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SUMMER SCHOOL ON PARTICLE PHYSICS

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GRAND UNIFICATION AND FERMION MASSES

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Grand Unification and Fermion Masses

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Hints in favor of GUTs:

(1) Remarkable fit of SM quark and lepton multiplets into multiplets of $SU(5)$, $SO(10)$

(2) Unification of gauge couplings at scale $\simeq 2 \times 10^{16}$ GeV $= M_G$ (in the MSSM).

(3) Neutrino masses

(a) ν_R predicted by Pati-Salam, $SO(10)$

(b) M_R as inferred from see-saw formula $\approx M_G$.

(4) Hints from q, ℓ masses, mixings

(a) $m_b^0 \cong m_\tau^0$, Similar q, ℓ hierarchies

(b) $\theta_{CKM} \ll 1$ ($\theta_{CKM} = 0$ in minimal $SO(10)$)

(c) Relationships among V_{cb}, m_s/m_b, $U_{\mu 3}$ etc (see lopsided models)

Areas of research

- Gauge hierarchy problem,
  2/3 splitting problem,
  $d=5$ proton decay operators

- Leptogenesis/baryogenesis
- Neutrino masses and mixings
- Quark and lepton masses and mixings (the flavor problem)
- Flavor changing processes ($\mu \to e\gamma$, $\tau \to \mu\gamma$ etc)
- Other low energy tests ?
- Higher dimensions ($d=5, 6$ brane models
- Strings ?

My goal in these lectures is to explain the basic

- schemes of unification

  (Pati-Salam, SU(5), SO(10), flipped SU(5), ...)

- issues and problems

  (gauge hierarchy, doublet-triplet splitting, $d=5$ proton decay operators, quark and lepton masses.)

and • practical group theory

Outline

I. SU(5) unification

long { II. SO(10) and other groups for unification (Pati-Salam, flipped SU(5), ...)

III. The gauge hierarchy problem }

short { IV. Fermion Masses }

LECTURE I   SU(5) UNIFICATION

A. Basic Group Theory Facts

Group $G = \{g\}$

closure $g_1 g_2 = g_3$
identity $eg = ge = g$
inverse $gg^{-1} = g^{-1}g = e$
assoc. $g_1(g_2 g_3) = (g_1 g_2) g_3$

Lie groups

"**Represent**" elements by unitary matrices

$$U(g) = e^{i \sum_a \theta^a \lambda^a}$$

generators
parameters associated with element g.

generators λ^a are hermitian matrices, satisfying the **group algebra**

$$[\lambda^a, \lambda^b] = i f^{abc} \lambda^c$$

structure constants

Number of generators = dim. of group.

If U is a $d \times d$ matrix representing an element of the group, it can be thought of as acting on a d-component object

$$\begin{bmatrix} & U & \end{bmatrix} \begin{pmatrix} T^1 \\ T^2 \\ \vdots \\ T^d \end{pmatrix} = \begin{pmatrix} T^{1\prime} \\ T^{2\prime} \\ \vdots \\ T^{d\prime} \end{pmatrix}$$

Strictly speaking, the U matrices form a "representation" of the group elements. But physicists often talk of the d-component **multiplet** as being a "representation".

Recall: always exists an <u>adjoint representation</u>

d×d matrices, where d = dimension of group
= # of generators

generators in the adjoint representation are a set of d hermitian d×d matrices

→ These act on d-component multiplets, which are called "adjoint representations" or "adjoint multiplets"

<u>Proof</u>: Jacobi identity

$$0 = [[\lambda^a, \lambda^b], \lambda^c] + [[\lambda^c, \lambda^a], \lambda^b] + [[\lambda^b, \lambda^c], \lambda^a]$$

$$0 = f^{abd} f^{dce} + f^{cad} f^{dbe} + f^{bcd} f^{dae}$$

Call $f^{abd} \equiv i(\lambda^a)^{bd}$

$$0 = i f^{abd} (\lambda^d)^{ce} - (\lambda^a)^{cd} (\lambda^b)^{de} + (\lambda^b)^{cd} (\lambda^a)^{de}$$

which is group algebra!

B) <u>SU(N) GROUP THEORY</u>

$N \qquad T^\alpha \longrightarrow T^{\alpha'} = U^{\alpha'}{}_\alpha T^\alpha$ <u>$\alpha = 1, \cdots, N$</u>

$\bar{N} \qquad T_\alpha \longrightarrow T_{\alpha'} = U_{\alpha'}{}^\alpha T_\alpha$

These multiplets are called "<u>fundamental</u> rep.", "<u>anti fundamental</u> rep."

$$T_\alpha = (T^\alpha)^* \quad , \quad U_{\alpha'}{}^\alpha = (U^{\alpha'}{}_\alpha)^*$$

$$\sum_{\alpha=1}^{N} T^\alpha T_\alpha = \text{invariant} \Rightarrow UU^\dagger = I = U^\dagger U$$

$U = N \times N$ unitary matrices, $\det U = 1$

$N\times N$, unitary, det = 1 matrices represent the elements of group $SU(N)$

Such matrices can be written in terms of N^2-1 parameters

$$\mathcal{U}^{\alpha'}{}_{\alpha} = \left(e^{i\sum_{a=1}^{N^2-1}\theta^a \lambda^a}\right)^{\alpha'}{}_{\alpha} \qquad \begin{array}{l} a=1,\cdots,N^2-1 \\ \alpha,\alpha'=1,\cdots,N \end{array}$$

<u>N^2-1 generators</u>. In this fundamental representation they are $N\times N$ traceless hermitian matrices.

[I will denote "adjoint indices" by $a, b, c, \cdots$
"fundamental indices" of $SU(N)$ by $\alpha, \beta, \gamma, \cdots$]

<u>Adjoint rep.</u>

For $SU(N)$ there exist (N^2-1)-component multiplets T^a that get acted upon by $(N^2-1)\times(N^2-1)$ unitary matrices. [Adjoint rep.]

$$\underbrace{T^{\alpha}{}_{\beta}} \equiv \sum_{a=1}^{N^2-1} \underbrace{T^a}(\lambda^a)^{\alpha}{}_{\beta}$$

Two ways of writing this multiplet

$\alpha,\beta=1,\cdots,N$; $a=1,\cdots,N^2-1$

$$T^{\alpha}{}_{\beta} \rightarrow T^{\alpha'}{}_{\beta'} = \mathcal{U}^{\alpha'}{}_{\alpha}\, \mathcal{U}_{\beta'}{}^{\beta}\, T^{\alpha}{}_{\beta}$$

$$T \rightarrow \mathcal{U} T \mathcal{U}^{\dagger}$$

[NOTE: CAN ALSO DENOTE λ^a BY $\lambda^{\alpha}{}_{\beta}$
eg. IN $SU(3)$ $\lambda^6 + i\lambda^7 = \begin{pmatrix} 0&0&0\\0&0&1\\0&0&0\end{pmatrix} \equiv \lambda^2{}_3$]

Let a Higgs field be in such an adjoint representation (i.e multiplet) of an SU(N) gauge sym.

$$H^\alpha{}_\beta = \sum_{a=1}^{N^2-1} H^a (\lambda^a)^\alpha{}_\beta$$

$$\langle H \rangle \to \mathcal{U}\langle H \rangle \mathcal{U}^\dagger = e^{i\sum_b \theta^b \lambda^b} \langle H \rangle e^{-i\sum_c \theta^c \lambda^c}$$

$$= \langle H \rangle + i\sum_b \theta^b [\lambda^b, \langle H \rangle] + \cdots$$

$\langle H \rangle$ is left invariant by "unbroken generators"

$\Rightarrow$ $\begin{cases} \text{Unbroken generators:} & [\lambda^a, \langle H \rangle] = 0 \\ \text{Broken generators:} & [\lambda^a, \langle H \rangle] \neq 0. \end{cases}$

c) SU(5) multiplets and symmetry breaking

$$\begin{cases} 5 & \psi^\alpha = \begin{pmatrix} \psi^1 \\ \psi^2 \\ \hline \psi^3 \\ \psi^4 \\ \psi^5 \end{pmatrix} \begin{matrix} \} \ \psi^i & i = 1,2 & SU(2)_L \\ \} \ \psi^a & a = 3,4,5 & SU(3)_c \end{matrix} \\ \bar{5} & \psi_\alpha \end{cases}$$

24 generators = 5×5 traceless, hermitian matrices (in fundamental rep.)

$$(\lambda^a)^\alpha{}_\beta = \left[\begin{array}{c|c} SU(2) & 0 \\ \hline 0 & 0 \end{array}\right], \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & SU(3) \end{array}\right], \left[\begin{array}{c|c} 0 & m \\ \hline m^\dagger & 0 \end{array}\right], \left[\begin{array}{cc|ccc} \frac{1}{2} & & & & \\ & \frac{1}{2} & & & \\ \hline & & -\frac{1}{3} & & \\ & & & -\frac{1}{3} & \\ & & & & -\frac{1}{3} \end{array}\right]$$

$$(1,3,0) \qquad (8,1,0) \qquad (3,2,\tfrac{5}{6}) + \text{h.c.} \qquad (1,1,0)$$

Now, let there be a Higgs in an adjoint multiplet of SU(5), and let its VEV be

$$\langle \Omega \rangle = \begin{bmatrix} 1/2 & & & & \\ & 1/2 & & & \\ & & -1/3 & & \\ & & & -1/3 & \\ & & & & -1/3 \end{bmatrix} \omega = \left(\frac{Y}{2}\right)\omega$$

$$\langle \Omega \rangle \text{ " } \langle \textstyle\sum_a \Omega_a \lambda^a \rangle$$

[Note: VEVs of adjoint Higgs multiplets always point in direction of some generator (or linear combination of gens.)]

Which generators are unbroken then?

$$[\lambda^a, \langle \Omega \rangle] = 0$$

Unbroken generators are those of

$$SU(3)_c \times SU(2)_L \times U(1)_Y = G_{SM}$$

<u>Embedding of SM multiplets in SU(5)</u> $\longrightarrow$

$$\bar{5}_L = \psi_\alpha = \begin{pmatrix} \psi_i \\ \hline \psi_a \end{pmatrix} = \begin{pmatrix} (1, 2, -1/2) \\ (\bar{3}, 1, 1/3) \end{pmatrix} \equiv \begin{pmatrix} \bar{2} \\ \bar{3} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} e_L \\ \nu_L \end{pmatrix} \\ d_L^c \end{pmatrix} \text{ etc}$$

$\uparrow$ my notation

$$10_L = \psi^{[\alpha\beta]} = \left[\begin{array}{c|c} \psi^{[12]} & \psi^{ia} \\ \hline \psi^{ai} & \psi^{ab} = \varepsilon^{abd}\psi_d \end{array}\right] = \left[\begin{array}{c|c} (1,1,+1) & (3,2,1/6) \\ \hline & (\bar{3},1,-2/3) \end{array}\right] = \left[\begin{array}{c|c} e_L^+ & \begin{pmatrix} u_L \\ d_L \end{pmatrix} \\ \hline & u_L^c \end{array}\right]$$

$$24_g = A^a = \left[\begin{array}{c|c} A^i{}_j & A^i{}_a \\ \hline A^a{}_i & A^a{}_b \end{array}\right] = \left[\begin{array}{c|c} (1,3,0) & (\bar{3},2,5/6) \\ \hline (3,2,-5/6) & (8,1,0) \end{array}\right] + (1,1,0)$$

$$A^a (\lambda^a)^\alpha{}_\beta = A^\alpha{}_\beta$$

Key point: $\psi_L(r) \xleftrightarrow{c} \psi_R(\bar{r})$

SM family (1st family)

$$(1,2,-1/2)_L = \begin{pmatrix} \nu_L \\ e^-_L \end{pmatrix} \equiv L$$

$$(1,1,-1)_R = e^-_R \xleftrightarrow{\quad c \quad} (1,1,+1)_L = e^+_L$$

$$(1,1,0)_R = N_R \xleftrightarrow{\quad c \quad} (1,1,0)_R = N^c_L$$

$$(3,2,1/6)_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \equiv Q$$

$$(3,1,2/3)_R = u_R \xleftrightarrow{\quad c \quad} (\bar{3},1,-2/3) = u^c_L$$

$$(3,1,-1/3)_R = d_R \longleftrightarrow (\bar{3},1,+1/3) = d^c_L$$

Two ways to write a mass term (or Yukawa term)

$$m\,\overline{\psi_R(r_2)}\,\psi_L(r_1) = m\left[\psi_R(r_2)\right]^\dagger \gamma^0 \psi_L(r_1)$$

← DIRAC BAR

or ϕ

$$\parallel$$

$$m\,\underbrace{\psi_L(\bar{r}_2)^T\, C\, \psi_L(r_1)}_{\text{More convenient for GUTs (also SUSY!)}}$$

eg electron mass:

$$m(\overline{e^-_R})\,e^-_L = m(e^+_L)^T\, C\, e^-_L$$

often do not bother to show

Arrows follow flow of "left-handedness"

Think of chirality as a charge $\chi(L) = -1$, $\chi(R) = +1$

$\Delta\chi = 2$

(D) Fermion Masses

$$\left\{\begin{array}{l} \text{FAMILY} = 10_L + \bar{5}_L + (1_L) \\ \quad\quad (\psi^{12}, \psi^{ia}, \psi^{ab})_L + (\psi_i, \psi_a)_L + \psi_L \\ \quad\quad (e^+_L \; \binom{u}{d}_L \; u^c_L) + (\binom{\nu}{l^-}_L, d^c_L) + N^c_L \end{array}\right.$$

HIGGS ARE IN $5_H + \bar{5}_H$

$$(H^i, H^a) + (H_i, H_a)$$

$$(H_u, H_{uC}) + (H_d, H_{dC})$$

[In non-SUSY: $\bar{5}_H = (5_H)^*$ i.e. $H_\alpha = (H^\alpha)^*$
In SUSY 5_H and $\bar{5}_H$ are distinct, L-H superfields.]

YUKAWA TERMS (minimal SU(5))

$$(Y_u)_{AB}\,(10_{LA}\, 10_{LB})\langle 5_H\rangle = (Y_u)_{AB}\,\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\left(\psi_A^{\alpha\beta}\,\psi_B^{\gamma\delta}\langle H^\varepsilon\rangle\right)$$
$$= (Y_u)_{AB}\left(\psi_A^{1a}\,\psi_B^{bc}\langle H^2\rangle\right) = (Y_u)_{AB}\, u_A\, u_B^c\,\langle H_u\rangle$$

NOTE: $Y_u = Y_u^T \Rightarrow \boxed{M_u = M_u^T \propto Y_u}$

$$(Y_D)_{AB}\,(10_{LA}\,\bar{5}_{LB})\langle\bar{5}_H\rangle = (Y_D)_{AB}\left(\psi_A^{\alpha\beta}\,\psi_{B\alpha}\langle H_\beta\rangle\right)$$
$$= (Y_D)_{AB}\left(\psi_A^{12}\,\psi_{B1} + \psi_A^{a2}\,\psi_{Ba}\right)\langle H_2\rangle$$
$$= (Y_D)_{AB}\left(e_A^+ e_B^- + d_A d_B^c\right)\langle H_d\rangle$$

NOTE: $\boxed{M_D = M_L^T \propto Y_D}$

$M_D = M_L^T$ (in minimal SU(5))

$\Rightarrow \begin{cases} m_b^0 = m_\tau^0 & \text{(Good within 20\% in SUSY GUTs)} \\ m_s^0 = m_\mu^0 & (m_s^0 \lesssim \frac{1}{3} m_\mu^0, \text{ from SUSY GUTs RGE}) \\ m_d^0 = m_e^0 & (m_d^0 \simeq 3 m_e^0, \quad " \quad " \quad " \quad ") \end{cases}$

<u>If</u> there exist N_L^c (Right-handed ν's):

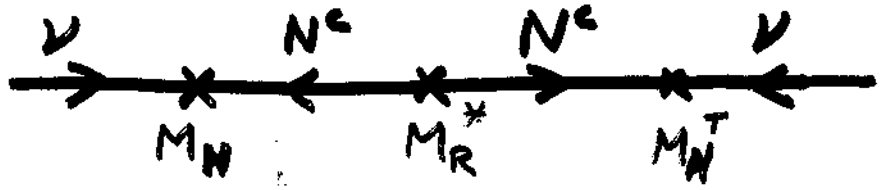
$$\left\{ \begin{aligned} (Y_N)_{AB}(\bar{5}_{LA}\, 1_{LB})\langle 5_H\rangle &= (Y_N)_{AB}(\psi_{A\alpha}\, \psi_B)\langle H^\alpha\rangle \\ &= (Y_N)_{AB}(\psi_{A2}\, \psi_B)\langle H^2\rangle = (Y_N)_{AB}\, \nu_A N_B^c \langle H_u\rangle \\ &\Rightarrow \boxed{M_N \propto Y_N} (\nu \text{ Dirac mass matrix}) \lesssim v_u \end{aligned} \right.$$

$$\left\{ \begin{aligned} (M_R)_{AB}(1_{LA}\, 1_{LB}) &= (M_R)_{AB}(N_A^c\, N_B^c) \\ &\boxed{M_R \ (\nu_R \text{ Majorana mass matrix})} \lesssim M_G \end{aligned} \right.$$

$$\Rightarrow (\nu \ \ N^c)^T \begin{pmatrix} 0 & M_N \\ M_N^T & M_R \end{pmatrix} \begin{pmatrix} \nu \\ N^c \end{pmatrix} \longrightarrow (\nu\, N^c)'^T \begin{pmatrix} M_\nu & 0 \\ 0 & M_R \end{pmatrix} \begin{pmatrix} \nu \\ N^c \end{pmatrix}'$$

$$\boxed{M_\nu \cong -M_N M_R^{-1} M_N^T}$$ SEE-SAW FORMULA

<u>DIAGRAMMATICALLY</u>:



$$M_\nu \cong M_N \frac{1}{\cancel{p} + M_R} M_R^* \frac{1}{\cancel{p} + M_R^*} M_N^T$$

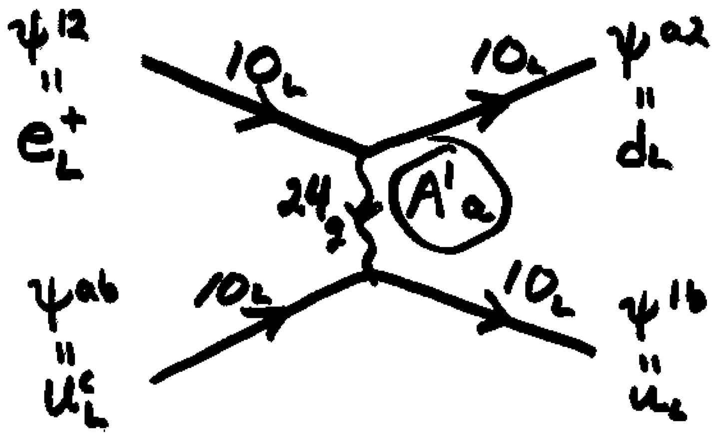
$$\cong M_N M_R^{-1} M_N^T$$

So, in minimal SU(5) there are four independent mass matrices for q and ℓ:

$$M_D = M_L^T \,,\quad M_u \,,\quad \underbrace{M_N \,,\quad M_R}_{\downarrow \; M_\nu}$$

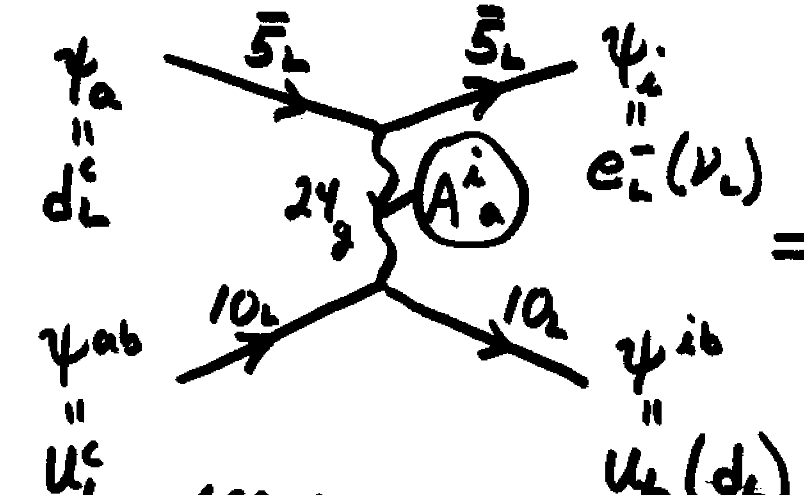
(E) PROTON DECAY OPERATORS

TWO KINDS OF DIAGRAMS FOR PDK IN NON-SUSY SU(5):
GAUGE BOSON MEDIATED, HIGGS BOSON MEDIATED



$$\Rightarrow \begin{cases} (10_L\, 10_L)^\dagger (10_L\, 10_L) \\ \quad\downarrow \\ (d_L\ u_L)^\dagger (u_L^c\, e_L^+) \\ \begin{pmatrix} \Delta L = +1 \\ \Delta B = +1 \end{pmatrix} \end{cases}$$

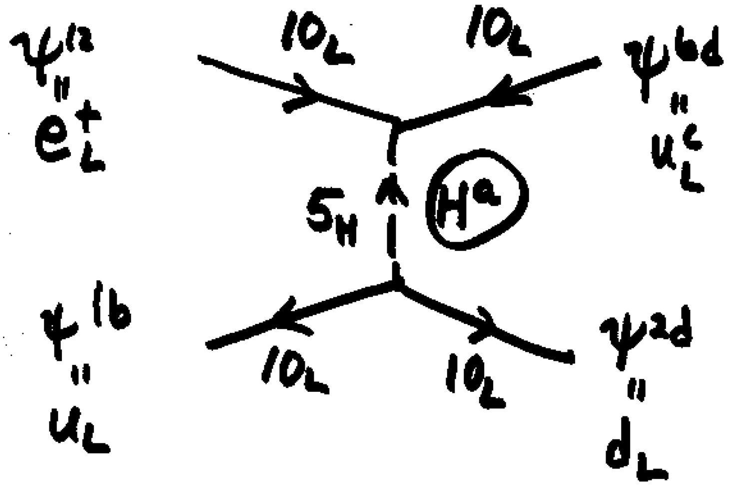
$$\Rightarrow \Delta(B-L) = 0$$



(GAUGE MEDIATED)

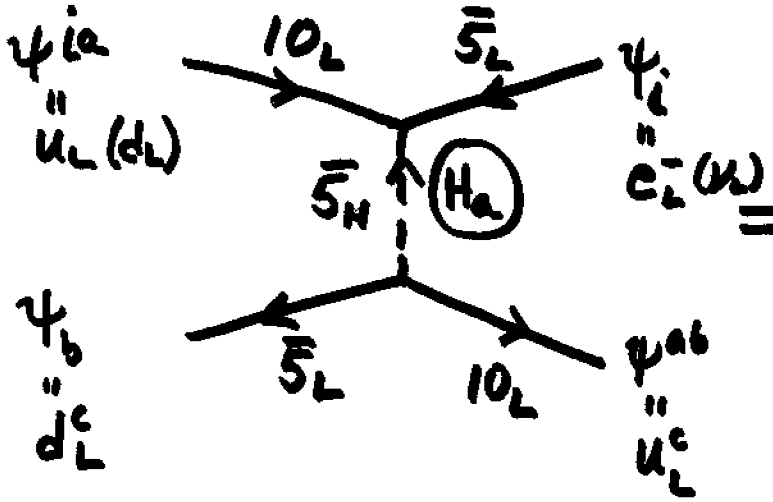
$$\Rightarrow \begin{cases} (\bar{5}_L\, 10_L)^\dagger (10_L\, \bar{5}_L) \\ \quad\downarrow \\ (\nu_L d_L + e_L^- u_L)^\dagger (u_L^c\, d_L^c) \\ \begin{pmatrix} \Delta L = +1 \\ \Delta B = +1 \end{pmatrix} \end{cases}$$

HIGGS BOSON MEDIATED



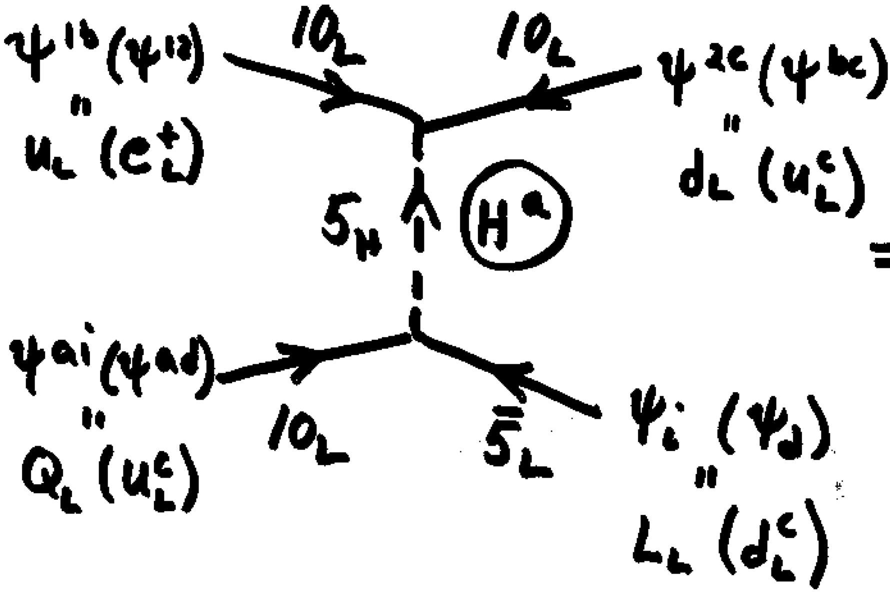
$$\Rightarrow \left\{ \begin{array}{c} (10_L\,10_L)^\dagger (10_L\,10_L) \\ \downarrow \\ (d_L u_L)^\dagger (u_L^c e^+) \end{array} \right.$$

(see previous page)



$$\Rightarrow \left\{ \begin{array}{c} (\bar{5}_L\,10_L)^\dagger (10_L\,\bar{5}_L) \\ \downarrow \\ (\nu_L d_L + e_L^- u_L)^\dagger (u_L^c d_L^c) \end{array} \right.$$

(see previous page)



different!

$$\Rightarrow \left\{ \begin{array}{c} (\bar{5}_L\,10_L\,10_L\,10_L) \\ \downarrow \\ (d_L^c\,u_L^c\,u_L^c\,e_L^+) \\ \text{or} \\ \underbrace{(e_L^- u_L - \nu_L d_L)}_{L\cdot Q}\underbrace{d_L u_L}_{Q\cdot Q} \end{array} \right.$$

$$\left(\begin{array}{c} \Delta L = +1 \\ \Delta B = +1 \end{array} \right)$$

The Higgs-mediated proton decay is caused by exchange of colored Higgs

These must be superheavy

$\Rightarrow$ $\boxed{\text{"DOUBLET – TRIPLET SPLITTING PROBLEM"}}$

$$5_H = H^\alpha = \begin{pmatrix} H^i \\ H^a \end{pmatrix} \begin{array}{l} \leftarrow \text{Weak-scale mass} \\ \leftarrow \text{GUT scale mass} \end{array}$$

THIS IS AN ASPECT OF THE INFAMOUS "GAUGE HIERARCHY PROBLEM"

THREE ASPECTS:

(a) 2/3 SPLITTING PROBLEM

(b) FINE-TUNING PROBLEM
(stability under radiative corrections)

(c) SMALL NUMBER PROBLEM
(why $M_W/M_G \sim 10^{-14}$)

(d)(In SUSY) "μ PROBLEM"

Why do all of these operators conserve B−L?

In minimal SU(5) model:

$$\begin{array}{cccccccccccc} (10_L & 10_L) & 5_H & + (10_L & \bar{5}_L) & \bar{5}_H & + (\bar{5}_L & 1_L) & 5_H & + M^2 & \bar{5}_H & 5_H \\ X= +1 & +1 & -2 & +1 & -3 & +2 & -3 & +5 & -2 & & +2 & -2 \end{array}$$

Can assign a global charge X that is conserved by couplings

Now $\langle 5_H \rangle, \langle \bar{5}_H \rangle$ break $\overbrace{U(1)_{I_{3L}} \times U(1)_Y}^{\text{local}} \times \overbrace{U(1)_X}^{\text{global}}$

by 't Hooft mechanism there remain

$$\underbrace{U(1)_Q}_{\text{local}} \times \underbrace{U(1)_{B-L}}_{\text{global}} \qquad Q = I_{3L} + Y/2$$
$$B-L = \frac{4}{5}\frac{Y}{2} + \frac{1}{5}X$$

In SO(10), as we shall see, $U(1)_X$ is contained in SO(10) and thus a <u>local</u> sym.

⇒ B−L is a generator of SO(10).

<u>NOTE</u> In SU(5), if there are right-handed neutrinos, they can have Majorana masses that break L and conserve B, thus ~~(B−L)~~.

$$M_R(\underset{+5}{1_L}\ \underset{+5}{1_L}) \Rightarrow \Delta X = -10 \Rightarrow \Delta(B-L) = -2$$

IMPORTANT FOR "LEPTOGENESIS".

PROTON DECAY OPERATORS IN NON SUSY SU(5)

gauge and higgs mediated

$$\begin{cases} 10_L^+ \, 10_L^+ \, 10_L \, 10_L \rightarrow (e_L^+ \, u_L^c)^\dagger u_L d_L = e_R^- u_R u_L d_L \\ 10_L^+ \, \bar{5}_L^+ \, 10_L \, \bar{5}_L \rightarrow (u_L^c \, d_L^c)^\dagger Q_L \cdot L_L = u_R d_R \, Q_L \cdot L_L \end{cases}$$

only higgs mediated

$$\begin{cases} 10_L \, 10_L \, 10_L \, \bar{5}_L \rightarrow e_L^+ u_L^c \, u_L^c \, d_L^c = (e_R^- u_R u_R d_L)^\dagger \\ \quad \text{and} \\ Q_L Q_L Q_L L_L \end{cases}$$

IN SUSY SU(5)

$$\mathcal{L} \supset \int d^2\theta \, d^2\bar{\theta} \, (10_L^+ 10_L^+ 10_L 10_L + 10_L^+ \bar{5}^+ 10_L \bar{5}_L)/M_G^2$$

Since both Φ and $\Phi^\dagger$ are involved, these are D-terms with $\int d^2\theta \, d^2\bar{\theta}$ thus by dimensional analysis $1/M_G^2$.

$\Rightarrow$ "$d=6$" proton decay operators caused by exchange of gauge and higgs <u>bosons</u>

But also allowed: "$d=5$" proton decay operators

$$\mathcal{L} \supset \int d^2\theta \, (10_L \, 10_L \, 10_L \, \bar{5}_L)/M_G$$

Term in superpotential W as only Φ so by dimensional analysis only suppressed by $1/M_G$.

NOTE: From <u>HIGGSINO</u> EXCHANGE. DANGEROUS!

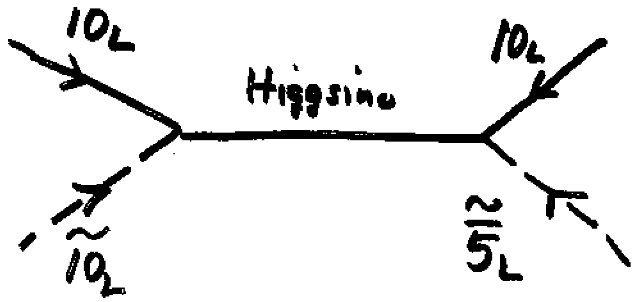
Also could have (disastrous) $d = 4$ operators" such as $\int d^2\theta\,(10_L\,\bar{5}_L\,\bar{5}_L')$, but these can be forbidden by imposing "matter parity" ($\approx$ R parity):

Z_2 under which $\begin{cases} \text{"matter" } (\equiv q + \ell) \text{ are odd} \\ \text{higgs, gauge are even.} \end{cases}$

(Such a Z_2 can arise automatically in SO(10) as a subgroup of SU(10))

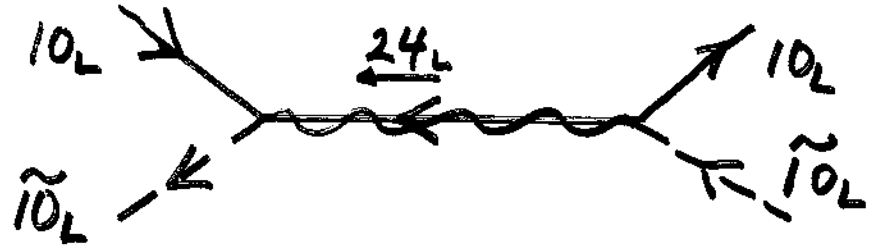
$d = 5$ p decay comes from (colored) Higgsino exchange

$$\frac{10_L\,10_L\,10_L\,\bar{5}_L}{M_G} \text{ from}$$



partner of Higgs boson exchange graph shown earlier

Why not $d = 5$ from exchange of gaugino?



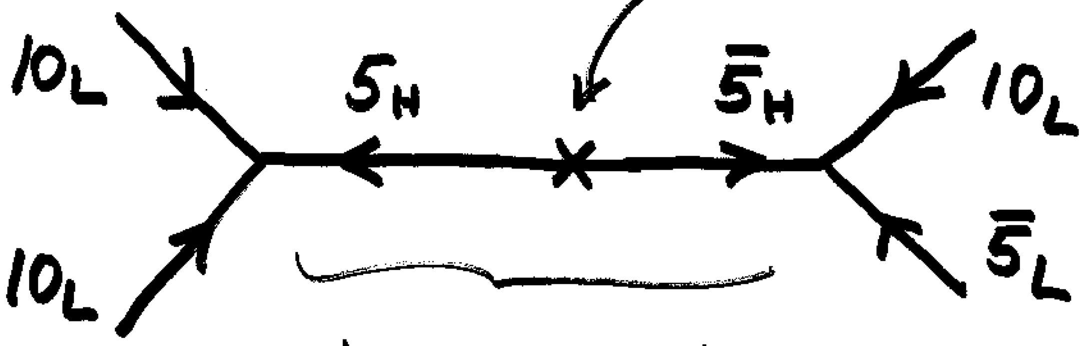
No "chirality flip" on gaugino line (since $m\,24_L\,24_L$, being a gaugino mass $\propto M_{SUSY} \ll M_G$).

$$\Rightarrow \quad \frac{1}{\cancel{p} + M_G} = \frac{\cancel{p} - \cancel{M_G}}{p^2 + M_G^2} \sim \frac{1}{M_G^2}$$

LOOK AT HIGGSINO EXCHANGE DIAGRAM AGAIN.

NEED A CHIRALITY FLIP

SUPERFIELD DIAGRAM:



$$\frac{1}{\cancel{p}+M_G}\, M_G\, \frac{1}{\cancel{p}+M_G} \sim \frac{1}{M_G}$$

NOTE: IN SUSY SU(5) $\bar{5}_H$, 5_H ARE NOT CONJUGATES OF EACH OTHER, BUT BOTH LEFT-HANDED CHIRAL SUPERFIELDS. SO "X" IN DIAGRAM IS FROM $\mu\, \bar{5}_H\, 5_H$

$$W \supset (10_L\, 10_L)\, 5_H + (10_L\, \bar{5}_L)\, \bar{5}_H + (\bar{5}_L\, 1_L)\, 5_H + \mu\, \bar{5}_H\, 5_H$$

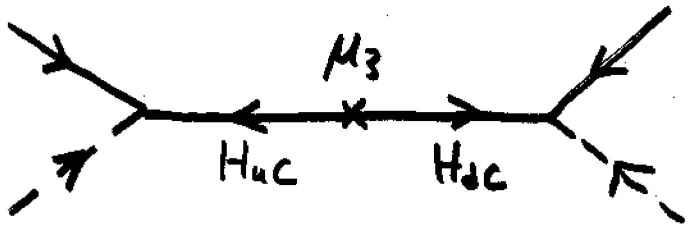
$\bar{5}_H = H_\alpha = \begin{pmatrix} H_i \\ H_a \end{pmatrix}$ $\begin{matrix} \longrightarrow H_d \text{ of MSSM} \\ \longrightarrow H_{dC} \text{ a } \bar{3}_L \end{matrix}$

$5_H = H^\alpha = \begin{pmatrix} H^i \\ H^a \end{pmatrix}$ $\begin{matrix} \longrightarrow H_u \text{ of MSSM} \\ \longrightarrow H_{uC} \text{ a } 3_L \end{matrix}$

$\boxed{\mu_2}$ μ_3

$\longrightarrow \mu_3\, H_{dC}\, H_{uC}$ $\qquad A_{p\,\text{decay}} \propto \dfrac{\mu_3}{M_{H_{uC}}\, M_{H_{dC}}}$

CRUCIAL POINT



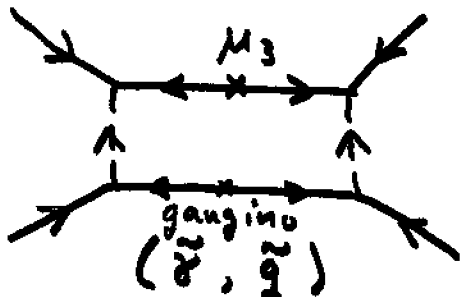
$\sim \frac{1}{M_G}$

WHY NOT DISASTROUS IF $\sim \frac{1}{M_G}$?

ADDITIONAL SMALL FACTORS (RELATIVE TO GAUGE MED)

(a) Couplings are Yukawa couplings of the light quarks and leptons $\sim \frac{m_{q,l}}{v}$

(b) These diagrams involve <u>scalar</u> quarks and leptons which must be turned into quarks and leptons by gaugino
$\Rightarrow$ loop



Calculation is complicated and involves hadronic matrix element.

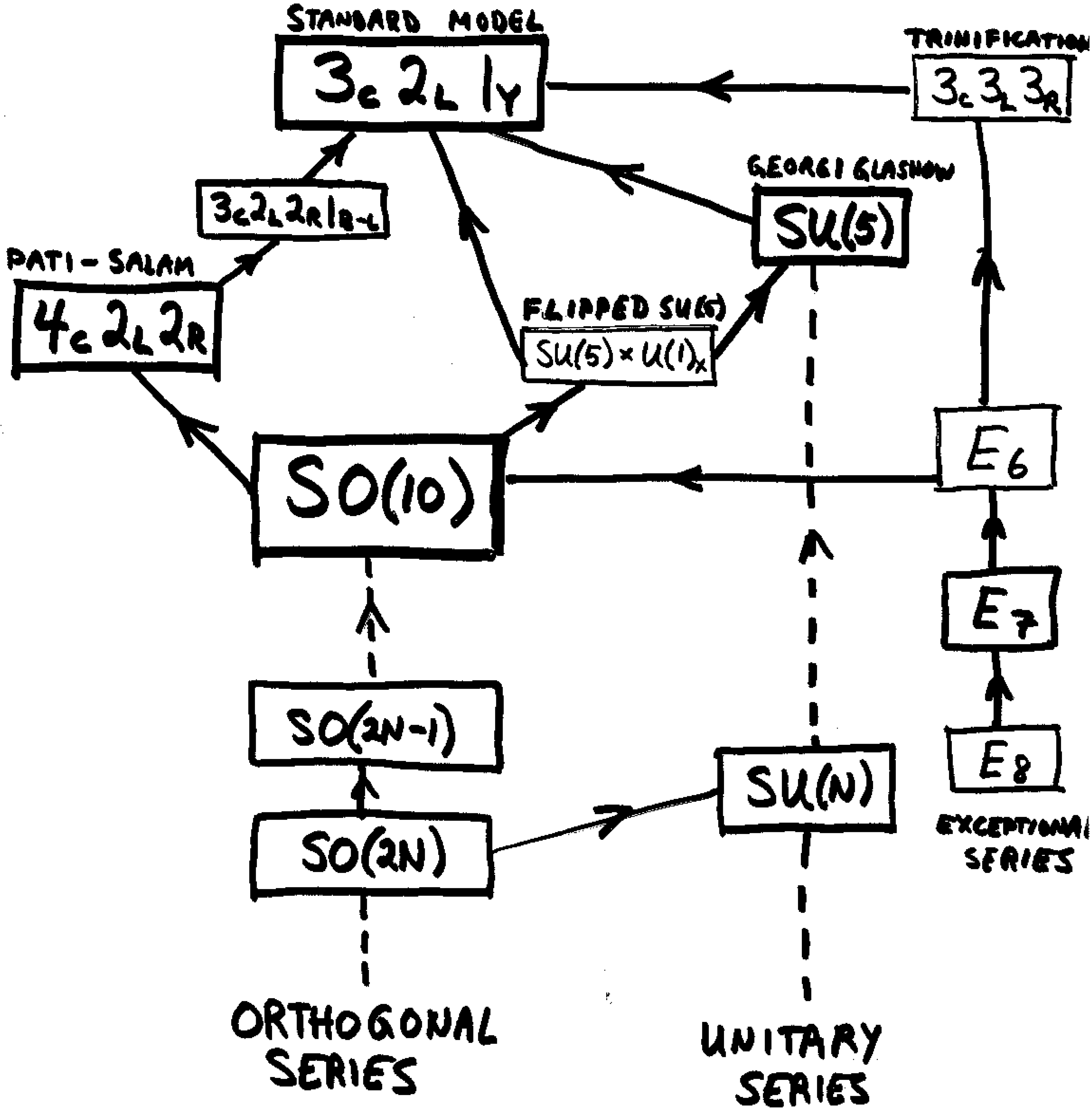
FOR SUSY SU(5) (minimal) [See H. Murayama and A. Pierce, hep-ph/0108104]

$3.5 \times 10^{14} \leq M_{H_c} \leq 3.6 \times 10^{15}$ GeV (90% c.l. from R.G.E.)

BUT $M_{H_c} \geq 7.6 \times 10^{16}$ GeV (from superK limit on $p \to K^+ \bar{\nu}$)

PROBLEM OF HIGGSINO-MEDIATED p DECAY (or d=5 p decay)

LECTURE II: SO(10) AND OTHER GROUPS FOR UNIFICATION



(A) MOST IMPORTANT GROUPS FOR UNIFICATION

PATI-SALAM ($SU(4)_c \times SU(2)_L \times SU(2)_R$)

First unification scheme, not "grand unified" but does have $q-\ell$ unification.

- Implies Right-handed neutrinos exist.
- Can have Left-Right symmetry.

SU(5) (GEORGI-GLASHOW)

- Smallest "grand unified" group

  [Only rank-4 unification group
  $\Rightarrow$ one stage of symmetry breaking
  $\rightarrow$ most predictive for p decay]

SO(10)

Contains **both** SU(5) and Pati-Salam groups

- Implies Right-handed neutrinos exist
- Has a Left-Right symmetry
- Is grand unified
- Complete $q-\ell$ unification of a family
- Most predictive for q, ℓ masses
- Automatic anomaly freedom.

(B)

PATI-SALAM $SU(4)_c \times SU(2)_L \times SU(2)_R$

$$\begin{array}{c} SU(2)_L \updownarrow \\ \end{array} \underbrace{\begin{pmatrix} u & u & u & \nu \\ d & d & d & e^- \end{pmatrix}_L}_{\longleftrightarrow\ SU(4)_c} \qquad \underbrace{\begin{pmatrix} u & u & u & N \\ d & d & d & e^- \end{pmatrix}_R}_{\longleftrightarrow\ SU(4)_c} \updownarrow SU(2)_R$$

$$\Updownarrow$$

$$\begin{pmatrix} u^c & u^c & u^c & N^c \\ d^c & d^c & d^c & e^+ \end{pmatrix}_L$$

$$(4,2,1)_L \qquad\qquad (\bar{4},1,2)_L$$

NOTE: $B-L = \lambda_{15} = \begin{pmatrix} 1/3 & & & \\ & 1/3 & & \\ & & 1/3 & \\ & & & -1 \end{pmatrix}$ of $SU(4)_c$

$$Q = \overset{SU(2)_L}{I_{3L}} + \underbrace{\overset{SU(2)_R}{I_{3R}} + \tfrac{1}{2}\overset{SU(4)_c}{(B-L)}}_{Y/2}$$

$$Q = I_{3L} + Y/2$$

Higgs of MSSM $\subset (1,2,2)$ "bidoublet"

$$SU(2)_L \updownarrow \langle [H_u, H_d] \rangle = \begin{pmatrix} v_u & 0 \\ 0 & v_d \end{pmatrix} \qquad \begin{array}{l} H_u = (1, 2, 1/2) \\ H_d = (1, 2, -1/2) \end{array}$$

$$\overset{\longleftrightarrow}{SU(2)_R}$$

Yukawa terms in <u>minimal</u> Pati Salam $\quad Y_{AB}\,(4,2,1)_{L\,A}\,(\bar{4},1,2)_{L\,B}\,(1,2,2)_H$

$$\Longrightarrow M_N = M_u \propto M_D = M_L \qquad \text{(not necess. symmetric)}$$

(c) SO(2N) GROUP THEORY:

VECTORS AND TENSORS

A VECTOR HAS 2N REAL COMPONENTS, SO WE DO NOT DISTINGUISH UPPER AND LOWER INDICES. CALL VECTOR INDICES i, j, k, etc.

$$T^i = \begin{pmatrix} T^1 \\ T^2 \\ \vdots \\ T^{2N} \end{pmatrix}$$

TRANSFORMATIONS THAT LEAVE $\sum_i T^i T^i$ INVARIANT ARE REAL UNITARY ($\equiv$ ORTHOGONAL) $2N \times 2N$ MATRICES.

$$O^T O = I$$

$$O = e^{i \sum_a \theta^a \lambda^a}$$

$\Rightarrow$ λ^a are purely imaginary, hermitian ($=$ anti symmetric) matrices.

$\Rightarrow$ $a = 1, \frac{1}{2}(2N)(2N-1)$ $\Rightarrow$ SO(10) has 45 generators

$i = 1, 2N$

ADJOINT REP:

$$T^{ij} = \sum_a T^a (\lambda^a)^{ij}$$

USEFUL TO WRITE THE ADJOINT INDEX a AS A PAIR OF VECTOR INDICES (ANTISYMMETRIZED). i.e. LABEL GENERATORS $\lambda^{[ij]}$ = GEN. OF ROTATION IN ij PLANE

$$(\lambda^{[ij]})^{kl} = -i\left[\delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk}\right]$$

FOR EXAMPLE:

$$\lambda^{[12]} = \left[\begin{array}{cc|c} 0 & -i & \\ +i & 0 & 0 \\ \hline \multicolumn{2}{c|}{0} & 0 \end{array}\right] \Rightarrow e^{i\theta\lambda^{[12]}} = \left[\begin{array}{cc|ccc} \cos\theta & \sin\theta & & & \\ -\sin\theta & \cos\theta & & & \\ \hline & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{array}\right]$$

IT IS EASY TO SEE THAT $SO(2N) \supset U(N) = SU(N) \times U(1)$.

$$T^i = \begin{pmatrix} T^1 \\ T^2 \\ \vdots \\ T^{2N} \end{pmatrix} \longrightarrow \underbrace{\begin{pmatrix} T^1 + iT^2 \\ T^3 + iT^4 \\ \vdots \\ T^{2N-1} + iT^{2N} \end{pmatrix}}_{\substack{= T^\alpha \\ \alpha = 1, \cdots, N}} \oplus \underbrace{\begin{pmatrix} T^1 - iT^2 \\ T^3 - iT^4 \\ \vdots \\ T^{2N-1} - iT^{2N} \end{pmatrix}}_{\substack{= T_\alpha \\ \alpha = 1, \cdots, N}}$$

So $\underset{\text{of } SU(N)}{T^\alpha} = \frac{1}{2}\left(T^{(2\alpha-1)} + \underset{\text{of } SO(2N)}{i\,T^{(2\alpha)}}\right) \equiv \frac{1}{2} T^{(2\alpha-1)+i(2\alpha)}$

$$T_\alpha = \frac{1}{2}\left(T^{(2\alpha-1)} - i\,T^{(2\alpha)}\right) \equiv \frac{1}{2} T^{(2\alpha-1)-i(2\alpha)}$$

$$\sum_{i=1}^{2N} T^i T^i = 4 \sum_{\alpha=1}^{N} T^\alpha T_\alpha$$

So $U(N)$ TRANSFORMATIONS THAT KEEP $\Sigma T^\alpha T_\alpha$ INVARIANT ARE ALSO $SO(2N)$ TRANSFORMATIONS.

For example, Cartan generators of $SO(2N)$ of form

$$-i\begin{bmatrix} \begin{matrix} & a_1 \\ -a_1 & \end{matrix} & & & \\ & \begin{matrix} & a_2 \\ -a_2 & \end{matrix} & & \\ & & \ddots & \\ & & & \begin{matrix} & a_N \\ -a_N & \end{matrix} \end{bmatrix} \quad \text{corresponds to} \quad \begin{bmatrix} a_1 & & & \\ & a_2 & & \\ & & \ddots & \\ & & & a_N \end{bmatrix} \text{ of } U(N)$$

eg. For $SO(2) = U(1)$

$$-i\begin{bmatrix} & 1 \\ -1 & \end{bmatrix} \quad \text{equals} \quad (1) \text{ of } U(1)$$

$$e^{i\theta\begin{pmatrix} & -i \\ i & \end{pmatrix}} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \quad \text{equals} \quad e^{i\theta} \text{ of } U(1)$$

We have seen $\begin{cases} \underline{\text{vector}} \text{ rep} & T^i \\ \underline{\text{adjoint}} \text{ rep} & T^{[ij]} \text{ "}{}^a\text{"} \end{cases}$

Of course there are an infinitely many types of tensor reps.

eg. $T^{(ij)}$ = traceless symmetric rank 2 tensor

$T^{[ijk]}$ etc.

(D) SO(2N) GROUP THEORY: SPINOR REPRESENTATIONS

ALGEBRA OF SO(2N)

$$\boxed{\left[\lambda^{[ij]}, \lambda^{[kl]}\right] = i\left(\delta^{ik}\lambda^{[jl]} - \delta^{il}\lambda^{[jk]} - \delta^{jk}\lambda^{[il]} + \delta^{jl}\lambda^{[ik]}\right)}$$

eg. $\left[\lambda^{[23]}, \lambda^{[31]}\right] = -i\lambda^{[21]} = i\lambda^{[12]}$

i.e. $\left[\lambda^1, \lambda^2\right] = i\lambda^3$

WE CAN FIND $2^N \times 2^N$ MATRICES THAT SATISFY THIS ALGEBRA.

FIRST, CONSTRUCT $2^N \times 2^N$ MATRICES

$$(\gamma^i)^\lambda{}_{\lambda'} \qquad \begin{array}{l} \lambda, \lambda' = 1, \cdots, 2^N \\ i = 1, \cdots, 2N \end{array}$$

THAT SATISFY "CLIFFORD ALGEBRA"

$$\boxed{\left\{\gamma^i, \gamma^j\right\} = 2\delta^{ij} I}$$

THEN DEFINE

$$\sigma^{[ij]} \equiv \frac{i}{2}\left[\gamma^i, \gamma^j\right]$$

THESE OBEY SO(2N) GROUP ALGEBRA (LEFT AS EXCERCIZE)

THEN ELEMENTS OF GROUP CAN BE REPRESENTED BY $2^N \times 2^N$ MATRICES

$$U = e^{i \sum_{ij} \theta^{ij} \sigma^{ij}}$$

THESE ARE <u>SPINOR REPRESENTION</u>. THESE MATRICES ACT ON 2^N - COMPONENT "SPINORS" (CALLED BY PHYSICISTS SPINOR REPRESENTATIONS).

THESE SPINORS CAN BE WRITTEN:

$$\psi^\lambda = | s_1\, s_2\, s_3 \cdots s_N \rangle$$

where $s_\alpha = \pm 1$.

: WILL WRITE

$$\gamma^{2k-1} = \underbrace{I \otimes I \otimes \cdots \otimes I}_{k-1} \otimes \underset{k^{th}}{\tau_1} \otimes \underbrace{\tau_3 \otimes \cdots \otimes \tau_3}_{N-k}$$

$$\gamma^{2k} = -\overbrace{I \otimes I \otimes \cdots \otimes I}\otimes \tau_2 \otimes \overbrace{\tau_3 \otimes \cdots \otimes \tau_3}$$

THESE ARE $2N$ HERMITIAN $2^N \times 2^N$ MATRICES. CAN CHECK THAT THEY SATISFY CLIFFORD ALGEBRA.

NOTE: γ^1, γ^2 FLIP FIRST "SPIN" s_1
γ^3, γ^4 FLIP SECOND "SPIN" s_2
$\vdots$
$\gamma^{2N-1}, \gamma^{2N}$ FLIP N^{TH} "SPIN" s_N

CHECK CLIFFORD ALGEBRA SATISFIED.

$$\begin{cases} \{\gamma^i, \gamma^i\} = 2\delta^{ii} I = 2I \Leftrightarrow (\gamma^i)^2 = I \\ (I \otimes I \otimes \cdots \otimes \tau_i \otimes \tau_3 \otimes \cdots \otimes \tau_3)^2 = I \checkmark \end{cases}$$

$$\begin{cases} \left\{ \overset{I}{\overline{\qquad\qquad}}\ \tau_i\ \overset{\tau_3}{\overline{\qquad\qquad\qquad}}\ ,\ \overset{I}{\overline{\qquad\qquad\qquad\qquad}}\ \tau_j\ \overset{\tau_3}{\overline{\qquad}} \right\} \\ = \overset{I}{\overline{\qquad\qquad}}\ \tau_i\ \overset{\tau_3}{\overline{\qquad}}\ \{\tau_3, \tau_j\}\ \overset{I}{\overline{\qquad}} = 0 \checkmark \end{cases}$$

$$\begin{cases} \left\{ \overset{I}{\overline{\qquad\qquad}}\ \tau_i\ \overset{\tau_3}{\overline{\qquad\qquad\qquad}}\ ,\ \overset{I}{\overline{\qquad\qquad}}\ \tau_j\ \overset{\tau_3}{\overline{\qquad\qquad\qquad}} \right\} \quad \begin{matrix} i \neq j \\ i,j = 1,2 \end{matrix} \\ = \overset{I}{\overline{\qquad\qquad}}\ \{\tau_i, \tau_j\}\ \overset{I}{\overline{\qquad\qquad}} = 0 \end{cases}$$

LATER WE WILL LOOK AT $\sigma^{ij} = \frac{i}{2}[\gamma^i, \gamma^j]$.

BUT NOTICE THAT σ^{ij} FLIPS <u>TWO</u> SPINS (UNLESS $i,j = (2k-1)(2k)$, WHEN FLIPS <u>ZERO</u> SPINS)

THUS $|s_1 \cdots s_N\rangle = \underbrace{|s_1 \cdots s_N\rangle_{\pi s = +1}}_{= \Psi_+} \oplus \underbrace{|s_1 \cdots s_N\rangle_{\pi s = -1}}_{= \Psi_-}$

= TWO IRREDUCIBLE SPINORS.

ONJUGATION: $\mathcal{U} \to \mathcal{U}^* \Rightarrow e^{i\theta^a\lambda^a} \longrightarrow e^{-i\theta^a\lambda^{a*}}$ 2.10

$\Rightarrow \lambda^a \to -\lambda^{a*} \Rightarrow \left(\frac{i}{2}[\gamma^i,\gamma^j]\right) \to \left(\frac{i}{2}[\gamma^{i*},\gamma^{j*}]\right)$

$\Rightarrow \gamma^i \to \gamma^{i*} \Rightarrow \tau_1 \to \tau_1,\ \tau_2 \to -\tau_2$

$\Rightarrow \tau^\pm \longrightarrow \tau^\mp \Longrightarrow$ SPIN UP $\longleftrightarrow$ SPIN DOWN

$\Rightarrow$ N = even $\quad \overline{\Psi_+} = \Psi_+ ,\quad \overline{\Psi_-} = \Psi_-$

N = odd $\quad \overline{\Psi_+} = \Psi_- ,\quad \overline{\Psi_-} = \Psi_+$

TABLE OF SPINORS FOR SO(2N)

QUIVALENT GP.	SO(2N)	Ψ_+	Ψ_-	vector
$\mathcal{U}(1)$	SO(2)	1	$\bar{1}$	$2 = 1 + \bar{1}$
$SU(2) \times SU(2)$	SO(4)	$2 = (2,1)$	$2' = (1,2)$	$4 = (2,2)$
SU(4)	SO(6)	$4 = T^\alpha$	$\bar{4} = T_\alpha$	$6 = T^{[\alpha\beta]}$
	SO(8)	8	8′	8
	SO(10)	16	$\overline{16}$	10
	SO(12)	32	32′	12
	SO(14)	64	$\overline{64}$	14
	SO(16)	128	128′	16
	SO(18)	256	$\overline{256}$	18

CONSIDER $SO(2p) \times SO(2N-2p)$ SUBGROUP OF $SO(2N)$.

VECTOR: $\quad 2N \longrightarrow (2p, 1) + (1, 2N-2p)$

SPINOR:

$$|s_1 \cdots s_N\rangle = |s_1 \cdots s_p\rangle |s_{p+1} \cdots s_N\rangle$$

$$\Psi_+ = |s_1 \cdots s_N\rangle_{\Pi s = +1} = |s_1 \cdots s_p\rangle_+ |s_{p+1} \cdots s_N\rangle_+ + |s_1 \cdots s_p\rangle_- |s_{p+1} \cdots s_N\rangle_-$$

$$\boxed{\begin{aligned} \Psi_+ &\longrightarrow (\Psi_+, \Psi_+) + (\Psi_-, \Psi_-) \\ \Psi_- &\longrightarrow (\Psi_+, \Psi_-) + (\Psi_-, \Psi_+) \end{aligned}}$$

IN PARTICULAR, CONSIDER

$$SO(10) \supset \underset{\substack{\| \\ SU(4)_c}}{SO(6)} \times \underset{\substack{\| \\ SU(2)_L \times SU(2)_R}}{SO(4)}$$

$$16 \longrightarrow \underset{\substack{\| \\ (4,2,1)}}{(4, 2)} + \underset{(\bar{4},1,2)}{(\bar{4}, 2')} = q_L + \ell_L$$

$$10 \longrightarrow \underset{\substack{\| \\ \underbrace{(6,1,1)}}}{(6, 1)} + \underset{(1,2,2)}{(1, 4)} = \text{Higgs}$$

PATI-SALAM FITS WITHIN $SO(10)$!

D.2 Family Unification (See Wilczek + Zee "Families from spinors")

$$SO(16) \supset SO(10) \times SO(6)$$

$$128_L \longrightarrow (16, 4)_L + (\overline{16}, \overline{4})_L$$

4 families — 4 mirror families

$\overline{16}_L \sim 16_R$

(V−A) Weak int. — (V+A) Weak int.

128 is self-conjugate (i.e. a "real representation")
So can have $M(128_L\ 128_L)$
"Naturally", 128_L would be superheavy unless some global (or local) symmetry prevents this mass term.

$$SO(18) \supset SO(10) \times SO(8)$$

$$256_L \longrightarrow (16, 8)_L + (\overline{16}, 8')_L$$

256 <u>not</u> self-conjugate, so $M\,256\,256$ not possible

Problem with such family unification schemes (also with E_8) is how to make V+A families heavy. Can be solved in higher dimensions. [K.S. Babu, S.M.B., and B. Kyae hep-ph/0202178]

D.3 How SO(10) contains SU(5) model

We know that a family

$$= 16 \text{ of } SO(10)$$

$$= (4,2,1) + (\bar{4},1,2) \text{ of Pati-Salam}$$

$$= 10 + \bar{5} + 1 \text{ of } SU(5)$$

We can see that under $SO(10) \to SU(5)$

$$16 \to 10 + \bar{5} + 1$$

directly, by looking more closely at spinor representations of $SO(2N)$.

SO(2N)

$$\lambda^{[ij]} \equiv \sigma^{ij} = \frac{i}{2}[\gamma^i, \gamma^j]$$

Recall that $U(N) = SU(N) \times U(1)$ embedded in $SO(2N)$ as follows:

$$T^\alpha = \frac{1}{2} T^{(2\alpha-1)+i(2\alpha)} = \frac{1}{2}\left(T^{(2\alpha-1)} + i\,T^{(2\alpha)}\right)$$

$$\underbrace{T_\alpha}_{U(N)} = \frac{1}{2} \underbrace{T^{(2\alpha-1)-i(2\alpha)}}_{SO(2N)} = \frac{1}{2}\left(T^{(2\alpha-1)} - i\,T^{(2\alpha)}\right)$$

So $SU(N)\times U(1)$ GENERATORS CAN BE WRITTEN AS $SO(2N)$ GENERATORS THUS:

$$\lambda^\alpha{}_\beta = \frac{-i}{4}\,\sigma^{[(2\alpha-1)+i(2\alpha),\,(2\beta-1)-i(2\beta)]}$$

$$= \frac{-1}{8}\left[\gamma^{2\alpha-1} + i\gamma^{2\alpha},\ \gamma^{2\beta-1} - i\gamma^{2\beta}\right]$$

$$= \frac{+1}{8}\Big[I\otimes\cdots\otimes I\otimes \underbrace{(\tau_1 - i\tau_2)}_{\alpha^{th}\text{ place} = 2\tau^-}\otimes \tau_3\otimes\cdots\otimes\tau_3,$$
$$I\otimes\cdots\otimes I\otimes \underbrace{(\tau_1 + i\tau_2)}_{\beta^{th}\text{ place} = 2\tau^+}\otimes \tau_3\otimes\cdots\otimes\tau_3\Big]$$

$\Rightarrow$ for $\alpha<\beta$

$$\lambda^\alpha{}_\beta = \frac{+1}{8}\Big(I\otimes\cdots\otimes I\otimes \overset{\alpha^{th}}{2\tau^+}\otimes\tau_3\otimes\cdots\otimes \overbrace{[\tau_3, 2\tau^+]}^{\beta^{th}}\otimes I\otimes\cdots\otimes I\Big)$$

$$= \Big(I\otimes\cdots\otimes I\otimes \underset{\alpha^{th}}{\underline{\underline{\tau^-}}}\otimes\tau_3\otimes\cdots\otimes \underset{\beta^{th}}{\underline{\underline{\tau^+}}}\otimes I\otimes\cdots\otimes I\Big)$$

So $\lambda^\alpha{}_\beta$ __raises__ __β^{th} spin__ and __lowers__ __α^{th} spin__.

For $\alpha=\beta$

$$\lambda^\alpha{}_\alpha = \frac{-1}{2}\Big(I\otimes\cdots\otimes I\otimes \underset{\alpha^{th}}{\underline{\underline{\tau_3}}}\otimes I\otimes\cdots\otimes I\Big) = \frac{-S_\alpha}{2}$$

(NO SUM)

$$\begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & \ddots & \\ & & & a_N\end{pmatrix} = \sum_{\alpha=1}^{N} a_\alpha \lambda^\alpha{}_\alpha = -\frac{1}{2}\sum_{\alpha=1}^{N} a_\alpha S_\alpha .$$

CONSIDER SPINOR $\Psi_+ = 16$ OF $SO(10)$.

$$= |s_1\ s_2\ s_3\ s_4\ s_5\rangle_{\prod s_\alpha = +1}$$

THIS CONTAINS

$$\left[\begin{array}{l} \{|+++++\rangle\} \longrightarrow 1 \text{ component} \\ \{|+++--\rangle, \text{ permutations}\} \longrightarrow 10 \text{ components} \\ \{|+----\rangle, \text{ permutations}\} \longrightarrow 5 \text{ components} \end{array}\right.$$

SINCE $SU(5)\times U(1)$ GENERATORS DO NOT CHANGE NUMBER OF + SIGNS $\Rightarrow$ THESE THREE TYPES DO NOT MIX WITH EACH OTHER UNDER $SU(5)\times U(1)$: IRREDUCIBLE REPS.

$$\lambda^1{}_2\,|\underbrace{+----}_{T_1}\rangle = |\underbrace{-+---}_{T_2}\rangle$$

$$\underbrace{\sum_\alpha a_\alpha \lambda^\alpha{}_\alpha}_{SU(5)\text{ GEN}}\,|+----\rangle = -\tfrac{1}{2}(a_1 - a_2 - a_3 - a_4 - a_5)|\ \rangle$$

$$= -a_1|\ \rangle \qquad \left(\text{since } \sum a_\alpha = 0 \text{ for } su(5)\right)$$

$$\Rightarrow \quad \{|+----\rangle, \text{ permutations}\} = \bar{5} \text{ of } su(5)$$

$$\{|+++--\rangle, \text{ permutations}\} = 10 \quad ''$$

$$\{|+++++\rangle\} = 1$$

$$\sum_\alpha a_\alpha \lambda^\alpha{}_\alpha\,|+++--\rangle = -\tfrac{1}{2}(a_1 + a_2 + a_3 - a_4 - a_5) = +(a_4+a_5)$$

$$= -(a_1+a_2+a_3)|\ \rangle \Longrightarrow |\ \rangle = T^{45} = T_{123}$$

CONSIDER THE $\mathcal{U}(1)$ GENERATOR

$$X \equiv -2\sum_{\alpha=1}^{5} \lambda^{\alpha}{}_{\alpha} = -2\begin{pmatrix}1&&&&\\&1&&&\\&&1&&\\&&&1&\\&&&&1\end{pmatrix} = \sum_{\alpha=1}^{5} S_\alpha$$

a common normalization

$$\left.\begin{aligned} X|{+}{+}{+}{+}{+}\rangle &= 5|{+}{+}{+}{+}{+}\rangle \\ X|{+}{+}{+}{-}{-}\rangle &= 1|{+}{+}{+}{-}{-}\rangle \\ X|{+}{-}{-}{-}{-}\rangle &= -3|{+}{-}{-}{-}{-}\rangle \end{aligned}\right\} \Rightarrow 16 = 1^{5} + 10^{1} + \bar{5}^{-3}$$

D.4 ANOTHER LOOK AT B−L

PATI−SALAM: $\quad SO(4)\times SO(6) \subset SO(10)$

$$\begin{array}{ccc} \| & & \| \\ {[SU(2)_L \times SU(2)_R]} & \times & SU(4)_c \\ \cup & & \cup \\ {[SU(2)_L \times U(1)_{I_{3R}}]} & \times & [SU(3)_c \times U(1)_{B-L}] \end{array}$$

LOOK AT $\mathcal{U}(3) = SU(3)_c \times U(1)_{B-L}$ GENERATOR EMBEDDED IN $SO(6)$ AS DESCRIBED ABOVE:

$$B-L = \frac{-2}{3}\begin{pmatrix}1&&\\&1&\\&&1\end{pmatrix} = \frac{-2}{3}\left(\lambda^3{}_3 + \lambda^4{}_4 + \lambda^5{}_5\right) = \frac{1}{3}(S_3+S_4+S_5)$$

$$\Rightarrow \begin{cases} (B-L)|{+}{+}{+}\rangle = \frac{1}{3}(1+1+1) = +1 & 1 \text{ of color} \\ (B-L)|{+}{-}{-}\rangle = \frac{1}{3}(1-1-1) = -1/3 & \bar{3} \text{ of color} \end{cases}$$

So Ψ_+ of $SO(6) = \bar{4}$ (fundamental of $SU(4)_c$)

$$= \underbrace{\bar{3}^{-1/3}}_{\text{antiquark}} + \underbrace{1^{+1}}_{\text{antilepton}}$$

As a $\mathcal{U}(5)$ GENERATOR: $\quad B-L = -\frac{2}{3}\begin{pmatrix}0&&&&\\&0&&&\\&&1&&\\&&&1&\\&&&&1\end{pmatrix}$

The Higgs of SU(5) model are in

$$\bar{5} + 5 = \begin{pmatrix} H_d \\ H_{dc} \end{pmatrix} + \begin{pmatrix} H_u \\ H_{uc} \end{pmatrix}$$

These combine into a 10 of SO(10).

$$X = -2 \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix} \Rightarrow 10 = \bar{5}^{+2} + 5^{-2}$$

$$B-L = -\frac{2}{3} \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix} \Rightarrow$$

H_d, H_u have $B-L = 0$

H_{dc} has $B-L = 2/3$

H_{uc} has $B-L = -2/3$

This is important for later discussion of <u>2/3 splitting.</u>

If $M_{Higgs} \propto B-L$

$$\Rightarrow \begin{cases} M(H_u, H_d) = 0 \\ M(H_{uc}, H_{dc}) \neq 0 \end{cases}$$

2.5 <u>COUPLING SPINORS TO VECTORS AND TENSORS</u>

Define $\gamma^{[ij\cdots k]} \equiv \left(\gamma^i \gamma^j \cdots \gamma^k\right)_{\text{antisymmetrized}}$.

γ^i FLIPS ONE "SPIN." $\Rightarrow$ $\gamma^{[ij\cdots k]}$ with even (odd) number of indices flips even (odd) number of spins.

C $\equiv$ charge conjugation matrix. C flips <u>all</u> spins.

One can construct an SO(2N) invariant as follows:

$$\boxed{\Psi^T C \left(\gamma^{[ij\cdots k]}\right) \Psi \; T^{[ij\cdots k]}}$$

[Note: if $\Psi \to e^{i\theta\sigma}\Psi$, then $(\Psi^T C) \to (\Psi^T C) e^{-i\theta\cdot\sigma}$.

Also only totally antisymmetrized product of γ^i need to be considered, since <u>symmetric</u> product $\gamma^{(i}\gamma^{j)} = \delta^{ij} I$ by Clifford algebra.]

For <u>N even</u>:
- $T^{[ij\cdots k]}$ with <u>even rank</u> couples to $\Psi_\pm^T C \Psi_\pm$
- $T^{[ij\cdots k]}$ with <u>odd rank</u> couples to $\Psi_\pm^T C \Psi_\mp$

For <u>N odd</u>:
- $T^{[ij\cdots k]}$ with <u>even rank</u> couples to $\Psi_\pm^T C \Psi_\mp$
- $T^{[ij\cdots k]}$ with <u>odd rank</u> couples to $\Psi_\pm^T C \Psi_\pm$

Define
$$\gamma_{\text{FIVE}} \equiv \eta \underbrace{\gamma^{[i}\gamma^j \cdots \gamma^{k]}}_{2N \text{ factors}} = \eta\, \gamma^1 \gamma^2 \gamma^3 \cdots \gamma^{2N}$$
$$= (\tau_3 \otimes \cdots \otimes \tau_3)$$

$$\gamma_{\text{FIVE}} \Psi_\pm = \pm \Psi_\pm$$

$$\underbrace{\gamma^{[i}\gamma^j \cdots \gamma^{k]}}_{p} = \pm \gamma_{\text{FIVE}} \underbrace{\gamma^{[l}\gamma^m \cdots \gamma^{n]}}_{2N-p}$$

$$\Rightarrow \begin{cases} \Psi^T_{\substack{\pm (N\text{ even}) \\ \mp (N\text{ odd})}} C\Big[T + \underbrace{\gamma^{[ij]}}_{-2i\sigma^{ij}} \underbrace{T^{[ij]}}_{\text{adjoint}} + \cdots + \underbrace{\gamma^{[i\cdots k]}}_{\substack{\text{largest even} \\ \le N}} T^{[i\cdots k]}\Big]\Psi_\pm \\ \Psi^T_{\substack{\mp (N\text{ even}) \\ \pm (N\text{ odd})}} C\Big[\gamma^i T^i + \gamma^{[ijk]} T^{[ijk]} + \cdots + \underbrace{\gamma^{[i\cdots k]}}_{\substack{\text{largest odd} \\ \le N}} T^{[i\cdots k]}\Big]\Psi_\pm \end{cases}$$

EXAMPLE : SO(10)

$$\begin{cases} \Psi_+ \; C\left(T + \gamma^{[ij]}T^{[ij]} + \gamma^{[ijkl]}T^{[ijkl]}\right)\Psi_- \\ 16 \quad (1 \;+\; 45 \;+\; 210)\; \overline{16} \end{cases}$$

$$\begin{cases} \Psi_\pm \; C\left(\gamma^i T^i + \gamma^{[ijk]}T^{[ijk]} + \gamma^{[ijklm]}T^{[ijklm]}\right)\Psi_\pm \\ 16 \quad (10 \;+\; 120 \;+\; \overline{126})\; 16 \\ (\overline{16}) \quad (10 \;+\; 120 \;+\; 126)\; (\overline{16}) \end{cases}$$

$$\Big[\text{Note} \quad T^{[ijklm]} = \pm\varepsilon^{ijklm\,i'j'k'l'm'}\, T^{[i'j'k'l'm']}$$

$$\text{Rank 5 tensor} = 252 = \underbrace{126 + \overline{126}}_{\text{irreducible}} \text{ tensors}\Big]$$

IN MINIMAL SO(10) MODEL, FERMION MASS TERMS

$$Y_{AB}(16_{LA}\,16_{LB})\,10_H$$

$$\begin{aligned} \longrightarrow Y_{AB}\big(& 10^1_A\; 10^1_B\; 5^{-2}_H && \to M_u \\ &+ 10^1_A\; \bar{5}^{-3}_B\; \bar{5}^{+2}_H + \bar{5}^{-3}_A\; 10^1_B\; \bar{5}^{+2}_H && \to M_d, M_L \\ &+ \bar{5}^{-3}_A\; 1^5_B\; 5^{-2}_H + 1^5_A\; \bar{5}^{-3}_B\; 5^{-2}_H\big) && \to M_\nu \end{aligned}$$

THE MAJORANA MASS MATRIX OF THE RIGHT-HANDED NEUTRINOS, CAN COME FROM:

$$\underset{}{(Y_R)_{AB}}(\underset{1^5}{16_{LA}}\,\underset{1^5}{16_{LB}})\,\underset{\langle 1^{-10}\rangle \sim M_{GUT}}{\overline{126}_H} \longrightarrow M_R$$

OR A HIGHER-DIMENSION OPERATOR:

$$(Y_R)_{AB}(\underset{1^5}{16_{LA}}\,\underset{1^5}{16_{LB}})\,\underset{\langle 1^{-5}\rangle}{\overline{16}_H}\,\underset{\langle 1^{-5}\rangle \sim M_{GUT}}{\overline{16}_H}/M_G$$

THUS, IN MINIMAL SO(10) ONE HAS

$$\underbrace{M_N = M_u \propto M_D = M_L}_{\text{SYMMETRIC}}, \quad \underbrace{M_R}_{\text{SYMMETRIC}}$$

IN MINIMAL PATI-SALAM, ONE HAS SAME RELATIONS, BUT THE DIRAC MATRICES NEED NOT BE SYMMETRIC.

IN MINIMAL SU(5)

$$M_N,\ \underbrace{M_u}_{\text{SYMMETRIC}},\ M_D = M_L^T,\ \underbrace{(M_R)}_{\text{SYMMETRIC}}$$

IN MINIMAL SO(10) (OR PATI SALAM) $M_\nu = M_u M_R^{-1} M_u^T$

$\Rightarrow$ (FOR ONE FAMILY MODEL) $m_3 = \dfrac{m_t^2}{M_R}$

$$M_R = \frac{m_t^2}{m_3} = \frac{(174\text{ GeV})^2}{0.06\text{ eV}} = \frac{1}{2}\times 10^{15}\text{ GeV}$$

Close to GUT scale!

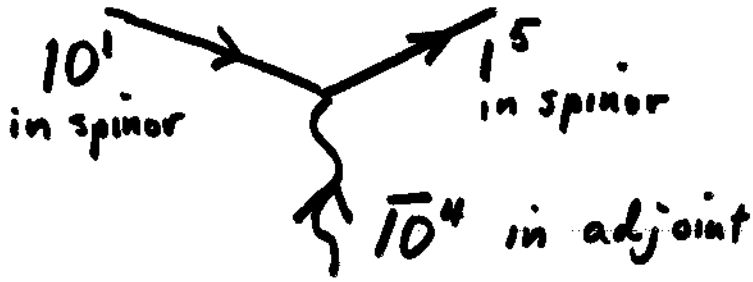
NOTE ALSO THAT MINIMAL SO(10) $\Rightarrow$ $\theta_{CKM} = 0$. (Approx. correct)

(E) BREAKING SO(10) TO SM.

NEED <u>TWO</u> <u>KINDS</u> OF HIGGS TO DO IT.

$$SO(10) \supset SU(5) \times U(1)_X$$

$$\begin{cases} 45 \rightarrow 24^0 + 1^0 + 10^{-4} + \overline{10}^{4} \\ 16 \rightarrow 10^1 + \bar{5}^{-3} + 1^5 \\ \overline{16} \rightarrow \overline{10}^{-1} + 5^3 + 1^{-5} \\ 10 \rightarrow 5^{-2} + \bar{5}^2 \end{cases}$$



etc.

We know that an SU(5) adjoint Higgs, 24_H, can be used to break $SU(5) \longrightarrow SM$.

In SO(10), an adjoint Higgs is not enough.

The 10^{-4} and $\overline{10}^{4}$ in a 45_H cannot get VEVs $\sim M_G$, since all components of them are electrically charged. (Even in "flipped SU(5)".) And 24^0 and 1^0 leave $U(1)_X$ unbroken, as well as $SU(3) \times SU(2) \times U(1) < SU(5)$.

Need another Higgs to break $U(1)_X$

Usual choices $16_H + \overline{16}_H$ <u>or</u> $126_H + \overline{126}_H$

(Need <u>both</u> 126 <u>and</u> $\overline{126}$, or both 16 and $\overline{16}$, so as not to break SUSY at M_G by D terms:

$$\langle D^a \rangle = \langle \sum_i \Phi_i^\dagger \lambda^a \Phi_i \rangle)$$

We saw that $16 \supset 1^5$, $\overline{16} \supset 1^{-5}$
and $126 \supset 1^{10}$, $\overline{126} \supset 1^{-10}$

and that these can give M_R. They also break $U(1)_X$.

SO:

45_H (and/or $54_H, \ldots$) $\longrightarrow$ Breaks SO(10) to rank 5 subgroup eg. $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$ — Does 2/3 splitting

$16_H + \overline{16}_H$ or $126_H + \overline{126}_H$ $\longrightarrow$ Breaks $U(1)_X$ ("breaks rank") — Gives mass to N^c (i.e. ν_R)

E.2 "Flipped SU(5)" [DeRujula, Georgi, Glashow 1980; S.M.B. 1982]

$U(1)_{em}$ can be embedded purely in $SU(5)$ (Georgi-Glashow) or partly in $SU(5)$ and partly in $U(1)_X$ (Flipped $SU(5)$).

$$SO(10) \supset SU(5) \times U(1)_X$$
$$\supset [SU(3)_c \times SU(2)_L \times \underbrace{U(1)_{Y_5}] \times U(1)_X}_{U(1)_Y}$$

Let $\boxed{Y/2 = \alpha\, Y_5/2 + \beta X}$ $(\alpha = 1, \beta = 0 \Rightarrow$ G.G.$)$

rep $[SU(3)_c \times SU(2)_L \times U(1)_{Y_5}]^{U(1)_X}$	G-G	Flipped	$\overbrace{\alpha Y_5/2 + \beta X}^{Y/2}$
$(1, 1, +1)^{1}$	e_L^+	N^c	$\alpha + \beta = 0, 1$
$(3, 2, 1/6)^{1}$	$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\boxed{\frac{1}{6}\alpha + \beta = \frac{1}{6}}$
$(\bar{3}, 1, -2/3)^{1}$	u_L^c	d_L^c	$-\frac{2}{3}\alpha + \beta = +1/3, -2/3$
$(1, 2, -1/2)^{-3}$	$L_L = \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}$	$L_L = \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}$	$\boxed{-\frac{1}{2}\alpha - 3\beta = -\frac{1}{2}}$
$(\bar{3}, 1, 1/3)^{-3}$	d_L^c	u_L^c	$\frac{1}{3}\alpha - 3\beta = -2/3, 1/3$
$(1, 1, 0)^{5}$	N_L^c	e_L^+	$0\alpha + 5\beta = 1, 0$

$$\left.\begin{aligned} \alpha + \beta &= 0 \\ \tfrac{1}{6}\alpha + \beta &= \tfrac{1}{6} \end{aligned}\right\} \Rightarrow \alpha = -1/5,\ \beta = 1/5 \qquad \underbrace{\boxed{Y/2 = +\frac{1}{5}\left(-Y_5/2 + X\right)}}_{\text{Flipped}}$$

Check: $-\frac{2}{3}\alpha + \beta = -\frac{2}{3}\left(-\frac{1}{5}\right) + \frac{1}{5} = \frac{1}{3}$ ✓

$\frac{1}{3}\alpha - 3\beta = \frac{1}{3}\left(-\frac{1}{5}\right) - 3\,\frac{1}{5} = -2/3$ ✓

HIGGS IN FLIPPED SU(5)

	$(3_c\ 2_L\ Y_5/2)^X$	$Y/2 = -\frac{1}{5}(Y_5/2) + \frac{1}{5}X$	$(3_c\ 2_L\ Y/2)$
10^1	$(1, 1, +1)^1$	$0 = -\frac{1}{5}(1) + \frac{1}{5}(1)$	$\langle(1, 1, 0)\rangle \neq 0$
	$(3, 2, 1/6)^1$	$1/6 = -\frac{1}{5}(1/6) + \frac{1}{5}(1)$	$(3, 2, 1/6)$
	$(\bar{3}, 1, -2/3)^1$	$1/3 = -\frac{1}{5}(-2/3) + \frac{1}{5}(1)$	$(\bar{3}, 1, 1/3)$
$\bar{5}^{-3}$	$(1, 2, -1/2)^{-3}$	$-1/2 = -\frac{1}{5}(-1/2) + \frac{1}{5}(-3)$	$(1, 2, -1/2)$
	$(\bar{3}, 1, 1/3)^{-3}$	$-2/3 = -1/5(1/3) + \frac{1}{5}(-3)$	$(\bar{3}, 1, -2/3)$
1^5	$(1, 1, 0)^5$	$1 = -\frac{1}{5}(0) + \frac{1}{5}(5)$	$(1, 1, +1)$
5^{-2}	$(1, 2, 1/2)^{-2}$	$-\frac{1}{2} = -\frac{1}{5}(\frac{1}{2}) + \frac{1}{5}(-2)$	$(1, 2, -1/2)$
	$(3, 1, -1/3)^{-2}$	$-\frac{1}{3} = -\frac{1}{5}(-\frac{1}{3}) + \frac{1}{5}(-2)$	$(3, 1, -1/3)$
$\bar{5}^2$	$(1, 2, -1/2)^2$	$\frac{1}{2} = -\frac{1}{5}(-\frac{1}{2}) + \frac{1}{5}(2)$	$(1, 2, 1/2)$
	$(\bar{3}, 1, 1/3)^2$	$\frac{1}{3} = -\frac{1}{5}(\frac{1}{3}) + \frac{1}{5}(2)$	$(\bar{3}, 1, 1/3)$

SO:
$$\begin{cases} \langle 1^0 (45_H) \rangle : & SO(10) \to SU(5) \times U(1)_X \\ \langle 10^1 (16_H) \rangle : & SU(5) \times U(1)_X \to SM. \end{cases}$$

IN "FLIPPED SU(5)" MODELS IT IS ASSUMED THAT STARTING GROUP IS $SU(5) \times U(1)_X$, <u>NOT</u> SO(10). AS WE SHALL SEE, THIS HAS ADVANTAGES. BUT IT LEAVES UNEXPLAINED UNIFICATION OF GAUGE COUPLINGS.

LECTURE III: THE GAUGE HIERARCHY PROBLEM IN SUSY GUTs

A) NON-SUSY GUTs, SU(5)

Adjoint Higgs (24_H): $\langle \Omega^\alpha{}_\beta \rangle = \Omega \begin{pmatrix} \frac{1}{2} & & & & \\ & \frac{1}{2} & & & \\ & & -\frac{1}{3} & & \\ & & & -\frac{1}{3} & \\ & & & & -\frac{1}{3} \end{pmatrix} = \Omega\, Y/2$

breaks $SU(5) \to SM$.

CONSIDER $V(5_H)$

$$V_{tree} = \lambda (5_H^\dagger 5_H)^2 + M^2 (5_H^\dagger 5_H) + \lambda' (\mathrm{tr}\, 24_H 24_H) 5_H^\dagger 5_H + \lambda'' 5_H^\dagger 24_H 24_H 5_H + M' 5_H^\dagger 24_H 5_H$$

$$= \lambda (H^\dagger_\alpha H^\alpha)^2 + M^2 (H^\dagger_\alpha H^\alpha) + \lambda' (\Omega^\alpha{}_\beta \Omega^\beta{}_\alpha) H^\dagger_\gamma H^\gamma + \lambda'' H^\dagger_\alpha \Omega^\alpha{}_\beta \Omega^\beta{}_\gamma H^\gamma + M' H^\dagger_\alpha \Omega^\alpha{}_\beta H^\beta$$

$$\to (H, H_c)^\dagger \left[\lambda'' \Omega^2 \begin{pmatrix} \frac{1}{4} I_2 & \\ & \frac{1}{9} I_3 \end{pmatrix} + (M^2 + \tfrac{5}{6}\lambda' \Omega^2) \begin{pmatrix} I_2 & \\ & I_3 \end{pmatrix} + M'\Omega \begin{pmatrix} \frac{1}{2} I_2 & \\ & -\frac{1}{3} I_3 \end{pmatrix} \right] \begin{pmatrix} H \\ H_c \end{pmatrix}$$

$$\Rightarrow \quad m^2(H) = \tfrac{1}{4}\lambda'' \Omega^2 + (M^2 + \tfrac{5}{6}\lambda' \Omega^2) + \tfrac{1}{2} M' \Omega \approx (10^2\,\mathrm{GeV})^2$$

$$m^2(H_c) = \tfrac{1}{9}\lambda'' \Omega^2 + (M^2 + \tfrac{5}{6}\lambda' \Omega^2) - \tfrac{1}{3} M' \Omega \approx (10^{15}\,\mathrm{GeV})^2$$

THREE ASPECTS:

(a) Doublet-Triplet (2/3) Splitting problem
[WHY CANCELLATION?]

(b) Fine tuning problem:
[Why true to all orders?]

(c) Small number problem: [where does $\left(\frac{10^2}{10^{15}}\right)^2 = 10^{-26}$ come from?]

(B) SUSY SU(5)

NOTE: IN SUSY WE CAN LOOK AT MASSES OF THE FERMIONIC PARTNERS OF THE HIGGS DOUBLETS AND TRIPLETS (i.e. THE HIGGSINOS). THESE CAN BE READ OFF DIRECTLY FROM THE SUPERPOTENTIAL

$$W = \lambda\, \bar{5}_H\, 24_H\, 5_H + M\, \bar{5}_H\, 5_H + W(24_H) + \cdots$$

NOTE: $\bar{5}_H$ AND 5_H ARE DISTINCT LEFT-HANDED CHIRAL SUPERFIELDS.

$$\Rightarrow \quad \underbrace{\bar{5}_H \left(\lambda \langle 24_H \rangle + M\right) 5_H}$$

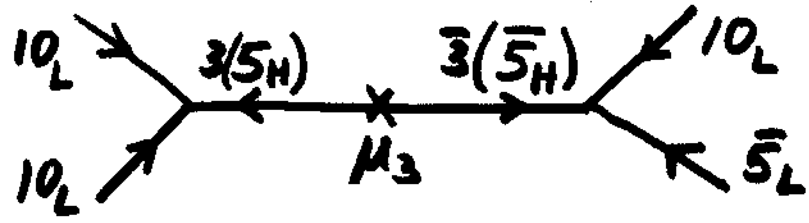
Higgsinos

$$\Rightarrow \quad (\bar{2}_H\ \bar{3}_H)\left[\lambda \Omega \begin{pmatrix} \frac{1}{2} I_2 & \\ & -\frac{1}{3} I_3 \end{pmatrix} + M \begin{pmatrix} I_2 & \\ & I_3 \end{pmatrix}\right]\begin{pmatrix} 2_H \\ 3_H \end{pmatrix}$$

$2 \equiv (1, 2, 1/2)$, $\quad \bar{2} \equiv (1, 2, -1/2)$
$3 \equiv (3, 1, -1/3)$, $\quad \bar{3} \equiv (\bar{3}, 1, 1/3)$

$$\Rightarrow \begin{cases} \mu_2 = \frac{1}{2}\lambda\Omega + M & \sim M_{weak} \\ \mu_3 = -\frac{1}{3}\lambda\Omega + M & \sim M_G \end{cases}$$

μ_2 is just "μ parameter" of MSSM
μ_3 is the parameter that comes into p decay amplitude (d=5 operator)



(C) SUSY MECHANISMS TO SOLVE THE 2/3 SPLITTING PROBLEM

SUSY AUTOMATICALLY SOLVES THE "FINE-TUNING" PROBLEM, BECAUSE W IS NOT RENORMALIZED — no radiative corrections to it.

FOUR MECHANISMS HAVE BEEN PROPOSED TO SOLVE 2/3 SPLITTING PROBLEM IN SUSY GUTS:

1. "SLIDING SINGLET MECHANISM" [Witten 1981, Nanopoulos + Tamvakis 1982]
2. "MISSING VEV MECHANISM" [Dimopoulos + Wilczek 1981] ("DIMOPOULOS WILCZEK MECHANISM")
3. "MISSING PARTNER MECHANISM" [Georgi 1982, Masiero, Nanopoulos, Tamvakis + Yanagida 1982]
4. "GIFT MECHANISM" [Berezhiani + Dvali 1989]

SLIDING SINGLET MECHANISM.

$$W \supset \lambda\, \bar{5}_H\, 24_H\, 5_H + \lambda'\, \bar{5}_H\, 1_H\, 5_H + \cancel{M\, \bar{5}_H\, 5_H}$$
$$+\, W(24_H) + \cdots$$

absorb into 1_H

SINGLET WHICH HAS NO OTHER COUPLINGS THAT GIVE IT POTENTIAL

$$\Rightarrow V(1_H) = \left|\left(\lambda\langle 24_H\rangle + \lambda'\langle 1_H\rangle\right)\langle 5_H\rangle\right|^2 + \left|\langle \bar{5}_H\rangle\left(\lambda\langle 24_H\rangle + \lambda'\langle 1_H\rangle\right)\right|^2 + \cancel{\text{SUSY}} \text{ terms.}$$

$$\langle 5_H\rangle = \begin{pmatrix} v_u \\ 0 \end{pmatrix} \qquad \langle \bar{5}_H\rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}$$

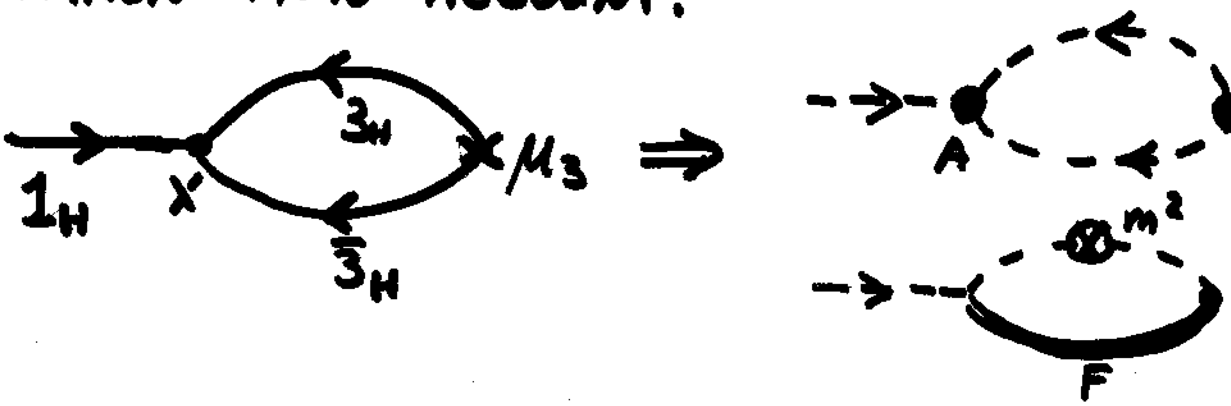
$$\Rightarrow V(1_H) = \left(\lambda \tfrac{1}{2}\Omega + \lambda'\langle 1_H\rangle\right)\left(|v_u|^2 + |v_d|^2\right) + \cancel{\text{SUSY}} \text{ terms.}$$

i.e. IN SUSY LIMIT,

$$V(1_H) = \mu_2 \left(|v_u|^2 + |v_d|^2 \right) = 0$$

$$\Rightarrow \boxed{\mu_2 = 0}, \quad \mu_3 \neq 0.$$

HOWEVER, IN SU(5) THE SLIDING SINGLET MECH. FAILS WHEN ~~SUSY~~ TAKEN INTO ACCOUNT.



$$\Rightarrow V(1_H) = \left| \mu_2(1_H)\, v \right|^2 + \frac{\lambda' \mu_3}{16\pi^2}\, m^2_{\cancel{SUSY}} \cdot \delta 1_H$$

$$0 = \frac{\partial V}{\partial(\delta 1_H)} = 2 \mu_2 v^2 \lambda' + \frac{\lambda' \mu_3}{16\pi^2}\, m^2_{\cancel{SUSY}}$$

$$\Rightarrow \quad \mu_2 = -\frac{1}{32\pi^2}\, \mu_3 \left(\frac{m^2_{\cancel{SUSY}}}{v^2} \right) \sim M_G$$

THE SLIDING SINGLET MECHANISM CAN BE MADE TO WORK IN SU(6) AND OTHER SU(N) GROUPS.

(Sen 1984, Barr 1998, Maekawa 2003)
+Yamashita hep-ph 0305116

CONSIDER SU(6)

Adjoint VEV:

$$\langle 35_H \rangle = \Omega \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & -1 & & \\ & & & & -1 & \\ & & & & & -1 \end{pmatrix}$$

Can arise from minima of potential

Fundamental VEV:

$$\langle 6_H \rangle = \begin{pmatrix} \Sigma \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$\Sigma \sim M_6$. THIS BREAKS $SU(6) \to SU(5)$. $\langle 35_H \rangle$ FURTHER BREAKS IT TO G_{SM}.

$$W \supset \lambda \bar{6}_H 35_H 6_H + \lambda' \bar{6}_H 1_H 6_H$$

$$\Rightarrow (\bar{\Sigma}, \bar{H}, \bar{H}_c) \begin{bmatrix} (\lambda\Omega + \lambda' 1_H) & & \\ & (\lambda\Omega + \lambda' 1_H) I_2 & \\ & & (-\lambda\Omega + \lambda' 1_H) I_3 \end{bmatrix} \begin{pmatrix} \Sigma \\ H \\ H_c \end{pmatrix}$$

$$\Rightarrow V(1_H) = (\mu_2 (1_H))^2 (v^2 + \Sigma^2 + \bar{\Sigma}^2)$$

+ $\cancel{\text{SUSY}}$ terms

$$\mu_2 \sim \frac{1}{32\pi^2} \mu_3 \frac{m^2_{\cancel{SUSY}}}{v^2 + \Sigma^2 + \bar{\Sigma}^2}$$

(There are some technical difficulties, but they can be overcome)

(2) MISSING VEV MECHANISM

(DIMOPOULOS - WILCZEK MECHANISM)

THE PROBLEM IN SU(5) IS THAT THE GENERATORS ARE TRACELESS:

$$\langle 24_H \rangle = \Omega (Y/2) = \Omega \begin{pmatrix} 1/2 & & & & \\ & 1/2 & & & \\ & & -1/3 & & \\ & & & -1/3 & \\ & & & & -1/3 \end{pmatrix}$$

SO NEED ANOTHER TERM TO CANCEL THE $\frac{1}{2}\Omega$ CONTRIBUTION TO μ_2.

IN $U(5)$ THE GENERATORS ARE NOT TRACELESS, BUT $U(5)$ IS REALLY $SU(5) \times U(1)$ AND THE ADJOINT "25" IS REALLY TWO IRREDUCIBLE REPS. WHOSE COUPLINGS ARE NOT RELATED BY SYMMETRY: $25 = 24 + 1$. SO CANCELLATION BETWEEN $\langle 24_H \rangle$ AND $\langle 1_H \rangle$ WOULD STILL BE ACCIDENTAL.

BUT $SO(10) \supset U(5)$

generator (in vector rep) of SO(10) $\longrightarrow$ generator (in fund. rep.) of U(5)

$$\begin{pmatrix} & a_1 & & & & & & & & \\ -a_1 & & & & & & & & & \\ & & & a_2 & & & & & & \\ & & -a_2 & & & & & & & \\ & & & & & a_3 & & & & \\ & & & & -a_3 & & & & & \\ & & & & & & & a_4 & & \\ & & & & & & -a_4 & & & \\ & & & & & & & & & a_5 \\ & & & & & & & & -a_5 & \end{pmatrix} \longrightarrow \begin{pmatrix} a_1 & & & & \\ & a_2 & & & \\ & & a_3 & & \\ & & & a_4 & \\ & & & & a_5 \end{pmatrix}$$

does not need to be traceless

B-L corresponds to $a_1 = a_2 = 0$

$a_3 = a_4 = a_5 = -2/3$

$\langle 45_H \rangle \propto B-L$ CAN ARISE FROM MINIMIZING A SIMPLE POTENTIAL.

eg.

$$W_{45} = \frac{\lambda}{M}\,\mathrm{tr}\,(45_H)^4 - \lambda' M\,\mathrm{tr}\,(45_H)^2$$

$$= \frac{2\lambda}{M}\sum_{i=1}^{5}(a_i)^4 - 2\lambda' M \sum_{i=1}^{5}(a_i)^2$$

$$\Rightarrow\quad 0 = \frac{\partial W}{\partial a_i} = \frac{8\lambda}{M}a_i^3 - 4\lambda' M a_i$$

$$\Rightarrow\quad a_i = 0 \quad \underline{\text{or}} \quad \sqrt{\frac{\lambda'}{2\lambda}}\,M \equiv \Omega$$

Multiply degenerate minimum, one of them is

$$\langle 45_H \rangle = \Omega \begin{pmatrix} \begin{smallmatrix} & 0 \\ 0 & \end{smallmatrix} & & & & \\ & \begin{smallmatrix} & 0 \\ 0 & \end{smallmatrix} & & & \\ & & \begin{smallmatrix} & 1 \\ -1 & \end{smallmatrix} & & \\ & & & \begin{smallmatrix} & 1 \\ -1 & \end{smallmatrix} & \\ & & & & \begin{smallmatrix} & 1 \\ -1 & \end{smallmatrix} \end{pmatrix} = \frac{3}{2}\Omega(B-L).$$

This solves 2/3 splitting problem:

$$W \supset 10_{1H}\, 45_H\, 10_{2H}$$

$$\longrightarrow \left(\bar{5}_{1H}\underbrace{(24_H + 1_H)}_{}5_{2H} + 5_{1H}\underbrace{(24_H+1_H)}_{}\bar{5}_{2H}\right)$$

only give mass to triplets (which have $B-L = \pm 2/3$)

[Need $10_{1H}, 10_{2H}$ different as otherwise term vanishes:

$T^a\, T^{[ab]}\, T^b$]

$$\underset{5_{1H}}{\underset{\shortparallel}{\begin{pmatrix}2_1\\3_1\end{pmatrix}}} - \underset{\bar{5}_{2H}}{\underset{\shortparallel}{\begin{pmatrix}\bar{2}_2\\\bar{3}_2\end{pmatrix}}} \qquad \underset{5_{2H}}{\underset{\shortparallel}{\begin{pmatrix}2_2\\3_2\end{pmatrix}}} - \underset{\bar{5}_{1H}}{\underset{\shortparallel}{\begin{pmatrix}\bar{2}_1\\\bar{3}_1\end{pmatrix}}}$$

NOTE: 4 LIGHT DOUBLET HIGGS, NOT 2.

(3) MISSING PARTNER MECHANISM

CONSIDER, IN $SU(5)$:

$$\begin{array}{cccc} \bar{5}_H & R_H & \bar{R}_H & 5_H \\ \begin{pmatrix} \bar{2} \\ \bar{3} \end{pmatrix} & \begin{pmatrix} \text{other} \\ 3 \end{pmatrix} & \begin{pmatrix} \text{other} \\ \bar{3} \end{pmatrix} & \begin{pmatrix} 2 \\ 3 \end{pmatrix} \end{array}$$

SUPPOSE "other" contains no $2 \equiv (1, 2, \tfrac{1}{2})$.
i.e. doublet "partner" of triplet in R is "missing"
$\Longrightarrow$ $\bar{2}$, 2 can remain light.

THIS CAN BE IMPLEMENTED IN $SU(5)$

$$\bar{5}_H \quad R_H \quad \langle V_H \rangle$$

Needs to leave SM unbroken, i.e. contain $(1, 1, 0)$

Must have $(3, 1, -\tfrac{1}{3})$ but not $(1, 2, \tfrac{1}{2})$

SIMPLEST POSSIBILITY is

$$R = 50 = T^{[\alpha\beta\gamma]}{}_{[\delta\varepsilon]} \quad \text{(traceless)}$$

$$V = 75 = T^{[\alpha\beta]}{}_{[\gamma\delta]} \quad \text{(traceless)}$$

We will now show that 50 contains $(3, 1, -\tfrac{1}{3})$ but not $(1, 2, \tfrac{1}{2})$.

$$10 \times \bar{5} = 45 + 5$$

$$T^{[\alpha\beta]} T_\gamma = \left[T^{[\alpha\beta]} T_\gamma - \tfrac{1}{4} T^{[\alpha\sigma} T_\sigma \delta^{\beta]}{}_\gamma \right] \quad \text{(Traceless)}$$
$$+ \tfrac{1}{4}\left[\underbrace{T^{[\alpha\sigma} T_\sigma}_{5} \delta^{\beta]}{}_\gamma \right]$$

$$\overline{10} \times \overline{10} = 50_S + 45_A + 5_S$$

$$T^{[\alpha\beta\gamma]} T_{[\delta\varepsilon]} = \left[T^{[\alpha\beta\gamma]}{}_{[\delta\varepsilon]} - (\text{traces}) \right] \quad \text{(Traceless)}$$
$$+ \left[\underbrace{T^{[\alpha\beta\sigma} T_{[\delta\sigma}}_{45} \delta^{\gamma]}{}_{\varepsilon]} - (\text{traces}) \right]$$
$$+ \left[\underbrace{T^{[\alpha\sigma\tau} T_{\sigma\tau}}_{5} \delta^{\beta}_{[\delta} \delta^{\gamma]}_{\varepsilon]} \right]$$

So $(\overline{10} \times \overline{10})_S = 50 + 5$

$$\begin{bmatrix} (1,1,-1) \\ (\bar{3},2,-1/6) \\ (3,1,2/3) \end{bmatrix} \times \left[(1,1,-1), (\bar{3},2,-1/6), (3,1,2/3) \right] = $$

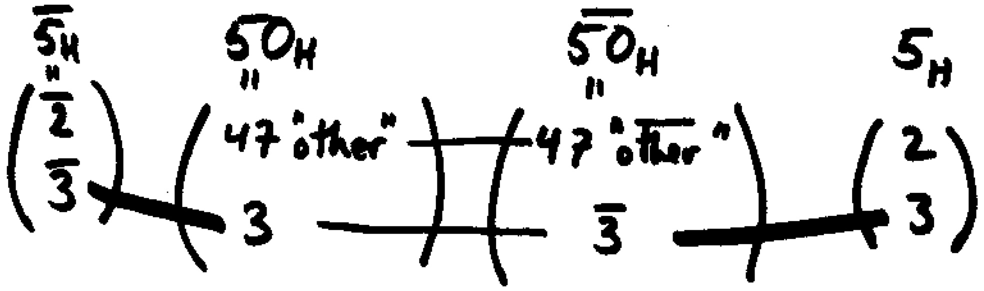
$(1,1,-2)_S$	$(\bar{3},2,-7/6)$	$(3,1,-1/3)$
$(\bar{3},2,-7/6)$	$(3,1,-1/3)_S$ $(6,3,-1/3)_S$ $(3,3,-1/3)_A$ $(6,1,-1/3)_A$	$(8,2,1/2)$ $(1,2,1/2)$
$(3,1,-1/3)$	$(8,2,1/2)$ $(1,2,1/2)$	$(\bar{6},1,4/3)_S$ $(3,1,4/3)_A$

$$50 = (1,1,-2) + \cancel{(3,1,-1/3)} + (6,3,-1/3) + (\bar{6},1,4/3)$$
$$+ (\bar{3},2,-7/6) + (3,1,-1/3) + (8,2,1/2) + \cancel{(1,2,1/2)}$$

(crossed out reps are $\in 5$)

So $50 \supset (3,1,-1/3)$ but $\not\supset (1,2,1/2)$

So one would have

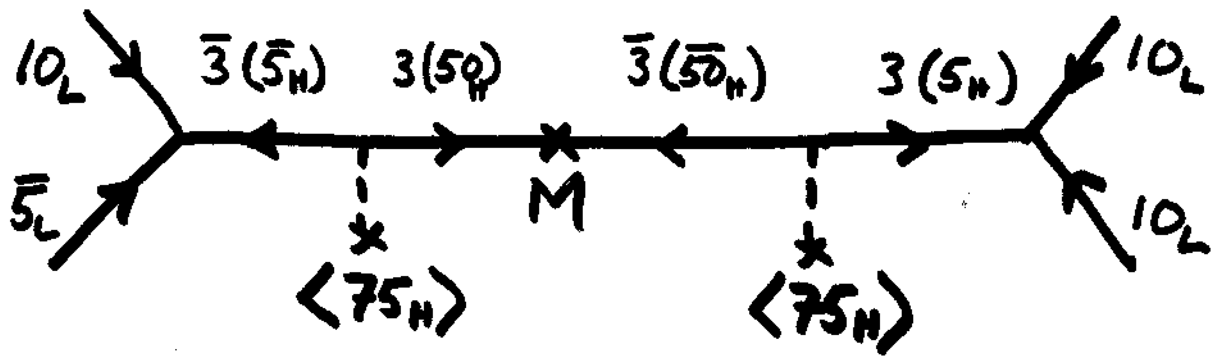


Triplet masses from $\underset{50_H}{H^{\alpha\beta\gamma}{}_{\delta\varepsilon}}\ \underset{\bar{5}_H}{H_\gamma}\ \underset{\langle 75_H\rangle}{\langle H^{\delta\varepsilon}_{\alpha\beta}\rangle}$

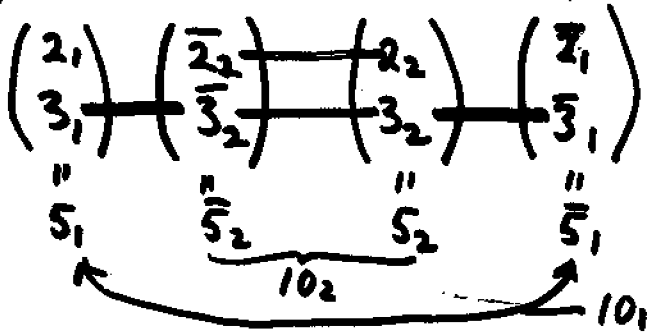
BUT, WHAT ABOUT 47 (other + $\overline{\text{other}}$)?

They must get superlarge mass. Only simple way to do this is by $M\,\overline{50}_H\,50_H$

THAT LEADS TO $d=5$ PROTON DECAY OPS.



THE SAME IS TRUE WITH MISSING PARTNER IN SO(10)



$10_{1H}\,\langle 45_H\rangle\,10_{2H}$

$M\,10_{2H}\,10_{2H}$

IN FLIPPED SU(5) THE MISSING PARTNER MECHANISM WORKS ELEGANTLY (IF <u>NOT</u> UNIFIED IN SO(10)!)

SUPPOSE

$$\left(5_H^{-2}\ 10_H^{1}\ 10_H^{1}\right) + \left(\bar 5_H^{2}\ \overline{10}_H^{-1}\ \overline{10}_H^{-1}\right)$$

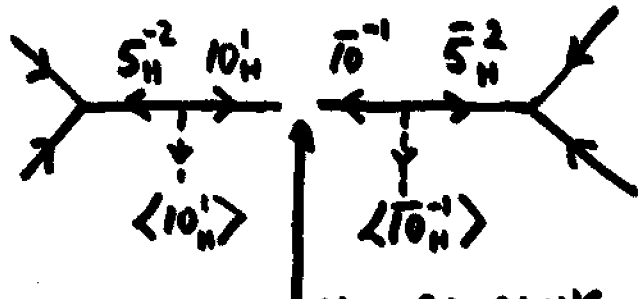
"other"

$$\begin{pmatrix}(1,2,-\tfrac12)\\(3,1,-\tfrac13)\end{pmatrix}\begin{pmatrix}(1,1,0)\\(3,2,\tfrac16)\\(\bar 3,1,\tfrac13)\end{pmatrix}\begin{pmatrix}\langle(1,1,0)\rangle\\(3,2,\tfrac16)\\(\bar 3,1,\tfrac13)\end{pmatrix}+\begin{pmatrix}(1,2,\tfrac12)\\(\bar 3,1,\tfrac13)\end{pmatrix}\begin{pmatrix}(1,1,0)\\(\bar 3,2,-\tfrac16)\\(3,1,-\tfrac13)\end{pmatrix}\begin{pmatrix}\langle(1,1,0)\rangle\\(\bar 3,2,-\tfrac16)\\(3,1,-\tfrac13)\end{pmatrix}$$

THE "OTHER" FIELDS GET <u>EATEN</u> BY HIGGS MECHANISM!

$\Rightarrow$ NO NEED TO COUPLE $M\,10_H^{1}\,\overline{10}_H^{-1}$

$\Rightarrow$ $d=5$ PROTON DECAY OPERATORS CAN BE SUPPRESSED.



HOWEVER, IF $SU(5)\times U(1)_X$ UNIFIED IN SO(10), THEN "MISSING PARTNER" IS <u>NOT</u> MISSING.

$$10_H^{1} \subset 16_H \quad \text{WHICH CONTAINS} \quad \bar 5^{-3} = \begin{pmatrix}(1,2,-\tfrac12)\\(\bar 3,1,-\tfrac23)\end{pmatrix}$$

$$\bar 5_H^{2}\ 5_H^{-3}\ \langle 10_H^{1}\rangle$$

(D) SO(10): GAUGE HIERARCHY AND PROTON DECAY

As we saw, the 2/3 splitting problem in SO(10) can be solved by assuming $\langle 45_H \rangle \propto B-L$ and having a term

$$\lambda\, 10_{1H}\, 45_H\, 10_{2H} \Rightarrow \lambda \Big[\bar{5}_{1H} \big(\underbrace{24_H + 1_H}_{=(B-L)\Omega} \big) 5_{2H} + 5_{1H} \left(24_H + 1_H \right) \bar{5}_{2H} \Big]$$

$$\Rightarrow \quad \begin{pmatrix} 2_1 \\ 3_1 \end{pmatrix} \begin{pmatrix} \bar{2}_2 \\ \bar{3}_2 \end{pmatrix} \begin{pmatrix} 2_2 \\ 3_2 \end{pmatrix} \begin{pmatrix} \bar{2}_1 \\ \bar{3}_1 \end{pmatrix}$$

To avoid having 4 light doublets, one needs to give mass to $\bar{2}_2, 2_2$. (<u>Assume MSSM doublets of Higgs H_u, H_d are $2_1, \bar{2}_1$</u>).

Simplest: $M\, 10_{2H}\, 10_{2H}$

Look at masses of color-triplet Higgsinos:

$$(\bar{3}_1, \bar{3}_2) \underbrace{\left(\begin{array}{c|c} 0 & \frac{2}{3}\lambda\Omega \\ \hline \frac{2}{3}\lambda\Omega & M \end{array} \right)}_{\equiv \mathcal{M}_3} \begin{pmatrix} 3_1 \\ 3_2 \end{pmatrix}$$

PROTON DECAY AMPLITUDE $\propto (\mathcal{M}_3^{-1})_{11}$

$$\mathcal{M}_3^{-1} = \begin{pmatrix} \frac{-M}{(\frac{2}{3}\lambda\Omega)^2} & \frac{1}{(\frac{2}{3}\lambda\Omega)} \\ \frac{1}{(\frac{2}{3}\lambda\Omega)} & 0 \end{pmatrix}$$

Can we make p decay amplitude small by setting $M \ll M_G$? YES, <u>BUT</u>

MAKING M small SUPPRESSES HIGGSINO-MEDIATED PROTON DECAY, BUT MAKES UNIFICATION OF GAUGE COUPLINGS WORSE:

$$\delta\epsilon_3\Big|_{2/3} = \frac{\delta\alpha_3(M_G)}{\alpha_3(M_G)} = \frac{3\alpha_G}{5\pi}\cdot\left[\ln|\det\mathcal{M}_3| - \ln|\det\mathcal{M}_2|\right]$$

[NOTE: ONLY "SPLIT MULTIPLETS" OF SU(5) CONTRIBUTE AT ONE LOOP TO RGE SPLITTING OF THE THREE SM GAUGE COUPLINGS. THUS, IF $\det\mathcal{M}_3 = \det\mathcal{M}_2 \Rightarrow \delta\epsilon_3 = 0$]

TRIPLET MASSES: $\mathcal{M}_3 = \begin{bmatrix} O(\mu) & \frac{2}{3}\lambda\Omega \\ \frac{2}{3}\lambda\Omega & M \end{bmatrix}$, $\mathcal{M}_2 = \begin{bmatrix} \mu & 0 \\ 0 & M \end{bmatrix}$

$$\Rightarrow \quad \delta\epsilon_3\Big|_{2/3} = -\underbrace{\frac{3\alpha_G}{5\pi}}_{\cong 0.008}\ln\Big(\mu\underbrace{\frac{M}{(\frac{2}{3}\lambda\Omega)^2}}_{(\mathcal{M}_3^{-1})_{11}}\Big)$$

$\leftarrow$ in p DECAY AMPLITUDE!

In Minimal SUSY SU(5), NEED $(\mathcal{M}_3^{-1})_{11}$ at least $\frac{1}{20}\times$ (and typically $\frac{1}{100}\times$) the value that gives good unification to sufficiently suppress p DECAY. $\Rightarrow \delta\epsilon_3 \gtrsim +0.04$

VERY GENERAL RESULT (S.M.B. hep-ph/9806217)

IF (1) ONLY MASSES OF THE FORM $(\bar{3}_H 3_H)$ OR $(\bar{2}_H 2_H + \bar{3}_H 3_H)$ $= (\bar{5}_H 5_H)$, BUT NOT $(\bar{2}_H 2_H)$, AND

(2) NO ARTIFICIAL CANCELLATIONS

$\Rightarrow$ SUPPRESSION OF p DECAY AMPLITUDE INCREASES ϵ_3.

HOWEVER, SUPPOSE: $10_{1H}\underbrace{45_H}_{B-L}10_{2H} + 10_{2H}(\underbrace{45'_H}_{I_{3R}})^2 10_{2H}/M_G$

$$H_u - \begin{pmatrix} 2_1 \\ 3_1 \end{pmatrix}\begin{pmatrix} \bar{2}_2 \\ \bar{3}_2 \end{pmatrix}\begin{pmatrix} 2_2 \\ 3_2 \end{pmatrix}\begin{pmatrix} \bar{2}_1 \\ \bar{3}_1 \end{pmatrix} - H_d$$

THIS REQUIRES 2 ADJOINT HIGGS

(E) STABILITY OF GAUGE HIERARCHY IN SO(10) MODELS [K.S. Babu + SMB 1993]

$$\mathcal{M}_2 = \begin{bmatrix} \mu & \lambda\langle 45_H\rangle \\ \lambda\langle 45_H\rangle & M \end{bmatrix}$$

Need this $\lesssim \sqrt{M_W M_G}$

Need this $\lesssim M_W$

- Must forbid $M_G(10_{1H}\,10_{1H})$ by some symmetry, Also $M_G(10_{1H}\,10_{2H})$.
- Must prevent terms that destabilize $\langle 45_H\rangle \propto B-L$

$$\langle 45_H\rangle = \begin{bmatrix} \boxed{\begin{smallmatrix} & b \\ -b & \end{smallmatrix}} & & & & \\ & \boxed{\begin{smallmatrix} & b \\ -b & \end{smallmatrix}} & & & \\ & & \boxed{\begin{smallmatrix} & a \\ -a & \end{smallmatrix}} & & \\ & & & \boxed{\begin{smallmatrix} & a \\ -a & \end{smallmatrix}} & \\ & & & & \boxed{\begin{smallmatrix} & a \\ -a & \end{smallmatrix}} \end{bmatrix} = \frac{3}{2}a(B-L) + 2b(I_{3R})$$

Must have $\quad b/a \lesssim \sqrt{M_W/M_G} \sim 10^{-7}$

THERE IS A TECHNICAL DIFFICULTY ASSOCIATED WITH COUPLING <u>ADJOINT HIGGS SECTOR</u> AND <u>SPINOR HIGGS SECTOR</u>

$$W_{\text{HIGGS}} \supset W(45_H) + W(\overline{16}_H, 16_H) + \underbrace{W_{45-16}}_{?} + 10_{1H}\,45_H\,10_{2H}$$

IF THERE ARE NO COUPLINGS (DIRECT OR INDIRECT) OF 45_H TO $\overline{16}_H + 16_H$, THEN BAD GOLDSTONE MODES RESULT.

WITHOUT 45_H COUPLING TO $\overline{16}_H + 16_H$, THE RELATIVE DIRECTION OF $\langle 45_H \rangle$ AND $\langle 16_H \rangle$ IS NOT DETERMINED BY $W \Rightarrow$ MASSLESS MODE.

WHAT ARE THE GOLDSTONE MODES?
LOOK AT WHICH GENERATORS ARE BROKEN BY <u>BOTH</u> $\langle 45_H \rangle$ AND $\langle 16_H \rangle$. THERE WOULD BE TWO "WOULD-BE GOLDSTONE BOSONS" FOR EACH SUCH GENERATOR, OF WHICH ONLY ONE CAN BE "EATEN".

$\langle 1^5(16_H) \rangle$ BREAKS $SO(10) \to SU(5)$

$$\Rightarrow \quad 45 \supset 24 + \overbrace{1 + 10 + \overline{10}}^{\text{broken}}$$

$$\begin{aligned}
= \; & (1,1,0) + \checkmark(1,1,0) + \checkmark(1,1,+1) + \checkmark(1,1,-1) \\
& +(3,2,-5/6) \qquad +\checkmark(3,2,1/6) + \checkmark(\bar{3},2,-1/6) \\
& +(\bar{3},2,5/6) \qquad +\checkmark(\bar{3},1,-2/3) + \checkmark(3,1,2/3) \\
& +(8,1,0)
\end{aligned}$$

$\langle 45_H \rangle \propto B-L$ BREAKS $SO(10) \to SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times SU(2)_R$

$$45 \supset (15,1,1) + (1,3,1) + (1,1,3) + (6,2,2) \text{ of P-S}$$

$$\begin{aligned}
= \; & (1^0,1,1) + (1^0,3,1) + (1^0,1,3) + \checkmark(3^{-2/3},2,2) \\
& (8^0,1,1) \qquad\qquad \checkmark(\bar{3}^{2/3},2,2) \\
& \checkmark(3^{4/3},1,1) \\
& \checkmark(\bar{3}^{-4/3},1,1)
\end{aligned}$$

$\Rightarrow$ GENERATORS IN $(\bar{3},2,-1/6) + (3,2,1/6)$
$(\bar{3},1,-2/3) + (3,1,2/3)$

ARE TWICE BROKEN

THE SIMPLEST WAY TO COUPLE 45_H TO $\overline{16}_H + 16_H$ IS NO GOOD:

$$\overline{16}_H \; 45_H \; 16_H$$

$$\langle 45_H \rangle = \tfrac{3}{2} a (B-L) + 2b (I_{3R})$$

$$\langle 1(16_H) \rangle \sim M_G \,, \quad \langle 1(\overline{16}_H) \rangle \sim M_G .$$

↑ has quantum numbers of N_L^c : $\begin{cases} B-L = +1 \\ I_{3R} = -\tfrac{1}{2} \end{cases}$

$$\Rightarrow \quad W \supset \underbrace{\# M_G \, b^2}_{\text{From } (45_H)^2} + \# \left[1(\overline{16}_H) \; b \; 1(16_H) \right]$$

$$0 = -F_b^* = \frac{\partial W}{\partial b} = \# M_G \, b + \# \langle 1(\overline{16}_H) \rangle \langle 1(16_H) \rangle$$

$$\Rightarrow \quad \boxed{b \sim M_G} \quad \text{(destroys hierarchy)}$$

There are various ways to couple 45_H to $\overline{16}_H, 16_H$ without destroying gauge hierarchy:

e.g.

(Babu Barr)

$$\overline{16}_H \; 45_X \; 16_H + \underbrace{45_X \; 45_{B-L} \; 45_{I_{3R}}}_{\text{THREE DISTINCT ADJOINT HIGGS}} + \cdots$$

(Barr Raby)

$$\overline{16}_H (45_H + 1_H) \underset{\substack{\uparrow \\ \text{VEV}=0}}{16_H'} + \underset{\substack{\uparrow \\ \text{VEV}=0}}{\overline{16}_H'} (45_H + 1_H') 16_H$$

LECTURE IV: FERMION MASSES IN SUSY GUTs

WE HAVE ALREADY SEEN THAT IN "MINIMAL SU(5)"

$$(Y_u)_{AB}\,(10_A\, 10_B)\,5_H \longrightarrow M_u = M_u^T = Y_u\, v_u$$

$$(Y_D)_{AB}\,(10_A\, \bar{5}_B)\,\bar{5}_H \longrightarrow M_D = M_L^T = Y_D\, v_d$$

$$(Y_N)_{AB}\,(\bar{5}_A\, 1_B)\,5_H \longrightarrow M_N = Y_N\, v_u$$

$$M_D = M_L^T \Rightarrow \begin{cases} m_b^0 = m_\tau^0 & \text{within } 20\% \\ m_s^0 = m_\mu^0 & (\text{expt} \simeq \tfrac{1}{3} m_\mu^0) \\ m_d^0 = m_e^0 & (\text{expt} \simeq 3\, m_e^0) \end{cases}$$

1) GEORGI - JARLSKOG

SU(5): $$(10_{1L}\; 10_{2L}\; 10_{3L}) \begin{bmatrix} 0 & \langle \bar{5}_H \rangle & 0 \\ \langle \bar{5}_H \rangle & \langle \overline{45}_H \rangle & 0 \\ 0 & 0 & \langle \bar{5}_H \rangle \end{bmatrix} \begin{pmatrix} \bar{5}_{1L} \\ \bar{5}_{2L} \\ \bar{5}_{3L} \end{pmatrix}$$ Yukawa constants not shown

$$(10_L\, \bar{5}_L)\,\bar{5}_H : \begin{cases} \psi^{a2}\, \psi_a\, \langle H_2 \rangle \longrightarrow d_L\, d_L^c \langle H_d \rangle \\ \psi^{12}\, \psi_1\, \langle H_2 \rangle \longrightarrow \ell_L^+\, \ell_L^- \langle H_d \rangle \end{cases}$$

$$(10_L\, \bar{5}_L)\,\overline{45}_H : \begin{cases} \psi^{a2}\, \psi_a\, \langle H_{a2}{}^a \rangle \rightarrow (-\tfrac{1}{3})\, d_L\, d_L^c \langle H_d' \rangle \\ \psi^{12}\, \psi_1\, \langle H_{12}{}^1 \rangle \rightarrow \ell_L^+\, \ell^- \langle H_d' \rangle \end{cases}$$

NOTE: $\overline{45}$ is traceless $\Rightarrow \sum_{a=3,4,5} H_{a2}{}^a + H_{12}{}^1 + \cancel{H_{11}{}^1} = 0$

$$\Rightarrow \underbrace{\langle H_{a2}{}^a \rangle}_{\text{no sum}} = -\tfrac{1}{3} \langle H_{12}{}^1 \rangle$$

$$M_L^\circ = \begin{bmatrix} 0 & C & \\ C & B & \\ & & A \end{bmatrix} \quad \xrightarrow{\text{diagonalize}} \quad \begin{bmatrix} -\frac{C^2}{B} & & \\ & B & \\ & & A \end{bmatrix}$$

$$M_D^\circ = \begin{bmatrix} & C & \\ C & \frac{1}{3}B & \\ & & A \end{bmatrix} \quad \longrightarrow \quad \begin{bmatrix} +\frac{3C^2}{B} & & \\ & -\frac{1}{3}B & \\ & & A \end{bmatrix}$$

LET $A \gg B \gg C$

NOTE: $\det M_L^\circ = \det M_L^\circ = -AC^2$

$$\Rightarrow \quad m_s^\circ \cong \tfrac{1}{3} m_\mu^\circ \qquad m_d^\circ \cong 3 m_e^\circ$$

These factors of $\frac{1}{3}$, 3 are called "Georgi Jarlskog factors."

SUCH FACTORS OF $\frac{1}{3}$, 3 ARISE VERY NATURALLY IN GUTs. THEY COME FROM FACT THAT THERE ARE 3 COLORS

ANOTHER WAY TO GET SUCH FACTORS

$$B-L = \begin{pmatrix} 1/3 & & & \\ & 1/3 & & \\ & & 1/3 & \\ & & & -1 \end{pmatrix} \quad (\text{generator of } SU(4)_c)$$

$= 1/3$ for quarks, -1 for leptons.

NOTE FORM OF 12 BLOCK ABOVE: $\begin{pmatrix} 0 & C \\ C & B \end{pmatrix}$.

Suggested independently by Weinberg, Wilczek + Zee, Fritzsch in 1979 to explain Cabibbo angle.

"TEXTURE"

) TEXTURES AND MIXING ANGLES

CONSIDER A TWO-FAMILY MODEL:

$$(\bar{d}_L\ \bar{s}_L)^{(0)} \begin{pmatrix} 0 & C' \\ C' & B' \end{pmatrix} \begin{pmatrix} d_R \\ s_R \end{pmatrix}^{(0)} + (\bar{u}_L\ \bar{c}_L)^{(0)} \begin{pmatrix} 0 & C \\ C & B \end{pmatrix} \begin{pmatrix} u_R \\ c_R \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} d_L \\ s_L \end{pmatrix} = \begin{pmatrix} \cos\theta_{dL} & \sin\theta_{dL} \\ -\sin\theta_{dL} & \cos\theta_{dL} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \end{pmatrix}^{(0)}$$ similar for d_R, s_R (θ_{dR})

$$\begin{pmatrix} u_L \\ c_L \end{pmatrix} = \begin{pmatrix} \cos\theta_{uL} & \sin\theta_{uL} \\ -\sin\theta_{uL} & \cos\theta_{uL} \end{pmatrix} \begin{pmatrix} u_L \\ c_L \end{pmatrix}^{(0)}$$ similar for u_R, c_R (θ_{uR})

$$\Rightarrow \quad \tan 2\theta_{dL} = \frac{2C'}{B'} \Rightarrow \theta_{dL} \cong \frac{C'}{B'} \cong \sqrt{\frac{m_d}{m_s}}$$

$$\tan 2\theta_{uL} = \frac{2C}{B} \Rightarrow \theta_{uL} \cong \frac{C}{B} \cong \sqrt{\frac{m_u}{m_c}}$$

bibbo:
$$\boxed{\underbrace{\theta_C}_{\simeq 0.2} = \theta_{dL} - \theta_{uL} \cong \underbrace{\sqrt{\frac{m_d}{m_s}}}_{\simeq 0.2} - e^{i\phi} \underbrace{\sqrt{\frac{m_u}{m_c}}}_{\simeq 0.07}}$$

(Old numerology: $\sin\theta_C \cong m_\pi/m_K \cong \sqrt{m_d/m_s}$)

Note that here matrices are assumed symmetric

In SU(5), justified for M_U, not M_D, M_L.

However, this assumption reduces # of parameters and thus allows a prediction.

Also gives $\theta_L = \theta_R$

(SI)MILAR TEXTURES FOR 23 BLOCK GIVE RELATIONS LIKE

$$\left\{ \begin{array}{|l|} \hline \underbrace{V_{cb}}_{\sim 0.04} = \sin\theta_{23}^{\text{quark}} \cong \underbrace{\sqrt{m_s/m_b}}_{\sim 0.14} - e^{i\phi'} \underbrace{\sqrt{m_c/m_t}}_{\sim 0.04} \\ \hline \underbrace{V_{\mu 3}}_{\sim 0.7} = \sin\theta_{23}^{\text{lepton}} \cong \underbrace{\sqrt{m_\mu/m_\tau}}_{\sim 0.25} - e^{i\phi''} \underbrace{\sqrt{m_2/m_3}}_{\sim 0.15} \end{array} \right.$$

3) AN SO(10) MODEL OF QUARK AND LEPTON MASSES

(Albright, Babu, Barr 1998)
(Albright, Barr 1999)
(Similar model by Babu, Pati, Wilczek)

THE VEVs:

$$O(M_G): \begin{cases} \langle 45_H \rangle \propto B-L \\ \langle 16_H \rangle \propto 1(16) \\ \langle \overline{16}_H \rangle \propto 1(\overline{16}) \end{cases}$$

$$O(M_W): \begin{cases} \langle 10_H \rangle \propto \begin{bmatrix} 2(5(10)) \\ \overline{2}(\overline{5}(10)) \end{bmatrix} \\ \langle 16'_H \rangle \propto \overline{2}(\overline{5}(16)) \end{cases}$$

THIRD FAMILY MASSES

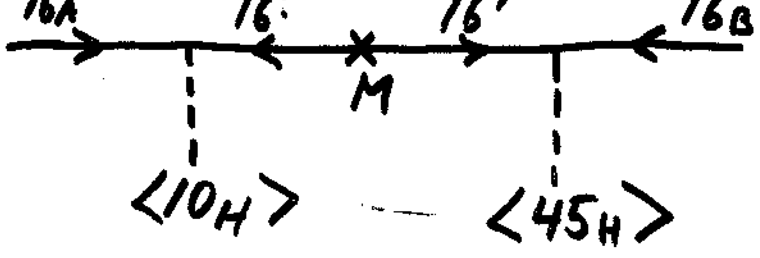
$$16_3\, 16_3 \langle 10_H \rangle \longrightarrow M_U, M_D, M_L, M_N \propto \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix}$$

SECOND FAMILY

Need Georgi-Jarlskog factors
This can be achieved if the $\langle 45_H \rangle$ is involved. Simplest effective operator is

$$16_A\, 16_B \langle 10_H \rangle \langle 45_H \rangle / M_G$$

This can arise from "integrating out" $\overline{16}' + 16'$



There are <u>two</u> ways to contract the SO(10) indices in the product $16\ 16\ 10\ 45$:

$$16 \times 16 = 10_S + 120_A + 126_S$$

$$10 \times 45 = 10 + 120 + 320$$

<u>So 16×16 must be contracted in 10 or 120.</u>

<u>If</u> $\langle 45_H \rangle \propto B-L$ <u>then only 120 channel contributes to quark and lepton masses</u>

<u>Thus this term is antisymmetric in flavor</u>

<u>PROOF</u> $\langle 10_H \rangle \sim (1,2,2)$ of Pati-Salam Group.

$\langle 45_H \rangle \sim (15,1,1)$ ($\propto B-L$ = a generator of $SU(4)_c$)

$$\Rightarrow \quad \langle 10_H \rangle \langle 45_H \rangle \sim (15,2,2)$$

But, this is not in 10 of SO(10).

$\Rightarrow$ <u>it must be in 120.</u>

CONSIDER, THEN,

$$(16_2\ 16_3) \langle 10_H \rangle \langle 45_H \rangle / M_G$$

$$\Rightarrow \quad M_u = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \varepsilon/3 \\ 0 & -\varepsilon/3 & 1 \end{bmatrix} m_u, \quad M_D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \varepsilon/3 \\ 0 & -\varepsilon/3 & 1 \end{bmatrix} m_D$$

$$M_N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\varepsilon \\ 0 & \varepsilon & 1 \end{bmatrix} m_u, \quad M_L = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\varepsilon \\ 0 & \varepsilon & 1 \end{bmatrix} m_D$$

Not realistic. <u>FOUR PROBLEMS</u>:

(1) Georgi-Jarlskog factors are 1/9 not 1/3

(2) $M_D \propto M_u \Rightarrow$ NO CKM MIXING.
e.g. $V_{cb} = 0$

(3) $M_D \propto M_u \Rightarrow \underset{\sim 0.0025}{m_c^0/m_t^0} = \underset{\sim 0.02}{m_s^0/m_b^0}$

(4) The atmospheric ν angle $U_{\mu 3} = O(\varepsilon) \ll 1$

Need another contribution to 23 block that makes M_u <u>not</u> proportional to M_D.

$\langle 45_H \rangle$ does not distinguish u from D.

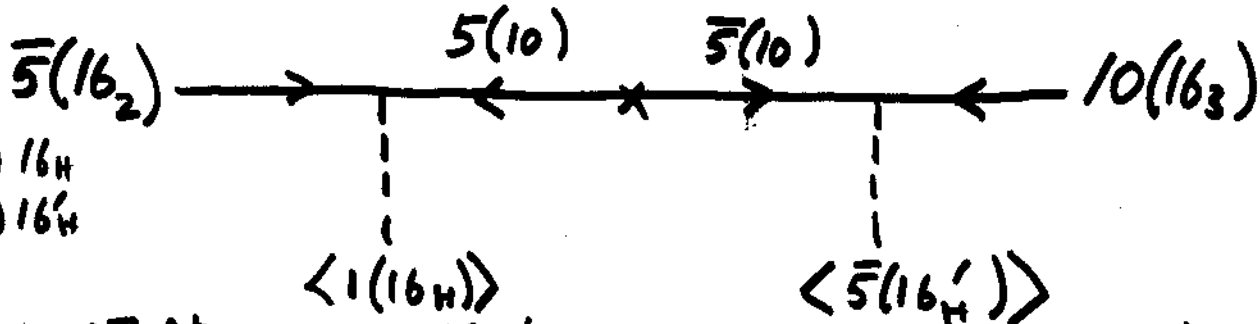
Simplest possibility: $\langle 16_H \rangle$ contributes.

Consider

$$\begin{array}{cccc} 16_2 & 16_3 & 16'_H & 16_H/M_G \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 10_2 & \bar{5}_3 & \langle \bar{5}'_H \rangle & \langle 1_H \rangle / M_G \end{array}$$

BUT $\underline{\underline{\text{NO}}}$ $10_2 \quad 10_3 \quad \cancel{5} \quad \langle 1_H \rangle / M_G$

THREE WAYS OF CONTRACTING INDICES. ASSUME OPERATOR COMES FROM DIAGRAM:



$_2 10) 16_H$
$_3 10) 16'_H$
$10\,10$

$\Rightarrow l_2^- l_3^+$ and $d_2^c d_3 \Rightarrow (M_L)_{23}$ and $(M_D)_{32}$

$$M_U = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{bmatrix} m_U \qquad M_D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & \sigma-\epsilon/3 & 1 \end{bmatrix} m_D$$

$$M_N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{bmatrix} m_U \qquad M_L = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma-\epsilon \\ 0 & \epsilon & 1 \end{bmatrix} m_D$$

ASSUME $\epsilon \ll \sigma \sim 1$

Then: (1) Georgi - Jarlskog factor is $\simeq 1/3$

(2) M_U not $\propto M_D \Rightarrow V_{cb} \neq 0$

in fact $\boxed{V_{cb}^{\circ} \simeq \frac{\epsilon}{3}\frac{\sigma^2}{1+\sigma^2}}$

(3) $\frac{m_c^{\circ}}{m_t^{\circ}} \simeq (\epsilon/3)^2 \ll \boxed{\frac{m_s^{\circ}}{m_b^{\circ}} \simeq \frac{\epsilon}{3}\frac{\sigma}{1+\sigma^2}}$

(4) $U_{\mu 3} \sim \mathcal{O}(\sigma) \sim \mathcal{O}\left(\frac{V_{cb}}{m_s/m_b}\right) \sim 1$

FIRST FAMILY CAN BE INCLUDED SIMPLY

$$M_U = \begin{bmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{bmatrix} m_U \qquad M_D = \begin{bmatrix} 0 & \delta & \delta' \\ \delta & 0 & \epsilon/3 \\ \delta' & \sigma-\epsilon/3 & 1 \end{bmatrix} m_D$$

$$M_N = \begin{bmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{bmatrix} m_U \qquad M_L = \begin{bmatrix} 0 & \delta & \delta' \\ \delta & 0 & \sigma-\epsilon \\ \delta' & \epsilon & 1 \end{bmatrix} m_D$$