpha\partial_\mu, \quad \bar{Q}_{\dot{\alpha}} = i\,\bar{\partial}_{\dot{\alpha}} - (\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu$$

Definition of covariant derivatives:

$$D_\alpha = -i\,\partial_\alpha - (\sigma^\mu \bar{\theta})_\alpha \partial_\mu, \quad \bar{D}_{\dot{\alpha}} = i\,\bar{\partial}_{\dot{\alpha}} + (\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu$$

Anticommutation relations of Q_α, D_α:

$$\begin{aligned}
\{Q_\alpha, D_\beta\} &= \{\bar{Q}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = \{Q_\alpha, \bar{D}_{\dot{\beta}}\} = 0 \\
\{Q_\alpha, Q_\beta\} &= \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0 \\
\{D_\alpha, D_\beta\} &= \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0 \\
\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} &= 2i(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu = 2(\sigma^\mu)_{\alpha\dot{\alpha}}P_\mu \\
\{D_\alpha, \bar{D}_{\dot{\alpha}}\} &= -2i(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu = -2(\sigma^\mu)_{\alpha\dot{\alpha}}P_\mu
\end{aligned}$$

$\Rightarrow$ D_α, $\bar{D}_{\dot{\alpha}}$ anticommute with SUSY generators

$\Rightarrow$ are invariant under SUSY transformations

$$[(\xi Q + \bar{\xi}\bar{Q}), D_\alpha] = 0, \quad [(\xi Q + \bar{\xi}\bar{Q}), \bar{D}_{\dot{\alpha}}] = 0$$

General superfield in component form

Most general form of field depending on x, θ, $\bar{\theta}$:

$$\begin{aligned}\Phi(x,\theta,\bar{\theta}) \;&=\; \varphi(x) + \theta\psi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta F(x) + \bar{\theta}\bar{\theta} H(x) + \theta\sigma^{\mu}\bar{\theta} A_{\mu}(x) \\ &\quad + (\theta\theta)\bar{\theta}\bar{\lambda}(x) + (\bar{\theta}\bar{\theta})\theta\xi(x) + (\theta\theta)(\bar{\theta}\bar{\theta})D(x)\end{aligned}$$

Further terms vanish because of $\theta\theta\theta = 0$

Components (can be complex):

ϕ, F, H, D: scalar fields

A_{μ}: vector field

ψ, $\bar{\chi}$, $\bar{\lambda}$, ξ: Weyl-spinorfields

$\Rightarrow$ Too many components in 4-dim. for irreducible representation of SUSY with spin ≤ 1 (chiral or vector multiplet)

$\Rightarrow$ representation is reducible
(not all component fields mix with each other under SUSY transf.)

⇒ Irreducible superfields (smallest building blocks) from imposing conditions on general superfield
conditions need to be invariant under SUSY transformations:

$\bar{D}_{\dot{\alpha}}\Phi = 0$: left-handed chiral superfield (LHχSF)

$D_{\alpha}\Phi = 0$: right-handed chiral superfield (RHχSF)

$\Phi = \Phi^{\dagger}$: vector superfield

Usefulness of two-component spinors:

SM fermions: left-handed and right-handed components transform differently

⇒ need superfields with only two fermionic degrees of freedom

⇒ chiral superfields describe left- or right-handed component of SM fermion + scalar partner

LHχSF in components:

$$\begin{aligned}\phi(x,\theta,\bar\theta) &= \varphi(x) + \sqrt{2}\theta\psi(x) - i\theta\sigma^\mu\bar\theta\partial_\mu\varphi(x) + \frac{i}{\sqrt{2}}(\theta\theta)(\partial_\mu\psi(x)\sigma^\mu\bar\theta) \\ &\quad - \frac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\partial^\mu\partial_\mu\varphi(x) - (\theta\theta)F(x)\end{aligned}$$

φ, F: scalar fields, ψ: Weyl-spinor field

F: auxiliary field, unphysical (has mass dimension 2)

RHχSF:

$$\bar\phi(x,\theta,\bar\theta) = \left(\phi(x,\theta,\bar\theta)\right)^\dagger$$

Transformation of component fields under infinitesimal SUSY transf.:

$$\delta\phi(x,\theta,\bar\theta) = i(\xi Q + \bar\xi\bar Q)\phi(x,\theta,\bar\theta)$$

Comparison with

$$\delta\phi(x,\theta,\bar\theta) = \delta\varphi + \sqrt{2}\theta\delta\psi - \ldots - (\theta\theta)\delta F$$

$\Rightarrow$ determination of $\delta\varphi$, $\delta\psi$, δF:

$$\begin{aligned} \delta\varphi &= \sqrt{2}\xi\psi && \text{boson} \to \text{fermion} \\ \delta\psi_\alpha &= -\sqrt{2}F\xi_\alpha - i\sqrt{2}(\sigma^\mu\bar\xi)_\alpha\partial_\mu\varphi && \text{fermion} \to \text{boson} \\ \delta F &= \partial_\mu(-i\sqrt{2}\psi\sigma^\mu\bar\xi) && F \to \text{total derivative} \end{aligned}$$

Vector superfield in components:

$$\begin{aligned} V(x,\theta,\bar\theta) &= c(x) + i\theta\chi(x) - i\bar\theta\bar\chi(x) + \theta\sigma^\mu\bar\theta v_\mu(x) \\ &+ \frac{i}{2}(\theta\theta)(M(x)+iN(x)) - \frac{i}{2}(\bar\theta\bar\theta)(M(x)-iN(x)) \\ &+ i(\theta\theta)\bar\theta\left(\bar\lambda(x) + \frac{i}{2}\partial_\mu\chi(x)\sigma^\mu\right) - i(\bar\theta\bar\theta)\theta\left(\lambda(x) - \frac{i}{2}\sigma^\mu\partial_\mu\bar\chi(x)\right) \\ &+ \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)\left(D(x) - \frac{1}{2}\partial^\mu\partial_\mu c(x)\right) \end{aligned}$$

$v_\mu(x)$: real vector field $\rightarrow$ describes gauge boson
(V must transform as adjoint representation of gauge group)

Number of components can be reduced by SUSY gauge transformation:

$$e^{gV} \longrightarrow e^{-ig\Lambda^\dagger} e^{gV} e^{ig\Lambda},$$

where $\Lambda(x,\theta,\bar\theta)$ is a chiral superfield and g is the gauge coupling

can perform a transformation such that $\chi(x) = c(x) = M(x) = N(x) \equiv 0$:
"Wess–Zumino gauge"

Wess–Zumino gauge removes many unphysical degrees of freedom

still leaves "ordinary" gauge freedom, e.g. $A_\mu(x) \longrightarrow A_\mu(x) + \partial_\mu \lambda(x)$ for abelian theory

Wess–Zumino gauge not preserved under SUSY transformations

Transformation of component fields under infinitesimal SUSY transf.:

$$\delta\lambda = \ldots, \quad \delta v_\mu = \ldots, \ldots, \quad \delta D = -\xi\sigma^\mu\partial_\mu\bar{\lambda}(x) - \partial_\mu\lambda(x)\sigma^\mu\bar{\xi}$$

$\Rightarrow$ D transforms into a total derivative

Supersymmetric Lagrangians

Aim: construct an action that is invariant under SUSY transformations:

$$\delta \int d^4x \mathcal{L}(x) = 0$$

Satisfied if $\mathcal{L} \longrightarrow \mathcal{L} +$ total derivative

F and D terms (the terms with the largest number of θ and $\bar{\theta}$ factors) of chiral and vector superfields transform into a total derivative under SUSY transformations

⇒ Use F-terms and D-terms to construct an invariant action:

$$S = \int d^4x \left(\int d^2\theta \mathcal{L}_f + \int d^2\theta d^2\bar{\theta} \mathcal{L}_d \right)$$

If Φ is a LHχSF ⇒ Φ^n is also a LHχSF (since $\bar{D}_{\dot{\alpha}}\Phi^n = 0$ for $\bar{D}_{\dot{\alpha}}\Phi = 0$)

⇒ products of chiral superfields are chiral superfields, products of vector superfields are vector superfields

F-term Lagrangian:

$$\mathcal{L}_F = \int d^2\theta \sum_{ijk} \left(a_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} \lambda_{ijk} \Phi_i \Phi_j \Phi_k \right) + \text{ h.c.}$$

Terms of higher order in Φ_i lead to non-renormalizable Lagrangians

$\Rightarrow$ F-term Lagrangian contains mass terms, scalar–fermion interactions ($\rightarrow$ superpotential), but no kinetic terms

D-term Lagrangian:

$$\mathcal{L}_D = \int d^2\theta d^2\bar{\theta} V$$

$\Rightarrow$ D-term Lagrangian contains kinetic terms

Example: the Wess–Zumino Lagrangian

Construction of Lagrangian from chiral superfields Φ_i

$\Rightarrow$ Φ_i, $\Phi_i\Phi_j$, $\Phi_i\Phi_j\Phi_k$

$\Phi_i^\dagger\Phi_i$: vector superfield, $(\Phi_i^\dagger\Phi_i)^\dagger = \Phi_i^\dagger\Phi_i$

$$\left[\Phi_i^\dagger\Phi_i\right]_{\theta\theta\bar\theta\bar\theta} = F^\dagger F + (\partial_\mu\varphi^*)(\partial^\mu\varphi) + \frac{i}{2}(\psi\sigma^\mu\partial_\mu\bar\psi - \partial_\mu\psi\sigma^\mu\bar\psi) + \partial_\mu(\ldots)$$

Auxiliary field F can be eliminated via equations of motion

$$\begin{aligned}\Rightarrow \quad \mathcal{L} \; = \; & \frac{i}{2}(\psi_i\sigma^\mu\partial_\mu\bar\psi_i - (\partial_\mu\psi_i)\sigma^\mu\bar\psi_i) - \frac{1}{2}m_{ij}(\psi_i\psi_j + \bar\psi_i\bar\psi_j) \\ & + (\partial_\mu\varphi_i^*)(\partial^\mu\varphi_i) - \sum_i \left| a_i + \frac{1}{2}m_{ij}\varphi_j + \frac{1}{3}\lambda_{ijk}\varphi_j\varphi_k \right|^2 \\ & - \lambda_{ijk}\varphi_i\psi_j\psi_k - \lambda^\dagger_{ijk}\varphi_i^*\bar\psi_j\bar\psi_k\end{aligned}$$

Lagrangian for scalar fields φ_i and spinor fields ψ_i with the same mass m_{ii} contains couplings of type $hf\bar{f}$ and $\tilde{h}\bar{\tilde{h}}\tilde{f}$ with the same strength

⇒ SUSY implies relations between masses and couplings

$\mathcal{L}$ can be rewritten as kinetic part + contribution of superpotential $\mathcal{V}$:

$$\mathcal{V}(\varphi_i) = a_i\varphi_i + \frac{1}{2}m_{ij}\varphi_i\varphi_j + \frac{1}{3}\lambda_{ijk}\varphi_i\varphi_j\varphi_k$$

$$\Rightarrow \quad \mathcal{L} = \frac{i}{2}(\psi_i\sigma^\mu\partial_\mu\bar{\psi}_i - (\partial_\mu\psi_i)\sigma^\mu\bar{\psi}_i) + (\partial_\mu\varphi_i^*)(\partial^\mu\varphi_i) - \sum_i\left|\frac{\partial\mathcal{V}}{\partial\varphi_i}\right|^2 - \frac{1}{2}\frac{\partial^2\mathcal{V}}{\partial\varphi_i\partial\varphi_j}\psi_i\psi_j - \frac{1}{2}\frac{\partial^2\mathcal{V}^*}{\partial\varphi_i^*\partial\varphi_j^*}\bar{\psi}_i\bar{\psi}_j$$

$\mathcal{V}$ determines all interactions and mass terms

Special case $a_i = 0$: Wess–Zumino model

In order to include vector bosons need to construct Lagrangian containing also vector superfields

Further requirement: gauge invariance

$$\int d^2\theta d^2\bar{\theta}\Phi^\dagger\Phi \longrightarrow \int d^2\theta d^2\bar{\theta}\Phi^\dagger e^{2gV}\Phi$$

Kinetic terms for gauge fields from field strength

⇒ Supersymmetric Lagrangian in the Wess–Zumino gauge for chiral superfields ϕ_i (with component fields $\varphi_i,\ \psi_i$) and vector superfields V^a (with component fields $v^a_\mu,\ \lambda^a$):

$$\begin{aligned}\mathcal{L} \ = \ & (D_\mu\varphi)^*_i(D^\mu\varphi)_i + \frac{i}{2}\psi_i\sigma^\mu(D_\mu\bar{\psi})_i - \frac{i}{2}(D_\mu\psi)_i\sigma^\mu\bar{\psi}_i \\ & -\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \frac{i}{2}\lambda^a\sigma^\mu(D_\mu\bar{\lambda})^a - \frac{i}{2}(D_\mu\lambda)^a\sigma^\mu\bar{\lambda}^a \\ & +\sqrt{2}ig\,\bar{\psi}_i\lambda^a T^a_{ij}\varphi_j - \sqrt{2}ig\,\varphi^*_i T^a_{ij}\psi_j\lambda^a \\ & -\frac{1}{2}\frac{\partial^2\mathcal{V}}{\partial\varphi_i\partial\varphi_j}\psi_i\psi_j - \frac{1}{2}\frac{\partial^2\mathcal{V}^*}{\partial\varphi^*_i\partial\varphi^*_j}\bar{\psi}_i\bar{\psi}_j - V(\varphi_i,\varphi^*_j)\end{aligned}$$

$\mathcal{V}(\varphi_i) = a_i\varphi_i + \frac{1}{2}m_{ij}\varphi_i\varphi_j + \frac{1}{3}\lambda_{ijk}\varphi_i\varphi_j\varphi_k$: holomorphic function of chiral sf.

The potential V is the sum of the F-terms and D-terms and reads

$$V(\varphi_i, \varphi_j^*) = \sum_i \left| \frac{\partial \mathcal{V}}{\partial \varphi_i} \right|^2 + \frac{1}{2} g^2 \sum_a (\varphi_i^* T_{ij}^a \varphi_j + \xi^a)^2$$

All couplings are determined either by gauge couplings or the superpotential

E.g.: D-term in potential V: quartic scalar interaction is given by the gauge coupling (in contrast to the SM case)

The ξ^a-term (called the Fayet-Iliopoulos term) can be present only for $U(1)$ gauge fields

The requirement of gauge invariance imposes constraints on the coefficients a_i, m_{ij} and λ_{ijk} of the superpotential $\mathcal{V}$

The (gauge) covariant derivatives are (f^{abc} are the structure constants of the gauge group)

$$(D_\mu \varphi)_i = \partial_\mu \varphi_i + ig\, v_\mu^a T_{ij}^a \varphi_j, \; (D_\mu \psi)_i = \partial_\mu \psi_i + ig\, v_\mu^a T_{ij}^a \psi_j, \; (D_\mu \lambda)^a = \partial_\mu \lambda^a - g f^{abc} v_\mu^b \lambda^c$$

Summary on construction of SUSY Lagrangians:

Construct Lagrangians for $N = 1$ SUSY from chiral superfields and vector superfields:

multiplets containing pairs of fields with the same mass, spin differs by $\frac{1}{2}$

Fermion Yukawa interactions and scalar quartic self-interactions are determined by the superpotential

Gauge interactions determine couplings of the gauge fields

$\Rightarrow$ Many relations between couplings

$N > 1$ SUSY

So far: $N = 1$ SUSY, simplest case, only one fermionic generator and its hermitean adjoint: Q_α, $\bar{Q}^{\dot{\beta}}$

$\Rightarrow$ one superpartner for photon: photino

N-extended SUSY: N generators Q_α^A, $\bar{Q}_B^{\dot{\beta}}$, $A = 1, \ldots, N$

$\Rightarrow$ N superpartners for the photon, . . .

Generalization of anticommutator relation:

$$\{Q_\alpha^A, Q_\beta^B\} = \epsilon_{\alpha\beta} X^{AB}$$

$X^{AB} = -X^{BA}$: "central charges"

Problem:

helicity $+\frac{1}{2}$ and helicity $-\frac{1}{2}$ fermions are in same supermultiplet (e.g.: hypermultiplet for $N = 2$ SUSY)

$\Rightarrow$ helicity $+\frac{1}{2}$ and $-\frac{1}{2}$ fermions need to transform in the same way under gauge transformations

not possible for chiral fermions of electroweak theory

$\Rightarrow$ $N > 1$ SUSY theories are 'non-chiral'

$\Rightarrow$ $N = 1$ SUSY theories are the best candidates for a realistic low-energy theory (extension of the SM)

However: $N > 1$ SUSY have interesting properties

e.g.: $N = 4$ SUSY field theory (flat space) is finite

Seiberg–Witten solution in $N = 2$ SUSY, ...

Soft SUSY breaking

Exact SUSY: $m_f = m_{\tilde{f}}, \ldots$

$\Rightarrow$ in a realistic model: SUSY must be broken

Only satisfactory way for model of SUSY breaking:
spontaneous SUSY breaking

Specific SUSY-breaking schemes (see below) in general yield effective Lagrangian at low energies, which is supersymmetric except for explicit soft SUSY-breaking terms

Soft SUSY-breaking terms: do not alter dimensionless couplings
(i.e. dimension of coupling constants of soft SUSY-breaking terms > 0)

$\Rightarrow$ no quadratic divergences (in all orders of perturbation theory)

scale of SUSY-breaking terms: $M_{\mathrm{SUSY}} \lesssim 1$ TeV

Classification of possible soft breaking terms:

[*L. Girardello, M. Grisaru '82*]

- scalar mass terms: $m^2_{\phi_i} |\phi_i|^2$
- trilinear scalar interactions: $A_{ijk}\phi_i\phi_j\phi_k + \text{h.c.}$
- gaugino mass terms: $\frac{1}{2} m \bar{\lambda}\lambda$
- bilinear terms: $B_{ij}\phi_i\phi_j + \text{h.c.}$
- linear terms: $C_i\phi_i$

$\Rightarrow$ relations between dimensionless couplings unchanged

no additional mass terms for chiral fermions

Unconstrained MSSM:

no particular SUSY breaking mechanism assumed, parameterization of possible soft SUSY-breaking terms

$\Rightarrow$ relations between dimensionless couplings unchanged
no quadratic divergencies

most general case:

$\Rightarrow$ 105 new parameters: masses, mixing angles, phases

Good phenomenological description for universal breaking terms (FCNC, . . .)

Scenarios for SUSY breaking $\Rightarrow$ prediction for soft SUSY-breaking terms in terms of small set of parameters

Experimental determination of SUSY parameters

$\Rightarrow$ Patterns of SUSY breaking

The Minimal Supersymmetric Standard Model (MSSM)

MSSM: superpartners for SM fields

SM matter fermions are in different representation of gauge group than gauge bosons

$\Rightarrow$ need to be placed in different superfields

$\Rightarrow$ no SM fermion is a gaugino

Fermions, sfermions:

use definition of chiral superfields via left-handed fermions
($\Rightarrow$ the conjugates of right-handed ones appear)

LHχSF Q: quark, squark SU(2) doublets

LHχSF U: up-type quark, squark singlets

LHχSF D: down-type quark, squark singlets

LHχSF L: lepton, slepton SU(2) doublets

LHχSF E: lepton, slepton singlets

$\Rightarrow$ one generation of SM fermions and their superpartners described by five LHχSFs

Gauge bosons, gauginos:

Vector superfields:

gluons g and gluinos $\tilde{g}$

W bosons $W^{\pm}$, W^0 and winos $\tilde{W}^{\pm}$, $\tilde{W}^0$

B boson B^0 and bino $\tilde{B}^0$

Higgs bosons, higgsinos:

LHχSF

In MSSM: two Higgs doublets needed $\Rightarrow$ two LHχSFs

Comparison with SM case:

$$\mathcal{L}_{\text{SM}} = \underbrace{m_d \bar{Q}_L H d_R}_{\text{d-quark mass}} + \underbrace{m_u \bar{Q}_L \tilde{H} u_R}_{\text{u-quark mass}}$$

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \tilde{H} = i\sigma_2 H^\dagger, \quad H \to \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \tilde{H} \to \begin{pmatrix} v \\ 0 \end{pmatrix}$$

In SUSY: term $\bar{Q}_L H^\dagger$ not allowed

Superpotential is holomorphic function of chiral superfields, i.e. depends only on φ_i, not on φ_i^*

No soft SUSY-breaking terms allowed for chiral fermions

$\Rightarrow$ H_d and H_u needed to give masses to down- and up-type fermions

Furthermore: two doublets also needed for cancellation of anomalies, quadratic divergences

Chiral supermultiplets of the MSSM:

		spin 0	spin $\frac{1}{2}$	(SU(3)$_c$, SU(2), U(1)$_Y$)
squarks and quarks	Q	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$(3, 2, \frac{1}{6})$
	U	$\tilde{u}^*_R$	$u^{\dagger}_R$	$(\bar{3}, 1, -\frac{2}{3})$
	D	$\tilde{d}^*_R$	$d^{\dagger}_R$	$(\bar{3}, 1, \frac{1}{3})$
sleptons and leptons	L	$(\tilde{\nu}, \tilde{e}_L)$	(ν, e_L)	$(1, 2, -\frac{1}{2})$
	E	$\tilde{e}^*_R$	$e^{\dagger}_R$	$(1, 1, 1)$
higgs and higgsinos	H_u	(h^+_u, h^0_u)	$(\tilde{h}^+_u, \tilde{h}^0_u)$	$(1, 2, \frac{1}{2})$
	H_d	(h^0_d, h^-_d)	$(\tilde{h}^0_d, \tilde{h}^-_d)$	$(1, 2, -\frac{1}{2})$

Vector supermultiplets:

	spin $\frac{1}{2}$	spin 1	(SU(3)$_c$, SU(2), U(1)$_Y$)
gluinos and gluons	$\tilde{g}$	g	$(8, 1, 0)$
winos and W-bosons	$\widetilde{W}^{\pm}, \widetilde{W}^0$	$W^{\pm}, W^0$	$(1, 3, 0)$
bino and B-boson	$\widetilde{B}$	B	$(1, 1, 0)$

Superpotential:

$$\mathcal{V}_{\mathsf{MSSM}} = U\mathbf{y_u}QH_u + D\mathbf{y_d}QH_d + E\mathbf{y_e}LH_d + \mu H_u H_d$$

$\mathbf{y_i}, \mathbf{a_i}$: Yukawa couplings, 3×3 matrices in family space

All terms have to be invariant under all gauge groups, SU(3)$_c$, SU(2), U(1)$_Y$

$\Rightarrow$ need $Y = 0$ for all terms

$H_u H_d \equiv (H_u)_a (H_d)_b \epsilon^{ab}$ (a, b: weak isospin indices), . . .

$\mathcal{V}_{\text{MSSM}}$ is not the most general gauge-invariant superpotential

contains only terms that are necessary to build a realistic model

Gauge interactions introduce only terms with even number of superpartners

The same holds for minimal version of $\mathcal{V}_{\text{MSSM}}$

$\Rightarrow$ MSSM has further symmetry: "R-parity"

all SM-particles and Higgs bosons: even R-parity, $P_R = +1$

all superpartners: odd R-parity, $P_R = -1$

Soft breaking terms:

$$\begin{aligned}\mathcal{L}_{\text{soft}} &= -\frac{1}{2}\Big(M_1\widetilde{B}\widetilde{B} + M_2\widetilde{W}\widetilde{W} + M_3\tilde{g}\tilde{g}\Big) + \text{h.c.} \\ &- m^2_{H_u}H_u^+H_u - m^2_{H_d}H_d^+H_d - (bH_uH_d + \text{h.c.}) \\ &- \Big(\tilde{u}_R\mathbf{a_u}\widetilde{Q}H_u - \tilde{d}_R\mathbf{a_d}\widetilde{Q}H_d - \tilde{e}_R\mathbf{a_e}\widetilde{L}H_d\Big) + \text{h.c.} \\ &- \widetilde{Q}^+\mathbf{m_Q^2}\widetilde{Q} - \widetilde{L}^+\mathbf{m_L^2}\widetilde{L} - \tilde{u}_R\mathbf{m_u^2}\tilde{u}_R^* - \tilde{d}_R\mathbf{m_d^2}\tilde{d}_R^* - \tilde{e}_R\mathbf{m_e^2}\tilde{e}_R^*\end{aligned}$$

Most general parameterization of SUSY-breaking terms that keep relations between dimensionless couplings unchanged ⇒ no quadratic divergences

Gaugino mass terms, scalar mass terms, and terms like in superpotential for all scalars

$\mathbf{m_i^2}$: 3×3 matrices in family space

⇒ many new parameters

Particle content of the MSSM:

Superpartners for Standard Model particles:

$$\left[u, d, c, s, t, b\right]_{L,R} \quad \left[e, \mu, \tau\right]_{L,R} \quad \left[\nu_{e,\mu,\tau}\right]_L \qquad \text{Spin } \tfrac{1}{2}$$

$$\left[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}\right]_{L,R} \quad \left[\tilde{e}, \tilde{\mu}, \tilde{\tau}\right]_{L,R} \quad \left[\tilde{\nu}_{e,\mu,\tau}\right]_L \qquad \text{Spin } 0$$

$$g \quad \underbrace{W^{\pm}, H^{\pm}} \quad \underbrace{\gamma, Z, H_1^0, H_2^0} \qquad \text{Spin 1 / Spin 0}$$

$$\tilde{g} \quad \tilde{\chi}_{1,2}^{\pm} \quad \tilde{\chi}_{1,2,3,4}^{0} \qquad \text{Spin } \frac{1}{2}$$

Enlarged Higgs sector:
Two Higgs doublets, physical states: $h^0, H^0, A^0, H^{\pm}$

Breaking of SU(2) $\times$ U(1)$_Y$ (electroweak symmetry breaking)

$\Rightarrow$ fields with different SU(2) $\times$ U(1)$_Y$ quantum numbers can mix if they have the same SU(3)$_c$, U(1)$_{\text{em}}$ quantum numbers

Squark mixing:

Stop, sbottom mass matrices ($X_t = A_t - \mu/\tan\beta$, $X_b = A_b - \mu\tan\beta$):

$$\mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} m^2_{\tilde{t}_L} + m^2_t + DT_1 & m_t X_t \\ m_t X_t & m^2_{\tilde{t}_R} + m^2_t + DT_2 \end{pmatrix} \Rightarrow \; m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}$$

$$\mathcal{M}^2_{\tilde{b}} = \begin{pmatrix} m^2_{\tilde{b}_L} + m^2_b + DB_1 & m_b X_b \\ m_b X_b & m^2_{\tilde{b}_R} + m^2_b + DB_2 \end{pmatrix} \Rightarrow \; m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_{\tilde{b}}$$

off-diagonal element prop. to mass of partner quark ($\tan\beta \equiv v_u/v_d$)

$\Rightarrow$ mixing important in stop sector (also in sbottom sector for large $\tan\beta$)

gauge invariance $\Rightarrow m_{\tilde{t}_L} = m_{\tilde{b}_L}$

$\Rightarrow$ relation between $m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_{\tilde{b}}$

Neutralinos and charginos:

Higgsinos and electroweak gauginos mix

charged:

$$\tilde{W}^+, \tilde{h}_u^+ \to \tilde{\chi}_1^+, \tilde{\chi}_2^+, \qquad \tilde{W}^-, \tilde{h}_d^- \to \tilde{\chi}_1^-, \tilde{\chi}_2^-$$

⇒ charginos: mass eigenstates

mass matrix given in terms of M_2, μ, $\tan\beta$

neutral:

$$\underbrace{\tilde{\gamma}, \tilde{Z},}_{\tilde{W}^0, \tilde{B}^0} \tilde{h}_u^0, \tilde{h}_d^0 \to \tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$$

mass matrix given in terms of M_1, M_2, μ, $\tan\beta$

⇒ only one new parameter

⇒ MSSM predicts mass relations between neutralinos and charginos

Summary of Lecture 2:

- Lagrangians for $N = 1$ SUSY are constructed from chiral superfields and vector superfields (multiplets containing pairs of fields with spin differing by $\frac{1}{2}$)

- $N = 1$ SUSY theories are the best candidates for a realistic low-energy theory (extension of the SM)

  $N > 1$ SUSY theories are 'non-chiral'

- Soft SUSY-breaking terms: do not alter dimensionless couplings, $M_{\text{SUSY}} \lesssim 1$ TeV

  $\Rightarrow$ no quadratic divergences

- Unconstrained MSSM: no particular SUSY breaking mechanism assumed, parameterization of possible soft SUSY-breaking terms