

***SUMMER SCHOOL ON PARTICLE PHYSICS***

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**GRAND UNIFICATION AND FERMION MASSES**

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# Grand Unification and Fermion Masses

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## Hints in favor of GUTs:

(1) Remarkable fit of SM quark and lepton multiplets into multiplets of  $SU(5)$ ,  $SO(10)$

(2) Unification of gauge couplings  
at scale  $\approx 2 \times 10^{16}$  GeV =  $M_G$   
(in the MSSM).

(3) Neutrino masses

(a)  $\nu_R$  predicted by Pati-Salam,  $SO(10)$

(b)  $M_R$  as inferred from see-saw  
formula  $\approx M_G$ .

(4) Hints from  $q, l$  masses, mixings

(a)  $m_b^0 \approx m_\tau^0$ , Similar  $q, l$  hierarchies

(b)  $\theta_{CKM} \ll 1$  ( $\theta_{CKM} = 0$  in minimal  $SO(10)$ )

(c) Relationships among  $V_{cb}$ ,  $m_s/m_b$ ,  
 $V_{\mu 3}$  etc (see lopsided models)

# Areas of research

- → Gauge hierarchy problem,  
     ↳  $\frac{2}{3}$  splitting problem,  
     ↳  $d=5$  proton decay operators
- ↳ • Leptogenesis/baryogenesis
- ↳ • Neutrino masses and mixings
- ↳ • Quark and lepton masses and mixings (the flavor problem)
- ↳ • Flavor changing processes  
     (  $\mu \rightarrow e\gamma$ ,  $\tau \rightarrow \mu\gamma$  etc )
- Other low energy tests ?
- Higher dimensions ( $d=5, 6$  brane models)
- Strings ?

(iii)

My goal in these lectures is to explain the basic

- Schemes of unification

(Pati-Salam,  $SU(5)$ ,  $SO(10)$ , flipped  $SU(5)$ , ...)

- issues and problems

(gauge hierarchy, doublet-triplet splitting,  $d=5$  proton decay operators, quark and lepton masses.)

and ● practical group theory

# Outline

I.  $SU(5)$  unification

long { II.  $SO(10)$  and other groups  
for unification  
(Pati-Salam, flipped  $SU(5)$ , ...)

III. The gauge hierarchy problem

short { IV. Fermion Masses

# LECTURE I SU(5) UNIFICATION

## A. Basic Group Theory Facts

Group  $G = \{g\}$

- closure  $g_1 g_2 = g_3$
- identity  $eg = ge = g$
- inverse  $gg^{-1} = g^{-1}g = e$
- assoc.  $g_1(g_2g_3) = (g_1g_2)g_3$

### Lie groups

"Represent" elements by unitary matrices

$$U(g) = e^{i \sum_a \theta^a \lambda^a}$$

↑ generators  
↑ parameters associated with element g.

generators  $\lambda^a$  are hermitian matrices, satisfying the group algebra

$$[\lambda^a, \lambda^b] = i f^{abc} \lambda^c$$

↑ structure constants

Number of generators = dim. of group.

If  $U$  is a  $d \times d$  matrix representing an element of the group, it can be thought of as acting on a  $d$ -component object

$$[U] \begin{pmatrix} T_1' \\ T_2' \\ \vdots \\ T_d' \end{pmatrix} = \begin{pmatrix} T_1' \\ T_2' \\ \vdots \\ T_d' \end{pmatrix}$$

Strictly speaking, the  $U$  matrices form a "representation" of the group elements. But physicists often talk of the  $d$ -component multiplet as being a "representation".

Recall: always exists an adjoint representation 1.2

$d \times d$  matrices, where  $d = \text{dimension of group}$   
 $= \# \text{ of generators}$

generators in the adjoint representation are a set of  $d$  hermitian  $d \times d$  matrices

→ These act on  $d$ -component multiplets, which are called "adjoint representations" or "adjoint multiplets"

Proof: Jacobi identity

$$0 = [[\lambda^a, \lambda^b], \lambda^c] + [[\lambda^c, \lambda^a], \lambda^b] + [[\lambda^b, \lambda^c], \lambda^a]$$

$$0 = f^{abd} f^{dce} + f^{cad} f^{dbe} + f^{bcd} f^{dae}$$

$$\text{Call } f^{abd} \equiv i(\lambda^a)^{bd}$$

$$0 = i f^{abd} (\lambda^d)^{ce} - (\lambda^a)^{cd} (\lambda^b)^{de} + (\lambda^b)^{cd} (\lambda^a)^{de}$$

which is group algebra!

## B) SU(N) GROUP THEORY

$$\left\{ \begin{array}{l} N \\ \bar{N} \end{array} \right. \begin{array}{l} T^\alpha \rightarrow T^{\alpha'} = U^{\alpha'}_\alpha T^\alpha \\ T_\alpha \rightarrow T_{\alpha'} = U_{\alpha'}^\alpha T_\alpha \end{array} \quad \alpha=1, \dots, N$$

These multiplets are called "fundamental rep.", "anti fundamental rep."

$$T_\alpha = (T^\alpha)^\dagger, \quad U_{\alpha'}^\alpha = (U^{\alpha'}_\alpha)^\dagger$$

$$\sum_{\alpha=1}^N T^\alpha T_\alpha = \text{invariant} \Rightarrow U U^\dagger = I = U^\dagger U$$

$U = N \times N$  unitary matrices,  $\det U = 1$



$N \times N$ , unitary,  $\det = 1$  matrices represent the elements of group  $SU(N)$

Such matrices can be written in terms of  $N^2 - 1$  parameters

$$U_{\alpha}^{\alpha'} = \left( e^{i \sum_{a=1}^{N^2-1} \theta^a \lambda^a} \right)_{\alpha}^{\alpha'} \quad \begin{matrix} a=1, \dots, N^2-1 \\ \alpha, \alpha'=1, \dots, N \end{matrix}$$

$N^2 - 1$  generators. In this fundamental representation they are  $N \times N$  traceless hermitian matrices.

[ I will denote "adjoint indices" by  $a, b, c, \dots$   
"fundamental indices" of  $SU(N)$  by  $\alpha, \beta, \gamma, \dots$  ]

Adjoint rep.

For  $SU(N)$  there exist  $(N^2 - 1)$ -component multiplets  $T^a$  that get acted upon by  $(N^2 - 1) \times (N^2 - 1)$  unitary matrices. [Adjoint rep.]

$$T^{\alpha}_{\beta} \equiv \sum_{a=1}^{N^2-1} T^a (\lambda^a)^{\alpha}_{\beta}$$

Two ways of writing this multiplet  
 $\alpha, \beta = 1, \dots, N$  ;  $a = 1, \dots, N^2 - 1$

$$T^{\alpha}_{\beta} \rightarrow T^{\alpha'}_{\beta'} = U^{\alpha'}_{\alpha} \times U_{\beta'}^{\beta} T^{\alpha}_{\beta}$$

$$T \rightarrow U T U^{\dagger}$$

[NOTE: CAN ALSO DENOTE  $\lambda^a$  BY  $\lambda^{\alpha}_{\beta}$   
eg. IN  $SU(3)$   $\lambda^6 + i\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \equiv \lambda^2_3$  ]

Let a Higgs field be in such an adjoint representation (i.e. multiplet) of an  $SU(N)$  gauge sym.

$$H^\alpha_\beta = \sum_{a=1}^{N^2-1} H^a (\lambda^a)^\alpha_\beta$$

$$\begin{aligned} \langle H \rangle \rightarrow U \langle H \rangle U^\dagger &= e^{i \sum_b \theta^b \lambda^b} \langle H \rangle e^{-i \sum_c \theta^c \lambda^c} \\ &= \langle H \rangle + i \sum_b \theta^b [\lambda^b, \langle H \rangle] + \dots \end{aligned}$$

$\langle H \rangle$  is left invariant by "unbroken generators"

$$\Rightarrow \begin{cases} \text{Unbroken generators: } [\lambda^a, \langle H \rangle] = 0 \\ \text{Broken generators: } [\lambda^a, \langle H \rangle] \neq 0. \end{cases}$$

c)  $SU(5)$  multiplets and symmetry breaking

$$\left\{ \begin{array}{l} 5 \\ \bar{5} \end{array} \right. \psi^\alpha = \begin{pmatrix} \psi^1 \\ \psi^2 \\ \hline \psi^3 \\ \psi^4 \\ \psi^5 \end{pmatrix} \left\{ \begin{array}{l} \psi^i \quad i=1,2 \quad SU(2)_L \\ \psi^a \quad a=3,4,5 \quad SU(3)_C \end{array} \right.$$

24 generators =  $5 \times 5$  traceless, hermitian matrices (in fundamental rep.)

$$(\lambda^a)^\alpha_\beta = \begin{bmatrix} SU(2) & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & SU(3) \end{bmatrix}, \begin{bmatrix} 0 & m \\ m^\dagger & 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix} \equiv \begin{matrix} (1, 3, 0) \\ (8, 1, 0) \\ (3, 2, \frac{1}{6}) + h.c. \\ (1, 1, 0) \end{matrix}$$

Now, let there be a Higgs in an adjoint multiplet of SU(5), and let its VEV be

$$\langle \Omega \rangle = \begin{bmatrix} 1/2 & & & & \\ & 1/2 & & & \\ & & -1/3 & & \\ & & & -1/3 & \\ & & & & -1/3 \end{bmatrix} \omega = \left(\frac{Y}{2}\right) \omega$$

$\left\langle \lambda^a \right\rangle$

[Note: VEVs of adjoint Higgs multiplets always point in direction of some generator (or linear combination of gens.)

Which generators are unbroken then?

$$[\lambda^a, \langle \Omega \rangle] = 0$$

Unbroken generators are those of

$$SU(3)_c \times SU(2)_L \times U(1)_Y = G_{SM}$$

Embedding of SM multiplets in SU(5)  $\longrightarrow$

$$\bar{5}_L = \psi_L = \begin{pmatrix} \psi_i \\ \psi_a \end{pmatrix} = \begin{pmatrix} (1, 2, -1/2) \\ (\bar{3}, 1, 1/3) \end{pmatrix} \equiv \begin{pmatrix} \bar{2} \\ 3 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} e_L^- \\ \nu_L \end{pmatrix} \\ d_L^c \end{pmatrix} \text{ etc}$$

$\uparrow$  my notation

$$10_L = \psi[\alpha\beta] = \begin{bmatrix} \psi[12] & \psi^{ia} \\ \psi_{ai} & \psi^{ab} = \epsilon^{abcd} \psi_d \end{bmatrix} = \begin{bmatrix} (1, 1, +1) & (\bar{3}, 2, 1/6) \\ & (\bar{3}, 1, -2/3) \end{bmatrix} = \begin{bmatrix} e_L^+ & (u_L^c) \\ & d_L^c \end{bmatrix}$$

$$24_g = A^a = \begin{bmatrix} A^i_j & A^i_a \\ A^i_a & A^a_b \end{bmatrix} = \begin{bmatrix} (1, 3, 0) & (\bar{3}, 2, 5/6) \\ (3, 2, 5/6) & (8, 1, 0) \\ & + (1, 1, 0) \end{bmatrix}$$

$A^a(A^b)^c = A^c$

Key point:  $\psi_L(r) \xleftrightarrow{C} \psi_R(\bar{r})$  1.6

SM family (1st family)

$$(1, 2, -\frac{1}{2})_L = \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix} \equiv L$$

$$(1, 1, -1)_R = e_R^- \quad \xleftrightarrow{C} \quad (1, 1, +1)_L = e_L^+$$

$$(1, 1, 0)_R = N_R \quad \xleftrightarrow{C} \quad (1, 1, 0)_L = N_L^c$$

$$(3, 2, \frac{1}{6})_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \equiv Q$$

$$(3, 1, \frac{2}{3})_R = u_R \quad \xleftrightarrow{C} \quad (\bar{3}, 1, -\frac{2}{3}) = u_L^c$$

$$(3, 1, -\frac{1}{3})_R = d_R \quad \xleftrightarrow{C} \quad (\bar{3}, 1, \frac{1}{3}) = d_L^c$$

Two ways to write a mass term (or Yukawa term)

$$m \overline{\psi_R(r_2)} \psi_L(r_1) = m [\psi_R(r_2)]^\dagger \gamma^0 \psi_L(r_1)$$

← DIRAC BAR

↑ =  $\psi$

or  $\psi$

$$\parallel$$

$$m \psi_L(\bar{r}_2)^T C \psi_L(r_1)$$

More convenient for GUTs (also SUSY!)

eg electron mass:

$$m(\overline{e_R^-}) e_L^- = m(e_L^+)^T C e_L^-$$

often do not bother to show



Arrows follow flow of "left-handedness"

Think of chirality as a charge  $\chi(L) = -1$ ,  $\chi(R) = +1$

$$\Delta\chi = 2$$

## (D) Fermion Masses

$$\left\{ \begin{aligned} \text{FAMILY} &= 10_L + \bar{5}_L + (1_L) \\ &\quad \parallel \\ &(\psi^{12}, \psi^{1a}, \psi^{ab})_L + (\psi_i, \psi_a)_L + \psi_L \\ &\quad \parallel \\ &(e_L^+, (u, d)_L, u_L^c) + ((\nu_{e^-}), d_L^c) + N_L^c \end{aligned} \right.$$

$$\left\{ \begin{aligned} \text{HIGGS ARE IN } &5_H + \bar{5}_H \\ &\quad \parallel \\ &(H^i, H^a) + (H_i, H_a) \\ &(H_u, H_{uc}) + (H_d, H_{dc}) \\ \text{[In non-SUSY: } &\bar{5}_H = (5_H)^* \text{ i.e. } H_{\alpha} = (H^{\alpha})^* \\ \text{In SUSY } &5_H \text{ and } \bar{5}_H \text{ are distinct, L-H superfields.]} \end{aligned} \right.$$

### YUKAWA TERMS (minimal SU(5))

$$\left\{ \begin{aligned} (Y_u)_{AB} (10_{LA} 10_{LB}) \langle 5_H \rangle &= (Y_u)_{AB} \epsilon_{\alpha\beta\gamma\delta} (\psi_A^{\alpha\beta} \psi_B^{\gamma\delta} \langle H^{\epsilon} \rangle) \\ &= (Y_u)_{AB} (\psi_A^{1a} \psi_B^{bc} \langle H^2 \rangle) = (Y_u)_{AB} u_A u_B^c \langle H_u \rangle \end{aligned} \right.$$

NOTE:  $Y_u = Y_u^T \Rightarrow \boxed{M_u = M_u^T \propto Y_u}$

$$\left\{ \begin{aligned} (Y_D)_{AB} (10_{LA} \bar{5}_{LB}) \langle \bar{5}_H \rangle &= (Y_D)_{AB} (\psi_A^{\alpha\beta} \psi_{B\alpha} \langle H_{\beta} \rangle) \\ &= (Y_D)_{AB} (\psi_A^{12} \psi_{B1} + \psi_A^{a2} \psi_{Ba}) \langle H_2 \rangle \\ &= (Y_D)_{AB} (e_A^+ e_B^- + d_A d_B^c) \langle H_d \rangle \end{aligned} \right.$$

NOTE:  $\boxed{M_D = M_D^T \propto Y_D}$

$$M_D = M_L^T \quad (\text{in minimal SU(5)}) \quad 1.8$$

$$\Rightarrow \begin{cases} m_b^0 = m_\tau^0 & (\text{Good within 20\% in SUSY GUTs}) \\ m_s^0 = m_\mu^0 & (m_s^0 \lesssim \frac{1}{3} m_\mu^0, \text{ from SUSY GUTs RGE}) \\ m_d^0 = m_e^0 & (m_d^0 \approx 3 m_e^0, \text{ " " " "}) \end{cases}$$

If there exist  $N_L^c$  (Right-handed  $\nu$ 's):

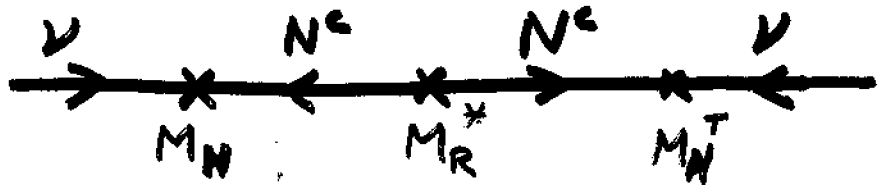
$$\begin{cases} (Y_N)_{AB} (\bar{5}_{LA} 1_{LB}) \langle S_H \rangle = (Y_N)_{AB} (\psi_{A1} \psi_B) \langle H^c \rangle \\ = (Y_N)_{AB} (\psi_{A2} \psi_B) \langle H^2 \rangle = (Y_N)_{AB} \nu_A N_B^c \langle H_u \rangle \\ \Rightarrow \boxed{M_N \propto Y_N} \text{ (}\nu \text{ Dirac mass matrix)} \approx V_u \end{cases}$$

$$\begin{cases} (M_R)_{AB} (1_{LA} 1_{LB}) = (M_R)_{AB} (N_A^c N_B^c) \\ \boxed{M_R} \text{ (}\nu_R \text{ Majorana mass matrix)} \approx M_G \end{cases}$$

$$\Rightarrow (\nu \ N^c)^T \begin{pmatrix} 0 & M_N \\ M_N^T & M_R \end{pmatrix} \begin{pmatrix} \nu \\ N^c \end{pmatrix} \rightarrow (\nu \ N^c)^T \begin{pmatrix} M_\nu & 0 \\ 0 & M_R \end{pmatrix} \begin{pmatrix} \nu \\ N^c \end{pmatrix}$$

$$\boxed{M_\nu \approx -M_N M_R^{-1} M_N^T} \quad \text{SEE-SAW FORMULA}$$

DIAGRAMMATICALLY:



$$\begin{aligned} M_\nu &\approx M_N \frac{1}{E + M_R} M_R^* \frac{1}{E + M_R} M_N^T \\ &\approx M_N M_R^{-1} M_N^T \end{aligned}$$

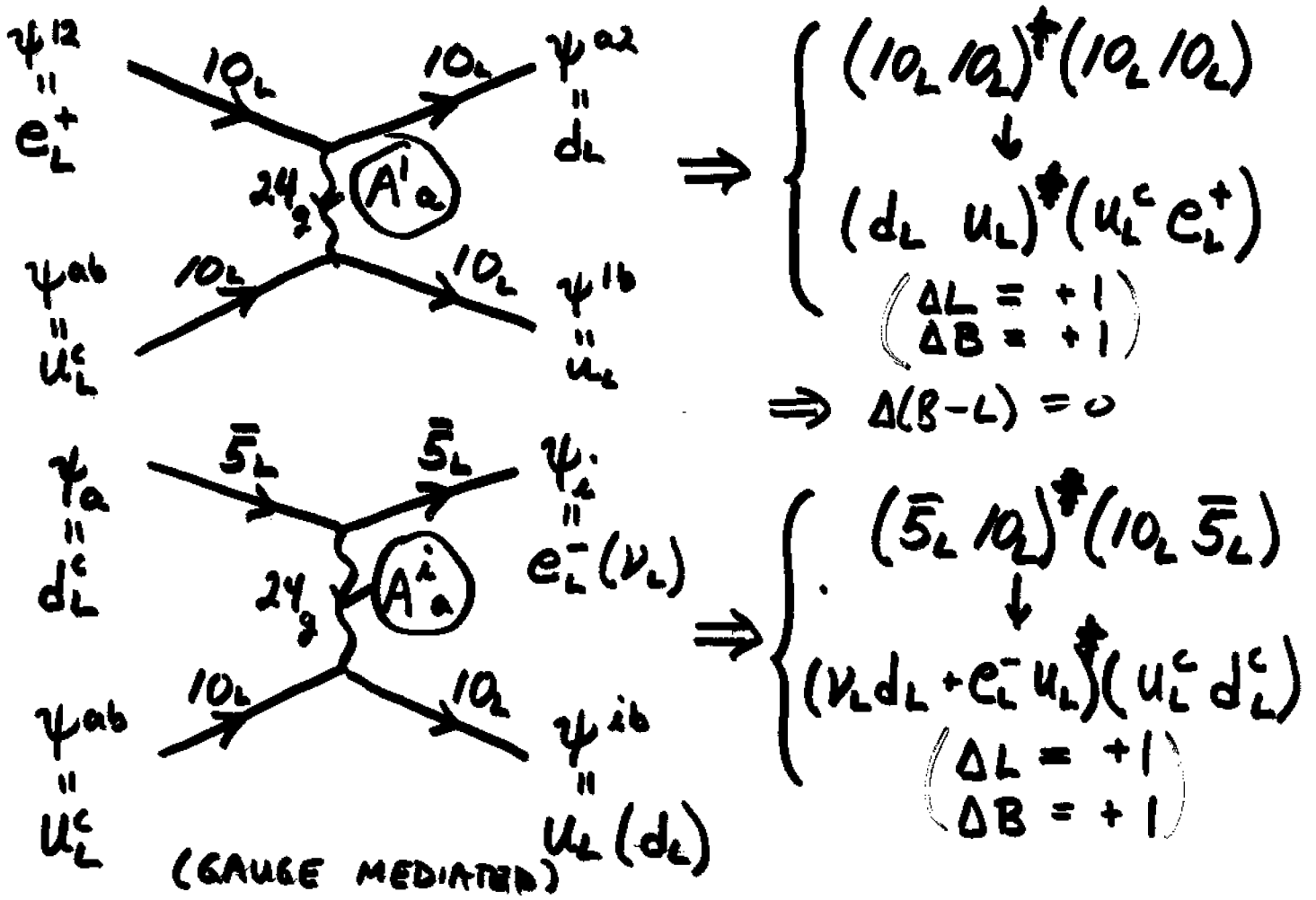
So, in minimal  $SU(5)$  there are four independent mass matrices for  $q$  and  $l$ :

$$M_D = M_L^T, M_u, M_N, M_R$$

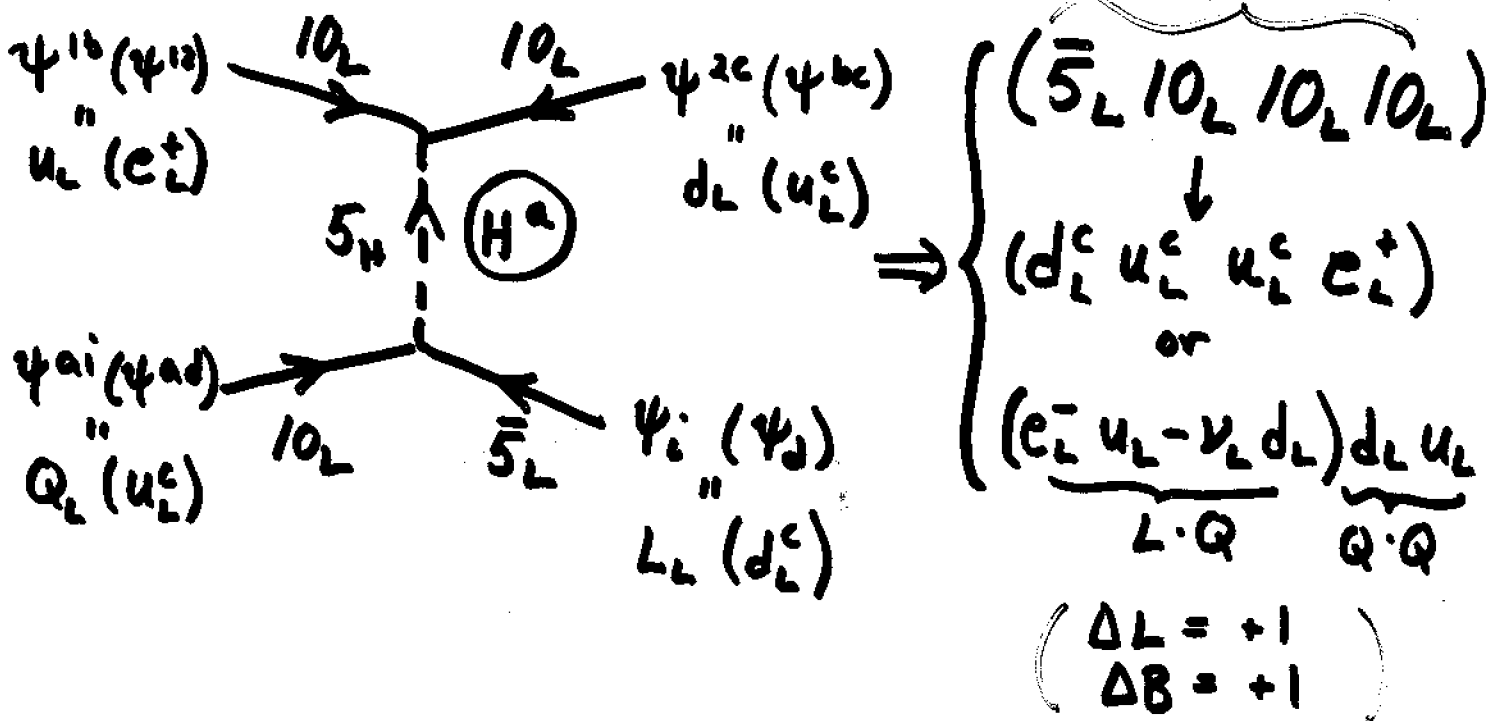
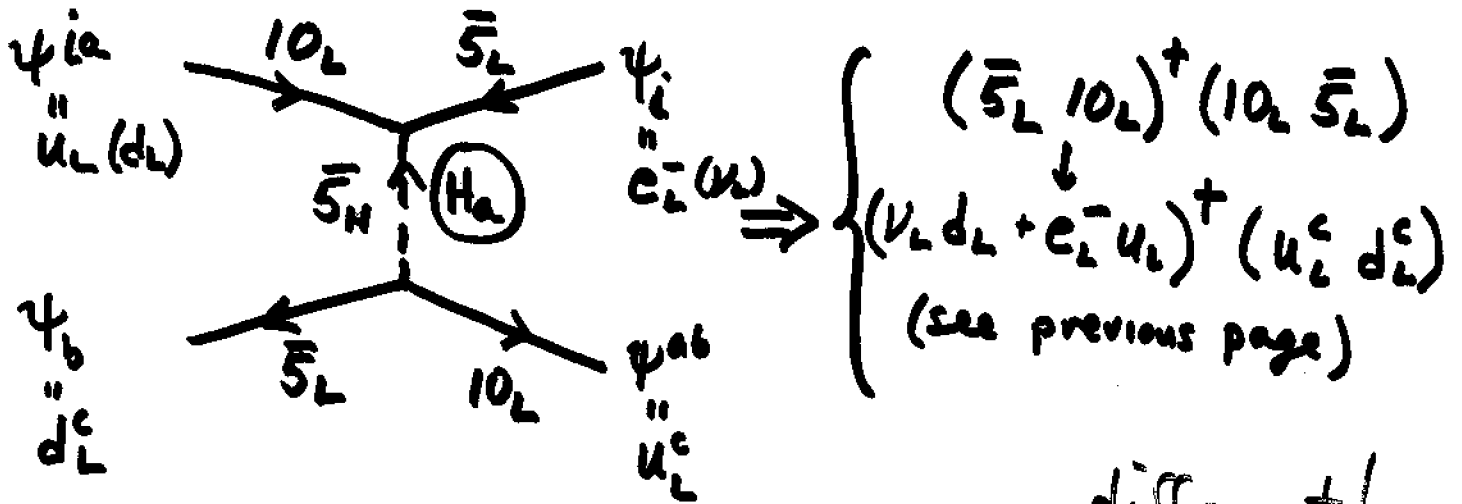
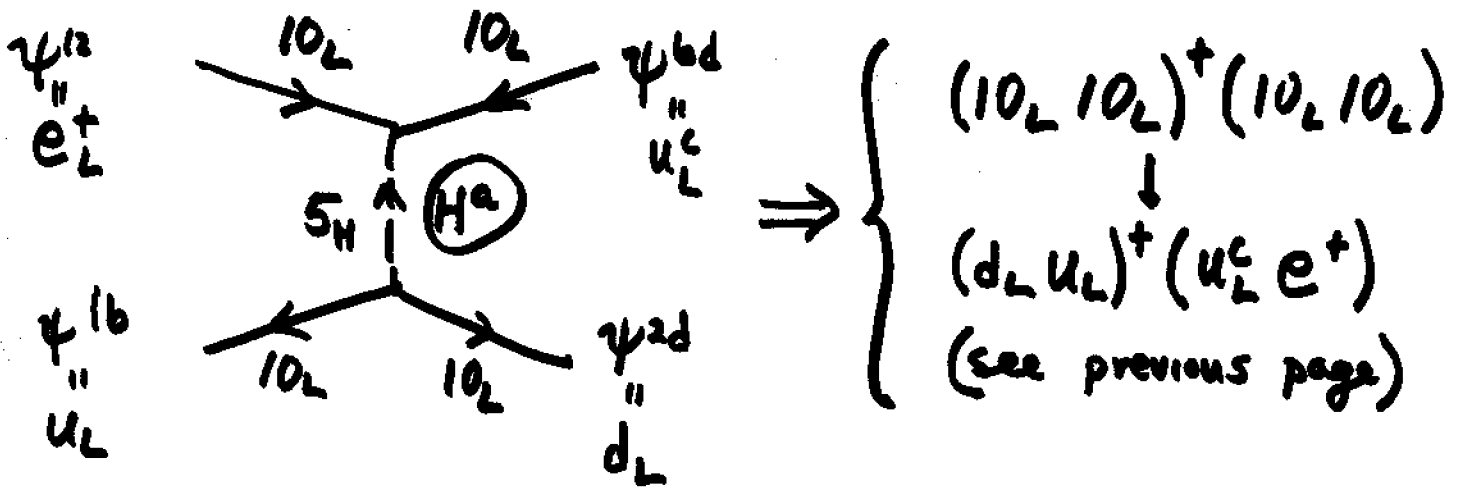
$\underbrace{M_N, M_R}_{M_\nu}$

### (E) PROTON DECAY OPERATORS

TWO KINDS OF DIAGRAMS FOR PDK IN NON-SUSY  $SU(5)$ :  
GAUGE BOSON MEDIATED, HIGGS BOSON MEDIATED



# HIGGS BOSON MEDIATED



different!



The Higgs-mediated proton decay is caused by exchange of colored Higgs

These must be superheavy

⇒ "DOUBLET-TRIPLET SPLITTING PROBLEM"

$$5_H = H^\alpha = \begin{pmatrix} H^i \\ H^a \end{pmatrix} \begin{matrix} \leftarrow \text{Weak-scale mass} \\ \leftarrow \text{GUT scale mass} \end{matrix}$$

THIS IS AN ASPECT OF THE INFAMOUS "GAUGE HIERARCHY PROBLEM"

- THREE ASPECTS:
- (a) 2/3 SPLITTING PROBLEM
  - (b) FINE-TUNING PROBLEM  
(stability under radiative corrections)
  - (c) SMALL NUMBER PROBLEM  
(why  $M_W/M_G \sim 10^{-14}$ )
- (d) (In SUSY) "μ PROBLEM"

Why do all of these operators conserve B-L?

In minimal SU(5) model:

$$\begin{array}{ccccccc}
 (10_L \ 10_L) \ 5_H & + & (10_L \ \bar{5}_L) \ \bar{5}_H & + & (\bar{5}_L \ 1_L) \ 5_H & + & M^2 \bar{5}_H \ 5_H \\
 X = +1 \ +1 \ -2 & & +1 \ -3 \ +2 & & -3 \ +5 \ -2 & & +2 \ -2
 \end{array}$$

Can assign a global charge X that is conserved by couplings

Now  $\langle 5_H \rangle, \langle \bar{5}_H \rangle$  break  $\underbrace{U(1)_{I_{3L}}}_{\text{local}} \times \underbrace{U(1)_Y}_{\text{local}} \times \underbrace{U(1)_X}_{\text{global}}$

by 't Hooft mechanism there remain

$$\begin{array}{l}
 \underbrace{U(1)_Q}_{\text{local}} \times \underbrace{U(1)_{B-L}}_{\text{global}} \qquad Q = I_{3L} + Y/2 \\
 \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad B-L = \frac{4}{5} \frac{Y}{2} + \frac{1}{5} X
 \end{array}$$

In SO(10), as we shall see,  $U(1)_X$  is contained in SO(10) and thus a local sym.  
 $\Rightarrow$  B-L is a generator of SO(10).

NOTE In SU(5), if there are right-handed neutrinos, they can have Majorana masses that break L and conserve B, thus ~~(B-L)~~.

$$M_R \begin{pmatrix} 1_L & 1_L \\ +5 & +5 \end{pmatrix} \Rightarrow \Delta X = -10 \Rightarrow \Delta(B-L) = -2$$

IMPORTANT FOR "LEPTOGENESIS".

# PROTON DECAY OPERATORS IN NON SUSY SU(5)

gauge  
and  
higgs  
mediated

$$\left\{ \begin{array}{l} 10_L^+ 10_L^+ 10_L 10_L \rightarrow (e_L^+ u_L^c)^t u_L d_L = e_R^- u_R u_L d_L \\ 10_L^+ \bar{5}_L^+ 10_L \bar{5}_L \rightarrow (u_L^c d_L^c)^t Q_L L_L = u_R d_R Q_L L_L \end{array} \right.$$

only  
higgs  
mediated

$$\left\{ \begin{array}{l} 10_L 10_L 10_L \bar{5}_L \rightarrow e_L^+ u_L^c u_L^c d_L^c = (e_R^- u_R u_R d_L)^t \\ \text{and} \\ Q_L Q_L Q_L L_L \end{array} \right.$$

## IN SUSY SU(5)

$$\mathcal{L} \supset \int d^2\theta d^2\bar{\theta} (10_L^+ 10_L^+ 10_L 10_L + 10_L^+ \bar{5}_L^+ 10_L \bar{5}_L) / M_G^2$$

Since both  $\Phi$  and  $\Phi^+$  are involved, these are D-terms with  $\int d^2\theta d^2\bar{\theta}$  thus by dimensional analysis  $1/M_G^2$ .

$\Rightarrow$  "d=6" proton decay operators  
caused by exchange of gauge and higgs bosons

But also allowed: "d=5" proton decay operators

$$\mathcal{L} \supset \int d^2\theta (10_L 10_L 10_L \bar{5}_L) / M_G$$

Term in superpotential  $W$  as only  $\Phi$  so  
by dimensional analysis only suppressed by  $1/M_G$ .

**NOTE: From HIGGS NO EXCHANGE. DANGEROUS!**

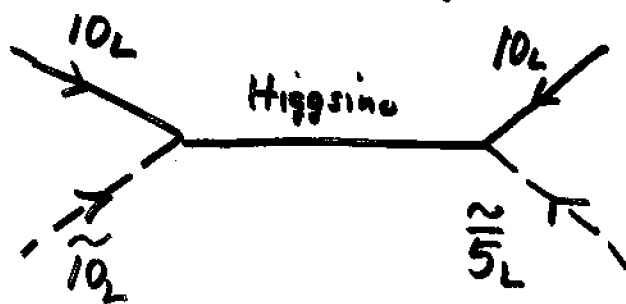
Also could have (disastrous)  $d = 4$  operators" 1.13  
 such as  $\int d^2\theta (10_L \bar{5}_L \bar{5}'_L)$ , but these can be  
 forbidden by imposing "matter parity" ( $\approx$  R parity):

$Z_2$  under which  $\begin{cases} \text{"matter"} (\equiv q + l) \text{ are odd} \\ \text{higgs, gauge} \text{ are even.} \end{cases}$

(Such a  $Z_2$  can arise automatically in  $SO(10)$  as a subgroup of  $SU(16)$ )

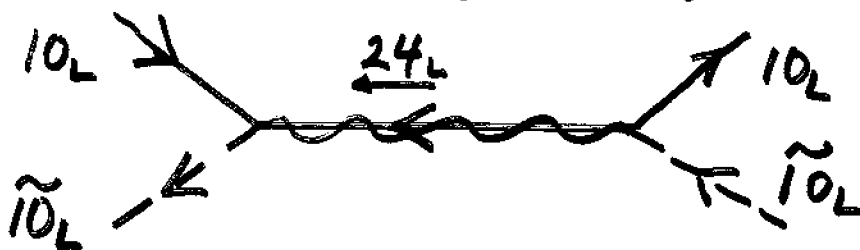
$d = 5$  p decay comes from (colored) Higgsino exchange

$10_L 10_L 10_L \bar{5}_L$  from  
 $M_G$



partner of Higgs boson exchange  
 graph shown earlier

Why not  $d = 5$  from exchange of gaugino?



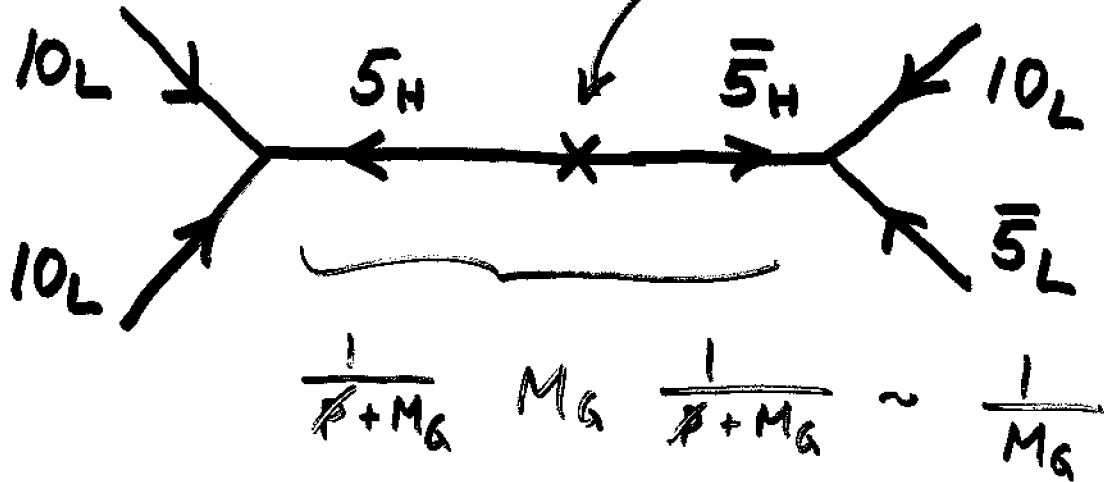
No "chirality flip" on gaugino line (since  $m_{24_L 24_L}$ ,  
 being a gaugino mass  $\propto M_{\text{sysy}} \ll M_G$ ).

$$\Rightarrow \frac{1}{p + M_G} = \frac{p - M_G}{p^2 + M_G^2} \sim \frac{1}{M_G^2}$$

LOOK AT HIGGSINO EXCHANGE DIAGRAM AGAIN.

NEED A CHIRALITY FLIP

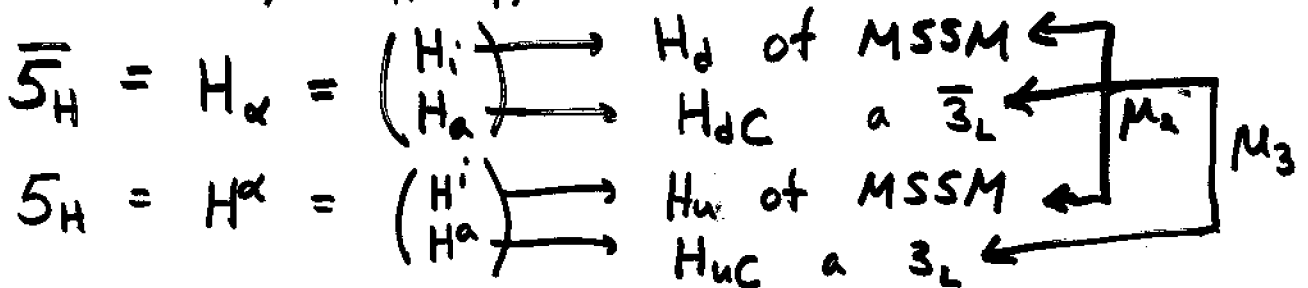
SUPERFIELD DIAGRAM:



NOTE: IN SUSY SU(5)  $\bar{5}_H, 5_H$  ARE NOT CONJUGATES OF EACH OTHER, BUT BOTH LEFT-HANDED CHIRAL SUPERFIELDS.

SO "X" IN DIAGRAM IS FROM  $\mu \bar{5}_H 5_H$

$$W \supset (10_L 10_L) 5_H + (10_L \bar{5}_L) \bar{5}_H + (\bar{5}_L 1_L) 5_H + \mu \bar{5}_H 5_H$$

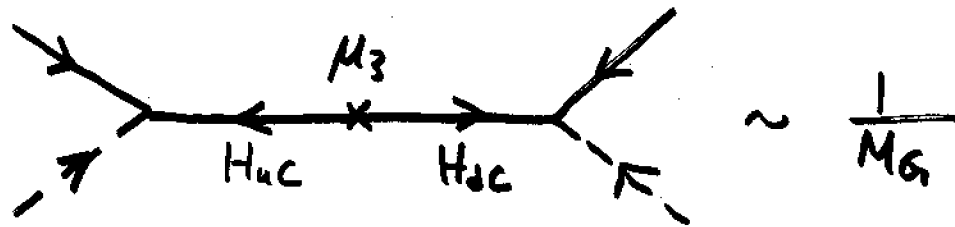


$\rightarrow \mu_3 H_{dc} H_{uc}$

$A_{p \text{ decay}} \propto \frac{\mu_3}{M_{H_{uc}} M_{H_{dc}}}$

# CRUCIAL POINT

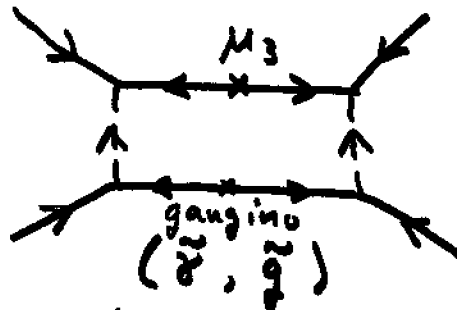
1.15



WHY NOT DISASTROUS IF  $\sim \frac{1}{M_G}$  ?

ADDITIONAL SMALL FACTORS (RELATIVE TO GAUGE MED)

- (a) Couplings are Yukawa couplings of the light quarks and leptons  $\sim \frac{m_{q,l}}{v}$
- (b) These diagrams involve scalar quarks and leptons which must be turned into quarks and leptons by gaugino  $\Rightarrow$  loop



Calculation is complicated and involves hadronic matrix element.

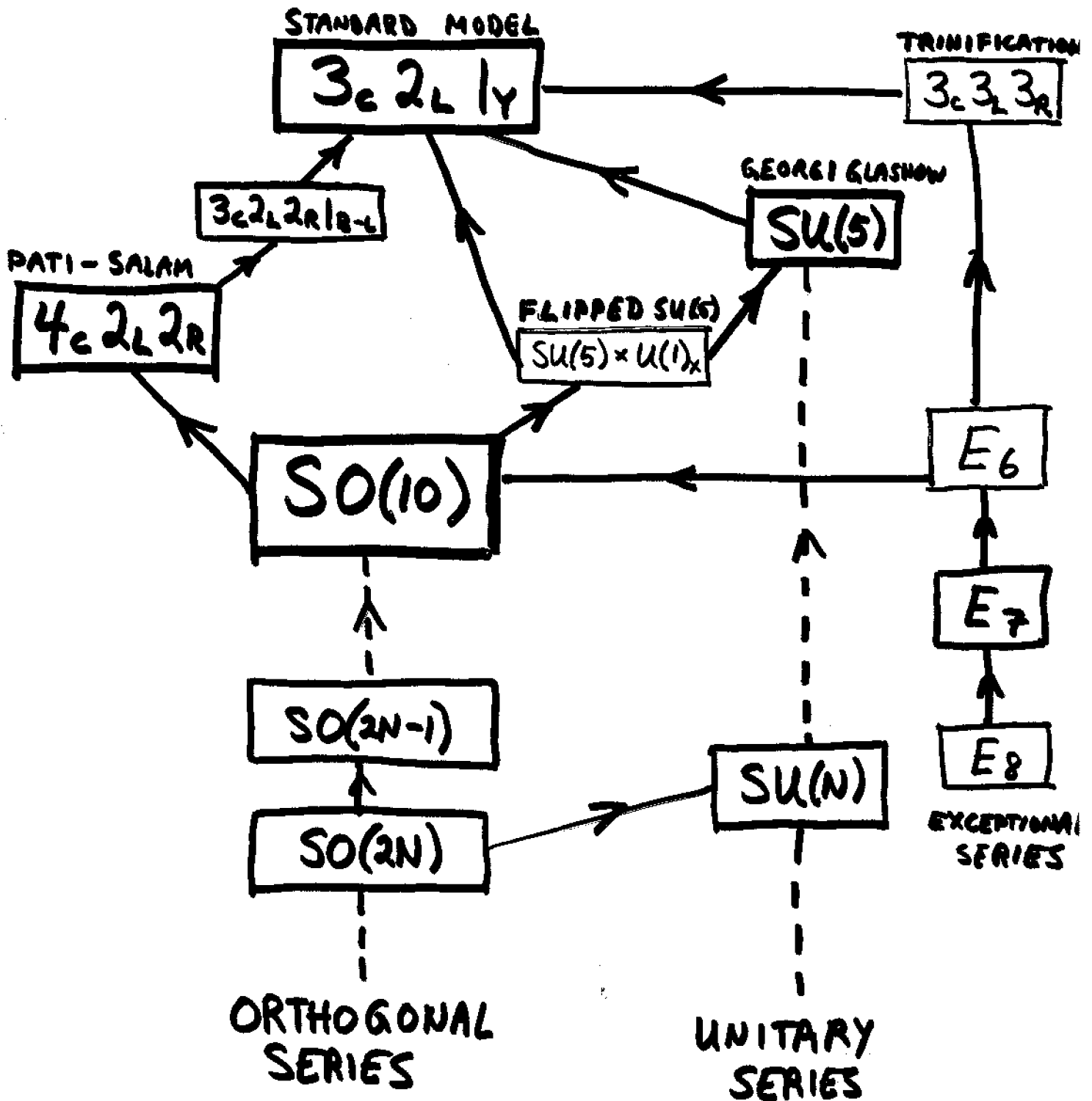
FOR SUSY SU(5) (minimal) [See H. Murayama and A. Pierce, hep-ph/0108104]

$$3.5 \times 10^{14} \leq M_{H_c} \leq 3.6 \times 10^{15} \text{ GeV} \quad (90\% \text{ c.l. from R.G.E.})$$

$$\text{BUT } M_{H_c} \geq 7.6 \times 10^{16} \text{ GeV} \quad (\text{from superK limit on } p \rightarrow K^+ \bar{\nu})$$

PROBLEM OF HIGGSINO-MEDIATED P DECAY (or  $d=5$  p decay)

# LECTURE II : $SO(10)$ AND OTHER GROUPS FOR UNIFICATION <sup>2.1</sup>



(A) MOST IMPORTANT GROUPS FOR UNIFICATION

## PATI-SALAM $(SU(4)_c \times SU(2)_L \times SU(2)_R)$

First unification scheme, not "grand unified" but does have  $g-l$  unification.

- Implies Right-handed neutrinos exist.
- Can have Left-Right symmetry.

## SU(5) (GEORGI-GLASHOW)

- Smallest "grand unified" group  
[Only rank-4 unification group  
⇒ one stage of symmetry breaking  
⇒ most predictive for  $p$  decay]

## SO(10)

Contains both  $SU(5)$  and Pati-Salam groups

- Implies Right-handed neutrinos exist
- Has a Left-Right symmetry
- Is grand unified
- Complete  $g-l$  unification of a family
- Most predictive for  $g, l$  masses
- Automatic anomaly freedom.



(B) PATI-SALAM  $SU(4)_c \times SU(2)_L \times SU(2)_R$

$$\begin{array}{ccc}
 \updownarrow SU(2)_L & \begin{pmatrix} u & u & u & \nu \\ d & d & d & e^- \end{pmatrix}_L & \begin{pmatrix} u & u & u & N \\ d & d & d & e^- \end{pmatrix}_R \updownarrow SU(2)_R \\
 & \leftarrow SU(4)_c \rightarrow & \leftarrow SU(4)_c \rightarrow
 \end{array}$$

$$\begin{array}{c}
 \updownarrow \\
 \begin{pmatrix} u^c & u^c & u^c & N^c \\ d^c & d^c & d^c & e^+ \end{pmatrix}_L \\
 \parallel \\
 (\bar{4}, 1, 2)_L
 \end{array}$$

$$\parallel \\
 (4, 2, 1)_L$$

NOTE:  $B-L = \lambda_{15} = \begin{pmatrix} 1/3 & 1/3 & 1/3 & -1 \end{pmatrix}$  of  $SU(4)_c$

$$\begin{aligned}
 Q &= I_{3L} + I_{3R} + \frac{1}{2}(B-L) \\
 Q &= I_{3L} + Y/2
 \end{aligned}$$

Higgs of MSSM  $\subset (1, 2, 2)$  "bidoublet"

$$\begin{array}{ccc}
 \updownarrow SU(2)_L & \langle [H_u, H_d] \rangle = \begin{pmatrix} v_u & 0 \\ 0 & v_d \end{pmatrix} & \begin{array}{l} H_u = (1, 2, 1/2) \\ H_d = (1, 2, -1/2) \end{array} \\
 & \leftarrow SU(2)_R \rightarrow &
 \end{array}$$

Yukawa terms in minimal Pati Salam  $Y_{AB} (4, 2, 1)_{LA} (\bar{4}, 1, 2)_{LB} (1, 2, 2)_H$

$$\Rightarrow M_N = M_U \propto M_D = M_L \quad (\text{not necess. symmetric})$$

# (C) $SO(2N)$ GROUP THEORY:

2.4

## VECTORS AND TENSORS

A VECTOR HAS  $2N$  REAL COMPONENTS, SO WE DO NOT DISTINGUISH UPPER AND LOWER INDICES. CALL VECTOR INDICES  $i, j, k$ , etc.

$$T^i = \begin{pmatrix} T^1 \\ T^2 \\ \vdots \\ T^{2N} \end{pmatrix}$$

TRANSFORMATIONS THAT LEAVE  $\sum T^i T^i$  INVARIANT ARE REAL UNITARY ( $\equiv$  ORTHOGONAL)  $2N \times 2N$  MATRICES.

$$O^T O = I$$

$$O = e^{i \sum_a \theta^a \lambda^a}$$

$\Rightarrow \lambda^a$  are purely imaginary, hermitian (= anti-symmetric) matrices.

$\Rightarrow a = 1, \frac{1}{2}(2N)(2N-1) \Rightarrow SO(10)$  has 45 generators  
 $i = 1, 2N$

ADJOINT REP:

$$T^{ij} = \sum_a T^a (\lambda^a)^{ij}$$

(USEFUL TO WRITE THE ADJOINT INDEX  $a$  AS A PAIR OF VECTOR INDICES (ANTISYMMETRIZED). i.e. LABEL

GENERATORS  $\lambda^{[ij]}$  = GEN. OF ROTATION IN  $ij$  PLANE

$$(\lambda^{[ij]})^{kl} = -i [\delta^{ik} \delta^{jl} - \delta^{il} \delta^{jk}]$$

FOR EXAMPLE:

2.5

$$\lambda^{[12]} = \left[ \begin{array}{cc|cc} 0 & -i & & \\ +i & 0 & & \\ \hline & & 0 & \\ & & & 0 \end{array} \right] \Rightarrow e^{i\theta \lambda^{[12]}} = \left[ \begin{array}{cc|cc} \cos\theta & \sin\theta & & \\ -\sin\theta & \cos\theta & & \\ \hline & & 1 & \\ & & & 1 \end{array} \right]$$

IT IS EASY TO SEE THAT  $SO(2N) \supset U(N)$   
 $= SU(N) \times U(1)$ .

$$-i = \begin{pmatrix} T^1 \\ T^2 \\ \vdots \\ T^{2N} \end{pmatrix} \rightarrow \begin{pmatrix} T^1 + iT^2 \\ T^3 + iT^4 \\ \vdots \\ T^{2N-1} + iT^{2N} \end{pmatrix} \oplus \begin{pmatrix} T^1 - iT^2 \\ T^3 - iT^4 \\ \vdots \\ T^{2N-1} - iT^{2N} \end{pmatrix}$$

$$\parallel \qquad \parallel$$

$$T^\alpha \qquad T_\alpha$$

$$\alpha = 1, \dots, N \qquad \alpha = 1, \dots, N$$

So  $T^\alpha$  of  $SU(N)$  =  $\frac{1}{2}(T^{(2\alpha-1)} + iT^{(2\alpha)})$  of  $SO(2N)$   $\equiv \frac{1}{2} T^{(2\alpha-1) + i(2\alpha)}$

$T_\alpha$  =  $\frac{1}{2}(T^{(2\alpha-1)} - iT^{(2\alpha)})$   $\equiv \frac{1}{2} T^{(2\alpha-1) - i(2\alpha)}$

$$\sum_{i=1}^{2N} T^i T^i = 4 \sum_{\alpha=1}^N T^\alpha T_\alpha$$

So  $U(N)$  TRANSFORMATIONS THAT KEEP  $\sum T^\alpha T_\alpha$  INVARIANT ARE ALSO  $SO(2N)$  TRANSFORMATIONS.

For example, Cartan generators of  $SO(2N)$  form

$$-i \begin{bmatrix} a_1 & & & \\ -a_1 & & & \\ & a_2 & & \\ & -a_2 & & \\ & & \dots & \\ & & & a_N \\ & & & -a_N \end{bmatrix} \text{ corresponds to } \begin{bmatrix} a_1 & & & \\ & a_2 & & \\ & & \dots & \\ & & & a_N \end{bmatrix} \text{ of } U(N)$$

eg. For  $SO(2) = U(1)$

$$-i \begin{bmatrix} & 1 \\ -1 & \end{bmatrix} \text{ equals } (1) \text{ of } U(1)$$

$$e^{i\theta} (i^{-1}) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \text{ equals } e^{i\theta} \text{ of } U(1)$$

We have seen  $\left\{ \begin{array}{l} \text{vector rep } T^i \\ \text{adjoint rep } T^{[ij]} \end{array} \right. = a$

Of course there are an infinitely many types of tensor reps.

eg.  $T^{(ij)}$  = traceless symmetric rank 2 tensor

$T^{[ijk]}$  etc.

# (D) $SO(2N)$ GROUP THEORY:

2.7

## SPINOR REPRESENTATIONS

### ALGEBRA OF $SO(2N)$

$$[\lambda^{[ij]}, \lambda^{[kl]}] = i \left( \delta^{ik} \lambda^{[jl]} - \delta^{il} \lambda^{[jk]} - \delta^{jk} \lambda^{[il]} + \delta^{jl} \lambda^{[ik]} \right)$$

eg.  $[\lambda^{[23]}, \lambda^{[31]}] = -i \lambda^{[21]} = i \lambda^{[12]}$

i.e.  $[\lambda^1, \lambda^2] = i \lambda^3$

WE CAN FIND  $2^N \times 2^N$  MATRICES THAT SATISFY THIS ALGEBRA.

FIRST, CONSTRUCT  $2^N \times 2^N$  MATRICES

$$(\gamma^i)^\lambda_{\lambda'} \quad \lambda, \lambda' = 1, \dots, 2^N$$
$$i = 1, \dots, 2N$$

THAT SATISFY "CLIFFORD ALGEBRA"

$$\{\gamma^i, \gamma^j\} = 2 \delta^{ij} I$$

THEN DEFINE

$$\sigma^{[ij]} \equiv \frac{i}{2} [\gamma^i, \gamma^j]$$

THESE OBEY  $SO(2N)$  GROUP ALGEBRA (LEFT AS EXERCISE)

THEN ELEMENTS OF GROUP CAN BE REPRESENTED BY  $2^N \times 2^N$  MATRICES

2.8

$$U = e^{i \sum_j \theta^j \sigma^j}$$

THESE ARE SPINOR REPRESENTATION. THESE MATRICES ACT ON  $2^N$ -COMPONENT "SPINORS" (CALLED BY PHYSICISTS SPINOR REPRESENTATIONS).

THESE SPINORS CAN BE WRITTEN :

$$\psi^\lambda = |s_1 s_2 s_3 \dots s_N\rangle$$

where  $s_\alpha = \pm 1$ .

∴ WILL WRITE

$$\gamma^{2k-1} = \underbrace{I \otimes I \otimes \dots \otimes I}_{k-1} \otimes \underbrace{\tau_1 \otimes \tau_3 \otimes \dots \otimes \tau_3}_{N-k}$$

$$\gamma^{2k} = -\underbrace{I \otimes I \otimes \dots \otimes I}_{k-1} \otimes \tau_2 \otimes \underbrace{\tau_3 \otimes \dots \otimes \tau_3}_{N-k}$$

THESE ARE 2N HERMITIAN  $2^N \times 2^N$  MATRICES. CAN CHECK THAT THEY SATISFY CLIFFORD ALGEBRA.

NOTE:  $\gamma^1, \gamma^2$  FLIP FIRST "SPIN"  $s_1$   
 $\gamma^3, \gamma^4$  FLIP SECOND "SPIN"  $s_2$   
 $\vdots$   
 $\gamma^{2N-1}, \gamma^{2N}$  FLIP  $N^{\text{TH}}$  "SPIN"  $s_N$

CHECK CLIFFORD ALGEBRA SATISFIED.

2.9

$$\begin{cases} \{\gamma^i, \gamma^i\} = 2\delta^{ii}I = 2I \Leftrightarrow (\gamma^i)^2 = I \\ (I \otimes I \otimes \dots \otimes \tau_2 \otimes \tau_3 \otimes \dots \otimes \tau_3)^2 = I \checkmark \end{cases}$$

$$\begin{cases} \left\{ \begin{array}{l} \frac{I}{\tau_2} \frac{\tau_3}{\tau_3} \\ \frac{I}{\tau_j} \frac{\tau_3}{\tau_3} \end{array} \right\} \\ = \frac{I}{\tau_2} \frac{\tau_3}{\tau_j} \{\tau_3, \tau_j\} \frac{I}{\tau_3} = 0 \checkmark \end{cases}$$

$$\begin{cases} \left\{ \begin{array}{l} \frac{I}{\tau_2} \frac{\tau_3}{\tau_3} \\ \frac{I}{\tau_j} \frac{\tau_3}{\tau_3} \end{array} \right\} \quad \begin{array}{l} i \neq j \\ ij = 1, 2 \end{array} \\ = \frac{I}{\tau_j} \{\tau_i, \tau_j\} \frac{I}{\tau_j} = 0 \end{cases}$$

LATER WE WILL LOOK AT  $\sigma^{ij} = \frac{i}{2} [\gamma^i, \gamma^j]$ .

BUT NOTICE THAT  $\sigma^{ij}$  FLIPS TWO SPINS  
(UNLESS  $ij = (2k-1)(2k)$ , WHEN FLIPS ZERO SPINS)

$$\begin{aligned} \text{THUS } |s_1 \dots s_N\rangle &= |s_1 \dots s_N\rangle_{\tau_s = +1} \oplus |s_1 \dots s_N\rangle_{\tau_s = -1} \\ &= \Psi_+ \oplus \Psi_- \\ &= \text{TWO IRREDUCIBLE SPINORS.} \end{aligned}$$

CONJUGATION:  $U \rightarrow U^* \Rightarrow e^{i\theta^a \lambda^a} \rightarrow e^{-i\theta^a \lambda^{a*}}$

$\Rightarrow \lambda^a \rightarrow -\lambda^{a*} \Rightarrow \left(\frac{i}{2} [\gamma^i, \gamma^j]\right) \rightarrow \left(\frac{i}{2} [\gamma^{i*}, \gamma^{j*}]\right)$

$\Rightarrow \gamma^i \rightarrow \gamma^{i*} \Rightarrow \tau_1 \rightarrow \tau_1, \tau_2 \rightarrow -\tau_2$

$\Rightarrow \tau^\pm \rightarrow \tau^\mp \Rightarrow \text{SPIN UP} \leftrightarrow \text{SPIN DOWN}$

$\Rightarrow \underline{N = \text{even}} \quad \overline{\Psi_+} = \Psi_+, \quad \overline{\Psi_-} = \Psi_-$

$\underline{N = \text{odd}} \quad \overline{\Psi_+} = \Psi_-, \quad \overline{\Psi_-} = \Psi_+$

### TABLE OF SPINORS FOR SO(2N)

<u>EQUIVALENT GP.</u>	<u>SO(2N)</u>	<u><math>\Psi_+</math></u>	<u><math>\Psi_-</math></u>	<u>vector</u>
$U(1)$	SO(2)	1	$\bar{1}$	$2 = 1 + \bar{1}$
$U(2) = SU(2)$	SO(4)	$2^{(2,1)}$	$2'^{(1,2)}$	$4^{(2,2)}$
$SU(4)$	SO(6)	$4 = T^a$	$\bar{4} = T_a$	$6 = T^{[AB]}$
	SO(8)	8	$8'$	8
	SO(10)	16	$\bar{16}$	10
	SO(12)	32	$32'$	12
	SO(14)	64	$\bar{64}$	14
	SO(16)	128	$128'$	16
	SO(18)	256	$\bar{256}$	18



CONSIDER  $SO(2p) \times SO(2N-2p)$  SUBGROUP 2.11  
OF  $SO(2N)$ .

VECTOR:  $2N \rightarrow (2p, 1) + (1, 2N-2p)$

SPINOR:

$$|s_1 \cdots s_N\rangle = |s_1 \cdots s_p\rangle |s_{p+1} \cdots s_N\rangle$$

$$\Psi_{\pm} = |s_1 \cdots s_N\rangle_{\pi s = \pm 1} = |s_1 \cdots s_p\rangle_{\pm} |s_{p+1} \cdots s_N\rangle_{\pm} + |s_1 \cdots s_p\rangle_{\mp} |s_{p+1} \cdots s_N\rangle_{\mp}$$

$$\begin{aligned} \Psi_{+} &\rightarrow (\Psi_{+}, \Psi_{+}) + (\Psi_{-}, \Psi_{-}) \\ \Psi_{-} &\rightarrow (\Psi_{+}, \Psi_{-}) + (\Psi_{-}, \Psi_{+}) \end{aligned}$$

IN PARTICULAR, CONSIDER

$$SO(10) \supset SO(6) \times SO(4) \\ \parallel \parallel \\ SU(4)_c \times SU(2)_L \times SU(2)_R$$

$$16 \rightarrow (4, 2) + (\bar{4}, 2') \\ \parallel \parallel \\ (4, 2, 1) + (\bar{4}, 1, 2) = 8_L + 8_R$$

$$10 \rightarrow (6, 1) + (1, 4) \\ \parallel \parallel \\ (6, 1, 1) + (1, 2, 2) = \text{Higgs}$$

PATI-SALAM FITS WITHIN  $SO(10)$ !

## D.2 Family Unification (See Wilczek + Zee "Families from spinors")

$$SO(16) \supset SO(10) \times SO(6)$$

$$128_L \longrightarrow (16, 4)_L + (\bar{16}, \bar{4})_L$$

4 families

4 mirror families

$$\bar{16}_L \sim 16_R$$

(V-A) Weak int.

(V+A) Weak int.

128 is self-conjugate (i.e. a "real representation")

So can have  $M(128_L, 128_L)$

"Naturally",  $128_L$  would be superheavy unless some global symmetry prevents this mass term.  
(or local)

$$SO(18) \supset SO(10) \times SO(8)$$

$$256_L \longrightarrow (16, 8)_L + (\bar{16}, 8')_L$$

256 not self-conjugate, so  $M 256 256$  not possible

Problem with such family unification schemes (also with  $E_8$ ) is how to make V+A families heavy. Can be solved in higher dimensions. [KS Babu, S.M.B., and B. Kyae hep-ph/0202178]

### D.3 How SO(10) CONTAINS SU(5) MODEL

2.13

We know that a family

$$= 16 \text{ of } SO(10)$$

$$= (4, 2, 1) + (\bar{4}, 1, 2) \text{ of Pati-Salam}$$

$$= 10 + \bar{5} + 1 \text{ of } SU(5)$$

We can see that under  $SO(10) \rightarrow SU(5)$   
 $16 \rightarrow 10 + \bar{5} + 1$   
directly, by looking more closely at spinor representations of  $SO(2N)$ .

#### SO(2N)

$$\lambda^{[ij]} \equiv \sigma^{ij} = \frac{i}{2} [\gamma^i, \gamma^j]$$

Recall that  $U(N) = SU(N) \times U(1)$  embedded in  $SO(2N)$  as follows:

$$T^\alpha = \frac{1}{2} T^{(2\alpha-1) + i(2\alpha)} = \frac{1}{2} (T^{(2\alpha-1)} + i T^{(2\alpha)})$$

$$\underbrace{T_\alpha}_{U(N)} = \frac{1}{2} \underbrace{T^{(2\alpha-1) - i(2\alpha)}}_{SO(2N)} = \frac{1}{2} (T^{(2\alpha-1)} - i T^{(2\alpha)})$$

So  $SU(N) \times U(1)$  GENERATORS CAN BE WRITTEN<sup>2.14</sup>  
AS  $SO(2N)$  GENERATORS THUS:

$$\begin{aligned} \lambda_{\beta}^{\alpha} &= -\frac{i}{4} \sigma[(2\alpha-1)+i(2\alpha), (2\beta-1)-i(2\beta)] \\ &= -\frac{1}{8} [\gamma^{2\alpha-1} + i\gamma^{2\alpha}, \gamma^{2\beta-1} - i\gamma^{2\beta}] \\ &= +\frac{1}{8} \left[ \underbrace{I \otimes \dots \otimes I}_{\alpha^{\text{th}} \text{ place} = 2\alpha^-} \otimes (\tau_1 - i\tau_2) \otimes \tau_3 \otimes \dots \otimes \tau_3, \right. \\ &\quad \left. I \otimes \dots \otimes I \otimes (\tau_1 + i\tau_2) \otimes \tau_3 \otimes \dots \otimes \tau_3 \right] \\ &\quad \underbrace{\hspace{10em}}_{\beta^{\text{th}} \text{ place} = 2\alpha^+} \end{aligned}$$

$\Rightarrow$  for  $\alpha < \beta$

$$\begin{aligned} \lambda_{\beta}^{\alpha} &= +\frac{1}{8} (I \otimes \dots \otimes I \otimes \underbrace{2\tau^+}_{\alpha^{\text{th}}} \otimes \tau_3 \otimes \dots \otimes \underbrace{[\tau_3, 2\tau^+]}_{\beta^{\text{th}}} \otimes I \otimes \dots \otimes I) \\ &= (I \otimes \dots \otimes I \otimes \underbrace{\tau^-}_{\alpha^{\text{th}}} \otimes \tau_3 \otimes \dots \otimes \underbrace{\tau^+}_{\beta^{\text{th}}} \otimes I \otimes \dots \otimes I) \end{aligned}$$

So  $\lambda_{\beta}^{\alpha}$  raises  $\beta^{\text{th}}$  spin and lowers  $\alpha^{\text{th}}$  spin.

For  $\alpha = \beta$

$$\lambda_{\alpha}^{\alpha} = \frac{1}{2} (I \otimes \dots \otimes I \otimes \tau_3 \otimes I \otimes \dots \otimes I) = -\frac{S}{2}$$

(no sum)

$$\begin{pmatrix} a_1 \\ a_2 \\ \dots \\ a_N \end{pmatrix} = \sum_{\alpha=1}^N a_{\alpha} \lambda_{\alpha}^{\alpha} = -\frac{1}{2} \sum_{\alpha=1}^N a_{\alpha} S_{\alpha}$$

CONSIDER SPINOR  $\Psi_+ = 16$  OF  $SO(10)$ . 2.16  
(NO 2.15)  
 $= |s_1 s_2 s_3 s_4 s_5\rangle_{\prod s_i = +1}$

THIS CONTAINS

$$\left[ \begin{array}{l} \{ |++++\rangle \} \longrightarrow 1 \text{ component} \\ \{ |+++--\rangle, \text{ permutations} \} \longrightarrow 10 \text{ components} \\ \{ |+----\rangle, \text{ permutations} \} \longrightarrow 5 \text{ components} \end{array} \right.$$

SINCE  $SU(5) \times U(1)$  GENERATORS DO NOT CHANGE NUMBER OF + SIGNS  $\Rightarrow$  THESE THREE TYPES DO NOT MIX WITH EACH OTHER UNDER  $SU(5) \times U(1)$ :  
 IRREDUCIBLE REPS.

$$\lambda'_2 | \underbrace{+----}_{T_1} \rangle = | \underbrace{-+----}_{T_2} \rangle$$

$$\underbrace{\sum_{\alpha} a_{\alpha} \lambda'_{\alpha}}_{SU(5) \text{ GEN}} | +---- \rangle = -\frac{1}{2} (a_1 - a_2 - a_3 - a_4 - a_5) | \rangle = -a_1 | \rangle \quad (\text{since } \sum a_{\alpha} = 0 \text{ for } su(5))$$

$$\Rightarrow \begin{array}{l} \{ | +---- \rangle, \text{ permutations} \} = \bar{5} \text{ of } su(5) \\ \{ | +++-- \rangle, \text{ permutations} \} = 10 \text{ " } \\ \{ | +++++ \rangle \} = 1 \end{array}$$

$$\sum_{\alpha} a_{\alpha} \lambda'_{\alpha} | +++-- \rangle = -\frac{1}{2} (a_1 + a_2 + a_3 - a_4 - a_5) = -(a_4 + a_5) | \rangle = -(a_4 + a_5) | \rangle \Rightarrow | \rangle = T^{45} = T_{45}$$

CONSIDER THE  $U(1)$  GENERATOR

$$X \equiv -2 \sum_{\alpha=1}^5 \lambda^\alpha \alpha = -2 \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix} = \sum_{\alpha=1}^5 S_\alpha$$

↑  
a common normalization

$$\left. \begin{aligned} X |++++\rangle &= 5 |++++\rangle \\ X |+++--\rangle &= 1 |+++--\rangle \\ X |+----\rangle &= -3 |+----\rangle \end{aligned} \right\} \Rightarrow 16 = 1^5 + 10^1 + 5^{-3}$$

#### D.4 ANOTHER LOOK AT B-L

PATI - SALAM:  $SO(4) \times SO(6) \subset SO(10)$

$$[SU(2)_L \times SU(2)_R] \times SU(4)_C$$

$$\cup \quad \cup$$

$$[SU(2)_L \times U(1)_{I_{3R}}] \times [SU(3)_C \times U(1)_{B-L}]$$

LOOK AT  $U(3) = SU(3)_C \times U(1)_{B-L}$  GENERATOR

EMBEDDED IN  $SO(6)$  AS DESCRIBED ABOVE:

$$B-L = \frac{-2}{3} \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix} = \frac{-2}{3} (\lambda^3_3 + \lambda^4_4 + \lambda^5_5) = \frac{1}{3} (S_3 + S_4 + S_5)$$

$$\Rightarrow \begin{cases} (B-L) |+++ \rangle = \frac{1}{3}(1+1+1) = +1 & 1 \text{ of color} \\ (B-L) |+- \rangle = \frac{1}{3}(1-1-1) = -\frac{1}{3} & \bar{3} \text{ of color} \end{cases}$$

$$\text{So } \Psi_+ \text{ of } SO(6) = \bar{4} \text{ (fundamental of } SU(4)_C)$$

$$= \underbrace{\bar{3}^{-1/3}}_{\text{antiquark}} + \underbrace{1^+}_{\text{antilepton}}$$

AS A  $U(5)$  GENERATOR:  $B-L = -\frac{2}{3} \begin{pmatrix} 0 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}$

The Higgs of  $SU(5)$  model are

$$\text{in } \bar{5} + 5 = \begin{pmatrix} H_d \\ H_{dc} \end{pmatrix} + \begin{pmatrix} H_u \\ H_{uc} \end{pmatrix}$$

These combine into a 10 of  $SO(10)$ .

$$X = -2 \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix} \Rightarrow 10 = \bar{5}^{+2} + 5^{-2}$$

$$B-L = -\frac{2}{3} \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix} \Rightarrow \begin{array}{l} H_d, H_u \text{ have } B-L = 0 \\ H_{dc} \text{ has } B-L = \frac{2}{3} \\ H_{uc} \text{ has } B-L = -\frac{2}{3} \end{array}$$

This is important for later discussion of  $\frac{2}{3}$  splitting.

$$\text{If } M_{\text{Higgs}} \propto B-L$$

$$\Rightarrow \begin{cases} M(H_u, H_d) = 0 \\ M(H_{uc}, H_{dc}) \neq 0 \end{cases}$$

# 2.5 COUPLING SPINORS TO VECTORS AND TENSORS

Define  $\gamma^{[ij \dots k]} \equiv (\gamma^i \gamma^j \dots \gamma^k)$  antisymmetrized.

$\gamma^i$  FLIPS ONE "SPIN."  $\Rightarrow \gamma^{[ij \dots k]}$  with even (odd) number of indices flips even (odd) number of spins.

$C \equiv$  charge conjugation matrix.  $C$  flips all spins.

We can construct an  $SO(2N)$  invariant as follows:

$$\Psi^T C (\gamma^{[ij \dots k]}) \Psi \quad T^{[ij \dots k]}$$

[Note: if  $\Psi \rightarrow e^{i\theta \sigma} \Psi$ , then  $(\Psi^T C) \rightarrow (\Psi^T C) e^{-i\theta \sigma}$ .

Also only totally antisymmetrized product of  $\gamma^i$  need to be considered, since symmetric product  $\gamma^{(i} \gamma^{j)} = \delta^{ij} I$  by Clifford algebra.]

For N even:  $\begin{cases} T^{[ij \dots k]} \text{ with } \underline{\text{even rank}} \text{ couples to } \Psi_{\pm}^T \sigma \Psi_{\pm} \\ T^{[ij \dots k]} \text{ with } \underline{\text{odd rank}} \text{ couples to } \Psi_{\pm}^T \sigma \Psi_{\mp} \end{cases}$

For N odd:  $\begin{cases} T^{[ij \dots k]} \text{ with } \underline{\text{even rank}} \text{ couples to } \Psi_{\pm}^T \sigma \Psi_{\mp} \\ T^{[ij \dots k]} \text{ with } \underline{\text{odd rank}} \text{ couples to } \Psi_{\pm}^T \sigma \Psi_{\pm} \end{cases}$

Define  $\gamma_{\text{FIVE}} \equiv \underbrace{\gamma^{[i_1 i_2 \dots i_{2N}]}}_{2N \text{ factors}} = \gamma^1 \gamma^2 \gamma^3 \dots \gamma^{2N}$

$$= (\tau_3 \otimes \dots \otimes \tau_3)$$

$$\gamma_{\text{FIVE}} \Psi_{\pm} = \pm \Psi_{\pm}$$

$$\underbrace{\gamma^{[i_1 i_2 \dots i_p]}}_p = \pm \gamma_{\text{FIVE}} \underbrace{\gamma^{[i_{p+1} i_{p+2} \dots i_{2N}]}}_{2N-p}$$



$$\Rightarrow \left[ \begin{array}{l} \Psi_{\pm}^T \quad C [T + \underbrace{\gamma^{[ij]} T^{[ij]}}_{-2i\sigma^{ij} \text{ adjoint}} + \dots + \underbrace{\gamma^{(i\dots k)} T^{(i\dots k)}}_{\substack{\text{largest even} \\ \leq N}}] \Psi_{\pm} \\ \pm (N \text{ even}) \\ \mp (N \text{ odd}) \end{array} \right. \left. \begin{array}{l} \Psi_{\pm}^T \quad C [\gamma^i T^i + \gamma^{[ijk]} T^{[ijk]} + \dots + \underbrace{\gamma^{(i\dots k)} T^{(i\dots k)}}_{\substack{\text{largest odd} \\ \leq N}}] \Psi_{\pm} \\ \mp (N \text{ even}) \\ \pm (N \text{ odd}) \end{array} \right]$$

EXAMPLE: SO(10)

$$\left\{ \begin{array}{l} \Psi_+ C (T + \gamma^{[ij]} T^{[ij]} + \gamma^{[ijkl]} T^{[ijkl]}) \Psi_- \\ 16 \quad (1 + 45 + 210) \overline{16} \end{array} \right.$$

$$\left\{ \begin{array}{l} \Psi_{\pm} C (\gamma^i T^i + \gamma^{[ijk]} T^{[ijk]} + \gamma^{[ijklm]} T^{[ijklm]}) \Psi_{\pm} \\ 16 \quad (10 + 120 + \overline{126}) 16 \\ (\overline{16}) \quad (10 + 120 + 126) (\overline{16}) \end{array} \right.$$

Note  $T^{[ijklm]} = \pm \epsilon^{ijklm i'j'k'l'm'} T^{[i'j'k'l'm']}$

Rank 5 tensor = 252 =  $126 + \overline{126}$   
irreducibly tensors

IN MINIMAL SO(10) MODEL, FERMION MASS TERMS

$$Y_{AB} (16_{LA} 16_{LB}) 10_H$$

$$\rightarrow Y_{AB} \left( \begin{array}{l} 10_A^1 \quad 10_B^1 \quad 5_H^{-2} \\ + 10_A^1 \quad \overline{5}_B^{-3} \quad \overline{5}_H^{+2} \\ + \overline{5}_A^{-3} \quad 10_B^1 \quad \overline{5}_H^{+2} \\ + \overline{5}_A^{-3} \quad \overline{15}_B^5 \quad 5_H^{-2} \\ + 10_A^1 \quad \overline{5}_B^{-3} \quad 5_H^{-2} \end{array} \right) \rightarrow \begin{array}{l} M_u \\ M_d, M_e \\ M_\nu \end{array}$$

THE MAJORANA MASS MATRIX OF THE RIGHT-HANDED NEUTRINOS, CAN COME FROM:

$$(Y_R)_{AB} (16_{L_A} / 16_{L_B}) \overline{126}_H \rightarrow M_R$$

$\langle 1^{-10} \rangle \sim M_{\text{GUT}}$

OR A HIGHER-DIMENSION OPERATOR:

$$(Y_R)_{AB} (16_{L_A} / 16_{L_B}) \overline{16}_H \overline{16}_H / M_G$$

$\langle 1^{-5} \rangle \langle 1^{-5} \rangle \sim M_{\text{GUT}}$

THUS, IN MINIMAL  $SO(10)$  ONE HAS

$$\underbrace{M_N = M_U \propto M_D = M_L}_{\text{SYMMETRIC}}, \quad \underbrace{M_R}_{\text{SYMMETRIC}}$$

IN MINIMAL PATI-SALAM, ONE HAS SAME RELATIONS, BUT THE DIRAC MATRICES NEED NOT BE SYMMETRIC.

IN MINIMAL  $SU(5)$

$$M_N, \quad \underbrace{M_U}_{\text{SYMMETRIC}}, \quad M_D = M_L^T, \quad \underbrace{(M_R)}_{\text{SYMMETRIC}}$$

---

IN MINIMAL  $SO(10)$  (OR PATI SALAM)  $M_\nu = M_U M_R^{-1} M_U^T$

$$\Rightarrow \text{(FOR ONE FAMILY MODEL)} \quad m_3 = \frac{m_\mu^2}{M_R}$$

$$M_R = \frac{m_\mu^2}{m_3} = \frac{(174 \text{ GeV})^2}{0.06 \text{ eV}} = \frac{1}{2} \times 10^{15} \text{ GeV}$$

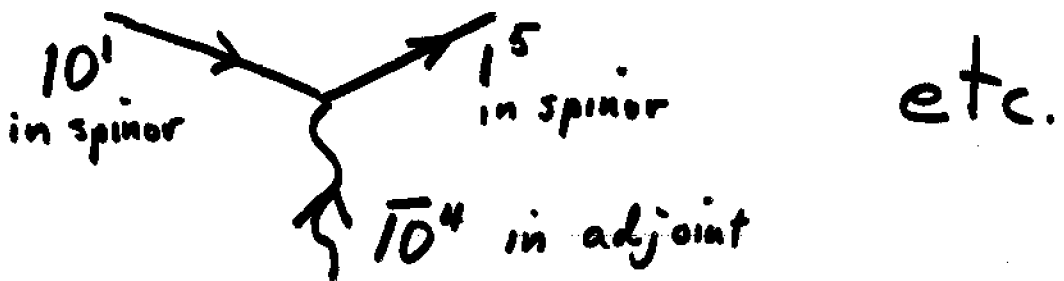
Close to GUT scale!

NOTE ALSO THAT MINIMAL  $SO(10) \Rightarrow \theta_{\text{CKM}} = 0$ . (Approx. correct)

(E) BREAKING  $SO(10)$  TO SM.NEED TWO KINDS OF HIGGS TO DO IT.

$$SO(10) \supset SU(5) \times U(1)_X$$

$$\left\{ \begin{array}{l} 45 \rightarrow 24^0 + 1^0 + 10^{-4} + \overline{10}^4 \\ 16 \rightarrow 10^1 + \overline{5}^{-3} + 1^5 \\ \overline{16} \rightarrow \overline{10}^{-1} + 5^3 + 1^{-5} \\ 10 \rightarrow 5^{-2} + \overline{5}^2 \end{array} \right.$$



We know that an  $SU(5)$  adjoint Higgs,  $24_H$ , can be used to break  $SU(5) \rightarrow SM$ .

In  $SO(10)$ , an adjoint Higgs is not enough.

The  $10^{-4}$  and  $\overline{10}^4$  in a  $45_H$  cannot get VEVs  $\sim M_G$ , since all components of them are electrically charged. (Even in "flipped  $SU(5)$ ".)

And  $24^0$  and  $1^0$  leave  $U(1)_X$  unbroken, as well as  $SU(3) \times SU(2) \times U(1) \subset SU(5)$ .

Need another Higgs to break  $U(1)_X$

Usual choices  $16_H + \overline{16}_H$  or  $126_H + \overline{126}_H$

(Need both 126 and  $\overline{126}$ , or both 16 and  $\overline{16}$ , so as not to break SUSY at  $M_G$  by D terms:

$$\langle D^a \rangle = \left\langle \sum_i \Phi_i^\dagger \lambda^a \Phi_i \right\rangle$$

We saw that  $16 = 1^5$ ,  $\overline{16} = 1^{-5}$   
and  $126 = 1^{10}$ ,  $\overline{126} = 1^{-10}$

and that these can give  $M_R$ . They also break  $U(1)_X$ .

So:

$45_H$  (and/or  $54_H, \dots$ )  $\longrightarrow$  Breaks  $SO(10)$  to rank 5 subgroup  
eg.  $SU(3)_L \times SU(2)_L \times U(1)_Y \times U(1)_X$  Does 2/3 splitting

$16_H + \overline{16}_H$   $\longrightarrow$  Breaks  $U(1)_X$  ("breaks rank") Gives mass to  $N^c$  (i.e.  $\nu_R$ )  
or

$126_H + \overline{126}_H$

E.2 "Flipped SU(5)" [DeRujck, Georgi, Glashow 1980 2.24  
S.M.B. 1982]

$U(1)_{em}$  can be embedded purely in  $SU(5)$  (Georgi-Glashow) or partly in  $SU(5)$  and partly in  $U(1)_X$  (Flipped  $SU(5)$ ).

$$SO(10) \supset SU(5) \times U(1)_X \\ \supset [SU(3)_C \times SU(2)_L \times U(1)_{Y_5}] \times U(1)_X$$

Let  $\boxed{Y/2 = \alpha Y_5/2 + \beta X}$   $U(1)_Y$   
( $\alpha=1, \beta=0 \Rightarrow$  G.G.)

rep $[SU(3)_C \times SU(2)_L \times U(1)_{Y_5}]^{U(1)_X}$	G-G	Flipped	$Y/2$ $\alpha Y_5/2 + \beta X$
$(1, 1, +1)^1$	$e_L^+$	$N^c$	$\alpha + \beta = 0, 1$
$(3, 2, 1/6)^1$	$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\boxed{\frac{1}{6}\alpha + \beta = \frac{1}{6}}$
$(\bar{3}, 1, -2/3)^1$	$u_L^c$	$d_L^c$	$\frac{-2}{3}\alpha + \beta = +1/3, -2/3$
$(1, 2, -1/2)^{-3}$	$L_L = \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}$	$L_L = \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}$	$\boxed{\frac{-1}{2}\alpha - 3\beta = \frac{-1}{2}}$
$(\bar{3}, 1, 1/3)^{-3}$	$d_L^c$	$u_L^c$	$\frac{1}{3}\alpha - 3\beta = -2/3, 1/3$
$(1, 1, 0)^5$	$N_L^c$	$e_L^+$	$0\alpha + 5\beta = 1, 0$

$$\left. \begin{aligned} \alpha + \beta &= 0 \\ \frac{1}{6}\alpha + \beta &= \frac{1}{6} \end{aligned} \right\} \Rightarrow \alpha = -1/5, \beta = 1/5$$

$$\boxed{Y/2 = +\frac{1}{5}(-Y_5/2 + X)}$$

Flipped

Check:  $\frac{-2}{3}\alpha + \beta = \frac{-2}{3}(-\frac{1}{5}) + \frac{1}{5} = \frac{1}{3} \checkmark$   
 $\frac{1}{3}\alpha - 3\beta = \frac{1}{3}(-\frac{1}{5}) - 3\frac{1}{5} = -\frac{2}{3} \checkmark$

# HIGGS IN FLIPPED SU(5)

$$(3_c 2_L \frac{1}{2})^X \quad \frac{1}{2} = \frac{-1}{5}(\frac{1}{2}) + \frac{1}{5}X \quad (3_c 2_L \frac{1}{2})$$

$10'$	$\left\{ \begin{array}{l} (1, 1, +1)^1 \\ (3, 2, \frac{1}{6})^1 \\ (\bar{3}, 1, -\frac{2}{3})^1 \end{array} \right.$	$0 = \frac{-1}{5}(1) + \frac{1}{5}(1)$	$\left\{ \begin{array}{l} \langle (1, 1, 0) \rangle \neq 0 \\ (3, 2, \frac{1}{6}) \\ (\bar{3}, 1, \frac{1}{3}) \\ (1, 2, -\frac{1}{2}) \\ (\bar{3}, 1, -\frac{2}{3}) \\ (1, 1, +1) \end{array} \right.$
		$\frac{1}{6} = \frac{-1}{5}(\frac{1}{6}) + \frac{1}{5}(1)$	
$\frac{1}{5}^{-3}$	$\left\{ \begin{array}{l} (1, 2, -\frac{1}{2})^{-3} \\ (\bar{3}, 1, \frac{1}{3})^{-3} \end{array} \right.$	$\frac{1}{3} = \frac{-1}{5}(-\frac{1}{3}) + \frac{1}{5}(1)$	$\left\{ \begin{array}{l} (1, 2, -\frac{1}{2}) \\ (3, 1, -\frac{1}{3}) \\ (1, 2, \frac{1}{2}) \\ (\bar{3}, 1, \frac{1}{3}) \end{array} \right.$
		$-\frac{1}{2} = \frac{-1}{5}(-\frac{1}{2}) + \frac{1}{5}(-3)$	
$1^5$	$\left\{ \begin{array}{l} (1, 1, 0)^5 \end{array} \right.$	$-\frac{2}{3} = \frac{-1}{5}(\frac{1}{3}) + \frac{1}{5}(-3)$	$\left\{ \begin{array}{l} (1, 2, -\frac{1}{2}) \\ (3, 1, -\frac{1}{3}) \\ (1, 2, \frac{1}{2}) \\ (\bar{3}, 1, \frac{1}{3}) \end{array} \right.$
		$1 = \frac{-1}{5}(0) + \frac{1}{5}(5)$	
$5^{-2}$	$\left\{ \begin{array}{l} (1, 2, \frac{1}{2})^{-2} \\ (3, 1, -\frac{1}{3})^{-2} \end{array} \right.$	$-\frac{1}{2} = \frac{-1}{5}(\frac{1}{2}) + \frac{1}{5}(-2)$	$\left\{ \begin{array}{l} (1, 2, -\frac{1}{2}) \\ (3, 1, -\frac{1}{3}) \\ (1, 2, \frac{1}{2}) \\ (\bar{3}, 1, \frac{1}{3}) \end{array} \right.$
		$-\frac{1}{3} = \frac{-1}{5}(-\frac{1}{3}) + \frac{1}{5}(-2)$	
$5^2$	$\left\{ \begin{array}{l} (1, 2, -\frac{1}{2})^2 \\ (\bar{3}, 1, \frac{1}{3})^2 \end{array} \right.$	$\frac{1}{2} = \frac{-1}{5}(\frac{1}{2}) + \frac{1}{5}(2)$	$\left\{ \begin{array}{l} (1, 2, -\frac{1}{2}) \\ (3, 1, -\frac{1}{3}) \\ (1, 2, \frac{1}{2}) \\ (\bar{3}, 1, \frac{1}{3}) \end{array} \right.$
		$\frac{1}{3} = \frac{-1}{5}(\frac{1}{3}) + \frac{1}{5}(2)$	

So:  $\left\{ \begin{array}{l} \langle 1^0 (45_H) \rangle : SO(10) \rightarrow SU(5) \times U(1)_X \\ \langle 10' (16_H) \rangle : SU(5) \times U(1)_X \rightarrow SM. \end{array} \right.$

IN "FLIPPED SU(5)" MODELS IT IS ASSUMED THAT STARTING GROUP IS  $SU(5) \times U(1)_X$ , NOT  $SO(10)$ . AS WE SHALL SEE, THIS HAS ADVANTAGES. BUT IT LEAVES UNEXPLAINED UNIFICATION OF GAUGE COUPLINGS.

# LECTURE III: THE GAUGE HIERARCHY PROBLEM IN SUSY GUTS

## A) NON-SUSY GUTS, SU(5)

Adjoint Higgs ( $24_H$ ):  $\langle \Omega^\alpha_\beta \rangle = \Omega \begin{pmatrix} \frac{1}{2} & & & & \\ & \frac{1}{3} & & & \\ & & -\frac{1}{3} & & \\ & & & -\frac{1}{3} & \\ & & & & -\frac{1}{3} \end{pmatrix} = \Omega \frac{1}{2}$   
breaks  $SU(5) \rightarrow SM$ .

CONSIDER  $V(5_H)$

$$V_{\text{tree}} = \lambda (5_H^\dagger 5_H)^2 + M^2 (5_H^\dagger 5_H) + \lambda' (\text{tr } 24_H 24_H) 5_H^\dagger 5_H + \lambda'' 5_H^\dagger 24_H 24_H 5_H + M' 5_H^\dagger 24_H 5_H$$

$$= \lambda (H_\alpha^\dagger H^\alpha)^2 + M^2 (H_\alpha^\dagger H^\alpha) + \lambda' (\Omega^\alpha_\beta \Omega^\beta_\alpha) H_\alpha^\dagger H^\alpha + \lambda'' H_\alpha^\dagger \Omega^\alpha_\beta \Omega^\beta_\gamma H^\gamma + M' H_\alpha^\dagger \Omega^\alpha_\beta H^\beta$$

$$\Rightarrow (H, H_c)^\dagger \left[ \lambda'' \Omega^2 \begin{pmatrix} \frac{1}{4} I_2 & \\ & \frac{1}{4} I_3 \end{pmatrix} + (M^2 + \frac{5}{8} \lambda' \Omega^2) \begin{pmatrix} I_2 & \\ & I_3 \end{pmatrix} + M' \Omega \begin{pmatrix} \frac{1}{2} I_2 & \\ & \frac{1}{3} I_3 \end{pmatrix} \right] \begin{pmatrix} H \\ H_c \end{pmatrix}$$

$$\Rightarrow m^2(H) = \frac{1}{4} \lambda'' \Omega^2 + (M^2 + \frac{5}{8} \lambda' \Omega^2) + \frac{1}{2} M' \Omega \approx (10^2 \text{ GeV})^2$$

$$m^2(H_c) = \frac{1}{9} \lambda'' \Omega^2 + (M^2 + \frac{5}{8} \lambda' \Omega^2) - \frac{1}{3} M' \Omega \approx (10^{15} \text{ GeV})^2$$

THREE ASPECTS:

(a) Doublet-Triplet ( $2/3$ ) Splitting problem  
[WHY CANCELLATION?]

(b) Fine tuning problem:  
[Why true to all orders?]

(c) Small number problem: [Where does  $(\frac{10^2}{10^{15}})^2 = 10^{-26}$  come from?]

(B) SUSY SU(5)

NOTE: IN SUSY WE CAN LOOK AT MASSES OF THE FERMIONIC PARTNERS OF THE HIGGS DOUBLETS AND TRIPLETS (I.E. THE HIGGSINOS). THESE CAN BE READ OFF DIRECTLY FROM THE SUPERPOTENTIAL

$$W = \lambda \bar{5}_H 24_H 5_H + M \bar{5}_H 5_H + W(24_H) + \dots$$

NOTE:  $\bar{5}_H$  AND  $5_H$  ARE DISTINCT LEFT-HANDED CHIRAL SUPERFIELDS.

$$\Rightarrow \bar{5}_H (\lambda \langle 24_H \rangle + M) 5_H$$

↑ Higgsinos

$$\Rightarrow (\bar{2}_H \bar{3}_H) \left[ \lambda \Omega \begin{pmatrix} \frac{1}{2} I_2 & \\ & -\frac{1}{3} I_3 \end{pmatrix} + M \begin{pmatrix} I_2 & \\ & I_3 \end{pmatrix} \right] \begin{pmatrix} 2_H \\ 3_H \end{pmatrix}$$

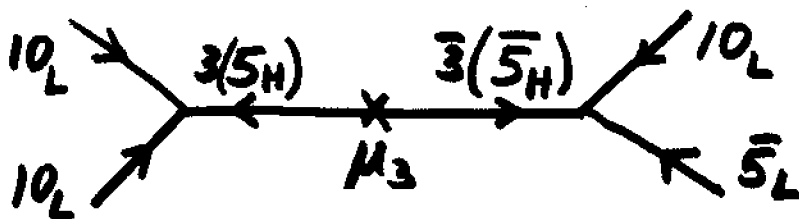
$$2 \equiv (1, 2, \frac{1}{2}), \quad \bar{2} \equiv (1, 2, -\frac{1}{2})$$

$$3 \equiv (3, 1, -\frac{1}{3}), \quad \bar{3} \equiv (\bar{3}, 1, \frac{1}{3})$$

$$\Rightarrow \begin{cases} \mu_2 = \frac{1}{2} \lambda \Omega + M \sim M_{\text{weak}} \\ \mu_3 = -\frac{1}{3} \lambda \Omega + M \sim M_G \end{cases}$$

$\mu_2$  is just " $\mu$  parameter" of MSSM

$\mu_3$  is the parameter that comes into  $p$  decay amplitude ( $d=5$  operator)





# (C) SUSY MECHANISMS TO SOLVE THE 2/3 SPLITTING PROBLEM

SUSY AUTOMATICALLY SOLVES THE "FINE-TUNING" PROBLEM, BECAUSE  $W$  IS NOT RENORMALIZED — no radiative corrections to it.

FOUR MECHANISMS HAVE BEEN PROPOSED TO SOLVE 2/3 SPLITTING PROBLEM IN SUSY GUTS:

1. "SLIDING SINGLET MECHANISM" [Litten 1981, Manopoulos + Tamvakis 1982]
2. "MISSING  $V_{EV}$  MECHANISM" [Dimopoulos + Wilczek 1981] ("DIMOPOULOS WILCZEK MECHANISM")
3. "MISSING PARTNER MECHANISM" [Georgi 1982, Masiero, Manopoulos, Tamvakis + Yanagida 1982]
4. "GIFT MECHANISM" [Berezhiani + Okuli 1989]

## SLIDING SINGLET MECHANISM.

$$W = \lambda \bar{5}_H 24_H 5_H + \lambda' \bar{5}_H 1_H 5_H + M \bar{5}_H 5_H + W(24_H) + \dots$$

$\uparrow$   
 SINGLET WHICH HAS NO OTHER COUPLINGS THAT GIVE IT POTENTIAL

absorb into  $1_H$

$$\Rightarrow V(1_H) = \left| (\lambda \langle 24_H \rangle + \lambda' \langle 1_H \rangle) \langle 5_H \rangle \right|^2 + \left| \langle \bar{5}_H \rangle (\lambda \langle 24_H \rangle + \lambda' \langle 1_H \rangle) \right|^2 + \text{SUSY terms.}$$

$$\langle 5_H \rangle = \begin{pmatrix} v_u \\ 0 \end{pmatrix} \Rightarrow V(1_H) = (\lambda \frac{1}{2} \Omega + \lambda' \langle 1_H \rangle) (|v_u|^2 + |v_d|^2) + \text{SUSY terms.}$$

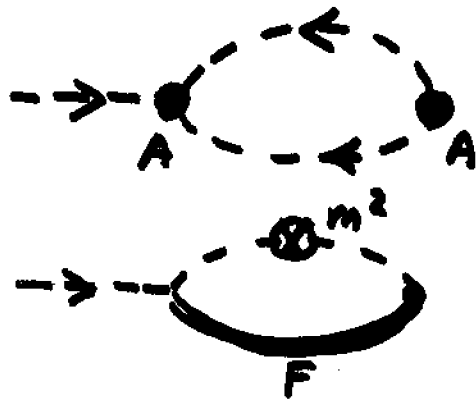
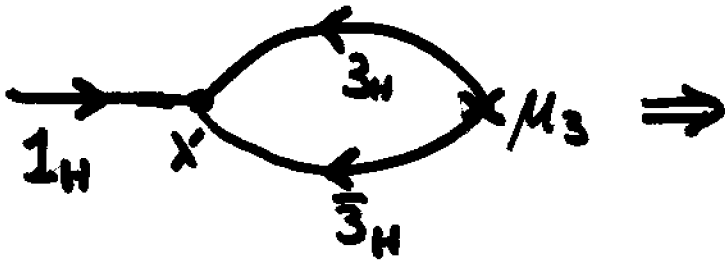
$$\langle \bar{5}_H \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}$$

i.e. IN SUSY LIMIT,

$$V(1_H) = \mu_2 (|v_u|^2 + |v_d|^2) = 0$$

$$\Rightarrow \boxed{\mu_2 = 0}, \mu_3 \neq 0.$$

HOWEVER, IN SU(5) THE SLIDING SINGLET MECH. FAILS WHEN SUSY TAKEN INTO ACCOUNT.



$$\Rightarrow V(1_H) = |\mu_2(1_H) V|^2 + \frac{\lambda' \mu_3}{16\pi^2} m_{\text{SUSY}}^2 \cdot \delta 1_H$$

$$0 = \frac{\partial V}{\partial(\delta 1_H)} = 2\mu_2 V^2 \lambda' + \frac{\lambda' \mu_3}{16\pi^2} m_{\text{SUSY}}^2$$

$$\Rightarrow \mu_2 = -\frac{1}{32\pi^2} \mu_3 \left( \frac{m_{\text{SUSY}}^2}{v^2} \right) \sim M_G$$

THE SLIDING SINGLET MECHANISM CAN BE MADE TO WORK IN SU(6) AND OTHER SU(N) GROUPS.

(Sen 1984, Barr 1998, Maekawa 2003)  
+ Yamashita hep-ph/0305116

CONSIDER  $SU(6)$

Adjoint VEV:

$$\langle 35_H \rangle = \Omega \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & -1 & & & \\ & & & -1 & & \\ & & & & -1 & \\ & & & & & -1 \end{pmatrix}$$

Can arise  
from minima  
of potential

Fundamental VEV:

$$\langle 6_H \rangle = \begin{pmatrix} \Sigma \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$\Sigma \sim M_6$ . THIS BREAKS  
 $SU(6) \rightarrow SU(5)$ .

$\langle 35_H \rangle$  FURTHER BREAKS  
IT TO  $G_{SM}$ .

$$W \supset \lambda \bar{6}_H 35_H 6_H + \lambda' \bar{6}_H 1_H 6_H$$

$$\Rightarrow (\bar{\Sigma}, \bar{H}, \bar{H}_c) \begin{bmatrix} (\lambda\Omega + \lambda' 1_H) \\ (\lambda\Omega + \lambda' 1_H) I_2 \\ (-\lambda\Omega + \lambda' 1_H) I_3 \end{bmatrix} \begin{pmatrix} \Sigma \\ H \\ H_c \end{pmatrix}$$

$$\Rightarrow V(1_H) = (\mu_2(1_H))^2 (v^2 + \Sigma^2 + \bar{\Sigma}^2) + \text{SUSY terms}$$

$$\mu_2 \sim \frac{1}{32\pi^2} \mu_3 \frac{m_{\text{SUSY}}^2}{v^2 + \Sigma^2 + \bar{\Sigma}^2}$$

There are some technical difficulties, but they can be overcome,

## (2) MISSING VEV MECHANISM

(DIMOPOULOS - WILCZEK MECHANISM)

THE PROBLEM IN  $SU(5)$  IS THAT THE GENERATORS ARE TRACELESS:

$$\langle 24_H \rangle = \Omega(\frac{1}{2}) = \Omega \begin{pmatrix} \frac{1}{2} & & & & \\ & \frac{1}{2} & & & \\ & & -\frac{1}{3} & & \\ & & & -\frac{1}{3} & \\ & & & & -\frac{1}{3} \end{pmatrix}$$

SO NEED ANOTHER TERM TO CANCEL THE  $\frac{1}{2}\Omega$  CONTRIBUTION TO  $\mu_2$ .

IN  $U(5)$  THE GENERATORS ARE NOT TRACELESS, BUT  $U(5)$  IS REALLY  $SU(5) \times U(1)$  AND THE ADJOINT "25" IS REALLY TWO IRREDUCIBLE REPS. WHOSE COUPLINGS ARE NOT RELATED BY SYMMETRY:  $25 = 24 + 1$ . SO CANCELLATION BETWEEN  $\langle 24_H \rangle$  AND  $\langle 1_H \rangle$  WOULD STILL BE ACCIDENTAL.

BUT  $SO(10) \supset U(5)$

generator (in vector rep) of  $SO(10)$

$$\begin{pmatrix} a_1 & & & & \\ -a_1 & a_2 & & & \\ & -a_2 & a_3 & & \\ & & -a_3 & a_4 & \\ & & & -a_4 & a_5 \\ & & & & -a_5 \end{pmatrix}$$



generator (in fund. rep.) of  $U(5)$

$$\begin{pmatrix} a_1 & & & & \\ & a_2 & & & \\ & & a_3 & & \\ & & & a_4 & \\ & & & & a_5 \end{pmatrix}$$

does not need to be traceless

$B-L$  corresponds to  $a_1 = a_2 = 0$   
 $a_3 = a_4 = a_5 = -\frac{2}{3}$

$\langle 45_H \rangle \propto B-L$  CAN ARISE FROM MINIMIZING A SIMPLE POTENTIAL.

eg.

$$W_{45} = \frac{\lambda}{M} \text{tr} (45_H)^4 - \lambda' M \text{tr} (45_H)^2$$

$$= \frac{2\lambda}{M} \sum_{i=1}^5 (a_i)^4 - 2\lambda' M \sum_{i=1}^5 (a_i)^2$$

$$\Rightarrow 0 = \frac{\partial W}{\partial a_i} = \frac{8\lambda}{M} a_i^3 - 4\lambda' M a_i$$

$$\Rightarrow a_i = 0 \text{ or } \sqrt{\frac{\lambda'}{2\lambda}} M \equiv \Omega$$

Multiply degenerate minimum, one of them is

$$\langle 45_H \rangle = \Omega \begin{pmatrix} \Omega & & & & \\ & \Omega & & & \\ & & -\Omega & & \\ & & & -\Omega & \\ & & & & -\Omega \end{pmatrix} = \frac{3}{2} \Omega (B-L).$$

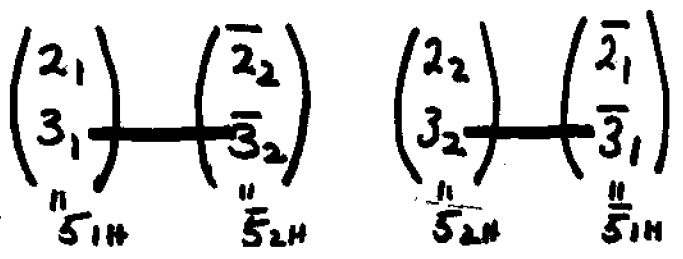
This solves 2/3 splitting problem:

$$W \supset 10_{1H} 45_H 10_{2H}$$

$$\rightarrow (\bar{5}_{1H} \binom{24+1}{H} 5_{2H} + 5_{1H} \binom{24+1}{H} \bar{5}_{2H})$$

only give mass to triplets (which have  $B-L = \pm 2/3$ )

[Need  $10_{1H}, 10_{2H}$  different as otherwise term vanishes:  
 $T^a T^b T^c$ ]



NOTE:  
 4 LIGHT  
 DOUBLET HIGGS,  
 NOT 2.

### (3) MISSING PARTNER MECHANISM

CONSIDER, IN  $SU(5)$ :

$$\begin{array}{cccc} \bar{5}_H & R_H & \bar{R}_H & 5_H \\ \left( \begin{array}{c} \bar{2} \\ \bar{3} \end{array} \right) & \left( \begin{array}{c} \text{other} \\ 3 \end{array} \right) & \left( \begin{array}{c} \overline{\text{other}} \\ \bar{3} \end{array} \right) & \left( \begin{array}{c} 2 \\ 3 \end{array} \right) \end{array}$$

SUPPOSE "other" contains no  $2 \equiv (1, 2, \frac{1}{2})$ .  
 .e. doublet "partner" of triplet in  $R$  is "missing"  
 $\Rightarrow \bar{2}, 2$  can remain light.

THIS CAN BE IMPLEMENTED IN  $SU(5)$

$$\bar{5}_H \quad R_H \quad \langle V_H \rangle$$

Needs to leave SM unbroken. i.e. contain  $(1, 1, 0)$   
 Must have  $(3, 1, -\frac{1}{3})$  but not  $(1, 2, \frac{1}{2})$

SIMPLEST POSSIBILITY is

$$R = 50 = T^{[\alpha\beta\gamma]}_{[8c]} \quad (\text{traceless})$$

$$V = 75 = T^{[\alpha\beta]}_{[8a]} \quad (\text{traceless})$$

We will now show that 50 contains  $(3, 1, -\frac{1}{3})$  but not  $(1, 2, \frac{1}{2})$ .

$$10 \times \bar{5} = 45 + 5$$

$$T^{[\alpha\beta]} T_\gamma = \left[ T^{[\alpha\beta]} T_\gamma - \frac{1}{4} T^{[\alpha\sigma} T_\sigma \delta^{\beta\gamma]} \right] \text{ (Traceless)} \\ + \frac{1}{4} \underbrace{\left[ T^{[\alpha\sigma} T_\sigma \delta^{\beta\gamma]} \right]}_5$$

$$\bar{10} \times \bar{10} = 50_S + 45_A + 5_S$$

$$T^{[\alpha\beta\gamma]} T_{[\delta\epsilon]} = \left[ T^{[\alpha\beta\gamma]}_{[\delta\epsilon]} - (\text{traces}) \right] \text{ (Traceless)} \\ + \left[ \underbrace{T^{[\alpha\beta\sigma} T_{[\delta\sigma} \delta^{\gamma\epsilon]}]}_{45} - (\text{traces}) \right] \\ + \left[ \underbrace{T^{[\alpha\sigma\tau} T_{\sigma\tau} \delta^{\beta\gamma]}_{[\delta\epsilon]}}_5 \right]$$

$$\bar{50} \quad (\bar{10} \times \bar{10})_S = 50 + 5$$

$$\begin{bmatrix} (1, 1, -1) \\ (\bar{3}, 2, -\frac{1}{6}) \\ (3, 1, \frac{2}{3}) \end{bmatrix} \times \begin{bmatrix} (1, 1, -1), (\bar{3}, 2, -\frac{1}{6}), (3, 1, \frac{2}{3}) \end{bmatrix} = \begin{bmatrix} (1, 1, -2)_S & (\bar{3}, 2, -\frac{7}{6}) & (3, 1, -\frac{1}{3}) \\ (\bar{3}, 2, -\frac{7}{6}) & (3, 1, -\frac{1}{3})_S & (8, 2, \frac{1}{2}) \\ (3, 1, -\frac{1}{3}) & (8, 2, \frac{1}{2}) & (\bar{6}, 1, \frac{4}{3})_S \end{bmatrix}$$

$(6, 3, -\frac{1}{3})_S$   
 $(3, 3, -\frac{1}{3})_A$   
 $(6, 1, -\frac{1}{3})_A$   
 $(8, 2, \frac{1}{2})$   
 $(1, 2, \frac{1}{2})$   
 $(3, 1, \frac{4}{3})_A$

$$50 = (1, 1, -2) + \cancel{(3, 1, -\frac{1}{3})} + (6, 3, -\frac{1}{3}) + (\bar{6}, 1, \frac{4}{3}) \\ + (\bar{3}, 2, -\frac{7}{6}) + (3, 1, -\frac{1}{3}) + (8, 2, \frac{1}{2}) + \cancel{(1, 2, \frac{1}{2})}$$

crossed out reps are = 5)

$$\text{So } 50 \supset (3, 1, -\frac{1}{3}) \text{ but } \not\supset (1, 2, \frac{1}{2})$$





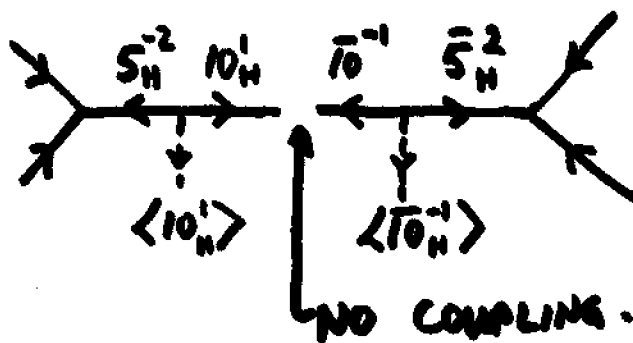
IN FLIPPED  $SU(5)$  THE MISSING PARTNER MECHANISM WORKS ELEGANTLY (IF NOT UNIFIED IN  $SO(10)$ !) 2.11

SUPPOSE

$$\begin{pmatrix} \bar{5}_H^{-2} & 10_H^1 & 10_H^0 \end{pmatrix} + \begin{pmatrix} \bar{5}_H^{-2} & \bar{10}_H^{-1} & \bar{10}_H^{-1} \end{pmatrix} \\
 \begin{pmatrix} (1, 2, -\frac{1}{2}) \\ (3, 1, -\frac{1}{3}) \end{pmatrix} \begin{pmatrix} (1, 1, 0) \\ (3, 2, \frac{1}{6}) \\ (\bar{3}, 1, \frac{1}{3}) \end{pmatrix} \begin{pmatrix} \langle (1, 1, 0) \rangle \\ (3, 2, \frac{1}{6}) \\ (\bar{3}, 1, \frac{1}{3}) \end{pmatrix} + \begin{pmatrix} (1, 2, \frac{1}{2}) \\ (\bar{3}, 1, \frac{1}{3}) \end{pmatrix} \begin{pmatrix} (1, 1, 0) \\ (3, 2, \frac{1}{6}) \\ (3, 1, -\frac{1}{3}) \end{pmatrix} \begin{pmatrix} \langle (1, 1, 0) \rangle \\ (\bar{3}, 2, -\frac{1}{6}) \\ (3, 1, -\frac{1}{3}) \end{pmatrix}$$

THE "OTHER" FIELDS GET EATEN BY HIGGS MECHANISM!

- ⇒ No NEED TO COUPLE  $M 10_H^1 \bar{10}_H^{-1}$
- ⇒  $d=5$  PROTON DECAY OPERATORS CAN BE SUPPRESSED.



HOWEVER, IF  $SU(5) \times U(1)_X$  UNIFIED IN  $SO(10)$ , THEN "MISSING PARTNER" IS NOT MISSING.

$$10_H^1 \subset 16_H \quad \text{WHICH CONTAINS } \bar{5}^{-3} = \begin{pmatrix} (1, 2, -\frac{1}{2}) \\ (\bar{3}, 1, -\frac{1}{3}) \end{pmatrix} \\
 \bar{5}_H^{-2} \bar{5}_H^{-3} \langle 10_H^1 \rangle$$

# (D) SO(10): GAUGE HIERARCHY AND PROTON DECAY

As we saw, the 2/3 splitting problem in SO(10) can be solved by assuming  $\langle 45_H \rangle \propto B-L$  and having a term

$$\lambda 10_{1H} 45_H 10_{2H} \Rightarrow \left[ \bar{5}_{1H} \overbrace{(24_H + 1_H)}^{= (B-L)\Omega} 5_{2H} + 5_{1H} (24_H + 1_H) \bar{5}_{2H} \right]$$

$$\Rightarrow \begin{pmatrix} 2_1 \\ 3_1 \end{pmatrix} \begin{pmatrix} \bar{2}_2 \\ \bar{3}_2 \end{pmatrix} \begin{pmatrix} 2_2 \\ 3_2 \end{pmatrix} \begin{pmatrix} \bar{2}_1 \\ \bar{3}_1 \end{pmatrix}$$

To avoid having 4 light doublets, one needs to give mass to  $\bar{2}_2, 2_2$ . (Assume MSSM doublets of Higgs  $H_u, H_d$  are  $2_1, \bar{2}_1$ ).

Simplest:  $M 10_{2H} 10_{2H}$

Look at masses of color-triplet Higgsinos:

$$(\bar{3}_1, \bar{3}_2) \begin{pmatrix} 0 & \frac{2}{3}\lambda\Omega \\ \frac{2}{3}\lambda\Omega & M \end{pmatrix} \begin{pmatrix} 3_1 \\ 3_2 \end{pmatrix}$$

$$\equiv \mathcal{M}_3$$

PROTON DECAY AMPLITUDE  $\propto (\mathcal{M}_3^{-1})_{11}$

$$\mathcal{M}_3^{-1} = \begin{pmatrix} \frac{-M}{(\frac{2}{3}\lambda\Omega)^2} & \frac{1}{(\frac{2}{3}\lambda\Omega)} \\ \frac{1}{(\frac{2}{3}\lambda\Omega)} & 0 \end{pmatrix}$$

Can we make p decay amplitude small by setting  $M \ll M_G$ ? YES, BUT

MAKING  $M$  SMALL SUPPRESSES HIGGSINO-MEDIATED PROTON DECAY, BUT MAKES UNIFICATION OF GAUGE COUPLINGS WORSE:

$$\delta E_3 \Big|_{\frac{2}{3}} = \frac{\delta \alpha_3(M_G)}{\alpha_3(M_G)} = \frac{3\alpha_G}{5\pi} \cdot [\ln|\det M_3| - \ln|\det M_2|]$$

NOTE: ONLY "SPLIT MULTIPLETS" OF  $SU(5)$  CONTRIBUTE AT ONE LOOP TO RGE SPLITTING OF THE THREE SM GAUGE COUPLINGS. THUS, IF  $\det M_3 = \det M_2 \Rightarrow \delta E_3 = 0$

TRIPLET MASSES:  $M_3 = \begin{bmatrix} 0 & \frac{2}{3}\lambda\Omega \\ \frac{2}{3}\lambda\Omega & M \end{bmatrix}, M_2 = \begin{bmatrix} \mu & 0 \\ 0 & M \end{bmatrix}$

$$\Rightarrow \delta E_3 \Big|_{\frac{2}{3}} = - \frac{3\alpha_G}{5\pi} \ln \left( \mu \frac{M}{(\frac{2}{3}\lambda\Omega)^2} \right)$$

$\approx 0.008$   $(M_3^{-1})_{11}$  ← in p decay AMPLITUDE!

IN MINIMAL SUSY  $SU(5)$ , NEED  $(M_3^{-1})_{11}$  at least  $1/20 \times$  (and typically  $1/100 \times$ ) the value that gives good unification to sufficiently suppress p decay.  $\Rightarrow \delta E_3 \approx +0.04$   
 VERY GENERAL RESULT (SM. B. hep-ph/9806217)

- F (1) ONLY MASSES OF THE FORM  $(\bar{3}_H 3_H)$  OR  $(\bar{2}_H 2_H + \bar{3}_H 3_H) = (\bar{3}_H 5_H)$ , BUT NOT  $(\bar{2}_H 2_H)$ , AND
- (2) NO ARTIFICIAL CANCELLATIONS

$\Rightarrow$  SUPPRESSION OF P DECAY AMPLITUDE INCREASES  $E_3$ .

HOWEVER, SUPPOSE:  $10_{1H} 45_{1H} 10_{2H} + 10_{2H} (45'_{1H})^2 10_{2H} / M_G$   
 $\uparrow$  B-L  $\uparrow$   $I_{3R}$

$$H_u \begin{pmatrix} 2_1 \\ 3_1 \end{pmatrix} \begin{pmatrix} \bar{2}_2 \\ \bar{3}_2 \end{pmatrix} \begin{pmatrix} 2_2 \\ 3_2 \end{pmatrix} \begin{pmatrix} \bar{2}_1 \\ \bar{3}_1 \end{pmatrix} H_u$$

THIS REQUIRES 2 ADJUNCT HIGGS

(E) STABILITY OF GAUGE HIERARCHY 3.14  
IN SO(10) MODELS [K.S. Babu + SMB 1993]

$$M_2 = \begin{bmatrix} \mu & \lambda \langle 45_H \rangle \\ \lambda \langle 45_H \rangle & M \end{bmatrix}$$

$\uparrow$  Need this  $\approx \sqrt{M_W M_G}$   
 $\uparrow$  Need this  $\approx M_W$

- Must forbid  $M_G(10_{1H} 10_{1H})$  by some symmetry, Also  $M_G(10_{1H} 10_{2H})$ .
- Must prevent terms that destabilize  $\langle 45_H \rangle \propto B-L$

$$\langle 45_H \rangle = \begin{bmatrix} b \\ -b \\ a \\ -a \\ a \\ -a \\ a \\ -a \end{bmatrix} = \frac{3}{2} a (B-L) + 2b (I_{3R})$$

Must have  $b/a \approx \sqrt{M_W/M_G} \sim 10^{-7}$

THERE IS A TECHNICAL DIFFICULTY ASSOCIATED WITH COUPLING ADJOINT HIGGS SECTOR AND SPINOR HIGGS SECTOR

$$W_{\text{Higgs}} = W(45_H) + W(\overline{16}_H, 16_H) + \underbrace{W_{45-16}}_{?} + 10_{1H} 45_H 10_{2H}$$

IF THERE ARE NO COUPLINGS (DIRECT OR INDIRECT) OF  $45_H$  TO  $\overline{16}_H + 16_H$ , THEN BAD GOLDSTONE MODES RESULT.

WITHOUT  $45_H$  COUPLING TO  $\bar{16}_H + 16_H$ , THE RELATIVE DIRECTION OF  $\langle 45_H \rangle$  AND  $\langle 16_H \rangle$  IS NOT DETERMINED BY  $W \Rightarrow$  MASSLESS MODE.

WHAT ARE THE GOLDSTONE MODES?

LOOK AT WHICH GENERATORS ARE BROKEN BY BOTH  $\langle 45_H \rangle$  AND  $\langle 16_H \rangle$ . THERE WOULD BE TWO "WOULD-BE GOLDSTONE BOSONS" FOR EACH SUCH GENERATOR, OF WHICH ONLY ONE CAN BE "EATEN".

$\langle 1^5(16_H) \rangle$  BREAKS  $SO(10) \rightarrow SU(5)$

$$\begin{aligned} \Rightarrow 45 &= 24 + \overbrace{1 + 10 + 10}^{\text{broken}} \\ &= (1, 1, 0) + \sqrt{2}(1, 1, 0) + \sqrt{2}(1, 1, +1) + \sqrt{2}(1, 1, -1) \\ &\quad + (3, 2, -5/6) \quad + \sqrt{2}(3, 2, 1/6) + \sqrt{2}(\bar{3}, 2, -1/6) \\ &\quad + (3, 2, 5/6) \quad + \sqrt{2}(3, 1, -2/3) + \sqrt{2}(3, 1, 2/3) \\ &\quad + (8, 1, 0) \end{aligned}$$

$\langle 45_H \rangle \propto B-L$  BREAKS  $SO(10) \rightarrow SU(3)_C \times U(1)_{B-L} \times SU(2)_L \times SU(2)_R$

$$\begin{aligned} 45 &= (15, 1, 1) + (1, 3, 1) + (1, 1, 3) + (6, 2, 2) \text{ of P-S} \\ &= (1^0, 1, 1) + (1^0, 3, 1) + (1^0, 1, 3) + \sqrt{2}(3^{-2/3}, 2, 2) \\ &\quad (8^0, 1, 1) \quad \sqrt{2}(3^{2/3}, 2, 2) \\ &\quad \sqrt{2}(3^{1/3}, 1, 1) \\ &\quad \sqrt{2}(\bar{3}^{-4/3}, 1, 1) \end{aligned}$$

$\Rightarrow$  GENERATORS IN  $(\bar{3}, 2, -1/6) + (3, 2, 1/6)$   
 $(\bar{3}, 1, -2/3) + (3, 1, 2/3)$

ARE TWICE BROKEN

THE SIMPLEST WAY TO COUPLE  $45_H$  TO  $\overline{16}_H + 16_H$  IS NO GOOD:

$$\overline{16}_H 45_H 16_H$$

$$\langle 45_H \rangle = \frac{2}{3} a (B-L) + 2 b (I_{3R})$$

$$\langle 1(16_H) \rangle \sim M_G, \quad \langle 1(\overline{16}_H) \rangle \sim M_G.$$

↑ has quantum numbers of  $N_L^c$  :  $\begin{cases} B-L = +1 \\ I_{3R} = -\frac{1}{2} \end{cases}$

$$\Rightarrow W \supset \underbrace{\# M_G}_{\text{From } (45_H)^2} b^2 + \# [1(\overline{16}_H) b 1(16_H)]$$

$$0 = -F_b^* = \frac{\partial W}{\partial b} = \# M_G b + \# \langle 1(\overline{16}_H) \rangle \langle 1(16_H) \rangle$$

$$\Rightarrow \boxed{b \sim M_G} \quad (\text{destroys hierarchy})$$

There are various ways to couple  $45_H$  to  $\overline{16}_H, 16_H$  without destroying gauge hierarchy:

$$\left\{ \begin{array}{l} \overline{16}_H 45_x 16_H + \underbrace{45_x 45_{B-L} 45_{I_{3R}}}_{\text{THREE DISTINCT ADJUNCT HIGGS}} + \dots \quad (\text{Babu Barr}) \\ \overline{16}_H (45_H + 1_H) \underset{\uparrow \text{VEV}=0}{16'_H} + \overline{16}'_H (45_H + 1'_H) \underset{\uparrow \text{VEV}=0}{16_H} \quad (\text{Barr Raby}) \end{array} \right.$$

# LECTURE IV : FERMION MASSES IN SUSY GUTS

WE HAVE ALREADY SEEN THAT IN "MINIMAL SU(5)"

$$(Y_U)_{AB} (10_A 10_B) 5_H \rightarrow M_U = M_U^T = Y_U V_U$$

$$(Y_D)_{AB} (10_A \bar{5}_B) \bar{5}_H \rightarrow M_D = M_D^T = Y_D V_D$$

$$(Y_N)_{AB} (\bar{5}_A 1_B) 5_H \rightarrow M_N = Y_N V_U$$

$$M_D = M_D^T \Rightarrow \begin{cases} m_b^0 = m_\tau^0 & \text{within 20\%} \\ m_s^0 = m_\mu^0 & (\text{expt} \approx \frac{1}{3} m_\mu^0) \\ m_d^0 = m_e^0 & (\text{expt} \approx 3 m_e^0) \end{cases}$$

## 1) GEORGI - JARLSKOG

$$SU(5): (10_{1L} 10_{2L} 10_{3L}) \begin{bmatrix} 0 & \langle \bar{5}_H \rangle & 0 \\ \langle \bar{5}_H \rangle & \langle \overline{45}_H \rangle & 0 \\ 0 & 0 & \langle \bar{5}_H \rangle \end{bmatrix} \begin{pmatrix} \bar{5}_{1L} \\ \bar{5}_{2L} \\ \bar{5}_{3L} \end{pmatrix} \quad \begin{array}{l} \text{Yukawa} \\ \text{constants} \\ \text{not shown} \end{array}$$

$$(10_L \bar{5}_L) \bar{5}_H : \begin{cases} \psi^{a2} \psi_a \langle H_2 \rangle \rightarrow d_L d_L^c \langle H_d \rangle \\ \psi^{i2} \psi_i \langle H_2 \rangle \rightarrow \ell_L^+ \ell_L^- \langle H_d \rangle \end{cases}$$

$$(10_L \bar{5}_L) \overline{45}_H : \begin{cases} \psi^{a2} \psi_a \langle H_{a2}^a \rangle \rightarrow (-\frac{1}{3}) d_L d_L^c \langle H_d' \rangle \\ \psi^{i2} \psi_i \langle H_{i2}^{i'} \rangle \rightarrow \ell_L^+ \ell_L^- \langle H_d' \rangle \end{cases}$$

NOTE:  $\overline{45}$  is traceless  $\Rightarrow \sum_{a=3,4,5} H_{a2}^a + H_{i2}^{i'} + \cancel{H_{11}} = 0$

$$\Rightarrow \langle H_{a2}^a \rangle = -\frac{1}{3} \langle H_{i2}^{i'} \rangle$$

NO SUM

$$M_L^0 = \begin{bmatrix} 0 & C & \\ C & B & \\ & & A \end{bmatrix}$$

$$M_D^0 = \begin{bmatrix} C & \frac{1}{3}B & \\ & \frac{1}{3}B & \\ & & A \end{bmatrix}$$

LET  $A \gg B \gg C$  ↓ diagonalize

$$\begin{bmatrix} \frac{-C^2}{B} & & \\ & B & \\ & & A \end{bmatrix}$$

$$\begin{bmatrix} \frac{+3C^2}{B} & & \\ & \frac{1}{3}B & \\ & & A \end{bmatrix}$$

NOTE:  $\det M_L^0 = \det M_D^0 = -AC^2$

$$\Rightarrow m_s^0 \cong \frac{1}{3} m_\mu^0 \quad m_d^0 \cong 3 m_e^0$$

These factors of  $\frac{1}{3}$ , 3 are called "Georgi Jarlskog factors."

SUCH FACTORS OF  $\frac{1}{3}$ , 3 ARISE VERY NATURALLY IN GUTS. THEY COME FROM FACT THAT THERE ARE 3 COLORS

ANOTHER WAY TO GET SUCH FACTORS

$$B-L = \begin{pmatrix} \frac{1}{3} & & & \\ & \frac{1}{3} & & \\ & & \frac{1}{3} & \\ & & & -1 \end{pmatrix} \quad (\text{generator of } SU(4)_c)$$

=  $\frac{1}{3}$  for quarks, -1 for leptons.

NOTE FORM OF 12 BLOCK ABOVE:  $\begin{pmatrix} 0 & C \\ C & B \end{pmatrix}$ .

Suggested independently by Weinberg, Wilczek + Zee, Fritzsche in 1979 to explain Cabibbo angle.

"TEXTURE"



# TEXTURES AND MIXING ANGLES

CONSIDER A TWO-FAMILY MODEL:

$$(\bar{d}_L \bar{s}_L)^{(0)} \begin{pmatrix} 0 & c' \\ c' & B' \end{pmatrix} \begin{pmatrix} d_R^{(0)} \\ s_R^{(0)} \end{pmatrix} + (\bar{u}_L \bar{c}_L)^{(0)} \begin{pmatrix} 0 & c \\ c & B \end{pmatrix} \begin{pmatrix} u_R \\ c_R \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} d_L \\ s_L \end{pmatrix} = \begin{pmatrix} \cos \theta_{dL} & \sin \theta_{dL} \\ -\sin \theta_{dL} & \cos \theta_{dL} \end{pmatrix} \begin{pmatrix} d_L^{(0)} \\ s_L^{(0)} \end{pmatrix} \quad \text{similar for } d_R, s_R \quad (\theta_{dR})$$

$$\begin{pmatrix} u_L \\ c_L \end{pmatrix} = \begin{pmatrix} \cos \theta_{uL} & \sin \theta_{uL} \\ -\sin \theta_{uL} & \cos \theta_{uL} \end{pmatrix} \begin{pmatrix} u_L^{(0)} \\ c_L^{(0)} \end{pmatrix} \quad \text{similar for } u_R, c_R \quad (\theta_{uR})$$

$$\Rightarrow \tan 2\theta_{dL} = \frac{2c'}{B'} \Rightarrow \theta_{dL} \approx \frac{c'}{B'} \approx \sqrt{\frac{m_d}{m_s}}$$

$$\tan 2\theta_{uL} = \frac{2c}{B} \Rightarrow \theta_{uL} \approx \frac{c}{B} \approx \sqrt{\frac{m_u}{m_c}}$$

bibbo:

$$\theta_C = \theta_{dL} - \theta_{uL} \approx \sqrt{\frac{m_d}{m_s}} - e^{i\phi} \sqrt{\frac{m_u}{m_c}}$$

$\approx 0.2 \qquad \qquad \qquad \approx 0.2 \qquad \qquad \qquad \approx 0.07$

Old numerology:  $\sin \theta_C \approx m_\mu / m_K \approx \sqrt{m_d / m_s}$

Note that here matrices are assumed symmetric in SU(5), justified for  $M_u$ , not  $M_D, M_L$ .

However, this assumption reduces # of parameters and thus allows a prediction.

Also gives  $\theta_L = \theta_R$

MILAR TEXTURES FOR 23 BLOCK GIVE RELATIONS LIKE

$$\left\{ \begin{array}{l} V_{cb} = \sin \theta_{23}^{\text{charm}} \approx \sqrt{m_s / m_b} - e^{i\phi'} \sqrt{m_c / m_e} \\ \sim 0.04 \qquad \qquad \sim 0.14 \qquad \qquad \sim 0.04 \\ V_{\mu 3} = \sin \theta_{23}^{\text{lepton}} \approx \sqrt{m_\mu / m_\tau} - e^{i\phi''} \sqrt{m_2 / m_3} \\ \sim 0.7 \qquad \qquad \sim 0.25 \qquad \qquad \sim 0.15 \end{array} \right.$$

### 3) AN SO(10) MODEL OF QUARK

4.4

#### AND LEPTON MASSES

(Albright, Babu, Barr 1998)  
(Albright, Barr 1999)

(Similar model by  
Babu, Pati, Wilczek)

THE VEVs:

$$O(M_G): \begin{cases} \langle 45_H \rangle \propto B-L \\ \langle 16_H \rangle \propto 1(16) \\ \langle \bar{16}_H \rangle \propto 1(\bar{16}) \end{cases}$$

$$O(M_W): \begin{cases} \langle 10_H \rangle \propto \begin{bmatrix} 2(5(10)) \\ \bar{2}(\bar{5}(10)) \end{bmatrix} \\ \langle 16'_H \rangle \propto \bar{2}(\bar{5}(16)) \end{cases}$$

#### THIRD FAMILY MASSES

$$16_3 \ 16_3 \ \langle 10_H \rangle \rightarrow M_U, M_D, M_L, M_N \propto \begin{pmatrix} 0 & \\ & 0 \\ & & 1 \end{pmatrix} \begin{matrix} m_1 \\ m_2 \\ m_3 \end{matrix}$$

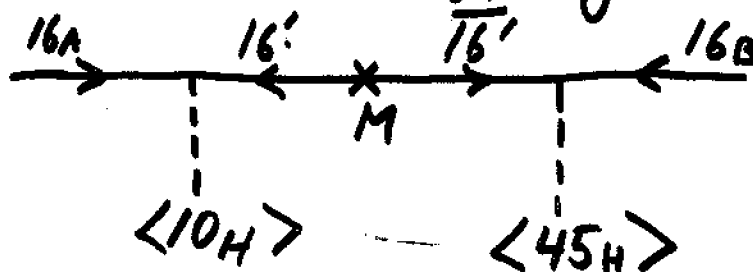
#### SECOND FAMILY

Need Georgi-Jarlskog factors

This can be achieved if the  $\langle 45_H \rangle$  is involved. Simplest effective operator is

$$16_A \ 16_B \ \langle 10_H \rangle \langle 45_H \rangle / M_G$$

This can arise from "integrating out"  $\bar{16}' + 16'$



There are two ways to contract the  $SO(10)$  indices in the product  $16 \ 16 \ 10 \ 45$ : 4.5

$$16 \times 16 = 10_S + 120_A + 126_S$$

$$10 \times 45 = 10 + 120 + 320$$

So  $16 \times 16$  must be contracted in  $10$  or  $120$ .

If  $\langle 45_H \rangle \propto B-L$  then only  $120$  channel contributes to quark and lepton masses

Thus this term is antisymmetric in flavor

PROOF  $\langle 10_H \rangle \sim (1, 2, 2)$  of Pati-Salam Group.

$\langle 45_H \rangle \sim (15, 1, 1)$  ( $\propto B-L =$  a generator of  $SU(4)_c$ )

$$\Rightarrow \langle 10_H \rangle \langle 45_H \rangle \sim (15, 2, 2)$$

But, this is not in  $10$  of  $SO(10)$ .

$\Rightarrow$  it must be in  $120$ .

CONSIDER, THEN,

$$(16_2 \ 16_3) \langle 10_H \rangle \langle 45_H \rangle / M_G$$

$$\Rightarrow M_U = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{bmatrix} m_u, \quad M_D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{bmatrix} m_D$$

$$M_N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{bmatrix} m_u, \quad M_L = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{bmatrix} m_D$$

Not realistic. FOUR PROBLEMS:

(1) Georgi-Jarlskog factors are  $1/9$  not  $1/3$

(2)  $M_D \propto M_U \Rightarrow$  No CKM MIXING.

eg.  $V_{cb} = 0$

(3)  $M_D \propto M_U \Rightarrow m_c^0/m_t^0 = m_s^0/m_b^0$   
 $\sim 0.0025 \quad \sim 0.02$

(4) The atmospheric  $\nu$  angle  $\theta_{13} = \mathcal{O}(\epsilon) \ll 1$

Need another contribution to 23 block that makes  $M_U$  not proportional to  $M_D$ .

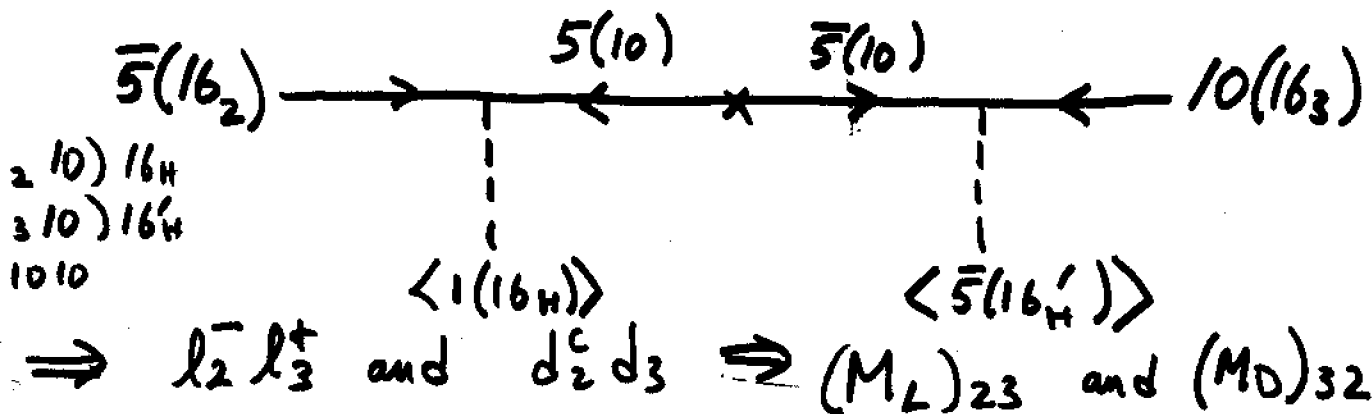
$\langle 45_H \rangle$  does not distinguish U from D.

Simplest possibility:  $\langle 16_H \rangle$  contributes.

Consider

$$\begin{array}{cccc}
 16_2 & 16_3 & 16'_H & 16_H / M_G \\
 \downarrow & \downarrow & \downarrow & \downarrow \\
 10_2 & \bar{5}_3 & \langle \bar{5}'_H \rangle & \langle 1_H \rangle / M_G \\
 \text{BUT } \underline{\text{NO}} & 10_2 & 10_3 & \cancel{\langle 1_H \rangle / M_G}
 \end{array}$$

THREE WAYS OF CONTRACTING INDICES. ASSUME OPERATOR COMES FROM DIAGRAM:



$$M_U = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{bmatrix} M_U \quad M_D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & \sigma^{-\epsilon/3} & 1 \end{bmatrix} M_D \quad \text{v.7}$$

$$M_N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{bmatrix} M_U \quad M_L = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma^{-\epsilon} \\ 0 & \epsilon & 1 \end{bmatrix} M_D$$

ASSUME  $\epsilon \ll \sigma \sim 1$

Then: (1) Georgi - Jarlskog factor is  $\approx 1/3$

(2)  $M_U \text{ not } \propto M_D \Rightarrow V_{cb} \neq 0$

in fact  $V_{cb} \approx \frac{\epsilon}{3} \frac{\sigma^2}{1+\sigma^2}$

(3)  $\frac{m_c^0}{m_t^0} \approx (\epsilon/3)^2 \ll \frac{m_s^0}{m_b^0} \approx \frac{\epsilon}{3} \frac{\sigma}{1+\sigma^2}$

(4)  $V_{ub} \sim \mathcal{O}(\sigma) \sim \mathcal{O}\left(\frac{V_{cb}}{m_s/m_b}\right) \sim 1$

FIRST FAMILY CAN BE INCLUDED SIMPLY

$$M_U = \begin{bmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{bmatrix} M_U \quad M_D = \begin{bmatrix} 0 & \delta & \delta' \\ \delta & 0 & \epsilon/3 \\ \delta' & \sigma^{-\epsilon/3} & 1 \end{bmatrix} M_D$$

$$M_N = \begin{bmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{bmatrix} M_U \quad M_L = \begin{bmatrix} 0 & \delta & \delta' \\ \delta & 0 & \sigma^{-\epsilon} \\ \delta' & \epsilon & 1 \end{bmatrix} M_D$$