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SMR.1508 - 29

SUMMER SCHOOL ON PARTICLE PHYSICS

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QDC PHASE TRANSITIONS

Lectures III & IV

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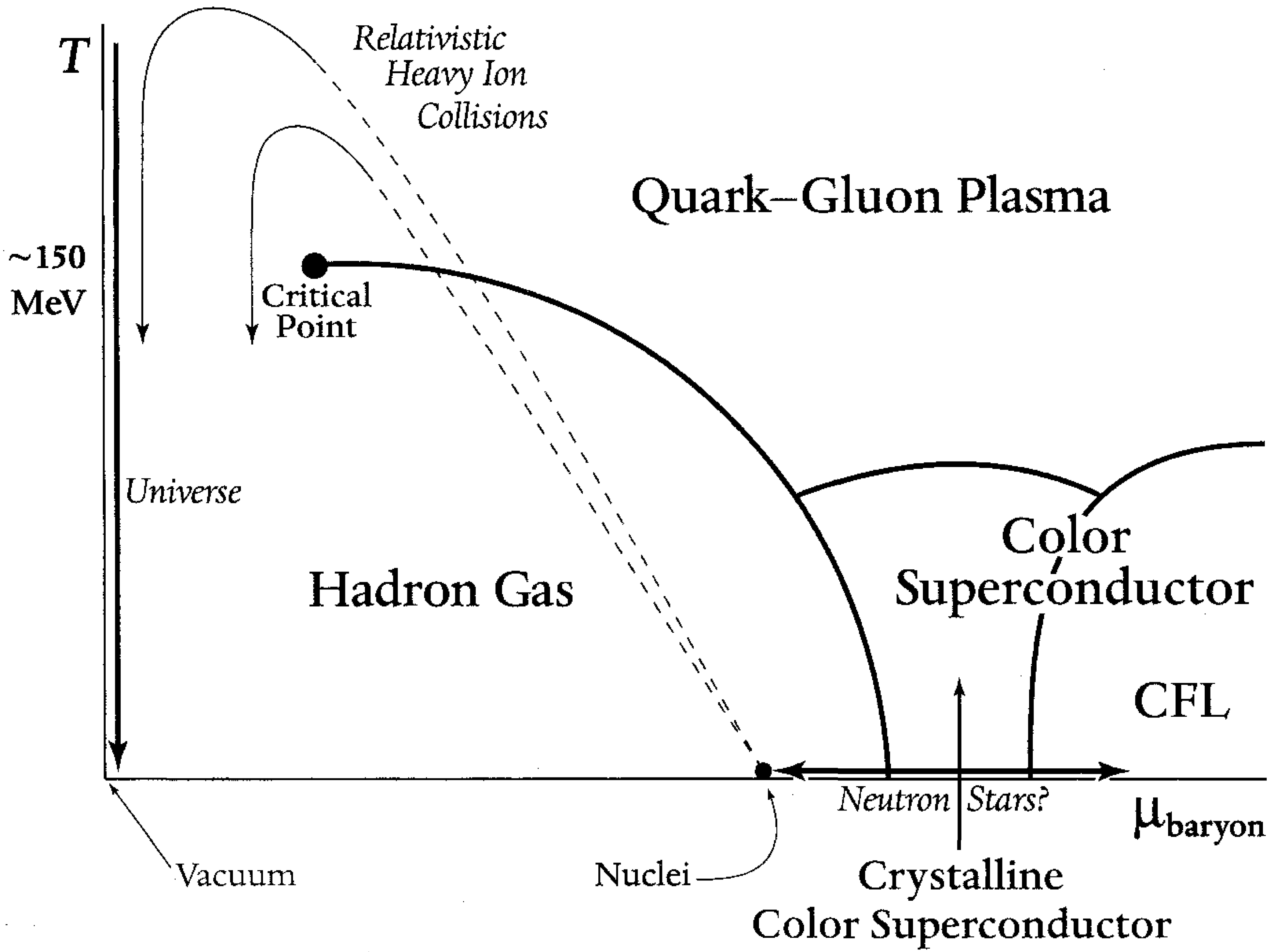
EXPLORING *the* PHASES *of* QCD



LARGE μ; SMALL T

Whereas at high T entropy wins
→ quark-gluon plasma with symmetries of the QCD Lagrangian manifest....

At large μ with small T we find quark matter with new patterns of order:

- Color superconductivity
- Color-Flavor Locking
- Crystalline Color Superconductivity

⋮

How can we use astrophysical observations of compact stars to determine the QCD phase diagram?

THE DIFFICULTY WITH DENSITY

Why are we still asking basic questions about QCD at high μ, low T, like "what is symmetry of ground state?"

NO LATTICE CALCULATIONS

$\mu \neq 0 \rightarrow$ complex Euclidean action
$\rightarrow$ sign problem that makes difficulty of standard Monte Carlo $\sim e^V$.

Equally nasty sign problems can be solved in simpler systems. Chandrasekharan Wiese

Sign problem may also be evaded:

- at small V, small μ/T Fodor Katz; Hands Karsch et al
- calculate at $\mathrm{Im}\,\mu$; continue observables. Works at $\mu/T < \pi/3$. V can be large. de Forcrand Philipsen; d'Elia Lombardo

$\rightarrow$ may be used to locate critical point.

- modify the theory. (color superconductivi studied on lattice for NJL & QCD $\bar{\omega}$ $N_c = 2$ Hands et al Kogut et al

NO EVASION POSSIBLE FOR QCD at $\mu \gg T$

- use smallness of g at $\mu \rightarrow \infty$
- use models at accessible μ.

WHY COLOR SUPERCONDUCTIVITY?

Large $\mu \rightarrow$ quarks filling Fermi sea up to a large Fermi energy. (E_F)

asymptotic freedom $\rightarrow$ weak interactions between quarks at Fermi surface.

BUT any attractive interaction, no matter how weak, $\rightarrow$ COOPER PAIRS ; $\langle qq \rangle$

One gluon exchange (& instanton interaction) attractive in color $\bar{3}$.

(no need to resort to phonons; $\therefore$ superconductivity more robust in QCD than in metals. Higher T_c/E_F.)

$\langle qq \rangle$, ie Cooper pairs of quarks,

$\Rightarrow$
- electric & color currents superconduc
- mass for photon & (some) gluons (?
- Meissner effects. (Magnetic & color magnetic fields excluded.)

INTRODUCTION TO SUPERCONDUCTIVITY

I will sketch the original BCS derivation of Cooper instability & the resulting BCS state.

Consider quarks interacting via a 4-fermion interaction. Ie replace 

by 

.

Drop all indices (L, R, color, flavor).
[ and indices will be restored later.]

BCS did the calculation using the variational method. Make an ansatz for the ground state and vary the parameters specifying the ansatz so as to minimize the free energy Ω.

$$\Omega = H - \mu N$$

where

$$N = \int d^3p \; \psi^\dagger(p)\, \psi(p)$$

$$H = \int d^3x \left(\bar{\psi}\, \partial\!\!\!/ \,\psi - G \underbrace{\bar{\psi}\psi\bar{\psi}\psi}_{\text{important indices dropped.}} \right)$$

Often, use 4-fermi interaction with quantum numbers of one-gluon exchange, but not momentum structure. Ie s-wave scattering vs. forward scattering dominated.

Or, use 4-fermion interaction with quantum numbers of 't Hooft vertex, induced by instanton.

Variational ansatz:

$$|\Psi_{BCS}\rangle = \prod_{\vec{p}} \left[\cos\theta_{\vec{p}} + \sin\theta_{\vec{p}}\, a^{\dagger}_{\vec{p}} a^{\dagger}_{-\vec{p}} \right] |0\rangle$$

where $\theta_{\vec{p}}$ is the variational parameter.
[One per $\vec{p}$ (and per helicity, color, flavor)]

- With probability $\sin^2\theta_{\vec{p}}$, the states with momenta $\vec{p}$ and $-\vec{p}$ are <u>both</u> filled. With prob. $\cos^2\theta_{\vec{p}}$, <u>both</u> empty. No probability that one filled & other empty.
- If $\theta_{\vec{p}} = \pi/2$ for $|\vec{p}| < P_F$
  and $\theta_{\vec{p}} = 0$ for $|\vec{p}| > P_F$

  Then state is simply a filled Fermi sea, with Fermi momentum P_F.
- Relative to this, $a^{\dagger}_{p} a^{\dagger}_{-p}$ makes Cooper pairs of particles or holes.
- Ansatz was motivated by Cooper's prior analysis of F.S. + one pair.

In this state,

$$\langle \Psi_{BCS} | \psi\psi | \Psi_{BCS} \rangle \equiv \Gamma \sim \int d^3p \sin\theta_p \cos\theta_p \quad (*)$$

Clearly, $\Gamma = 0$ for Fermi sea.

$\Gamma \neq 0$ is one of the hallmarks of superconductivity. Breaks symmetries – eg $U(1)_B$. Need to restore all the missing indices to see what symmetries are broken.

Now, evaluate

$$\langle \Psi_{BCS} | \Omega | \Psi_{BCS} \rangle \sim \int d^3p (\overset{|\vec{p}|}{p} - \mu) \sin^2\theta_p - G\Gamma^2$$

Now, vary with respect to θ_p:

$$\rightarrow 2 \sin\theta_p \cos\theta_p (p - \mu) = 2G\Gamma (\cos^2\theta_p - \sin^2\theta_p)$$

$$\rightarrow \boxed{\tan 2\theta_p = \frac{\Delta}{p - \mu} \quad \text{where } \Delta \equiv G\Gamma} \quad (**)$$

State is now fully specified in terms of a quantity Δ that we must interpret and determine.

Interpreting Δ:

Evaluate energy cost of an elementary excitation with momentum p relative to the energy of the BCS state.
(Ie: "remove the pair" with momentum $p, -p$ and just fill one of the two states.)

Find:

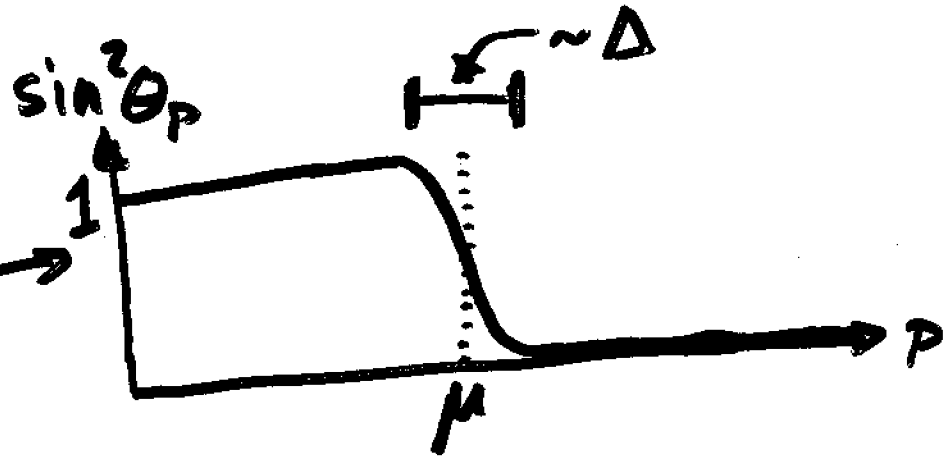
Quasiparticle energy $= \sqrt{(p-\mu)^2 + \Delta^2}$

$\therefore \boxed{\Delta \equiv \text{Gap}}$ ← hallmark of supercond.

Takes energy $\geq \Delta$ to excite a single excitation above the correlated BCS ground state. [Note: $\Delta = 0$ for F.S.]

Also,

$\tan 2\theta_p = \frac{\Delta}{p-\mu}$ →



Determining Δ:

Equations (*) and (**) are consistent only if:

$$\Delta = G \int d^3p \frac{\Delta}{\sqrt{(p-\mu)^2 + \Delta^2}}$$

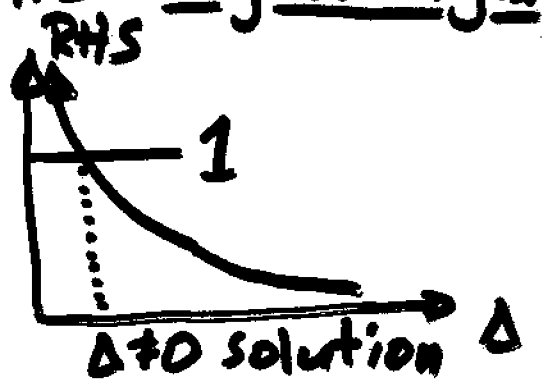
so, $\Delta = 0$ or

$$\boxed{1 = G \int d^3p \frac{1}{\sqrt{(p-\mu)^2 + \Delta^2}}}$$

"the gap equation"

If $G<0$ (repulsive) then $\Delta = 0$ is only solution.

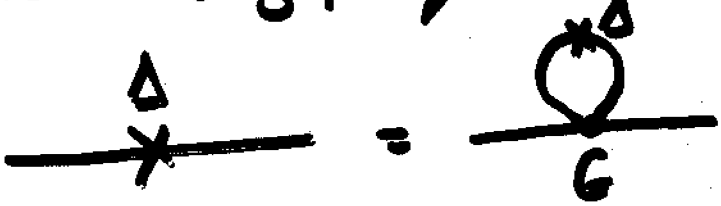
If $G>0$ (attractive) then gap equation has $\Delta \neq 0$ solution no matter how small G is, because RHS log divergent at $p=\mu$ as $\Delta \to 0$:



Cooper's Instability!

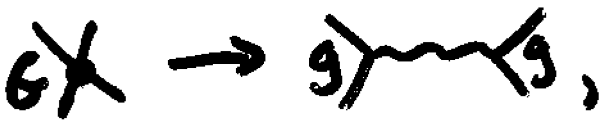
Solution: $\Delta \sim \mu e^{-1/\mu^2 G}$

Can rederive gap equation diagrammatically:



$$\Delta = G\int d^4p\,\frac{\Delta}{p_0^2+(p-\mu)^2+\Delta^2} = G\int d^3p\,\frac{\Delta}{\sqrt{(p-\mu)^2+\Delta^2}}$$

Can now sketch how gap equation (and its solution) change when



, which is the correct interaction at asymptotically high densities, where $g(\mu)$ is small.

θ: angle by which quark scatters
θμ: gluon mom.

$$\Delta \sim g^2 \int d\epsilon\,\underbrace{\frac{\Delta}{\sqrt{\epsilon^2+\Delta^2}}}_{\text{fermion prop.}} \int \sin\theta\, d\theta\, \underbrace{\frac{\mu^2}{\theta^2\mu^2+\Delta^2}}_{\text{gluon prop.}}$$

$$\epsilon = p-\mu$$

$$\rightarrow \Delta \sim g^2\Delta \ln\frac{\Delta}{\mu}\ln\frac{\Delta}{\mu} \rightarrow \boxed{\frac{\Delta}{\mu} \sim e^{-\mathrm{const}/g}}$$

GAP AND T_c

Much work (that I will not review)

suggests that @ $\mu_q \sim 500$ MeV $\lceil \sim 10 \times$ nuclear density$\rfloor$

$$\Delta \lesssim 100 \text{ MeV}$$

$$T_c \lesssim 50 \text{ MeV}$$

Note: $T_c / E_F \sim 1/10 \rightarrow$ THIS is high T_c s.c.!

Two classes of methods ~ agree:

i) models normalized to $\mu = 0$ physics

(Alford, K.R., Wilczek, Rapp, Schäfer, Shuryak, Velkovsky, Berges, Carter, Diakonov, Evans, Hsu, Schwetz, ...

ii) weak-coupling QCD calculations, valid for $\mu \to \infty$; $g \to 0$. (Quantitatively, valid for $g \lesssim 1$ which means $\mu \gtrsim 10^9$ MeV K.R., Shuster)

$$\frac{\Delta}{\mu} \sim \underbrace{256\pi^4 e^{-\frac{\pi^2+4}{8}} \left(\frac{N_f}{2}\right)^{5/2}}_{} \; \underbrace{\frac{1}{g^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}\, g}\right)}_{\text{Son}}$$

Schaefer, Wilczek; Pisarski, Rischke; Hong, Miransky, Shovkovy, Wijewardhana; Evans, Hsu, Schwetz;

Brown, Liu, Ren; Beane, Bedaque, Savage; K.R., Shuster; Rischke, Wang; ...

$\lceil \Delta \sim \exp(-1/g)$ comes from divergence in small angle scattering via exchange of unscreened magnetic gluons:

$$\rightarrow 1 = g^2 \underbrace{\ln\frac{\Delta}{\mu}}_{\text{BCS}} \; \underbrace{\ln\frac{\Delta}{\mu}}_{\text{collinear divergence}}$$

COLOR-FLAVOR LOCKED QUARK MATTER

Alford, K.R., Wilczek; Schaefer, Wilczek; ...

- occurs for $\mu \to \infty$, and at any μ if $M_s = M_{u,d}$
- all 9 quarks pair*, and ∴ are gapped
- Cooper pairs antisymmetric in
  color*, spin*, and ∴ flavor
  *: the factors making CFL most favorable
- superfluid. ($\langle qq \rangle \neq 0$)
- chiral symmetry spontaneously broken
  → pseudo-Nambu-Goldstone mesons
- unbroken gauged $U(1)$ → massless photon
- <u>transparent insulator</u> (neutral without electrons) K.R., Wilczek
  - index of refraction and Litim, Manuel
    reflection/refraction coeffs. known Manuel, K.R.
- occurs in quark matter in Alford, Berges, K.R.; Schaefer, Wilczek; K.R., Wilczek; Alford, K.R., Reddy, Wilczek
  nature wherever $\mu > M_s^2/4\Delta$,
  possibly augmented by K^0-condensate
  Bedaque, Schaefer; Kaplan, Reddy
- M_s and Δ both μ-dependent and uncertain
- could be single nuclear/CFL transition
  → sharp interface with charged
    boundary layers Alford, K.R., Reddy, Wilczek
- OR less symmetrically paired quark matter
  may intervene, between nuclear and
  CFL matter. To this we now turn....

PUTTING THE INDICES BACK IN

The condensate takes the form:

$$\langle \psi^\alpha_a C\gamma_5 \psi^\beta_b \epsilon_{\alpha\beta\gamma} \epsilon^{abc} \rangle \sim \Delta^c_\gamma \neq 0$$

Greek indices: color
- antisymmetric in color because QCD interaction is only attractive between pairs of quarks that are antisymmetric in color

$C\gamma_5$: Lorentz scalar
- antisymmetric in Dirac indices
- favored because rotationally sym., letting whole Fermi surface pair

Latin indices: flavor
- antisymmetric in flavor by Pauli.

<u>CFL</u>: $\Delta^c_\gamma = \Delta\, \delta^c_\gamma$. All 9 quarks pair equally. Leaves a large color-flavor sym. unbroken.

$N_f = 3$: COLOR-FLAVOR LOCKING

Condensate pairs quarks of all colors + flavors: Alford KR Wilczek

$$\langle u_L d_L - d_L u_L - u_L d_L + d_L u_L + d_L s_L - d_L s_L - s_L d_L + s_L d_L + s_L u_L - u_L s_L - s_L u_L + u_L s_L \rangle \neq 0$$

Locks $SU(3)_{color}$ to $SU(3)_L$.

ie $SU(3)_{color+L}$ is a symmetry.

Similarly, condensate of R-quarks locks $SU(3)_{color}$ to $SU(3)_R$.

Result:

$SU(3)_{color+L+R}$ unbroken. ⟷ Use these "EM" + "isospin" to classify excitations

Chiral symmetry broken.

$U(1)_{\tilde{Q}}$ unbroken. ⟷

All other gauge symmetries broken.

$U(1)_B$ broken. ∴ superfluid.

$N_f = 3$: COLOR-FLAVOR LOCKING

$$\langle q^{\alpha}_{La} q^{\beta}_{Lb} \rangle = -\langle q^{\alpha}_{Ra} q^{\beta}_{Rb} \rangle$$

color

flavor

$$= \Delta\, \epsilon^{\alpha\beta A} \epsilon_{abA}$$

not quite correct.
(symmetries correct)

Locks $SU(3)_L$ to $SU(3)_{color}$

Locks $SU(3)_R$ to $SU(3)_{color}$

Result:

$$SU(3)_{color} \times \underbrace{SU(3)_L \times SU(3)_R}_{\text{contains } U(1)_{EM}} \times U(1)_B$$

$$\rightarrow \underbrace{SU(3)_{color+L+R}}_{\text{contains unbroken gauged } U(1)_{\tilde{Q}}}$$

ie gauge symmetries: $SU(3)_{color} \times U(1)_{EM} \rightarrow U(1)_{\tilde{Q}}$

$\therefore$ Classify excitations using $\tilde{Q}$ charge (unbroken, but modified, "photon") and "isospin".

CFL IN PICTURES

Goal: to understand what

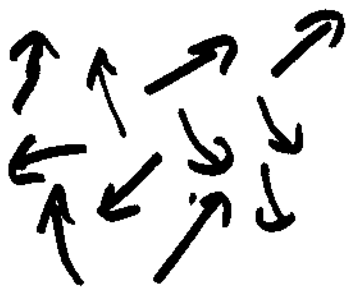
$$SU(3)_A \times SU(3)_B \times SU(3)_C \to SU(3)_{A+B+C}$$

means by understanding

$$U(1) \times U(1) \times U(1) \to U(1)$$

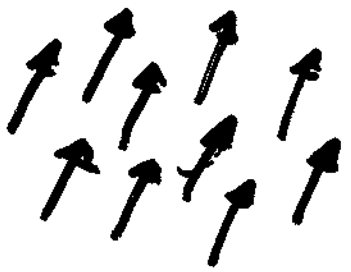
in pictures....

First, understand $U(1) \to$ nothing.



$U(1)$ associated with rotation of $\nearrow$'s is <u>unbroken</u>.

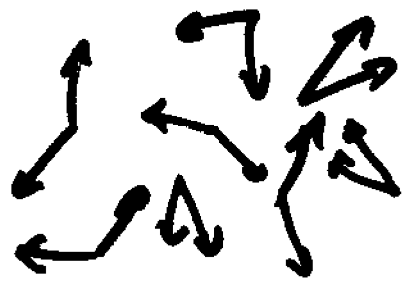
cool the system.
system orders, or condenses



$U(1)$ spontaneously broken by a condensate.
$\uparrow$'s ordered

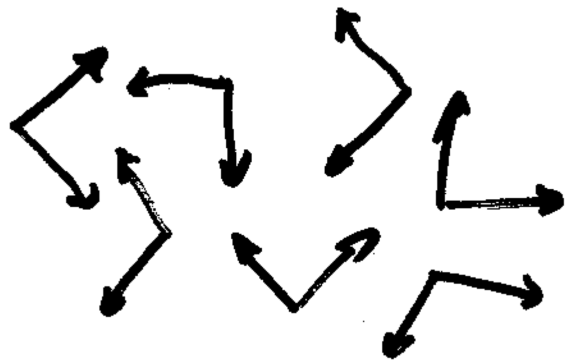
Goldstone boson: because $\nearrow$'s could equally well have ordered in another direction, long wavelength oscillations are massless.

Now ... $U(1) \times U(1) \to U(1)$ cf ^{3}He-B



$U(1) \times U(1)$ unbroken.

$\downarrow$ cool



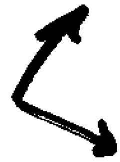
angle fixed. "locked".

$\therefore$ $U(1)$ not a symmetry.
$U(1)$ " " "

$U(1)_{g+r}$ is an unbroken symmetry.

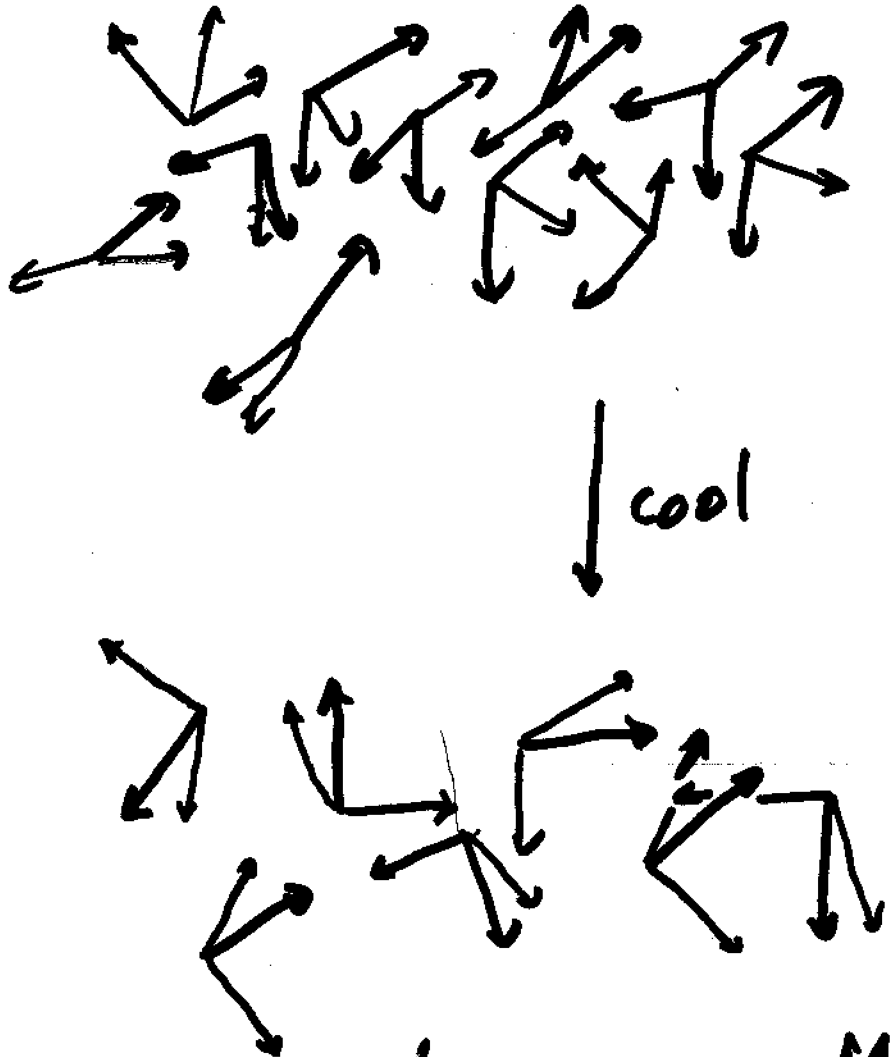
One goldstone boson ....

Describe it to me ...

long wavelength oscillations of the angle between  .

Ie I could have chosen any angle. I broke the symmetry by choosing 90° angle  .

Now... $U(1) \times U(1) \times U(1) \to U(1)_{g+r+p}$



$U(1)$ $U(1)$
$U(1)$
all unbroken

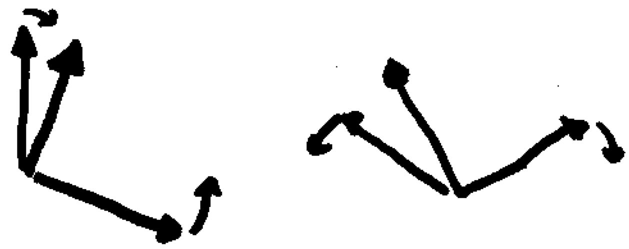
Two angles ($\uparrow\!\rightarrow$ and $\downarrow\uparrow$) locked.

$\therefore$ $U(1)$, $U(1)$, $U(1)$ not symmetries

$U(1)_{g+r+p}$ <u>is</u> an unbroken symmetry.

Describe the two goldstone bosons ...

Convenient to describe the two Goldstone bosons as long wavelength oscillations of:

i) 

with ↗ unaffected

and ii) 

with ↗ ↖ unaffected.

Now, gauge $U(1)$.

(ii) becomes longitudinal component of massive vector meson

(i) remains a goldstone boson.

$\tilde{Q}$ IN THE CFL PHASE

$$\langle q_a^\alpha q_b^\beta \rangle \sim \Delta_1 \delta_a^\alpha \delta_b^\beta + \Delta_2 \delta_b^\alpha \delta_a^\beta$$

$$\tilde{Q} = Q_{EM} + \frac{1}{\sqrt{3}} T_8$$

Q_{EM}:
- $\frac{2}{3}$ for u
- $-\frac{1}{3}$ for d
- $-\frac{1}{3}$ for s

$\frac{1}{\sqrt{3}} T_8$:
- $-\frac{2}{3}$ for b
- $\frac{1}{3}$ for r
- $\frac{1}{3}$ for g

$\tilde{Q}$ charges of quarks:

u	+1
u	+1
u	0
d	0
d	0
d	−1
s	0
s	0
s	−1

Similarly, $\tilde{Q}$ charges of gluons all integer-valued. Also for $\tilde{Q}$ charges of Goldstone bosons.

EXCITATIONS

Alford RR Wilczek
Schaefer Wilczek

- 9 massless Nambu-Goldstone bosons
  - 8 "pions" ; 1 singlet (∴ superfluid)
- 8 gluons → octet of <u>massive</u>
  vector bosons. (Meissner / Higgs)
- 9 quarks form "isospin" octet + singlet
  ↳ or, are they baryons ?
  - all have a gap.

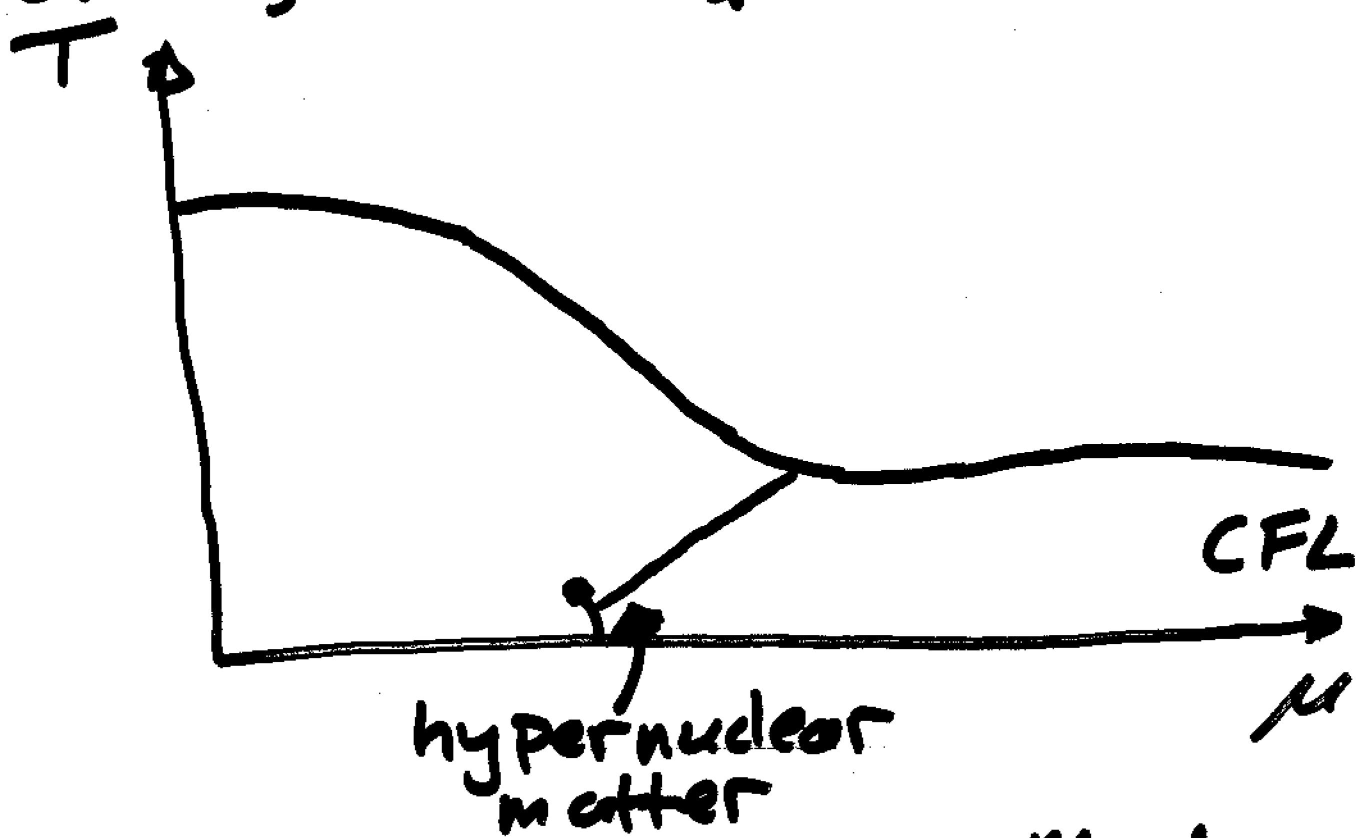
ALL excitations have INTEGER $\tilde{Q}$!

- Dense quark matter with same symmetries
  and similar excitations as superfluid
  <u>hyper</u> NUCLEAR MATTER.
- QUARK MATTER and NUCLEAR MATTER
  may be continuously connected. ($N_f = 3$)
- BROKEN CHIRAL SYMMETRY ;
  "CONFINEMENT" (described complementarily)
  arising at arbitrarily WEAK COUPLING.

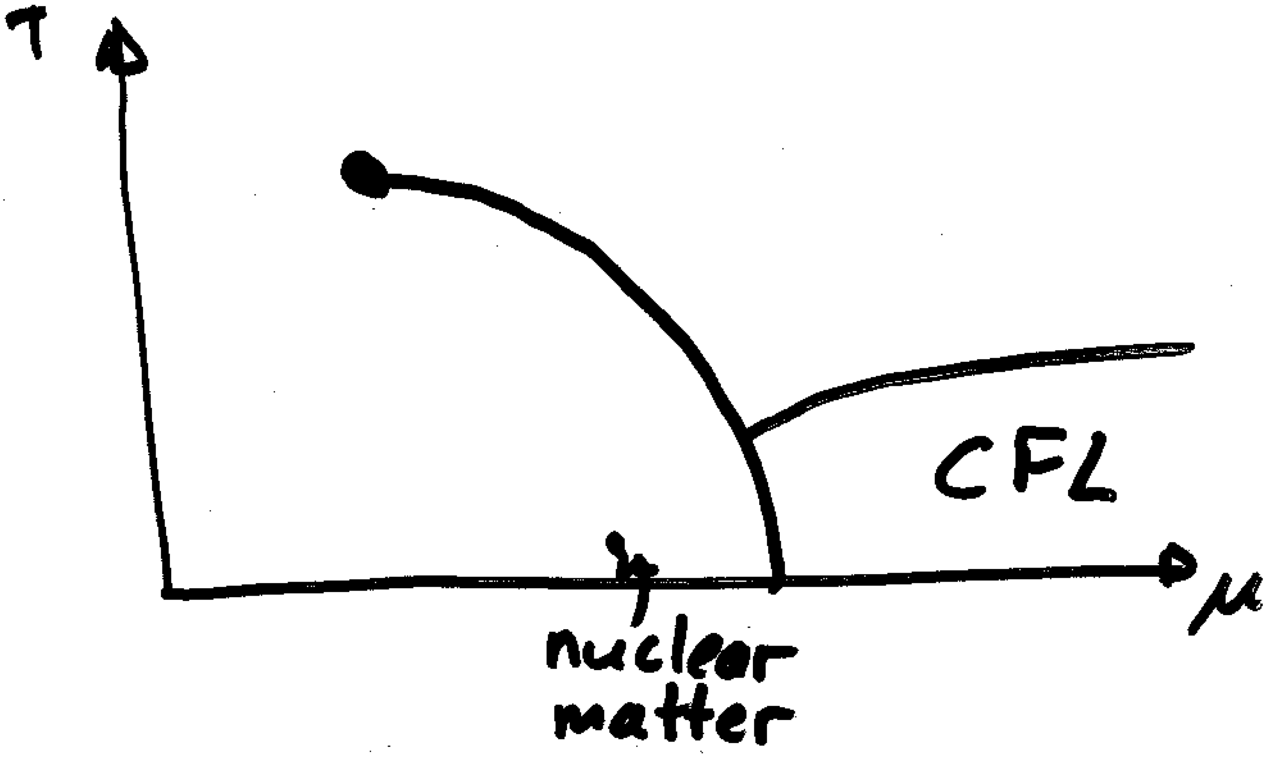
⇒ a weak coupling <u>understanding</u> of what
were thought to be strong coupling phenomena.

"MINIMAL" PHASE DIAGRAMS

i) If $m_s = m_u = m_d$



ii) Real world, with $m_s \gg m_{u,d}$



By "minimal" I mean:

- We know nuclear matter is the phase at nuclear matter density.*
- We know CFL is the stable phase at asymptotic density.**
- Assume just a single phase transition between them.***

*: Almost certainly correct. We will not discuss the (improbable) alternatives.

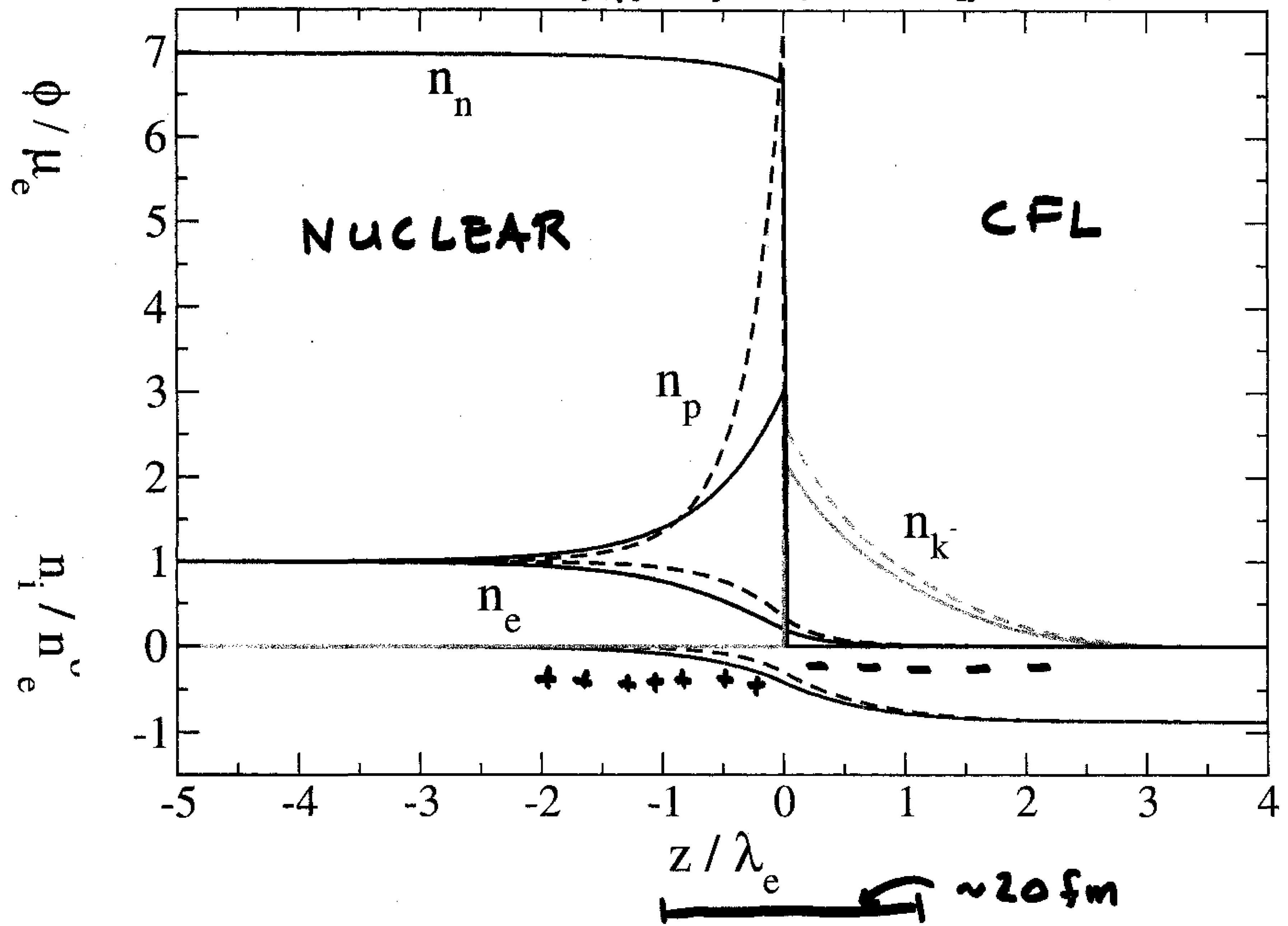
**: CERTAINLY correct.

***: Quite possibly incorrect, as we will discuss later.

Minimal phase diagram leads to....

MINIMAL CFL - NUCLEAR INTERFACE

Alford, KR, Reddy, Wilczek

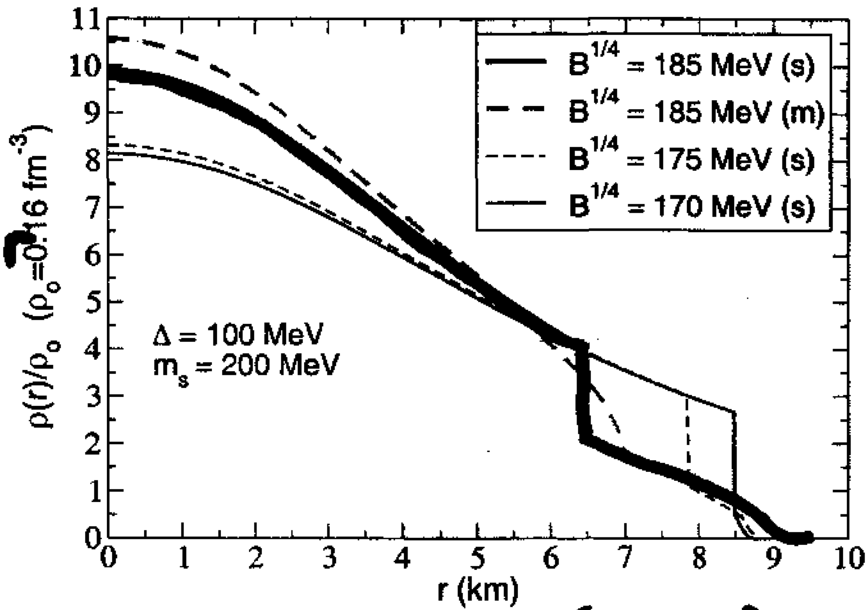


NEUTRON STAR WITH CFL CORE

One example below. Varying parameters & varying nuclear E.O.S. → varying position of interface.

The profiles of the maximum mass superconducting stars for different values of the bag constant, $\Delta = 100$ MeV and $m_s = 200$ MeV are shown in Fig. 4. For $B^{1/4} = 185$ MeV results for the sharp interface (denoted as (s)) and the mixed phase (denoted as (m)) scenario are shown. Here the maximum masses correspond to 1.33 $M_\odot$ and 1.35 $M_\odot$, respectively. The maximum mass for $B^{1/4} = 175$ MeV and $B^{1/4} = 170$ MeV are $M_{max} = 1.44\ M_\odot$ and $M_{max} = 1.52\ M_\odot$, respectively. Fig. 4 shows that the typical density discontinuity in the sharp interface scenario is $\approx 3\rho_o$. It also shows that for smaller values of B, the $NM \rightleftharpoons QM$ phase transition occurs very close to the surface of the star (at lower density as discussed earlier). The denser exterior regions of these stars (despite a less dense inner core) are primarily responsible for the increase in the maximum mass observed as one decreases B.

Density in units of nuclear density



radius (km)

Figure 4: Profile of the maximum mass star for bag constant $B^{1/4}$ = 185, 175, 170 MeV with m_s = 200 MeV and Δ = 100 MeV. The mixed phase (dashed) and the sharp interface curves are shown. The Walecka model was used to describe the nuclear part of the equation of state.

Fig. 4 indicates that in the mixed phase scenario there are no discontinuities in the density profile of the star. However, this is not true in general. It is interesting to note that even when mixed phases are allowed, there can still be discontinuities in energy density within them. In a small range of parameters, we find stars that have a crust of nuclear matter surrounding a mixed NM-QM core, but the mixed phase has an outer part which is a mixture of unpaired QM with NM, and an inner part that is a mixture of CFL QM with NM. At the interface between the two there

Alford & Reddy

ILLUMINATING CFL QUARK MATTER

Manuel, KR

In CFL quark matter

$$SU(3)_L \times SU(3)_R \times SU(3)_{color} \to SU(3)_{L+R+c}$$

But: $\because$ one $U(1) \in SU(3)_L \times SU(3)_R$ is gauged (electromagnetism)

$\therefore$ one unbroken $U(1)_{\tilde{Q}} \in SU(3)_{C+L+R}$ is <u>gauged</u>. We have seen

$$\tilde{Q} = Q + \frac{1}{\sqrt{3}} T_8$$

$$Q = \frac{2}{3}, \frac{-1}{3}, \frac{-1}{3} \text{ for } u, d, s$$

$$\frac{1}{\sqrt{3}} T_8 = -\frac{2}{3}, \frac{1}{3}, \frac{1}{3} \text{ for } r, g, b$$

We have seen: condensate is $\tilde{Q}$-neutral.

Now, let's find the $\tilde{Q}$-photon.

Analyze

$$\left| \left(\partial_\mu + eA_\mu Q + gG_\mu^8 T_8 \right) \langle q_a^\alpha q_b^\beta \rangle \right|^2$$

and find combination of A_μ and G_μ for which this is zero. ($\Rightarrow$ No Meissner effect / Higgs mechanism for that "photon")

One finds... ordinary photon gluon

$$A_\mu^{\tilde{Q}} = \cos\theta \, A_\mu + \sin\theta \, G_\mu^8$$

$$A_\mu^X = -\sin\theta \, A_\mu + \cos\theta \, G_\mu^8$$

where

$A_\mu^{\tilde{Q}}$ is massless. This $\tilde{Q}$-photon satisfies Maxwell's equations with dielectric const $\epsilon \neq \epsilon_0$. (medium is polarizable.) To the $\tilde{Q}$-photon, CFL matter is a transparent dielectric medium

A_μ^X is massive. Like Z-boson.

θ is analogue of Weinberg angle

$$\sin\theta = \frac{e/\sqrt{3}}{\sqrt{g^2 + e^2/3}} \simeq \frac{e}{g\sqrt{3}} \sim \frac{1}{20}$$

$\tilde{Q}$-photon is "mostly ordinary photon."

Alford KR Wilczek; Alford Berges KR

A TRANSPARENT INSULATOR

What can the $\tilde{Q}$-photon scatter off?

- the CFL condensate itself is $\tilde{Q}$-neutral.
- once you include nonzero quark masses, all excitations with $\tilde{Q} \neq 0$ are massive.
- $\therefore$ for $T \ll M_{\text{lightest excitation with } \tilde{Q} \neq 0}$,

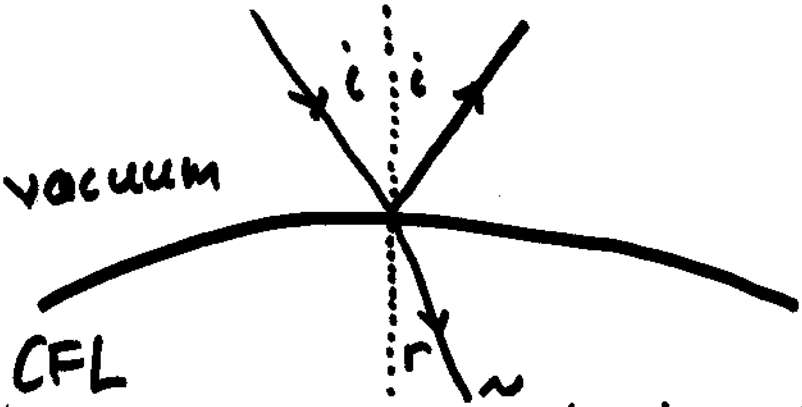
likely a kaon

the CFL phase is transparent to the $\tilde{Q}$-photon. It is a $\tilde{Q}$-insulator, with some index of refraction $n_{CFL} \neq 1$.

ILLUMINATING CFL QUARK MATTER

Manuel, KR

Suppose (just for fun) you had a quark star in CFL phase, and shone light on it:



$\tilde{Q}$-light: $A_\mu^{\tilde{Q}} = \cos\theta\, A_\mu^{EM} + \sin\theta\, G_\mu^8$

$$n_{CFL} = 1 + \frac{4\alpha}{9\pi}\frac{\mu^2}{\Delta^2}\cos^2\theta \quad \text{(Litim, Manuel)}$$

Find: $\dfrac{\sin i}{\sin r} = n_{CFL}$

Explicit expressions in terms of n, θ for reflection & refraction coefficients for light of either possible polarization.

Its fun to think of 10 km lenses in space, but more likely applicable version of this is in the static limit:

Suppose core of a neutron star is CFL. How does it respond to the large static $\vec{B}$ it finds itself in?

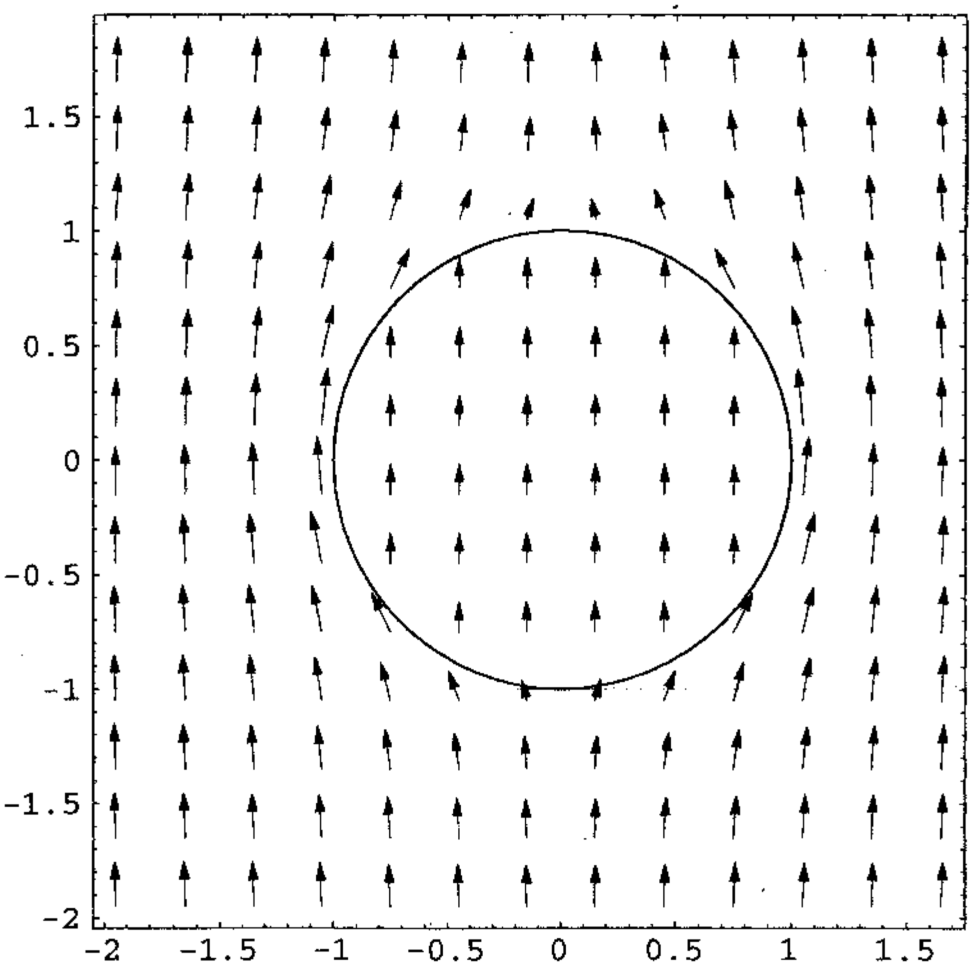
ANSWER: (Alford, Berges, KR)

(found via magnetic b.c.'s ...)

Partial Meissner effect...

Magnetic field solution (sharp boundary)

Stitching together the inside and outside solutions, we find the solution. For $\cos\alpha_0 = 0.5$



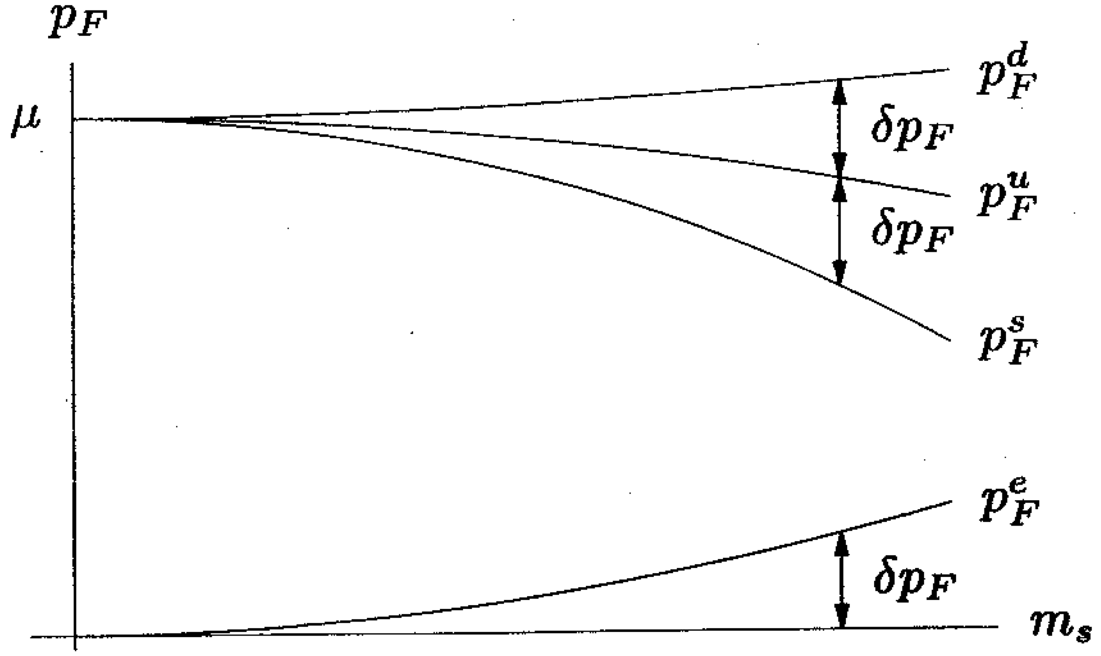
In the real world α_0 is small, so the field is mostly converted into $\tilde{Q}$ flux by the supercurrents and monopoles, and penetrates the interior. Only a weak field is excluded.

8

Alford, Berges, KR

INTERMEDIATE DENSITY QUARK MATTER

- M_s important
- For orientation, consider noninteracting quarks, $M_u = M_d = 0$ $M_s \neq 0$, impose electrical neutrality and weak eqbm:



- In noninteracting quark matter, $\delta p_F \simeq \dfrac{M_s^2}{4\mu}$
- Motivates result that CFL pairing "breaks" when $\dfrac{M_s^2}{4\mu} > \Delta$
- Also, when CFL "breaks", no residual <ud> pairing either. Alford, KR

LESS SYMMETRIC PAIRING

Recall: $\langle \psi^\alpha_a C\gamma_5 \psi^\beta_b \epsilon_{\alpha\beta\gamma} \epsilon^{abc} \rangle \sim \Delta^c_\gamma$

where the CFL phase has $\Delta^c_\gamma = \Delta \begin{pmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{pmatrix}$

In response to effects of m_s,

try:

- $\Delta^c_\gamma = \Delta \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
  - only pairs u, d and r, g
  - called "2SC" because this is only option in $N_f = 2$ QCD
  - stable phase, but seems* always (ie at all m_s) to be less favorable than either CFL or $\Delta = 0$. Alford, KR

    *: model dependent, ∴ not certain
- If not that, given the pattern of p_F's why not try....

THE KOUVARIS PHASE*

*: we'll call it "gapless CFL" in the literature

$$\Delta^c_\gamma = \Delta \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

u,d and u,s pairs, but no d,s.

- same U(1) symmetries as CFL, including unbroken $U(1)_{\tilde{Q}}$
- two gapless quarks: d, s
  - both those have $\tilde{Q}=0$, so still a $\tilde{Q}$-insulator
- we have to date not succeeded in solving the coupled gap and neutrality equations for such a phase, indicating that it is not stable. However, we are unsure <u>why</u> we cannot find a solution. So, stay tuned.

Kouvaris, Alford, KR, work in progress

CRYSTALLINE COLOR SUPERCONDUCTIVITY

Alford Bowers K.R.; Bowers Kundu K.R. Shuster; Leibovich K.R. Shuster; Casalbuoni Gatto Mannarelli Nardulli; Giannakis Liu Ren; Bowers K.R.;

As $\mu \downarrow$, if CFL "breaks" before you get to hadronic matter, quark matter at intermediate density may have:

Pairing between quarks with <u>different p_F</u>

<u>GOAL</u>: both quarks in a pair on respective Fermi surfaces

<u>IDEA</u>: Cooper pairs with momentum!

$$(\vec{p}+\vec{q}, -\vec{p}+\vec{q}) \text{ for any } \vec{p}.$$

Each pair has total momentum $2\vec{q}$

- $|\vec{q}| \simeq 1.2\, \delta p_F$ determined energetically
- "pattern" of $\{\hat{q}_i\}$ " " Bowers K.R.

$$\langle \psi\psi \rangle \sim \Delta \sum_i e^{i\vec{q}_i \cdot \vec{x}}$$

- spontaneous breaking of rotational and translational symmetry.

<u>LOFF</u>: Larkin Ovchinnikov Fulde Ferrell (1964) considered this state for $\langle e_\uparrow e_\downarrow \rangle$ pairing with Zeeman splitting. State not seen in condensed matter. Problem is that $\vec{B} \to$ orbital effects, not just Zeeman. QCD, with its "flavor Zeeman splitting" turns out to be the natural context for LOFF's idea!

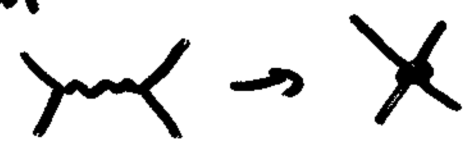
SIMPLIFICATIONS, FOR NOW

- two flavors, with Fermi surfaces split by a $\delta\mu$ introduced by hand:

$$\mu_u = \mu + \delta\mu$$
$$\mu_d = \mu - \delta\mu$$

(instead of 3 flavors with F.S. splitting from M_s, neutrality.)

See work by Kundu + KR for how to use M_s instead of $\delta\mu$.

- point-like 4-fermion interaction between quarks,  with quantum #s of g.

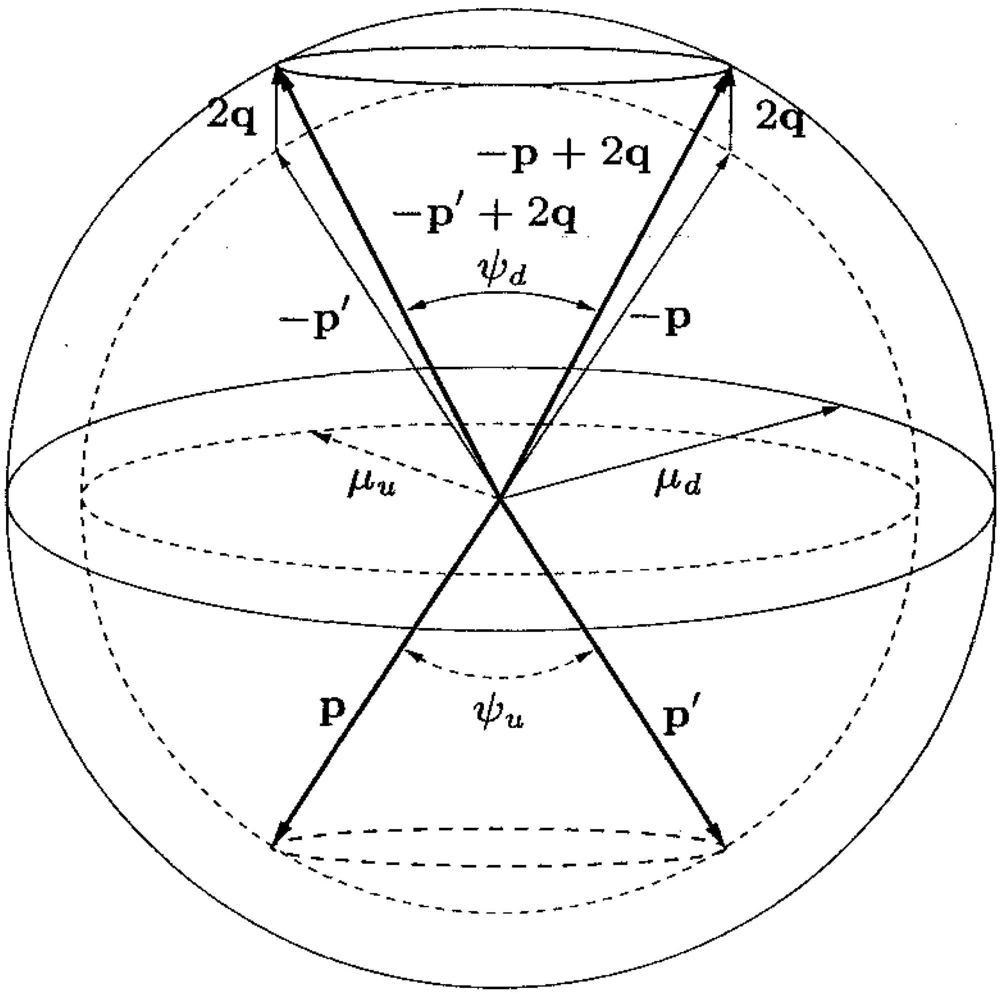
See Leibovich, KR, Shuster; Giannakis, Liu, Ren for 

- $\Delta \ll \mu$

Basic LOFF idea

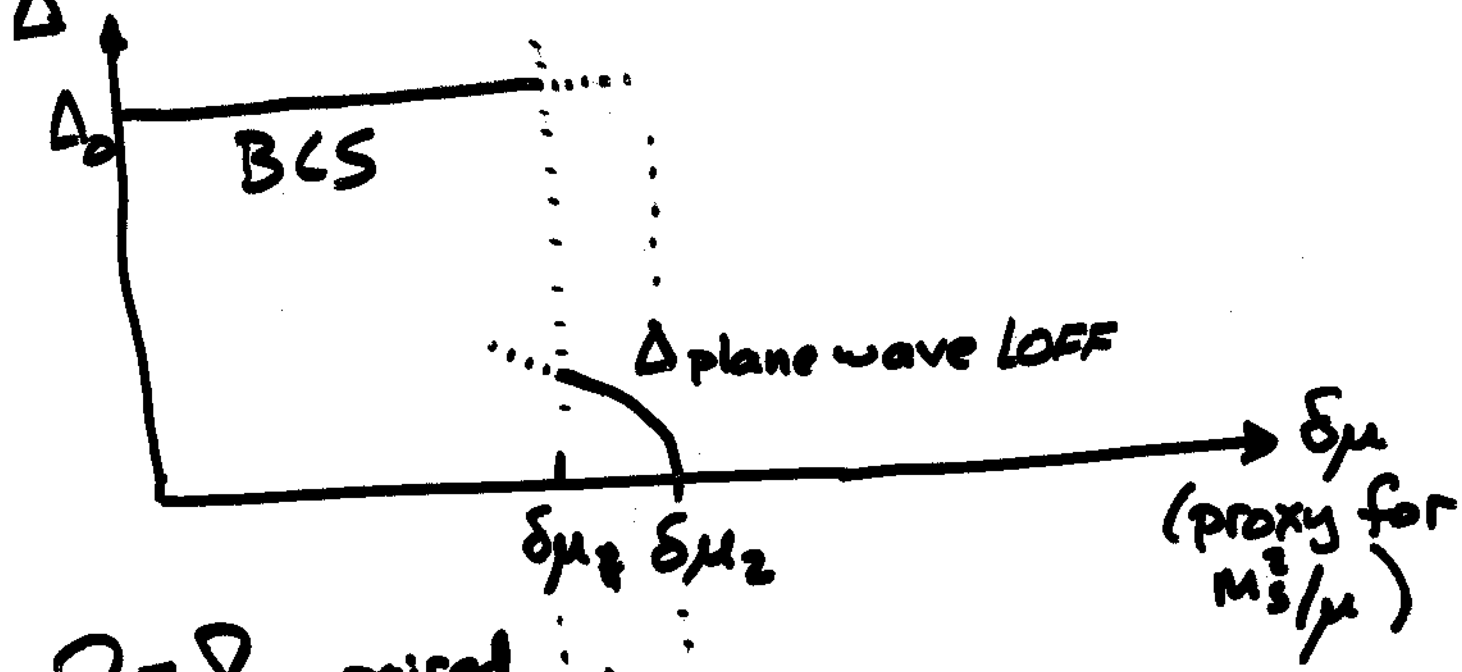
Try Cooper pairs $(\mathbf{p}, -\mathbf{p}+2\mathbf{q})$

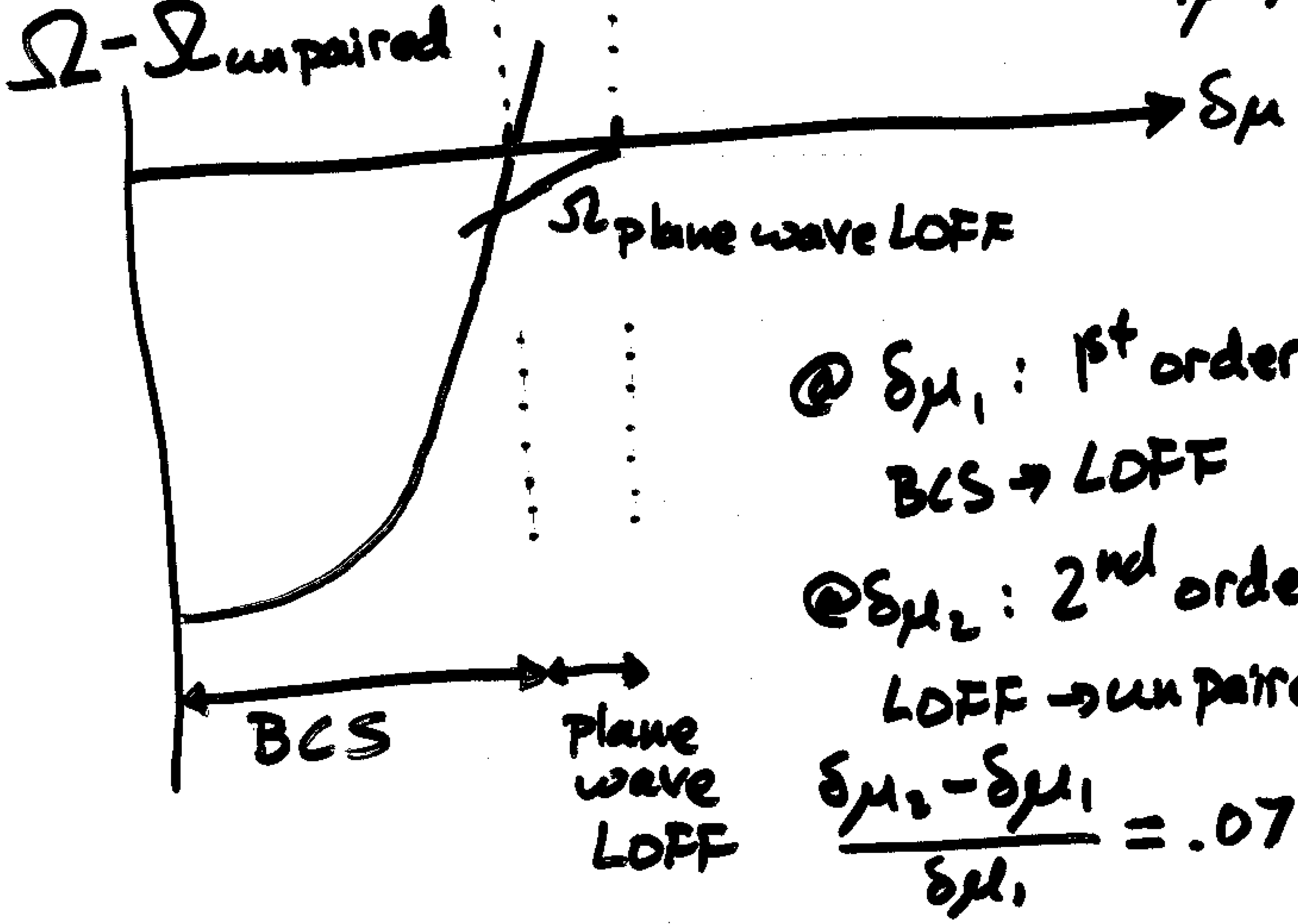
- total momentum $2\mathbf{q}$ for each and every pair
- each quark at its Fermi surface, even with $p_F^u \neq p_F^d$
- $\hat{q}$ chosen spontaneously, $|\mathbf{q}|$ determined variationally (result is $|\mathbf{q}| = q_0 \approx 1.20\delta\mu$)
- condensate forms a ring on each Fermi surface, with opening angle $\psi_u \approx \psi_d \approx 2\cos^{-1}(\delta\mu/q_0) \approx 67.1°$



SINGLE PLANE WAVE

$$\langle \psi(x)\,\psi(x)\rangle \sim \Delta\, e^{i2\vec{q}\cdot\vec{x}}$$





@ $\delta\mu_1$: 1st order

BCS → LOFF

@ $\delta\mu_2$: 2nd order

LOFF → unpaired

$$\frac{\delta\mu_2 - \delta\mu_1}{\delta\mu_1} = .07$$

MULTIPLE PLANE WAVES

If system unstable to formation of 1 plane wave, this allows quarks lying on one ring on each F.S. to pair. Much of F.S. remains unpaired....

Why not multiple $\vec{q}$'s? i.e. multiple rings?

Want to compare many different possible $\{\vec{q}_i\}$;

$$\langle \psi(x)\,\psi(x)\rangle = \sum_{\{\vec{q}_i\}} \Delta\, e^{i 2\vec{q}_i\cdot\vec{x}}$$

and for each $\{\vec{q}_i\}$ calculate Δ and Ω

$\{q_i\}$, ie crystal structure, with lowest Ω wins.

GINZBURG - LANDAU

For $\Delta \ll \Delta_0$, ie for $\delta\mu \to \delta\mu_2$,

the free energy Ω can be evaluated order-by-order in Δ, for <u>many</u> crystal structures.

<u>Order Δ^2</u>: $|\vec{q}_i| = 1.2\,\delta\mu$ for all q_i's

→ each q_i gives pairing on a ring with opening angle 67°.

- the more q_i's, the better.

<u>Order Δ^4 and Δ^6</u>: "interaction between rings"

- intersecting rings costs a lot
  ⇒ at most 9 plane waves
- "regularity" (lots of different ways of making closed 4-, 6-, ... sided figures from q_i's) strongly favored.
- indicates that best choice is.....

FCC Crystal

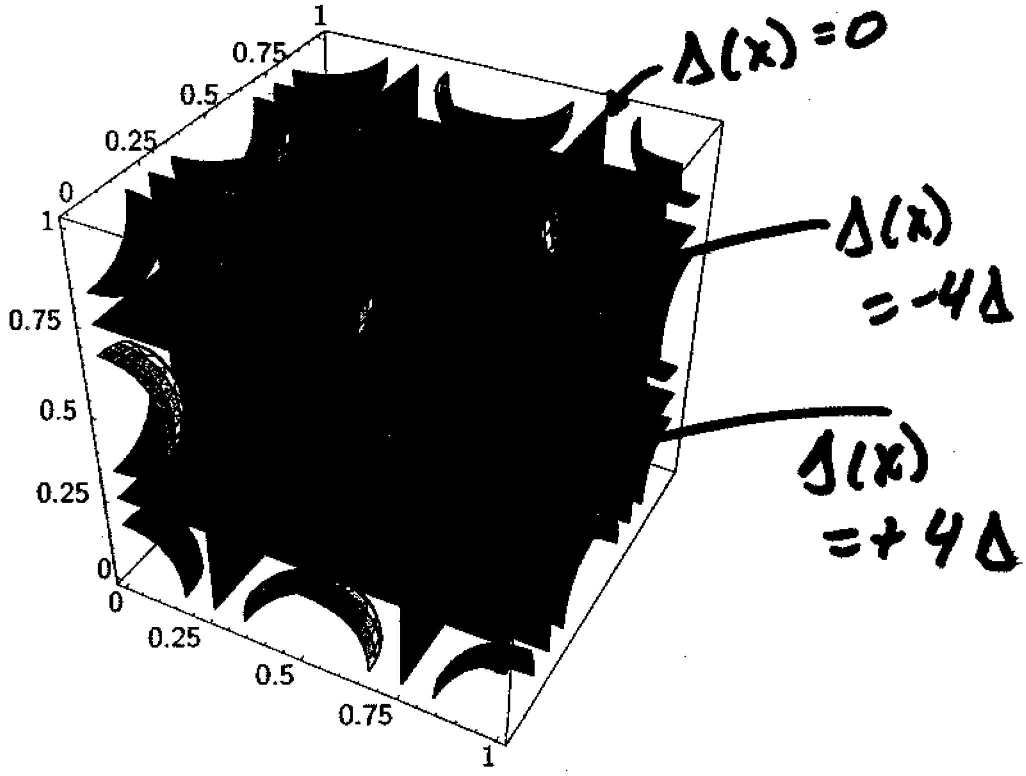
Favored according to Ginzburg Landau analysis, that is not yet quantitatively reliable. Bowers KR

- The cube structure is the favored ground state: eight wave vectors pointing towards the corners of a cube, forming the eight shortest vectors in the reciprocal lattice of a face-centered-cubic crystal. The gap function is

$\Delta \sim \Delta_{CFL}$

$$\Delta(\mathbf{x}) = 2\Delta\left[\cos\frac{2\pi}{a}(x+y+z)+\cos\frac{2\pi}{a}(x-y+z) + \cos\frac{2\pi}{a}(x+y-z)+\cos\frac{2\pi}{a}(-x+y+z)\right]$$

A unit cell:



with contours $\Delta(\mathbf{x}) = +4\Delta$ (black), 0 (gray), -4Δ (white). Lattice constant is $a = \sqrt{3}\pi/|\mathbf{q}| \simeq 6.012/\Delta_0$.

Crystal structures

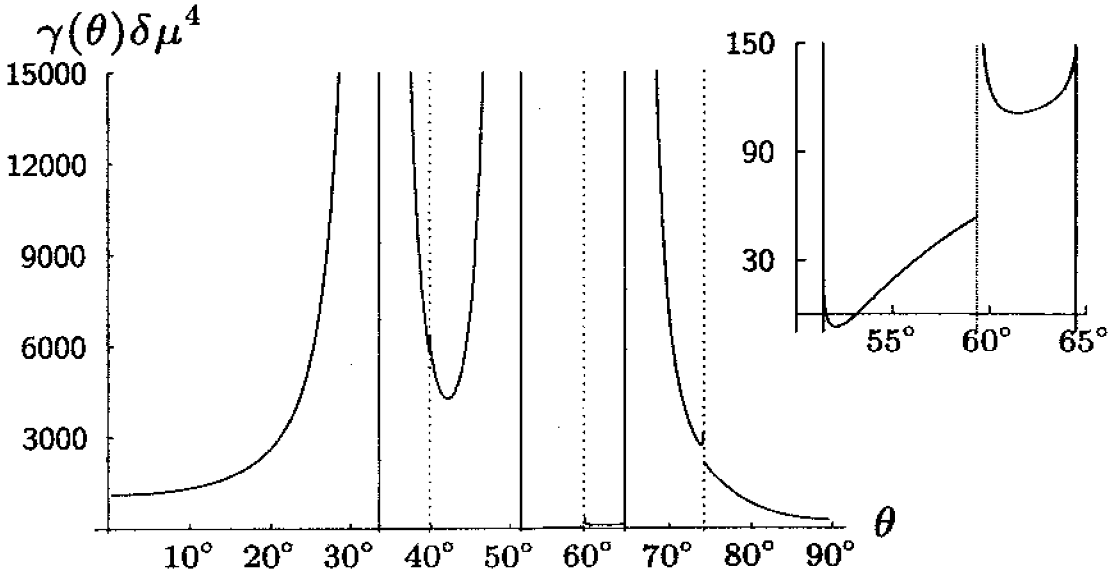
Candidate crystal structures with P plane waves, specified by their symmetry group $\mathcal{G}$ and Föppl configuration. Bars denote dimensionless equivalents: $\bar{\beta} = \beta\,\delta\mu^2$, $\bar{\gamma} = \gamma\,\delta\mu^4$, $\bar{\Omega} = \Omega/(\delta\mu_2^2 N_0)$ with $N_0 = 2\bar{\mu}^2/\pi^2$. $\bar{\Omega}_{\min}$ is the (dimensionless) minimum free energy at $\delta\mu = \delta\mu_2$. The phase transition (first order for $\bar{\beta} < 0$ and $\bar{\gamma} > 0$, second order for $\bar{\beta} > 0$ and $\bar{\gamma} > 0$) occurs at $\delta\mu_*$.

Structure	P	$\mathcal{G}$(Föppl)	$\bar{\beta}$	$\bar{\gamma}$	$\bar{\Omega}_{\min}$	$\delta\mu_*/\Delta_0$
point	1	$C_{\infty v}(1)$	0.569	1.637	0	0.754
antipodal pair	2	$D_{\infty v}(11)$	0.138	1.952	0	0.754
triangle	3	$D_{3h}(3)$	-1.976	1.687	-0.452	0.872
tetrahedron	4	$T_d(13)$	-5.727	4.350	-1.655	1.074
square	4	$D_{4h}(4)$	-10.350	-1.538	–	–
pentagon	5	$D_{5h}(5)$	-13.004	8.386	-5.211	1.607
trigonal bipyramid	5	$D_{3h}(131)$	-11.613	13.913	-1.348	1.085
square pyramid	5	$C_{4v}(14)$	-22.014	-70.442	–	–
octahedron	6	$O_h(141)$	-31.466	19.711	-13.365	3.625
trigonal prism	6	$D_{3h}(33)$	-35.018	-35.202	–	–
hexagon	6	$D_{6h}(6)$	23.669	6009.225	0	0.754
pentagonal bipyramid	7	$D_{5h}(151)$	-29.158	54.822	-1.375	1.143
capped trigonal antiprism	7	$C_{3v}(13\bar{3})$	-65.112	-195.592	–	–
cube	8	$O_h(44)$	-110.757	-459.242	–	–
square antiprism	8	$D_{4d}(4\bar{4})$	-57.363	-6.866	–	–
hexagonal bipyramid	8	$D_{6h}(161)$	-8.074	5595.528	-2.8×10^{-6}	0.755
augmented trigonal prism	9	$D_{3h}(3\bar{3}\bar{3})$	-69.857	129.259	-3.401	1.656
capped square prism	9	$C_{4v}(144)$	-95.529	7771.152	-0.0024	0.773
capped square antiprism	9	$C_{4v}(14\bar{4})$	-68.025	106.362	-4.637	1.867
bicapped square antiprism	10	$D_{4d}(14\bar{4}1)$	-14.298	7318.885	-9.1×10^{-9}	0.755
icosahedron	12	$I_h(15\bar{5}1)$	204.873	145076.754	0	0.754
cuboctahedron	12	$O_h(4\bar{4}\bar{4})$	-5.296	97086.514	-2.6×10^{-9}	0.754
dodecahedron	20	$I_h(5555)$	-527.357	114166.566	-0.0019	0.772

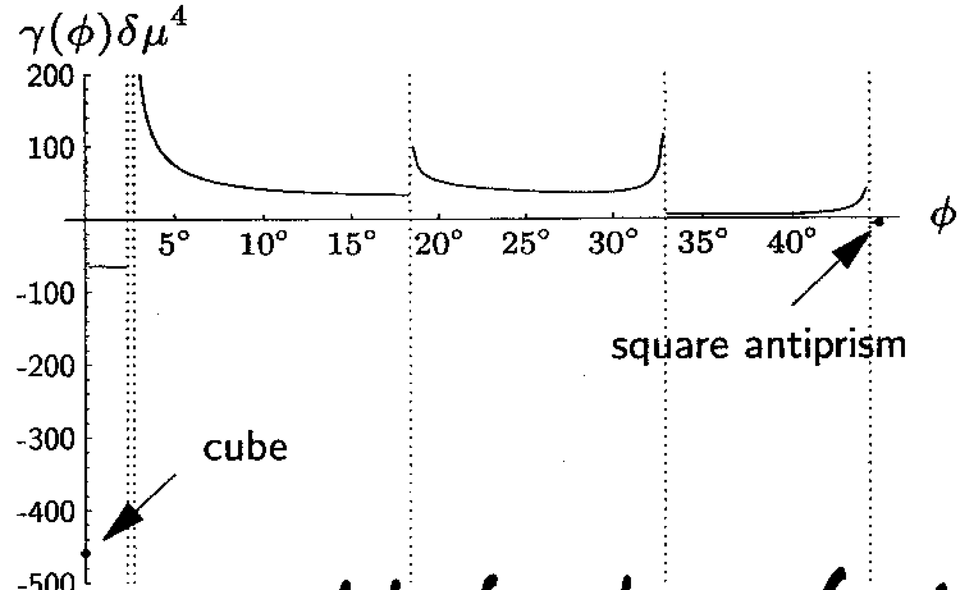
Continuous variations $\Omega = \alpha \Delta^2 + \beta \Delta^4 + \gamma \Delta^6 + \dots$

γ FOR DIFFERENT CRYSTALS WITH 8 WAVES

- Varying the "height" of a square antiprism



- Varying the "twist" of a square prism



of all 23 crystal structures (and their continuous variations) we investigated,

– Typeset by FoilTEX – CUBE has most negative β and γ.

CONCLUSIONS

- FCC cube is favored structure. Ginzburg-Landau analysis has taught us what features of a crystal structure are favored, and thus why FCC best.
- BUT: $\Omega = \alpha\Delta^2 + \beta\Delta^4 + \gamma\Delta^6 + \ldots$ with β, γ large and negative, and $\alpha \sim (\delta\mu - \delta\mu_2)$
  - $\Rightarrow$ Strong 1st order crystalline $\to$ unpaired transition at a $\delta\mu_* \gg \delta\mu_2$
  - crystalline "window" in phase diagram not small
  - Δ not small

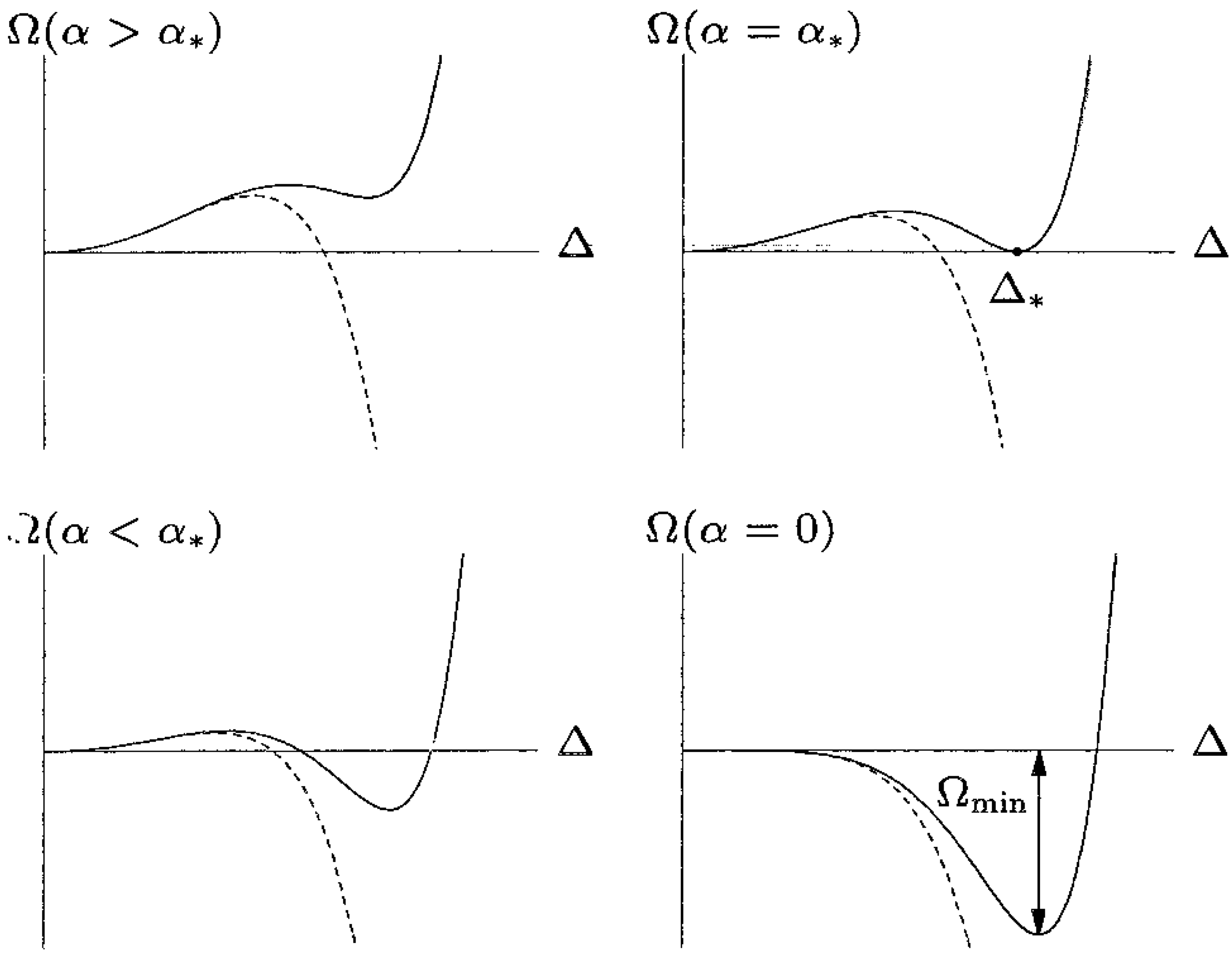
cf G-L analysis of liquid-solid transition

$\Rightarrow$ Ginzburg-Landau cannot provide quantitative calculation of Δ, Ω.

- Make FCC ansatz, calculate Ω, Δ variationally. (In progress)

Unstable structures?

- Ginzburg-Landau instability guarantees a strong first-order transition at some $\delta\mu = \delta\mu_* \gg \delta\mu_2$
- Δ_*, $\Omega_{\rm min}$ are large, but cannot be predicted by the Ginzburg-Landau method
- Larger instability $\Rightarrow$ more robust ground state (cube has the most unstable Ginzburg-Landau free energy)



OUTLOOK AND IMPLICATIONS

- variational calculation for FCC crystal, now that we know this is the favored one.
- three-flavor analysis

CRYSTALLINE SUPERFLUIDITY

- this phase may be created in gases of ultracold fermionic atoms (Combescot)
  - trap 2 hyperfine states of atom;
  - arrange strong attractive interaction between 2 "species"
  - arrange different number densities for 2 "species"

VORTEX PINNING & PULSAR GLITCHES:

- rotate crystal; what happens? vortices? pinned at intersections of crystal planes?
- if so, presence of a layer of crystalline color superconducting quark matter within neutron star ⇒ glitch

IMPLICATIONS FOR COMPACT STARS

or, flipping that around, ... How can we use observations of compact stars to determine the high density region of the phase diagram?

- If core of "neutron" star is quark matter, then it <u>IS</u> a color superconductor.

  $$T_{star} \sim \text{keV} \lll T_c$$

- Not known whether neutron stars have quark matter cores. Goal: understand observational consequences, so we can find out.

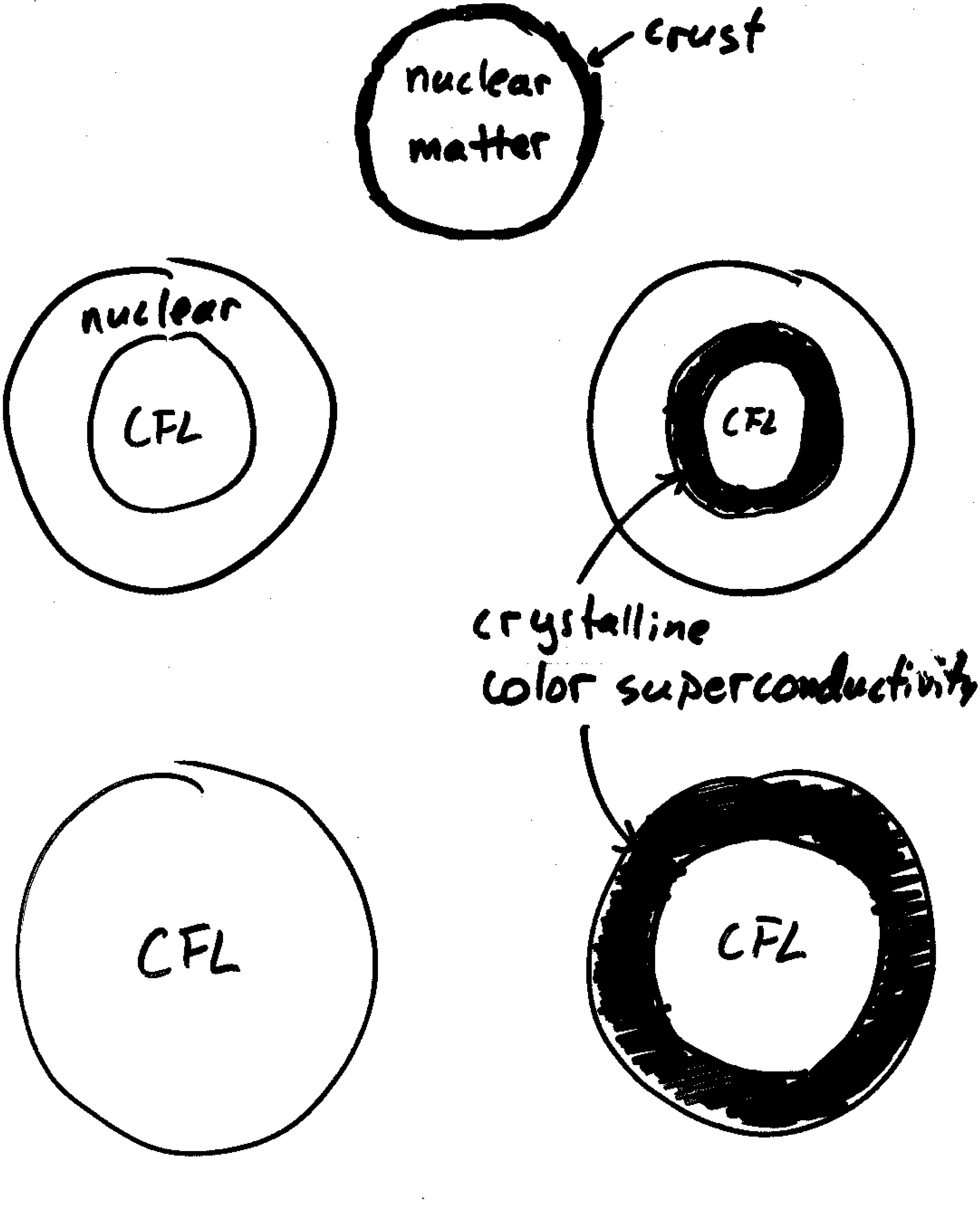
<u>FIRST</u>: Can we discover whether there is a crystalline color superconductivity window?

- As a function of increasing depth, $M_s^2/\mu\Delta$ decreases.

  ∴ LOFF WINDOW → LOFF SHELL

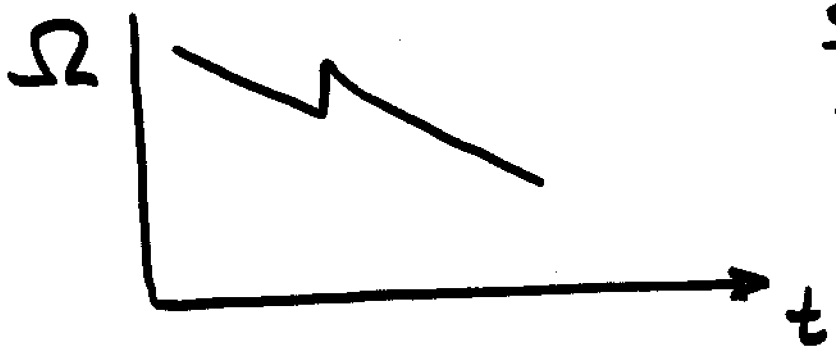
<u>THEN</u>: List other examples of ways to answer this question.

SEVERAL SCENARIOS



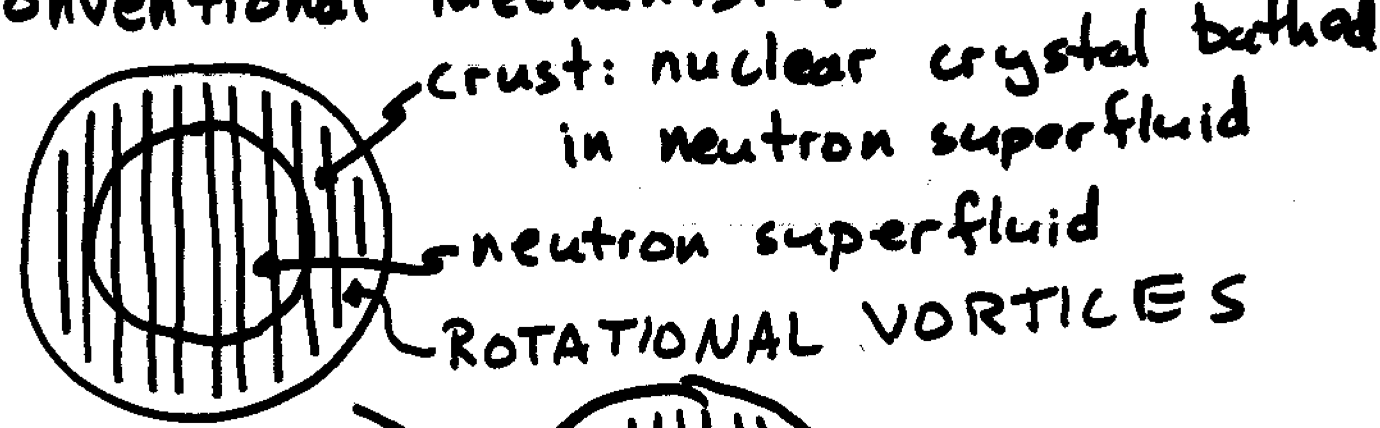
GLITCHES

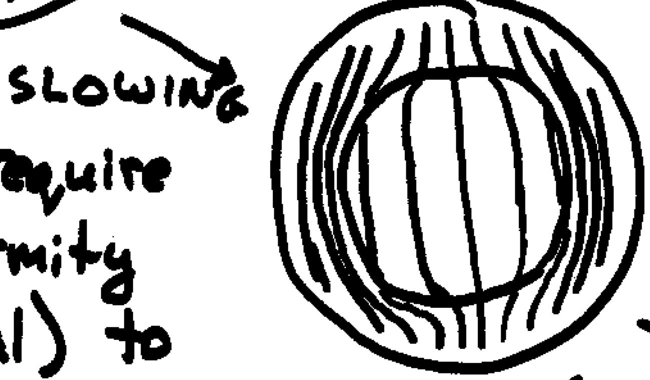
Pulsars glitch:



$$\frac{\delta\Omega}{\Omega} \sim 10^{-9} \to 10^{-6}$$

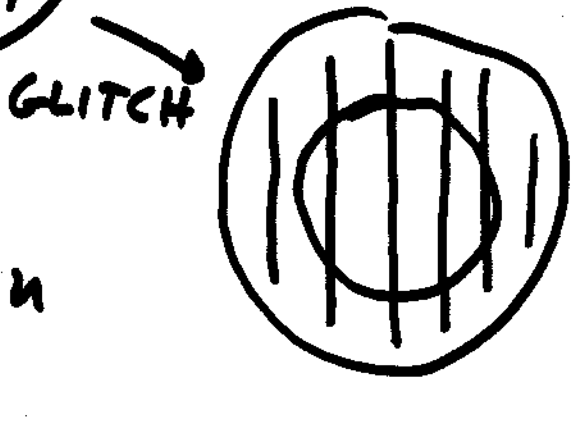
Conventional mechanism:





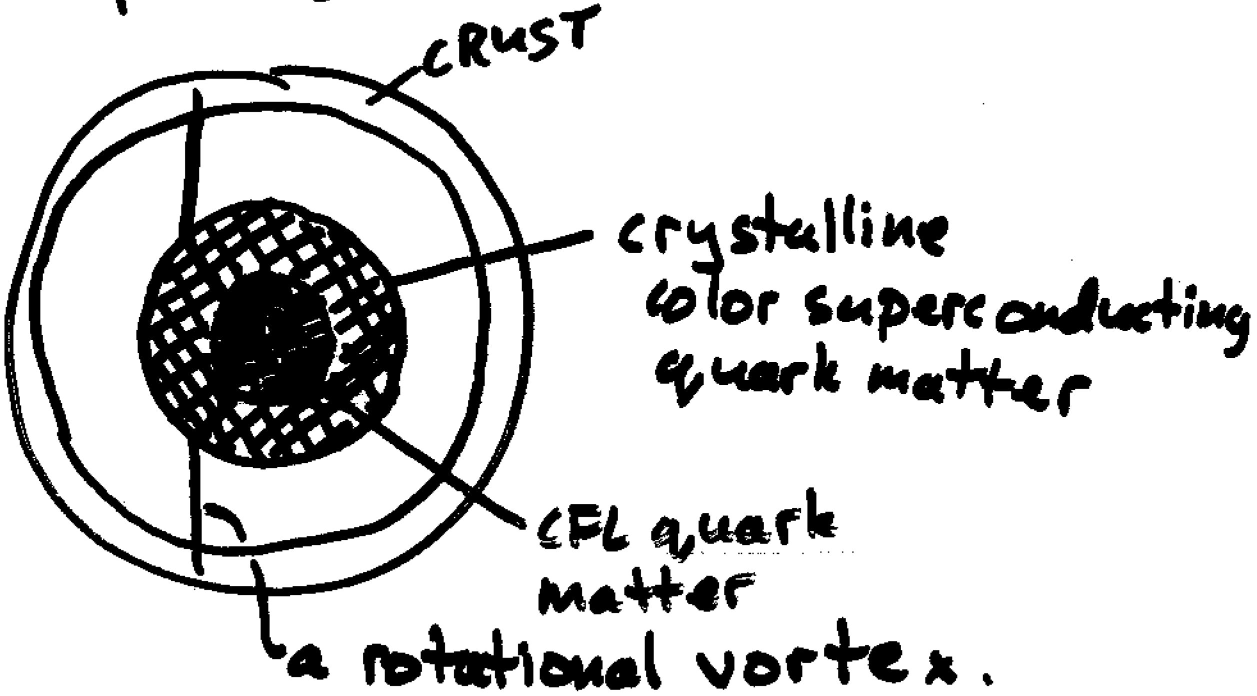
Glitches require non-uniformity (ie crystal) to impede (pin) motion of vortices.

$\therefore$ thought impossible in QM.



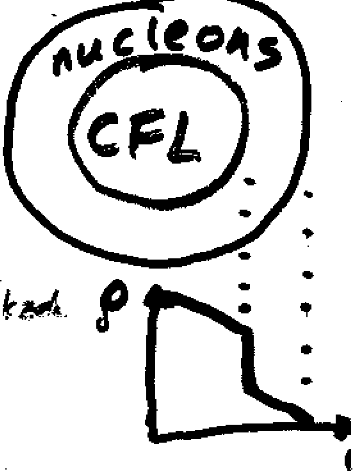
GLITCHES IN QUARK MATTER?

- crystalline condensate may pin vortices, since they will prefer to follow intersections of nodal planes



- could some (eg the smaller?) glitches originate in crystalline layer in core?
- could observed features of glitches rule out existence of crystalline layer?
- serious glitch phenomenology awaits calculation of pinning force. This is in progress.

ASTROPHYSICAL CONSEQUENCES IF NEUTRON STARS HAVE CFL CORES



- For given M, R a little smaller.
  But, uncertainty in R still *Alford Reddy*
  dominated by nuclear outer layer.
- At a sharp interface, big *Alford K.R. Reddy Cirkovic*
  density step. $\xrightarrow{?}$ LIGO signal
- If spherical stars have CFL cores
  but oblate stars do not, → unusual spin-up
  history. *Glendenning, Weber; Blaschke, Grigorian, Poghosyan*
- Transparent insulator. → $\vec{B}$ in core not in
  flux tubes; not frozen. → $\vec{B}$ evolution
  governed by outer layer.
- For $T <$ few MeV: *Page Prakash Lattimer Steiner*
  - very small specific heat, neutrino *Jaikumar Prakash Schaefer*
    emissivity, neutrino opacity.
  - superfluidity → very large *Shovkovy Ellis*
    thermal conductivity

  ⇒ cooling of star controlled by nuclear outer layer
- During supernova, $T \sim$ tens of MeV > meson mass
  → mesons emit and scatter neutrinos *Reddy Sadzikowski Tachibana;*
  - and, also, may be phase transitions *Carter Reddy*

  $\xrightarrow{?}$ signals in time distribution of supernova ν
- Bare quark star would be nice. NOT seen...

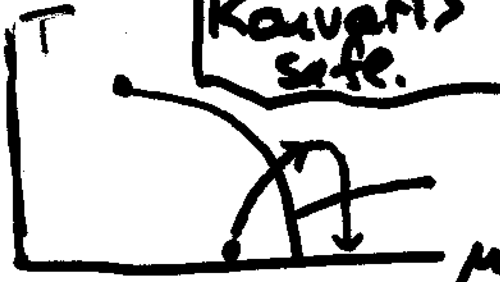
EQUATION OF STATE: Quark matter has effects; color superconductivity affects pressure only at order $\Delta^2/\mu^2 \sim 5-10\%$.
BUT: sharp density discontinuity may be seen in gravitational waves emitted during inspiral? (a LIGO signal?)

Alford, KR Reddy, Wilczek

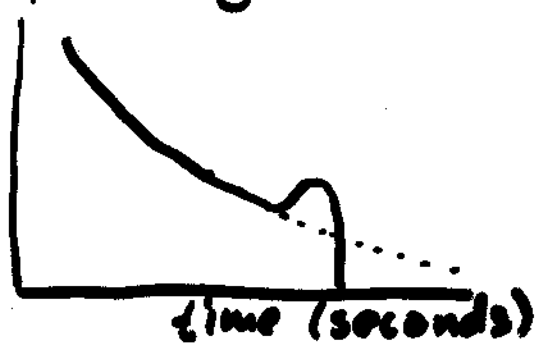
NEUTRON STAR COOLING: rapid ν-emission if some quarks have $\Delta \lesssim T$. Fully gapped quark matter (ie CFL) is inert. This can teach us value of smallest gap.
(Page Prakash Lattimer Steiner)

Could rule out x-tal phase? Kouvaris safe.

SUPERNOVA NEUTRINOS:



Transition to c.s. during first seconds of cooling of proto-neutron star → sudden ν-transparency → sudden burst of ν's ....
$\xrightarrow{?}$ bump in ν time distribution @ SNO, SuperK? (Carter, Reddy)



CFL EFFECTIVE THEORY

- an important, and well-developed, aspect of the subject that I have not had time to treat.
- CFL phase has broken chiral symmetry and ∴ light Goldstone bosons. And yet, for large μ the coupling is weak and so we can calculate, eg, parameters in the e.f.t. describing the goldstone bosons. Thus,

  $m_{\pi, K, \eta}$ & $f_{\pi, K, \eta}$ now calculated from first principles for $g \to 0$, $\mu \to \infty$.

Casalbuoni, Gatto; Son, Stephanov; Rho, Wirzba, Zahed; Hong, Lee, Min; Manuel, Tytgat; Zarembo; Beane, Bedaque, Savage; Bedaque, Schäfer; Kaplan, Reddy; ....

OUTSTANDING QUESTIONS

"LITTLE" ≡ people are working on them

- construct vortices in crystalline phase
- crystalline phase beyond Ginzburg-Landau
- is the Kouvaris phase stable? favorable?
- astrophysics of signatures I described
- microscopic understanding of K^0-rotation of CFL condensate
- lattice calculation of analogue phenomena in $N_c = 2$ QCD or $N_f = 2$ QCD at large μ_{isospin}.

"BIG" (≡ new ideas needed before work begins)

- new astrophysical signatures
- reformulate weak coupling calculation of gap to make it systematic and, perhaps, better convergent.
- solve the sign problem! then, answer all questions on lattice. [Need to reorganize lattice path integral to avoid need for huge cancellations at $\mu \neq 0$.] Solving this would revolutionize CMT too....