united nations
educational, scientific
and cultural
organization

international atomic
energy agency

the
abdus salam
international centre for theoretical physics

SMR.1508 - 32

SUMMER SCHOOL ON PARTICLE PHYSICS

16 June - 4 July 2003

ASTROPARTICLE PHYSICS

Lecture I

V. BEREZINSKY
I.N.F.N.
Laboratorio Nazionale del Gran Sasso (L.N.G.S.)
Assergi, L'Aquila
ITALY

COSMIC RAYS:

ACCELERATION AND PROPAGATION

book "ASTROPHYSICS OF COSMIC RAYS"

V.S. BEREZINSKY, S.V. BULANOV, V.A. DOGIEL, V.L. GINZBURG, V.S. PTUSKIN   ELSEVIER 1990

BASIC EXP DATA

- POWER-LAW SPECTRUM $F(E) \sim E^{-\gamma}$ ($\gamma \simeq 2.7$)
  UP TO $E_{knee} \sim 3 \cdot 10^{15}$ eV AND
  $F(E) \sim E^{-\gamma_1}$ ($\gamma_1 \simeq 3.0$) AT $E > E_{knee}$

  SOME STEEPENING AT $E = (6 \pm 2) \cdot 10^{17}$ eV:
  SECOND KNEE

- CHEMICAL COMPOSITION:
  P : He : (CNO) : (Ne, Mg, Si) : Fe = 1 : 0.72 : 0.33 : 0.29 : 0.47

- TOTAL FLUX AND ENERGY DENSITY

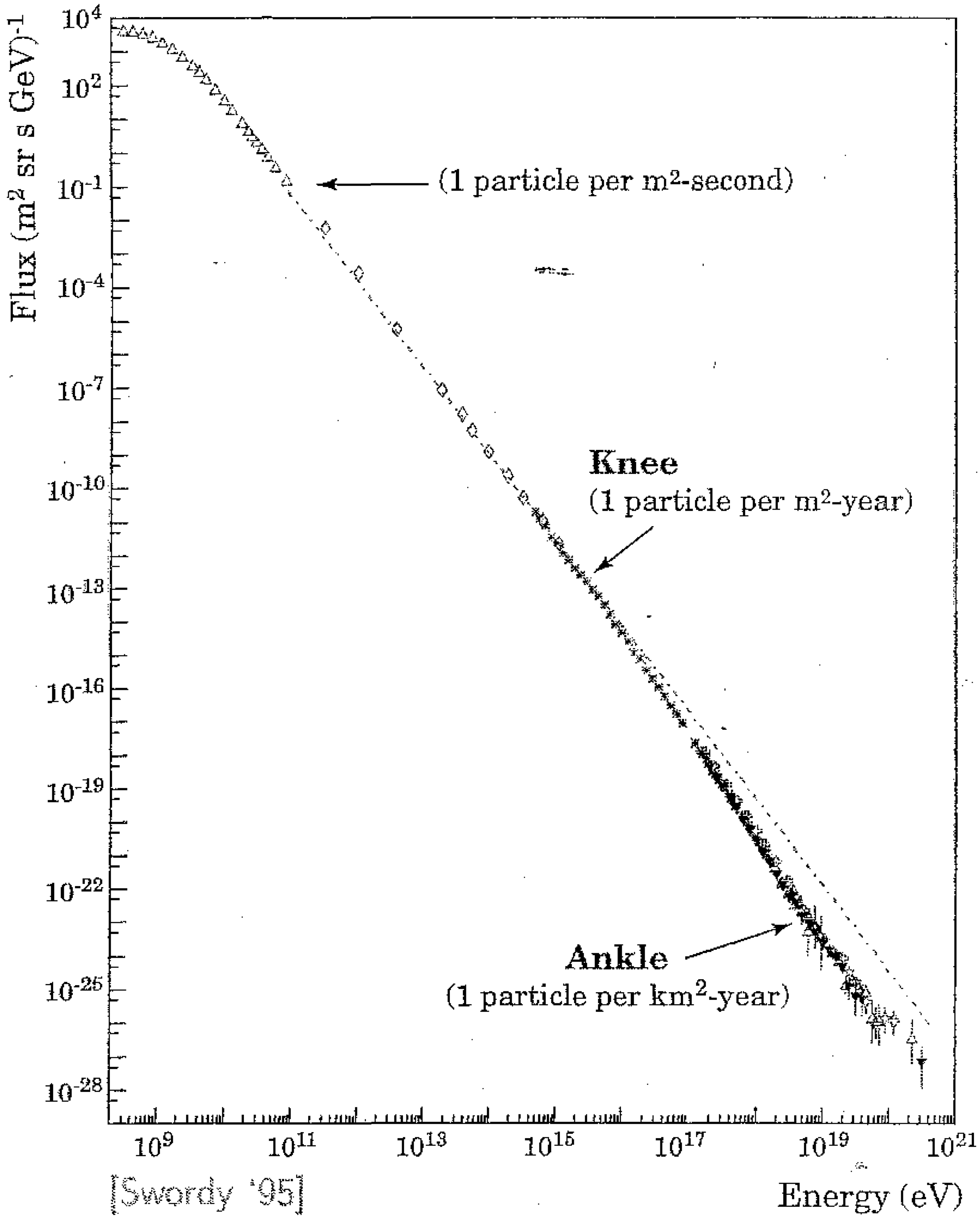
$$F_{CR}(>1\,\text{GeV}) \sim 1\ \text{cm}^{-2}\text{s}^{-1}\text{sr}^{-1}$$

$$\omega_{CR} \simeq 0.5\ \text{eV/cm}^3$$

$$\omega_e \sim 10^{-2}\omega_p$$

- ANISOTROPY

$$\delta = \frac{I_{max} - I_{min}}{\frac{1}{2}(I_{max} + I_{min})} \sim 10^{-3} \quad \text{AT } 10\text{-}100\ \text{GeV}$$



Flux (m² sr s GeV)⁻¹
10⁴
10²
10⁻¹
10⁻⁴
10⁻⁷
10⁻¹⁰
10⁻¹³
10⁻¹⁶
10⁻¹⁹
10⁻²²
10⁻²⁵
10⁻²⁸
(1 particle per m²-second)
Knee
(1 particle per m²-year)
Ankle
(1 particle per km²-year)
10⁹
10¹¹
10¹³
10¹⁵
10¹⁷
10¹⁹
10²¹
[Swordy '95]
Energy (eV)

PROPAGATION OF CR

BASIC MODE: DIFFUSION ON TURBULENT MAGNETIC INHOMOGENEITIS

$\theta \sim \frac{\lambda}{R_H}$ $\quad R_H = \frac{cp}{ZeH_\perp}$; $\omega_H = \frac{ZeH_\perp}{p}$

EXCITATION OF PLASMA IS DESCRIBED BY WAVES

$$\exp i(\omega t - kz)$$

SCATTERING OF PARTICLE OFF WAVES IS DOMINATED BY RESONANCE $\omega \sim \omega_H$ OR MORE PRECISILY

$$\omega - kv\cos\theta = \omega_H$$

SPECTRUM OF EXCITATIONS $W(k) \sim k^{-s}$

$s = 5/3$ (KOLMOGOROV), $s = 3/2$ (KRAICHNAN),

$s = 2$ (SHOCK WAVES: TOPTYGIN, BYKOV)

$W(k)$ IS ENERGY DENSITY PER UNIT WAVE NUMBER

KOLMOGOROV SPECTRUM OF TURBULENT EXCITATIONS:

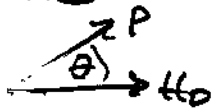
DECAYING WHIRLS WITH BASIC (LARGEST) SCALE $\lambda_0 = 2\pi/k_0$

$\lambda_0 \sim 100\,pc$ IN OUR GALAXY (OBSERVATIONS)

$$W(k) = \frac{s-1}{k_0}\left(\frac{k}{k_0}\right)^{-s} \frac{H_{e\perp}^2}{4\pi}$$

RATE (FREQUENCY) OF SCATTERING FOR PARTICLES WITH ω_H OFF THE EXCITATIONS

$$\nu(\cos\theta) = 2\pi^2 \omega_H \frac{K_{res} W(K_{res})}{H_0^2}$$



WITH $K_{res} \sim \frac{\omega_H}{v\cos\theta}$

DIFFUSION COEFFICIENT $D = \frac{1}{3} \ell_s v$, WHERE SCATTERING LENGTH $\ell_s = v/\nu(\cos\theta)$

$$\langle D \rangle = \frac{1}{3} v^2 \frac{H_0^2}{\omega_H} \int \frac{d\cos\theta}{K_{res} W(K_{res})}$$

$$\langle D \rangle = \frac{1}{3\pi^2} \frac{v\lambda_0}{s-1} \frac{H_0^2}{H_{e\perp}^2} \left(\frac{2\pi r_H}{\lambda_0}\right)^{2-s} \int \frac{d\cos\theta}{(\cos\theta)^{2-s}} \sim 3 \cdot 10^{28} \left(\frac{E/Z}{10\,\mathrm{GeV}}\right)^{1/3}$$

- D DEPENDS ON RIGIDITY $R \equiv E/Z$
- $D \sim R^{1/3}$ FITS RATIO OF SECONDARY/PRIMARY NUCLEI (Li, Be, B)/CNO
- ABSOLUTE VALUE $D \sim 10^{28} - 10^{29}$ cm²/s FITS CR AGE AND ANISOTROPY
- SPECTRUM $F(E) \sim Q_{gen}(E) \cdot T(E) \sim \frac{Q_{gen}(E)}{D(E)} \sim E^{-(\gamma_g + \frac{1}{3})}$

  NEEDS $\gamma_g = 2.4$. SHOCK ACCELERATION GIVES $\gamma_g = 2.0$

OPEN PROBLEMS OF PROPAGATION

- ROLE OF CONVECTION AND OTHER WAYS OF PROPAGATION

- D IN THE HOT, WARM AND COLD GAS.

  DIFFUSION IN THE HALO

  ROLE OF DIFFERENT EXCITATIONS IN PLASMA: MHD WAVES, SHOCK WAVES etc

  ROLE OF OTHER TYPES OF DIFFUSION: HALL DIFF, MOVEMENT OF MAGN FIELD LINES

  DIFFUSION TENSOR

  $$D_{ij} = D_{\parallel} e_i e_j + D_{\perp} \delta_{ij} + D_A \epsilon_{ijk} e_k$$

  where $\vec{e} = \vec{H}_0/H_0$, ϵ_{ijk} IS ANTISYMMETRIC TENSOR

  $D_A = \frac{1}{3} r_H V \sim E$ DESCRIBES THE HALL (DRIFT) DIFFUSION (THIS IS REGIME OF THE BOHM DIFFUSION $D \sim r_H c$)

  ROLE OF $D_{\perp}$

- BACKREACTION OF COSMIC RAYS

THE KNEE

RIGIDITY MODEL:

$$D(R) \sim \begin{cases} \sim R^{1/3} & \text{at } R \le R_c \\ \sim R^{n} & \text{at } R \ge R_c \end{cases}$$

IT IS CONFIRMED BY KASCADE

	P	He	C	Fe
E_{obs} (GeV)	$2.5 \cdot 10^6$	$5.5 \cdot 10^6$	$1.8 \cdot 10^7$	$7.0 \cdot 10^7$ (?)
$E = R_p Z$	$2.5 \cdot 10^6$	$5.0 \cdot 10^6$	$1.5 \cdot 10^7$	$6.5 \cdot 10^7$

PREDICTION OF STANDARD MODEL

$$\nu(\theta) = 2\pi^2 \omega_H \frac{k_{res} W(k_{res})}{H_0^2}$$

$$\langle \nu \rangle = \frac{\pi}{2} \omega_H \frac{s-1}{s} \frac{H_{e\perp}^2}{H_0^2} \left(\frac{\lambda_0}{2\pi r_H} \right)^{1-s}$$

AT $\nu \ll \omega_H$ PARTICLES ARE MAGNETIZED

$\nu \sim \omega_H$ IS CONDITION OF THE KNEE, i.e. $l_{sc} \sim r_H$, where $l_{sc} \sim V/\nu$)

FOR $s = \frac{5}{3}$, $H_{e\perp} \sim 3H_0$, $H_0 = 2 \cdot 10^{-6}$ G AND $\lambda_0 \sim 100$ pc

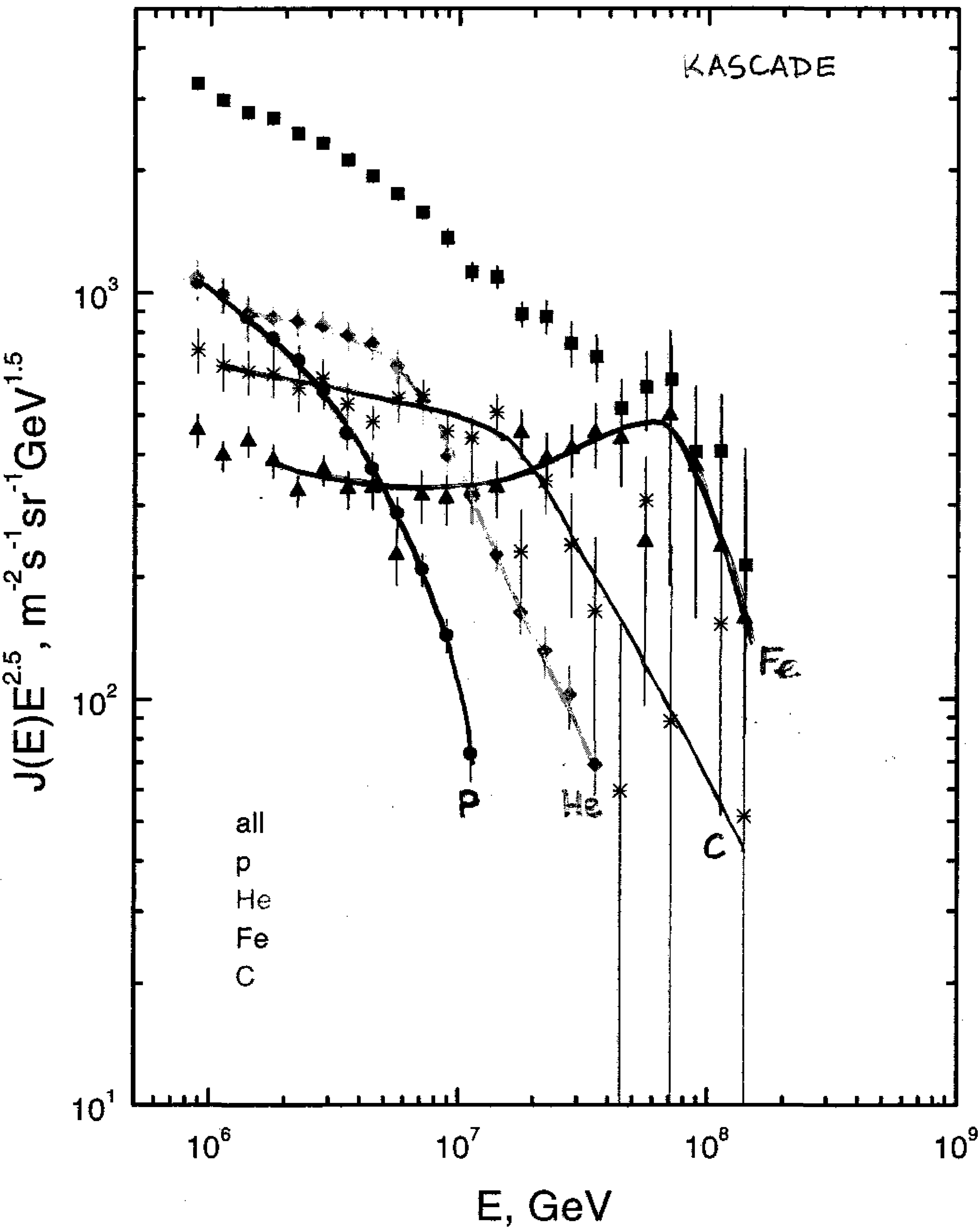
$$E_Z = 2 \cdot 10^6 Z \text{ GeV}$$

DIFFUSION ABOVE THE KNEE ($r_H \gg \lambda_0$)

 $\theta \sim \frac{\lambda_0}{r_H}$ $\quad \langle \theta^2 \rangle = \left(\frac{\lambda_0}{r_H}\right)^2 n = \left(\frac{\lambda_0}{r_H}\right)^2 \frac{r}{\lambda_0}$

AT $r \sim \lambda_{sc}$ $\langle \theta^2 \rangle \sim 1$, IT GIVES $l_{sc} \sim r_H^2 / \lambda_0$

$D \sim \frac{1}{3} l_{sc} V \sim \frac{1}{3} V \frac{r_H^2}{\lambda_0} \sim E^2$ FLUX $F(E) \sim \frac{Q_{gen}(E)}{D(E)} \sim E^{-(\gamma_g + 2)}$


KASCADE
$J(E)E^{2.5}$, $m^{-2}s^{-1}sr^{-1}GeV^{1.5}$
E, GeV
10^1
10^2
10^3
10^6
10^7
10^8
10^9
p
He
C
Fe
all
p
He
Fe
C

STATUS OF THE KNEE

- EXPERIMENTAL DATA ARE STILL INSUFFICIENT ESPECIALLY FOR HEAVY NUCLEI.
  RIGIDITY CHARACTER OF THE KNEES IS NOT RELIABLY ESTABLISHED.

- THE THEORETICAL MODEL OF THE KNEE IS NOT ESTABLISHED.
  IN PRINCIPLE THERE ARE THREE EXPLANATIONS

(i) RIGIDITY MODELS

STANDARD MODEL AND ONE WITH THE HALL DIFFUSION WHEN CONDITION OF THE KNEE $D_{\parallel}(E) \sim D_{A}(E)$

$$(D_{\parallel} \sim E^{1/3}, \; D_A \sim E)$$

(ii) ACCELERATION MODELS

WHERE $E_Z = E_{max}$ FOR A GIVEN SOURCE (TYPE OF SN)

DIFFERENT SOURCES (SN) ARE ENRICHED BY DIFFERENT NUCLEI

(iii) KNEE IS PRODUCED DUE TO INTERACTION OF NUCLEONS IN THE ATMOSPHERE.

ACCELERATION

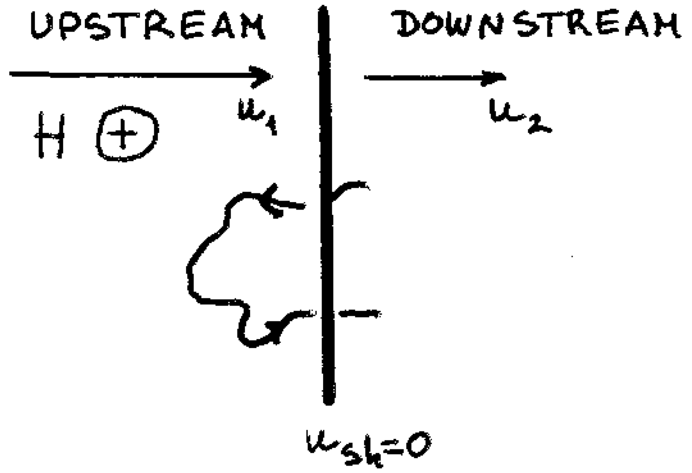
1. SHOCK ACCELERATION

PRINCIPLE AND MAXIMUM ENERGY

$\rho_1 u_1 = \rho_2 u_2$

$\frac{\rho_2}{\rho_1} \equiv \sigma$

$\sigma = 4$: strong shock



$$\frac{\Delta E}{E} \sim \frac{u_1}{c}$$ c is particle velocity

ACCELERATION TIME t_a

TIME OF ACCELERATION IS TIME OF DOWNSTREA ESCAPE

$$D_1 t_a \sim u_1^2 t_a^2$$

$$t_a \sim \frac{D_1}{u_1^2}$$

MORE ACCURATELY: $t_a = \frac{3}{u_1 - u_2}\left(\frac{D_1}{u_1} + \frac{D_2}{u_2}\right)$

MAXIMUM ENERGY

- ENERGY LOSSES $t_{en.\,loss} = \left(\frac{1}{E}\frac{dE}{dt}\right)^{-1}$: $t_a \sim t_{en.\,loss}$
- SHOCK WAVE IS SPHERICAL R_{sh}:

  UPSTREAM ESCAPE LIMITS THE ENERGY

$$t_a \sim t_{diff}$$

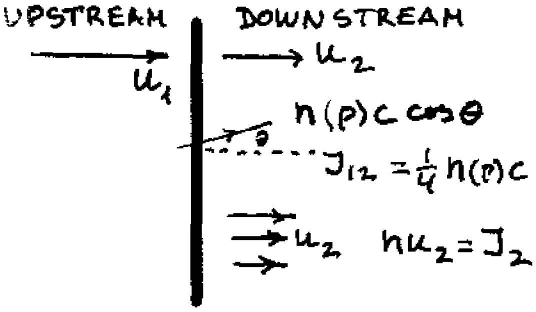
$$D_1 / u_1^2 \sim R_{sh}^2 / D_1$$

$$D_1 = R_{sh} u_1$$

$$\hookrightarrow r_H c \equiv (E/300H)c$$

$$E_{max} = 300 H R_{sh} \frac{u_1}{c} \text{ eV}$$

SHOCK ACCELERATION IN ELEMENTARY CONSIDERATION



KINEMATICS: $\langle \Delta p \rangle = \frac{4}{3} \frac{u_1 - u_2}{c} p$

INTEGRAL NUMBER OF ACC. PARTICLES

$$N(p) \equiv n(>p)$$

$$N(p + \Delta p) = P_{acc} N(p)$$

P_{acc} IS A PROBABILITY TO PERFORM A CYCLE OF ACCELERATION

$$N(p) + \Delta p \frac{dN(p)}{dp} = P_{acc} N(p)$$

$$\frac{dN}{dp} = \frac{P_{acc} - 1}{\langle \Delta p \rangle} N$$

$$P_{acc} = \frac{J_{12} - J_2}{J_{12}} = \frac{\frac{1}{4} nc - n u_2}{\frac{1}{4} nc} = 1 - 4 \frac{u_2}{c}$$

$$p \frac{dN}{dp} = - \frac{3 u_2}{u_1 - u_2} N$$

$$n(p) = - \frac{dN}{dp}$$

$$p n(p) = \frac{3 u_2}{u_1 - u_2} N(p)$$

$$\frac{d}{dp} (p n(p)) + \frac{3 u_2}{u_1 - u_2} n(p) = 0$$

$$n(p) \sim p^{-\gamma} \quad \gamma = \frac{\sigma + 2}{\sigma - 1} = 2 \quad \text{at } \sigma \equiv \frac{u_1}{u_2} = 4 \text{ (strong shock)}$$

THIS MECHANISM CAN BE INTERPRETED AS FERMI I BETWEEN TWO "CLOUDS": FIRST CLOUD IS UPSTREAM REGION FROM WHICH A PARTICLE IS REFLECTED WITH PROBABILITY $P = 1$ AND THE SECOND IS DOWNSTREAM REGION FROM WHICH A PARTICLE IS REFLECTED WITH $P = 1 - 4 \frac{u_2}{c}$.

EXACT SOLUTION

$f(p)$ IS A DENSITY IN MOMENTUM SPACE

$$N(p)\,dp = f(p)\, 4\pi p^2\, dp$$

$$\frac{\partial f(\vec{p},t)}{\partial t} = \underbrace{\mathrm{div}\left(D\, \mathrm{grad}\, f(p,t)\right)}_{\text{diffusion}} \underbrace{- \mathrm{div}(\vec{u} f)}_{\text{convection}} + \underbrace{\frac{1}{3}\, \mathrm{div}\, \vec{u}\, \frac{1}{p^2} \frac{\partial}{\partial p}(p^2 f)}_{\text{adiabatic energy losses}} + Q$$

strong shock: $N(p) \sim p^{-2}$

SN SHOCKS

- CR IN OUR GALAXY ARE ACCELERATED BY SN SHOCKS ($L_{CR} \sim 3\cdot 10^{40}$ erg/s CAN BE PROVIDED BY THESE SHOCKS)

- SHOCKS IN ISM CANNOT PROVIDE OBSERVED E_{max}

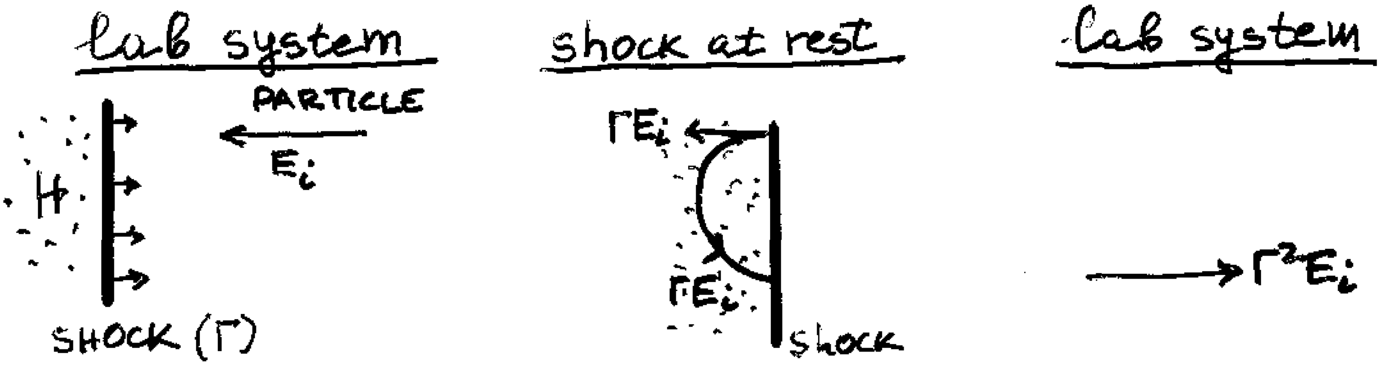
$$E_{max} = 300 H R_{sh} \frac{u}{c} \lesssim 10^{15} eV$$

- SN EXPLOSIONS IN STELLAR WIND OR TURBULENCE IN YOUNG SN REMNANT CAN INCREASE THIS ENERGY UP TO $E_{max} \sim 10^{17}$ eV FOR PROTONS

ULTRA RELATIVISTIC SHOCKS

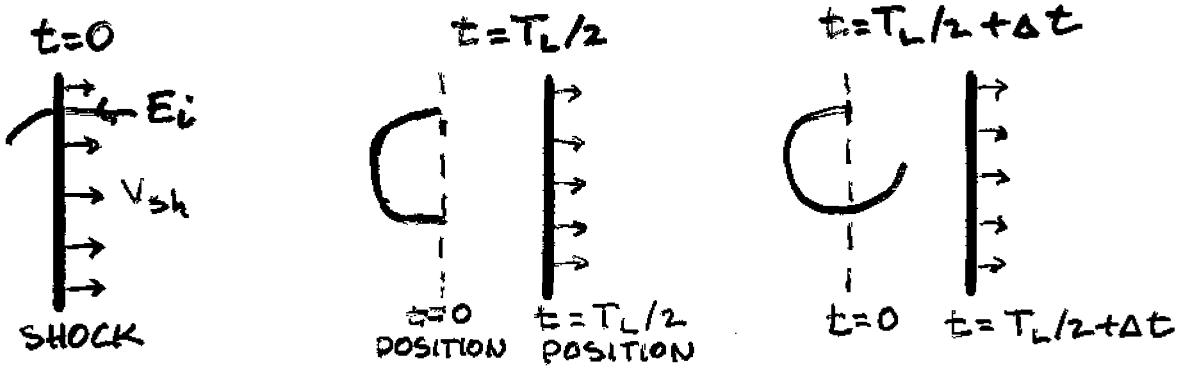
(Γ^2-ACCELERATION)

BASIC IDEA



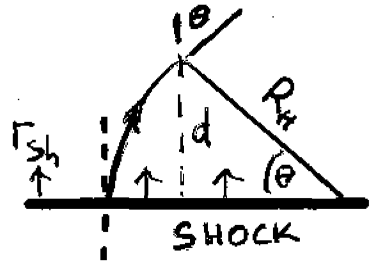
PROBLEMS WITH Γ^2-ACCELERATION

- CAPTURING BEHIND THE SHOCK



ACCELERATED PARTICLE IS CAPTURED BEHIND THE SHOCK

- ESCAPING PARTICLES



LENGTH OF PROTON TRAJECTORY

$$\ell_p = R_H\,\theta$$

DISTANCE TRAVERSED BY SHOCK

$$\ell_{sh} = R_H \sin\theta$$

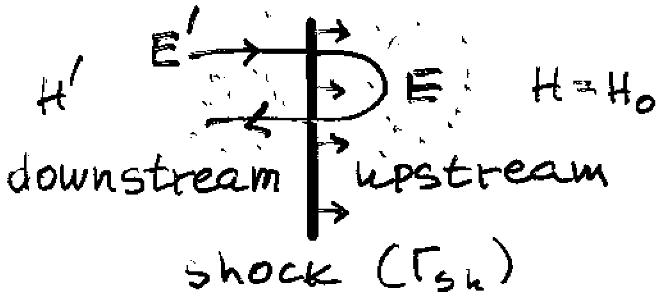
SHOCK CATCHES UP THE PARTICLE WHEN $\ell_p = \ell_{sh}$

$$\ell_p / v_p = \ell_{sh} / v_{sh}, \text{ or } \theta = \sqrt{3}/\Gamma_{sh}$$

$$t \sim \frac{R_H\,\theta}{c} \sim \frac{E}{ecH\,\Gamma_{sh}}$$

ONE-LOOP REGIME

Gallant, Achterberg 2000



E_{max} FROM CONDITION: $t_{acc} \leq t_{age}$

$$t_{age} = R_{sh}/c$$

$$t_{acc} = t_d + t_u$$

- **UPSTREAM**

$$t_u = \frac{E}{ecH_\perp \Gamma_{sh}}$$

- **DOWNSTREAM**

$$t'_d \sim \frac{R'_H}{c} = \frac{E'}{ecH'} = \frac{E'}{ec\Gamma_{sh}H_\perp} = \frac{E/\Gamma_{sh}}{ec\Gamma_{sh}H_\perp}$$

$H'_\perp = \Gamma_{sh}H_\perp$ $\qquad$ $E \sim \Gamma_{sh}E'$

$$t_d = \Gamma_{sh}t'_d = \frac{E}{ecH_\perp\Gamma_{sh}} = t_u$$

$$E_{max} \sim eH_0R_{sh}\Gamma_{sh}$$

FOR GRB: $H_0 \sim 1\,\mu G$, $R_{sh} \sim 10^{16}\,cm$, $\Gamma_{sh} \sim 300$

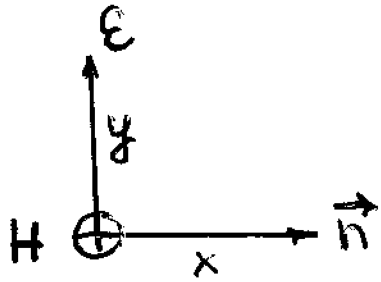
$$E_{max} \sim 10^{15}\,eV$$

POSSIBILITY: REACCELERATION

ACCELERATION IN THE STRONG E-M WAVES

(THE GUNN-OSTRIKER MECHANISM)

$\mathcal{E} = H_\sim \sin \Omega t$  $\dot{\vec{P}}_0 = 0$



$$\frac{d\vec{P}}{dt} = e\vec{\mathcal{E}} + e\vec{v}\times\vec{H} \qquad \dot{P}_x = eHV_y$$

$$\frac{dE_{kin}}{dt} = e\vec{\mathcal{E}}\vec{v} \qquad \dot{E}_{kin} = eHV_y$$

$$\Big\} \; E_{kin} = P_x$$

$$P_x^2 + P_y^2 + m^2 = (E_{kin}+m)^2$$

$$P_y = \sqrt{2mP_x} \qquad V_y = \sqrt{2V_x/\Gamma} \qquad \Gamma \text{ is Lorentz-factor}$$

$$V_x \to 1$$

FROM $\dot{E}_{kin} = eHV_y$: $E_{kin} = eHV_y t = \frac{1}{\sqrt{\Gamma}}\, eHt$

$E_{kin} \simeq \Gamma m$: $\quad \Gamma^{3/2} \simeq \frac{eH}{mc}\, t$

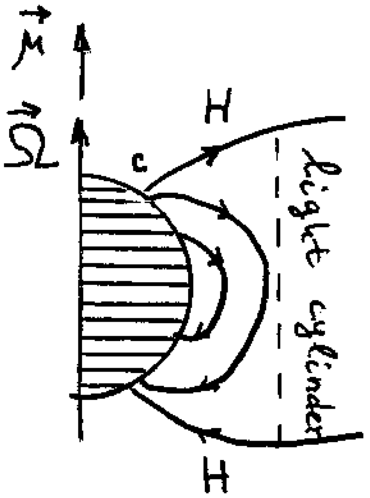
ENERGY OBTAINED DURING $t \sim 1/\Omega$

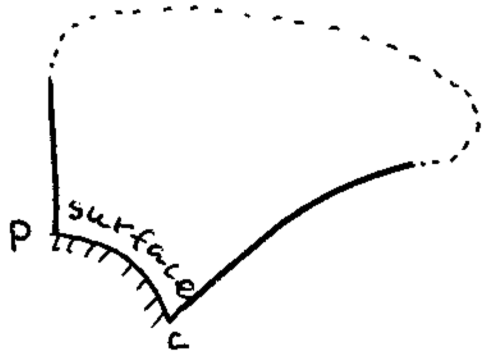
$$\Gamma \sim \left(\frac{\omega_H}{\Omega}\right)^{2/3}$$

IN THE STRONG (LARGE H) LOW-FREQUENCY (SMALL Ω) WAVE Γ IS VERY LARGE.

UNIPOLAR INDUCTION

PULSAR





$$\varphi_c - \varphi_p = \int_c^p \vec{\mathcal{E}}\, d\vec{l} = -\int \frac{1}{c}(\vec{V}\times\vec{H})\, d\vec{l} = \frac{\Omega\mu}{cR_s}\left(\sin^2\theta_c - \sin^2\theta_p\right)$$

$$\vec{V} = \vec{\Omega}\times\vec{r} \qquad \vec{H} = 3\mu\cos\theta\,\frac{\vec{n}}{r^3} - \frac{\vec{\mu}}{r^3} \qquad \sin^2\theta_c = \frac{\Omega R_s}{c}$$

$$\vec{n} = \vec{r}/r$$

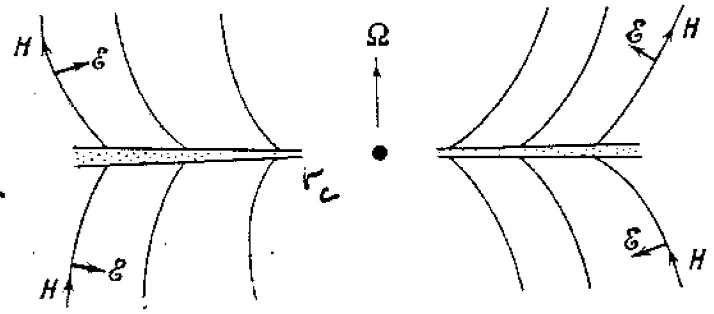
$$\varphi_c - \varphi_p = \frac{1}{2c^2}\Omega^2 H_s R_s^3 = 6.6\cdot 10^{17}\left(\frac{10\,\mathrm{ms}}{T}\right)^2 \frac{H_s}{10^{13}} \ \mathrm{V}$$

$$L \sim \frac{1}{c^3} H_s^2 \Omega^4 R^6 = 5.8\cdot 10^{41}\left(\frac{P_s}{10\,\mathrm{ms}}\right)^{-4}\left(\frac{H_s}{10^{13}\,\mathrm{G}}\right)^2 \ \mathrm{erg/s}$$

UNIPOLAR INDUCTION IN ACCRETION DISCS

Blandford MN 176, 465, 1976

Lovelace, Nature 262, 649, 1976



$$r_c = 6\,GM_h/c^2 = 9\cdot 10^5 \frac{M_h}{M_\odot}$$ last stable orbit

$$V_\varphi(r) = \left(G_N M_h / r\right)^{1/2}$$

$$H_z(r) = H_c\left(\frac{r}{r_c}\right)^{-n} \qquad n = \tfrac{1}{2}\,,\ 1$$

$$\varphi_c = \frac{1}{c}\int_{r_c}^{R} dr\, V_\varphi(r)\, H_z(r) = \frac{1}{\sqrt{6}} H_c r_c \ln\frac{R}{r_c}$$

$$M \sim 10^8 M_\odot\,,\quad H_c \sim 10^3\,G\,,\quad \varphi \sim 10^{20}\,V$$

$$L \sim 2J\varphi_c = 2\kappa c\varphi_c^2 \sim 10^{44}\left(\frac{H_c}{10^3}\right)^2\left(\frac{M_h}{10^8 M_\odot}\right)^2 \ln^2\frac{R}{r_c}\ \frac{\text{erg}}{\text{s}}$$