

SUMMER SCHOOL ON PARTICLE PHYSICS

16 June - 4 July 2003

EXTRA DIMENSIONS

Lectures I, II, III & IV

**R. SUNDRUM
John Hopkins University
Baltimore, MD
U.S.A.**

EXTRA DIMENSIONS
AND
WARPED HIERARCHIES

Raman Sundrum
Johns Hopkins University

What's at stake

- Higher Dimensions \Rightarrow Higher dimensional Lorentz invariance & translational symmetry.
- Hiding extra dimensions \Rightarrow Breaking higher dim. symmetries at some level.
- Higher spacetime symmetries \Rightarrow extra dimensional locality.
- Higher Dimensions \Rightarrow Non-renormalizable effective field theory
- Extra-dimensional locality \Rightarrow hidden finiteness (predictivity).
- Hidden dimensions + General Relativity
 \Rightarrow hidden spacetime curvature,
 \Rightarrow Warped Hierarchies "WARPING"
 \Rightarrow Realistic models of Nature
(without supersymmetry).

Deep connections with String Theory

- String theory \Rightarrow
Extra dimensions
+ ways of hiding them
- AdS/CFT Correspondence
 \Rightarrow Crucial insights into
physics of warped
higher-dimensional
spacetimes.
- Warping in String Theory
generic, useful way to
arrange hierarchies.

OUTLINE

HIDING EXTRA DIMENSIONS

- Compactification
- Branes

GENERAL RELATIVITY review

GRAVITY LOCALIZATION - RS2

WARPED WEAK/PLANCK HIERARCHY
- RS1

RADIUS STABILIZATION

- "Holographic" Renormalization Group

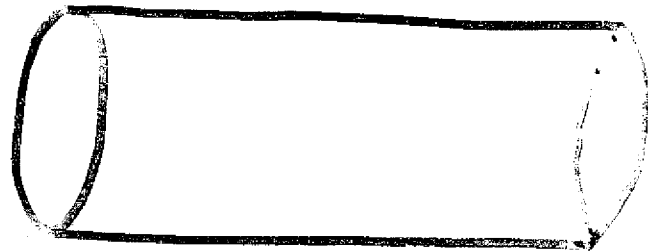
ADS/CFT, RS/CFT Correspondence

RS1 & GUTs

RS1 & Electroweak Precision Tests.

COMPACTIFICATION

String motivation for extra dimensions: Green, Schwarz



Witten Ch.1, ...
Polchinski's Book too
↑ 5th
dim., ϕ

→ 3+1D, x^μ

distances: $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\phi^2$

↑
Compactification
Radius

5D Lorentz invariance

globally,

but not broken locally.

Scalar Field

$$S = \int d^4x \int_{-\pi}^{\pi} d\phi r_c \left\{ \frac{1}{2} (\partial_\mu \chi)^2 - \frac{1}{2r_c^2} (\partial_\phi \chi)^2 - \frac{m_5^2}{2} \chi^2 - \lambda_5 \chi^4 \right\},$$

$\chi(x^\mu, \phi)$

↑ 2π -periodic

$[\chi] = 3/2$

$[\lambda_5] = -1$

ie. non-renormalizable

Non-renormalizable

Effective Field Theory

see Georgi, "Weak Interactions"

UV divergences are local,

$$S_{\text{div.}} = \int d^5 X \sum_{\mathcal{O}} \lambda_5^N \mathcal{O}_{\text{local}}(x, \partial) \Lambda_{\text{UV}}^{5+N-[O]}$$

$X^M \equiv$ all coordinates

for $5+N-[O] \geq 0$.

Renormalizability $\equiv \sum_{\mathcal{O}} ::$ Finite no. terms

so that $S_{\text{div.}}$ has same structure as $S_{\text{c.t.}}$

& $S_{\text{c.t.}}$ " " " " S_{ren}

\equiv finite no. input couplings

But here, $[\lambda_5] < 0 \Rightarrow \infty$ couplings as input!

Cure for $E \ll \Lambda_{\text{UV}} \ll \sqrt{\lambda_5}$:

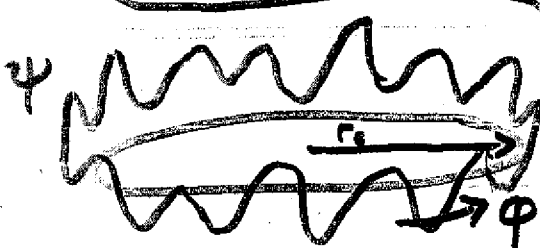
Work to fixed order in λ_5 .

\equiv Fixed order in $(E\lambda_5), (\Lambda_{\text{UV}}\lambda_5)$.

After "renormalization", we are expanding in $(E\lambda_5)$

$$S = \int d^4x \int_{-\pi}^{\pi} d\phi \tau_c \left\{ \chi \left[\frac{\partial_\mu \partial^\mu}{2} + \frac{\partial_\phi^2}{2\tau_c^2} - \frac{m_s^2}{2} \right] \chi \right. \\ \left. - \lambda_s \chi^4 \right\}$$

■ Analog 1D QM Problem:

$$\underbrace{\left[-\frac{1}{2m_{QM}} \frac{\partial_\phi^2}{\tau_c^2} + V_{QM}(\phi) \right]}_{H_{QM}} \psi_n(\phi) = E_{QM}^{(n)} \psi_n(\phi)$$


with $m_{QM} \equiv \frac{1}{2}$, $V_{QM} \equiv m_s^2$

$$\Rightarrow \psi_n(\phi) = \frac{e^{in\phi}}{\sqrt{2\pi\tau_c}}, \quad E_{QM}^{(n)} = \frac{n^2}{\tau_c^2} + m_s^2$$

Expand $\chi(x^\mu, \phi) = \sum_n \chi_n(x) \psi_n(\phi)$

Real $\chi \Rightarrow \chi_{-n} = \chi_n^*$

~~BL = ...~~

Kaluza-Klein (KK)

Decomposition

$$S = \int d^4x \int d\varphi r_c \left\{ \frac{1}{2} \chi [-\partial_\mu \partial^\mu - H_{QM}] \chi - \lambda_5 \chi^4 \right\}$$

$$= \int d^4x \sum_n \left\{ \frac{1}{2} (\partial_\mu \chi_0)^2 - \frac{1}{2} E_{QM}^{(0)} \chi_0^2 + |\partial_\mu \chi_n|^2 - E_{QM}^{(n)} |\chi_n|^2 \right\}$$

$$- \frac{\lambda_5}{2\pi r_c} \int d^4x \sum_{m,n,k,l} c_{mnlk} \chi_m \chi_n \chi_k \chi_l$$

$$c_{mnlk} = \delta_{m+n+k+l, 0}$$

extra-dimensional
angular momentum
conservation

$$m_{4D}^2 \equiv E_{QM} = \frac{n^2}{r_c^2} + m_5^2 \leftarrow \text{Mass gap if } m_5 \ll r_c$$

$$\lambda_{4D} \equiv \lambda_5 / 2\pi r_c \text{ "renormalizable" !}$$

LOW-ENERGY 4D EFFECTIVE THEORY

$$\text{Ella } E, m_5 \ll \frac{1}{r_c}, m_{4D}^{(n \neq 0)}$$

$$S_{\text{eff}} = \int d^4x \left\{ \frac{1}{2} (\partial_\mu \chi_0)^2 - \frac{1}{2} m_5^2 \chi_0^2 - \frac{\lambda_5}{2\pi r_c} \chi_0^4 \right\}$$

at tree-level.

When quantum loops considered
4D theory "matches" 5D theory
at low energies only with more
general couplings, masses:

$$S_{\text{eff}} = \int d^4x \left\{ \frac{1}{2} (\partial_\mu \chi_0)^2 - \frac{1}{2} m_{\text{eff}}^2 \chi_0^2 - \lambda_{\text{eff}} \chi_0^4 \right\}$$

4D Renormalizability \equiv Low-energy
insensitivity to UV details such as
extra dimensions, r_c , $\chi_n^{(2c)}$...

General Relativity (GR)

Newton's Law in
Gauss Law form:

$$\nabla^2 V_{\text{grav}}(x) = T^{00}(x) \times G_{\text{Newton}}$$

↑
Newt. potential

↑
energy density
 $\in T^{\mu\nu}$ Energy-momentum
Tensor

Special Relativity \Rightarrow

$$V_{\text{grav}}(x) \equiv h_{00}(x)$$

for some tensor $h_{\mu\nu}(x)$.

Equation of Motion (EOM):

$$G_{\mu\nu} = 8\pi G_N \times T_{\mu\nu}$$

↑
Made From $h_{\mu\nu}$ & ∂
"Einstein Tensor"

↑
Conserved
in absence of
gravity, $\partial^\mu T_{\mu\nu} = 0$

Analogy: EM

see "Feynman Lectures"

Coulomb Law

$$\nabla^2 V_{\text{Electrostatic}} = J_0$$

↑
Charge density
 $\in J_\mu$ 4-current

+ Special Relativity \Rightarrow

$$V_{\text{Elec}} \equiv A_0, \text{ some } A_\mu.$$

$$\text{EOM: } [\alpha \partial^\mu \partial_\nu + \beta \partial^2 \delta^\mu_\nu] A_\mu = J_\mu$$

$$\text{Current Conservation } \partial_\mu J^\mu = 0$$

$$\Rightarrow \alpha = -\beta. \text{ ie. } \beta \partial^\mu F_{\mu\nu} = J_\nu$$

gauge invariant:

$$A_\mu \rightarrow A_\mu + \partial_\mu \lambda$$

For $\beta=1$, ∇_μ gauge: Recover Coulomb Law

Curved Spacetime

see Weinberg "Gravitation & Cosmology"

Wald "General Relativity"

S. Weinberg '64-'67

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$

$$\rightarrow g_{\mu\nu}(x) dx^\mu dx^\nu$$

$g_{\mu\nu} \rightarrow$ still raises/lowers indices

$$\begin{aligned} & \xrightarrow{x \rightarrow x'(x)} \underbrace{g_{\mu\nu}(x) \frac{\partial x^\mu}{\partial x'^{\mu'}} \frac{\partial x^\nu}{\partial x'^{\nu'}}}_{\equiv g'_{\mu'\nu'}(x')} dx'^{\mu'} dx'^{\nu'} \end{aligned}$$

General Coordinate Inv. (GCI)

defines new "gauge symmetry"

on $g_{\mu\nu}(x) \equiv \eta_{\mu\nu} + h_{\mu\nu}(x)$

Curvature: $R^{\sigma}_{\mu\nu\rho} \equiv \Gamma^{\sigma}_{\mu\rho,\nu} - \Gamma^{\sigma}_{\nu\rho,\mu} + \Gamma^{\alpha}_{\mu\rho} \Gamma^{\sigma}_{\alpha\nu} - \Gamma^{\alpha}_{\nu\rho} \Gamma^{\sigma}_{\alpha\mu}$

Christoffel symbol

Riemann Tensor

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \{ g_{\nu\sigma,\mu} + g_{\mu\sigma,\nu} - g_{\mu\nu,\sigma} \}$$

$$R_{\mu\rho} \equiv R_{\mu\nu\rho}{}^{\nu}, \quad R \equiv R_{\mu\rho} g^{\mu\rho}$$

Ricci Tensor

Ricci Scalar

Scalars: ~~$\phi(x)$~~

$$\phi'(x'(x)) \stackrel{x \rightarrow x'(x)}{=} \phi(x)$$

Tensors:

$$T^{\mu_1 \dots \mu_N}_{\nu_1 \dots \nu_M}(x'(x)) \stackrel{x \rightarrow x'(x)}{=} \dots$$

$$\frac{\partial x^{\mu_1}}{\partial x'^{\nu_1}} \dots \frac{\partial x^{\mu_N}}{\partial x'^{\nu_N}} T^{\mu_1 \dots \mu_N}_{\nu_1 \dots \nu_M}(x) \frac{\partial x^{\nu_1}}{\partial x'^{\nu_1}} \dots \frac{\partial x^{\nu_M}}{\partial x'^{\nu_M}}$$

Contractions of tensors ~~are~~ are tensors

Invariant Integration Measure

$$\int d^4x \sqrt{-g} \dots = \int d^4x' \sqrt{-g'}$$

where $\sqrt{-g} \equiv \sqrt{-\det(g_{\mu\nu}^{(x)})}$

~~...~~

$\therefore \int d^4x \sqrt{-g} \mathcal{L}(x)$ is GCI if $\mathcal{L}(x)$ is a scalar

Relativity + Newtonian Limit

$$+ \partial_\mu T^{\mu\nu} = 0 \quad \underbrace{\text{in flat space limit}}_{\hbar \omega_{\mu\nu} \rightarrow 0}$$

\Rightarrow Unique Gravity EOM: "Einstein Equations"

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \stackrel{\downarrow}{=} 8\pi G_N \times T_{\mu\nu}$$

\uparrow "Einstein Tensor" \nearrow Matter obtained from GCI action.

Action

$$S = \int d^4x \sqrt{-g} \left\{ + \frac{1}{16\pi G_N} R + \frac{g^{\mu\nu}}{2} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right\}$$

$\equiv 2 \frac{\delta S_{\text{matter}}}{\sqrt{-g} \delta g^{\mu\nu}}$

Then $T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left[\frac{g^{\alpha\beta}}{2} \partial_\alpha \phi \partial_\beta \phi - V \right]$

& Einstein's Equations $\equiv \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = 0$

Linearized GR

Treat $h_{\mu\nu}$ as infinitesimal,
as well as ξ_ν in

$$x'(x) \equiv x + \xi(x).$$

Then, $h'_{\mu\nu}(x) \equiv h_{\mu\nu}(x) + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$
to 1st order in infinitesimals.

Gauge-Fixing (Axial)

Choose a spatial direction, y .

$$h_{\mu y} = h_{y\mu} = 0 \quad \& \quad h_{\mu\nu} \text{ arbitrary}$$

For any μ For $\mu, \nu \neq y$.

To go to this gauge choose $\partial_y \xi_y \equiv -\frac{h_{yy}}{2}$

$$\& \quad \partial_y \xi_{\mu \neq y} \equiv -h_{y(\mu \neq y)} - \partial_{\mu \neq y} \xi_y \quad \blacksquare$$

"Planck Scale" and
field normalization

$$S_{\text{quad}} \ni \int d^4x \frac{h_{ij}^2}{64\pi G} + \dots$$

Canonical Field for QFT:

$$\cancel{h_{ij}} = \frac{\overset{\text{canonical}}{h_{ij}}}{M}$$

↑
Planck scale

$$M \equiv \frac{1}{32\pi G}$$

$$\therefore S = \int d^4x \sqrt{-g} \{ 2M^2 R + \dots \}$$

Compactifying GR in Higher Dimensions (Linearized Analysis, quadratic in action)

Axial gauge with respect to φ obstructed by $\xi^M(x, \varphi)$ not being 2π -periodic.

Instead use

$$\partial_\varphi \xi_\varphi \equiv -\left(\frac{h_{\varphi\varphi}}{2} - \bar{h}_{\varphi\varphi}\right)$$

$$\partial_\varphi \xi_\mu \equiv -\left(h_{\varphi\mu} - \bar{h}_{\varphi\mu}\right) - \partial_\mu \left(\frac{\xi_\varphi}{2} - \bar{\xi}_\varphi\right)$$

where $\bar{h}_{MN}^{(x)} \equiv \frac{1}{2\pi} \int_{-\pi}^{\pi} d\varphi h_{MN}(x, \varphi)$, $\bar{\xi}_\varphi^{(x)} \equiv \frac{1}{2\pi} \int_{-\pi}^{\pi} d\varphi \xi_\varphi(x, \varphi)$

so $h'_{\mu\nu}(x, \varphi)$ arbitrary, but $h'_{\mu\varphi} = \bar{h}_{\mu\varphi}$, $h'_{\varphi\varphi} = \bar{h}_{\varphi\varphi}$

$$\begin{aligned}
S_{\text{quad}} = \int d^4x \int_{-\pi}^{\pi} d\varphi r_c \left\{ \frac{1}{2} (\partial_\sigma h_{\mu\nu})^2 - (\partial_\nu h^{\mu\nu})^2 \right. \\
+ \partial_\nu h^{\mu\nu} \partial_\mu h_\sigma^\sigma - \frac{1}{2} (\partial_\mu h_\nu{}^\nu)^2 \\
- \frac{1}{2} h_{\mu\nu} H_{\text{QM}} h^{\mu\nu} + \frac{1}{2} h_\mu{}^\mu H_{\text{QM}} h_\nu{}^\nu \\
+ \frac{1}{r_c^2} (\partial_\mu \bar{h}_{\nu\varphi})^2 - \frac{1}{r_c^2} (\partial^\mu \bar{h}_{\varphi\mu})^2 \\
\left. - \frac{1}{r_c^2} \partial_\mu \bar{h}_{\varphi\varphi} \partial_\nu h^{\mu\nu} \right\}
\end{aligned}$$

where $H_{\text{QM}} \equiv -\frac{\partial^2 \varphi}{r_c^2}$

More compactly,

$$\begin{aligned}
S_{\text{quad}} = \int d^4x \int_{-\pi}^{\pi} d\varphi r_c \left\{ \frac{1}{2} h_{\mu\nu} D^{\mu\nu\rho\sigma} h_{\rho\sigma} \right. \\
- \frac{1}{2} h_{\mu\nu} H_{\text{QM}} h^{\mu\nu} + \frac{1}{2} h_\mu{}^\mu H_{\text{QM}} h_\nu{}^\nu \\
\left. + \mathcal{O}(\bar{h}) \right\}
\end{aligned}$$

~~Handwritten scribble~~

$$= \int d^4x \sum_n \left\{ \frac{1}{2} h_{\mu\nu}^{(n)} D^{\mu\nu\rho\sigma} h_{\rho\sigma}^{(n)} \right. \\ \left. - \frac{1}{2} h_{\mu\nu}^{(n)} \frac{n^2}{c^2} h^{(n)\mu\nu} + \frac{1}{2} h_{\mu}^{(n)\mu} \frac{n^2}{c^2} h^{(n)\nu}_{\nu} \right. \\ \left. + \dots + \mathcal{O}(\bar{h}, h^{(0)}) \right\}$$

where $h_{\mu\nu}^{(n)} \equiv h_{\mu\nu}^{(n)} \psi_n(\varphi)$

$n \neq 0 \equiv$ Massive spin-2 EOM:

In momentum space in rest frame

$$(E^2 - m^2) h_{ij}^{\text{traceless}} = 0$$

$$-\frac{2}{3} (E^2 - m^2) h_{kk} = m^2 h_{00}$$

$$m^2 h_{kk} = 0$$

$$m^2 h_{0i} = 0$$

$$\text{where } m^2 = \frac{n^2}{c^2}$$

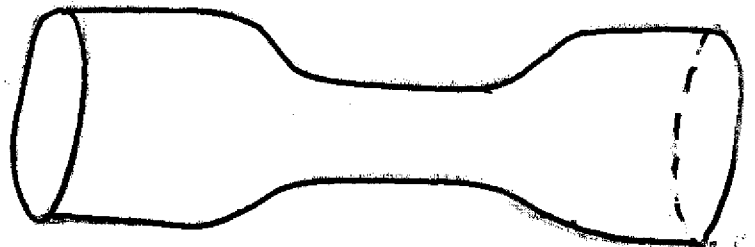
4D LOW-ENERGY EFFECTIVE FIELD THEORY

$$S_{\text{quad}}^{\text{eff}} = \int d^4x \left\{ \frac{1}{2} h_{\mu\nu}^{(0)} D^{\mu\nu\rho\sigma} h_{\rho\sigma}^{(0)} + \frac{1}{4} F_{\mu\nu}^2 - \frac{\sqrt{2\pi}}{r_c^{3/2}} \partial_\mu \bar{h}_{\varphi\varphi}^{(x)} \partial_\nu h^{\mu\nu} \right\}$$

$h_{\mu\nu}^{(0)}$ is effective 4D graviton

$A_\mu^{(x)} \equiv \sqrt{\frac{4\pi}{r_c}} \bar{h}_{\mu\varphi}^{(x)}$ is "KK gauge boson"

Recalling $ds^2 \ni r_c^2 d\varphi^2 + h_{\varphi\varphi}^{(x)} d\varphi^2$
we see that $\bar{h}_{\varphi\varphi}^{(x)}$
is the "Radion"



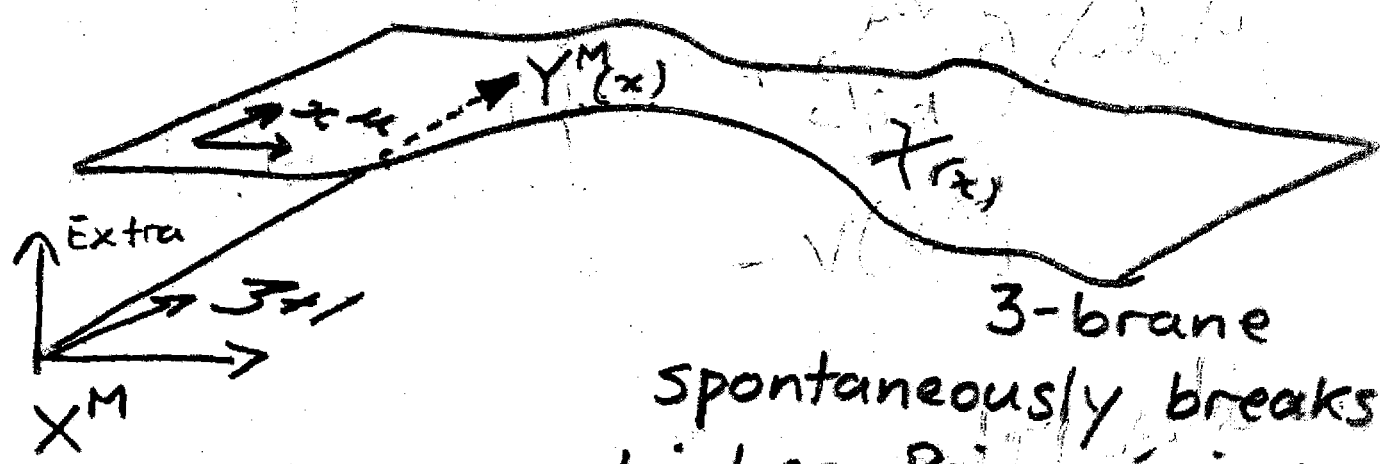
Its kinetic mixing with 4D graviton can be undone with field redefinitions.

Interactions: $S_{\text{eff}} = \int d^4x 4\pi r_c M_5^3 \bar{R} + \dots$
 $\equiv \int d^4x 2M_4^2 \bar{R} + \dots$ where \bar{R} is 4D
curvature of $\bar{g}_{\mu\nu}^{(x)} \equiv \eta_{\mu\nu} + h_{\mu\nu}^{(0)}/M_4$

BRANES

EFT - Sundrum hep-ph/9806
 Stringy motivation -
 Polchinski - hep-th/9812198

"The Bulk"



3-brane
 spontaneously breaks
 higher Poincaré inv.
 → 4D

Symmetries of the action:

5D Poincaré inv. in $Y^M(x)$.

4D reparametrization inv. in x^μ

Building Blocks:

$\partial_\mu Y^M$ is vector under both sym.

$\chi(x)$ " scalar " " " "

Vacuum state: $Y^M(x) = x^\mu$, $Y^5 = \text{const}$

$\chi = 0$

Induced metric: $ds_{\text{brane}}^2 = \underbrace{\eta_{MN}}_{\equiv g_{\mu\nu}^{\text{ind.}}} \frac{\partial Y^M}{\partial x^\mu} \frac{\partial Y^N}{\partial x^\nu} dx^\mu dx^\nu$

Action

constant "tension"

$$S = \int d^4x \sqrt{-g_{\text{ind}}} \left\{ -\overbrace{f^4}^{\text{constant "tension"}} + g_{\text{ind}}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - V(\chi) \right\}$$

Gauge-fixing reparametrizations

$$Y^\mu(x) = x^\mu, \quad Y^5(x) \text{ arbitrary.}$$

$$\int d^4x \sqrt{-g_{\text{ind}}} \{-F^4\}$$

$$\approx \int d^4x \left(-\det[\eta_{\mu\nu} - \partial_\mu Y^5 \partial_\nu Y^5] \right)^{\frac{1}{2}} \{-F^4\}$$

$$\approx \int d^4x \left\{ -F^4 + \frac{F^4}{2} \partial_\mu Y^5 \partial^\mu Y^5 + \mathcal{O}(Y^4) \right\}$$

↑
+ve kinetic term.

$Y^5(x)$ is Goldstone boson of spontaneous breaking of extra-dimensional translations & boosts, & rotations into x, y, z

Coupling Gravity to the Brane.

Symmetry: 5D Poincaré inv.
→ 5D GCI.

Induced metric: $\equiv dY^M dY^N$

$$ds_{\text{Brane}}^2 = \underbrace{G_{MN}(Y(x)) \frac{\partial Y^M}{\partial x^\mu} \frac{\partial Y^N}{\partial x^\nu} dx^\mu dx^\nu}_{\equiv g_{\mu\nu}^{\text{ind}}(x)}$$

$$S = \int d^5x \sqrt{-G} \{ 2M_5^3 R$$

$$+ \int d^4x \sqrt{-g_{\text{ind}}} \{ -F^4 + g_{\text{ind}}^{\mu\nu} \partial_\mu X \partial_\nu X - V(X) \}$$

~~Being~~ Bulk-Brane couplings do not conserve extra-dimensional momentum.

GRAVITY LOCALIZATION

— RANDALL-SUNDRUM II (RS2) hep-th/9906064

$$S = \int d^5x \sqrt{-G} \{ 2M_5^3 R - \Lambda \}$$

$$+ \int d^4x \sqrt{-g_{\text{ind}}} \{ g_{\text{ind}}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - V(\chi) - F^4 \}$$

Λ is 5D "cosmological constant".
Think of it as $V_{\text{min}}^{\text{Bulk}}$ for heavy bulk fields.

Take extra dimension to be infinite
 \therefore can work in full axial gauge.
Instead of angle φ , we have "y".

Vacuum Solution with 4D Poincaré Invariance

General form: $ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$
 $\chi = 0, Y^\mu = x^\mu, Y^5 = 0$

5D Einstein Equations:

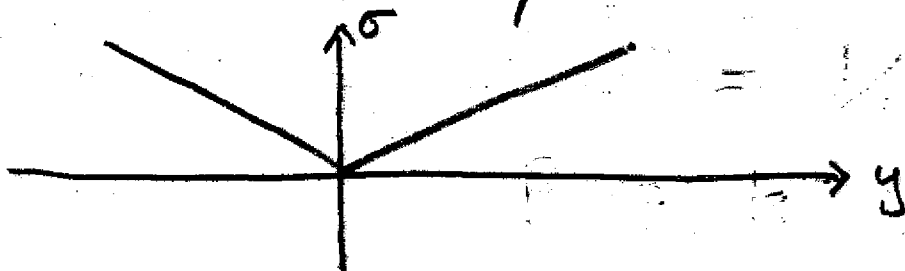
$$6\sigma'^2 = \frac{-\Lambda}{4M_5^3}$$

$$3\sigma'' = \frac{F^4}{4M_5^3} \delta(y)$$

$$S_{\text{brane}}(x, Y^5=0) = \int d^4x dy \delta(y) \sqrt{-g_{\text{ind}}} \{-F^4\}$$

$$\text{or } g_{\mu\nu}^{\text{ind}} = G_{\mu\nu}(x, y)$$

Self-consistency: $F^4 = 24M_5^3 k, \Lambda = -24M_5^3 k$



Y^M EOM satisfied, noting $\sigma'(y=0)=0$

Vacuum metric:

$$ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$

↑
"Warp" factor

Compare with Anti-de Sitter
5D spacetime (AdS_5),
maximally symmetric spacetime
of negative curvature:

$$ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$

"Radius of curvature" = $1/k$

$$R = k^2$$

KK decomposition

of gravitational fluctuation

In axial gauge,

$$ds^2 = \left[e^{-2k|y|} \eta_{\mu\nu} + h_{\mu\nu}(x, y) \right] dx^\mu dx^\nu + dy^2$$

Substituting into action, and
working to $\mathcal{O}(h^2)$,

$$S = \int d^4x \int_{-\infty}^{\infty} dy \left\{ \frac{e^{2k|y|}}{2} h_{\mu\nu} D^{\mu\nu\rho\sigma} h_{\rho\sigma} \right. \\ \left. - \frac{1}{2} h_{\mu\nu} (\partial_y^2 + 4k\delta(y) - 4k^2) h^{\mu\nu} \right. \\ \left. + \frac{1}{2} h_\mu{}^\nu (\partial_y^2 + 4k\delta(y) - 4k^2) h_\nu{}^\mu \right\}$$

Warp factor has stopped us
achieving separation of variables.

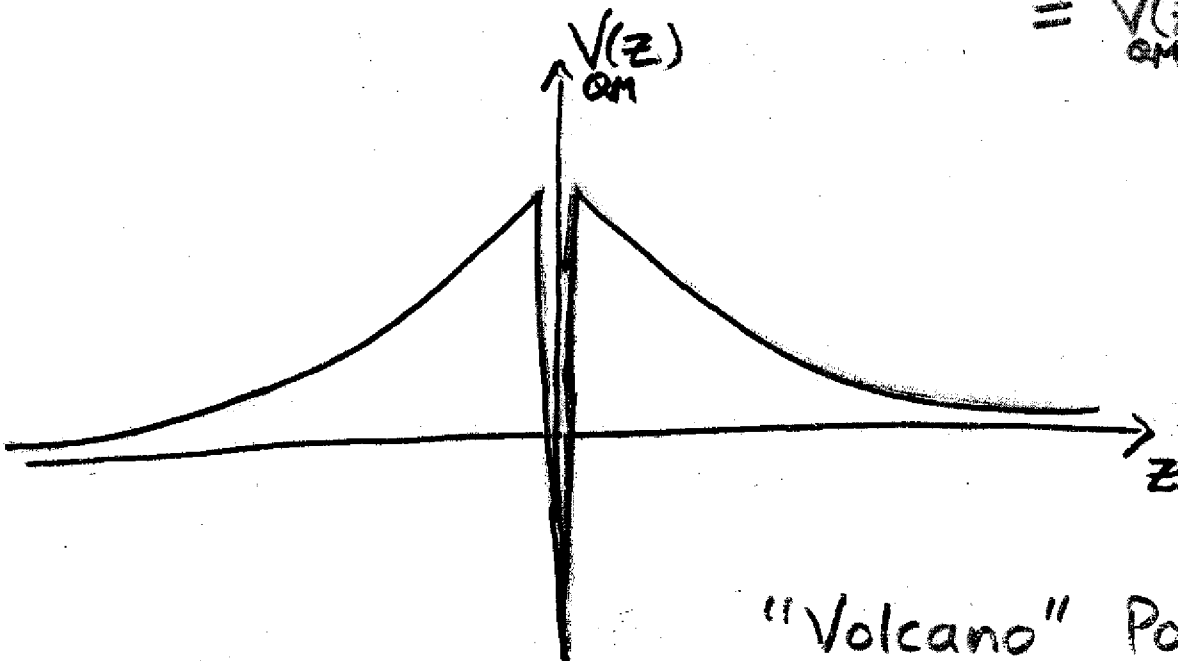
Change variables:

$$z \equiv \frac{\text{sgn}(y) (e^{k|y|} - 1)}{k}$$

$$\hat{h}_{\mu\nu}(x, y) \equiv h_{\mu\nu}(x, y) e^{k|y|/2}$$

$$S = \int d^4x \int_{-\infty}^{\infty} dz \left\{ \frac{1}{2} \hat{h}_{\mu\nu} D^{\mu\nu\rho\sigma} \hat{h}_{\rho\sigma} - \frac{1}{2} \hat{h}_{\mu\nu} H_{QM} \hat{h}^{\mu\nu} + \frac{1}{2} \hat{h}_{\mu}^{\mu} H_{QM} \hat{h}^{\nu}_{\nu} \right\}$$

where $H_{QM} = -\frac{1}{2} \partial_z^2 + \underbrace{\frac{15k^2}{8(k|z|+1)^2} - \frac{3k}{2} \delta(z)}_{\equiv V_{QM}(z)}$



"Volcano" Potential

General considerations

\Rightarrow 1 bound state

+ continuum ~~where~~ $E_{QM} > 0$.

Bound state $\psi_0(z) = \frac{(k|z|+1)^{-3/2} k^{1/2}}{\cancel{\text{norm}}}$

has $E_{QM} = 0$, corresponding to massless 4D graviton.

Continuum: any $E_{QM} = m^2 > 0$

$$\psi_m^{\text{even}}(z) \doteq N_m (|z| + \frac{1}{k})^{1/2}$$

$$\times \left[Y_2(m|z| + \frac{1}{k}) + \frac{4k^2}{\pi m^2} J_2(m|z| + \frac{1}{k}) \right]$$

for $m \ll k$

N_m is normalization constant
 $= \frac{\pi m^{5/2}}{4k^2}$ in plane-wave normalization.

N.B $\sqrt{\frac{z|m}{2}} J_2(mz) \underset{|z| \text{ large}}{\sim} \cos(mz - \frac{5\pi}{4}), \sqrt{\frac{z|m}{2}} Y_2(mz) \sim \sin(mz - \frac{5\pi}{4})$

4D Effective Field Theory

Retain only $\hat{h}_{\mu\nu}(x, y) = h_{\mu\nu}^{(0)} \psi_0(z)$

ie. $h_{\mu\nu}(x, y) = e^{-2k|y|} h_{\mu\nu}^{(0)}$

ie. ~~ds^2~~ $ds^2 = e^{-2k|y|} \underbrace{\left(\eta_{\mu\nu} + h_{\mu\nu}^{(0)} \right)}_{g_{\mu\nu}^{ind}(x)} dx^\mu dx^\nu + dy^2$

~~$S_{eff} = \int d^4x \int_{-\infty}^{\infty} dy e^{-2k|y|} \left[2M_5^3 \bar{R} + \delta(y) \left[g^{\mu\nu}_{ind} \partial_\mu \chi \partial_\nu \chi - V(\chi) \right] \right]$~~

$$S_{eff} = \int d^4x \int_{-\infty}^{\infty} dy e^{-2k|y|} \sqrt{-g_{ind}} \left\{ 2M_5^3 \bar{R} + \delta(y) \left[g^{\mu\nu}_{ind} \partial_\mu \chi \partial_\nu \chi - V(\chi) \right] \right\}$$

$$= \int d^4x \sqrt{-g_{ind}} \left\{ 2M_4^2 \bar{R} + g^{\mu\nu}_{ind} \partial_\mu \chi \partial_\nu \chi - V(\chi) \right\}$$

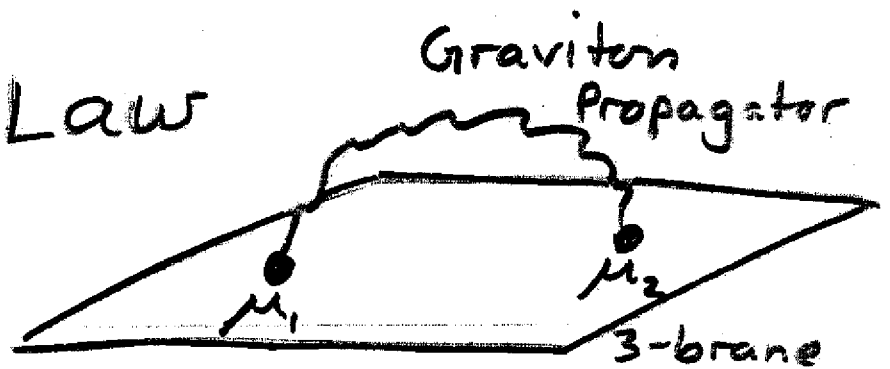
where \bar{R} is 4D curvature of g_{ind}

& $M_4^2 \equiv \frac{2M_5^3}{k}$. Compare with unwarped case.

Non-observability of KK excitations

Requires explanation since they have arbitrarily small 4D mass.

Newton's Law



$$V(r) \approx G_N \frac{\mu_1 \mu_2}{r}$$

corresponding to M_4

$$+ \int_0^\infty dm \left(\frac{G_N}{k} \right) \left(\frac{m}{k} \right) \frac{e^{-mr}}{r} \mu_1 \mu_2$$

5D Newton constant $|\psi_m(0)|^2$ Massive static potential.

$$= G_N \frac{\mu_1 \mu_2}{r} \left(1 + \frac{1}{k^2 r^2} \right)$$

Thus KK effects
negligible at large
distances, $r \gg \frac{1}{k}$.

Typically, we consider

$$k \lesssim M_5 \lesssim M_4 \sim 10^{18} \text{ GeV.}$$

WHY GRAVITY IS WEAK

— RANDALL-SUNDRUM I (RSI)

$(G_{\text{Newton}} \ll G_{\text{Fermi}})$

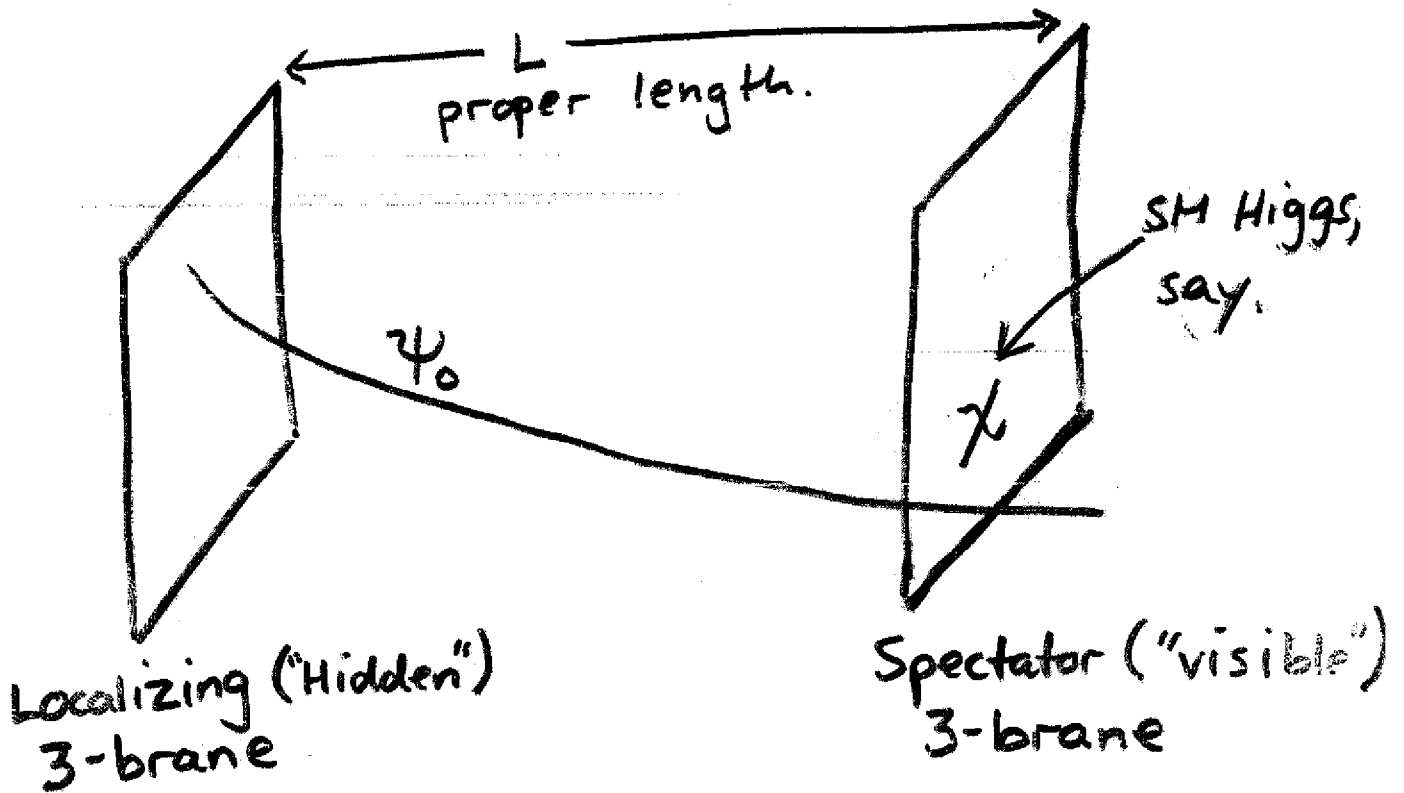
see Witten '96 on M-theory

Arkani-Hamed, Dimopoulos, Dvali '98

Randall, Sundrum — hep-ph/9905221

Stringy: Lukas, Ovrut, Stelle, Waldram hep-th/9803235

Intuition:



Standard Model particles ~~see~~ couple to $\sim e^{-kL}$ tail of massless 4D graviton ~~see~~ profile. The strength of gravity is thereby exponentially diminished.

$$S = \int d^5 X \sqrt{-G} \left\{ 2M_5^3 R + 24M_5^3 k^2 \right\}$$

$$+ \int d^4 x \sqrt{-g_{\text{hid}}} \left\{ -24M_5^3 k \right\}$$

$$+ \int d^4 x \sqrt{-g_{\text{vis}}} \left\{ g_{\text{vis}}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - V(\chi) - f_{\text{vis}}^4 \right\}$$

4D Poincare invariant ansatz:

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$

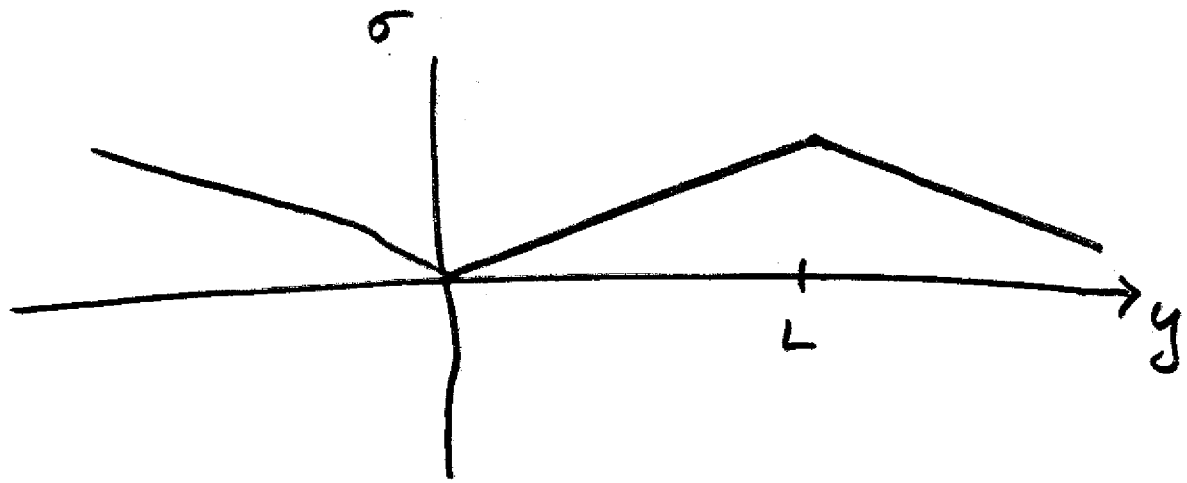
$$g_{\mu\nu}^{\text{hid}} = \eta_{\mu\nu}, \quad \text{~~g}_{\mu\nu}^{\text{vis}} = e^{-2\sigma(L)} \eta_{\mu\nu}~~$$

$$g_{\mu\nu}^{\text{vis}} = e^{-2\sigma(L)} \eta_{\mu\nu}$$

$e^{-2\sigma(0)} = 1$ is just a convention

Einstein Equations: $\sigma'^2 = k^2$

$$3\sigma'' = 6k\delta(y) + \frac{f^4}{4M_5^3} \delta(y-L)$$



is only consistent possibility.

But even here, we require

$$F_{vis}^4 = \cancel{24 M_s^3 k} - 24 M_s^3 k$$

Normally this is bad:

$$\int d^4x \sqrt{-g_{ind}} \{-F^4\}$$

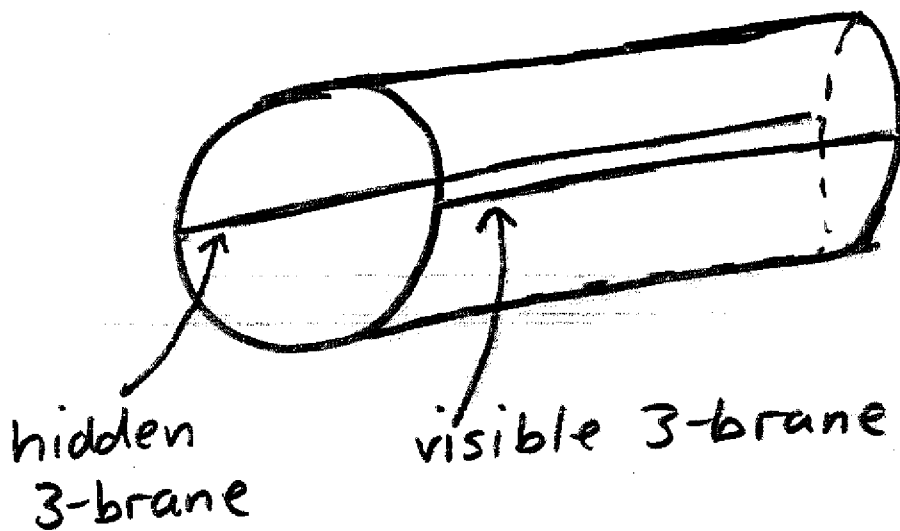
small brane
fluctuations $\int d^4x \left\{ -F^4 + \frac{F^4}{2} \partial_\mu Y^S \partial^\mu Y^S \right\}$

correct sign kinetic term
 $\equiv F^4 > 0.$

Wrong sign kinetic term \equiv
unbounded below energy (Hamiltonian)

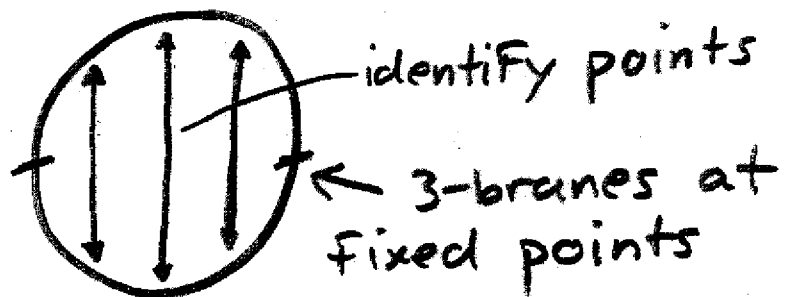
THE CURE — ORBIFOLD SYMMETRY

Compactify extra dimension to circle, S^1 :



Dynamics is symmetric ~~about~~ between upper & lower hemispheres.

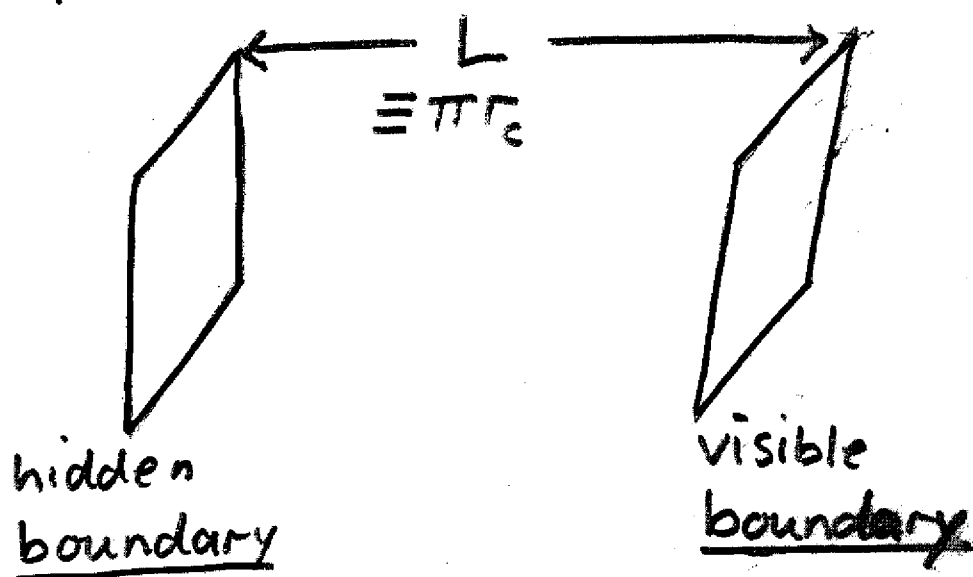
We can therefore identify the two hemispheres ("gauge" the \mathbb{Z}_2 symmetry):



Orbifold symmetry forbids brane fluctuations which threaten vacuum stability.

Under \mathbb{Z}_2 -identification we throw out all field fluctuations which are \mathbb{Z}_2 -odd, retaining only \mathbb{Z}_2 -even.

Physically, we are left with



The orbifolding procedure generates sensible generally covariant boundary conditions.

\mathbb{Z}_2 -transformation of fields

$G_{\mu\nu}(x, \phi)$ is parity-even

ie. ~~para~~ orbifold projects out odd functions of $\phi \in [-\pi, \pi]$

$G_{\mu\phi}(x, \phi)$ is parity-odd

ie. orbifold projects out even functions of ϕ

\therefore Axial gauge - $G_{\mu\phi}(x, \phi) \rightarrow \overline{G_{\mu\phi}}(x)$
is projected out!

$G_{\phi\phi}(x, \phi)$ is parity even.

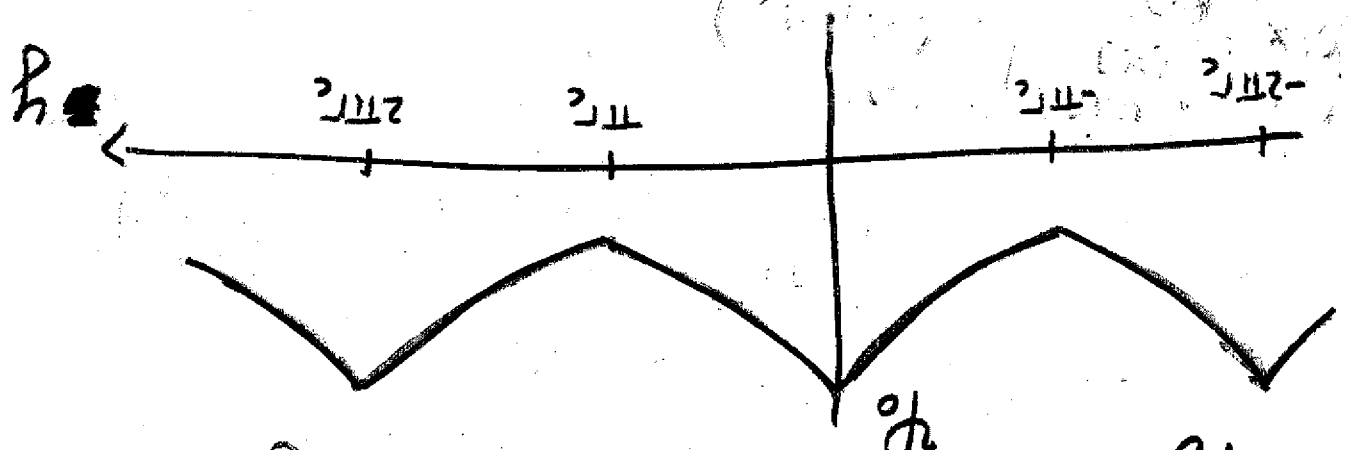
\therefore axial gauge "radion" field is
~~not~~ retained

$\chi(x)$ brane fields are parity-even
as they sit at \mathbb{Z}_2 fixed points.

4D LOW-ENERGY

EFFECTIVE FIELD THEORY

$\psi_0(z)$ remains $m_{4D}^2 = 0$ eigenfunction:



∴ massless 4D graviton mode

$$\equiv ds^2 = e^{-2k|y|} (g_{\mu\nu}^{(4)} + h_{\mu\nu}^{(4)}) dx^\mu dx^\nu + (r_c dy)^2$$

Noting that $g_{\mu\nu}^{vis} = e^{-2k\pi r_c y} g_{\mu\nu}^{hid}$

$$S_{4T} = \int d^4x \sqrt{g_{hid}} \left\{ 2M_4^2 R + e^{-2k\pi r_c y} \right\} \chi^2 dx^\nu$$

$$+ e^{-4k\pi r_c y} \lambda (\chi^2 - v^2)^2$$

$$M_4^2 = \int_{-\pi r_c}^{\pi r_c} dy e^{-2k|y|/3} = \frac{2\pi r_c}{3} M_5^2$$

WARPED HIERARCHY

Suppose all scales in our set-up have no large hierarchies

$$M_5 \gtrsim k \gtrsim v, \frac{1}{\pi r_c}$$

& dimensionless couplings are order one: $\lambda \approx 1$.

Field re-defining $\hat{\chi} \equiv e^{-k\pi r_c} \chi$,

$$S_{\text{eff}} = \int d^4x \sqrt{-g_{\text{hid}}} \left\{ 2M_4^2 \bar{R} + g_{\text{hid}}^{\mu\nu} \partial_\mu \hat{\chi} \partial_\nu \hat{\chi} + \lambda \left(\hat{\chi}^2 - (e^{-k\pi r_c} v)^2 \right)^2 \right\}$$

$$\therefore M_4^2 = \frac{(1 - e^{-2k\pi r_c}) M_5^3}{k} \approx \frac{M_5^3}{k}$$

while ~~the~~ Weak scale = $v_{\text{eff}} = e^{-k\pi r_c} v \lesssim e^{-k\pi r_c} M_4$.

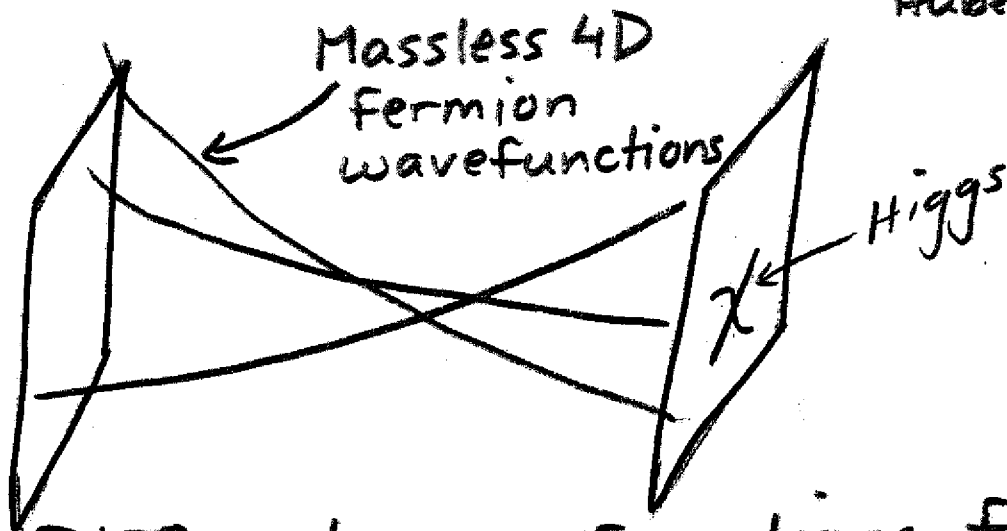
GENERAL WARPED HIERARCHIES

$$m_{4D} \sim e^{-ky} m_{5D}$$

\uparrow
 5D mass parameter
 \uparrow
 Dominant location
 of associated physics

Fermion Hierarchies

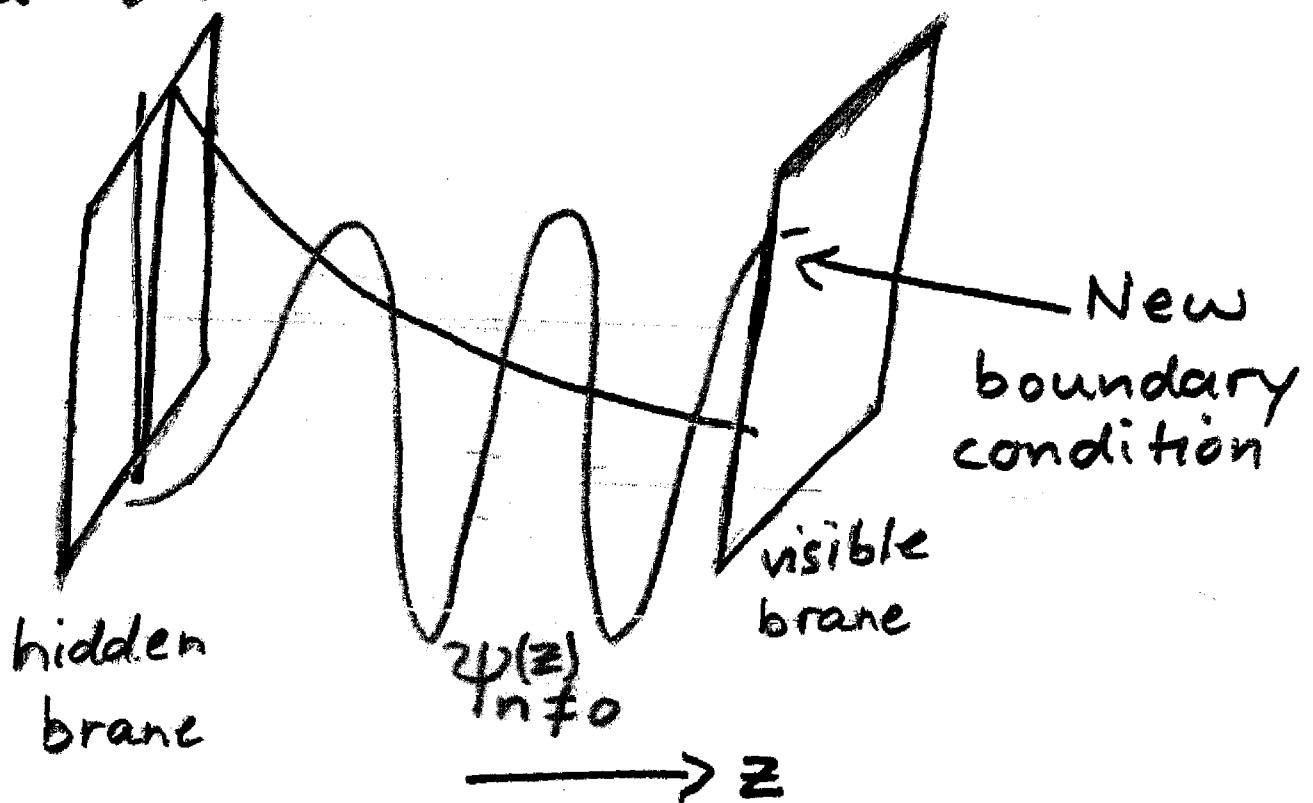
Arkani-Hamed, Schmaltz &
 Gherghetta, Pomarol '00
 Huber, Shafi '01, '02
 Huber '02



Different wavefunctions for
 different flavors produce hierarchies
 in Yukawa couplings.

KK GRAVITONS

Compactification puts
(half) the volcano potential in
a box:



KK continuum \rightarrow quantized,
discrete modes.

Plane wave approximation for ψ_n
near $z_c \gg 1 \Rightarrow \sim \frac{\pi}{z_c}$ splittings
of KK states.

$$z_c \sim \frac{e^{k\pi r_c}}{k} \Rightarrow$$

$$\text{KK splittings} \sim k e^{-k\pi r_c} \\ \sim \text{Weak scale}$$

\therefore KK gravitons kinematically accessible to expt.

Coupling to visible matter

$$\int d^4x T_{\mu\nu}^{\text{vis}} \underbrace{\hat{h}^{\mu\nu}(x, z_c)}_{\sum_n h_n^{\mu\nu}(x) \psi_n(z_c)} \text{ in linearized approximation}$$

$$\text{Now } \int d^4x T_{\mu\nu}^{\text{vis}}(x) \hat{h}_{(0)}^{\mu\nu}(x) \psi_0(z_c)$$

\equiv usual massless 4D graviton coupling.

$$\therefore \int d^4x T_{\mu\nu}^{\text{vis}}(x) \hat{h}_{n \neq 0}^{\mu\nu}(x) \psi_{n \neq 0}(z_c) \text{ is stronger} \\ \text{by } \frac{\psi_{n \neq 0}(z_c)}{\psi_0(z_c)}$$

In plane wave approximation

$$|\psi_{n \neq 0}(z_0)| \sim \frac{1}{\sqrt{z_0}}$$

while $\psi_0(z_0) \underset{z \gg 1/k}{\sim} \frac{1}{k^{3/2} |z|^{3/2}}$

\therefore KK gravitons couple to $T_{vis}^{\mu\nu}$

$$\sim \cancel{k} k z_0 \sim e^{k\pi r_c} \lesssim \frac{M_{pl}^4}{\sigma}$$

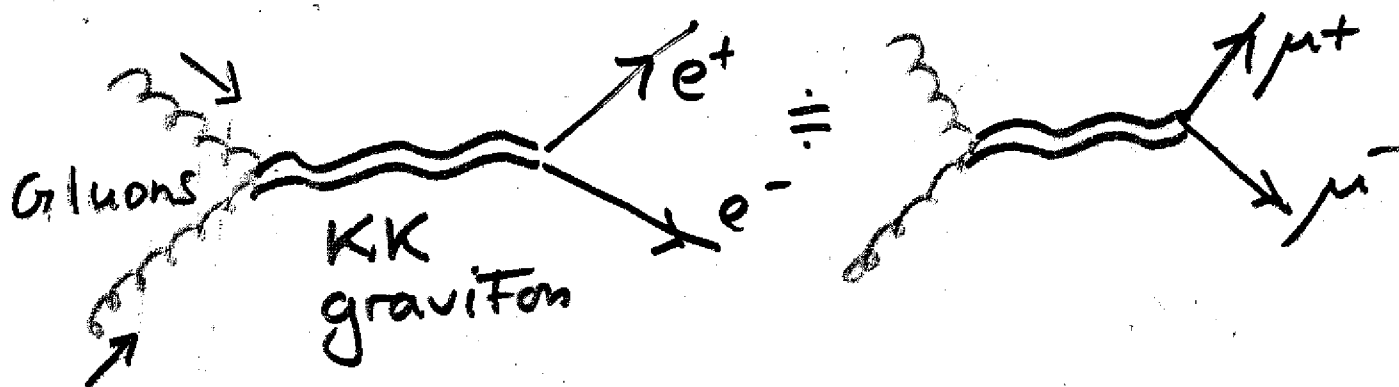
more strongly than massive gravitons!

$$\text{i.e. } \frac{T_{vis}^{\mu\nu} h_{\mu\nu}^{(0)}}{M_{pl}^2} \longrightarrow \lesssim \frac{T_{vis}^{\mu\nu} h_{\mu\nu}^{(n \neq 0)}}{\sigma}$$

TeV energies \Rightarrow unsuppressed
KK production.

KEY FEATURES

- Gravitational universality of couplings



- Narrow (-ish) KK resonances if

$$k < M_{KK}, \text{ since } k e^{-k\pi r_c} \equiv \text{KK mass scale}$$

$$\text{while } E \times \frac{e^{k\pi r_c}}{M_{KK}} \text{ sets coupling.}$$

- Spin-2 nature of resonance visible in angular distribution of decay products

See eg. Davoudiasl, Hewett, Rizzo hep-ph/9909255

RADIUS STABILIZATION

Goldberger, Wise hep-ph/9907218; Lewandowski, May, Sundrum hep-th/0209050

As in unwarped case, \exists massless scalar radion with NO potential, i.e. a "modulus" or "flat direction" in field space.

\therefore radion VEV, r_c , is not dynamically determined. Let's rectify this by adding more physics, a bulk scalar field χ :

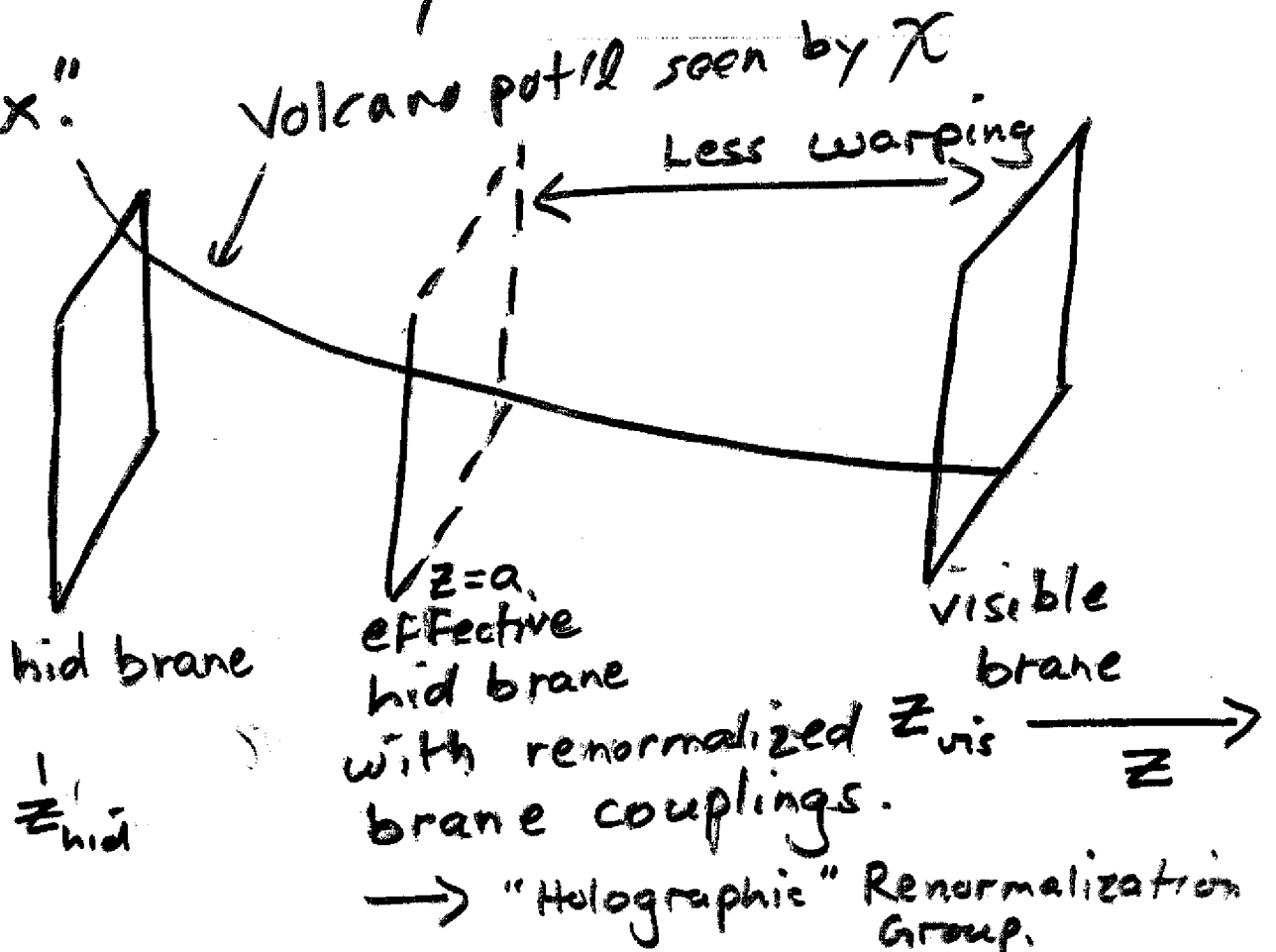
$$S = \int d^5x \sqrt{-G} \left(\frac{1}{2} G^{MN} \partial_M \chi \partial_N \chi - \frac{m^2}{2} \chi^2 + \dots \right) \\ + \int d^4x \sqrt{g_{hid}} \left\{ \hat{\lambda} \chi + \frac{1}{2} \chi k \lambda \chi + \dots \right\} \\ + \int d^4x \sqrt{g_{vis}} \left\{ \hat{\sigma} \chi + \frac{1}{2} \chi k \sigma \chi + \dots \right\}$$

~~This will lead to the stabilization mechanism of Goldberger, Wise hep-ph/9907218.~~

Effect of χ on radius potential like many physical effects in highly warped spaces, is NOT OBVIOUS. Look for a more insightful calculational tool.

THE BIG IDEA

For low-energy questions replace RS "box" by a smaller "effective box".



Study χ dynamics in gravitational vacuum (neglecting gravitational backreaction).

$$\therefore ds^2 = \frac{1}{(kz)^2} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right) \quad z \in [z_{hid}, z_{vis}]$$

$$\lambda \equiv \lambda_0 + \frac{\lambda_2 g_{hid}^{\mu\nu} D_\mu D_\nu}{k^2} + \frac{\lambda_4 g^{\mu\nu} g^{\alpha\beta} D_\mu D_\nu D_\alpha D_\beta}{k^4}$$

+ ...

$$\stackrel{\text{in above metric}}{=} \sum_{n \geq 0} \lambda_n \left(z^2 \partial^2 \right)^n$$

\uparrow
 i.e. 4D ∂^2

Similarly for σ .

However, $\hat{\lambda}, \hat{\sigma} = \text{constants necessarily.}$

χ EOM:

$$\left[\partial_M \sqrt{G} G^{MN} \partial_N + \sqrt{G_{vis}} m^2 + \sqrt{g_{vis}} k \sigma \delta(z - z_{vis}) + \sqrt{g_{hid}} k \lambda \delta(z - z_{hid}) \right] \chi(x, z) = 0$$

Orbifolding + δ -function matching
 & Fourier transforming $x^\mu \rightarrow q^\mu$
 (but NOT z) \Rightarrow EOM:

$$\left[-q^2 - \partial_z^2 + \frac{3}{z} \partial_z + \frac{m^2}{(kz)^2} \right] \chi(q, z) = 0$$

with boundary conditions

$$z \partial_z \chi(q, z) \Big|_{z \rightarrow z_{hid}} = \frac{\lambda(qz)}{z} \chi(q, z) + \frac{\hat{\lambda}}{z} \times \delta^4(q)$$

$$z \partial_z \chi(q, z) \Big|_{z \rightarrow z_{vis}} = - \frac{\lambda(qz)}{z} \chi(q, z) - \frac{\hat{\lambda}}{z} \delta^4(q)$$

where $\lambda(qz) \equiv \sum_{n \geq 0} \lambda_n (z^2 q^2)^n$.

EFFECTIVE THEORY

On effective brane at $z=a$, choose effective brane couplings

$$\lambda \rightarrow \lambda_{eff}(qa, a) = \sum_n \lambda_n^{eff}(a) (a^2 q^2)^n$$

$$\hat{\lambda} \rightarrow \hat{\lambda}_{eff}(a), \dots$$

bulk & vis couplings unchanged:

IF $\chi(x, z)$ solves original EOM in $[z_{hid}, z_{vis}]$
 then " " " " " effective " " " $[a, z_{vis}]$
 subject to effective boundary condition,

$$(*) \quad z \partial_z \chi(q, z) \Big|_{z \rightarrow a_+} = \frac{\lambda_{eff}(q, a)}{2} \chi(q, a) + \frac{\hat{\lambda}_{eff}}{2} \delta(q)$$

Also require $\lambda_{eff}(q, z_{hid}, z_{hid}) = \lambda(q, z_{hid})$.

RG FLOW FOR $\lambda_{eff}, \hat{\lambda}_{eff}, \dots$
 $\frac{d}{da} (*) \Rightarrow$ (Using bulk EOM & $(*)$ to eliminate $\partial_z^2 \chi$ & $\partial_z \chi$)

$$\left(-a \frac{d \hat{\lambda}_{eff}}{da} + 4 \hat{\lambda}_{eff} - \frac{\hat{\lambda}_{eff} \lambda_{eff}}{2} \right) \delta(q) + \left(-a \frac{d \lambda_{eff}}{da} + 4 \lambda_{eff} - \frac{\lambda_{eff}^2}{2} + 2m^2 - 2q^2 \right) \chi = 0$$

in $k=1$ units.

$$\Leftarrow \quad a \frac{d \hat{\lambda}_{eff}}{da} = 4 \hat{\lambda}_{eff} - \frac{1}{2} \hat{\lambda}_{eff} \lambda_{eff}$$

$$\sum_n (q^2 a^2)^n a \frac{d \lambda_n^{eff}}{da} = -2 \sum_n n (q^2 a^2)^n \lambda_n^{eff} + 4 \lambda_{eff} - \frac{\lambda_{eff}^2}{2} + 2m^2 - 2q^2$$

Comparing powers of $q^2 \Rightarrow \infty$ set of coupled
 RG equations for $\hat{\lambda}_{eff}(a)$ & $\lambda_n^{eff}(a)$.

RG FIXED POINT

We have 4D RG "as if"
we're studying a 4D QFT.

The RG has a fixed point:

$$\lambda_{\text{eff}}^*(qa) = 4 - 2\nu + 2qa \frac{J_{\nu-1}(qa)}{J_{\nu}(qa)},$$

$$\nu \equiv \sqrt{4+m^2}$$

$$\hat{\lambda}_{\text{eff}}^* = 0$$

\equiv local $\mathcal{L}_{\text{brane}}$ in x -space since

λ_{eff}^* has series expansion in q^2 .

Exercise: Check.

The fixed point is attractive as $a \rightarrow 2\nu$.
Linearizing about fixed point RG is

$$a \frac{\partial}{\partial a} \lambda_n = \delta_{nm} (\lambda - \lambda_n)_m$$

\uparrow lower triangular

δ has eigenvalues $4 - 2j - \frac{1}{2}(4 + 2\nu) < 0$,
Harder Exercise: Check this. $n \in \mathbb{Z}_x$, $j \in \mathbb{Z}_x$

SOLVING THE RG

Suppose "initially" (at z_{hid}) we are moderately near fixed point, so can use linearized RG.

Since here we're interested in potential energy, drop all q^2 terms in RG.

~~$$a \frac{\partial}{\partial a} \hat{\lambda}_{eff}$$~~

$$a \frac{\partial}{\partial a} \hat{\lambda}_{eff} = \left(4 - \frac{\lambda_{eff}^*}{2}\right) \hat{\lambda}_{eff} = (2 - \nu) \hat{\lambda}_{eff}$$

$$a \frac{\partial}{\partial a} \lambda_0^{eff} = \left(4 - \lambda_0^{eff*}\right) (\lambda_0^{eff} - \lambda_0^{eff*})$$

$$= -2\nu (\lambda_0^{eff} - \lambda_0^{eff*})$$

Solving & "running" down to $a \sim z_{vis}$

$$\Rightarrow \hat{\lambda}_{eff}(\sim z_{vis}) = \hat{\lambda}_{eff} \left(\frac{z_{vis}}{z_{hid}} \right)^{2-\nu}$$

$$\lambda_0^{eff}(\sim z_{vis}) = \lambda_0^{eff*} + (\lambda_0 - \lambda_0^{eff*}) \left(\frac{z_{vis}}{z_{hid}} \right)^{-2\nu}$$

For such small effective dimension
 can neglect "bulk" & treat χ
 as constant in extra dimension.

$$\therefore S_{\text{eff}} = S_{\text{hid eff}} + S_{\text{vis}} + \cancel{S_{\text{bulk}}}$$

$$= \int \frac{d^4 x}{z_{\text{vis}}^4} \left[\hat{\sigma} + \hat{\lambda} \left(\frac{z_{\text{vis}}}{z_{\text{hid}}} \right)^{2-\nu} \right] \chi$$

$$+ \left[\hat{\sigma}_0 + \lambda_0^{\text{eff}*} + \left(\lambda_0 - \lambda_0^{\text{eff}*} \right) \left(\frac{z_{\text{vis}}}{z_{\text{hid}}} \right)^{-2\nu} \right] \frac{\chi^2}{2}$$

Self-consistently
 neglect for large $\frac{z_{\text{vis}}}{z_{\text{hid}}}$
 & moderately small m^2 .

$$\therefore \langle \chi \rangle = \frac{\hat{\sigma} + \hat{\lambda} \left(\frac{z_{\text{vis}}}{z_{\text{hid}}} \right)^{-m^2/8}}{\hat{\sigma}_0 + \lambda_0^{\text{eff}*}}$$

Plugging back into action \Rightarrow

$$S_{\text{eff}} = \int d^4 x \frac{\left[\hat{\sigma} + \hat{\lambda} \left(\frac{z_{\text{vis}}}{z_{\text{hid}}} \right)^{-m^2/8} \right]^2}{2 z_{\text{vis}}^4 (\hat{\sigma}_0 + \lambda_0^{\text{eff}*})}$$

EFFECTIVE RADION POTENTIAL

$$S_{4F} = \int d^4x \left(-V_{\text{eff}} \left(\frac{z_{\text{vis}}}{z_{\text{hid}}} \right) \right)$$

The VEV of the Radion degree of freedom when gravity is turned back on

Minimizing this $V_{\text{eff}} \Rightarrow$

$$\frac{z_{\text{vis}}}{z_{\text{hid}}} = \left(\frac{-\hat{\sigma}}{\hat{\lambda}} \right)^{-8/m^2}$$

\therefore LARGE ratio can be generated from modest $\frac{\hat{\sigma}}{\hat{\lambda}}, m^2!$

This ratio is Planck/Visible hierarchy!! i.e. accumulated

warp factor, $z \sim e^{ky}$
 RG approach v. powerful way to show stability of RS hierarchy under general (quantum) interaction

AdS/CFT CORRESPONDENCE (DUALITY)

Bulk AdS_5 dynamics described by 4D Fixed point RG is not coincidence:

5D Gravity + ... on AdS_5 is DUAL to 4D theory (without gravity), "the holographic dual", which is scale invariant, i.e. a conformal field theory.

Review: Aharony, Gubser, MALDACENA, Ooguri, Oz hep-th/9905111

also Witten hep-th/9802150.

"RS/CFT"

Take a look at

Arkani-Hamed, Porrati, Randall hep-th/0011081
0012148

Rattazi, Zaffaroni hep-th/0012248.

- 5D Gravity in RS dual to 4D CFT coupled to 4D gravity.
- CFT is strongly coupled & to be consistent with an unusual γ_{mn} "critical exponents/anomalous dimensions".
- Conformal invariance spontaneously broken at TeV scale.
- Radion dual to Goldstone of $U(1)$!
- KK modes & visible particles are composites of strong dynamics.

- Weakly coupled SD theory
 $\equiv \frac{1}{N_{\text{color}}}$ - like expansion of
 strong dynamics.
- RSI dual to type of
 composite Higgs theory.
- \exists many checks of all this.
- Warped effective field theory
 is a powerful approach to studying
 Nature's non-supersymmetric option
 above weak scale via compositeness.
 Direct attack is too hard.
- This is approach now being taken
 with RSI phenomenology applied to
 GUTs, precision tests, Flavor, ...