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"Are Interaction Effects Responsible for Temperature and Magnetic Field Dependent Conductivity in Si-MOSFETSs?"

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These are preliminary lecture notes, intended only for distribution to participants.

Физический институт имени П.Н.Лебедева
ФИАН

Are Interaction Effects Responsible for Temperature and Magnetic Field Dependent Conductivity in Si-MOSFETs ?

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Outline

Puzzle of the metallic-like conduction

Renormalization of χ^*, m^* with r_s in the liquid phase

Comparison of $\rho(T)$ with interaction quantum corrections

Advancements and Yet unsolved problems

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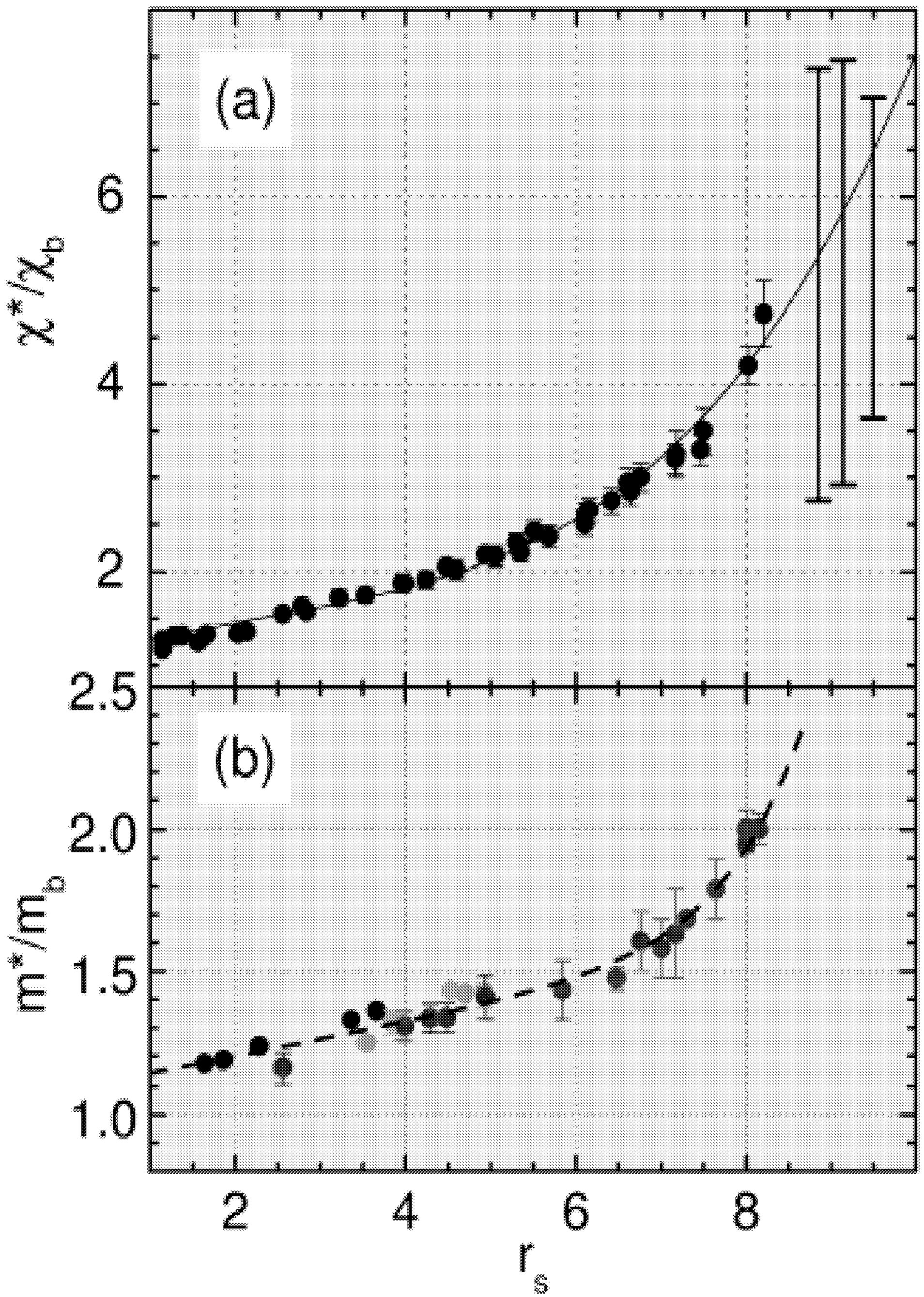
Recent important progress

In theory (Zala, Narozhny, Aleiner, 2001):
quantum corrections to σ calculated beyond the diffusive regime in terms of the FL coupling constants and τ

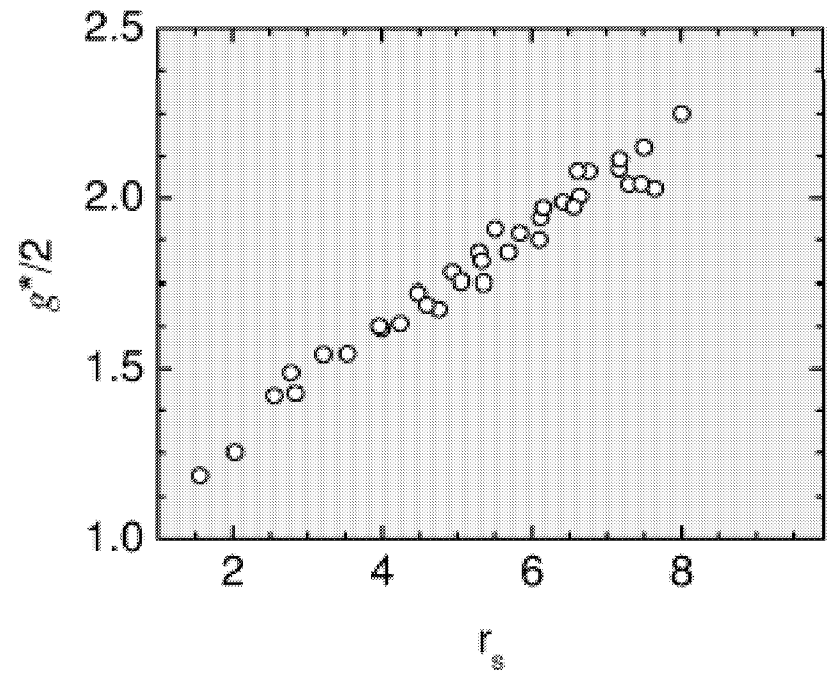
In experiment (Okamoto et al. 1999; Pudalov et al. 2001) :
FL coupling constants F_0^a and F_1^s versus density determined from measurements of SdH oscillations

These advances allow to understand the metallic-like conduction in the regime $\rho << h/e^2$

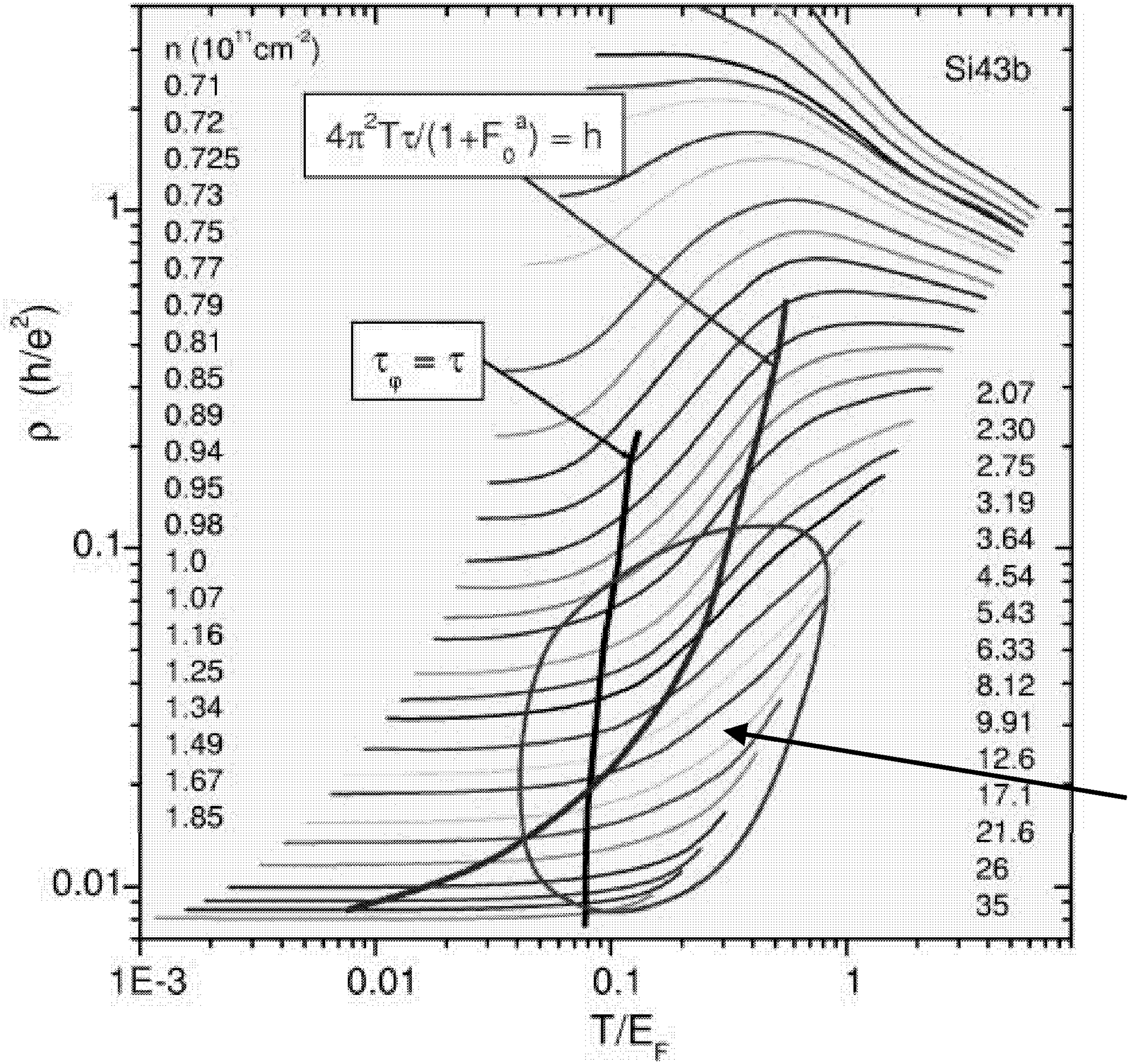


Renormalized χ^*, m^*, and g^* versus r_s determined from SdH oscillations in crossed fields

Pudalov et al. PRL (2002)



ρ(*T*) in a wide *T*-range



Field of the talk

Physical picture of the coherent e-e interaction in the ballistic regime (after: Zala, Narozhny, Aleiner, 2001)

Quantum corrections to the Drude conductivity :

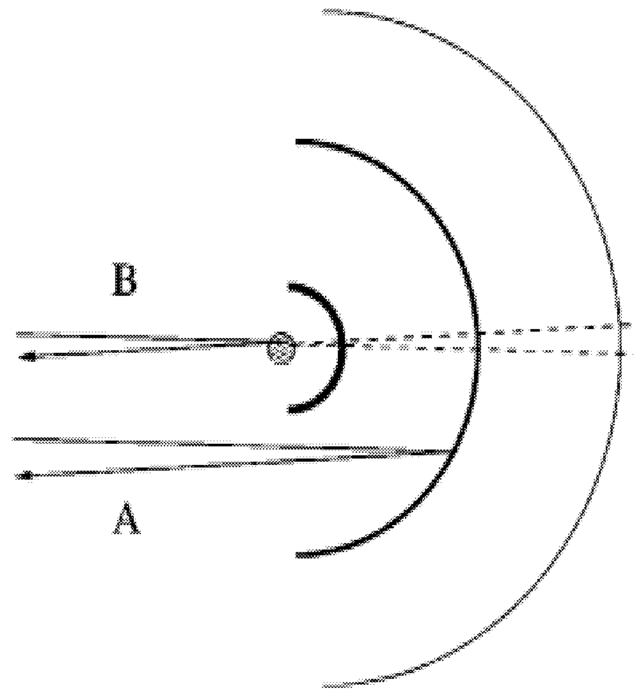
$$\sigma\left(T,B_{\|}\right)-\sigma_D=\delta\sigma_{\mathrm{C}}+15\delta\sigma_{\mathrm{T}}+\delta\sigma_{\mathrm{loc}}\left(T\right).$$

Here (in units of e^2/h):

$$\delta\sigma_{\mathrm{C}}\approx\left(T\tau\right)\left[1-\frac{3}{8}f\left(T\tau\right)\right]$$ **- singlet channel**

$$\delta\sigma_{\mathrm{T}}\approx\frac{F_0^a}{1+F_0^a}\left(T\tau\right)\left[1-\frac{3}{8}t\left(T\tau,F_0^a\right)\right]$$ **- triplet channel**

$$\delta\sigma_{\mathrm{loc}}=\frac{1}{2\pi}\ln\left(\tau/\tau_\varphi(T)\right)$$ **- weak localization correction.**



Due to 2-valley (100)-Si, there are 4x4-1= 15 triplets. $d\sigma/dT$ becomes < 0 for $F_0^a<0$!

How it works:

(a) <u>ballistic regime $T\tau \gg 1$</u>

$\Delta\sigma \cong T\tau\,[1+15F_0^a/(1+F_0^a)]$

b) <u>diffusive regime $T\tau \ll 1$</u>

$\Delta\sigma \cong [1+1-15x(1-\ln(1+F_0^a)/F_0^a]\,\ln T$

<u>None parameter</u>
comparison with theory:

F_0^a – from g^*,

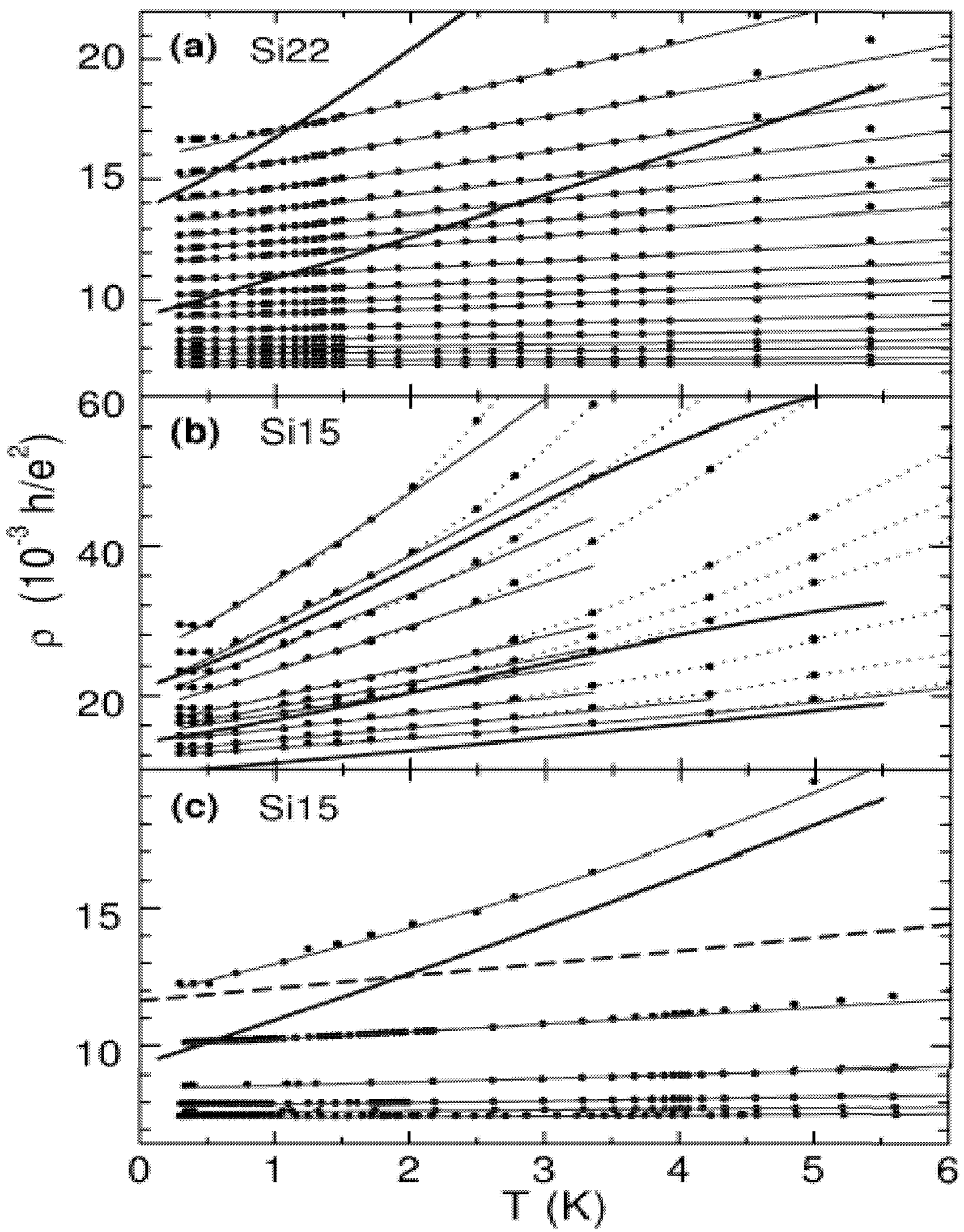
τ – m^* and ρ_D

ρ_D – from $\rho(T)$ extrapolation to T=0

Dots – experimental data.

<u>Red</u> lines - 1st order in T & high orders in interaction.

Blue lines – numerical RPA, to all order in T.

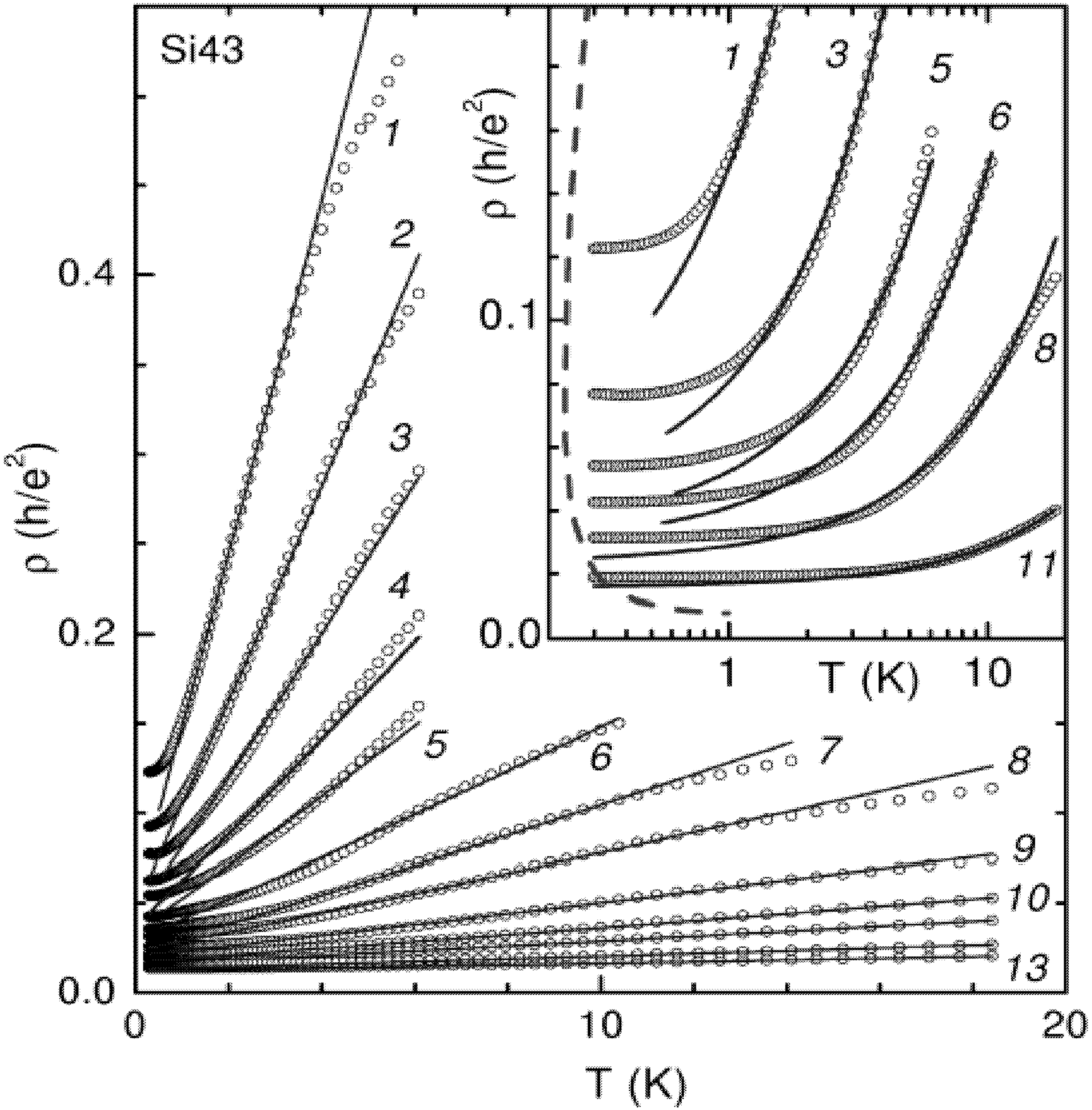


For some samples, the agreement holds up to

$\delta\rho/\rho > 1$

<u>Disagreements:</u>

1) in the duffusive regime $T\tau << 1$;
2) for $T \sim T_F$



So far seems good.

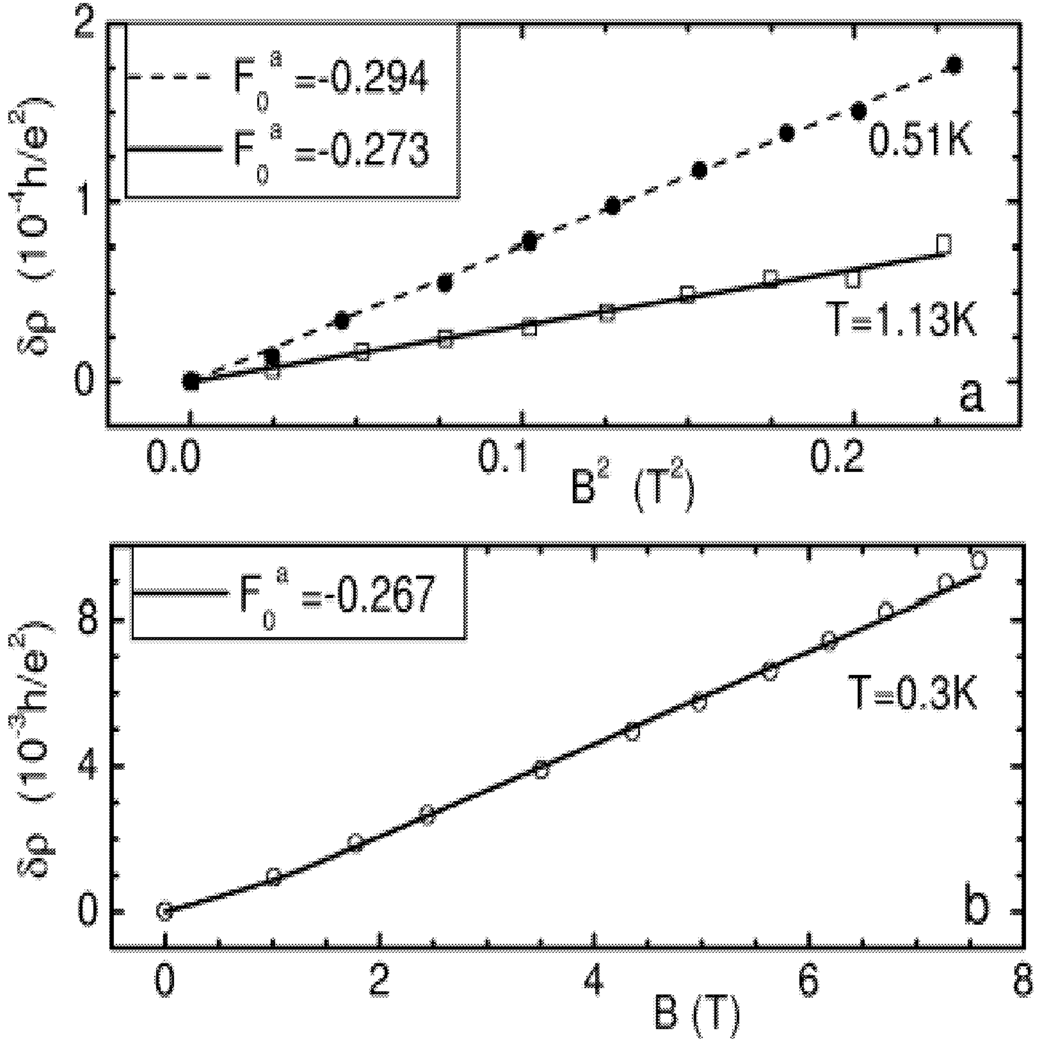
<u>Drawbacks:</u>
magnetoresistance scales somewhat different

$\sigma(B_{||}, T) \sim (B^2/T^{\alpha})$

with $\alpha \sim 1.1$

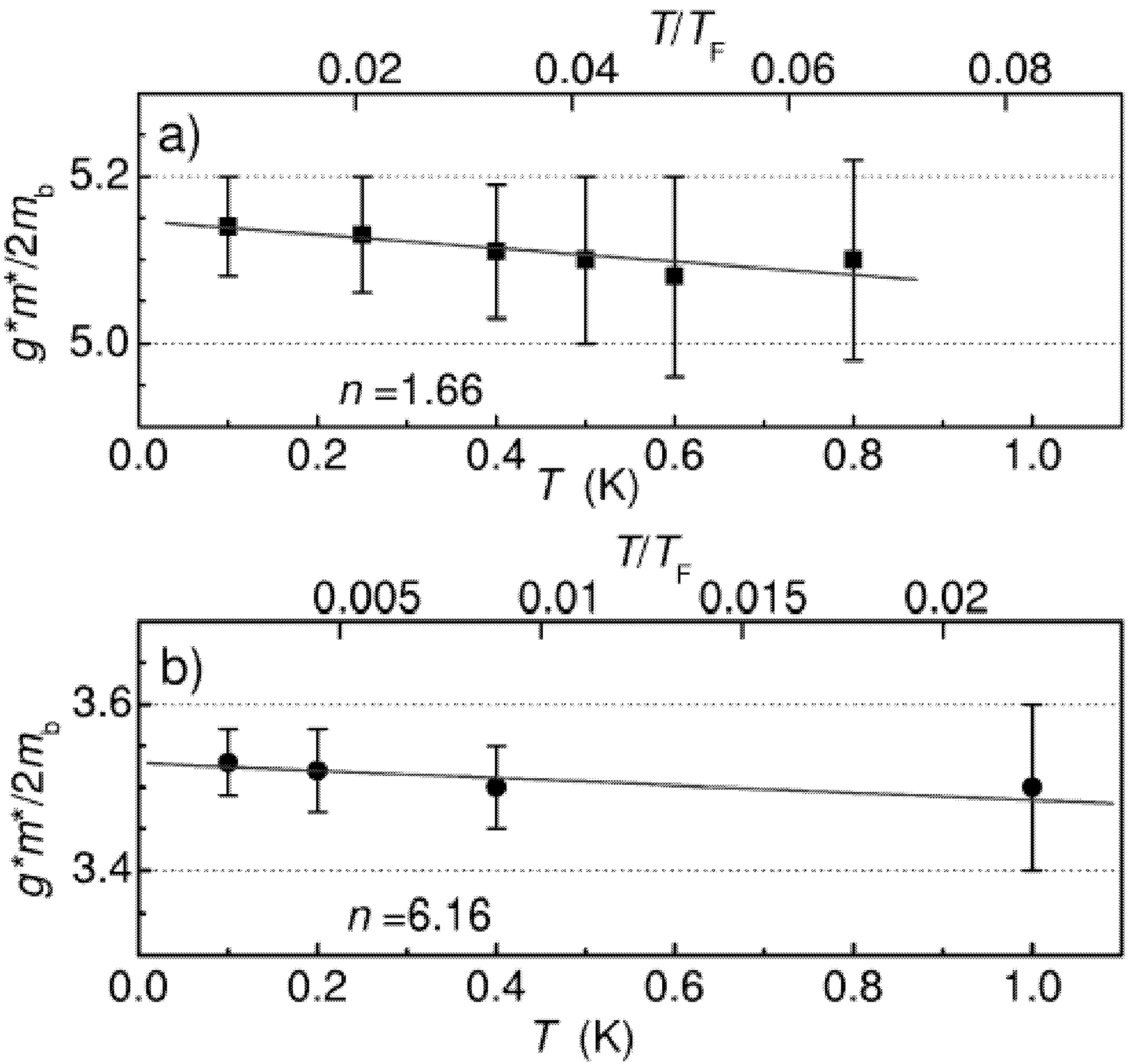
Can F_0^a strongly depend on T?

Very unlikely, because $\chi^* \sim g^* m^*$ varies < 5%



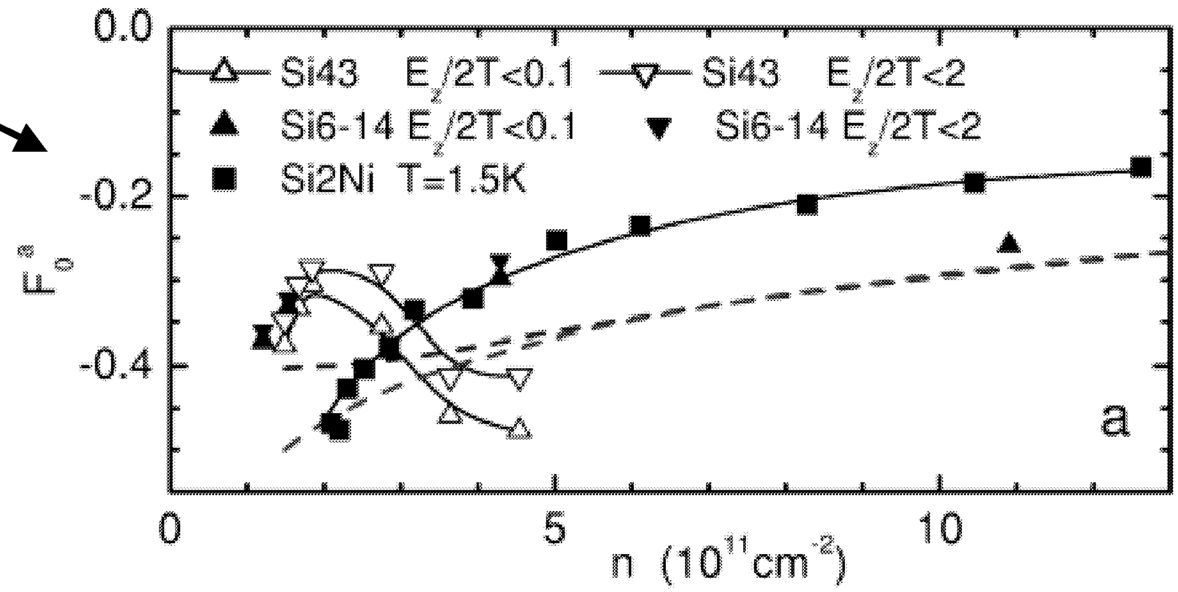
Direct data from SdH interference pattern, for two densities (in units of $10^{11} cm^{-2}$):

T - dependence of χ^* is weaker than T/T_F

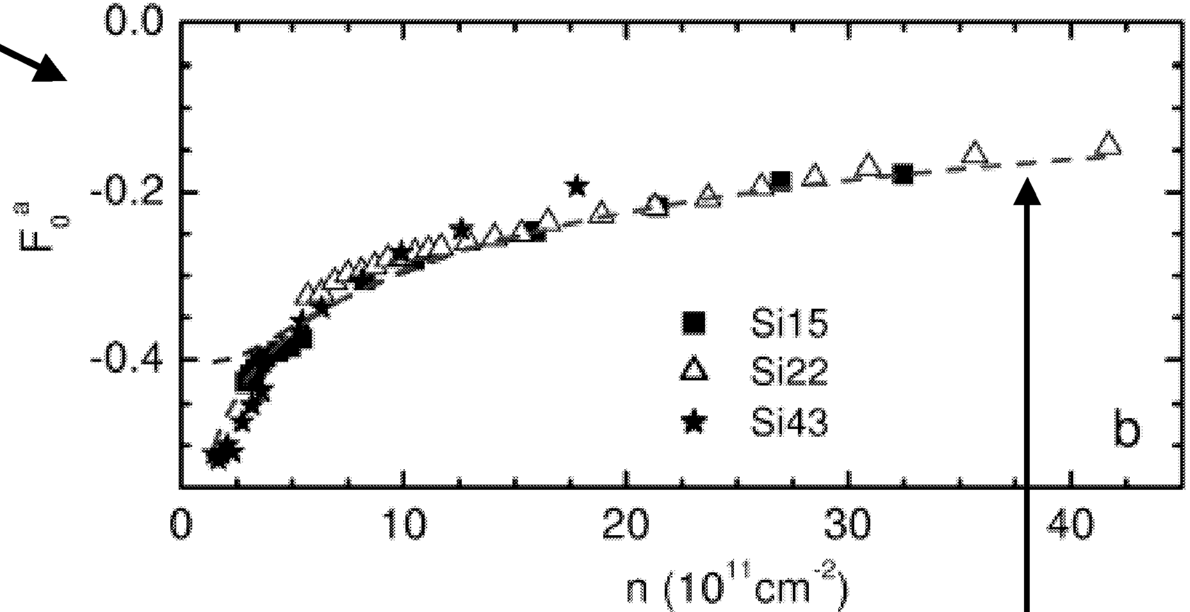


When F_0^a is treated as a fitting parameter:

(a) $\sigma(B)$ is sample-dependent and poorly described by the theory even in the regime $\sigma >> e^2/h$ and $g\mu B_{||} < T$



(b) $\sigma(T)$ is sample independent and in a good agreement with theory



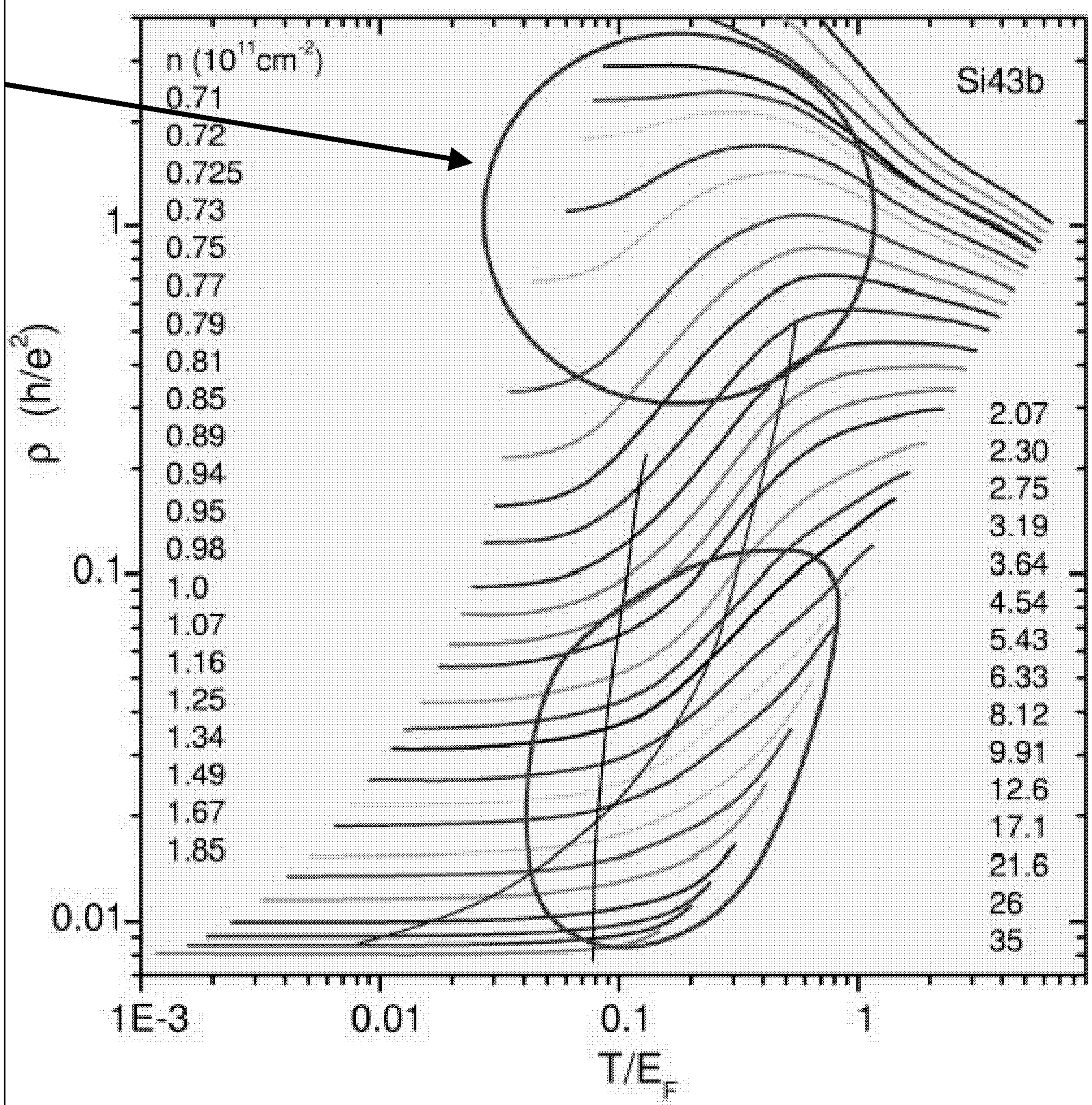
Sample-dependent MR signals influence of the localized states and associated magnetic moments

From SdH measurements

If theory works so nicely, it seems reasonable to apply it to the critical regime

Two-parameter scaling by RG equations (Finkelstein, and Punnoose, 2002).

<u>Main problem:</u> sample-specific bare disorder effects



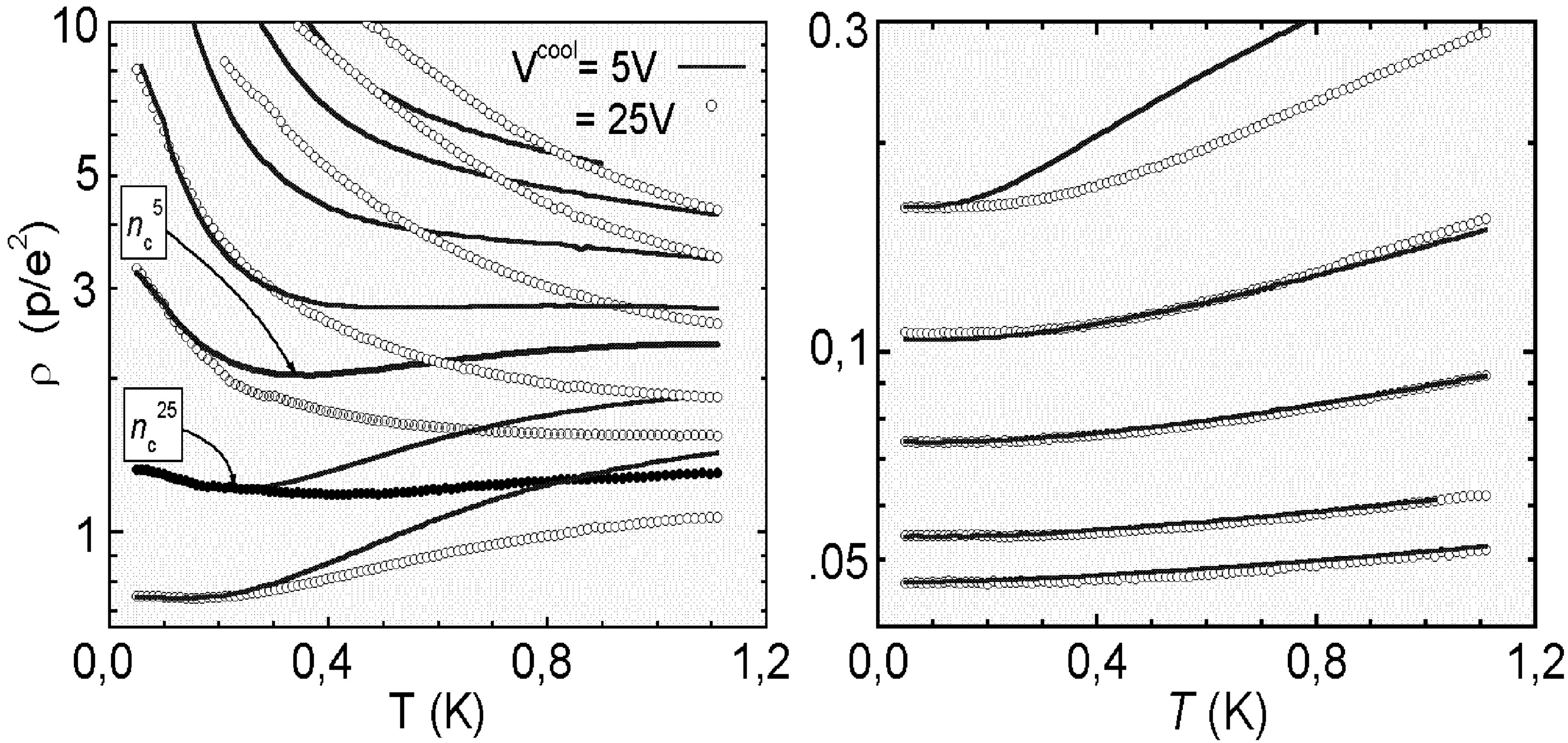
Definitions:

Insulator ⇒ Solid Phase

Metallic conduction ⇒ Liquid phase

Caution: All measurements are done at $T \neq 0$!

Note: *In the $T \rightarrow 0$ limit, transport becomes "universal" and sample-specific disorder effects vanish*



Summary

A good agreement between $\rho(T)$ and calculated quantum corrections in the ballistic regime over a wide range of $n = (1.5 - 40)\ 10^{11}\text{cm}^{-2}$

Substantial disagreement with theories in the diffusive regime at $T\tau < 1$, and at $T \sim T_F$.

In-plane magnetoresistance is sample dependent and deviates from the theory, especially for $g^*\mu B > T$, and for $g^*\mu B \sim E_F$.

Transport in the vicinity of $\rho \sim h/e^2$ is non-universal and depends on details of disorder

The sample-specific effects vanish as $T \to 0$

Interaction Effects in Conductivity of Si Inversion Layers at Intermediate Temperatures

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We compare the temperature dependence of resistivity $\rho(T)$ of Si-metal-oxide-semiconductor field-effect transistors with the recent theory by Zala *et al.* In this comparison, the effective mass m^* and g^* factor for mobile electrons have been determined from independent measurements. An anomalous increase of ρ with temperature, which has been considered as a signature of the "metallic" state, can be described quantitatively by the interaction effects in the ballistic regime. The in-plane magnetoresistance $\rho(B_\|)$ is only qualitatively consistent with the theory; the lack of quantitative agreement indicates that the magnetoresistance is more sensitive to sample-specific effects than $\rho(T)$.

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The theory of quantum corrections due to single-particle (weak localization) and interaction effects has been very successful in describing the low-temperature electron transport in low-dimensional conductors (see, e.g., [1]). Serious quantitative discrepancies between the experimental data and the theory of interaction corrections were noticed two decades ago (see, e.g., [2–4]); this disagreement became dramatic for high-mobility Si devices [5,6]. The attempts to attribute the "anomalous metallic behavior" of Si-metal-oxide-semiconductor field-effect transistors (MOSFETs) in the low-density (dilute) regime to the temperature dependent screening [7–9] could not reconcile the disagreement between the theory and experimental data (for reviews, see [10,11]).

Recently, important progress has been made in both experiment and theory, which allows solving this long-standing problem. First, the interaction corrections to the conductivity have been calculated beyond the diffusive regime in terms of the Fermi-liquid (FL) interaction parameters [12]. This is crucial because the most pronounced increase of ρ with temperature is observed in the ballistic regime (see below). Second, FL parameters for a 2D electron system have been found from measurements of Shubnikov-de Haas (SdH) oscillations in Si-MOSFETs [13,14]. All this allows one to perform quantitative comparison of experimental data with theory.

In this Letter, we show that the most prominent feature of the metallic state in Si-MOSFETs, the strong temperature dependence of the conductivity $\sigma(T)$ at large values of $\sigma \gg e^2/h$, can be accounted for by the theory [12] over a wide range of carrier densities and temperatures. Thus, the metallic conductivity of high-mobility Si-MOSFETs, which was considered anomalous for a decade, can now be explained by the interaction effects in the ballistic regime. The interaction effects in Si-MOSFETs are strongly enhanced due to the valley degeneracy [15] and renormalization of the FL parameters at low n. The experiment, however, still deviates from the theory at very low temperatures and in strong magnetic fields $B_\| \gg k_BT/g\mu_B$, as well as at low electron densities, where sample-specific effects come into play [16].

The ac (13 Hz) transport measurements have been performed on five (100) Si-MOS samples from different wafers: Si15b (peak mobility $\mu^{\rm peak} = 4$ m^2/V s), Si2Ni (3.4 m^2/V s), Si22 (3.3 m^2/V s), Si6-14 (2.4 m^2/V s), and Si43 (1.96 m^2/V s); more detailed description of the samples can be found elsewhere [16,17].

The theory introduced recently by Zala *et al.* [12] considers backscattering of electrons at the scattering centers and at the Friedel oscillations of the density of surrounding electrons. The interference between these scattering processes gives rise to quantum corrections to the Drude conductivity $\sigma_D = e^2n\tau/m^*$ (τ is the momentum relaxation time, m^* is the effective mass of carriers), which can be expressed (in units of $e^2/\pi\hbar$) as follows [12]:

$$\sigma(T, B_\|) - \sigma_D = \delta\sigma_C + 15\delta\sigma_T + 2[\sigma(E_Z, T) - \sigma(0, T)] + 2[\sigma(\Delta_v, T) - \sigma(0, T)] + [\sigma(E_Z + \Delta_v, T) - \sigma(0, T)] + [\sigma(E_Z - \Delta_v, T) - \sigma(0, T)] + \delta\sigma_{\rm loc}(T). \quad (1)$$

Here $\delta\sigma_C = x[1 - \frac{3}{8}f(x)] - \frac{1}{2\pi}\ln(\frac{E_F}{T})$ and $\delta\sigma_T = A(F_0^a)x[1 - \frac{3}{8}t(x, F_0^a)] - [1 - B(F_0^a)]\frac{1}{2\pi}\ln(\frac{E_F}{T})$ are the interaction contributions in the singlet and triplet channels, respectively; $\delta\sigma_{\rm loc}(T) = \frac{1}{2\pi}\ln[\tau/\tau_\varphi(T)]$ is the weak localization contribution. The terms $\sigma(Z, T) - \sigma(0, T)$ reduce the triplet contribution when the Zeeman energy ($Z = E_Z = 2\mu_BB_\|$), the valley splitting ($Z = \Delta_v$), or a combination of these factors ($Z = E_Z \pm \Delta_v$) exceed the temperature. The prefactor 15 to $\delta\sigma_T$ reflects the enhanced number of triplet components due to the valley-degenerate electron spectrum in (100) Si-MOSFETs [15].

Because of this enhancement, the "negative" correction to the conductivity due to the triplet channel, $d\delta\sigma_T/dT < 0$, overwhelms the "positive" correction due to the singlet channel and weak localization, $d(\delta\sigma_C + \delta\sigma_{loc})/dT > 0$. Equation (1) predicts the linear dependence $\sigma(T)$ in the ballistic regime $T\tau \gg 1$ and the logarithmic dependence in the diffusive regime $T\tau \ll 1$; the crossover between the two regimes occurs at $T \approx (1 + F_0^a)/(2\pi\tau)$ [12].

The terms in Eq. (1) are functions of $x = T\tau/\hbar$, Z, and F_0^a; their explicit expressions are given in Ref. [12]. The FL interaction constant $F_0^a \equiv F_0^\sigma$, which controls the renormalization of the g^* factor [$g^* = g_b/(1 + F_0^a)$, where $g_b = 2$ for Si], has been independently determined in Ref. [14]. The momentum relaxation time τ is found from the Drude resistivity $\rho_D \equiv \sigma_D^{-1}$ using the renormalized effective mass m^* determined in Ref. [14]. Earlier, there were attempts to apply the theory [12] for fitting the experimental data on $\rho(T)$ and $\rho(B)$ in p-type GaAs [18] and Si-MOSFETs [19] using a number of fitting parameters. In contrast, our approach provides a rigorous test of applicability of the theory [12] to Si-MOSFETs, as we determined the two FL constants in independent measurements.

Figure 1 illustrates the central result of this paper: the experimental dependences $\rho(T)$ can be quantitatively described by the theory of electron-electron corrections in the ballistic regime [12]. For the samples studied, the ballistic regime extends down to $T \lesssim 0.2$ K [12]. The solid lines show the $\rho(T)$ dependences calculated according to Eq. (1). Throughout the paper, we assume $\Delta_v = 0$; small values $\Delta_v \lesssim 1$ K do not affect the theoretical curves at intermediate temperatures.

In the comparison, the Drude resistivity is needed for both, calculating the magnitude of ρ and determining τ in Eq. (1). The theory [12] suggests a recipe for finding the classical ρ_D value by extrapolating the high-temperature quasilinear $\rho(T)$ dependence in the ballistic regime to $T = 0$. One needs to do an accurate nonlinear extrapolation, according to Eq. (1), to account for the contribution of the $\ln(T/E_F)$ terms and nonlinear crossover functions $t(T\tau)$ and $f(T\tau)$. Both nonlinear terms extend far into the ballistic regime. The obtained ρ_D values differ from the result of the simplified linear extrapolation by ~1 to 10% as n changes from 40×10^{11} cm^{-2} to 2×10^{11} cm^{-2}. We note that this difference is important only for finding the magnitude of ρ, whereas the slope of the $\rho(T)$ dependence in the ballistic regime is not sensitive to such small variations in ρ_D. The measured and calculated resistivities are in good agreement for all samples over broad intervals of T and n. For sample Si43, the agreement with the theory holds up to such high temperatures ($T \sim 0.3E_F$) that $\delta\rho/\rho \sim 1$ (see Fig. 2 of Ref. [21][21]). In this case, which is beyond the applicability of the theory [12], we still calculated the corrections to the resistivity according to $\delta\rho = -\delta\sigma\rho_D^2$. For much more disordered sample Si46, the agreement with the theory is worse: the theoretical $\rho(T)$ curves are consistent with the data only at $T < 10$ K, which correspond for this sample mostly to the diffusive regime.

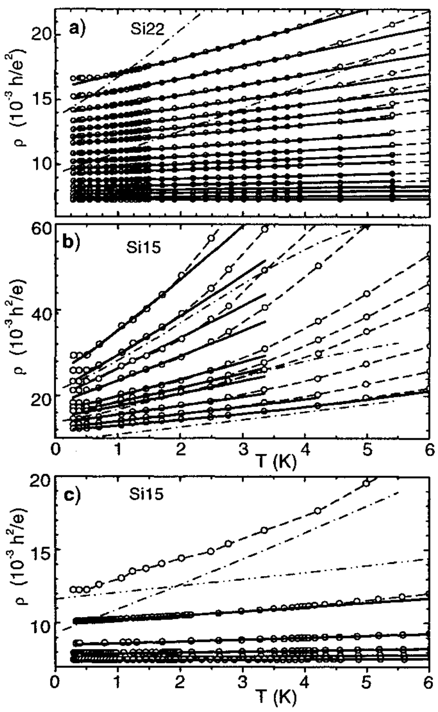

FIG. 1 (color online). ρ vs T for two samples. Circles show the data; thick solid lines correspond to Eq. (1) with $\Delta_v = 0$ and F_0^a from Ref. [14]. The densities, from top to bottom, are (in units of 10^{11} cm^{-2}): in (a) 5.7, 6.3, 6.9, 7.5, 8.1, 8.7, 9.3, 10.5, 11.7, 12.9, 14.1, 16.5, 18.9, 21.3, 23.7, 28.5, 35.7; in (b) 2.23, 2.46, 2.68, 2.90, 3.34, 3.56, 3.78, 4.33, 4.88, 5.44; and in (c) 5.44, 10.45, 15.95, 21.4, 26.9, 32.4. Dash-dotted lines reproduce calculated $\rho(T)$ dependences for sample Si-15 from Fig. 1b of Ref. [9], dash-dot-dotted line — full RPA calculations for $n = 5.6$ from Ref. [20].

For comparison, Fig. 1 also shows the $\rho(T)$ dependences, calculated for Si-MOSFETs within the model of the temperature dependent screening: (i) according to Ref. [9](dash-dotted lines), taking into account the density-dependent collisional broadening, and (ii) according to the full RPA results [20] (dash-dot-dotted line).

The temperature range, in which ρ varies quasilinearly with T extends for approximately a decade up to $T \approx 0.1E_F$; it shrinks, however, towards low densities, $n \sim 1 \times 10^{11}$ cm^{-2}, and high densities, $n \sim 4 \times 10^{12}$ cm^{-2}. The linear $\rho(T)$ dependence is only a part of the overall nonlinear $\rho(T)$ dependence [11,22]; beyond this temperature range, the $\rho(T)$ data depart from the theory (see Fig. 1). Manifestly, the numerical calculations to all orders in T [9,20] do not provide a better fit than the linear T corrections [12]. At $T \rightarrow E_F$, the deviation from the theories might be caused by thermal activation of the interface localized states, which are ignored in the theory.

Weakening of the $\rho(T)$ dependence at low temperatures might be caused by a nonzero valley splitting Δ_v. Indeed, for samples Si22, Si15b, and Si6-14, the temperature of the $\rho(T)$ "saturation" (0.2–0.5 K, see Figs. 1) is of the order of valley splitting estimated from SdH measurements, ~(0.6–0.8) K. However, for sample Si43, the saturation temperature is too high, (1–8) K depending on the density (see Fig. 2 of Ref. [21]), which makes this interpretation of the saturation dubious. The saturation at such high temperatures cannot be caused by electron overheating. One of the reasons for diminishing the interaction contribution might be strong (and sample-specific) intervalley scattering. A theory which takes the intervalley scattering into account is currently unavailable.

We now turn to the magnetoresistance (MR) data in the in-plane field $B_{\|}$. In contrast to the temperature dependences of ρ, the magnetoresistance agrees with the theory [12] only qualitatively. To quantify deviations from the theory, we treated F_0^a as an adjustable parameter in fitting the MR data. Figure 2 shows that the F_0^a values, found for sample Si6-14 from fitting at low ($E_Z/2T < 1$) and high ($E_Z/2T \gg 1$) fields, agree with each other within 10%; at the same time, these values differ by 30% from the values determined in SdH studies [14]. Similar situation is observed for Si43 in weak [Figs. 3(c) and 3(f)] and moderate [Figs. 3(b) and 3(e)] fields.

Fitting the weak-field MR at different temperatures provides an apparent T-dependent F_0^a [Fig. 2(a)]. On the other hand, our SdH data for the same sample do not confirm such a dependence: g^*m^* is constant within 2% over the same temperature range [14]. This discrepancy is consistent with our observation (see also Ref. [3]) that the experimental low-field dependence, $\rho(B_{\|}, T) \propto B^2/T^{\alpha}$ with $\alpha = 0.7$–1, differs from the theoretical one, Eq. (1).

The discrepancy between the theory and the MR data is much more pronounced for sample Si43 in strong fields $E_Z/2T \gg 1$, where even the sign of deviations becomes density dependent [compare Figs. 3(a) and 3(e)]. The nonuniversal behavior of the MR has been reported earlier [16,17]. It might be caused by the interaction of mobile electrons with field-dependent and sample-specific localized electron states. The low lying localized states, which are expected to be singly occupied and carry a nonzero spin, exhibit a substantial T-dependent magnetization [23]; it is natural therefore to expect the effect of localized states to be most pronounced in the MR.

The $F_0^a(n)$ values obtained from fitting the low-field MR for three samples are summarized in Fig. 4(a). The nonmonotonic density dependence of F_0^a and scattering of data for different samples indicate that the MR is more susceptible to the sample-specific effects than $\rho(T)$. For comparison and for testing our analysis, we performed an additional fit of the $\rho(T)$ data for three samples where we also treated F_0^a as an adjustable parameter. The corresponding F_0^a values obtained from $\rho(T)$ fitting are presented in Fig. 4(b). In contrast to Fig. 4(a), there is an excellent agreement between the F_0^a values extracted from SdH measurements and from fitting the $\rho(T)$ dependences. The agreement is observed over a wide density range $n = (1.5$–$40) \times 10^{11}$ cm^{-2} and confirms our conclusion that the theory [12] with FL parameters determined from SdH [14] agrees with the $\rho(T)$ data.

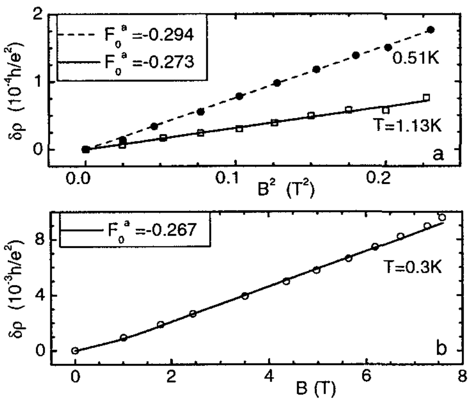

FIG. 2. Magnetoresistivity for sample Si6-14 versus $B_{\|}^2$ at low fields $E_Z/T < 1$ (a), and versus $B_{\|}$ at high fields $E_Z/T \gg 1$ (b). The electron density $n = 4.94 \times 10^{11}$ cm^{-2} is the same for both panels. Lines are the best fits with the F_0^a values shown in the panels.

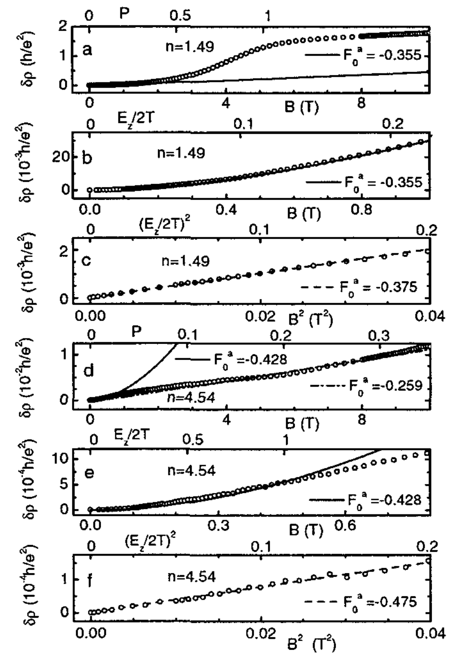

FIG. 3 (color online). Magnetoresistivity for sample Si43 vs $B_{\|}$ and $B_{\|}^2$ for two densities: $n = 1.49 \times 10^{11}$ cm^{-2} [(a),(b),(c)] and 4.54×10^{11} cm^{-2} [(d),(e),(f)]. The upper horizontal scales show $P \equiv g^*\mu_B B/2E_F$ in (a) and (d), and $E_Z/2T$ (b), (c), (e), and (f). $T = 0.31$ K.

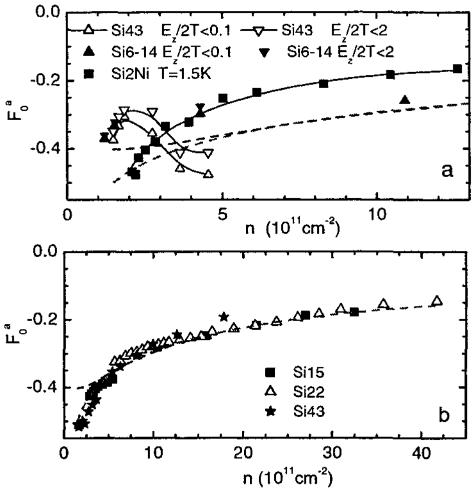

FIG. 4 (color online). Comparison of the $F_0^a(n)$ values determined from (a) fitting $\rho(B_{\parallel})$ for three samples, and (b) fitting $\rho(T)$ for three samples. Dashed lines depict the upper and lower limits for F_0^a from SdH measurements [14].

In summary, we performed a rigorous experimental test of the applicability of the theory [12] to electron transport in Si inversion layers. For high-mobility samples, we found an excellent agreement between $\rho(T)$ and the theory in the ballistic regime over a wide range of temperatures and electron densities $n = (1.5–40) \times 10^{11}$ cm^{-2}; for the comparison, we used independently measured renormalized effective mass and g factor. Our experiments strongly support the theory attributing the anomalous metallic behavior of high-mobility Si-MOSFETs [6] to the interaction effects in the intermediate (ballistic) temperature regime [24]. The existing numerical RPA calculations to all orders in temperature [9,20] do not fit well the nonlinear $\rho(T)$ dependences, especially at high $T > 0.1E_F$ and over a broad density range. Sample-dependent deviations from the theory [12] have been observed in the slope $d\rho/dT$ for both the lowest temperatures and high temperatures ($T \lesssim E_F$). The deviations are more pronounced in the in-plane magnetoresistance, especially in high fields ($2\mu_B B/T > 1$). We attribute this nonuniversality to interaction of the mobile electrons with the field-affected interface localized electron states, which are ignored in the existing theories.

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[1] B. L. Altshuler, A. G. Aronov, and P. Lee, Phys. Rev. Lett. **44**, 1288 (1980).

[2] D. J. Bishop, R. C. Dynes, and D. C. Tsui, Phys. Rev. B **26**, 773 (1982).

[3] V. T. Dolgopolov, S. I. Dorozhkin, and A. A. Shashkin, Solid State Commun. **50**, 273 (1984).

[4] M. S. Burdis and C. C. Dean, Phys. Rev. B **38**, 3269 (1988).

[5] M. D'Iorio, V. M. Pudalov, and S. G. Semenchinsky, Phys. Lett. A **150**, 422 (1990).

[6] S. V. Kravchenko, G. V. Kravchenko, J. E. Furneaux, V. M. Pudalov, and M. D'Iorio, Phys. Rev. B **50**, 8039 (1994).

[7] F. Stern, Phys. Rev. Lett. **44**, 1469 (1980); F. Stern and S. Das Sarma, Solid State Electron. **28**, 158 (1985).

[8] A. Gold and V. T. Dolgopolov, Phys. Rev. B **33**, 1076 (1986).

[9] S. Das Sarma and E. H. Hwang, Phys. Rev. Lett. **83**, 164 (1999).

[10] E. Abrahams, S. V. Kravchenko, and M. P. Sarachik, Rev. Mod. Phys. **73**, 251 (2001).

[11] B. L. Altshuler, D. L. Maslov, and V. M. Pudalov, Physica (Amsterdam) **9E**, 209 (2001).

[12] G. Zala, B. N. Narozhny, and I. L. Aleiner, Phys. Rev. B **64**, 214204 (2001); **65**, 020201 (2002).

[13] T. Okamoto, K. Hosoya, S. Kawaji, and A. Yagi, Phys. Rev. Lett. **82**, 3875 (1999).

[14] V. M. Pudalov, M. Gershenson, H. Kojima, N. Butch, E. M. Dizhur, G. Brunthaler, A. Prinz, and G. Bauer, Phys. Rev. Lett. **88**, 196404 (2002). The upper and lower limits for the g^* factor over the range $r_s = 1.6–8.4$ are $g^*_{\rm high} = 1.976 + 0.3485r_s - 0.01068r_s^2 + 0.00048r_s^3$ and $g^*_{\rm low} = 2.013 + 0.1694r_s + 0.1233r_s^2 - 0.03107r_s^3 + 0.002r_s^4$.

[15] A. Punnoose and A. M. Finkelstein, Phys. Rev. Lett. **88**, 016802 (2002).

[16] V. M. Pudalov, M. E. Gershenson, and H. Kojima, cond-mat/0201001.

[17] V. M. Pudalov, G. Brunthaler, A. Prinz, and G. Bauer, Phys. Rev. Lett. **88**, 076401 (2002).

[18] Y. Y. Proskuryakov, A. K. Savchenko, S. S. Safonov, M. Pepper, M. Y. Simmons, and D. A. Ritchie, Phys. Rev. Lett. **89**, 076406 (2002).

[19] A. A. Shashkin, S. V. Kravchenko, V. T. Dolgopolov, and T. M. Klapwijk, Phys. Rev. B **66**, 073303 (2002); S. A. Vitkalov, K. James, B. N. Narozhny, M. P. Sarachik, and T. M. Klapwijk, Phys. Rev. B **67**, 113310 (2003).

[20] S. Das Sarma and E. H. Hwang, cond-mat/0302047.

[21] V. M. Pudalov, M. E. Gershenson, H. Kojima, G. Brunthaler, A. Prinz, and G. Bauer, cond-mat/0205449.

[22] G. Brunthaler, A. Prinz, G. Bauer, and V. M. Pudalov, Phys. Rev. Lett. **87**, 096802 (2001).

[23] O. Prus, Y. Yaish, M. Reznikov, U. Sivan, and V. M. Pudalov, Phys. Rev. B **67**, 205407 (2003).

[24] In Ref. [22], the relevance of coherent e-e interactions was underestimated, on the basis of the Altshuler-Aronov model of interactions in diffusive regime. In Ref. [12], a new type of e-e interactions (coherent, but in the ballistic regime) was introduced.

Low-Density Spin Susceptibility and Effective Mass of Mobile Electrons in Si Inversion Layers

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We studied the Shubnikov–de Haas (SdH) oscillations in high-mobility Si-MOS samples over a wide range of carrier densities $n \simeq (1-50) \times 10^{11}$ cm^{-2}, which includes the vicinity of the apparent metal-insulator transition in two dimensions (2D MIT). Using a novel technique of measuring the SdH oscillations in superimposed and independently controlled parallel and perpendicular magnetic fields, we determined the spin susceptibility χ^*, the effective mass m^*, and the g^* factor for mobile electrons. These quantities increase gradually with decreasing density; near the 2D MIT, we observed enhancement of χ^* by a factor of $\sim$4.7.

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Many two-dimensional (2D) systems exhibit an apparent metal-insulator transition (MIT) at low temperatures as the electron density n is decreased below a critical density n_c (for reviews, see, e.g., Refs. [1–3]). The phenomena of the MIT and "metallic" conductivity in 2D attract a great deal of interest, because it addresses a fundamental problem of the ground state of strongly correlated electron systems. The strength of electron-electron interactions is characterized by the ratio r_s of the Coulomb interaction energy to the Fermi energy ϵ_F. The 2D MIT is observed in Si MOSFETs at $n_c \sim 1 \times 10^{11}$ cm^{-2}, which corresponds to $r_s \sim 8$ [for 2D electrons in (100)-Si, $r_s = 2.63\sqrt{10^{12}\text{ cm}^{-2}/n}$].

In the theory of electron liquid, the electron effective mass m^*, the g^* factor, and the spin susceptibility $\chi^* \propto g^*m^*$ are renormalized depending on r_s [4]. Though the quantitative theoretical results [5–7] vary considerably, all of them suggest enhancement of χ^*, m^*, and g^* with r_s. Earlier experiments [8–11] have shown growth of m^* and g^*m^* at relatively small r_s values, pointing to a ferromagnetic type of interactions in the explored range $1 \lesssim r_s < 6.5$. Potentially, strong interactions might drive an electron system towards ferromagnetic instability [4]. Moreover, it has been suggested that the metallic behavior in 2D is accompanied by a tendency to a ferromagnetic instability [12]. Thus, in relation to the still open question of the origin of the 2D MIT, direct measurements of these quantities in the dilute regime near the 2D MIT are crucial.

In this Letter, we report the *direct* measurements of χ^*, m^*, and, hence, g^* over a wide range of carrier densities ($1 \lesssim r_s \lesssim 8.4$), which extends for the first time down to and across the 2D MIT. The data were obtained by a novel technique of measuring the interference pattern of Shubnikov–de Haas (SdH) oscillations in *crossed magnetic fields*. The conventional technique of measuring g^*m^* in tilted magnetic fields [8,10] is not applicable when the Zeeman energy is greater than half the cyclotron energy [13]. The crossed field technique removes this restriction and allows us to extend measurements over the wider range of r_s. We find that, for small r_s, the g^*m^* values increase slowly in agreement with the earlier data by Fang and Stiles [8] and Okamoto *et al.* [10]. For larger values of r_s, g^*m^* grows faster than it might be extrapolated from the earlier data. At our highest value of $r_s \simeq 8.4$, the measured g^*m^* is greater by a factor of 4.7 than that at low r_s.

Our resistivity measurements were performed by ac (13 Hz) technique at the bath temperatures 0.05–1.6 K, on seven (100) Si-MOS samples selected from four wafers: Si12 (peak mobility $\mu \simeq 3.4$ m^2/V s at $T = 0.3$ K), Si22 ($\mu \simeq 3.2$ m^2/V s), Si57, Si3-10, Si6-14/5, Si6-14/10, and Si6-14/18 ($\mu \simeq 2.4$ m^2/V s for the latter five samples) [14]. The gate oxide thickness was 190 ± 20 nm for all the samples; the 2D channel was oriented along [011] for sample Si3-10, for all other samples—along [010]. The crossed magnetic field system consists of two magnets, whose fields can be varied independently. A split coil, producing the field $B_\perp \leq 1.5$ T normal to the plane of the 2D layer, is positioned inside the main solenoid, which creates the in-plane field $B_\parallel$ up to 8 T. The electron density was determined from the period of SdH oscillations.

Typical traces of the longitudinal resistivity ρ_{xx} as a function of $B_\perp$ are shown in Fig. 1. Because of the high electron mobility, oscillations were detectable down to 0.25 T, and a large number of oscillations enabled us to extract the fitting parameters g^*m^* and m^* with a high accuracy. The oscillatory component $\delta\rho_{xx}$ was obtained by subtracting the monotonic "background" magnetoresistance (MR) from the $\rho_{xx}(B_\perp)$ dependence (the background MR is more pronounced for lower n; compare Figs. 1 and 3).

The theoretical expression for the oscillatory component of the magnetoresistance is as follows [15]:

$$\frac{\delta\rho_{xx}}{\rho_0} = \sum_s A_s \cos\left[\pi s\left(\frac{\hbar\pi n}{eB_\perp} - 1\right)\right]Z_s, \qquad (1)$$

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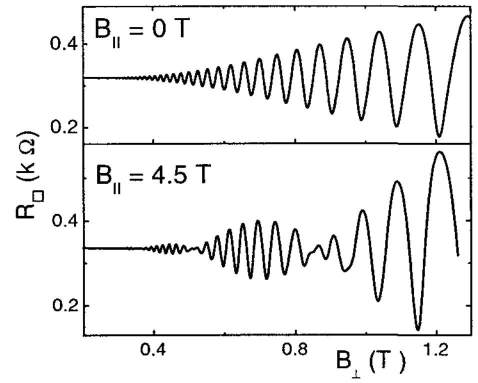

FIG. 1. Shubnikov–de Haas oscillations for $n = 10.6 \times 10^{11}$ cm^{-2} (sample Si6-14/10) at $T = 0.35$ K.

where

$$A_s = 4\exp\left(-2\pi^2 s\frac{k_B T_D}{\hbar\omega_c}\right)\frac{2\pi^2 s k_B T/\hbar\omega_c}{\sinh(2\pi^2 s k_B T/\hbar\omega_c)}. \quad (2)$$

Here $\rho_0 = \rho_{xx}(B_\perp = 0)$, $\omega_c = eB_\perp/m^* m_e$ is the cyclotron frequency, m^* is the dimensionless effective mass, m_e is the free electron mass, and T_D is the Dingle temperature. We take the valley degeneracy $g_v = 2$ in Eqs. (1) and (2) and throughout the paper. The Zeeman term $Z_s = \cos[\pi s\hbar\pi(n_\uparrow - n_\downarrow)/(eB_\perp)]$ reduces to a field-independent constant for $B_\parallel = 0$.

Application of $B_\parallel$ induces beating of SdH oscillations, which are observed as a function of $B_\perp$. The beat frequency is proportional to the spin polarization of the interacting 2D electron system [16]:

$$P \equiv \frac{n_\uparrow - n_\downarrow}{n} = \frac{\chi^* B_{\mathrm{tot}}}{g_b \mu_B n} = g^* m^* \frac{eB_{\mathrm{tot}}}{nh}, \quad (3)$$

where $n_\uparrow$ ($n_\downarrow$) stands for the density of spin-up (spin-down) electrons, $g_b \simeq 2$ is the bare g factor for Si, and $B_{\mathrm{tot}} = \sqrt{B_\perp^2 + B_\parallel^2}$. Equations (2) and (3) imply that the spin polarization of the electron system is linear in B_{tot} (relevance of this assumption to our experiment will be justified below).

In the experiment, we observed a well-pronounced beating pattern at a nonzero $B_\parallel$ (Figs. 1 and 2), in agreement with Eq. (1). The phase of SdH oscillations remains the same between the adjacent beating nodes, and changes by π through the node. The interference pattern (including positions of the nodes) is controlled by Z_s in Eq. (1) and is defined solely by $g^* m^*$. Systematic study of this pattern enables us to determine $g^* m^*$ with high accuracy (~2%–5%). The $g^* m^*$ values are independent of T (at $T < 1$ K) within our accuracy. We have observed a $B_\parallel$ dependence of $g^* m^*$ in strong parallel fields; it is more pronounced near the 2D MIT, where $g^* m^*$ varies with $B_\parallel$ by ~15%. To determine $g^* m^*$ in the linear regime, we systematically measured, for each n, the beating pattern at decreasing values of $B_\parallel$ until $g^* m^*$ becomes independent of $B_\parallel$. Typically, the linear regime corresponded to $B_\parallel \sim 1$ T at high densities, $n \sim 10^{12}$ cm^{-2}, and to $B_\parallel \simeq 0.1$–0.4 T at low densities, $n \sim 10^{11}$ cm^{-2}.

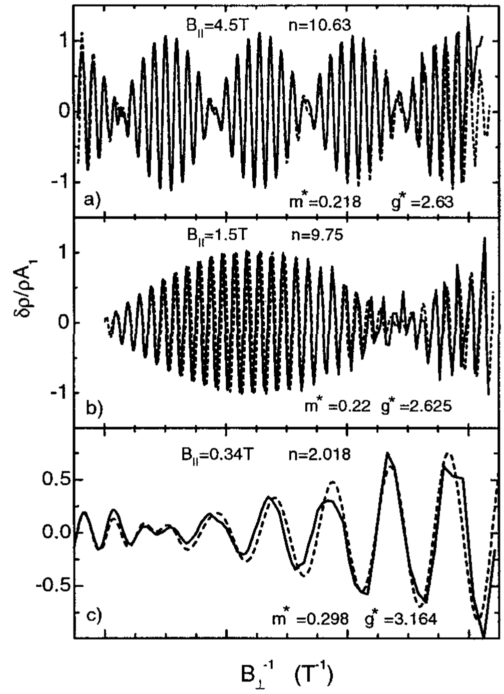

FIG. 2. Examples of fitting with Eq. (1): (a) $n = 10.6 \times 10^{11}$ cm^{-2}, $T = 0.35$ K, $B_\parallel = 4.5$ T (the data correspond to Fig. 1); (b) $n = 9.75 \times 10^{11}$ cm^{-2}, $B_\parallel = 1.5$ T; (c) $n = 2.02 \times 10^{11}$ cm^{-2}, $T = 0.2$ K, $B_\parallel = 0.34$ T. The data for sample Si6-14/10 are shown as the solid lines, the fits (with parameters shown) as dashed lines. All are normalized by $A_1(B_\perp)$.

Comparison between the measured and calculated dependences $\delta\rho_{xx}/\rho_0$ versus $B_\perp$, both normalized by the amplitude of the first harmonic A_1, is shown in Fig. 2 for three carrier densities. The normalization assigns equal weights to all oscillations. We analyzed SdH oscillations over the low-field range $B_\perp \leq 1$ T; this limitation arises from the assumption in Eq. (1) that $\hbar\omega_c \ll \epsilon_F$ and $\delta\rho_{xx}/\rho_0 \ll 1$. The latter condition also allows us to neglect the interlevel interaction which is known to enhance g^* in stronger fields [18].

The amplitude of SdH oscillations at small $B_\perp$ can be significantly enhanced by applying $B_\parallel$ (see Fig. 3), which is another advantage of the cross-field technique. Indeed, for low $B_\perp$ and n, the electron energy spectrum is complicated by crossing of levels corresponding to different spins/valleys. By applying $B_\parallel$, we can control the energy separation between the levels, and enhance the amplitude of low-B oscillations (see Fig. 3). We have verified that application of $B_\parallel$ (up to the spin polarization ~20%) does not affect the extracted m^* values (within 10% accuracy), provided the sample remains in the metallic regime (the

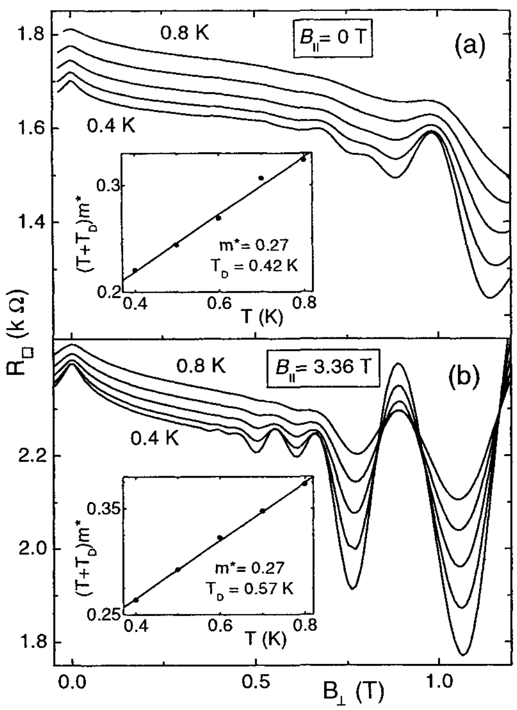

FIG. 3. Shubnikov–de Haas oscillations versus $B_\perp$ for sample Si6-14/10 ($n = 2.2 \times 10^{11}$ cm^{-2}) at $T = 0.4, 0.5, 0.6, 0.7$, and 0.8 K: (a) $B_\parallel = 0$ and (b) $B_\parallel = 3.36$ T. The insets show the temperature dependences of fitting parameters $(T + T_D)m^*$.

insets of Fig. 3 show that the values of m^* measured at $B_\parallel = 0$ and 3.36 T do coincide).

Fitting of the data provides us with two combinations of parameters: g^*m^* and $(T + T_D)m^*$. The measured values of g^*m^*, as well as m^* which are discussed below, were similar for different samples. Figure 4a shows that, for small r_s, our g^*m^* values agree with the earlier data by Fang and Stiles [8] and Okamoto *et al.* [10]. For $r_s \geq 6$, g^*m^* increases with r_s faster than it might be expected from extrapolation of the earlier results [10]. The MIT occurs at the critical r_s value ranging from $r_c = 7.9$ to 8.8 for different samples; in particular, $r_c \simeq 8.2$ for samples Si6-14/5, 10, 18 which have been studied down to the MIT.

An interesting question is whether the measured dependence $\chi^*(r_s)$ shown in Fig. 4a represents a critical behavior as might be expected if χ^* diverges at n_c. Analysis of the $B_\parallel$-induced magnetoresistance led the authors of Refs. [20,21] to the conclusion that there is a ferromagnetic instability at $n \approx n_c$. Our results, which do not support the occurrence of spontaneous spin polarization and divergence of χ^* at $n = n_c$, are presented in Ref. [17].

The second combination, $(T + T_D)m^*$, controls the amplitude of oscillations. In order to disentangle T_D and m^*, we analyzed the temperature dependence of oscillations over the range $T = 0.3$–1.6 K (for some samples 0.4–0.8 K) [19]. The conventional procedure of calculating the effective mass for low r_s values ($\lesssim 5$), based on the assumption that T_D is T independent, is illustrated by the insets of Fig. 3. In this small-r_s range, our results are in good agreement with the earlier data by Smith and Stiles [9], and by Pan *et al.* [11]. The assumption $T_D \neq f(T)$, however, becomes dubious near the MIT, where the resistance varies significantly over the studied temperature range; in this case, the two parameters T_D and m^* become progressively more correlated. The open dots in Fig. 4b were obtained by assuming that T_D is T independent over the whole explored range of n: m^* increases with r_s, and the ratio m^*/m_b becomes ~ 2.5 at $r_s = 8$ ($m_b = 0.19$ is the band mass). As another limiting case, one can attribute the change in $R(T)$ solely to the temperature dependence of the short-range scattering and request T_D to be proportional to $R(T)$. In the latter case, the extracted dependence $m^*(r_s)$ is weaker (the solid dots in Fig. 4b). To reduce the uncertainty of m^* at large r_s, it is necessary to separate the effects of T-dependent scattering and "smearing" of the Fermi distribution in the dependence $R(T)$; an adequate theory is currently unavailable.

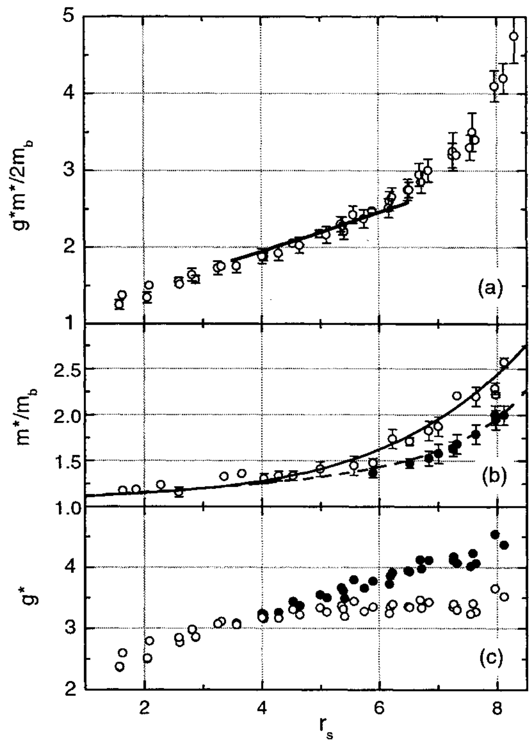

FIG. 4. Parameters g^*m^*, m^*, and g^* for different samples as a function of r_s (dots). The solid line in (a) shows the data by Okamoto *et al.* [10]. The solid and open dots in (b) and (c) correspond to two different methods of finding m^* (see the text). The solid line and the dashed line in (b) are polynomial fits for the two dependences $m^*(r_s)$. The values of g^* shown in (c) were obtained by dividing the g^*m^* data by the smooth approximations of the experimental dependences $m^*(r_s)$ shown in (b).

Our data shows that the combination $(T + T_D)m^*$ is almost the same for electrons in both spin-up and spin-down subbands [e.g., for $n = 3.76 \times 10^{11}$ cm^{-2} and $B_{\parallel} = 2.15$ T ($P = 20\%$), the T_D values for "spin-up" and "spin-down" levels differ by $\leq 3\%$]. This is demonstrated by the observed almost 100% modulation of SdH oscillations (see, e.g., Figs. 2a and 2b). Thus, the carriers in the spin-up and spin-down subbands have nearly the same mobility; this imposes some constraints on theoretical models of electron transport in the 2D metallic state.

In conclusion, using a novel cross-field technique for measuring SdH oscillations, we performed direct measurements of the spin susceptibility, effective mass, and g factor of conducting electrons over a wide range of carrier densities. By studying the temperature dependence of SdH oscillations, we disentangled three unknown parameters (g^*, m^*, and T_D) and obtained their values in the limit of small magnetic fields. We found that both g^*m^* and m^* are almost independent of the temperature over the range 0.3–1 K. At the 2D metal-insulator transition, we observed a finite value of the spin susceptibility, enhanced by a factor of $\sim$4.7 with respect to the high-density limit.

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[1] E. Abrahams, S. V. Kravchenko, and M. P. Sarachik, Rev. Mod. Phys. **73**, 251 (2001).

[2] B. L. Altshuler, D. L. Maslov, and V. M. Pudalov, Physica (Amsterdam) **9E**, 209 (2001).

[3] V. M. Pudalov, G. Brunthaler, A. Prinz, and G. Bauer, cond-mat/0103087.

[4] A. A. Abrikosov, *Fundamentals of the Theory of Metals* (North-Holland, Amsterdam, 1988); A. Isihara, *Electron Liquids* (Springer-Verlag, Berlin, 1997).

[5] N. Iwamoto, Phys. Rev. B **43**, 2174 (1991).

[6] Y. Kwon, D. M. Ceperley, and R. M. Martin, Phys. Rev. B **50**, 1684 (1994).

[7] G.-H. Chen and M. E. Raikh, Phys. Rev. B **60**, 4826 (1999).

[8] F. F. Fang and P. J. Stiles, Phys. Rev. B **174**, 823 (1968).

[9] J. L. Smith and P. J. Stiles, Phys. Rev. Lett. **29**, 102 (1972).

[10] T. Okamoto, K. Hosoya, S. Kawaji, and A. Yagi, Phys. Rev. Lett. **82**, 3875 (1999).

[11] W. Pan, D. C. Tsui, and B. L. Draper, Phys. Rev. B **59**, 10 208 (1999).

[12] A. M. Finkel'stein, Z. Phys. Cond. Matter **56**, 189 (1984); Sov. Sci. Rev. A, Phys. Rev. **14**, 1 (1990); C. Castellani, C. DiCastro, P. A. Lee, M. Ma, S. Sorella, and E. Tabet, Phys. Rev. B **30**, 1596 (1984).

[13] The common technique of measuring g^*m^* in a tilted field is based on detecting the disappearance of the first harmonic of oscillations when $2g^*\mu_B B_{\rm tot} = \hbar e B_{\perp}/(m^* m_e)$. This technique is applicable only up to $r_s = 6.3$ [10].

[14] Samples Si6-14/5, 10, 18 correspond to the same chip Si6-14 cooled from 300 down to 4.2 K, while holding the gate voltage V_g at 5, 10, and 18 V. These "samples" showed different parallel-field MR at low T.

[15] I. M. Lifshitz and A. M. Kosevich, Zh. Eksp. Teor. Fiz. **29**, 730 (1955); A. Isihara and L. Smrčka, J. Phys. C **19**, 6777 (1986).

[16] Yu. A. Bychkov and L. P. Gor'kov, Sov. Phys. JETP **14**, 1132 (1982).

[17] V. M. Pudalov, M. E. Gershenson, and H. Kojima, cond-mat/0110160.

[18] The oscillatory behavior of the g^* factor in high $B_{\perp}$ is related to the exchange interaction between Landau levels (see, e.g., Isihara [4]). This interaction leads to the enhancement of the g^* value averaged over the period of oscillations [for the Si data, see V. M. Pudalov *et al.*, J. Exp. Theor. Phys. **62**, 1079 (1985)]. In low fields and the weak SdH effect regime, the interlevel interaction can be ignored.

[19] Below 0.3 K, we observed the trend of simultaneous saturation of the temperature dependences of ρ_0, $\delta\rho_{xx}$, and the dephasing rate, which is presumably caused by electron overheating.

[20] A. A. Shashkin, S. V. Kravchenko, V. T. Dolgopolov, and T. M. Klapwijk, Phys. Rev. Lett. **87**, 086801 (2001).

[21] S. A. Vitkalov, H. Zheng, K. M. Mertes, M. P. Sarachik, and T. M. Klapwijk, Phys. Rev. Lett. **87**, 086401 (2001).