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ETERNAL INFLATION: DOES IT NEED A BEGINNING?

Special Lecture

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ETERNAL INFLATION: DOES IT NEED A BEGINNING?

— Alan Guth (MIT)

gr-qc/0110012, PRL 90, 151301 (2003), with Arvind Borde and Alex Vilenkin
Inflationary Spacetimes Are Incomplete in Past Directions

OUTLINE

1. Background
   - A) Eternal inflation
   - B) $ds^2 = dt^2 - e^{2Ht} d\vec{x}^{\,2}$, and geometric incompleteness
   - C) Borde & Vilenkin, PRL **72**, 3305 (1994).
2. Our Argument
   - A) Warmup — Nonrelativistic calculation
   - B) Real Thing — Relativistic calculation
3. Conclusions

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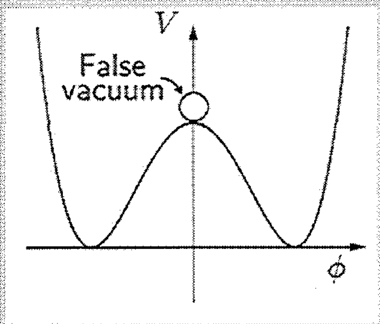
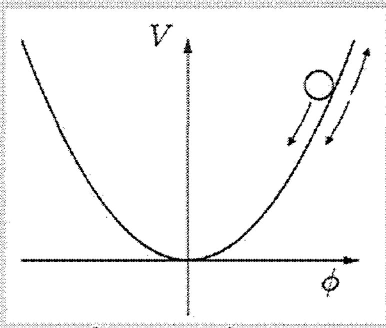
ETERNAL INFLATION



New Inflation



Chaotic Inflation

- New Inflation: False vacuum decays exponentially, but much slower than the exponential expansion. ∴ the volume of false vacuum increases exponentially with time. (Steinhardt, Vilenkin, 1983)

- Chaotic Inflation: Random quantum fluctuations are superimposed on the classical downward motion of the field. In a Hubble time, each Hubble volume expands to $e^3 \approx 20$ Hubble volumes, each of which behaves independently. If P(upward fluctuation) $> 1/20$, then the volume with $\phi > \phi_{\text{initial}}$ increases with time. (Linde, 1986)

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De Sitter Space in Flat Coordinates

$$ds^2 = dt^2 - e^{2Ht} d\vec{x}^{\,2}$$

Is this a universe with no initial singularity? No. This spacetime really has a past boundary.

Can be seen most easily by embedding in a (4+1)-D Minkowski space:

$$ds^2 = d\vec{X}^{\,2} + dW^2 - dV^2 \,, \qquad \vec{X}^{\,2} + W^2 - V^2 = H^{-2}$$

$$t = H^{-1} \ln[H(V + W)] \,, \qquad \vec{x} = e^{-Ht} \vec{X} \,.$$

So $t = -\infty$ is not really infinitely far away!

For timelike geodesics, $\vec{x} = const$ has infinite length. Any other backward-going timelike or null geodesic has finite length. (One could complete the spacetime, but the added region would be contracting, and would decay if the de Sitter space is supported by a false vacuum.)

Our theorem will show that this is always the case for "globally expanding" spacetimes: comoving backwards-going geodesics can have infinite length, but all others are bounded.

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Borde and Vilenkin, 1994

They proved that eternally-inflating spacetimes must be past-incomplete, by making a number of assumptions, including:

- Past volume difference condition: If $\hat{P}$ is in the future of P, then the set of points that are in the past of $\hat{P}$ but not in the past of P have a spacetime volume that is finite.
- Weak energy condition: $\rho + p > 0$. (More generally, the condition is that $T_{00} > 0$ in all frames.)

PROBLEM: The weak energy condition is violated by quantum fluctuations.

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Our Argument: Simple Summary

- When a geodesic observer moves through an expanding space, she slows down (redshift).
- The redshift is a purely kinematical effect, independent of the dynamics of GR, and therefore independent of any energy conditions.

  To see this, imagine a universe with 1D space, and imagine that the comoving particles are cars. Consider a geodesic observer moving relative to them. In observer's frame, the cars are moving by. If the first car passes at 70 km/hr, and the cars are moving apart, then the next car will not pass at 80 km/hr!
- If we follow the observer backwards in an expanding universe, she speeds up. But, the calculation shows that if $H > H_{\min} > 0$ in the past, then she will reach the speed of light in a finite proper time.

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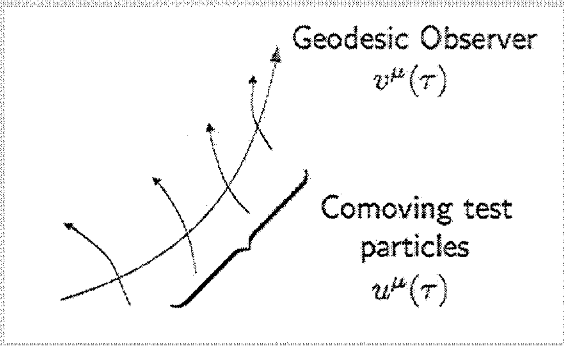
Local Definition of Hubble Parameter H

Consider a geodesic observer (timelike or null trajectory) moving through an expanding universe:



The observer measures the velocities $u^\mu(\tau)$ of the comoving geodesic test particles that she passes, and from their motion she infers a local, unidirectional Hubble parameter

$$H \equiv \frac{\Delta v_{\rm radial}}{\Delta r} .$$

The test particles need not exist: we can define their trajectories arbitrarily, but acceleration must vanish.

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Warmup: The Nonrelativistic Argument

- Work in rest frame of observer. Velocity of passing test particles is $\vec{u}(\tau)$. Particle 1 passes at time τ_1, particle 2 passes at time $\tau_2 = \tau_1 + \Delta\tau$.
- Consider standing on particle 2, using particle 1 to infer a Hubble parameter:

$$\Delta\vec{r} = \vec{r}_1 - \vec{r}_2 = \vec{u}\,\Delta\tau .$$

$$\Delta\vec{u} = \vec{u}_1 - \vec{u}_2 = -\frac{d\vec{u}}{d\tau}\,\Delta\tau .$$

$$\text{Radial velocity} \equiv \Delta u_r = \Delta\vec{u}\cdot\frac{\Delta\vec{r}}{|\Delta\vec{r}|} .$$

$$H \equiv \frac{\Delta u_r}{|\Delta\vec{r}|} = -\frac{1}{|\vec{u}|^2}\,\vec{u}\cdot\frac{d\vec{u}}{d\tau} .$$

- Let $v_{\rm rel} = |\vec{u}|$. Then

$$\boxed{H = -\frac{d}{d\tau}\ln\left(v_{\rm rel}\right) .}$$

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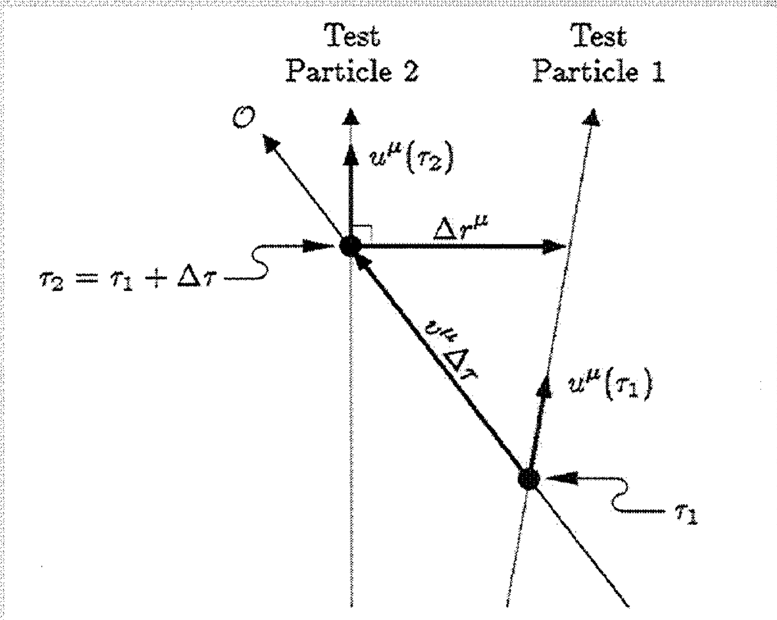
The Real Thing: The Relativistic Argument



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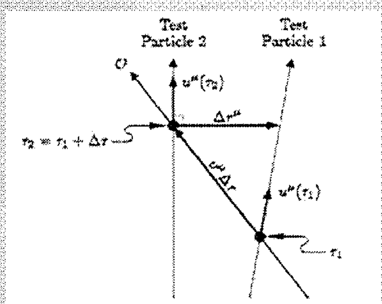

Let

$$\gamma \equiv u_\mu \, v^\mu = \frac{1}{\sqrt{1 - v_{\rm rel}^2}} \,,$$

where the 2nd equality holds only for timelike geodesics.

$$\Delta r^\mu = -v^\mu \, \Delta\tau + \text{const}\, u^\mu$$

Find *const* such that $u^\mu \, \Delta r_\mu = 0$:

$$\Delta r^\mu = -v^\mu \, \Delta\tau + \gamma u^\mu \, \Delta\tau \,.$$

$$\Delta r^2 \equiv |\Delta r^\mu|^2 = (\gamma^2 - \kappa)\Delta\tau^2 \,,$$

where

$$\kappa \equiv v^\mu v_\mu = \begin{cases} 1 & \text{timelike} \\ 0 & \text{null} \end{cases} \,.$$

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Separation velocity:

$$\Delta u^\mu = -\frac{Du^\mu}{d\tau}\,\Delta\tau\ ,$$

where $D/d\tau$ denotes the covariant derivative along the wordline of the observer. The radial velocity is then

$$\Delta u_r = -\frac{\Delta u^\mu\,\Delta r_\mu}{|\Delta r|} = -\frac{v_\mu\,\frac{Du^\mu}{d\tau}\,\Delta\tau^2}{|\Delta r|}$$

$$H \equiv \frac{\Delta u_r}{\Delta r} = -\frac{v_\mu\,\frac{Du^\mu}{d\tau}}{\gamma^2-\kappa}\ .$$

But $Dv_\mu/d\tau = 0$, so

$$H = -\frac{\frac{d\gamma}{d\tau}}{\gamma^2-\kappa} = \frac{d}{d\tau}F(\gamma),$$

where

$$F(\gamma) = \begin{cases} 1/\gamma & \text{for null observers} \\ \mathrm{arctanh}(1/\gamma) & \text{for timelike observers} \end{cases}\ .$$

$$F(\gamma) = \text{“slowness”}\ .$$

So, for geodesic observers moving at relative speed $v_{\rm rel}$ at time τ_f,

$$\int^{\tau_f} H\,d\tau \le \mathrm{arctanh}\left(\frac{1}{\gamma}\right) = \mathrm{arctanh}\left(\sqrt{1-v_{\rm rel}^2}\right)\ .$$

For null observers, if we normalize the affine parameter τ by $d\tau/dt = 1$ at τ_f, then

$$\int^{\tau_f} H\,d\tau \le 1\ .$$

CONCLUSIONS

Average expansion condition: We use the term ***average expansion condition*** to describe the property that a congruence of comoving test particle trajectories can be found so that $H_{av} > 0$ everywhere along at least one past-directed geodesic. This criterion excludes, for example, the complete de Sitter space.

The theorem can then be summarized by saying that any spacetime region satisfying the average expansion condition cannot be geodesically complete in the past.

Disclaimers: There is of course no conclusion that an eternally inflating model must have a unique beginning, and no conclusion that there is an upper bound on the length of all backwards-going geodesics from a given point. There may be models with regions of contraction embedded within the expanding region that could evade our theorem. Aguirre & Gratton have proposed a model that evades our theorem, in which the arrow of time reverses at the $t = -\infty$ hypersurface, so the universe "expands" in both halves of the full de Sitter space.

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Claim: An eternally inflating model of the type usually assumed, which would obey the average expansion condition, cannot be complete. Some physics other than inflation would be needed to describe the past boundary of the inflating region. One possibility would be a quantum origin.

Cyclic Model of Steinhardt and Turok: This is an example of a simple eternally inflating model, for which our theorem applies. Along the branes their model produces a net expansion for each cycle. This implies that backward-going lightlike geodesics have a finite affine length, and hence some initial boundary condition has to be specified. The theorem does not exclude, however, the possibility that there are an infinite number of cycles in our past.

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