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SUMMER SCHOOL ON PARTICLE PHYSICS

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LECTURES ON SUPERSYMMETRY

Lecture III

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Lectures on Supersymmetry

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Lecture 3:

Models for SUSY breaking

• Properties of SUSY theories

Models for SUSY breaking

Phenomenological constraints on soft-breaking terms:

E.g.:
$$(m^2)_{ij}\varphi_j^*\varphi_i = \tilde{e}_R m_{ee}^2 \tilde{e}_R^\dagger + \tilde{e}_R m_{e\mu}^2 \tilde{\mu}_R^\dagger + \dots$$

 $(m^2)_{ij}$: 3 × 3 matrix in family (flavor) space

no a priori reason why off-diagonal terms $m_{e\mu}^2$ should be much smaller than diagonal terms m_{ee}^2 , $m_{ij}^2\sim 1~{\rm TeV^2}$

but: if off-diagonal terms are large

 \Rightarrow observable lepton number violation (e.g. $\mu \rightarrow e\gamma$)

Strong experimental constraints:

if $m_{e\mu}^2 \approx m_{ee}^2$

 \Rightarrow SÚSY contribution 5–6 orders of magnitude above experimental limit

 \Rightarrow need $m_{e\mu}^2 \ll m_{ee}^2$

Similarly for squark mass matrices; also strong constraints on $\mathcal{CP}\mbox{-violating}$ complex phases

- \Rightarrow experimental constraints satisfied if SUSY breaking is suitably 'universal', \approx diagonal in flavour space
- ⇒ small flavour-changing neutral currents (FCNC) no new complex phases in this limit
- \Rightarrow hints of an "organizing principle" for soft breaking terms?

Simplest ansatz:

Assume universality at high energy scale (GUT scale, M_{PI} , ...)

- All scalar masses are the same: $\tilde{m}^2 = m_0^2$ (assumption of "universality")
- The gaugino masses are the same: $M_i = m_{1/2}$ ("GUT relation")
- Soft-breaking trilinear terms are universal:

 $\mathcal{L}_{tri} = A_0(H_UQy_u\bar{u} + H_DQy_d\bar{d} + H_DLy_l\bar{e}), y_u, y_d, \dots$ are the same matrices which appear in Yukawa couplings (assumption of "proportionality")

Results in five parameters, if possible phases are ignored:

$$m_0^2, m_{1/2}, A_0, B\mu, \mu$$

Require correct value of M_Z

 \Rightarrow $|\mu|$, B given in terms of tan β , sign μ

 \Rightarrow Scenario characterized by

 $m_0^2, m_{1/2}, A_0, \tan\beta, \operatorname{sign}\mu$

Usually called 'CMSSM' (constrained MSSM) or 'mSUGRA' (see below)

In agreement with all phenomenological constraints (see below)

Spontaneous breaking of global SUSY

If global SUSY is spontaneously broken $\Rightarrow \langle 0|H|0 \rangle = E_{Vac} > 0$ $\Rightarrow V > 0$ required

$$V(\varphi_i, \varphi_j^*) = \underbrace{\sum_{i} \left| \frac{\partial \mathcal{V}}{\partial \varphi_i} \right|^2}_{F\text{-term}} + \underbrace{\frac{1}{2} g^2 \sum_{a} (\varphi_i^* T_{ij}^a \varphi_j + \xi^a)^2}_{D\text{-term}}$$

 \Rightarrow either $\langle F \rangle > 0$ (*F*-term breaking) or $\langle D \rangle > 0$ (*D*-term breaking)

 \Rightarrow requires that $F_i = 0$, $D^a = 0$ cannot be simultaneously satisfied for any values of the fields

F-term breaking:

need linear term in superpotential $\mathcal{V}(\phi_i) = a_i \phi_i + \frac{1}{2} m_{ij} \phi_i \phi_j + \frac{1}{3} \lambda_{ijk} \phi_i \phi_j \phi_k$ \Rightarrow requires a chiral superfield that is a singlet under all gauge groups \Rightarrow not possible within the MSSM

D-term breaking:

If $\langle \varphi_i \rangle = 0 \Rightarrow \xi^a$ -term (Fayet-Iliopoulos term) leads to spontaneous SUSY breaking

Does not work in the MSSM (leads to charge and color-breaking minima)

Further problems of spontaneous breaking of global SUSY:

• Mass sum rules for SM particles and superpartners (at tree level), e.g.

 $\sum_{i} (-1)^f m_i^2 = 0$

 $(-1)^f = 1$ for bosons, $(-1)^f = -1$ for fermions

 \Rightarrow not all superpartners can be heavier than SM particles

 \Rightarrow spectra not realistic

- difficult to give masses to gauginos (SUSY doesn't allow scalar-gauginogaugino couplings)
- hierarchy problem, if characteristic SUSY-breaking scale enters as parameter in the Lagrangian
- cosmological constant problem

 \Rightarrow Very difficult to build realistic model

Problems can be overcome if SUSY breaking happens in a 'hidden sector', i.e. by fields which have only very small couplings to ordinary matter

SUSY breaking in the hidden sector: tree-level (like F- and D-term breaking) or dynamical breaking (similar to chiral symmetry breaking in QCD), ...

SUSY-breaking terms in the MSSM arise radiatively via interaction that communicates SUSY breaking rather than through tree-level couplings to SUSY breaking v.e.v.s

⇒ phenomenology depends mainly on mechanism for communicating SUSY breaking rather than on SUSY-breaking mechanism itself

If mediating interactions are \approx flavor-diagonal

⇒ universal soft-breaking terms

"Hidden sector": \longrightarrow Visible sector:SUSY breakingMSSM

"Gravity-mediated": mSUGRA "Gauge-mediated": GMSB "Anomaly-mediated": AMSB "Gaugino-mediated"

. . .

SUGRA: mediating interactions are gravitational

GMSB: mediating interactions are ordinary electroweak and QCD gauge interactions

AMSB, Gaugino-mediation: Gaugino-mediation:

SUSY breaking happens on a different brane in a higher-dimensional theory

Gravity-mediated SUSY breaking

Local SUSY \Leftrightarrow supergravity:

Spontaneous breaking of global SUSY \rightarrow massless neutral Weyl fermion: "goldstino"

Spontaneous breaking of local SUSY \rightarrow goldstino absorbed ("eaten") by gravitino

- \Rightarrow gives longitudinal components to gravitino
- \Rightarrow gravitino acquires a mass, graviton stays massless

"Super-Higgs mechanism"

only known consistent way of breaking local SUSY

It is possible to have spontaneous breaking of local SUSY and also $\langle V \rangle = 0$

Quantum field theory of supergravity:

QFT with spin 2 (and spin $\frac{3}{2}$) field is not renormalizable

- \Rightarrow cannot be extended to arbitrarily high energies
- \Rightarrow QFT of supergravity has to be interpreted as effective theory contains non-renormalizable terms prop. to inverse powers of $M_{\rm Pl}$
- Best candidate for fundamental theory: string theory

SUSY breaking in hidden sector:

 \Rightarrow supergravity Lagrangian contains non-renormalizable terms that communicate between hidden and visible sector $\sim 1/M_{\rm Pl}^n$

Dimensional analysis:

SUSY breaking in hidden sector by v.e.v. $\langle F \rangle$, coupling $\sim 1/M_{\text{Pl}}$ require $m_{\text{soft}} \rightarrow 0$ for $\langle F \rangle \rightarrow 0$ (no SUSY breaking) and for $M_{\text{Pl}} \rightarrow \infty$ (vanishing gravitational interaction)

$$\Rightarrow \quad m_{\rm soft} \approx \frac{\langle F \rangle}{M_{\rm PI}}$$

For $m_{\rm soft} \lesssim 1$ TeV (hierarchy problem) $\Rightarrow \sqrt{\langle F \rangle} \approx 10^{11}$ GeV: scale of SUSY breaking in hidden sector

In general:
$$m_{\frac{3}{2}} \approx \frac{\langle F \rangle}{M_{Pl}}$$

 $\Rightarrow m_{\frac{3}{2}} \approx m_{soft}$, gravitational interactions
 \Rightarrow gravitino not important for collider phenomenology

Non-renormalizable terms in supergravity Lagrangian:

$$\mathcal{L}_{\text{NR}} = -\frac{1}{M_{\text{Pl}}} F_X \sum_a \frac{1}{2} f_a \lambda^a \lambda^a + \text{h.c.} - \frac{1}{M_{\text{Pl}}^2} F_X F_X^* k_j^i \varphi_i \varphi^{*j}$$
$$- \frac{1}{M_{\text{Pl}}} F_X (\frac{1}{6} y'^{ijk} \varphi_i \varphi_j \varphi_k + \frac{1}{2} \mu'^{ij} \varphi_i \varphi_j) + \text{h.c.}$$

 F_X : auxiliary field for a chiral supermultiplet X in the hidden sector φ_i , λ^a : scalar and gaugino fields in the MSSM

If $\langle F_X \rangle \sim 10^{10} \text{--} 10^{11} \text{ GeV}$

 \Rightarrow yields soft SUSY-breaking terms of MSSM with $m_{soft} \approx 10^2 - 10^3$ GeV

Assumption of a "minimal" form of the supergravity Lagrangian (theoretically difficult to justify; supergravity doesn't require flavor-universal terms)

⇒ soft-breaking terms which obey "universality' and "proportionality"

⇒ "supergravity inspired scenario", "mSUGRA", characterized by five parameters

$$m_0^2, m_{1/2}, A_0, \tan\beta, \operatorname{sign}\mu$$

 m_0 : universal scalar mass parameter

 $m_{1/2}$: universal gaugino mass parameter

 A_0 : universal trilinear coupling

 $\tan \beta$: ratio of Higgs vacuum expectation values

 $sign(\mu)$: sign of supersymmetric Higgs parameter

 $m_0, m_{1/2}, A_0$: GUT scale parameters \Rightarrow particle spectra from renormalization group running to weak scale

Lightest SUSY particle (LSP) is usually lightest neutralino gaugino masses run in same way as gauge couplings ⇒ gluino heavier than charginos, neutralinos



Gauge-mediated SUSY breaking (GMSB)

New chiral supermultiplets, "messengers", couple to SUSY breaking in hidden sector

Couple indirectly to MSSM fields via gauge interactions

- ⇒ mediation of SUSY breaking via electroweak and QCD gauge interactions
- $\Rightarrow \approx$ flavour-diagonal

SUSY breaking in messenger spectrum

 \Rightarrow masses of SUSY particles from loop diagrams with messenger particles, gauge-interaction strength

$$\Rightarrow \quad m_{\rm soft} \approx \frac{\alpha_i}{4\pi} \frac{\langle F \rangle}{M_{\rm mes}}, \quad M_{\rm mes} \sim \sqrt{\langle F \rangle}$$

For $m_{\rm soft} \lesssim 1~{\rm TeV} \Rightarrow \sqrt{\langle F \rangle} \approx 10^4 \text{--} 10^5~{\rm GeV}$

 \Rightarrow scale of SUSY breaking in hidden sector much lower than in SUGRA

Gravitino mass: $m_{\frac{3}{2}} \approx \frac{\langle F \rangle}{M_{\text{Pl}}} \approx 10^{-9} \text{ GeV}$

 \Rightarrow Gravitino is always the lightest SUSY particle (LSP)

Gaugino masses generated at one-loop order, $m_\lambda\approx\frac{\alpha_i}{4\pi}$



Scalar masses generated at two-loop order, $m_{\varphi}^2 \approx \left(\frac{\alpha_i}{4\pi}\right)^2$



⇒ Typical mass hierarchy in GMSB scenario between strongly interacting and weakly interacting particles $\sim \alpha_3/\alpha_2/\alpha_1$

GMSB scenario characterized by

 $M_{\text{mes}}, N_{\text{mes}}, \Lambda, \tan\beta, \operatorname{sign}(\mu)$

 M_{mes} : messenger mass scale

 N_{mes} : messenger index (number of messenger multiplets)

∧: universal soft SUSY breaking mass scale felt by low-energy sector

LSP is always the gravitino, next-to-lightest SUSY particle (NLSP): $\tilde{\chi}_1^0$ or $\tilde{\tau}_1$

can decay into LSP inside or outside the detector



GMSB scenario with $\tilde{\tau}$ NLSP (SPS 7 benchmark scenario):

AMSB scenario characterized by

 $m_{aux}, m_0, \tan\beta, \operatorname{sign}(\mu)$



Properties of SUSY theories:

Gauge coupling unification:

Running

of gauge couplings:
$$\frac{1}{g^2(\mu^2)} = \frac{1}{g^2(\mu_0^2)} + \beta \ln\left(\frac{\mu^2}{\mu_0^2}\right)$$

Unification of couplings at high scale \Leftrightarrow "Grand unified theories" (GUTs) E.g.: SO(10) GUTs, can naturally accomodate right-handed neutrinos

Unification of the Coupling Constants in the SM and the MSSM



[U. Amaldi, W. de Boer, H. Fürstenau '91]

 \Rightarrow coupling constant unification in MSSM for $M_{\rm SUSY} \lesssim 1~{\rm TeV}$

Radiative electroweak symmetry breaking:

Universal boundary conditions at GUT scale, renormalization group running down to weak scale

400 (GeV)300 Sparticle Mass 200 100 W B 0 10¹² $10^{\overline{3}}$ 10⁶ 10⁹ 10^{15} Q (GeV)

large corrections from top-quark Yukawa coupling $\Rightarrow m_{H_u}^2$ driven to negative values \Rightarrow ew symmetry breaking

emerges naturally at scale $\sim 10^2~{\rm GeV}$ for 100 GeV $\lesssim m_{\rm t} \lesssim$ 200 GeV

 $M_0 = 300 \text{ GeV}, M_{1/2} = 100 \text{ GeV}, A_0 = 0$

R parity

Most general gauge-invariant and renormalizable superpotential with chiral superfields of the MSSM:

$$\mathcal{V} = \mathcal{V}_{\text{MSSM}} + \underbrace{\frac{1}{2} \lambda^{ijk} L_i L_j E_k}_{2} + \lambda^{\prime ijk} L_i Q_j D_k + \mu^{\prime i} L_i H_u}_{2} + \underbrace{\frac{1}{2} \lambda^{\prime\prime ijk} U_i D_j D_k}_{2}$$

violate lepton number violates baryon number

If both lepton and baryon number are violated

 \Rightarrow rapid proton decay

Minimal choice (MSSM) contains only terms in the Lagrangian with even number of SUSY particles

 \Rightarrow additional symmetry: "R parity"

 \Rightarrow all SM particles have even R parity, all SUSY particles have odd R parity

Phenomenological consequences of R parity conservation:

- SUSY particles can only be produced in pairs, e.g. $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$
- Decay of SUSY particles into SM particles
 + odd number of SUSY particles

 \Rightarrow Lightest SUSY particle (LSP) is stable

- All SUSY particles will eventually decay into LSP
 - \Rightarrow Decay cascades of heavy SUSY particles
- LSP stable
 - \Rightarrow some LSP must have survived from Big Bang
 - ⇒ Weakly interacting massive particle ("WIMP") (otherwise ruled out from bounds on exotic isotopes etc.)
 - \Rightarrow Candidate for cold dark matter in the Universe (if $m_{\text{LSP}} \lesssim 1 \text{ TeV}$)
- LSP neutral, uncolored ⇒ leaves no traces in collider detectors
 ⇒ Typical SUSY signatures: "missing energy"

Relations between SUSY parameters

Symmetry properties of MSSM Lagrangian (SUSY, gauge invariance) give rise to coupling and mass relations

Soft SUSY breaking does not affect SUSY relations between dimensionless couplings

E.g.:

gauge boson-fermion coupling

gaugino-fermion-sfermion coupling

for U(1), SU(2), SU(3) gauge groups

In SM: all masses are free input parameters (except $M_W - M_Z$ interdependence)

MSSM:

- Upper bound on mass of lightest $\mathcal{CP}\text{-}\mathsf{even}$ Higgs boson
- Relations between neutralino and chargino masses
- Sfermion mass relations, e.g.

$$m_{\tilde{e}_L}^2 = m_{\tilde{\nu}_L}^2 - M_{\mathsf{W}}^2 \cos(2\beta)$$

All relations receive corrections from loop effects

 $\Leftrightarrow \text{ effects of soft SUSY breaking, electroweak symmetry breaking}$

 \Rightarrow Experimental verification of parameter relations is a crucial test of SUSY!

The Higgs sector of the MSSM

Two Higgs doublets:

$$H_{1} = \begin{pmatrix} H_{1}^{1} \\ H_{1}^{2} \end{pmatrix} = \begin{pmatrix} v_{1} + (\phi_{1}^{0} + i\chi_{1}^{0})/\sqrt{2} \\ \phi_{1}^{-} \end{pmatrix}$$
$$H_{2} = \begin{pmatrix} H_{2}^{1} \\ H_{2}^{2} \end{pmatrix} = \begin{pmatrix} v_{1} + (\phi_{1}^{0} + i\chi_{1}^{0})/\sqrt{2} \\ \phi_{1}^{-} \end{pmatrix}$$

Higgs potential:

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$

+
$$\frac{g'^2 + g^2}{8} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g^2}{2} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM

Mixing of CP-even, CP-odd, charged fields:

$$\begin{pmatrix} H^{0} \\ h^{0} \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \phi_{1}^{0} \\ \phi_{2}^{0} \end{pmatrix}$$

$$\begin{pmatrix} G^{0} \\ A^{0} \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \chi_{1}^{0} \\ \chi_{2}^{0} \end{pmatrix}, \quad \begin{pmatrix} G^{\pm} \\ H^{\pm} \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_{1}^{\pm} \\ \phi_{2}^{\pm} \end{pmatrix}$$
$$\tan(2\alpha) = \tan(2\beta) \frac{M_{A}^{2} + M_{Z}^{2}}{M_{A}^{2} - M_{Z}^{2}}$$

Three Goldstone bosons (as in SM): G^0 , G^{\pm}

 \longrightarrow longitudinal components of W^{\pm} , Z

 \Rightarrow Five physical states: h^0, H^0, A^0, H^{\pm}

h, *H*: neutral, CP-even, A^0 : neutral, CP-odd, H^{\pm} : charged

Gauge-boson masses:

$$M_{\rm W}^2 = \frac{1}{2}g'^2(v_1^2 + v_2^2), \quad M_Z^2 = \frac{1}{2}(g^2 + g'^2)(v_1^2 + v_2^2), \quad M_\gamma = 0$$

Problem:

MSSM contains term $\mu H_1 H_2$ in superpotential

 μ : dimensionful parameter

For ew symmetry breaking required: $\mu \sim$ electroweak scale

But: no a priori reason for $\mu \neq 0$, $\mu \ll M_{\mathsf{P}}$

(problem mainly in GMSB svenario, easier to overcome in mSUGRA)

Possible solution:

 μ related to v.e.v. of additional field

 \Rightarrow Introduction of extra singlet field S, v.e.v. $s \Rightarrow$ "NMSSM"

Superpotential:
$$\mathcal{V} = \lambda H_1 H_2 S + \frac{1}{3} \kappa S^3 + \dots$$

Physical states in NMSSM Higgs-sector:

 S_1, S_2, S_3 (CP-even), P_1, P_2 (CP-odd), H^{\pm}

Parameters in MSSM Higgs potential V (besides g, g'):

 $v_1, v_2, m_1, m_2, m_{12}$

relation for M_W^2 , $M_Z^2 \Rightarrow 1$ condition

minimization of V w.r.t. neutral Higgs fields H_1^1 , $H_2^2 \Rightarrow 2$ conditions

 \Rightarrow only two free parameters remain in V, conventionally chosen as $\tan \beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2(\tan \beta + \cot \beta)$

 \Rightarrow $m_{\rm h}$, $m_{\rm H}$, mixing angle α , $m_{\rm H^{\pm}}$: no free parameters, can be predicted

In lowest order:

$$M_{\mathsf{H}^{\pm}}^2 = M_\mathsf{A}^2 + M_\mathsf{W}^2$$

Predictions for m_h , m_H from diagonalization of tree-level mass matrix:

 $\phi_1 - \phi_2$ basis:

Tree-level result for m_h , m_H :

$$m_{\rm H,h}^2 = \frac{1}{2} \left[M_{\rm A}^2 + M_{\rm Z}^2 \pm \sqrt{(M_{\rm A}^2 + M_{\rm Z}^2)^2 - 4M_{\rm Z}^2 M_{\rm A}^2 \cos^2 2\beta} \right]$$

 $\Rightarrow m_{\mathsf{h}} \leq M_{\mathsf{Z}}$ at tree level

$$\Rightarrow$$
 Light Higgs boson h required in SUSY

Measurement of $m_{\rm h}$, Higgs couplings

 \Rightarrow test of the theory (more directly than in SM)

Higgs couplings, tree level:

$$g_{\rm HVV} = \sin(\beta - \alpha) g_{\rm HVV}^{\rm SM}, \quad V = W^{\pm}, Z$$

$$g_{\rm HVV} = \cos(\beta - \alpha) g_{\rm HVV}^{\rm SM}$$

$$g_{\rm hAZ} = \cos(\beta - \alpha) \frac{g'}{2\cos\theta_{\rm W}}$$

$$g_{\rm h\pi^{\pm}\pi^{-}} = -\frac{\sin\alpha}{2} g_{\rm Hb\bar{b}}^{\rm SM} H_{\pi^{\pm}\pi^{-}}$$

$$g_{\rm hb\bar{b}}, g_{\rm h\tau^+\tau^-} = -\frac{6}{\cos\beta} g_{\rm Hb\bar{b}, H\tau^+\tau^-}^{\rm SM}$$
$$g_{\rm ht\bar{t}} = \frac{\cos\alpha}{\sin\beta} g_{\rm Ht\bar{t}}^{\rm SM}$$
$$g_{\rm Ab\bar{b}}, g_{\rm A\tau^+\tau^-} = \gamma_5 \tan\beta g_{\rm Ab\bar{b}}^{\rm SM}$$

 $\Rightarrow g_{\rm hVV} \leq g_{\rm HVV}^{\rm SM}, \quad g_{\rm hVV}, g_{\rm HVV}, g_{\rm hAZ} \text{ cannot all be small}$ $g_{\rm hb\bar{b}}, g_{\rm h\tau^+\tau^-} \text{: significant suppression or enhancement w.r.t. SM coupling possible}$

Summary of Lecture 3:

• SUSY breaking mechanism not well understood

Most attractive framework: SUSY breaking happens in 'hidden sector', can be communicated to MSSM fields in different ways

- Gravity mediation (local SUSY includes gravity), gauge mediation, ...
- Properties of SUSY theories: allow gauge coupling unification, radiative electroweak symmetry breaking
- LSP: attractive candidate for cold dark matter in the Universe
- SUSY requires light Higgs boson
- SUSY parameter relations ⇒ crucial test of SUSY