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SMR.1508 - 8

SUMMER SCHOOL ON PARTICLE PHYSICS

16 June - 4 July 2003

LONG TERM PERSPECTIVES OF NEUTRINO PHYSICS

Special Lecture

M. LINDNER
Technical University Munich
Munich
GERMANY

Long Term Perspectives of ν Physics

M. Lindner

Technical University Munich

Physics Beyond the Standard Model

Theoretical Reasons:

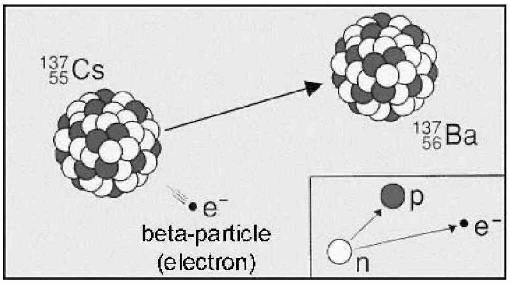
- Strictly speaking: SM does not exist without cutoff
- Higgs-doublet = most simple extension for correct elektro-weak symmetry breaking
- Explanation for fermion generations missing
- Unexplained parameters: (9+? masses- und 4+? mixing parameters)
- Charge quantization
- Gauge hierarchy problem
- Gravity not yet included, ...

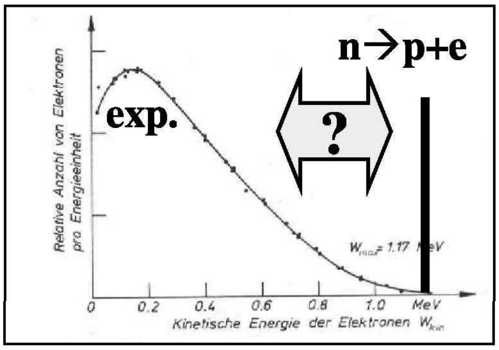
Experimental Reasons = Facts:

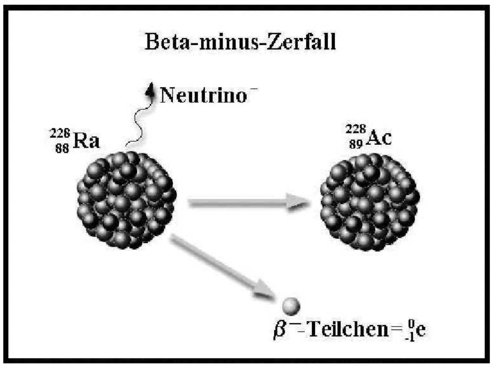
- Dark matter and dark energy exists in the cosmos!
- Baryon asymmetry of the universe ⇔ much too small in SM
- Neutrino masses discovered .... in the midst of a discovery phase

⇒ **new physics** beyond SM implications for models beyond SM!

The Birth of the Neutrino



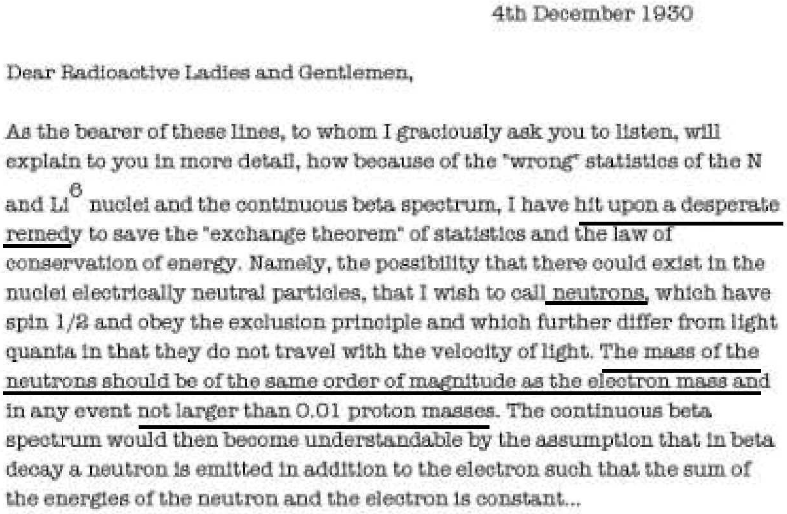




ν

Energy - momentum conservation

→ postulate a new particle
→ invisible, since Q=0
→ spin ½, ...



4th December 1930

Dear Radioactive Ladies and Gentlemen,

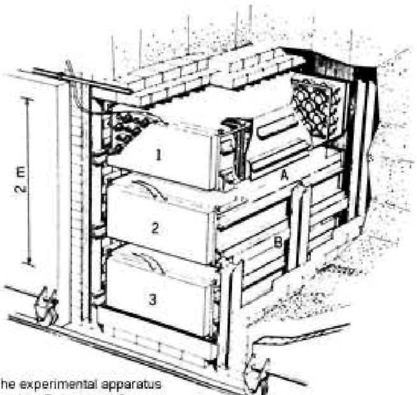
As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and Li^6 nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...

Pauli: Neutrinos will never be detected ...

- neutral ν are „ghost particles“
- screening of solar ν : 1000 ly lead !
- Cowen & Reines 1954-56 → detection of reactor ν

Project

„Poltergeist“



The experimental apparatus used by Reines and Cowan to detect the electron neutrino

- solar ν studied since 1967:
  → Nobel price 2002 for R. Davis

Majorana Mass Terms

- Pair of left- and right-handed fields $L, R \Rightarrow$ $\boxed{\mathcal{L}_m = -m_D(\overline{L}R + \overline{R}L)}$
- Charge conjugation for χ-ral fields: $\boxed{L_i = \text{left} \Leftrightarrow R_i = \text{right}}$

$$R' = L^c\ ; \quad L' = R^{\,c} \qquad \Leftrightarrow \qquad L = (R')^c\ ; \quad R = (L')^c$$

$$\left.\begin{array}{ll} \text{L-fields:} & \mathbb{L} = (L_1\ ,\ L_2\ ,\ L_3\ ,\ L_1' = R_1^c\ ,\ L_2' = R_2^c\ ,\ L_3' = R_3^c) \\ \text{R-fields:} & \mathbb{R} = (R_1' = L_1^c\ ,\ R_2' = L_2^c\ ,\ R_3' = L_3^c\ ,\ R_1\ ,\ R_2\ ,\ R_3) \end{array}\right\rangle \Leftarrow \text{every } \chi\text{-ral field 2x !}$$

Lorentz-invariant fermion mass terms (3 generations $\Rightarrow$ 6x6 Matrix):

$$\boxed{\begin{aligned} -\mathcal{L}_m &= \overline{\mathbb{L}}\mathbb{M}\mathbb{R} + \text{h.c.} \\ &= \left(\ \overline{L}_i,\ \ \overline{R_i}^{\,c}\ \right) \begin{pmatrix} M_L & m_D \\ m_D^T & M_R \end{pmatrix}_{ij} \begin{pmatrix} L_j^c \\ R_j \end{pmatrix} + \text{h.c.} \end{aligned}}$$

SM: $M_L = M_R = 0$ (charge conservation) $\quad m_D$ allowed besides for neutrinos (no ν_R)
BSM: Neutrinos may have $m_D \neq 0$, $M_L \neq 0$, $M_R \neq 0$ $\quad M_L$, M_R $\Leftrightarrow$ L violation

Neutrino Masses and Mixings

Some new physics to allow for neutrino mass terms $\Rightarrow$

- **Interaction states:**
  Active Flavour States ν_{e_f} = 3 electro-weak partners of e, μ, τ (LEP: Z-line shape)
  Sterile States ν_{N_s} = N electro-weak singlets

General mass matrix
$(\nu_e, \nu_\mu, \nu_\tau, \nu_{N_1}, \nu_{N_2}, \ldots)$

$$\begin{pmatrix} M_L & m_D \\ m_D^T & M_R \end{pmatrix}$$

$M_L \simeq 0$ with standard Higgs content
$m_D \simeq$ mass scale of charged leptons
$M_R \simeq$ embedding scale (LR, GUT, ...)

- **Physical states:** $\Leftrightarrow$ propagation as mass eigenstate ν_i with mass m_i

Diagonalization:$\Rightarrow$ See–Saw Mechanism

Heavy sterile ν's: $m_{heavy} \simeq M_R$ ($\simeq$ right-handed)
Light active ν's: $m_{light} \simeq M_L - m_D^T M_R^{-1} m_D$ ($\simeq$ left-handed)

For $\mathrm{m_{heavy}} \gg \mathrm{m_{light}}$: $$\begin{pmatrix} \nu_{e_f} \\ \nu_{N_s} \end{pmatrix} = \begin{pmatrix} U^{\mathrm{light}}_{\mathrm{mix}} & \approx 0 \\ \approx 0 & U^{\mathrm{heavy}}_{\mathrm{mix}} \end{pmatrix} \cdot \begin{pmatrix} \nu_i^{\mathrm{light}} \\ \nu_j^{\mathrm{heavy}} \end{pmatrix}$$

Mass hierarchy: $\Rightarrow$ Consider sub-space of light neutrinos $U^{\mathrm{light}}_{\mathrm{mix}}$ $\simeq$ unitary

Leptonic mixing matrix in basis where charged leptons are diagonal:

$$\boxed{\mathrm{U_{MNS}} := \mathrm{U^{light}_{mix}} = \mathrm{U_{Dirac}} \cdot \mathbf{diag}\left(\mathrm{e}^{\mathrm{i}\alpha_1}, \mathrm{e}^{\mathrm{i}\alpha_2}, \ldots, \mathrm{e}^{\mathrm{i}\alpha_{n-1}}, 1\right)}$$

Oscillations depend only on $\mathrm{U_{Dirac}}$:

2 Neutrinos: 1 angle + 0 phase (1 Majorana-phase; does not enter osc.)
3 Neutrinos: 3 angles + 1 phase (θ_{12}, θ_{23}, θ_{13}, δ) (+2 further Majorana phases)

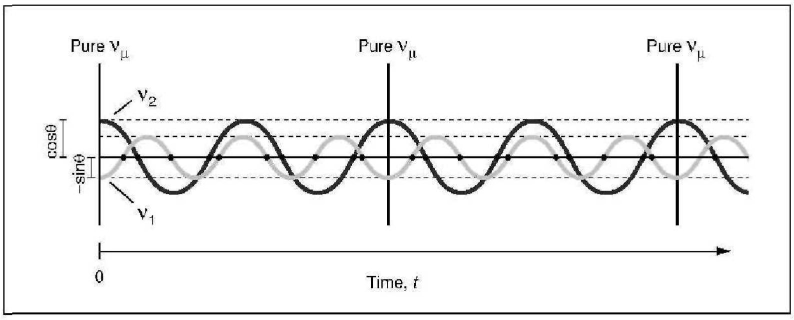
$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

The Status Quo

➔ neutrinos are routinely observed
➔ 4 ways to measure / limit neutrino masses

1) Neutrino oscillations
2) Lepton number violating processes
3) Kinematical mass measurements
4) Astrophysics & cosmology

1) Neutrino Oscillation



$$|\nu_e(0)\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$
$$|\nu_\mu(0)\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

$$|\nu_\mu(t)\rangle = -\sin\theta\ \exp[-\frac{iE_1 t}{\hbar}]\ |\nu_1\rangle + \cos\theta\ \exp[-\frac{iE_2 t}{\hbar}]\ |\nu_2\rangle$$

$$E_i = \sqrt{p_i^2 + m_i^2} \quad \xrightarrow{p_i = p \gg m_i} \quad \simeq p + \frac{m_i^2}{2p} \quad \simeq p + \frac{m_i^2}{2E}$$

$$L = c \cdot t \qquad \Delta m^2 = m_2^2 - m_1^2 \Rightarrow \quad E_2 - E_1 = \frac{\Delta m^2}{2E}$$

Transition probability:
$$P(\nu_\mu \to \nu_e) = |\langle\nu_\mu(t)|\nu_e(0)\rangle|^2 = \sin^2 2\theta \cdot \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

- 18 different appearance or disappearance channels

$$\left.\begin{array}{c}\nu_e\\ \nu_\mu\\ \nu_\tau\end{array}\right\} \xrightarrow{oscillation} \left\{\begin{array}{ccc}\nu_e & \Rightarrow & e^-\\ \nu_\mu & \Rightarrow & \mu^-\\ \nu_\tau & \Rightarrow & \tau^-\end{array}\right.$$

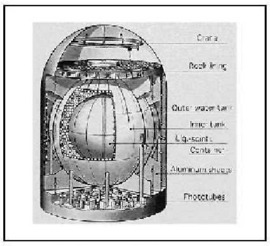
$$\left.\begin{array}{c}\overline{\nu}_e\\ \overline{\nu}_\mu\\ \overline{\nu}_\tau\end{array}\right\} \xrightarrow{oscillation} \left\{\begin{array}{ccc}\overline{\nu}_e & \Rightarrow & e^+\\ \overline{\nu}_\mu & \Rightarrow & \mu^+\\ \overline{\nu}_\tau & \Rightarrow & \tau^+\end{array}\right.$$

2 flavour approximation:

$P_{ab} = \sin^2(2\theta)\ \ \sin^2(\Delta m^2 L/4E)$

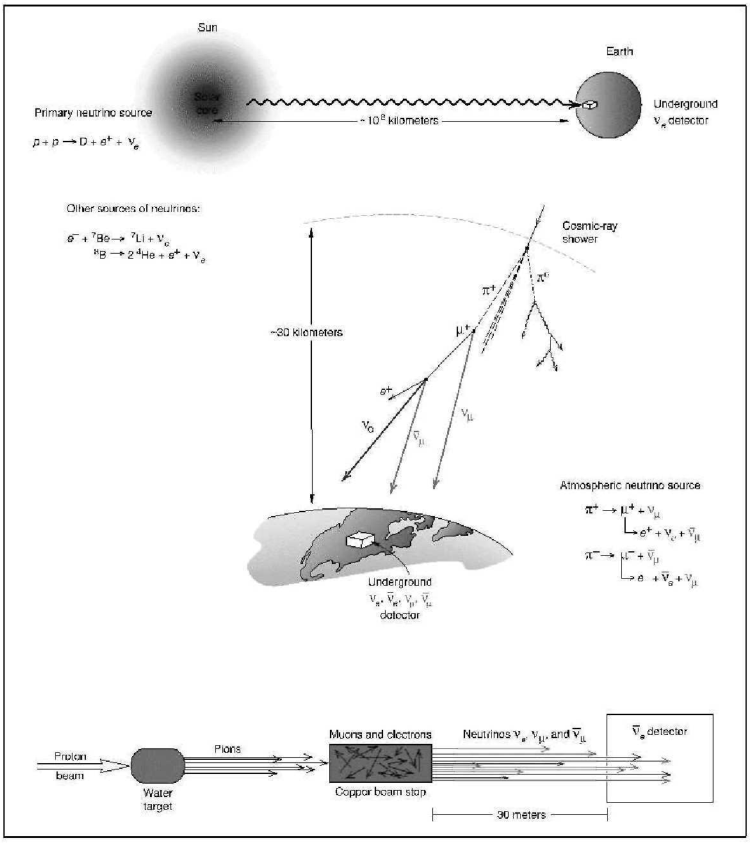
$P_{aa} = 1 - P_{ab}$

Neutrino Oscillation Signals



KamLAND

⇔



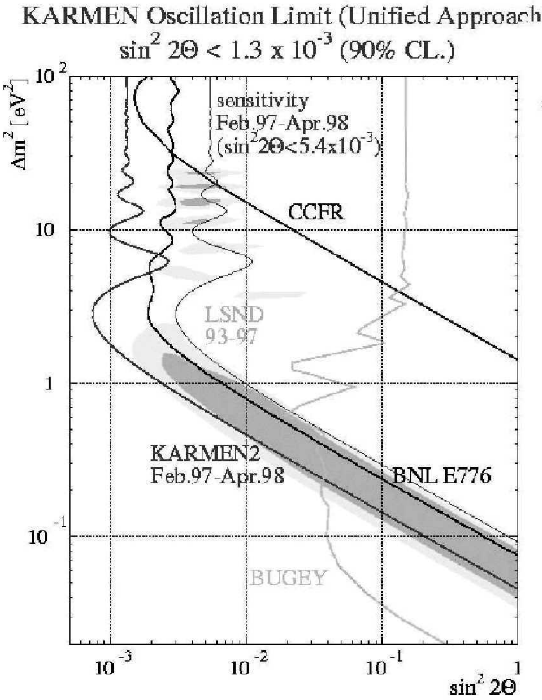
Solar

Atmospheric

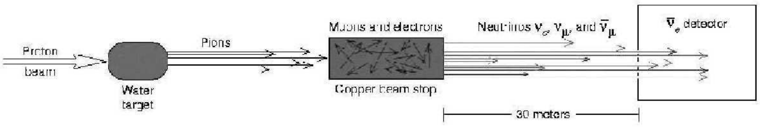
neutrino oscillation signals

LSND ?

LSND claims evidence for oscillations



unified approach: R.D. Cousins and G.J. Feldman
Phys. Rev. D57 (1998) 3873



- LSND claims $\overline{\nu}_\mu \to \overline{\nu}_e$ and $\nu_\mu \to \nu_e$ osc.

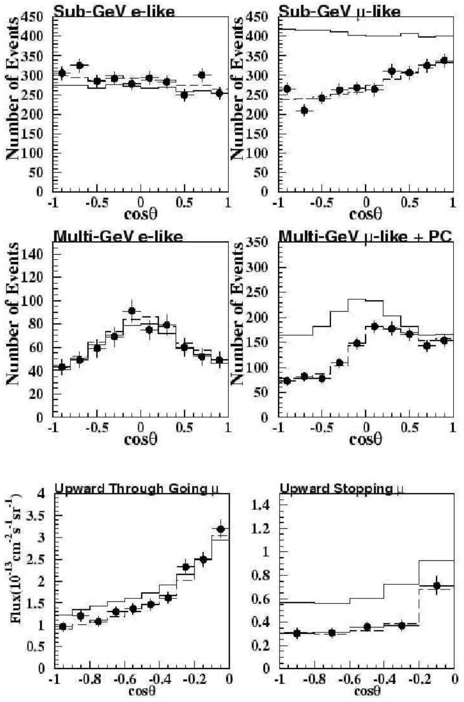
  1) Third $\Delta m^2 \simeq 1$ eV⇒ 4 light neutrinos?
  ⇔ Z line-shape

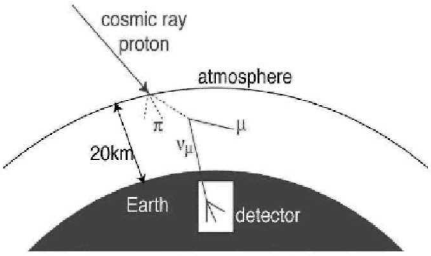
  2) CPT violation = different param. for ν and $\overline{\nu}$
  ⇔ local QFT!?

- Partly ruled out by KARMEN
- Partly ruled out by →cosmology
- Will be testet soon by ⇒ **MiniBooNE**

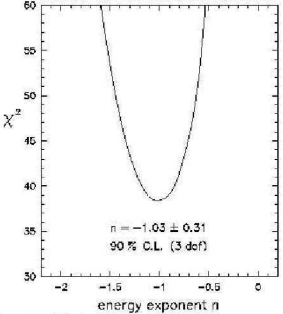
⇒ ruled out or very exciting!

Atmospheric Oscillations





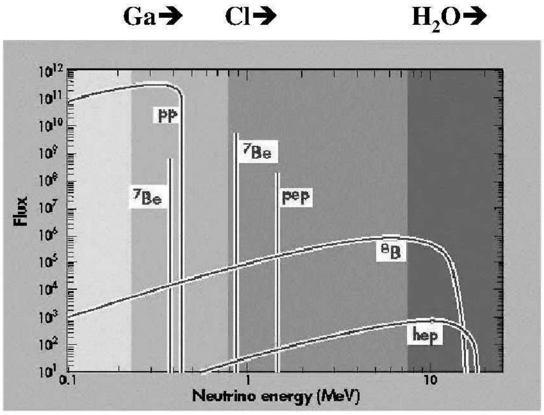
- **Atmospheric oscillation fit:**
  $|\Delta m^2_{31}| \simeq (1.5\text{--}4)\cdot 10^{-3}\,\mathrm{eV}^2,\quad \sin^2 2\theta_{23} \geq 0.88$
- 8 sigma signal for ν_μ disappearance
- Not $\nu_\mu \to \nu_e \Rightarrow \quad \nu_\tau$ (some $\tau's$ seen, Chooz **,no ν_e)**
- Not $\nu_\mu \to \nu_s$ from NC/CC comparison
- L/E confirmed by K2K ($\simeq 2\sigma$)
- Sensitivity to $(L/E)^n$

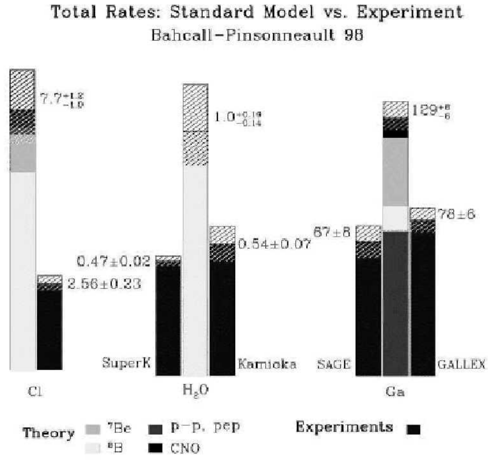


$\Rightarrow \quad n = -1.03 \pm 0.31$ 90%CL

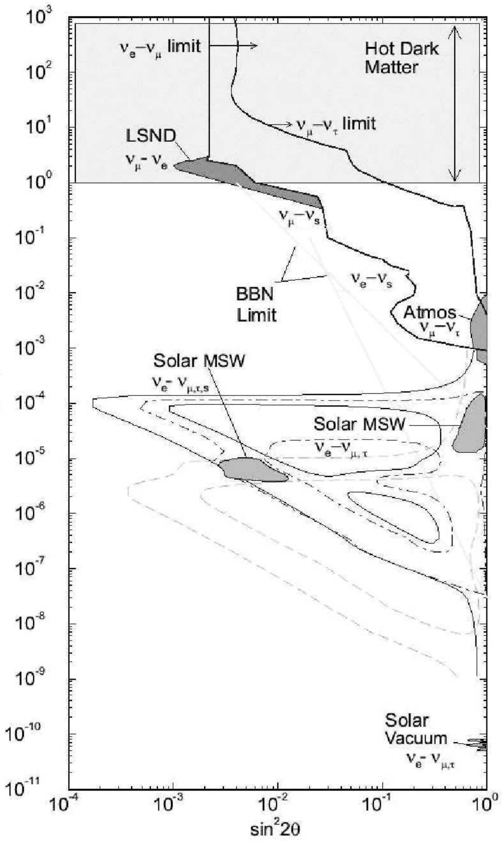
Fogli et al.

SOLAR PHASE 1: Rates compared to Solar Models: ≤ 2000

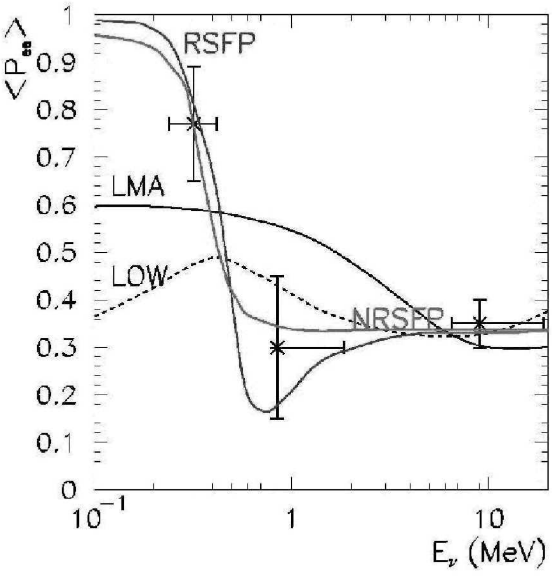




- Consistent deficit of total CC neutrino rates
- Combining $Cl/H_2O/Ga/D_2O \Rightarrow$ some spectral information
- Depends on solar modelling, but should be robust
- Can be explained by oscillation $\Rightarrow$ different regimes $\Rightarrow$

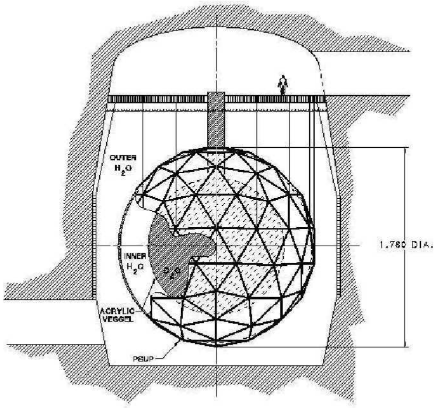


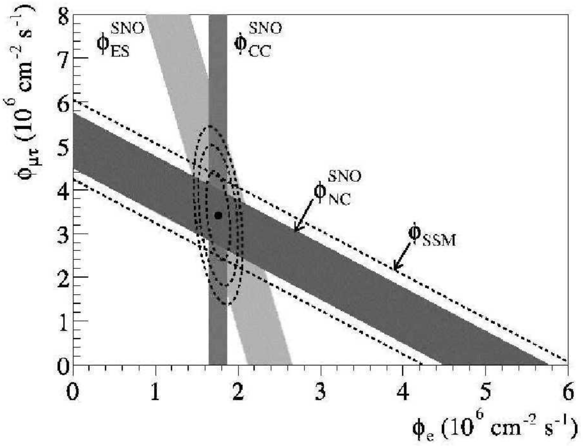
oscillation fits (≃3 years ago)



- **Which oscillation solution?**
  - SNO day/night ⇒ LMA best
- **RSFP worked even better!**
  - Reqires large magnetic moments
  - ... and large magnetic fields

SOLAR PHASE 2: NC, CC and ES Rates from SNO 2002

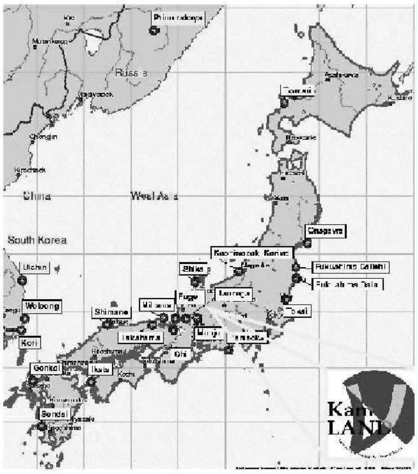


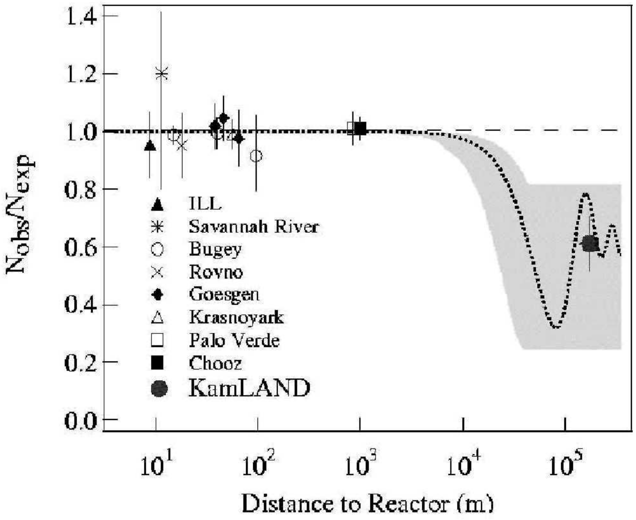


$$\Phi_{CC} = \Phi_e$$
$$\Phi_{ES} = \Phi_e + \frac{1}{6}\,\Phi_{\mu\tau}$$
$$\Phi_{NC} = \Phi_e + \quad \Phi_{\mu\tau}$$

- Clear proof of flavour conversion
- Independent of SSM (⇒ SSM test)
- Both no conversion and conversion to sterile excluded by more than 5σ
- ≃ no L/E dependence
- LMA best oscillation fit, LOW ≃ OK, SMA and VAC out
- RSFP worked best!

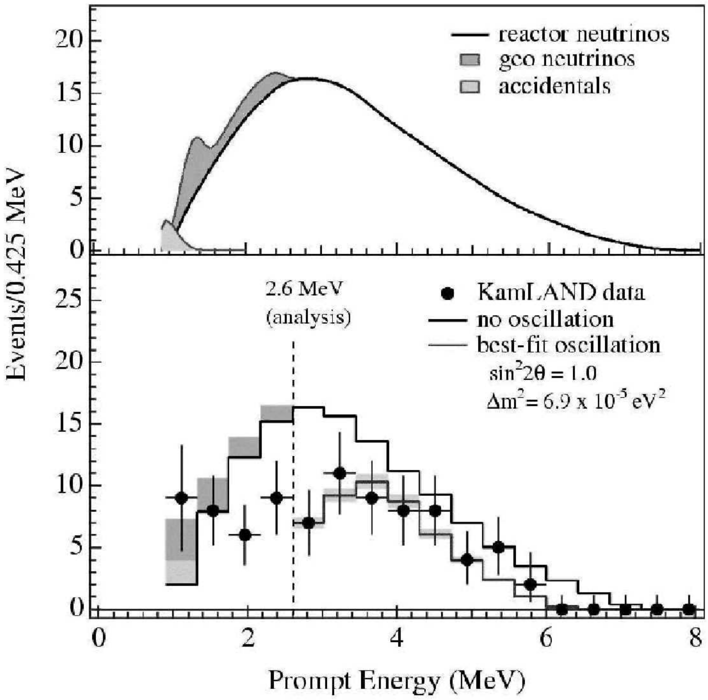
SOLAR/REACTOR PHASE 3: ⇒ KamLAND 2002





- Dominated by 70GW (= 7% of world total) at 175 ± 30 km
- Tests solar oscillation with reactor antineutrinos
- Different L, E and a controlled source
- Rules out RSFP (no magnetic fields)

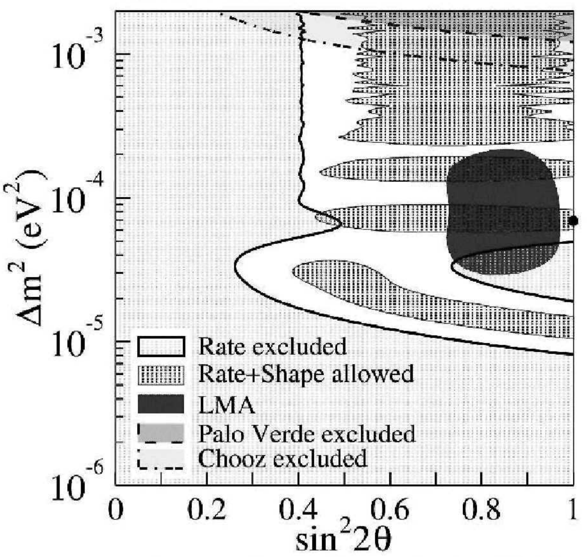
KamLAND
event spectrum:



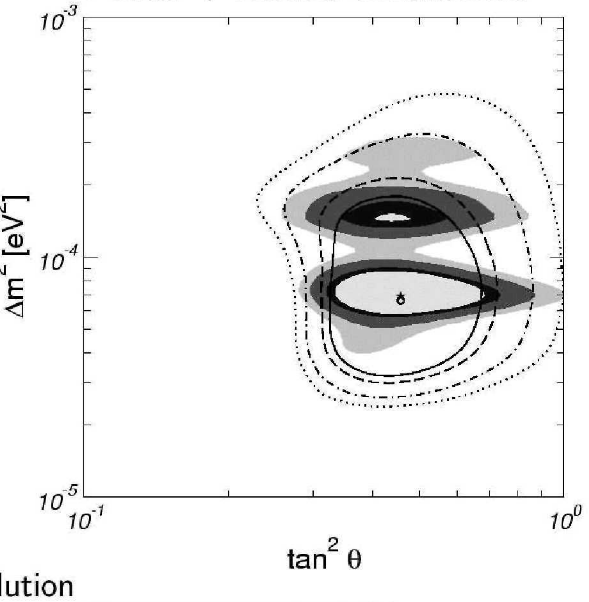
- Mostly a rate effect ⇒ flavour transition
- Already mild L/E sensitivity
- Geological $\nu's$: ≃40TW at lower energy ⇒ $E \geq 2.6$ MeV

Combining solar and reactor data:

- KamLAND:



- Solar + reactor oscillations:

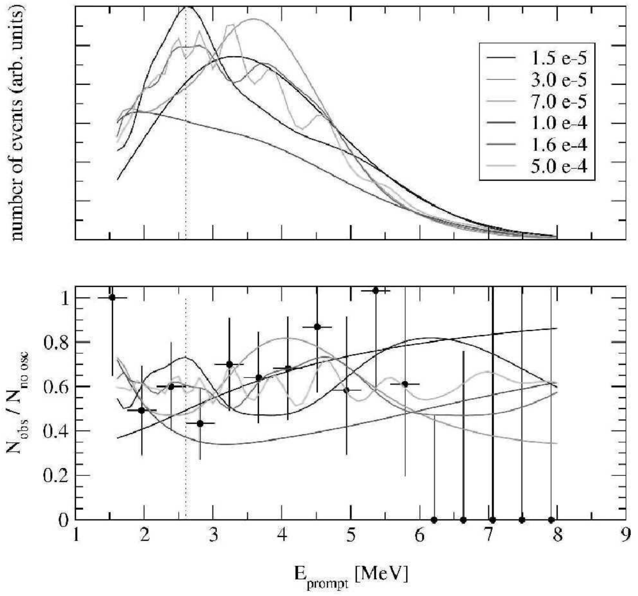


- Impressive confirmation of solar LMA solution
- Combined fit ⇒ LMA-I: $\Delta m^2 \simeq 6.9 \cdot 10^{-5}\mathrm{eV}^2 \qquad \tan^2\theta = 0.46$

  LMA-II: $\Delta m^2 \simeq 1.4 \cdot 10^{-4}\mathrm{eV}^2 \qquad \tan^2\theta = 0.42$

LMA ←→ leptonic CP violation!

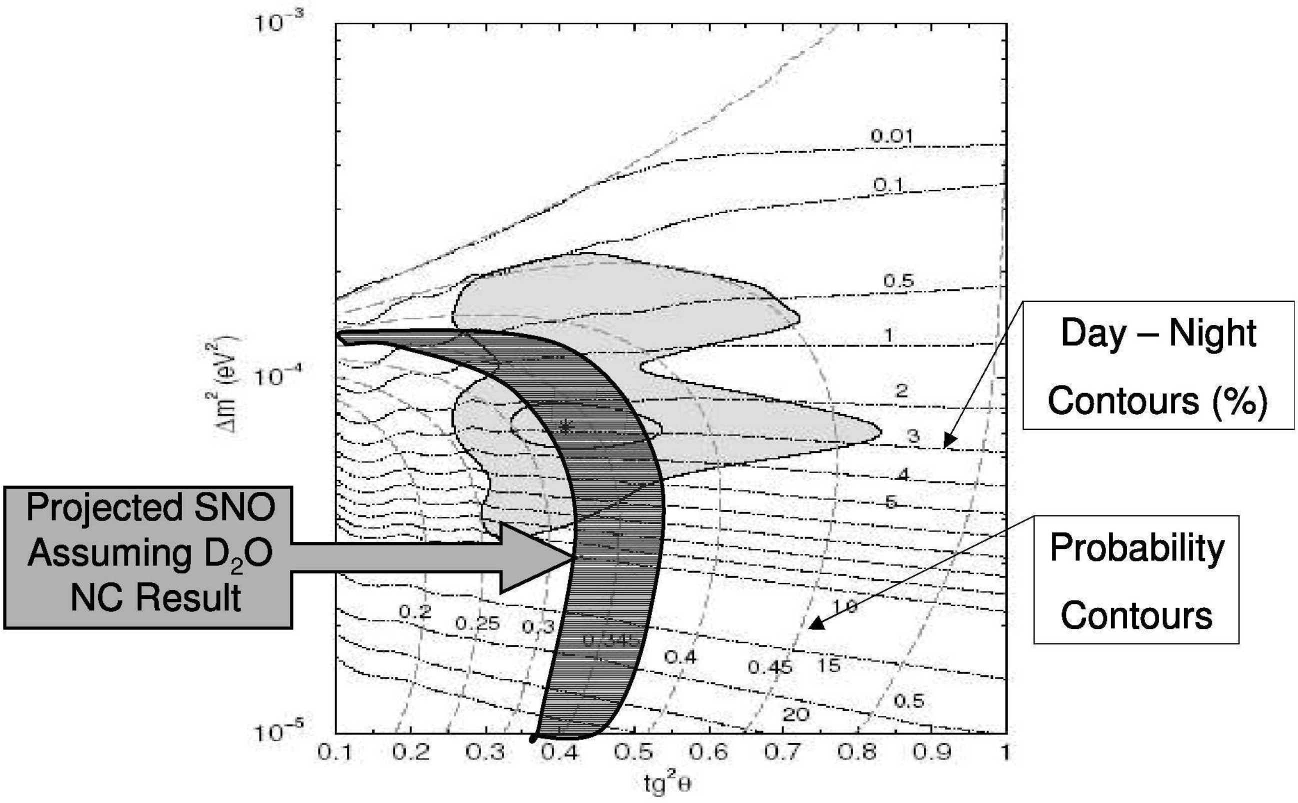
SOLAR/REACTOR PHASE 4: ⇒ KamLAND 2003+2004



- L/E dependence of oscillations ⇒ establish oscillations without doubt
  ⇒ solar oscillation is QM on largest scales
- Δm^2 dependence of oscillations ⇒ improved Δm^2
- Inclusion of low energy bins ⇔ geological neutrinos

Soon expected results from SNO

De Holanda, Smirnov

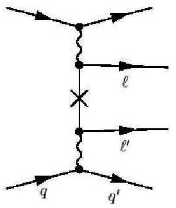


Day – Night Contours (%)

Projected SNO Assuming D_2O NC Result

Probability Contours

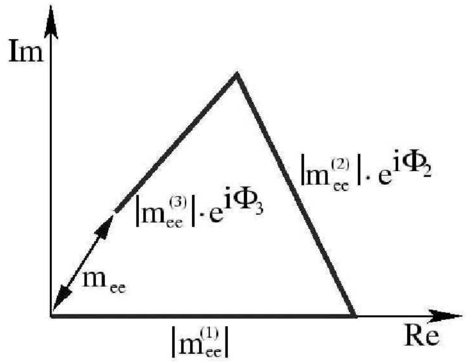
2) Lepton Number Violation ⇒ $0\nu2\beta$ Decay



$$\propto |\langle \mathbf{m_{ee}} \rangle| = |\sum \mathbf{m_i U_{ei}^2}| \overset{(?=)}{\leq} 0.28 \text{ eV}$$

$$m_{ee} = |m_{ee}^{(1)}| + e^{i\Phi_2}|m_{ee}^{(2)}| + e^{i\Phi_3}|m_{ee}^{(3)}|$$

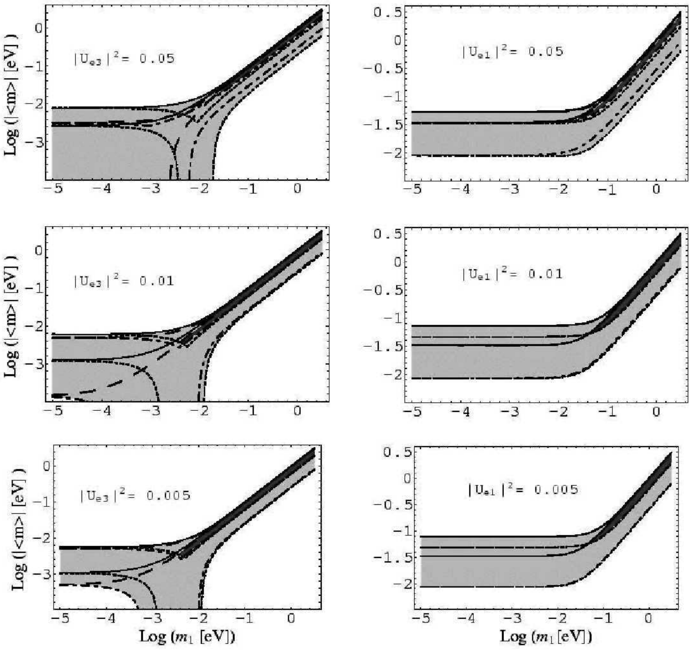
$$\begin{aligned} |m_{ee}^{(1)}| &= |U_{e1}|^2 m_1 \\ |m_{ee}^{(2)}| &= |U_{e2}|^2 \sqrt{m_1^2 + \Delta m_{21}^2} \\ |m_{ee}^{(3)}| &= |U_{e3}|^2 \sqrt{m_1^2 + \Delta m_{31}^2} \end{aligned}$$



solar ⇒ $|U_{e1}|^2, |U_{e2}|^2, \Delta m_{21}^2$ atmosph. ⇒ Δm_{31}^2 CHOOZ ⇒ $|U_{e3}|^2 < 0.1$
FREE: m_1, CP-phases Φ_2, Φ_3

Petcov,
...

$\Delta m^2_{31} > 0$



$\Delta m^2_{31} < 0$

- Compensations possible
- Model dependent: LR, SUSY R-partity violation, ... nuclear matrix elements ...
- Only $|m_{ee}|$ measured ⇔ one Majorana CP-phase comb. ⇒ 2nd phase?
- **Better experiments are very important** ⇔ L-violating effects

CUORE: 0.1eV, MOON: 0.03eV, EXO,GENIUS(1t,10t): 0.02eV,0.002eV ⇔ $\sqrt{\Delta m^2_{13}} \simeq 0.06$ eV

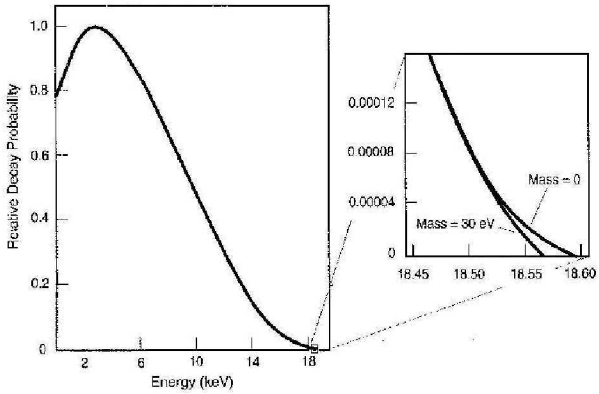
Very fary future: if |<m$_{ee}$>| < 10^{-3} eV → normal hierarchy

3) Kinematical Mass Effects

$$E^2 = p^2 + m^2 \; ; \quad \sum p_i^\mu = \sum p_f^\mu$$

Measure neutrino masses in the endpoint of decays

Tritium → $He^3 + e^- + \overline{\nu}_e$



Current Bounds:

"Electron neutrino":	$m < 2.2$ eV	(Mainz, Troitsk)
"Muon neutrino":	$m < 170$ keV	
"Tau neutrino":	$m < 15.5$ MeV	

Better: $\sum m_i |U_{ei}|^2 < 2.2$ eV

Oscillations: $\Delta m^2_{ij} \ll m^2_i$ ⇒ improve m_1 ⇒ **KATRIN:** $\longrightarrow 0.35$ **eV**

⇔ cosmologically interesting range (WMAP; 2df galaxy survey, Lyman α, ...)

4) Cosmological Effects of Neutrino Mass

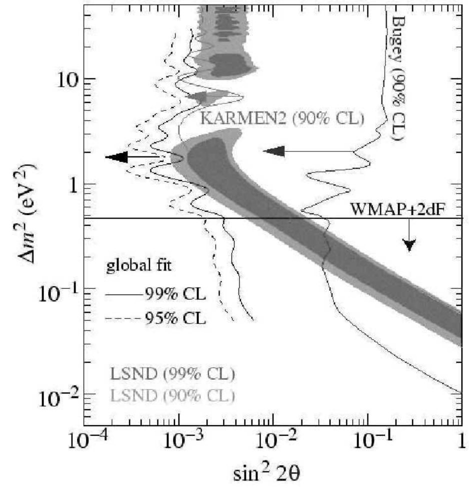
Effects:

- Neutrinos are degrees of freedom in hot early universe
  $\Rightarrow$ counting neutrino species with masses below $\simeq$ MeV
- 330 neutrinos/cm^3 carry significant energy and momentum
  $\Rightarrow$ gravitational and streaming effects in structure formation
- ...further astronomical / cosmological effects

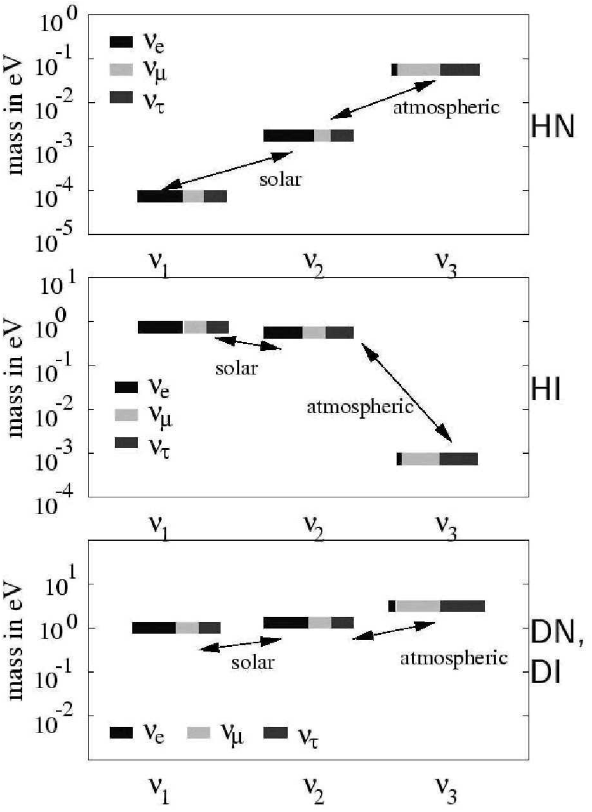
Recent CMBR results of WMAP

...combined with 2df galaxy survey, Lyman α, ...

- Small scale structure $\Rightarrow$ $m_\nu \leq\ 0.7\ eV$ $(?1.2\ eV)$
- 3 degenerate neutrinos $\Rightarrow$ $m_\nu \leq 0.23$ eV
- Indication against additional sterile ν in LSND range
  ... will soon be tested directly by MiniBooNE
- Cosmological data do not prefer $0\nu2\beta$ mass range claimed by Heidelberg–Moscow experiment
  ....unless the nuclear matrix element is very large



The Neutrino Mass Spectrum



Different 3ν scenarios:

- **atmospheric $\nu's$:**

$$\sqrt{3.5\cdot10^{-3}\ eV^2} \simeq \pm \boxed{\mathbf{0.06\ eV}}$$

- **solar $\nu's$:**

$$\sqrt{\boxed{7}\cdot10^{-5}\ eV^2} \simeq \boxed{\mathbf{0.008\ eV}}\ \text{(+sign)}$$

$\Rightarrow$ **hierarchy of mass splittings**

- total mass scale: $\mathrm{m} \leq \mathcal{O}(1\ \mathrm{eV})$

- spectrum:
  - hierarchical (H) or degenerate (D)
  - normal (N) or inverted (I)?

- questions:
  - more than 3 neutrinos?
  - how precise are future measurements?
  - is precision valuable?

The near future

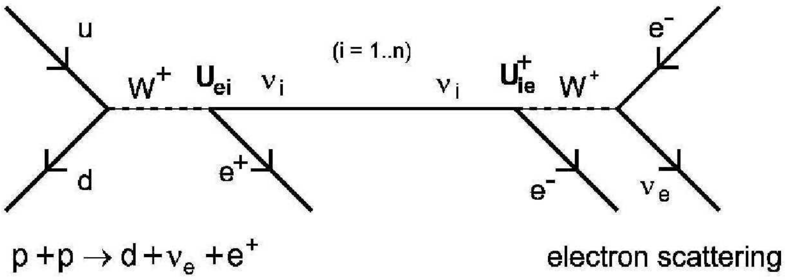
Progress / results on the horizon:

- **MiniBooNE ←→** LSND, 4 neutrinos
- **KamLAND ←→** spectral info. on solar Δm^2 ←→ oscillation
- **SNO ←→** NC/CC, day/night ←→ improve LMA fits
- **Borexino, KamLAND ←→** Be flux ←→ consistency, sun
- **K2K, CNGS, MINOS ←→** establish/test atm. Δm^2 with beam
- **KATRIN ←→** lower absolute neutrino mass
- **0ν2β ←→** lower Majorana mass limit / value

Precise description required →

Neutrino Oscillations for $N = 2$

Production
Propagation
Detection



2 Flavours ν_e, ν_μ:

$$\underbrace{\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}}_{\text{flavour states}} = \underbrace{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}_{\text{mixing matrix}} \cdot \underbrace{\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}}_{\text{mass eigenstates}}$$

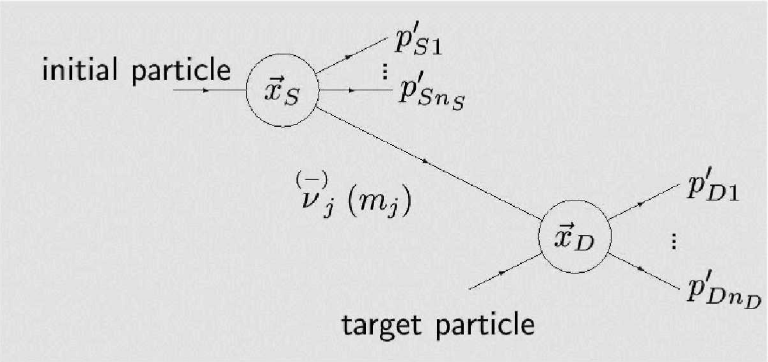
- Production as flavour eigenstate from W-exchange
- Detection via W-exchange $\equiv$ projection on flavour state
- Propagation as mass eigenstate $\Rightarrow$ use mixing matrix at vertex
- Is a simple QM treatment justified?

Neutrino Oscillations in QFT

QFT description of a neutrino produced in a decay at rest:

- localized source and detector
- $L = |\vec{x}_D - \vec{x}_S|$
- initial particle at rest
- target particle at rest

... DIF similar



Transition probability from Feynman diagram:

$$\boxed{\left\langle P_{\overset{(-)}{\nu}_\alpha\to\overset{(-)}{\nu}_\beta}\right\rangle_{\mathcal{P}} \propto \int dP_S \int_{\mathcal{P}} \frac{d^3p_{D1}}{2E_{D1}}\cdots\frac{d^3p_{Dn_D}}{2E_{Dn_D}} \left|\mathcal{A}_{\overset{(-)}{\nu}_\alpha\to\overset{(-)}{\nu}_\beta}\right|^2}$$

$\Rightarrow$ **leads to neutrino oscillation + avoids confusion ...**

Kinematics: Equal Energy or equal Momenta?

- Consider e.g. pion decay at rest: $\pi^+ \to \mu^+ + \nu_\mu$
- Neutrino energy and momentum determined by energy-momentum conservation

$$p_k^2 = \frac{m_\pi^2}{4}\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2 - \frac{m_k^2}{2}\left(1+\frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_k^4}{4\,m_\pi^2}$$

$$E_k^2 = \frac{m_\pi^2}{4}\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2 + \frac{m_k^2}{2}\left(1-\frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_k^4}{4\,m_\pi^2}$$

- **For $E \gg m$:** $\boxed{p_k \simeq E - \xi\,\frac{m_k^2}{2E}\,, \qquad E_k \simeq E + (1-\xi)\,\frac{m_k^2}{2E}}$

with $E = \frac{m_\pi}{2}\left(1-\frac{m_\mu^2}{m_\pi^2}\right) \simeq 30\,\mathrm{MeV}\,, \qquad \xi = \frac{1}{2}\left(1+\frac{m_\mu^2}{m_\pi^2}\right) \simeq 0.8$

$\Rightarrow$ **neither equal energy nor equal momentum!**

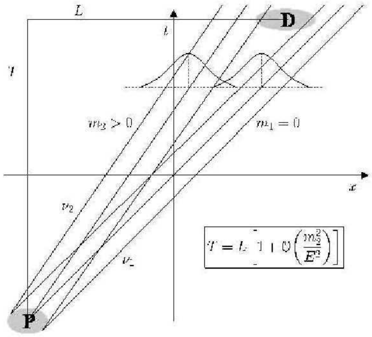
$e^{ipx} \Rightarrow$ $\boxed{p_\mu \cdot x^\mu = p_k L - E_k T = -\frac{m_k^2 L}{2E}}$ for $L = T$

$\Rightarrow$ ξ drops out of the oscillation formulae $\Leftrightarrow$ naive treatment correct

- Shown for π-decay, but valid in general (DIF, N-body, ..., different ξ)

Localized Source and Detector:

- Feynman rules for particles of given momentum ($\simeq$ on-shell)
  $\Rightarrow$ this corresponds to an infinitely extended (non-localized) plane wave
- Localized source (wave packet) and detector in space-time $(\Delta x_S, \Delta t_S)$, $(\Delta x_D, \Delta t_D)$:
  $\Rightarrow$ Source: Fourier superposition of momenta with $\sigma_S^2 \simeq min(\Delta x_S^2, \Delta t_S^2)$
  $\Rightarrow$ Detector: projection on a superposition of momenta with $\sigma_D^2 \simeq min(\Delta x_D^2, \Delta t_D^2)$
- Different masses and momenta $\Rightarrow$ dispersion $\Rightarrow$ loss of coherence



- separation of wave packets: $\boxed{\Delta x_{ij} = \frac{|\Delta m_{ij}^2| L}{2E^2}}$
- coherence condition: $\Delta x_{ij} < \sigma := \sqrt{\sigma_S^2 + \sigma_D^2}$
- coherence length: $L < \frac{2E^2 \sigma}{|\Delta m_{ij}^2|}$

- Oscillations from QFT $\Rightarrow$ $P_{\nu_\alpha \to \nu_\beta}(L, T) = \left|\sum_k U_{\alpha k}^* \, e^{ip_k L - iE_k T} \, U_{\beta k}\right|^2$
- Very interesting QM effects (σ, decay)

$N \geq 2$ with CP and Matter Effects

Precision: $N = 2$ description insufficient $\Rightarrow$ modifications

- **$2 \to 3$ neutrino framework $\Rightarrow$ more parameters & CP effects**
- **MSW: parameter mapping in matter**

Quantum mechanical treatment for N ultra-relativistic neutrinos:

$$P_{\nu_{e_l} \to \nu_{e_m}}(L/E) = \left| \sum_j U_{mj} U_{lj}^* \exp\left(\frac{-i \; m_j^2 L}{2E} \right) \right|^2$$

- masses m_j associated to mass eigenfields ν_j
- flavour eigenstates ν_{e_l}, ν_{e_m}, ...
- neutrino energy E
- source and detector distance L
- unitary mixing matrix U

General 3x3 neutrino mixing matrix:

has (up to) 3 angles + 1 Dirac-phase +2 Majorana-phases: θ_{12}, θ_{23}, θ_{13}, δ, Φ_1, Φ_2

$$\boxed{U_{MNS} = U \cdot \mathbf{diag}\,(\exp[\mathbf{i}\Phi_1], \exp[\mathbf{i}\Phi_2], 1)}$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

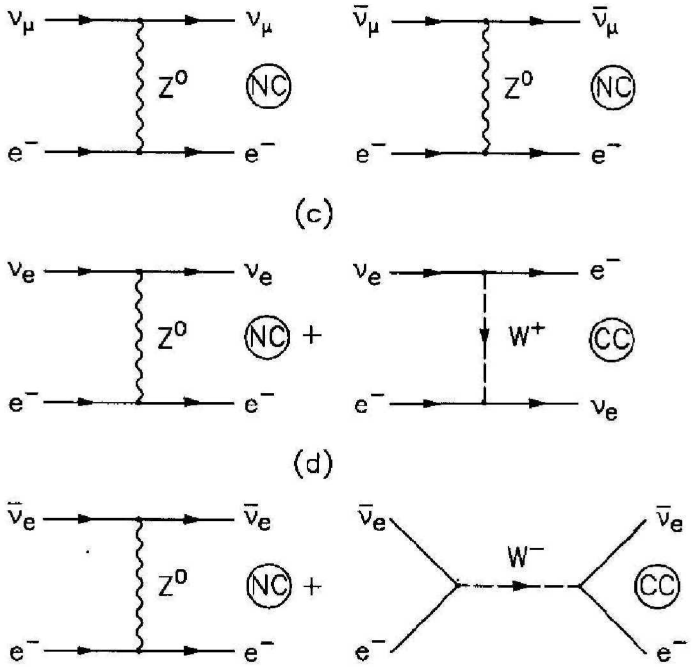
- Only U enters in neutrino oscillations: $\boxed{J_{ij}^{e_l e_m} := U_{li}U_{lj}^{*}U_{mi}^{*}U_{mj}}$
- All oscillation frequencies show up: $\boxed{\Delta_{ij} := \frac{\Delta m_{ij}^2 L}{4E} = \frac{(m_i^2 - m_j^2)L}{4E}}$

$$P(\nu_{e_l} \rightarrow \nu_{e_m}) = \underbrace{\delta_{lm} - 4\sum_{i>j} \mathrm{Re}J_{ij}^{e_l e_m} \sin^2\Delta_{ij}}_{P_{CP}} \underbrace{-2\sum_{i>j} \mathrm{Im}J_{ij}^{e_l e_m} \sin 2\Delta_{ij}}_{P_{\cancel{CP}}}$$

⇒ **Leptonic CP violation, genuine 3 flavour and matter effects**

Matter Effects and MSW Resonance

Mikheyev-Smirnov-Wolfenstein: coherent forward scattering



$\mathcal{L}_{NC}$ = flavour universal

$\mathcal{L}_{CC} = \sqrt{2}G_F n_e$ ⇔ only ν_e

MSW-resonance energy(Δm_{31}^2)

Earth: $\mathbf{E_{res} \simeq 10}$ **GeV**

for beams
dominated by average density

$\rho = \rho_{\text{average}} + \delta\rho$

Analytic Description

- full numerical simulation
- $\Delta = \Delta m_{31}^2 L/4E$
- qualitative understanding $\Rightarrow$ expand in $\alpha = \Delta m_{21}^2/\Delta m_{31}^2$ and $\sin^2 2\theta_{13}$
- matter effects $\hat{A} = A/\Delta m_{31}^2 = 2VE/\Delta m_{31}^2;\ \ V = \sqrt{2} G_F n_e$

$$P(\nu_\mu \to \nu_\mu) \approx 1 - \cos^2\theta_{13} \sin^2 2\theta_{23} \sin^2\Delta + 2\,\alpha\, \cos^2\theta_{13} \cos^2\theta_{12} \sin^2 2\theta_{23} \Delta \cos\Delta$$

$$\begin{aligned} P(\nu_e \to \nu_\mu) \approx\ & \sin^2 2\theta_{13} \sin^2\theta_{23} \frac{\sin^2((1-\hat{A})\Delta)}{(1-\hat{A})^2} \\ & \pm \sin\delta_{\rm CP}\ \alpha \sin 2\theta_{12} \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \sin(\Delta) \frac{\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})} \\ & + \cos\delta_{\rm CP} \alpha \sin 2\theta_{12} \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \cos(\Delta) \frac{\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})} \\ & + \alpha^2 \sin^2 2\theta_{12} \cos^2\theta_{23} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2} \end{aligned}$$

➔ **degeneracies, correlations, ...**

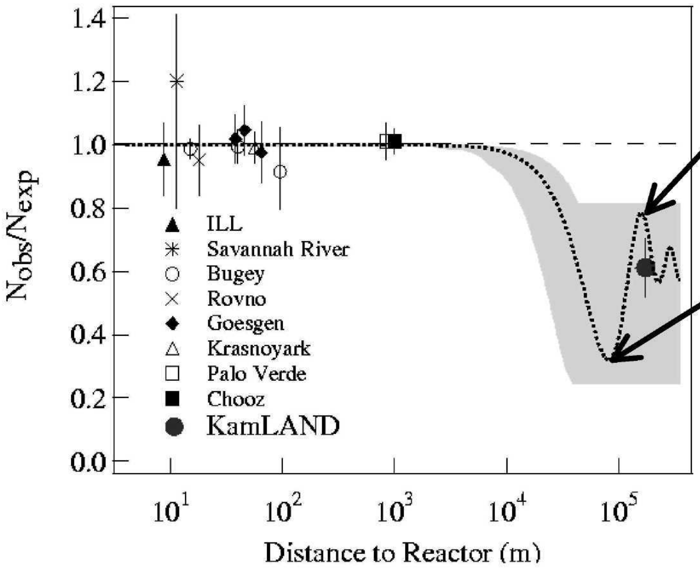
More in the future

... or what needs to be done, but is not yet under construction

Absolute neutrino mass:
- 0ν2β
- improved endpoint measurements

Oscillations:
- New reactor experiments
- Superbeams
- Neutrino factories

Implications of Kamland measurements



poor θ_{12} sensitivity

good θ_{12} sensitivity

➔KamLAND is not in the ideal place!

LMA-I ➔ 70 km
LMA-II ➔ 20-30 km

- **New reactor experiment at ideal location → better θ_{12} sensitivity**
- **HLMA ➔ even some θ_{13} sensitivity** **Petcov, Choubey**

Another new reactor idea

$\overset{\overline{\nu}_e}{\Longrightarrow}$ **near detector** (170m) $\overset{\overline{\nu}_e}{\Longrightarrow}$ **far detector** (1700m)

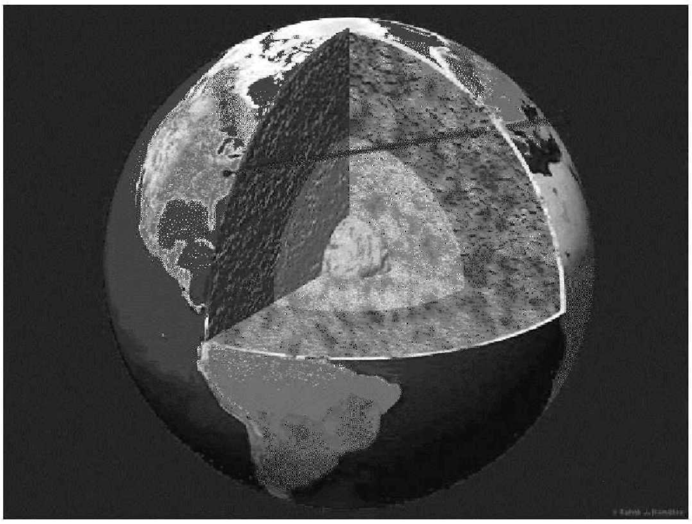
- The survival probability:
  - expand in small quantities

$$P_{ee} \approx 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{31} L}{4E_\nu} + \left(\frac{\Delta m^2_{21} L}{4E_\nu}\right)^2 \cos^4 \theta_{13} \sin^2 2\theta_{12}$$

  - last term negligible for $\frac{\Delta m^2_{31} L}{4E_\nu} \sim \pi/2$ and $\sin^2 2\theta_{13} \gtrsim 10^{-3}$
  - atmospheric frequency is dominant
  - most important:

- **No degeneracies!**
- **Practically no correlations!**
- **No matter effects!**

Long baseline plans for superbeams & neutrino factory



Precision!

Source ⊗	**Oscillation** ⊗	**Detector**
- neutrino energy E	- oscillation channels	- effective mass, material
- flux and spectrum	- realistic baselines	- threshold, resolution
- flavour composition	- MSW matter profile	- particle ID (flavour, charge, event reconstruction, ...)
- contamination		- backgrounds
- symmetric $\nu/\overline{\nu}$ operation		- x-sections (at low E)

Existing Projects and Plans (partly)

Existing	K2K	(running)			establish atm. osc. in lab.
Construction	MINOS	(2005)	CNGS	(2005?)	$\simeq$10% for Δm^2_{31}, θ_{23}, improve θ_{13}?
Approval	JHF-SK	(2007)			$\simeq$few % for Δm^2_{31}, θ_{23}, improve θ_{13}!
LOIs	NuMI off-axis	(200x)	JHF-HK	(201x)	$\simeq$%, improve θ_{13}, CP?, $sgn(\Delta m^2)$?
Long term	neutrino factory	(201y)			CP violation
...and then	muon collider	(20xx)			

- Continuos line of improvements for
  - beams
  - detectors
  - physics

➔ Very precise oscillation parameters (% precision)
➔ Sensitivity to leptonic CP violation

Why should we measure neutrino parametrers so precisely?

⇓

Impact on our understanding of physics beyond the Standard Model

---

Fermion Mass Models

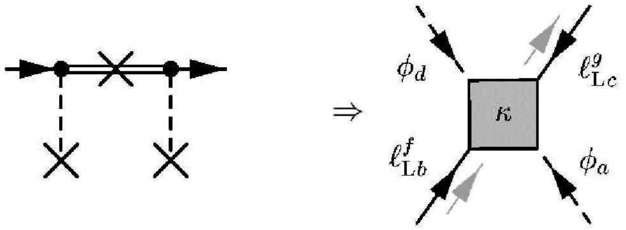
- Structure of m_{LL} from **GUT physics** and/or **discrete symmetries**

m_{LL}	scheme	$0\nu2\beta$-decay
$\frac{m}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & 1 & 1\\ 0 & 1 & 1\end{pmatrix}$ + corr.	hierarchical, normal	small
$\frac{m}{\sqrt{2}}\begin{pmatrix}0 & 1 & 1\\ 1 & 0 & 0\\ 1 & 0 & 0\end{pmatrix}$ + corr.	hierarchical, inverted	small
$m\begin{pmatrix}1 & 0 & 0\\ 0 & \frac{1}{2} & \frac{1}{2}\\ 0 & \frac{1}{2} & \frac{1}{2}\end{pmatrix}$ + corr.	hierarchical, inverted	large
$m\begin{pmatrix}0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2}\end{pmatrix}$ + corr.	degenerate	small
$m\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$ + corr.	degenerate	large

Examples: ...many

$d = 5$ Neutrino Mass Operator

- Integrating out all heavy degrees of freedom ⇒ only light degrees of freedom
- Graphically for the example of M_R:



⇔ $\mathcal{L}_{mass} = \kappa \cdot \overline{\nu}_L^C \nu_L \Phi^T \Phi$

d=5 Mass Operator:

- Light degrees of freedom ⇒ (ν_L, Φ)
- κ is dimension-full
- Example: $\kappa \simeq \frac{1}{M_R}$
- For $\langle \Phi \rangle = v$ ⇒ neutrino masses $\sim \kappa v^2$

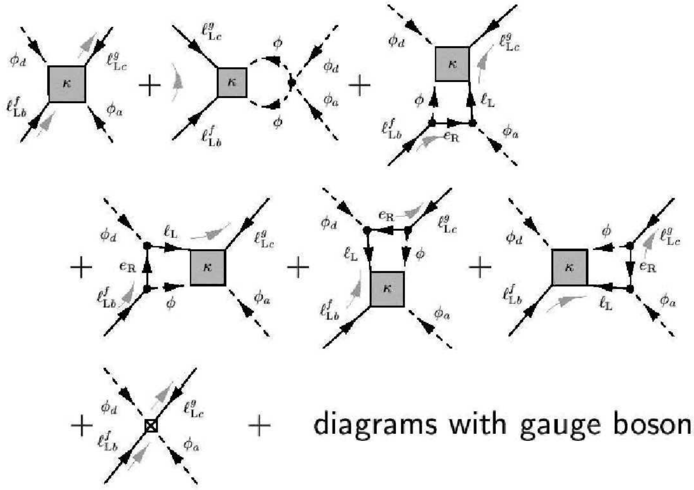
Quantum Corrections to Neutrino Masses and Mixings

Experiment: Masses, mixings at low energies (in the future very precise!)
Theory: ⇒ Models for Y_D and M_R at high energies

⇓

Quantum corrections ⇒ RGE-evolution of κ to the GUT-scale

1-loop corrections:

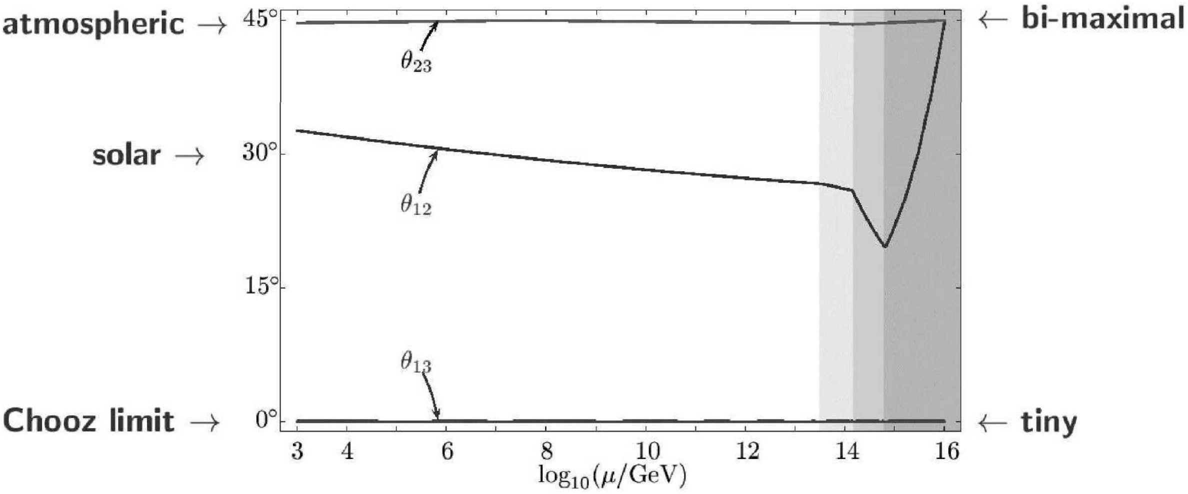


+ diagrams with gauge bosons

Example for RGE Effects

RG-evolution of bi-maximal mixings at the GUT-scale (MSSM example)

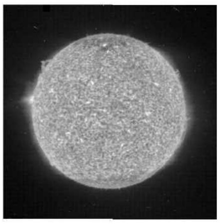
Antusch, Kersten, **ML**, Ratz, Phys.Lett. B544 (2002) 1



Rather general property:

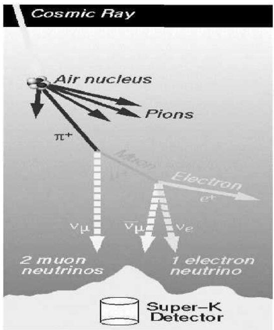
- θ_{12} runs stronger than $\theta_{23}, \theta_{13} \simeq konst.$
- Compare with data ⇔ **deviation from bi-maximality can be explained by RGE**

Learn about other things:

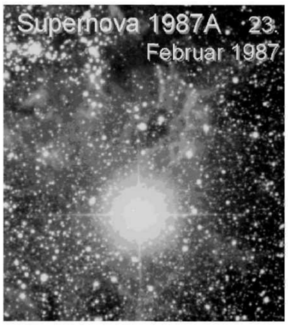


←**Sun**

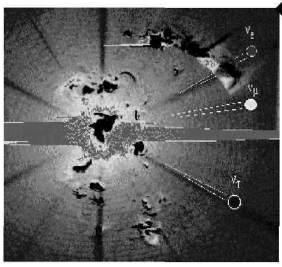
Earth→



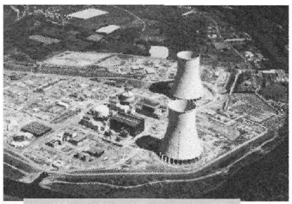
←**Atmosphere**



Supernovae→



←**Big Bang**



Reactors→

Accelerators→

Learn about / from

- Sun (learn about fusion cylce...)
- Earth (geo neutrinos, density profile with beams & SN...)
- Atmosphere ...

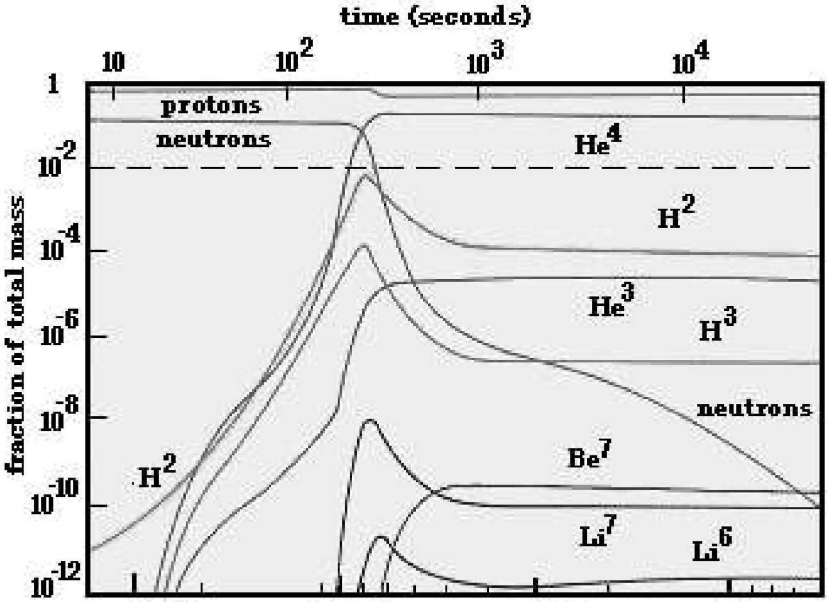
Neutrinos in Astrophysics and Cosmology

⇓

Staus Quo and Evolution of the Universe depends on Neutrinos

- **Big Bang Nucleosynthesis (BBN)**
- **Supernovae**
- **Baryon asymmetry**
- **...**

BBN predicts light element abundances very nicely



more neutrinos etc. Tend to make this worse...

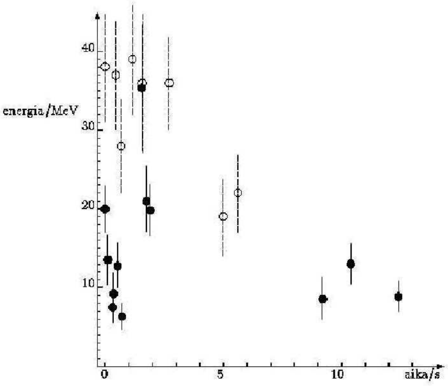
Neutrinos and Supernovae

- Collapse of a typical star ⇒ $\simeq 10^{57}\nu's$, $\simeq 99\%$ of released energy in $\nu's$
- Essential for explosion – simulations do not (yet) explode

SN1987A neutrino burst

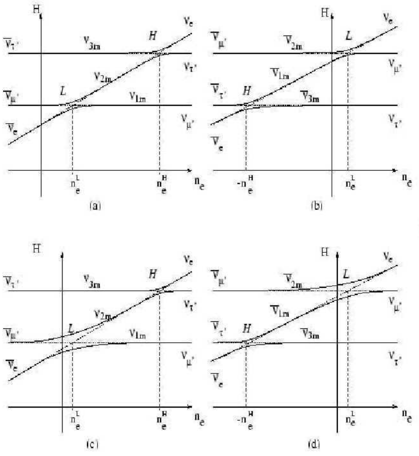
Progenitor:
Sandulaek -69 202 in LMC
15-18 solar masses

⇒ $m_\nu \leq 15$ eV
⇒ $(E_\nu/m_v)\tau_{\overline{\nu}_e} \geq 5 \cdot 10^{12}$ s



- Next galactic SN in 20..100 years
  - Many events ⇒ details of the explosion mechanism

- Next galactic SN in 20..100 years ⇒ neutrino mass limits $\mathcal{O}(\mathrm{eV})$



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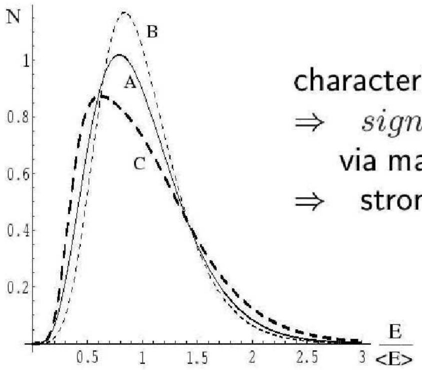


characteristic effects in spectrum
⇒ $sign(\Delta m^2_{21})$ and $sign(\Delta m^2_{31})$
via matter effects (SN and Earth)
⇒ strong limits on θ_{13}

Dighe and Smirnov, Phys.Rev. D62 (2000) 033007

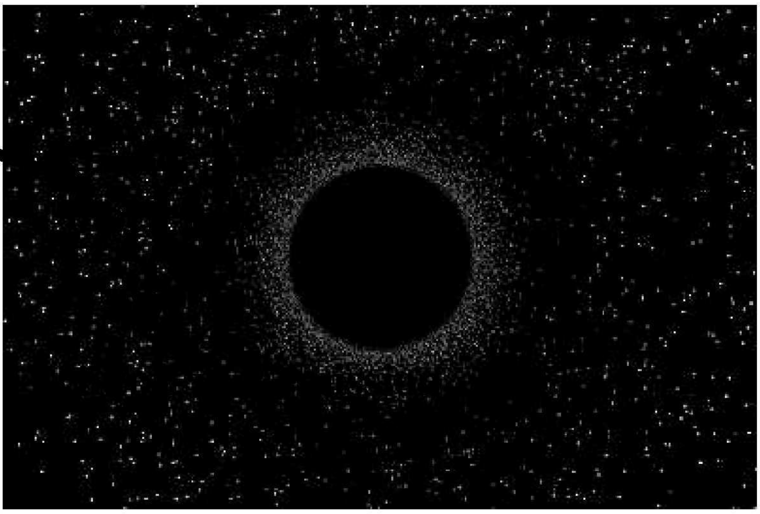
- Time scales: neutrino masses from other experimenst
  - ⇒ learn more about SN explosion mechanism
  - ⇒ improved understanding of SN, neutron star, black hole,...
  - ⇒ consistency test and redundancy

SN➔ 2 possibilities:

Supernovae

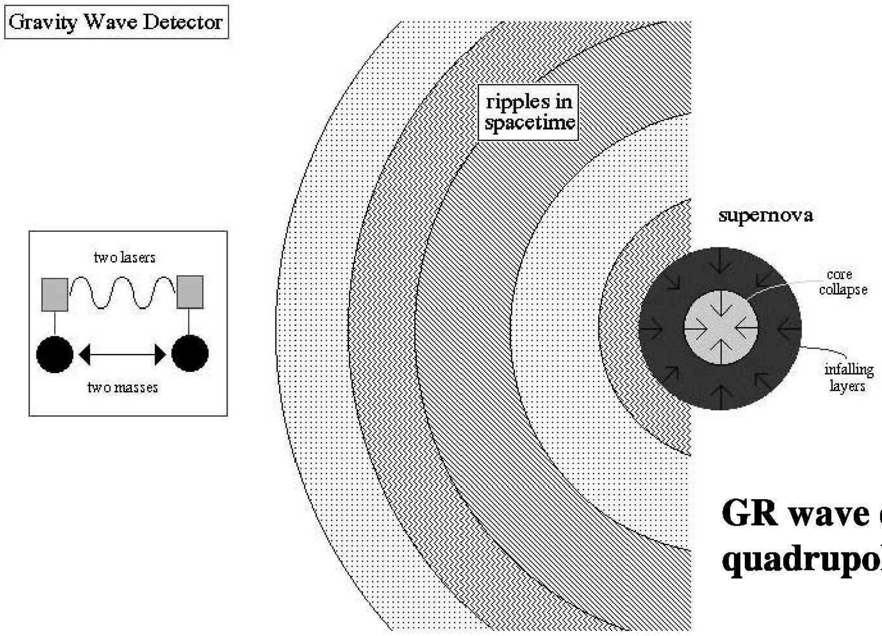
neutron star or

keeps cooling



➔ Extremely impressive signal of BH & ν physics

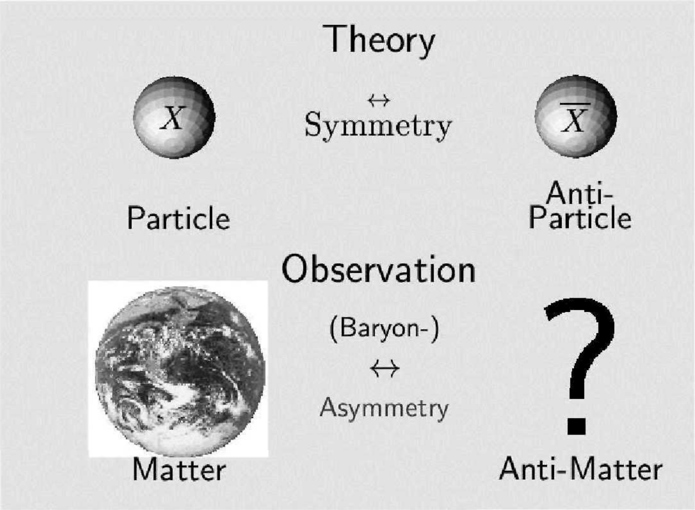
Gravitational waves & SN:



GR wave emission depends on quadrupol moment of explosion

➔ Adds to understanding of SN ➔ global fits

Baryon Asymmetry & Neutrino Physics



Theory

X ↔ Symmetry $\overline{X}$

Particle | Anti-Particle

Observation

Matter (Baryon-) ↔ Asymmetry ? Anti-Matter

Measured baryon asymmetry:

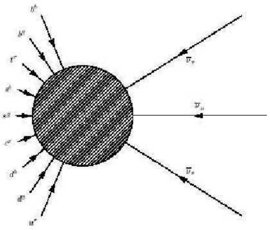
$\eta = \frac{\mathbf{n_B}}{\mathbf{n_\gamma}} = \mathbf{(4-7)\cdot 10^{-10}}$

Early Universe $n_B = 0$ → **Baryogenesis** → Today $n_B \neq 0$

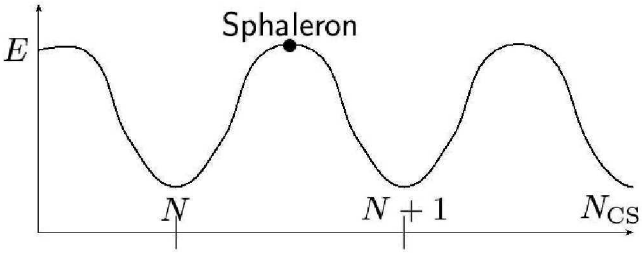
- **Baryogenesis ⇒ Sakharov conditions:**

1. **C & CP violation** ↔ complex couplings in $\mathcal{L}$
2. **Departure from thermal equilibrium** ↔ $\Gamma < H$
3. **B violation** ↔ B+L violating Sphalerons

- **Sphaleron Processes:** ⇔ topology of electro-weak vacuum

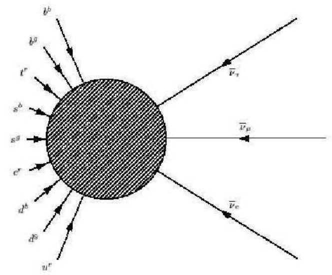


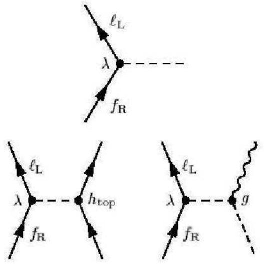
⇒

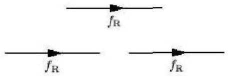


- Sphaleron processes are in equilibrium for $T \gtrsim T_{\rm EW}$
- Sphaleron processes stop for $T < T_{\rm EW}$
- Provide $\not{B}$
- Change $(B+L)$, but not $(B-L)$
- Only lefthanded particles are affected ⇔ lepton number assignment & equilibration

Wash-Out and Equilibration







+ $\mathcal{L} = \lambda \overline{f}_R (\varphi \ell_L)$+h.c.

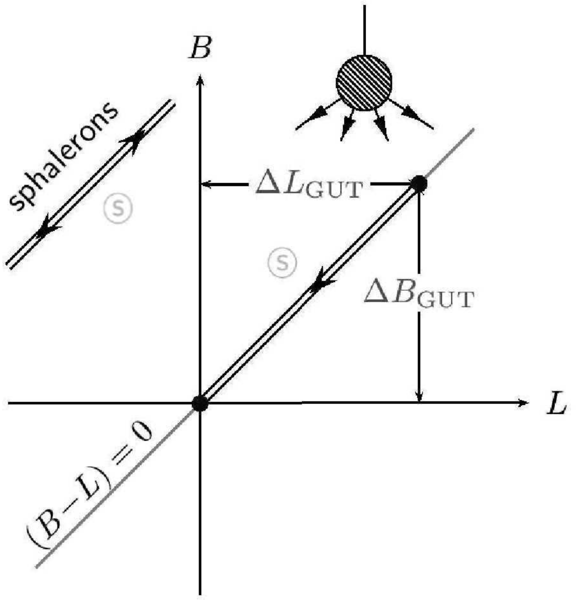
Only lefthanded particles are affected.

LR-conversion ↔ Higgs-Yukawa-couplings.

- Equilibration very fast for sizable Yukawa couplings ⇒ ignore
- Small Yukawa couplings ⇒ treat $SU(2)$ singlets seperately

GUT Baryogenesis & Wash-Out

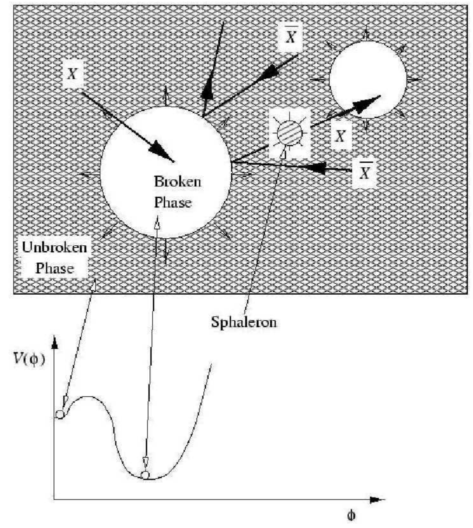
Kuzmin, Rubakov & Shaposhnikov



- Initial B asymmetry, $B-L$ conserved
- Sphalerons: B asymmetry is washed out!

Electroweak Baryogenesis

- SM fulfills Sakharov conditions
- CP violation tiny
- Requires first order phase transition

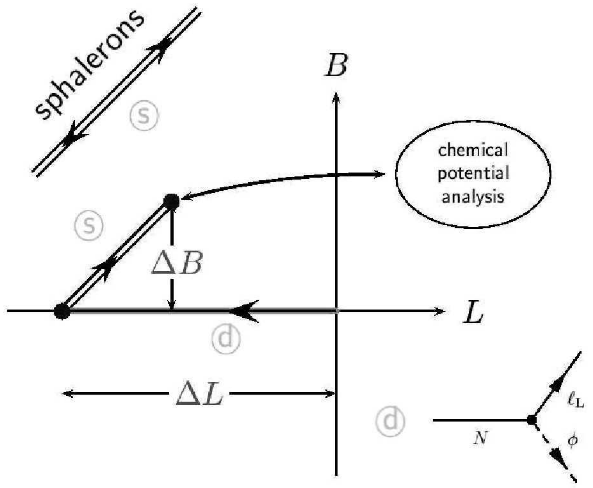


- Electroweak Baryogenesis does not work within the Standard Model
- 1st order phase transition ⇒ one tiny λ ⇒ tendency to predict light Higgses
  ⇒ almost excluded in extensions like 2HDM, MSSM, ...

Leptogenesis

Fukugita & Yanagida, Luty, Buchmüller et al

- Righthanded Majorana neutrinos decays ⓓ produce ΔL ($\leftrightarrow \not{L}$)
- Sphalerons ⓢ convert ΔL partially into ΔB

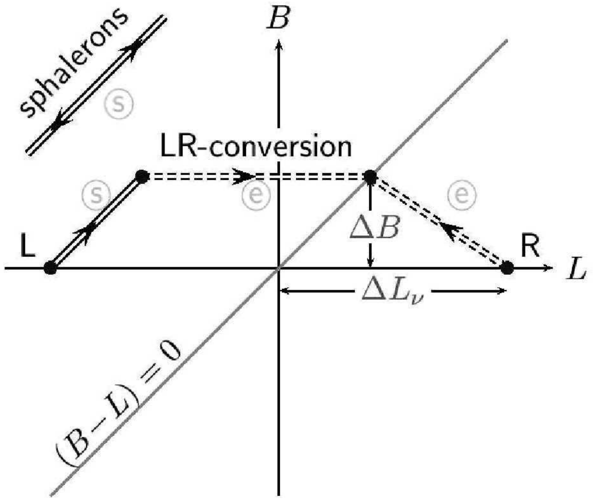


⇒ Sphalerons convert a L asymmetry into a B asymmetry
⇒ upper bound on $m_\nu < \mathcal{O}(0.1\ \mathrm{eV})$ Buchmuller, Plumacher, Int.J.Mod.Phys.A15 (2000)5047

Neutrinogenesis

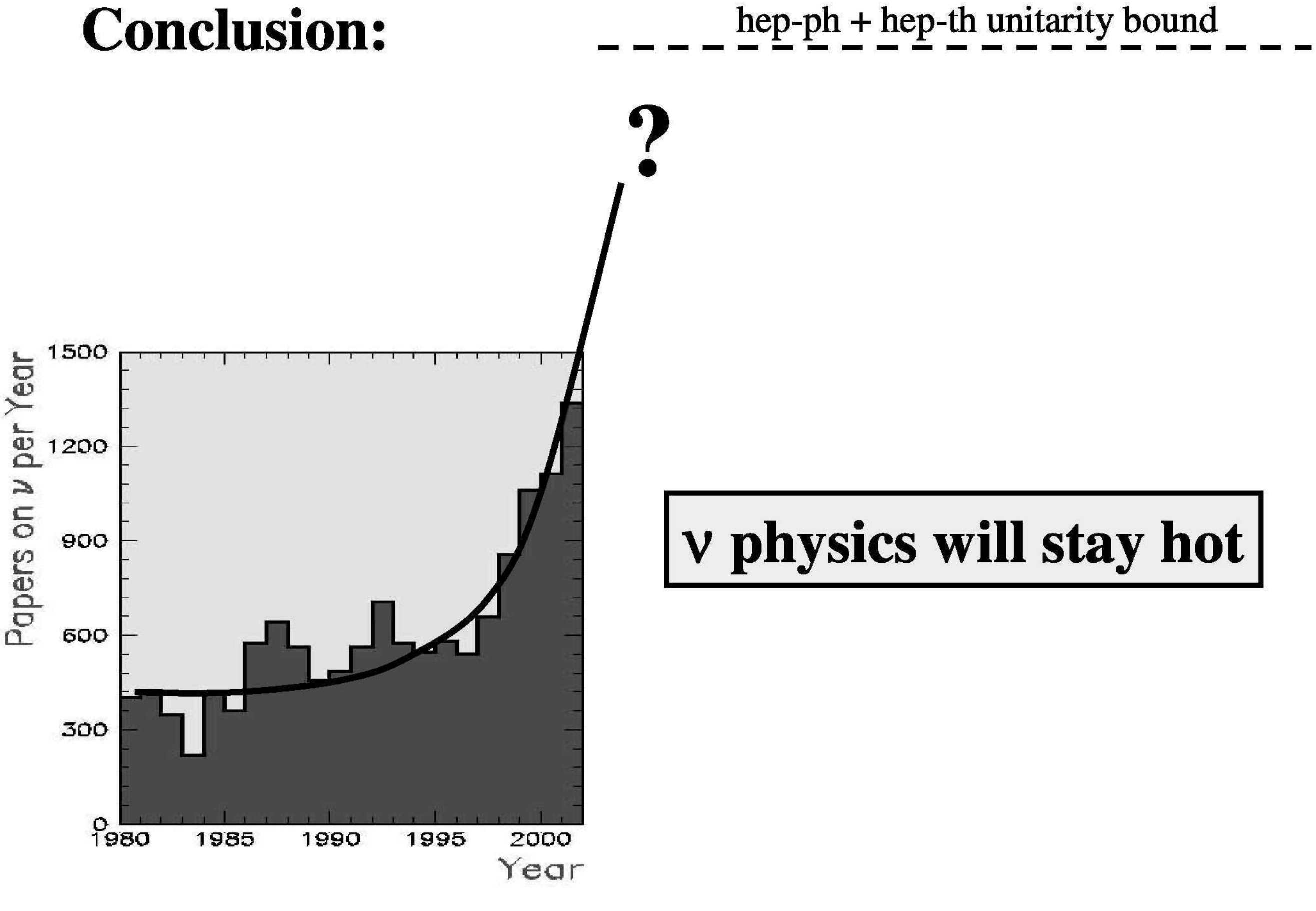
Dick, Lindner, Ratz, Wright, Phys. Rev. Lett. 84 (2000) 4039

- Small Yukawa couplings ⇒ LR-conversion too slow
- Equilibration occures after sphalerons switch off



⇒ Baryon asymmetry with Dirac neutrinos

Conclusion:



ν physics will stay hot