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"Is the Conductivity of 2D Systems Hopping at $\sigma < e^2/h$?"

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Is the conductivity of 2D systems hopping at $\sigma<e^2/h$?

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If a beast looks like a tiger,
smells as a tiger,
roars as a tiger,
prey on as a tiger
may be it is really tiger?

It is clear that the increase of disorder sooner or later has to lead at low temperature to the change of the conductivity mechanism from the diffusive conductivity to the hopping one. **The question is: when does it happen in 2D?**

It is commonly accepted that the transition to the hopping conductivity occurs when the conductivity becomes lower than e^2/h and the strong enough temperature dependence arises. One of the reason for such conclusion is the well known Landauer formula which shows that the conductivity of one open channel is e^2/h. For the first sight the 2D network of the open 1D channels will have the conductivity e^2/h also. This is true without taking into account interference. The interference of the waves propagated over the different channels has to lead to decreasing of the conductivity value. Therefore it seems that the value of conductivity e^2/h is not lower limit for the diffusive conductivity.

Usually, the conclusion on the conductivity mechanism comes from an analysis of the temperature dependence of the conductivity. When $\sigma<e^2/h$, the experimental dependences ρ (T) can be well described by the expression

$$\rho(T)=\rho_0(n,T)\exp(T_0/T)^p \qquad \text{with} \quad 0.3<p<0.8.$$

Such value of power p leads to the conclusion that the variable range hopping regime takes place. However some unusual features are observed in 2D, compared with the well-studied 3D hopping conductivity:

2D	3D
The power p remains less than unity up to high temperatures so that the transition to nearest neighbor hopping regime or to ε_1 regime does not observed.	The power p changes at increasing T from the value less than unity for variable range hoping to p=1 for nearest neighbor hopping regime or ε_1 regime.
The extrapolation of the resistivity to $T\rightarrow\infty$ gives the value of ρ_0 close to h/e^2 for different carriers density and different structures.	In hopping regime $\rho(n, \infty)$ is determined by overlapping of the wave functions of the localized states and therefore exponentially depends on density of localized states.
The Hall coefficient practically does not depend on the temperature down to σ = (0.1-0.01) e^2/h and gives true density of carriers.	In hopping regime, the Hall effect either can not be measured or has strong temperature dependence and does not give true carrier density.
The negative magnetoresistance is observed within wide conductivity range down to σ = (0.1-0.01) e^2/h. The shape of the magnetoresistance at low conductivity is very close to that for high conductivity values σ=(10-100) e^2/h.	The negative magnetoresistance is observed within narrow electron density range after merger of the impurity and conduction bands.

We believe there is another way to understand not only strong temperature dependence of the conductivity at $k_F l$ close to unity but the magnetic field and temperature dependences of the Hall effect and magnetoresistance without the contradictions mentioned above.
This way is consideration of the diffusive conductivity taking into account the quantum correction.

Really, the quantum corrections to the conductivity in the main determine the temperature and magnetic field dependences of the both diagonal and off-diagonal conductivity tensor components of 2D systems at $k_F l >> 1$. These corrections are negative and increase when the temperature decreases.

In principle, at low enough temperature these corrections can be comparable with the Drude conductivity and thus lead to strong temperature dependence of the conductivity.
Indeed, practically all theoretical results for the quantum corrections were obtained for large $k_F l$ values and small value of the quantum corrections, however these mechanisms remain in force at $k_F l$ close to unity also.

The conventional theories of quantum corrections give numerous predictions:
Low magnetic field (weak localization correction)
1. The specific shape of the negative magnetoresistance at low magnetic field which results from suppression of the interference.
2. The absolute value of the phase breaking time which can be found from the negative magnetoresistance.
3. The conductivity and temperature dependences of the phase breaking time.
High enough magnetic field (e-e inetraction correcton)
4. Logarithmic temperature dependence of σ_{xx}.
5. An absence of the temperature dependence of σ_{xy}.
6. An absence of the magnetic field dependence of $\Delta\sigma_{xx}$ while $g\mu_B B<kT$.
Zero magnetic field
7. The logarithmic temperature dependence of $\sigma(0)$.
8. The slope of this dependence is determined by sum of the interference correction and correction due to e-e interaction which can be found separately.

From our point of view, to get the reliable conclusion on the conductivity mechanism it is not enough to analyze the dependence $\sigma(T)$ alone. It is essentially to analyze at least the temperature and magnetic field dependencies of the conductivity and Hall effect at low and high magnetic field.

We have made such measurements over wide conductivity range for 2D structures with simplest, well known energy spectrum and carried out consequent analysis starting from the well understandable from theoretical point of view case $k_F l >> 1$.

Samples
Three types of single well GaAs/InGaAs/GaAs heterostructures were studied. They were the gated structures with doped quantum well, with doped barriers, and undoped structures which conductivity was changed by illumination. The measurements of the ohmic conductivity and Hall effect were carried out within the temperature range 0.4-10 K and magnetic fields up to 6 T. Nonohmic conductivity was analyzed within the temperature range 0.4 – 4 K.

Results
I. The electron-electron interaction contribution to the conductivity was obtained from analysis of the temperature and magnetic field dependences of the conductivity tensor components σ_{xx}, σ_{xy}. The conventional theory predicts that e-e interaction contributes to σ_{xx} only. When it is the case, one can find the e-e correction from the temperature dependence of σ_{xx}

$$\Delta\sigma_{ee} = K_{ee}\ln(k_B T\hbar/\tau_p)$$

Such analysis was carried out for different types of the structures over the wide $k_F l$ range. Fig.1 shows the $k_F l$ dependence of the e-e interaction correction for one of the structure investigated.

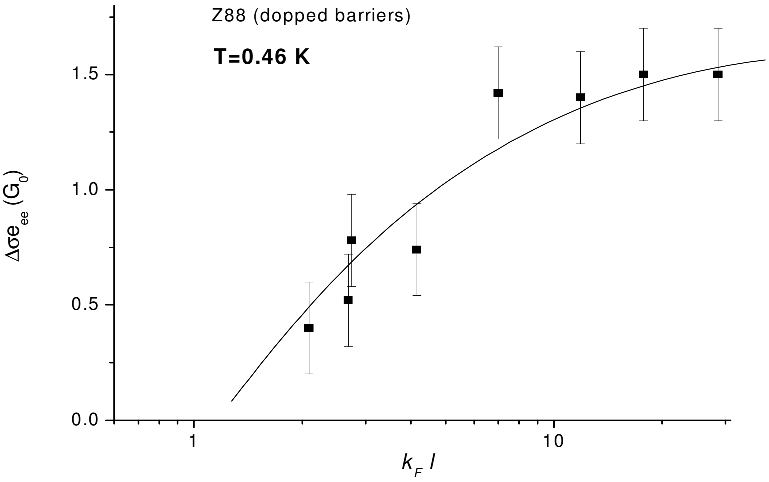

Fig.1 The $k_F l$ dependence of the interaction correction.

So, the correction to the conductivity due to e-e interaction decreases with lowering $k_F l$.
(These results were published in PRB **67**, 205307 (2003))

II. The interference correction to the conductivity was determined from the detailed studies of the negative magnetoresistance and temperature dependences of the conductivity at zero magnetic field. The theory [I. L. Aleiner, et al Waves Random Media **9** 201 (1999)] taking into account the second order terms in $1/\sigma$ was used for data treatment. These terms do not change the temperature dependences of the conductivity at zero magnetic field and shape of the magnetoresistance curve but vary the value of magnetoresistance (i.e. lead to decreasing prefactor in $\sigma(B)$ expression).

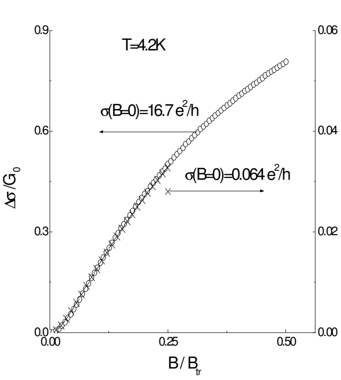

Fig.2 The shape of the negative magnetoresistance at different σ.

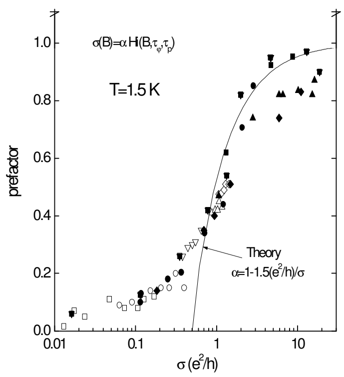

Fig.3 The conductivity dependence of the prefactor α.

Fig.3 shows that at σ=16.7 e^2/h ($k_F l$=18) and σ=0.06 e^2/h ($k_F l$ is about 1-1.2) the shape of the negative magnetoresistance curves is very similar and is well described by the Hikami-Larkin-Nagaoka (HLN) expression. The theory [I. L. Aleiner, et al] describes the value and shape of the negative magnetoresistance quantitatively down to σ=0.6 e^2/h ($k_F l$ is about 2) and qualitatively at lower conductivity.

III. The value and the conductivity dependence of the phase breaking time τ_φ found from the negative magnetoresistance is well described by theoretical expression down to very low conductivity value as seen from Fig.4.

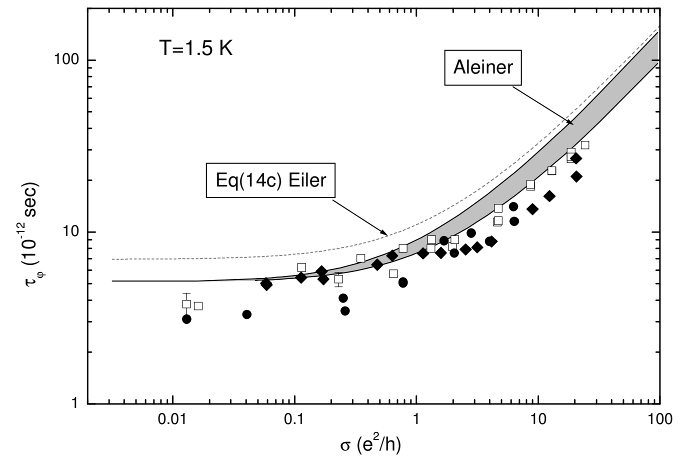

Fig.4 The conductivity dependence of the phase breaking time.

The interference correction to the conductivity is three – five times larger than the correction due to the e-e interaction within whole $k_F l$ range.
Fig.5 shows the $k_F l$ dependence of both quantum corrections at T=0.46 K.
At $k_F l$ =2 the value of the interference correction is not small and is about 70% of the Drude conductivity.

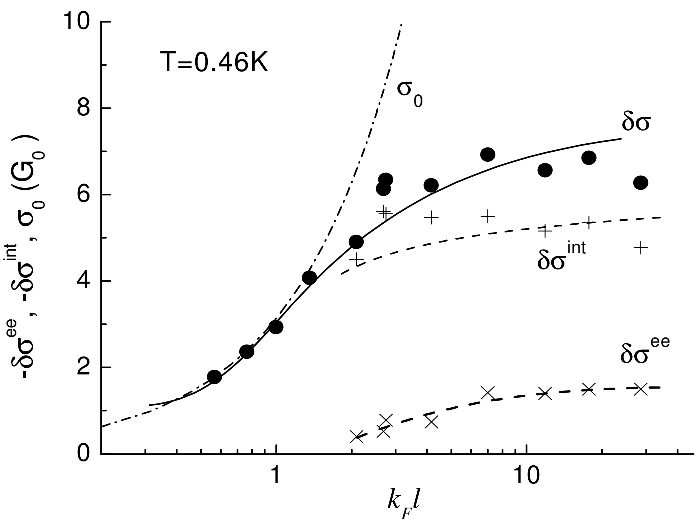

Fig.5 The kFl dependence of the conductivity, interferece and interaction corrections.

IV. The temperature dependences of the conductivity at B=0 are well described by self-consistent theory of the "weak localization" [D. Vollhardt and P.Woelfle, Phys. Rev. B **22**, 4666 (1980)] down to σ=0.01e^2/h. This theory predicts that σ(T) is solution of the equation

$$\sigma(T)+\ln(\sigma(T))=\sigma_{Dr}+\ln(\sigma_{Dr})-\ln(\tau_\varphi/\tau_p)$$

(in this equation σ is measured in G_0).
When τ_φ is power function of T, the *(σ(T)+ln(σ(T))*-versus-*ln(T)* dependence has to be linear.
Fig.6 demonstrates dependences *σ(T)* plotted in coordinates which correspond to: **a** – diffusive conductivity when the value of quantum corrections is small compared with the Drude conductivity; **b** - diffusive conductivity when the value of quantum corrections is comparable in magnitude with the Drude conductivity.

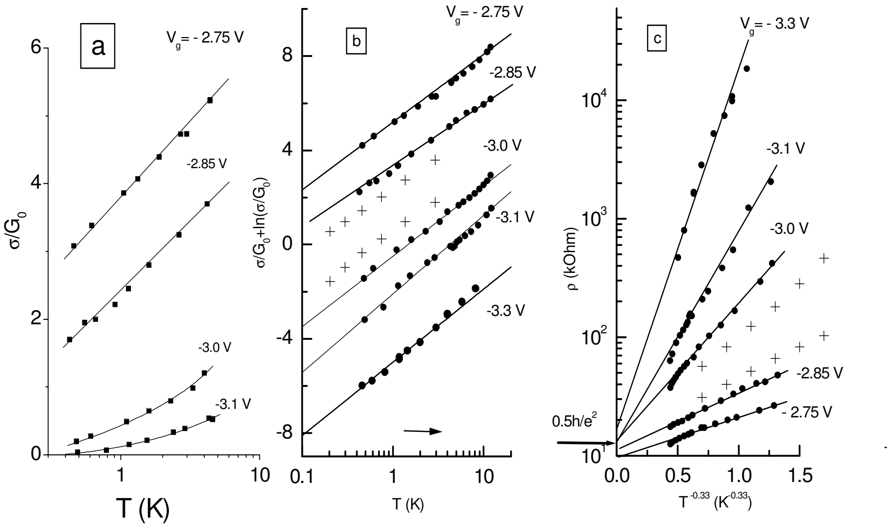

Fig.6 The temperature dependencies of the conductivity. The ponts are our data, the crosses are the data from S.I.Khondaker et al, PRB **59**, 4580 (1999)

Good agreement is evident down to conductivity $0.01e^2/h$.

Thus all the predictions of the conventional theories for diffusive conductivity with the quantum corrections are fulfilled quantitatively down to conductivity about $0.1e^2/h$ and qualitatively down to conductivity about $0.01e^2/h$.

Note, that as shows Fig. 6b,c, the data at $\sigma<e^2/h$ can be described equally well within the framework of self-consistent theory of weak localization and variable range hopping conductivity.

So, the temperature dependence of the conductivity alone does not allow us to specify the conductivity mechanism.

An additional information on the conductivity mechanism can be obtained from the study of the nonohmic (nonlinear) conductivity. Really, at the diffusive conductivity the nonlinearity results from electrons heating, while for the hopping conductivity additional mechanisms of non-linearity occur, namely, increasing of the hopping probability in electric field and impact ionization of localized states.

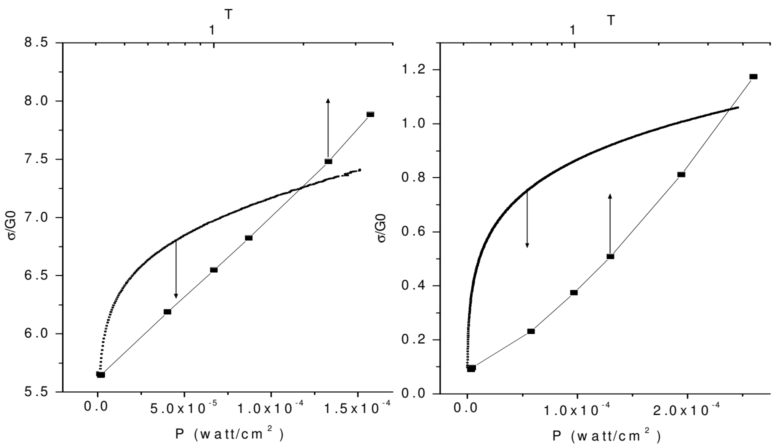

Fig.7 The temperature and power dependencies of the conductivity for two conductivity values.

In Fig.7 we have plotted the temperature and power dependences of the conductivity for conductivity values higher and lower than e^2/h. To compare the nonlinearity over wide conductivity range we have introduced it as $F(P,T_L) \equiv (\Delta\sigma_P/P)/(\Delta\sigma_T/\Delta T)$ at $\Delta\sigma_P=\Delta\sigma_T$, where $\Delta\sigma_P$ is conductivity variation under imparted power P and $\Delta\sigma_T$ is the conductivity variation with increasing temperature by ΔT. For diffusive conductivity there is a simple relation between F(P,T) and energy relaxation rate P: $P=F(P,T)^{-1}(T_e-T_L)$.

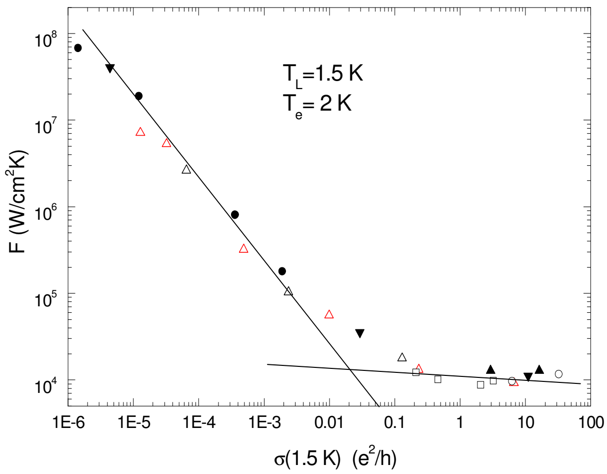

Fig.8 The conductivity dependemce of nonlinearity at T_L=1.5 K.

Theory for energy relaxation rate in the diffusive regime predicts only slight dependence of nonlinearity F on the conductivity. The conductivity dependence of nonlinearity plotted in Fig.8 shows that it is the case down to conductivity $\sigma(1.5K)\sim(0.1\text{-}0.01)\ e^2/h$ and only at lower conductivity values F strongly increases.

Thus, the conductivity dependence of the nonlinearity shows that down to the conductivity (0.1-0.01) e^2/h the conductivity mechanism remains unchanged.
(In more detail the results on the conductivity nonlinearity is presented in Poster.)

Thus, we have traced the value of quantum correction to the conductivity at decreasing k_Fl.
We show that at $k_Fl \approx 1\text{-}1.5$ the value of interference correction at low temperature becomes comparable with the Drude conductivity and, as sequence, the value of low temperature conductivity becomes significantly lower than e^2/h .
Down to this k_Fl value the temperature and magnetic field dependences at low and high magnetic field, the nonohmic conductivity look like ones for the diffusive conductivity,
namely:
the shape of the negative magnetoresistance;
the value of the negative magnetoresistance;
the value of the phase breaking time found from the negative magnetoresistance;
the conductivity dependence of the phase breaking time;
the temperature dependence of the conductivity at zero magnetic field;
the value and temperature dependence of the nonlinearity of the conductivity.

We conclude
If a beast looks like a tiger,
smells as a tiger,
roars as a tiger,
prey on as a tiger
most probably it is tiger.

We believe the crossover to the hopping conductivity occurs at $k_Fl < 1$ and at low temperature conductivity less than $10^{-3}\ e^2/h$.