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SUMMER SCHOOL ON PARTICLE PHYSICS

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EXTRA DIMENSIONS

Lectures I, II, III & IV

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EXTRA DIMENSIONS AND WARPED HIERARCHIES

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What's at stake

- Higher Dimensions ⇒ Higher dimensional Lorentz invariance & translational symmetry.
- Hiding extra dimensions ⇒ Breaking higher dim. symmetries at some level.
- Higher spacetime symmetries ⇒ extra dimensional locality.
- Higher Dimensions ⇒ Non-renormalizabl effective field theory
- Extra-dimensional locality ⇒ hidden finiteness (predictivity).
- Hidden dimensions + General Relativity ⇒ hidden spacetime curvature, "WARPING".
  ⇒ Warped Hierarchies
  ⇒ Realistic models of Nature (without supersymmetry).

Deep connections with String Theory

- String theory ⇒ Extra dimensions + ways of hiding them
- AdS/CFT Correspondence ⇒ Crucial insights into physics of warped higher-dimensional spacetimes.
- Warping in String Theory generic, useful way to arrange hierarchies.

OUTLINE

HIDING EXTRA DIMENSIONS

- Compactification
- Branes

GENERAL RELATIVITY review

GRAVITY LOCALIZATION – RS2

WARPED WEAK/PLANCK HIERARCHY – RS1

RADIUS STABILIZATION

- "Holographic" Renormalization Group

ADS/CFT, RS/CFT Correspondence

RS1 & GUTs

RS1 & Electroweak Precision Tests.

COMPACTIFICATION

String motivation for extra dimensions: Green, Schwar
Witten Ch.1, ...,
Polchinski's Book too

↑ 5^{th} dim, φ

$\rightarrow$ 3+1D, x^μ

distances: $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\varphi^2$

Compactification Radius

~~5D Lorentz invariance~~ globally,

but not broken locally.

Scalar Field

$$S = \int d^4x \int_{-\pi}^{\pi} d\varphi \, r_c \left\{ \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{2r_c^2}(\partial_\varphi \chi)^2 - \frac{m_5^2}{2}\chi^2 - \lambda_5 \chi^4 \right\},$$

$\chi(x^\mu, \phi)$ — ϕ 2π-periodic

$[\chi] = 3/2$.

$[\lambda_5] = -1$

ie. non-renormalizable

Non-renormalizable Effective Field Theory

see Georgi, "Weak Interactions"

UV divergences are local,

$$S_{div.} = \int \underbrace{d^5X}_{X^M \equiv \underline{all}\text{ coordinates}} \sum_{\mathcal{O}_i} \lambda_5^N \, \mathcal{O}^{(x)}_{local}[x,\partial] \, \Lambda_{UV}^{5+N-[\mathcal{O}]}$$

for $5+N-[\mathcal{O}] \geqslant 0$.

Renormalizability $\equiv$ $\sum_{\mathcal{O}_i}$: Finite no. ter

so that $S_{div.}$ has same structure as $S_{c.}$

& $S_{c.t.}$ " " " " S_{ren}

$\equiv$ finite no. input couplings

But here, $[\lambda_5] < 0 \Rightarrow \infty$ couplings as input!

Cure for $E \ll \Lambda_{UV} \ll 1/\lambda_5$:

Work to $\underline{\text{fixed}}$ order in λ_5.

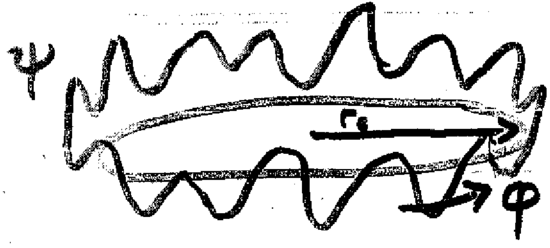
$\equiv$ fixed order in $(E\lambda_5)$, $(\Lambda_{UV}\lambda_5)$.

After "renormalization," we are expanding in $(E\lambda_5$

$$S = \int d^4x \int_{-\pi}^{\pi} d\phi \, r_c \left\{ \chi \left[-\frac{\partial_\mu \partial^\mu}{2} + \frac{\partial_\phi^2}{2 r_c^2} - \frac{m_5^2}{2} \right] \chi - \lambda_5 \chi^4 \right\}$$

- Analog 1D QM Problem:

$$\underbrace{\left[-\frac{1}{2 m_{QM}} \frac{\partial_\phi^2}{r_c^2} + V_{QM}(\phi) \right]}_{H_{QM}} \psi_n(\phi) = E_{QM}^{(n)} \psi_n$$



with $m_{QM} \equiv \frac{1}{2}$, $V_{QM} \equiv m_5^2$

$$\Rightarrow \quad \psi_n(\phi) = \frac{e^{in\phi}}{\sqrt{2\pi r_c}}, \quad E_{QM}^{(n)} = \frac{n^2}{r_c^2} + m_5^2$$

Expand $\chi(x^\mu, \phi) = \sum_n \chi_n(x) \, \psi_n(\phi)$

Real $\chi \Rightarrow \chi_{-n} = \chi_n^*$

Kaluza-Klein (KK) Decomposition

$$S = \int d^4x \int d\varphi \, r_c \left\{ \frac{1}{2} \chi \left[-\partial_\mu \partial^\mu - H_{QM} \right] \chi - \lambda_5 \chi^4 \right\}$$

$$= \int d^4x \sum_n \left\{ \frac{1}{2} \left(\partial_\mu \chi_0 \right)^2 - \frac{1}{2} E^{(0)}_{QM} \chi_0^2 + \left| \partial_\mu \chi_n \right|^2 - E^{(n)}_{QM} \left| \chi_n \right|^2 \right\}$$

$$- \frac{\lambda_5}{2\pi r_c} \int d^4x \sum_{m,n,k,l} c_{mnkl} \, \chi_m \chi_n \chi_k \chi_l$$

$$c_{mnkl} = \underbrace{\delta_{m+n+k+l,0}}_{\text{extra-dimensional angular momentum conservation}}$$

[redacted]

Mass ga

$$m^2_{4D} \equiv E_{QM} = \frac{n^2}{r_c^2} + m_5^2 \Leftarrow \text{if } m_5 \ll$$

$\lambda_{4D} \equiv \lambda_5 / 2\pi r_c$ "renormalizable"!

LOW-ENERGY 4D
EFFECTIVE THEORY

$E, m_5 \ll 1/r_c \, , \, m_{4D}^{(n \neq 0)}$

$$S_{eff} = \int d^4x \left\{ \frac{1}{2}(\partial_\mu \chi_0)^2 - \frac{1}{2} m_5^2 \chi_0^2 - \frac{\lambda_5}{2\pi r_c} \chi_0^4 \right\}$$

at tree-level.

When quantum loops considered 4D theory "matches" 5D theory at low energies only with more general couplings, masses:

$$S_{eff} = \int d^4x \left\{ \frac{1}{2}(\partial_\mu \chi_0)^2 - \frac{1}{2} m_{eff}^2 \chi_0^2 - \lambda_{eff} \chi_0^4 \right\}$$

4D Renormalizability $\equiv$ Low-energy insensitivity to UV details such as extra dimensions, r_c, $\chi_n(x)$ ...

General Relativity (GR)

Newton's Law in
Gauss Law form :

$$\nabla^2 V_{grav}(x) = T^{00}(x) \times G_{Newton}$$

↑ Newt. potential

↑ energy density $\in T^{\mu\nu}$ Energy-momentu
Tensor

Special Relativity $\Rightarrow$

$$V_{grav}(x) \equiv h_{00}(x)$$

for some tensor $h_{\mu\nu}(x)$.

Equation of Motion (EOM) :

$$\mathcal{G}_{\mu\nu} = 8\pi G_N \times T_{\mu\nu}$$

↑ Made from $h_{\mu\nu}$ & ∂
"Einstein Tensor"

Conserved in absence of gravity, $\partial^\mu T_{\mu\nu} = 0$

Analogy: EM

see "Feynman Lectures"

Coulomb Law

$$\nabla^2 V_{\text{Electrostatic}} = J_0$$

$\uparrow$ Charge density

$\in J_\mu$ 4-current

+ Special Relativity ⇒

$$V_{\text{Elec}} \equiv A_0 \text{, some } A_\mu .$$

EOM: $$\left[\alpha\, \partial^\mu \partial_\nu + \beta\, \partial^2 \delta^\mu{}_\nu \right] A_\mu = J_\mu$$

Current Conservation $\partial_\mu J^\mu = 0$

⇒ $\alpha = -\beta$. ie. $\beta \underbrace{\partial^\mu F_{\mu\nu}}_{\text{gauge invariant:}} = J_\nu$

$$A_\mu \rightarrow A_\mu + \partial_\mu \lambda$$

For $\beta = 1$, $\exists_\lambda$ Coulomb gauge: Recover Coulomb Law

Curved Spacetime

see Weinberg "Gravitatio
& Cosmology"
Wald "General Relat
S.Weinberg '64–'67

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$

$$\longrightarrow \underset{\searrow \text{ still raises/lowers indices}}{g_{\mu\nu}(x)} dx^\mu dx^\nu$$

$$\xrightarrow[x \to x'(x)]{} \underbrace{g_{\mu\nu}(x) \frac{\partial x^\mu}{\partial x'^{\mu'}} \frac{\partial x^\nu}{\partial x'^{\nu'}}}_{\equiv\, g'_{\mu'\nu'}(x')} dx'^{\mu'} dx'^{\nu'}$$

General Coordinate Inv. (GCI)

defines new "gauge symmetry"

on $g_{\mu\nu}(x) \equiv \eta_{\mu\nu} + h_{\mu\nu}(x)$ [redacted]

Curvature: $\underset{\uparrow \text{ Riemann Tensor}}{R_{\mu\nu\rho}{}^{\sigma}} \equiv \Gamma^\sigma{}_{\mu\rho,\nu} - \Gamma^\sigma{}_{\nu\rho,\mu} + \Gamma^\alpha{}_{\mu\rho}\Gamma^\sigma{}_{\alpha\nu} - \Gamma^\alpha{}_{\nu\rho}\Gamma^\sigma{}_{\alpha\mu}$

Christoffel symbol

$$\overset{\downarrow}{\Gamma^\rho{}_{\mu\nu}} = \frac{1}{2} g^{\rho\sigma} \left\{ g_{\nu\sigma,\mu} + g_{\mu\sigma,\nu} - g_{\mu\nu,\sigma} \right\}$$

$$\underset{\uparrow \text{ Ricci Tensor}}{R_{\mu\rho}} \equiv R_{\mu\nu\rho}{}^{\nu}, \quad \underset{\uparrow \text{ Ricci Scalar}}{R} \equiv R_{\mu\rho} g^{\mu\rho}.$$

Scalars:

$$\phi'(x'(x)) \underset{x\to x'(x)}{=} \phi(x)$$

Tensors:

$$T'^{\mu_1'\cdots\mu_N'}_{\nu_1'\cdots\nu_M'}(x'(x)) \underset{x\to x'(x)}{=}$$

$$\frac{\partial x'^{\mu_1'}}{\partial x^{\mu_1}}\cdots\frac{\partial x'^{\mu_N'}}{\partial x^{\mu_N}} T^{\mu_1\cdots\mu_N}_{\nu_1\cdots\nu_M}(x) \frac{\partial x^{\nu_1}}{\partial x'^{\nu_1'}}\cdots\frac{\partial x^{\nu_M}}{\partial x'^{\nu_M'}}$$

Contractions of tensors are tensors

Invariant Integration Measure

$$\int d^4x\sqrt{-g}\ldots = \int d^4x'\sqrt{-g'}$$

where $\sqrt{-g} \equiv \sqrt{-\det\left(g_{\mu\nu}^{(x)}\right)}$

$\therefore \int d^4x\sqrt{-g}\,\mathcal{L}(x)$ is GCI if $\mathcal{L}(x)$ is a scalar

Relativity + Newtonian Limit

$$+ \ \partial_\mu T^{\mu\nu} = 0 \ \underbrace{\text{in flat space limit}}_{h^{(x)}_{\mu\nu} \to 0}$$

$\Longrightarrow$ Unique Gravity EOM:

$$\underset{\uparrow \atop \text{"Einstein Tensor"}}{\mathcal{G}_{\mu\nu}} \equiv R_{\mu\nu} - \tfrac{1}{2} R\, g_{\mu\nu} \overset{\text{"Einstein Equations"} \atop \downarrow}{=} 8\pi G_N \times \underset{\nearrow \atop \equiv \text{ Matter obtained from GCI action.}}{T_{\mu\nu}}$$

Action

$$S = \int d^4x \sqrt{-g} \left\{ + \frac{1}{16\pi G_N} R \right.$$
$$\left. + \frac{g^{\mu\nu}}{2} \partial_\mu \chi \, \partial_\nu \chi - V(\chi) \right\}$$

$$\text{Then } \overset{\equiv \frac{2}{\sqrt{-g}} \frac{\delta S_{\text{matter}}}{\delta g^{\mu\nu}}}{T_{\mu\nu}} = \partial_\mu \chi \, \partial_\nu \chi - g_{\mu\nu} \left[\frac{g^{\alpha\beta}}{2} \partial_\alpha \chi \, \partial_\beta \chi - V \right]$$

$$\& \text{ Einstein's Equations} \equiv \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = 0$$

Linearized GR

Treat $h_{\mu\nu}$ as infinitesimal, as well as ξ_ν in

$$x'(x) \equiv x + \xi(x).$$

Then, $h'_{\mu\nu}(x) \equiv h_{\mu\nu}(x) + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

to 1st order in infinitesimals.

Gauge-Fixing (Axial)

Choose a spatial direction, y.

$h_{\mu y} = h_{y\mu} = 0$ For <u>any</u> μ & $h_{\mu\nu}$ arbitrary for $\mu, \nu \neq y$.

To go to this gauge choose $\partial_y \xi_y \equiv -\frac{h_{yy}}{2}$

& $\partial_y \xi_{\mu \neq y} \equiv -h_{y(\mu \neq y)} - \partial_{\mu \neq y} \xi_y$ ■

"Planck Scale" and field normalization

$$S_{quad} \ni \int d^4x \, \frac{\dot{h}_{ij}^2}{64\pi G} + \cdots$$

Canonical Field for QFT:

$$h_{ij} = \frac{\overset{\text{canonical}}{h_{ij}}}{\underset{\nwarrow \text{ Planck scale}}{M}}$$

$$M \equiv \frac{1}{32\pi G}$$

$$\therefore \; S = \int d^4x \sqrt{-g} \left\{ 2M^2 R + \cdots \right\}$$

Compactifying GR in Higher Dimensions

(Linearized Analysis, quadratic in action)

Axial gauge with respect to φ obstructed by $\xi^M(x,\varphi)$ not being 2π-periodic.

Instead use

$$\partial_\varphi \xi_\varphi \equiv -\left(\frac{h_{\varphi\varphi}}{2} - \frac{\bar{h}_{\varphi\varphi}}{2}\right)$$

$$\partial_\varphi \xi_\mu \equiv -\left(h_{\varphi\mu} - \bar{h}_{\varphi\mu}\right) - \partial_\mu\left(\xi_\varphi - \bar{\xi}_\varphi\right)$$

where $\bar{h}_{MN}(x) \equiv \frac{1}{2\pi}\int_{-\pi}^{\pi} d\varphi\, h_{MN}(x,\varphi)$, $\bar{\xi}_\varphi(x) = \frac{1}{2\pi}\int d\varphi\, \xi_\varphi(x,$

so $h'_{\mu\nu}(x,\varphi)$ arbitrary, but $h'_{\mu\varphi} = \bar{h}_{\mu\varphi}$, $h'_{\varphi\varphi} = \bar{h}_{\varphi\varphi}$

$$S_{quad} = \int d^4x \int_{-\pi}^{\pi} d\varphi\, r_c \Big\{ \tfrac{1}{2}(\partial_\sigma h_{\mu\nu})^2 - (\partial_\nu h^{\mu\nu})^2$$

$$+ \partial_\nu h^{\mu\nu} \partial_\mu h_\sigma{}^\sigma - \tfrac{1}{2}(\partial_\mu h_\nu{}^\nu)^2$$

$$- \tfrac{1}{2} h_{\mu\nu} H_{QM} h^{\mu\nu} + \tfrac{1}{2} h_\mu{}^\mu H_{QM} h_\nu{}^\nu$$

$$+ \frac{1}{r_c^2}(\partial_\mu \bar{h}_{\nu\varphi})^2 - \frac{1}{r_c^2}(\partial^\mu \bar{h}_{\varphi\mu})^2$$

$$- \frac{1}{r_c^2} \partial_\mu \bar{h}_{\varphi\varphi} \partial_\nu h^{\mu\nu} \Big\}$$

where $H_{QM} \equiv -\frac{\partial_\varphi^2}{r_c^2}$

More compactly,

$$S_{quad} = \int d^4x \int_{-\pi}^{\pi} d\varphi\, r_c \Big\{ \tfrac{1}{2} h_{\mu\nu} D^{\mu\nu\rho\sigma} h_{\rho\sigma}$$

$$- \tfrac{1}{2} h_{\mu\nu} H_{QM} h^{\mu\nu} + \tfrac{1}{2} h_\mu{}^\mu H_{QM} h_\nu{}^\nu$$

$$+ \mathcal{O}(\bar{h}) \Big\}$$

$$= \int d^4x \sum_n \Big\{ \tfrac{1}{2} h^{(n)}_{\mu\nu} D^{\mu\nu\rho\sigma} h^{(n)}_{\rho\sigma}$$

$$- \tfrac{1}{2} h^{(n)}_{\mu\nu} \frac{n^2}{r_c^2} h^{\mu\nu}_{(n)} + \tfrac{1}{2} h^{(n)\mu}_{\mu} \frac{n^2}{r_c^2} h^{(n)\nu}_{\nu}$$

$$+ \mathcal{O}(\bar{h}, h^{(0)}) \Big\}$$

where $h_{\mu\nu}(x,\varphi) \equiv h^{(n)}_{\mu\nu}(x)\, \psi_n(\varphi)$

$n \neq 0$ $\equiv$ Massive spin-2 EOM:

In momentum space in rest fram

$$(E^2 - m^2)\, h_{ij}^{\text{traceless}} = 0$$

$$-\frac{2}{3}(E^2 - m^2)\, h_{kk} = m^2 h_{00}$$

$$m^2 h_{kk} = 0$$

$$m^2 h_{0i} = 0$$

where $m^2 = \dfrac{n^2}{r_c^2}$.

4D LOW-ENERGY EFFECTIVE FIELD THEORY

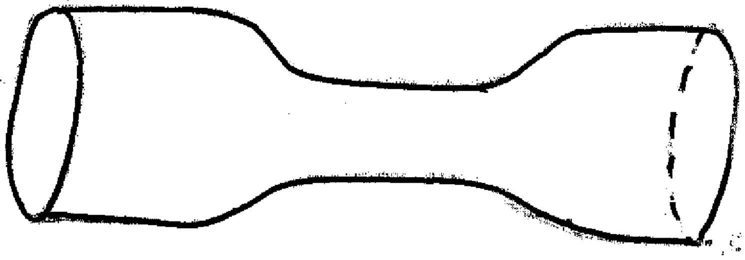
$$S^{eff}_{quad} = \int d^4x \left\{ \frac{1}{2} h^{(0)(x)}_{\mu\nu} D^{\mu\nu\rho\sigma} h^{(0)(x)}_{\rho\sigma} + \frac{1}{4} F_{\mu\nu}^2 - \frac{\sqrt{2\pi}}{r_c^{3/2}} \partial_\mu \bar{h}^{(x)}_{\varphi\varphi} \partial_\nu h^{\mu\nu}_{(0)} \right\}$$

$h^{(0)(x)}_{\mu\nu}$ is effective 4D graviton

$A^{(x)}_\mu \equiv \sqrt{\frac{4\pi}{r_c}}\, \bar{h}^{(x)}_{\mu\varphi}$ is "KK gauge boson"

Recalling $ds^2 \ni r_c^2 d\varphi^2 + h^{(x)}_{\varphi\varphi} d\varphi^2$

we see that $\bar{h}^{(x)}_{\varphi\varphi}$ is the "Radion"



It's kinetic mixing with 4D graviton can be undone with field redefinitions.

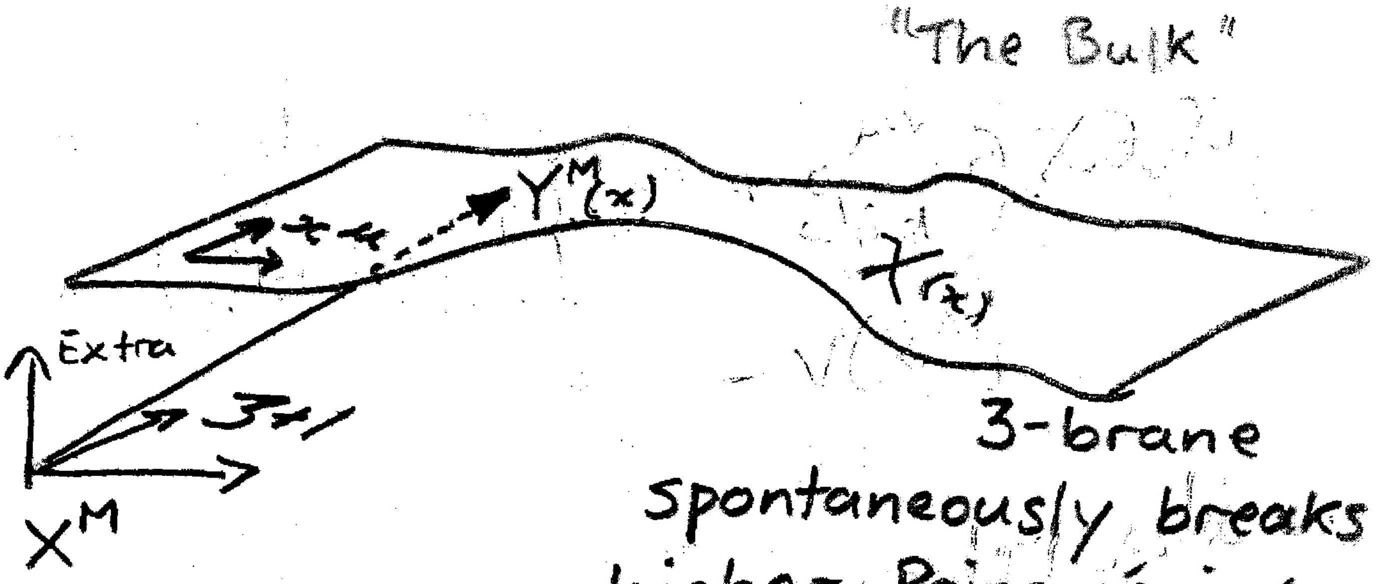
Interactions: $S_{eff} = \int d^4x\, 4\pi r_c M_5^3 \bar{R} + \ldots$

$\equiv \int d^4x\, 2M_4^2 \bar{R} + \ldots$ where $\bar{R}$ is 4D curvature of $\bar{g}^{(x)}_{\mu\nu} \equiv \eta_{\mu\nu} + h^{(0)}_{\mu\nu}/M_4$

BRANES

EFT — Sundrum hep-ph/980

Stringy motivation —
Polchinski — hep-th/9611



spontaneously breaks
higher Poincaré inv.
→ 4D " "

Symmetries of action:

5D Poincaré inv. in $Y^M(x)$.

4D reparametrization inv. in x^μ

Building Blocks:

$\partial_\mu Y^M$ is vector under both sym.

$\chi(x)$ " scalar " " "

Vacuum state: $Y^\mu(x) = x^\mu$, $Y^5 = \text{const}$.

$\chi = 0$

Induced metric: $ds^2_{\text{Brane}} = \overbrace{\eta_{MN} \frac{\partial Y^M}{\partial x^\mu} \frac{\partial Y^N}{\partial x^\nu}}^{\equiv g^{\text{ind.}}_{\mu\nu}} dx^\mu d$

Action

$$S = \int d^4x \sqrt{-g_{ind.}} \left\{ \overbrace{-f^4}^{\text{constant ``tension''}} + g^{\mu\nu}_{ind} \partial_\mu \chi \partial_\nu \chi - V(\chi) \right\}$$

Gauge-fixing reparametrizations

$Y^\mu(x) = x^\mu$, $Y^5(x)$ arbitrary.

$$\int d^4x \sqrt{-g_{ind}} \left\{ -F^4 \right\}$$

$$= \int d^4x \left(-\det\left[\eta_{\mu\nu} - \partial_\mu Y^5 \partial_\nu Y^5 \right] \right)^{\frac{1}{2}} \left\{ -F^4 \right\}$$

$$\approx \int d^4x \left\{ -F^4 \underset{\uparrow}{+} \frac{F^4}{2} \partial_\mu Y^5 \partial^\mu Y^5 + \mathcal{O}(Y^4) \right\}$$

+ve kinetic term.

$Y^5(x)$ is Goldstone boson of spontaneous breaking of extra-dimensiona translations, boosts, & rotations into x, y, z

Coupling Gravity to the Brane.

Symmetry : 5D Poincaré inv.
→ 5D GCI.

Induced metric :

$$ds^2_{\text{Brane}} = \underbrace{G_{MN}(Y(x))\,\overbrace{\frac{\partial Y^M}{\partial x^\mu}\frac{\partial Y^N}{\partial x^\nu}\,dx^\mu dx^\nu}^{\equiv dY^M dY^N}}_{\equiv g^{\text{ind}}_{\mu\nu}(x)}$$

$$S = \int d^5X \sqrt{-G}\,\{2M_5^3 R$$

$$+ \int d^4x \sqrt{-g_{\text{ind}}}\,\{-f^4 + g^{\mu\nu}_{\text{ind}}\,\partial_\mu X \partial_\nu X - V(X)\}$$

Bulk-Brane couplings do not conserve extra-dimensional momentum.

GRAVITY LOCALIZATIO

— RANDALL-SUNDRUM II (RS2)

hep-th/990606

$$S = \int d^5X \sqrt{-G} \left\{ 2M_5^3 R - \Lambda \right\}$$

$$+ \int d^4x \sqrt{-g_{ind}} \left\{ g^{\mu\nu}_{ind} \partial_\mu \chi \partial_\nu \chi - V(\chi) - f^4 \right\}$$

Λ is 5D "cosmological constant".
Think of it as V^{Bulk}_{min} for heavy bulk fields.

Take extra dimension to be infini
$\therefore$ can work in full axial gauge.
Instead of angle φ, we have "y".

Vacuum Solution with 4D Poincaré Invariance

General form: $ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$

$\chi = 0, \quad Y^\mu = x^\mu, \quad Y^5 = 0.$

5D Einstein Equations:

$$6\sigma'^2 = \frac{-\Lambda}{4M_5^3}$$

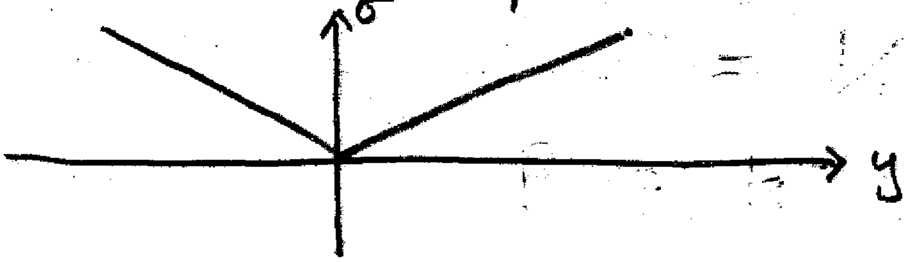
$$3\sigma'' = \frac{F^4}{4M_5^3}\,\delta(y)$$

$$\uparrow$$

$$S_{\text{brane}}(\chi, Y^5=0) = \int d^4x\, dy\ \delta(y)\sqrt{-g_{\text{ind}}}\left\{-F^4\right\}$$

$$g^{\text{ind}}_{\mu\nu}(x) = G_{\mu\nu}(x,y)$$

Self-consistency: $F^4 = 24M_5^3 k, \Lambda = -24M_5^3$



Y^M EOM satisfied, noting $\sigma'(y=0)=0$

Vacuum metric:

$$ds^2 = \underset{\substack{\uparrow \\ \text{"Warp" factor}}}{e^{-2k|y|}} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$

Compare with Anti-de Sitter 5D spacetime (AdS_5), maximally symmetric spacetime of negtive curvature:

$$ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$

"Radius of curvature" $= 1/k$

$$\mathbf{R} = k^2$$

KK decomposition of gravitational fluctuation

In axial gauge,

$$ds^2 = \left[e^{-2k|y|}\eta_{\mu\nu} + h_{\mu\nu}(x,y)\right]dx^\mu dx^\nu + dy^2$$

Substituting into action, and working to $\mathcal{O}(h^2)$,

$$S = \int d^4x \int_{-\infty}^{\infty} dy \left\{ \frac{e^{2k|y|}}{2} h_{\mu\nu} D^{\mu\nu\rho\sigma} h_{\rho\sigma} \right.$$
$$- \frac{1}{2} h_{\mu\nu}\left(\partial_y^2 + 4k\delta(y) - 4k^2\right) h^{\mu\nu}$$
$$\left. + \frac{1}{2} h_\mu{}^\mu \left(\partial_y^2 + 4k\delta(y) - 4k^2\right) h_\nu{}^\nu \right\}$$

Warp factor has stopped us achieving separation of variables.

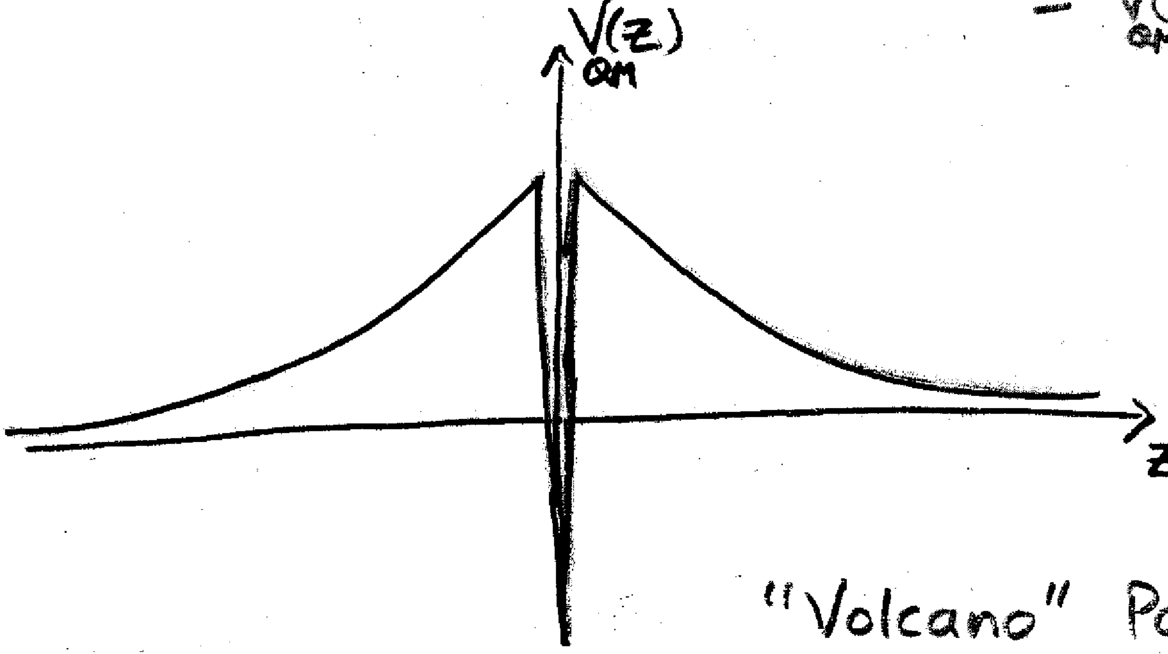
Change variables:

$$z \equiv \mathrm{sgn}(y) \frac{(e^{k|y|}-1)}{k}$$

$$\hat{h}_{\mu\nu}(x,y) \equiv h_{\mu\nu}(x,y)\, e^{k|y|/2}$$

$$S = \int d^4x \int_{-\infty}^{\infty} dz \left\{ \frac{1}{2} \hat{h}_{\mu\nu} D^{\mu\nu\rho\sigma} \hat{h}_{\rho\sigma} - \frac{1}{2} \hat{h}_{\mu\nu} H_{QM} \hat{h}^{\mu\nu} + \frac{1}{2} \hat{h}_{\mu}{}^{\mu} H_{QM} \hat{h}_{\nu}{}^{\nu} \right\}$$

where $$H_{QM} = -\frac{1}{2} \partial_z^2 + \underbrace{\frac{15k^2}{8(k|z|+1)^2} - \frac{3k}{2} \delta(z}_{\equiv V_{QM}(z)}$$



"Volcano" Potential

General considerations

$\Rightarrow$ 1 bound state
+ continuum $E_{QM} > 0$.

Bound state $\psi_0(z) = \underline{(k|z|+1)}^{-3/2} k^{1/2}$

has $E_{QM} = 0$, corresponding to massless 4D graviton.

Continuum: any $E_{QM} = m^2 > 0$

$$\psi_m^{even}(z) \doteq N_m \left(|z| + 1/k\right)^{1/2}$$
$$\times \left[Y_2(m|z| + 1/k) + \frac{4k^2}{\pi m^2} J_2(m|z| + 1/k) \right]$$

for $m \ll k$

N_m is normalization constant
$= \frac{\pi m^{5/2}}{4k^2}$ in plane-wave normalization

N.B $\sqrt{\frac{z\pi m}{2}} J_2(mz) \underset{|z|\text{ large}}{\sim} \cos(mz - \frac{5}{4}\pi)$, $\sqrt{\frac{z\pi m}{2}} Y_2(mz) \sim \sin(mz - \frac{5}{4}\pi)$

4D Effective Field Theory

Retain only $\hat{h}_{\mu\nu}(x, z) = \hat{h}^{(0)}_{\mu\nu}(x)\, \psi_0(z)$

ie. $h_{\mu\nu}(x,y) = e^{-2k|y|}\, \hat{h}^{(0)}_{\mu\nu}(x)$

ie. $$ds^2 = e^{-2k|y|} \underbrace{\left(\eta_{\mu\nu} + \hat{h}^{(0)}_{\mu\nu}(x)\right)}_{g^{ind}_{\mu\nu}(x)} dx^\mu dx^\nu + dy^2$$

$$\cancel{S_{eff} = \int d^4x \int_{-\infty}^{\infty} dy\, 2M_5^3\, e^{-2k|y|} k}$$

$$S_{eff} = \int d^4x \int_{-\infty}^{\infty} dy\, e^{-2k|y|} \sqrt{-g_{ind}} \Big\{ 2M_5^3 \bar{R}$$
$$+ \delta(y) \left[g^{\mu\nu}_{ind}\, \partial_\mu X\, \partial_\nu X - V(X) \right] \Big\}$$
$$= \int d^4x \sqrt{-g^{ind}} \left\{ 2M_4^2 \bar{R} + g^{\mu\nu}_{ind}\, \partial_\mu X\, \partial_\nu X - V(X) \right\}$$

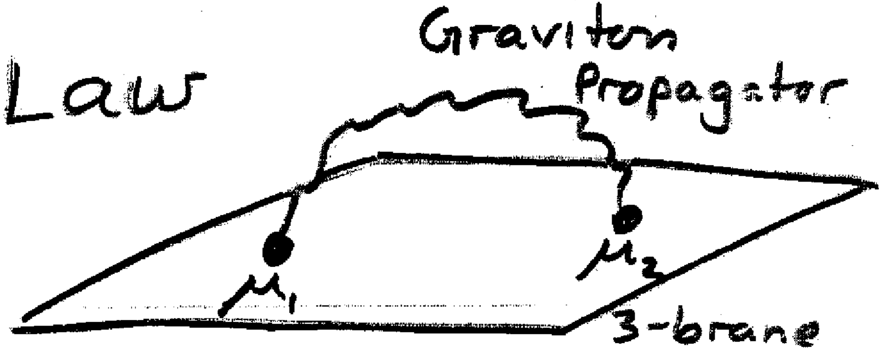
where $\bar{R}$ is 4D curvature of g_{ind}

& $M_4^2 \equiv \frac{2M_5^3}{k}$. Compare with unwarped case.

Non-observability of KK excitations

Requires explanation since they have arbitrarily small 4D mass.

Newton's Law



$$V(r) \doteqdot G_N \frac{\mu_1 \mu_2}{r}$$

Corresponding to M_4

$$+ \int_0^\infty dm \left(\frac{G_N}{k}\right)\left(\frac{m}{k}\right) \frac{e^{-mr}}{r} \mu_1 \mu_2$$

5D Newton constant

$|\psi_m(0)|^2$

Massive stati
potential.

$$= G_N \frac{\mu_1 \mu_2}{r} \left(1 + \frac{1}{k^2 r^2} \right)$$

Thus KK effects negligible at large distances, $r \gg 1/k$.

Typically, we consider

$$k \lesssim M_5 \lesssim M_4 \sim 10^{18}\,\text{GeV}.$$

WHY GRAVITY IS WEAK

— RANDALL-SUNDRUM I (RSI)

($G_{\text{Newton}} \ll G_{\text{Fermi}}$)

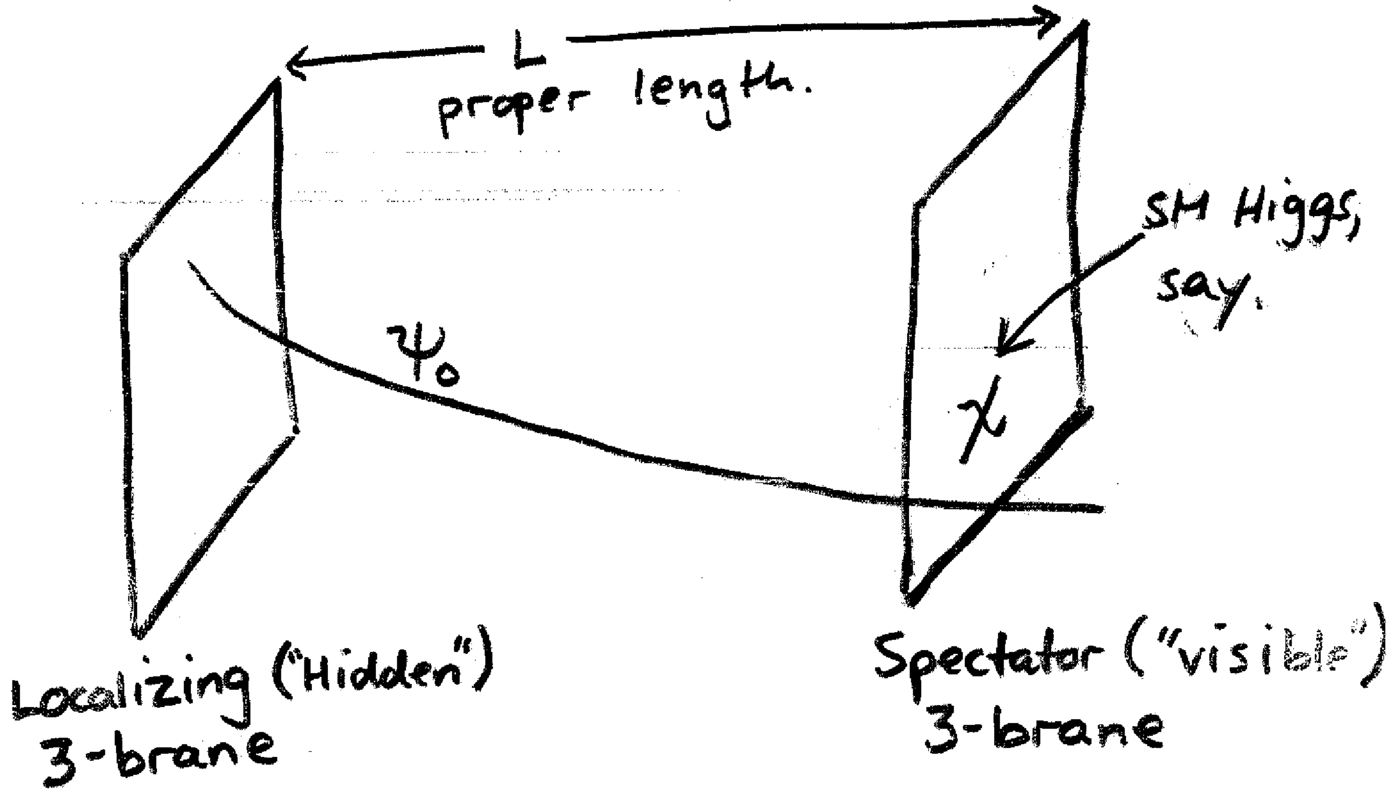
see Witten '96 on $M_{GUT} \ll M_{11}$

Arkani-Hamed, Dimopoulos, Dvali '98

Randall, Sundrum — hep-ph/9905221

Stringy: Lukas, Ovrut, Stelle, Waldram hep-th/9803235

Intuition:



Standard Model particles couple to ~ e^{-kL} tail of massless 4D graviton profile. The strength of gravity is thereby exponentially diminished.

$$S = \int d^5x \sqrt{-G} \left\{ 2M_5^3 R + 24 M_5^3 k^2 \right\}$$

$$+ \int d^4x \sqrt{-g_{hid}} \left\{ -24 M_5^3 k \right\}$$

$$+ \int d^4x \sqrt{-g_{vis}} \left\{ g^{\mu\nu}_{vis} \partial_\mu \chi \partial_\nu \chi - V(\chi) - f^4_{vis} \right\}$$

4D Poincare invariant ansatz:

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$

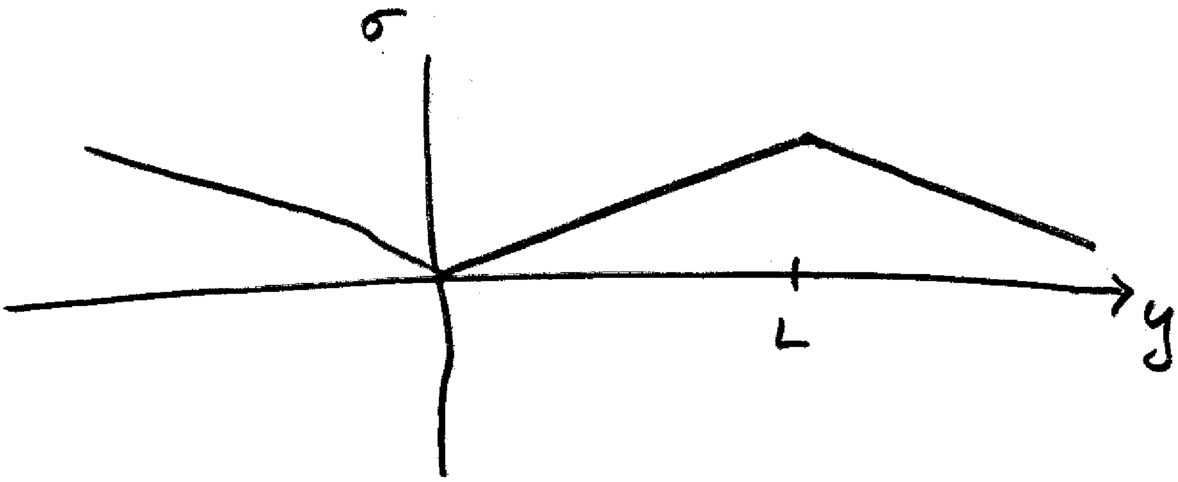
$g^{hid}_{\mu\nu} = \eta_{\mu\nu}$, ~~$g^{vis}_{\mu\nu} = e^{-2kL}\eta_{\mu\nu}$~~

$$g^{vis}_{\mu\nu} = e^{-2\sigma(L)} \eta_{\mu\nu}$$

$e^{-2\sigma(0)} = 1$ is just a convention

Einstein Equations: $\sigma'^2 = k^2$

$$3\sigma'' = 6k\delta(y) + \frac{f^4}{4M_5^3}\delta(y-L)$$



is only consistent possibility.

But even here, we require

$$f_{vis}^4 = -24M_5^3 k$$

Normally this is bad:

$$\int d^4x \sqrt{-g_{ind}}\{-F^4\}$$

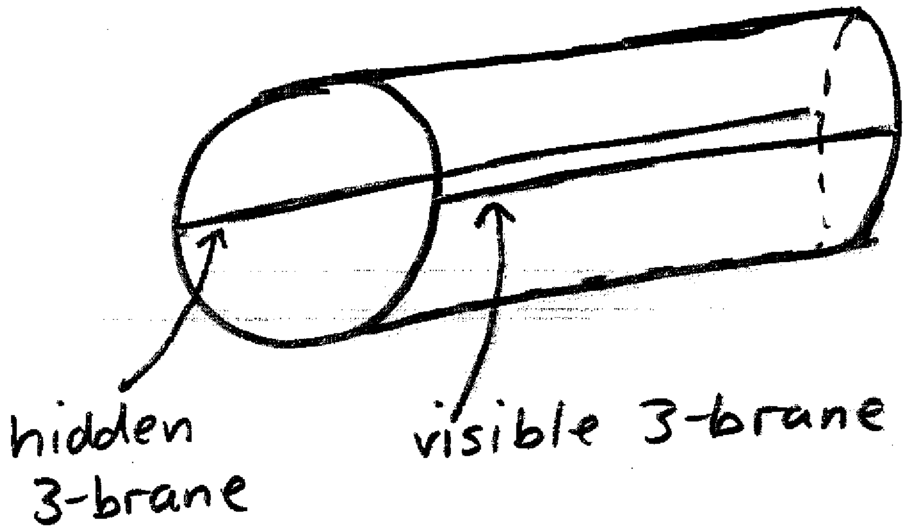
$$\underset{\text{small brane fluctuations}}{\approx} \int d^4x \left\{-F^4 + \frac{F^4}{2}\partial_\mu Y^5 \partial^\mu Y^5\right\}$$

correct sign kinetic term $\equiv f^4 > 0$.

Wrong sign kinetic term ≡ unbounded below energy (Hamiltonian)

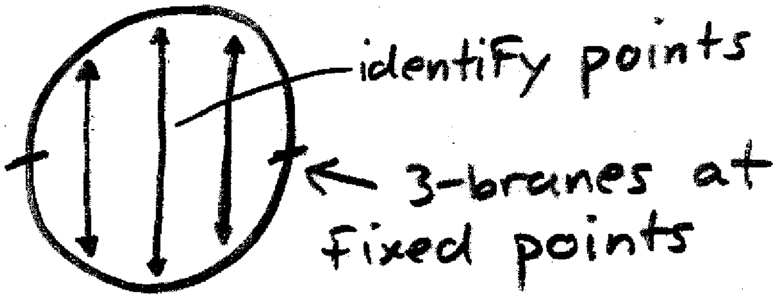
THE CURE — ORBIFOLD SYMMETRY

Compactify extra dimension to circle, S^1:



Dynamics is symmetric between upper & lower hemispheres.

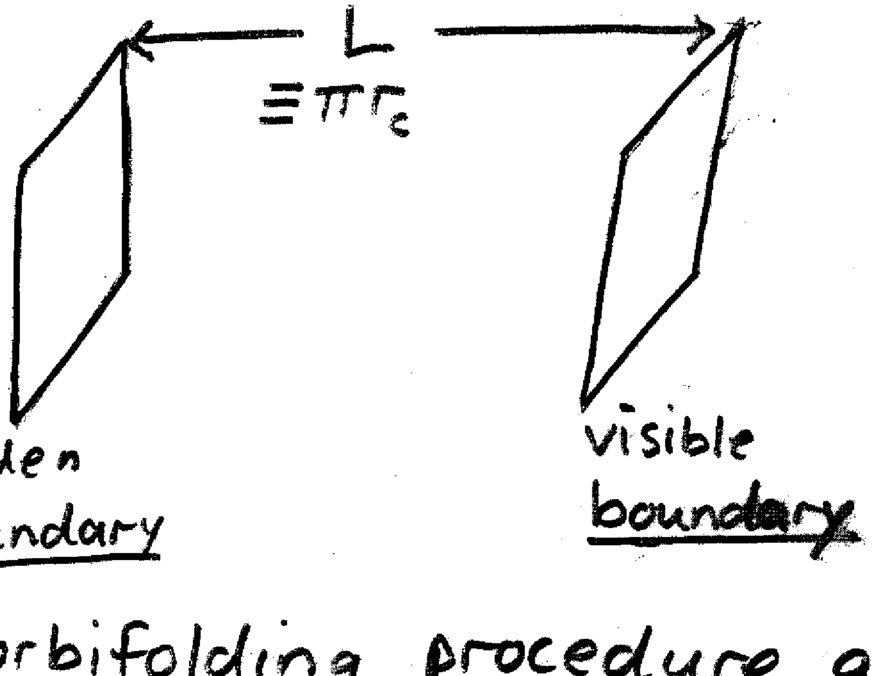
We can therefore identify the two hemispheres ("gauge" the $\mathbb{Z}_2$ symmetry):



Orbifold symmetry forbids brane fluctuations which threaten vacuum stability.

Under $\mathbb{Z}_2$-identification we throw out all field fluctuations which are $\mathbb{Z}_2$-odd, retaining only $\mathbb{Z}_2$-even.

Physically, we are left with



The orbifolding procedure generates sensible generally covariant boundary conditions.

$\mathbb{Z}_2$ - transformation of fields

$G_{\mu\nu}(x,\varphi)$ is parity-even

ie. orbifold projects out odd functions of $\phi \in [-\pi, \pi]$

$G_{\mu\varphi}(x,\phi)$ is parity-odd

ie. orbifold projects out even functions of φ

$\therefore$ Axial gauge - $G_{\mu\varphi}(x,\varphi) \to \overline{G}_{\mu\varphi}(x)$ is projected out!

$G_{\varphi\varphi}(x,\varphi)$ is parity even.

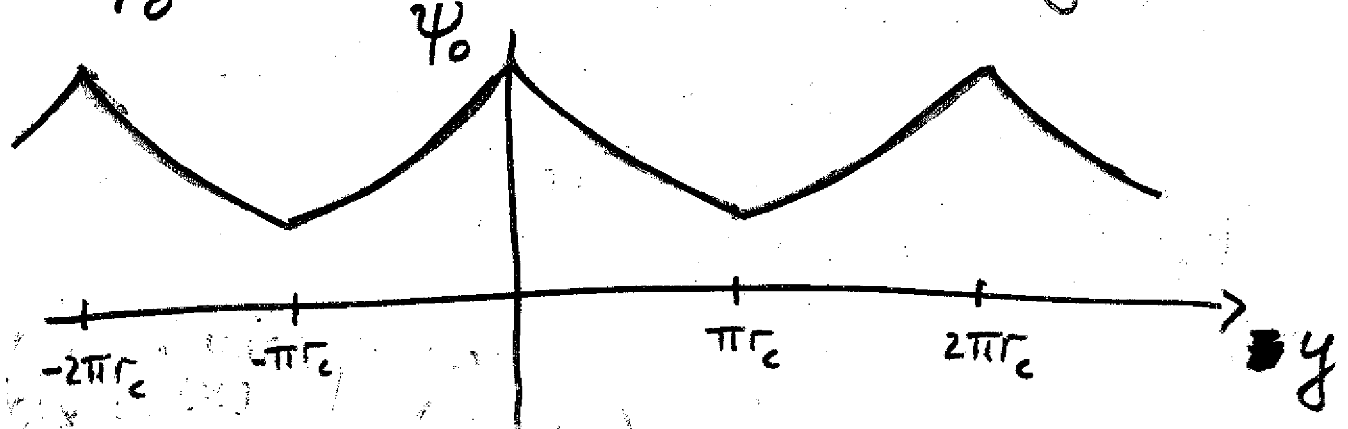
$\therefore$ axial gauge "radion" field is retained

$\chi(x)$ brane fields are parity-even as they sit at $\mathbb{Z}_2$ fixed points.

4D LOW-ENERGY

EFFECTIVE FIELD THEORY

$\psi_0(z)$ remains $m^2_{4D}=0$ eigenfunction:



$\therefore$ massless 4D graviton mode

$$\equiv ds^2 = e^{-2k|y|}\underbrace{\left(\eta_{\mu\nu} + h^{(0)}_{\mu\nu}(x)\right)}_{g^{hid}_{\mu\nu}}dx^\mu dx^\nu + \underbrace{(r_c d\phi)}_{\equiv dy}{}^2$$

Noting that $g^{vis}_{\mu\nu} = e^{-2k\pi r_c} g^{hid}_{\mu\nu}$,

$$S_{eff} = \int d^4x \sqrt{-g^{hid}} \Big\{ 2M_4^2 \bar{R} + e^{-2k\pi r_c} g^{\mu\nu}_{hid} \partial_\mu \chi \partial_\nu \chi + e^{-4k\pi r_c} \underbrace{\lambda(\chi^2 - v^2)^2}_{V(\chi)} \Big\}, \quad M_4^2 = \int_0^{\pi r_c} dy\, e^{-2k|y|} 2M_5^3$$

WARPED HIERARCHY

Suppose all scales in our set-up have no large hierarchies

$$M_5 \gtrsim k \gtrsim v, \frac{1}{\pi r_c}$$

& dimensionless couplings are order one : $\lambda \lesssim 1$.

Field re-defining $\hat{\chi} \equiv e^{-k\pi r_c}\chi$,

$$S_{eff} = \int d^4x \sqrt{-g_{hid}} \left\{ 2M_4^2 \bar{R} + g_{hid}^{\mu\nu} \partial_\mu \hat{\chi} \partial_\nu \hat{\chi} + \lambda \left(\hat{\chi}^2 - \left(e^{-k\pi r_c} v\right)^2 \right)^2 \right\}$$

$$\therefore \quad M_4^2 = \frac{\left(1 - e^{-2k\pi r_c}\right) M_5^3}{k} \approx \frac{M_5^3}{k}$$

while Weak scale $= v_{eff} = e^{-k\pi r_c} v$

$$\lesssim e^{-k\pi r_c} M_4 .$$

GENERAL WARPED HIERARCHIES

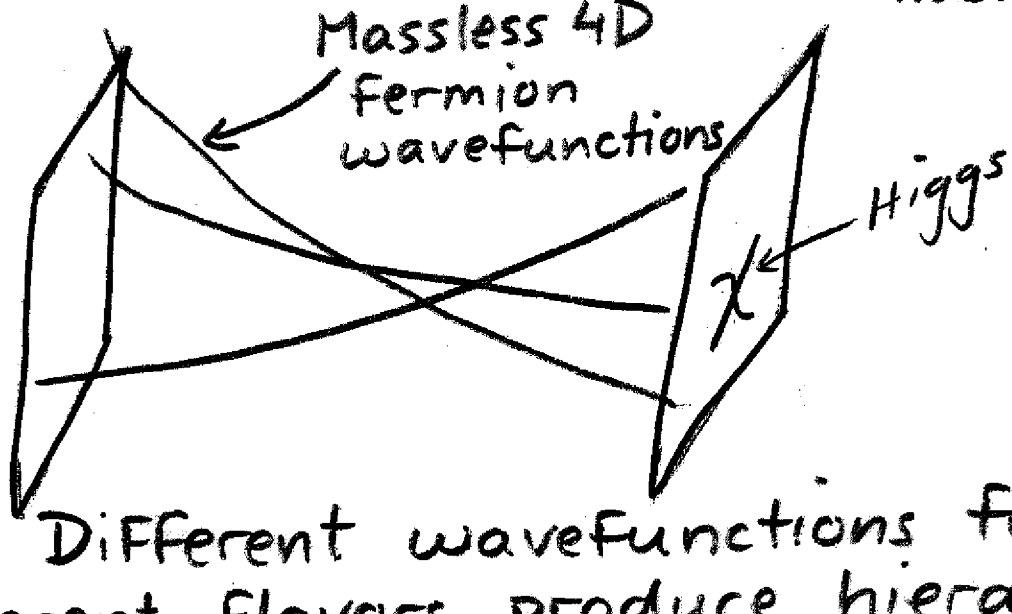
$$m_{4D} \sim e^{-ky} m_{5D}$$

5D mass paramete

Dominant location of associated physics

Fermion Hierarchies

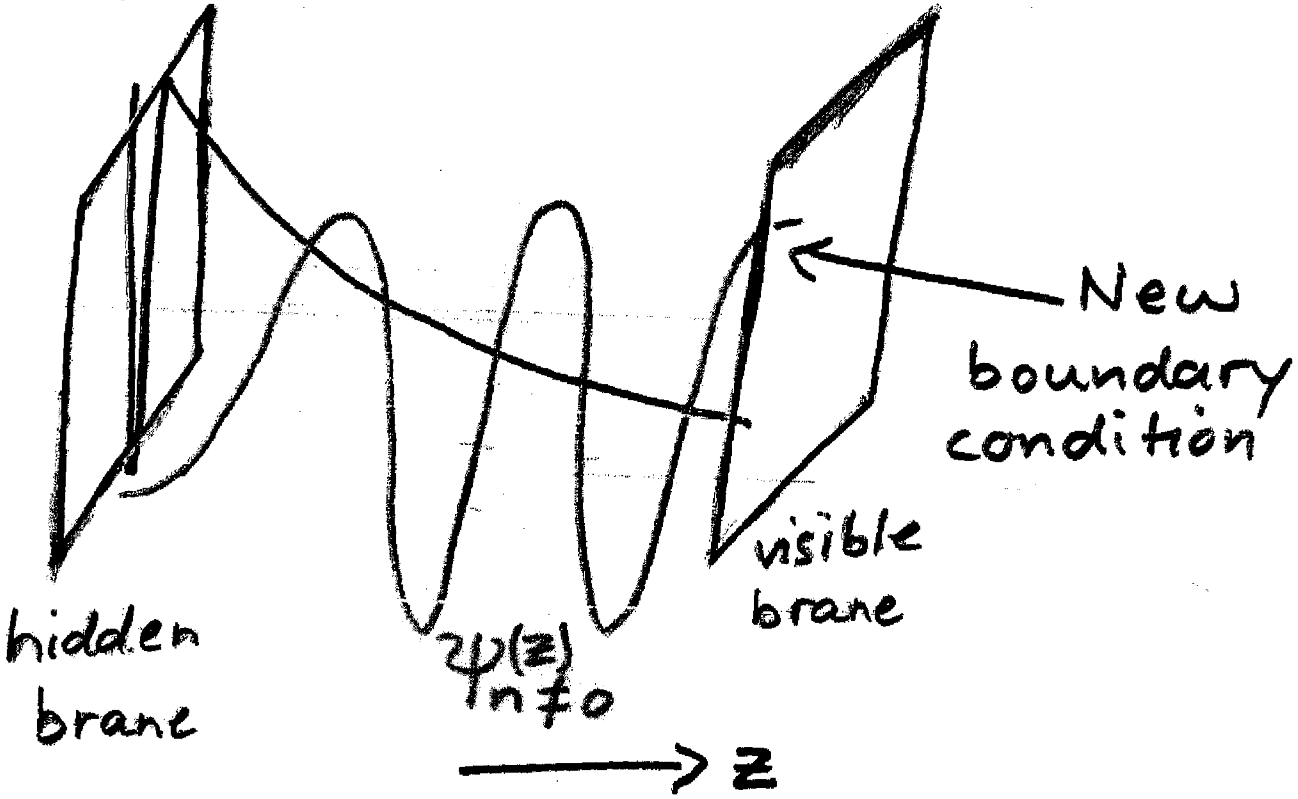
Arkani-Hamed, Schmaltz &
Gherghetta, Pomarol '00
Huber, Shafi '01, '02
Huber '02



Different wavefunctions for different flavors produce hierarchies in Yukawa couplings.

KK GRAVITONS

Compactification puts (half) the volcano potential in a box:



KK continuum $\longrightarrow$ quantized, discrete modes.

Plane wave approximation for ψ_n near $z_c \gg 1 \Rightarrow \sim \frac{\pi}{z_c}$ splittings of KK states.

$$z_c \sim \frac{e^{k\pi r_c}}{k} \Rightarrow$$

KK splittings $\sim k e^{-k\pi r_c}$

$\sim$ Weak scale

$\therefore$ KK gravitons kinematically accessible to expt.

Coupling to visible matter

$$\int d^4_x \, T^{vis}_{\mu\nu}(x) \underbrace{\hat{h}^{\mu\nu}(x, z_c)}_{\sum_n h^{\mu\nu}_n(x) \psi_n(z_c)}$$ in linearized approximation

Now $\int d^4x \, T^{vis}_{\mu\nu}(x) \, \hat{h}^{\mu\nu}_{(0)}(x) \, \psi_0(z_c)$

$\equiv$ usual massless 4D graviton coupling.

$\therefore \int d^4x \, T^{vis}_{\mu\nu}(x) \, \hat{h}^{\mu\nu}_{n\neq 0}(x) \, \psi_{n\neq 0}(z_c)$ is stronger

by $\dfrac{\psi_{n\neq 0}(z_c)}{\psi_0(z_c)}$

In plane wave approximation

$$|\psi_{n\neq 0}(z_{\bullet})| \sim \frac{1}{\sqrt{z_c}}$$

while $\psi_0(z_{\bullet}) \underset{z\gg 1/k}{\sim} \frac{1}{k^{\bullet}|z|^{3/2}}$

$\therefore$ KK gravitons couple to $T^{\mu\nu}_{vis}$

$\sim kz_c \sim e^{k\pi r_c} \lesssim \frac{M_{\bullet 4}}{v}$

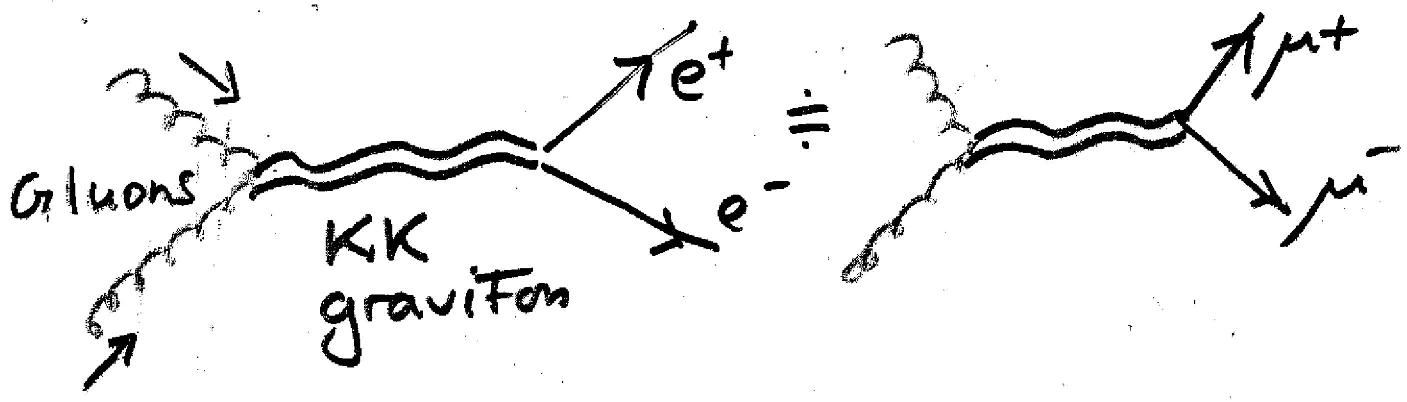
more strongly than massive gravitons!

i.e. $\frac{T^{\mu\nu}_{vis}h^{(0)}_{\mu\nu}}{M_{Pl}} \longrightarrow \lesssim \frac{T^{\mu\nu}_{vis}h^{(n\neq 0)}_{\mu\nu}}{v}$

TeV energies ⇒ unsuppressed KK production.

KEY FEATURES

- Gravitational universality of couplings



- Narrow (-ish) KK resonances if $k < M_4$, since $k e^{-k\pi r_c} \equiv$ KK mass scale while $E \times \frac{e^{k\pi r_c}}{M_4}$ sets coupling.

- Spin-2 nature of resonance visible in angular distribution of decay products

See eg. Davoudiasl, Hewett, Rizzo hep-ph/9909255

RADIUS STABILIZATION

Goldberger, Wise hep-ph/9907218 ; Lewandowski, May, Sundrum hep-th/0209050

As in unwarped case, $\exists$ massless scalar radion with NO potential, ie. a "modulus" or "flat direction" in field space.

$\therefore$ radion VEV, r_c, is not dynamically determined. Let's rectify this by adding more physics, a bulk scalar field χ:

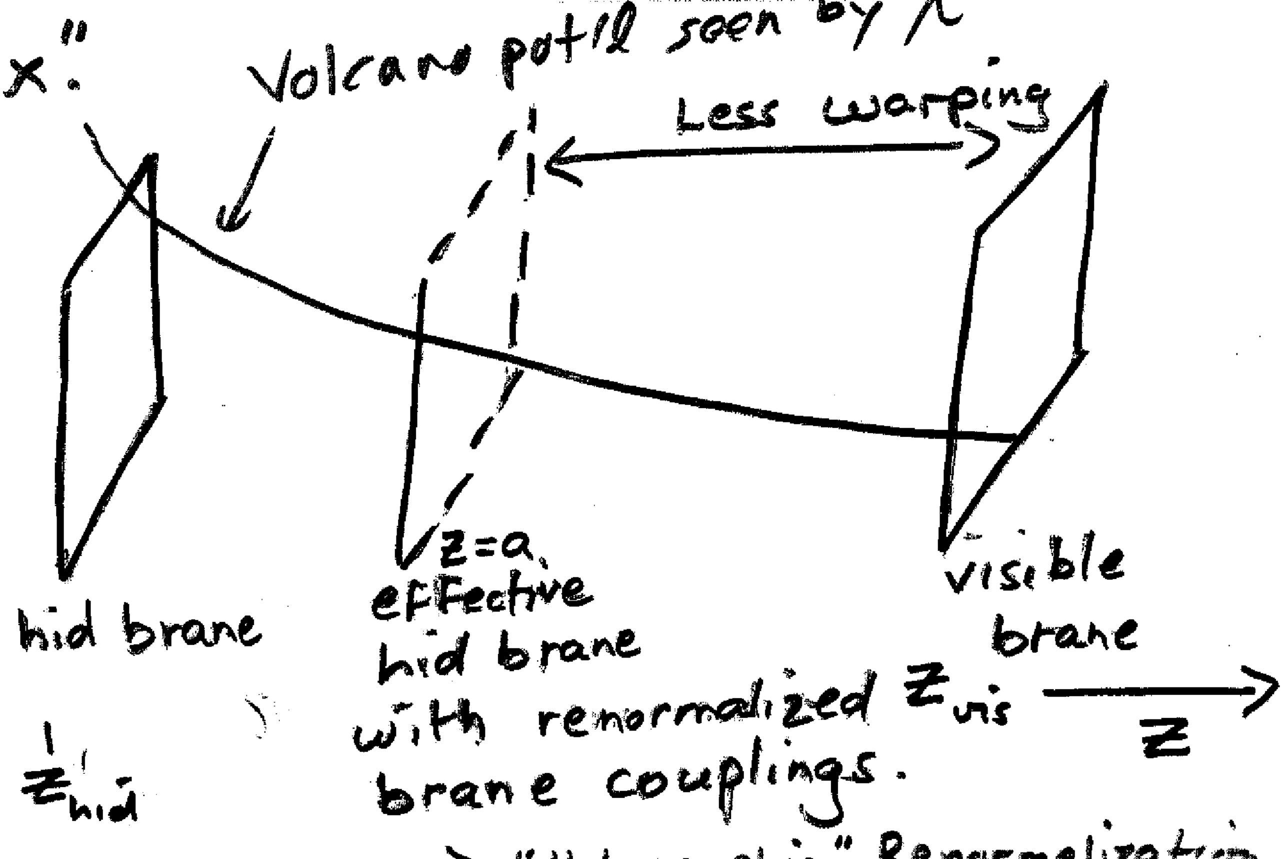
$$S = \int d^5x \sqrt{-G} \left(\frac{1}{2} G^{MN} \partial_M \chi \partial_N \chi - \frac{m^2}{2} \chi^2 + \cdots \right)$$
$$+ \int d^4x \sqrt{g_{hid}} \left\{ \text{[illegible]} \hat{\lambda} \chi + \frac{1}{2} \chi k \lambda \chi + \cdots \right\}$$
$$+ \int d^4x \sqrt{g_{vis}} \left\{ \hat{\sigma} \chi + \frac{1}{2} \chi k \sigma \chi + \cdots \right\}$$

~~This will lead to the stabilization mechanism of Goldberger, Wise hep-ph/9907218.~~

Effect of χ on radius potential like many physical effects in highly warped spaces, is NOT OBVIOUS. Look for a more insightful calculational tool.

THE BIG IDEA

For low-energy questions replace RS "box" by a smaller "effective box".



→ "Holographic" Renormalization Group.

Study χ dynamics in gravitational vacuum (neglecting gravitational backreaction).

$$\therefore\ ds^2 = \frac{1}{(kz)^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right) \qquad z \in [z_{hid}, z_{vis}].$$

$$\lambda \equiv \lambda_0 + \lambda_2 \frac{g^{\mu\nu}_{hid} D_\mu D_\nu}{k^2} + \lambda_4 \frac{g^{\mu\nu} g^{\alpha\beta} D_\mu D_\nu D_\alpha D_\beta}{k^4} + \cdots$$

$$\underset{\text{in above metric}}{=} \sum_{n \geq 0} \lambda_n \left(z^2 \underset{\text{ie. } \underline{\underline{4D}}\ \partial^2}{\overset{\uparrow}{\partial^2}}\right)^n$$

Similarly for σ.

However, $\hat{\lambda}, \hat{\sigma}$ = constants necessarily.

χ EOM:

$$\left[\partial_M \sqrt{-G}\, G^{MN} \partial_N + \sqrt{G}\, m^2 + \sqrt{g_{vis}}\, k\sigma\, \delta(z - z_{vis}) + \sqrt{g_{hid}}\, k\lambda\, \delta(z - z_{hid})\right] \chi(x,z) = 0$$

Orbifolding + δ-function matching & Fourier transforming $x^\mu \to q_\mu$ (but NOT z) $\Rightarrow$ EOM:

$$\left[-q^2 - \partial_z^2 + \frac{3}{z}\partial_z + \frac{m^2}{(kz)^2}\right]\chi(q,z) = 0$$

with boundary conditions

$$z\,\partial_z \chi(q,z)\Big|_{z\to z_{hid}} = \frac{\lambda}{2}(qz)\,\chi(q,z) + \frac{\hat{\lambda}}{2} \times \delta^4(q)$$

$$z\,\partial_z \chi(q,z)\Big|_{z\to z_{vis}} = -\frac{\sigma(qz)}{2}\,\chi(q,z) - \frac{\hat{\sigma}}{2}\delta^4(q)$$

where $\lambda(qz) \equiv \sum_{n\geq 0} \lambda_n \,(z^2q^2)^n$.

EFFECTIVE THEORY

On effective brane at $z=a$, choose effective brane couplings

$$\lambda \to \lambda_{eff}(qa, a) = \sum_n \lambda_n^{eff}(a)\,(a^2q^2)^n$$

$$\hat{\lambda} \to \hat{\lambda}_{eff}(a), \ \ldots$$

bulk & vis couplings unchanged:

IF $\chi(x,z)$ solves original EOM in $[z_{hid}, z_{vis}]$
then " " effective " " $[a, z_{vis}]$
subject to effective boundary condition,

$$(*) \quad z\partial_z \chi(q,z)\Big|_{z\to a_+} = \frac{\lambda_{eff}(qq,a)}{2}\chi(q,a) + \frac{\hat{\lambda}_{eff}}{2}\delta^4(q)$$

Also require $\lambda_{eff}(qz_{hid}, z_{hid}) = \lambda(qz_{hid})$.

RG FLOW FOR $\lambda_{eff}, \hat{\lambda}_{eff}, \ldots$

$$a\frac{d}{da}(*) \Rightarrow \begin{pmatrix}\text{using bulk EOM \& } (*) \text{ to eliminate} \\ \partial_z^2\chi \ \& \ \partial_z\chi\end{pmatrix}$$

$$\left(-a\frac{d\hat{\lambda}_{eff}}{da} + 4\hat{\lambda}_{eff} - \frac{\hat{\lambda}_{eff}\lambda_{eff}}{2}\right)\delta^4(q) + \left(-a\frac{d\lambda_{eff}}{da} + 4\lambda_{eff} - \frac{\lambda_{eff}^2}{2} + 2m^2 - 2q^2\right)\chi = 0$$

in $k=1$ units.

$$\Leftarrow \quad a\frac{d}{da}\hat{\lambda}_{eff} = 4\hat{\lambda}_{eff} - \frac{1}{2}\hat{\lambda}_{eff}\lambda^{eff}_{0}$$

$$\sum_n (q^2a^2)^n a\frac{\partial\lambda_n^{eff}}{\partial a} = -2\sum_n n(q^2a^2)^n\lambda_n^{eff} + 4\lambda_{eff} - \frac{\lambda_{eff}^2}{2} + 2m^2 - 2q^2$$

Comparing powers of q^2 => ∞ set of coupled RG equations for $\hat{\lambda}_{eff}(a)$ & $\lambda_n^{eff}(a)$.

RG FIXED POINT

We have <u>4D</u> RG "as if" we're studying a 4D QFT.

The RG has a fixed point:

$$\lambda^*_{eff}(qa) = 4-2\nu + 2qa \frac{J_{\nu-1}(qa)}{J_\nu(qa)},$$

$$\nu \equiv \sqrt{4+m^2}$$

$$\hat{\lambda}^*_{eff} = 0$$

$\equiv$ <u>local</u> $\mathcal{L}_{brane}$ in x-space since λ^*_{eff} has series expansion in q^2.

Exercise: Check.

The fixed point is attractive as $a \to z_\nu$:

Linearizing about fixed point RG is

$$a\frac{\partial}{\partial a}\lambda_n = \underset{\uparrow \text{lower triangular}}{\gamma_{nm}}(\lambda-\lambda_*)_m$$

γ has eigenvalues $4-2j-\frac{n}{2}(4+2\nu) < 0$, $n \in \mathbb{Z}_{>}$ $j \in \mathbb{Z}_{>}$

Harder Exercise: Check this.

SOLVING THE RG

Suppose "initially" (at z_{hid}) we are moderately near fixed point, so can use linearized RG.

Since here we're interested in potential energy, drop all q^2 terms in RG.

$$a\frac{\partial}{\partial a}\hat{\lambda}_{eff} = \left(4 - \frac{\lambda^*_{eff\,0}}{2}\right)\hat{\lambda}_{eff} = (2-\nu)\hat{\lambda}_{eff}$$

$$a\frac{\partial}{\partial a}\lambda_0^{eff} = (4 - \lambda_0^{eff*})(\lambda_0^{eff} - \lambda_0^{eff*})$$
$$= -2\nu(\lambda_0^{eff} - \lambda_0^{eff*})$$

Solving & "running" down to $a \sim z_{vis}$

$$\Rightarrow \quad \hat{\lambda}_{eff}(\sim z_{vis}) = \hat{\lambda}\left(\frac{z_{vis}}{z_{hid}}\right)^{2-\nu}$$

$$\lambda_0^{eff}(\sim z_{vis}) = \lambda_0^{eff*} + (\lambda_0 - \lambda_0^{eff*})\left(\frac{z_{vis}}{z_{hid}}\right)^{-2\nu}$$

For such small effective dimension can neglect "bulk" & treat χ as constant in extra dimension.

$$\therefore S_{eff} = S^{(a)}_{hid\,eff} + S_{vis} + \cancel{S_{bulk}}$$

$$= \int \frac{d^4x}{z_{vis}^4}\left[\left(\hat{\sigma} + \hat{\lambda}\left(\frac{z_{vis}}{z_{hid}}\right)^{2-\nu}\right)\chi\right.$$

$$\left. + \left(\sigma_0 + \lambda_0^{eff*} + \left(\lambda_0 - \lambda_0^{eff*}\right)\left(\frac{z_{vis}}{z_{hid}}\right)^{-2\nu}\right)\frac{\chi^2}{2}\right]$$

Self-consistently neglect for v. large $\frac{z_{vis}}{z_{hid}}$. & moderately small m^2.

$$\therefore \quad \langle\chi\rangle \doteqdot \frac{\hat{\sigma} + \hat{\lambda}\left(\frac{z_{vis}}{z_{hid}}\right)^{-m^2/8}}{\sigma_0 + \lambda_0^{eff*}}$$

Plugging back into action $\Rightarrow$

$$S_{eff} = \int d^4x \, \frac{\left[\hat{\sigma} + \hat{\lambda}\left(\frac{z_{vis}}{z_{hid}}\right)^{-m^2/8}\right]^2}{2 z_{vis}^4 \left(\sigma_0 + \lambda_0^{eff*}\right)}$$

EFFECTIVE RADION POTENTIAL

$$S_{eff} = \int d^4x \left(- V_{eff}\left(\frac{z_{vis}}{z_{hid}} \right) \right)$$

The VEV of the Radion degree of freedom when gravity is turned back on

Minimizing this V_{eff} $\Longrightarrow$

$$\frac{z_{vis}}{z_{hid}} = \left(\frac{-\hat{\sigma}}{\hat{\lambda}} \right)^{-8/m^2}$$

$\therefore$ LARGE ratio can be generate from modest $\frac{\hat{\sigma}}{\hat{\lambda}}$, m^2!

This ratio is Planck/Visible hierarchy!! ie. accumulated warp factor, $z \sim e^{ky}$.

RG approach v. powerful way to show stabilit of RS hierarchy under general (quantum) interaction

AdS/CFT CORRESPONDENCE (DUALITY)

Bulk AdS_5 dynamics described by 4D Fixed point RG is not coincidence:

5D Gravity + ... on AdS_5 is DUAL to 4D theory (without gravity), "the holographic dual", which is scale invariant, ie. a confomal field theory.

Review: Aharony, Gubser, MALDACENA, Ooguri, Oz hep-th/9905111

also Witten hep-th/9802150.

"RS/CFT"

Take a look at

Arkani-Hamed, Porrati, Randall hep-th/
0012148

Rattazi, Zaffaroni hep-th/0012248.

- 5D Gravity in RS dual to 4D CFT coupled to 4D gravity.
- CFT is strongly coupled to be consistent with unusual γ_{mn} "critical exponents/anomalous dimensio
- Conformal invariance spontaneously broken at TeV scale.
- Radion dual to Goldstone of !
- KK modes & visible particles are composites of strong dynamics.

- Weakly coupled 5D theory $\equiv \frac{1}{N_{color}}$ - like expansion of strong dynamics.

- RS1 dual to type of composite Higgs theory.

- $\exists$ many checks of all this.

- Warped effective field theory is a powerful approach to studying Nature's non-supersymmetric option above weak scale via compositeness. Direct attack is too hard.

- This is approach now being taken with RS1 phenomenology applied to GUTs, precision tests, Flavor, ...