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SMR.1508 - 3

SUMMER SCHOOL ON PARTICLE PHYSICS

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FLAVOUR PHYSICS

Part 2

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② Path integral:

$$\sim \int [dW]\, e^{i\int dx\, \mathcal{L}[W,q]}$$

$$\rightarrow \quad S_{eff} \cong \int dx\,dy\, g^2 J_\mu(x)\, \Delta^\mu_\nu\, J^\nu(y)$$

$$\Delta^\mu_\nu = \int \frac{d^4k}{(2\pi)^4}\, e^{-ik(x-y)} \frac{g^\mu_\nu}{k^2 - M_W^2}$$

$$k^2 \ll M_W^2 \rightarrow \quad \Delta^\mu_\nu \sim \delta(x-y)$$

same result

need to consider $J_\mu(x)\, J^\nu(y)$
$x \rightarrow y$

Operator product expansion

③ general

$$\mathcal{L}[a, b, .. ; A, B, C ..] \quad m_{a..} \ll m_{A...}$$

effective theory for $a, b ..$

$$\mathcal{H}_{eff}\,(\mathcal{L}_{eff}) = \sum_n C_n(A...)\, O_n[a, ..]$$

effective Hamiltonian

of interest: $\langle \mathcal{H}_{eff} \rangle$ matrixelements

$$\langle \mathcal{H}_{eff} \rangle = \sum C_n \langle O_n \rangle$$

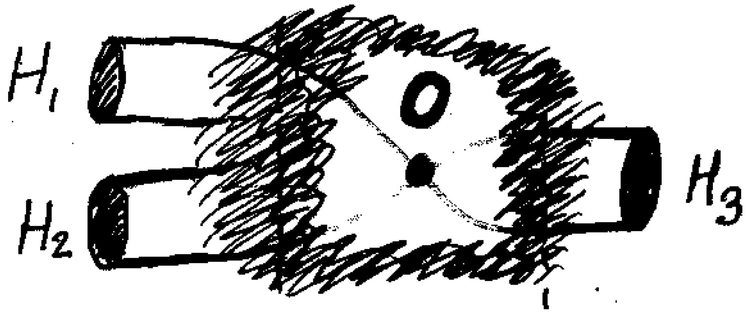
examples:

leptons : $\langle e\bar{\nu} | (\bar{e}\gamma_\mu \nu) | 0 \rangle \simeq \bar{u}_e \gamma_\mu v_\nu$

perturbative

quarks : $\langle H_1, H_2 \,|\, O[q] \,|\, H_3 \rangle$

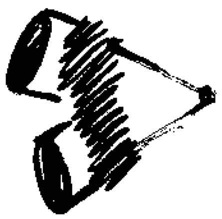
hadrons quarks



Very difficult

Special cases:

1)  $\langle M | J_\mu | 0 \rangle = f_M \cdot p_{M\mu}$

2)  $\langle M_1 | J_\mu | M_2 \rangle = f_1 p^1_\mu + f_2 p^2_\mu$

decay constant, form factor

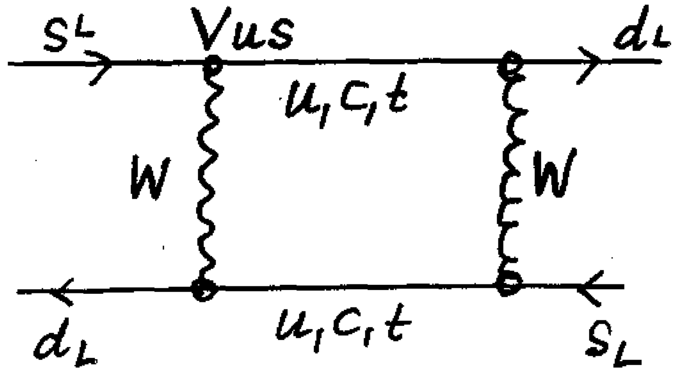
- often obtained from experiment
- non-perturbative calculations

3)  factorization

Input in unitarity triangle

① $K^\circ - \bar{K}^\circ$ mixing: $\langle \underset{\bar{s}d}{K^\circ} | \underset{\Delta S=2}{\mathcal{H}_{eff}} | \underset{s\bar{d}}{\bar{K}^\circ} \rangle$

needs 2 W's



$$\Rightarrow f(g, m_u, m_c, m_t \ldots) \cdot (V_{ij})^4 \cdot \underbrace{(\bar{d}_L \gamma^\mu s_L)(\bar{d} \gamma_\mu s_L)}_{O_{eff}^{\Delta S=2}}$$

$$f \sim g^4 \int \frac{d^4k}{(2\pi)^4} \frac{(\gamma_\mu \not{k} \gamma_\lambda)(\gamma^\mu \not{k} \gamma^\lambda)}{(k^2 - M_W^2)^2 (k^2 - m_i^2)(k^2 - m_j^2)} \left(V_{is} V_{id}^*\right)^2$$

$$\mathcal{H}_{eff} = \frac{G_F^2}{16\pi^2} M_W^2 \left\{ (V_{cs} V_{cd}^*)^2 S_0(x_c) \eta_1 + (V_{ts} V_{td}^*)^2 S_0(x_t) \eta_2 + \cdots \right\} O_{eff}^{\Delta S=2}$$

$$x_c = \frac{m_c^2}{m_W^2} \qquad x_t = \frac{m_t^2}{M_W^2}$$

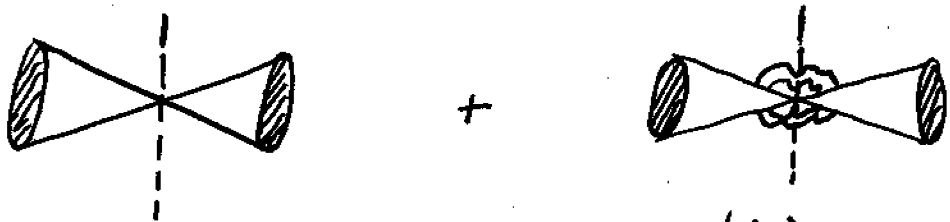
$$\eta_1 \simeq 1.4 \pm 0.2 \qquad \eta_2 = 0.57$$ QCD corrections

$$S_0 = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x^2 \log x}{2(1-x)^3}$$

$$S(x \simeq 0) = x$$

Need $\langle K^0 | O_{eff}^{\Delta S=2} | \bar{K}^0 \rangle$

$$= \sum_i \langle K^0 | \bar{d}_L \gamma^\mu s_L \underbrace{| i \rangle \langle i |}_{\sum |i\rangle\langle i| = 1} \bar{d}_L \gamma_\mu s_L | \bar{K}^0 \rangle$$



$|i\rangle = |0\rangle$ $\qquad$ $|i\rangle = \dots$

$$\left(\langle K^0 | \bar{d}_L \gamma^\mu s_L | 0 \rangle\right)^2 \qquad ? \ \text{(Lattice, } \chi PT \dots\text{)}$$

$\frac{1}{2} f_K p^{K\mu}$ ← from $K \to \mu\nu$

$$\langle \quad \rangle \cong \frac{1}{4} f_K^2 (p^K)^2 \underbrace{\left[1 + \sum_i A_i \right]}_{B_K(\mu)}$$

$$\langle \dots \rangle = \frac{8}{3} f_K^2 m_K^2 B_K^*(\mu)$$

$$\Delta m_K = \frac{G_F^2}{16\pi^2} M_W^2 \{\dots\} f_K^2 m_K \frac{8}{3} \hat{B}_K$$

$$* \ \hat{B}_K = B_K(\mu) (\alpha_s)^{-2/9} \left(1 + J_3 \frac{\alpha_s}{4\pi} \right)$$

Similar calculation for ϵ_K

$$\epsilon_K = \frac{G_F^2 f_K^2 m_K}{6\pi^2\sqrt{2}(\Delta m_K)} \hat{B}_K \, \mathrm{Im}(V_{ts}V_{td}^*) e^{i\pi/4}$$

$$\cdot \left\{ \mathrm{Re}(V_{cs}V_{cd}^*)\eta_1 S_0(x_c) - \eta_3 S_0(x_t) - \mathrm{Re}(V_{ts}V_{td}^*)\eta_2 S(m_c, m_t) \right\}$$

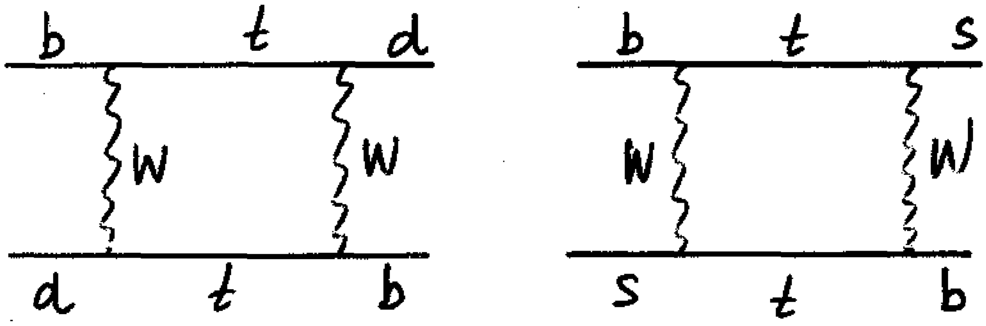
$$= (2.26)\, 10^{-3} e^{i\pi/4}$$

$$0.26 = A^2 \bar{\eta} \left((1-\bar{\rho}) A^2 \eta_2 S_0(x_t) + 0.3 \right) \hat{B}_K$$

in $\bar{\rho}, \bar{\eta}$: hyperbola

$\hat{B}_K \simeq 0.86 \pm 0.06 \pm 0.14$ Lattice

② $B - \bar{B}$ ($B_s - \bar{B}_s$) mixing



mostly t

$\sim |V_{td}|^2 \sim (\lambda^3)^2$ $\qquad$ $\sim |V_{ts}|^2 \sim (\lambda^2)^2$

0.55

$$\Delta m_{B_d} = \frac{G_F}{6\pi^2} M_{B_d} B_{B_d} f_{B_d}^2 \eta_B M_W^2 S_0(t) |V_{td}|^2$$

$$\Delta m_{B_s} = \dots\dots\dots B_{B_s}^2 f_{B_s}^2 \dots\dots\dots |V_{ts}|^2$$

$$f_{B_d} = 203 \pm 27 {}^{+0}_{-20} \quad (U\text{-}Q)$$

$$B_{B_d} = 1.34 \pm 0.12$$

Lattice

$$f_{B_s}/f_{B_d} = 1.18 \pm 4 {}^{+12}_{-0}$$

$$B_{B_s}/B_{B_d} = 1.00 \pm 0.03$$

$$\xi = f_{B_s}\sqrt{B_{B_s}} / f_{B_d}\sqrt{B_{B_d}} = 1.24 \pm 0.04 \pm 0.06$$

$$\Delta m_{B_d} = 0.5\,\mathrm{ps}^{-1} \left(\frac{f\sqrt{B}}{200M}\right)\left(\frac{\bar{m}_t(m_t)}{170}\right)\left(\frac{V_{td}}{0.008}\right)^2\left(\frac{\eta_B}{0.55}\right)$$

$$\Delta m_{B_s} = 16\,\mathrm{ps}^{-1} \left(\quad \right)\left(\quad \right)\left(\frac{V_{ts}}{0.04}\right)^2\left(\quad \right)$$

time evolution : $\dfrac{P(B^0)}{P(\bar{B}^0)} = \dfrac{x^2}{2+x^2}$ $\qquad x = \dfrac{\Delta m}{\Gamma}$

$$\Delta m_d = 0.47\ ps^{-1} \quad (\pm 0.03)$$

$$\Delta m_s \geqslant 15\ ps^{-1}$$

$$\frac{\Delta m_s}{\Delta m_d} \approx \left|\frac{V_{ts}}{V_{td}}\right|^2 \Rightarrow V_{td} = V_{ts}\sqrt{\frac{\Delta m_d}{\Delta m_s}}$$

$$\Delta m_s > \dots \longrightarrow V_{td} < \dots$$

best method for V_{td}, since $\sqrt{\frac{\Delta m_d}{\Delta m_s}}$ safe and V_{ts} very well known by unitarity

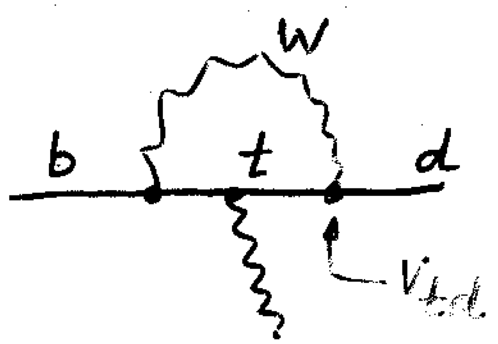
but: New physics in Δm_s

$$\Delta m_s = (\Delta m_s)_{SM} + F$$

$$V_{td} = V_{ts}\sqrt{\frac{\Delta m_d}{\Delta m_s - F}}$$

Complementary measurements of V_{td}

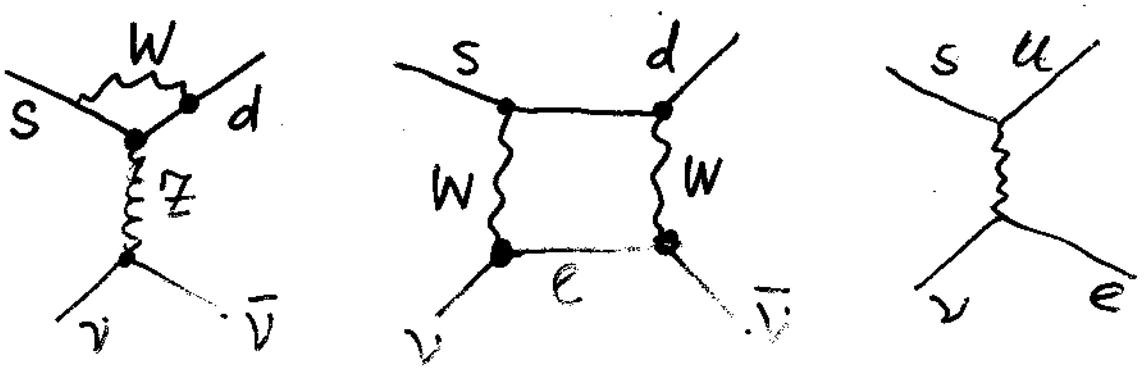
Rare decays: $b \to d\gamma$, $b \to d\ell\ell, \dots$
$B \to \rho\gamma$, $B \to \pi\ell\ell \dots$

- 

$BR \sim 10^{-6}$

- Also in Kaons

$K^+ \to \pi^+ \nu\bar\nu$



$$\mathcal{H} = \frac{G_F \alpha}{\sqrt{2}\, 2\pi \sin^2\theta_W} \sum_{l=e,\mu,\tau} \left(V_{cs}^* V_{cd} X_{NL}^{l} + V_{ts}^* V_{td} X(m_t) \right) \cdot \underbrace{(\bar{s}\gamma_\mu d_L)(\bar{\nu}\gamma^\mu \nu)}_{\mathcal{O}}$$

$X(x)$: loop function

X_{NL} : charm

$\mathcal{O} \longrightarrow \mathcal{O}_e = (\bar{s}\gamma_\mu u_L)(\bar{\nu}\gamma^\mu e_L)$

relate to $BR(K^+ \to \pi^0 e^+ \nu)$

$$BR(K^+ \to \pi^+\nu\bar\nu) = 4.11 \cdot 10^{-11} A^4 X^2(m_t) \cdot \left(\frac{1}{1-\lambda^2/2}\right)^2 \left[\left(\frac{1}{1-\frac{\lambda^2}{2}}\right)^2 \bar\eta^2 + (\rho_0 - \bar\rho)^2 \right]$$

$$\rho_0 = 1 + \frac{P_0(X)}{A^2 X(t)} \qquad P_0 = \frac{1}{4}\left(\frac{2}{3} X_{NL}^{e} + \frac{1}{3} X_{NL}^{\tau} \right)$$

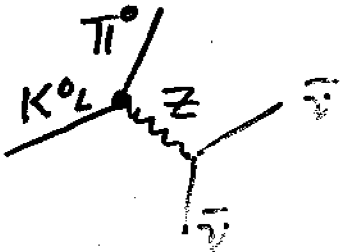
* little L.D. effects
* strong m_t dependence

Very good for V_{td}!

two events seen : $BR(K^+ \to \pi^+ \bar\nu\nu) \simeq (4 + 9.7 - 3.5)10^{-10}$

more events!

$K_L^0 \to \pi^0 \bar\nu\nu$



$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta} (V_{ts}^* V_{td}) X(t)$$
$$(\bar{s}\gamma_\mu d_L)(\bar{\nu}\gamma^\mu \nu) + h.c.$$

¡ Main contribution CP-violating ! show it !

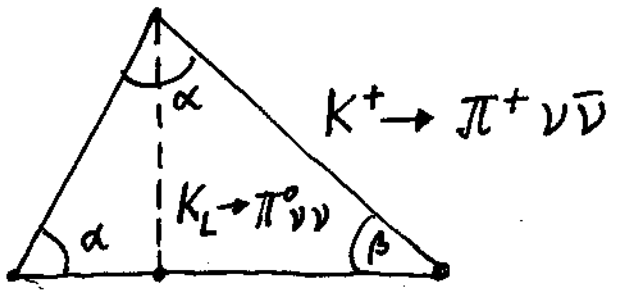
$$\rightarrow \quad \frac{BR(K_L \to \pi^0 \nu\bar\nu)}{BR(K^+ \to \pi^0 e^+ \nu)} = \ldots \; \mathrm{Im}(V_{ts}^* V_{td})$$

$$\rightarrow \quad BR(K_L \to \pi^0 \nu\bar\nu) = (3.0) 10^{-11} \left(\frac{\eta}{0.39}\right) \left(\frac{\bar{m}_t(m_t)}{170}\right)^{2.3} \left(\frac{V_{cb}}{0.04}\right)^4$$

or

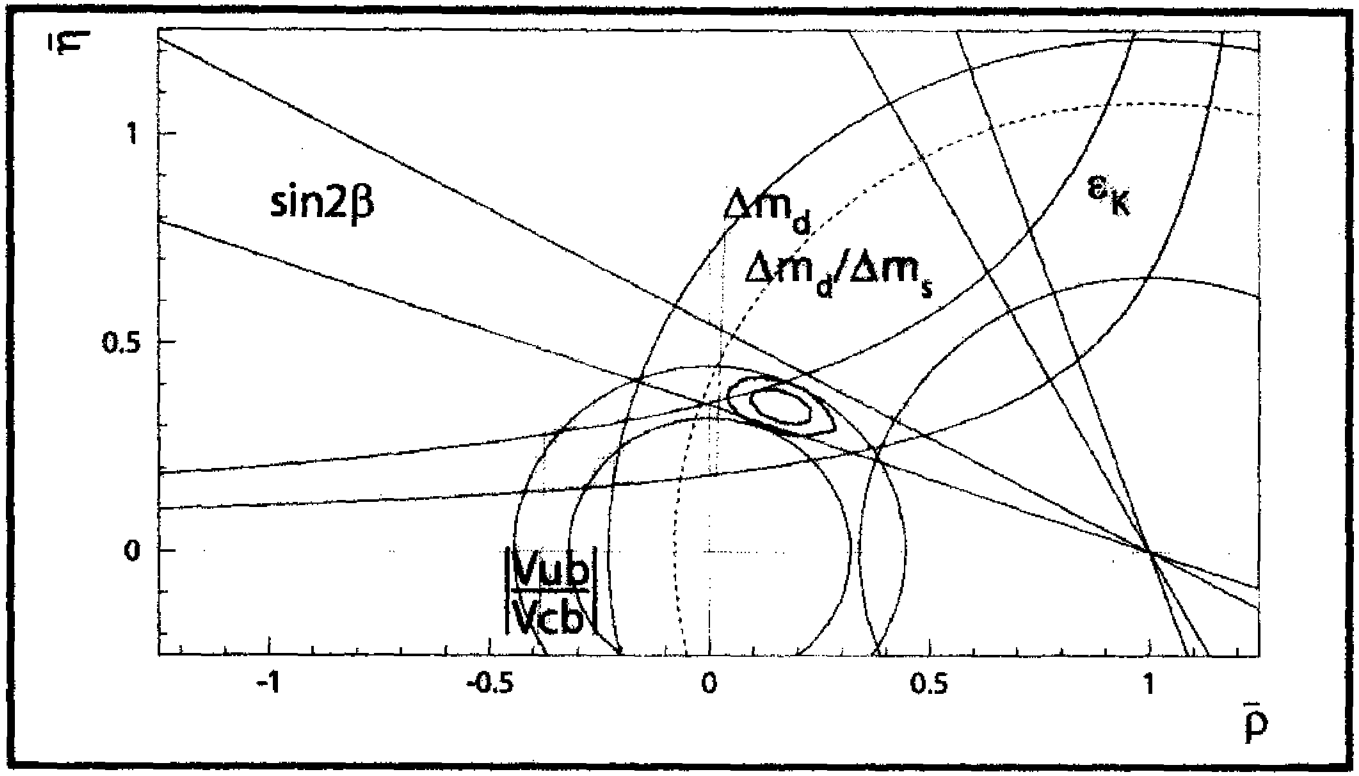
$$\mathrm{Im}(V_{ts}^* V_{td}) = (1.36 \cdot 10^{-4}) \left(\frac{170}{m_t}\right)^{1.15} \left(\frac{BR(K_L \to \pi^0 \nu\nu)}{10^{-11} \cdot 3}\right)^{1/2}$$

Buras et. al.
NLL



interesting
hard experimentally

Fit of the Unitarity Triangle in SM



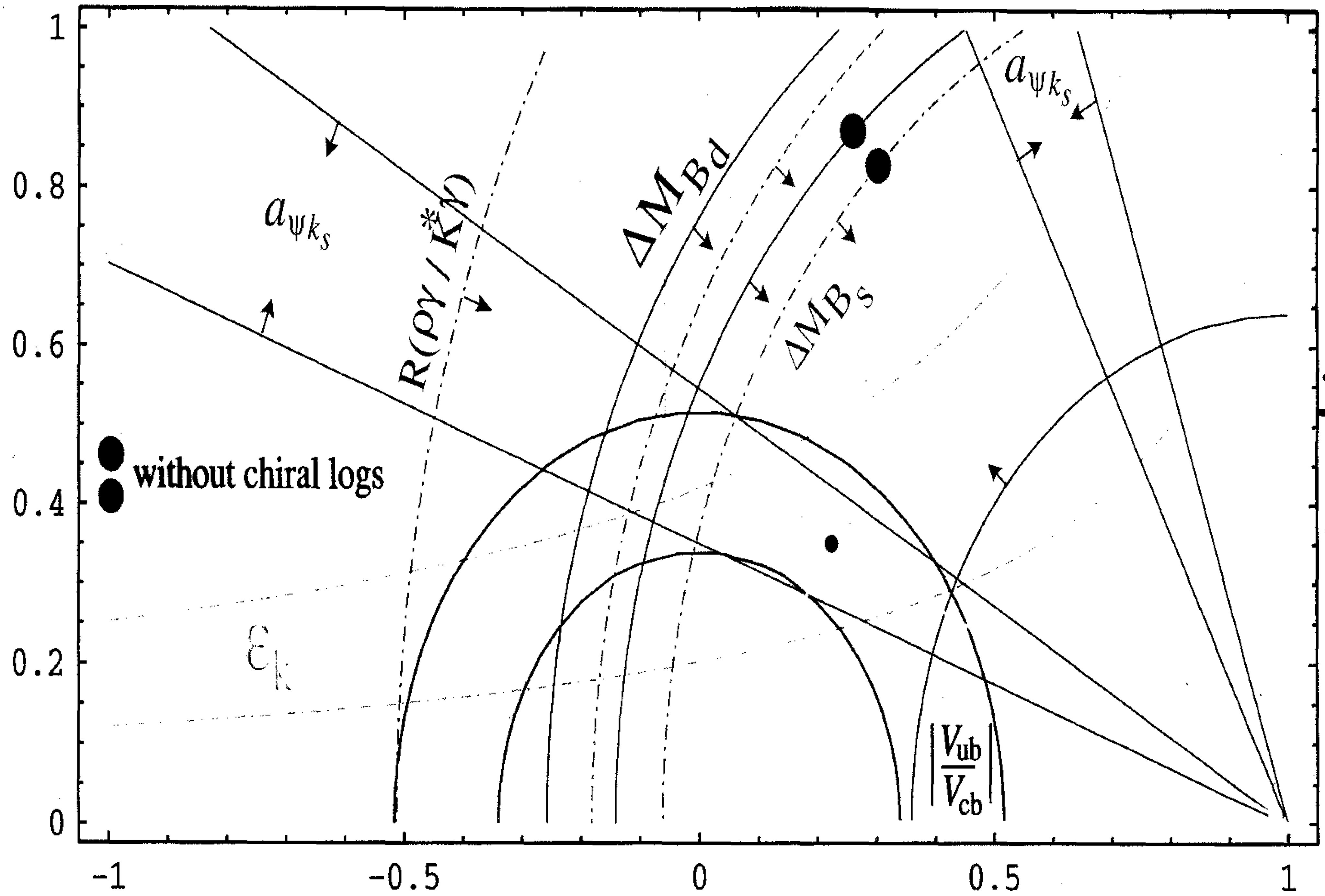
$\bar{\rho} = 0.162 \pm 0.046$

[0.067 – 0.250] @95% C.L.

$\bar{\eta} = 0.347^{+[illegible]}_{-0.025}$

[0.291 – 0.403] @95% C.L.

Stocchi


$a_{\psi k_s}$
$R(\rho\gamma / K^*\gamma)$
ΔM_{Bd}
ΔM_{Bs}
$a_{\psi k_s}$
without chiral logs
ε_K
$\left|\frac{V_{ub}}{V_{cb}}\right|$
1
0.8
0.6
0.4
0.2
0
-1
-0.5
0
0.5
1

Lunghi

Parameter	Value	Error(Gaussian)	Error(Flat)
λ	0.2240	0.036	
V_{cb} ($\times 10^{-3}$) (excl.)	42.1	2.1	
V_{cb} ($\times 10^{-3}$) (incl.)	41.6	0.7	0.6
V_{ub} ($\times 10^{-4}$) (excl.)	33.0	2.4	4.6
V_{ub} ($\times 10^{-4}$) (incl.)	40.9	4.6	3.6
Δm_d (ps^{-1})	0.503	0.006	
Δm_s (ps^{-1})	$> 14.4\ ps^{-1}$ at 95% CL		
m_t (GeV)	167	5	
m_c (GeV)	1.3		0.1
$f_{B_d}\sqrt{\hat{B}_{B_d}}$ (MeV)	223	33	12
ξ	1.24	0.04	0.06
B_K	0.86	0.06	0.14