Stellar dynamics for LISA: Stars captured by a massive black hole

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SFB 439: "Galaxies in young universe"
A5: "Formation and growth of black holes in galactic nuclei"

LISA sources: Compact stars orbiting MBHs

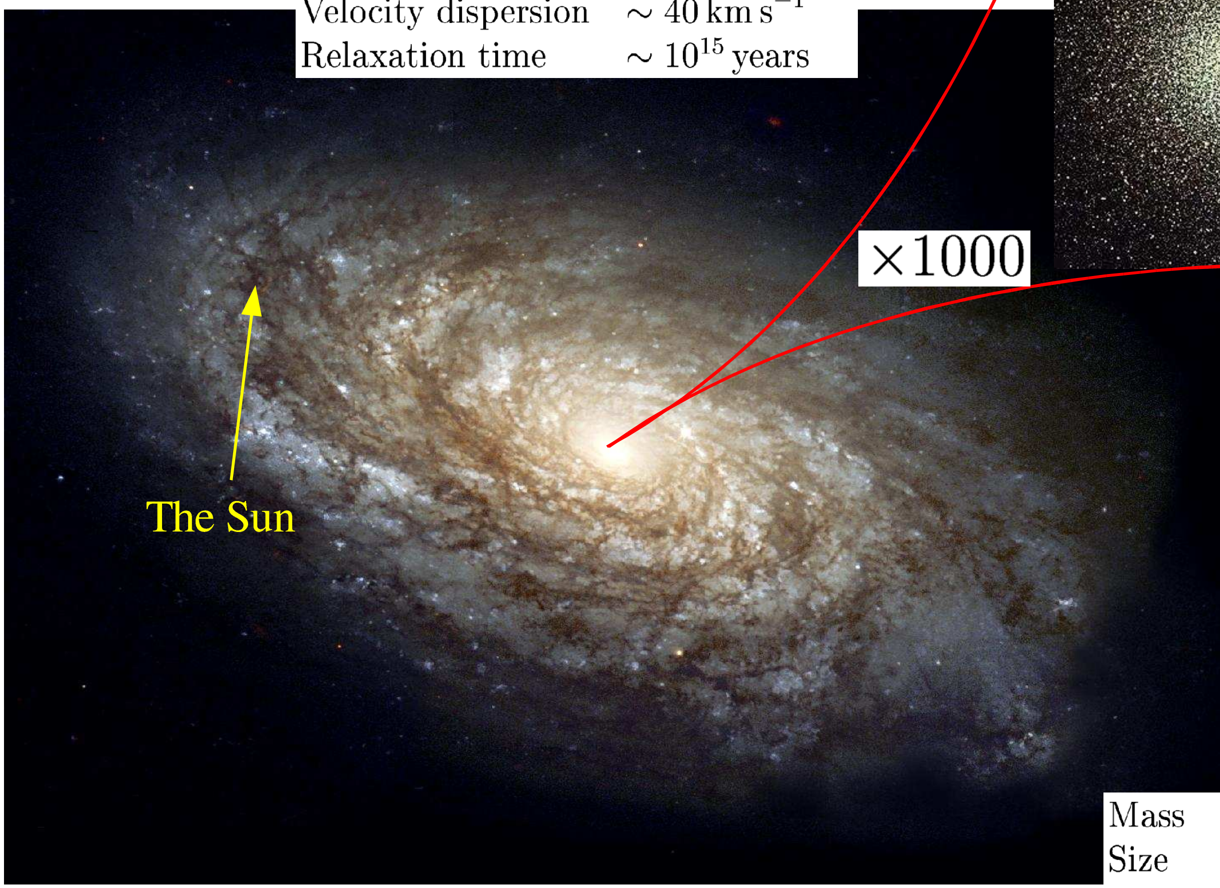
- Dark compact objects are present at the centre of galaxies
- $1 - 10^4\,M_\odot(?)$ spiralling into $10^5 - 10^7\,M_\odot$ MBH
  - Only compact stars; MS stars disrupted early
  - Stellar BH detectable to few 100s Mpc, IMBHs to few Gpc!!
  - Only 3 MBH known in this range!
- "Geo"desic map of central object space-time ($\Rightarrow M$ & a if MBH)
- Theoretical difficulties:
  - **Rate of captures?** (literature: from 10^{-8} to $10^{-4}\,\mathrm{yr}^{-1}$ per galaxy)
    Hils & Bender 95; Sigurdsson & Rees 97; Freitag 01, 03; Ivanov 02; Sigurdsson 03 (review)

    Stellar dynamical problem Stars must be sent onto $e \simeq 1$ orbits through 2-body relaxation (and other processes...
  - Orbital evolution / waveforms computations (but extreme mass-ratio allows perturbative computations; full GR not required)
    Glampedakis & Kennefick 02; Glampedakis, Hughes & Kennefick 02
  - **Data analysis** (Complicated waveforms)

Setting the stage. . .

Galaxy

Size	$\sim 10^4$ pc
Density	$\sim 0.05\,\mathrm{M}_\odot \mathrm{pc}^{-3}$
Velocity dispersion	$\sim 40\,\mathrm{km\,s}^{-1}$
Relaxation time	$\sim 10^{15}$ years



Galactic nucleus

Size	$\sim 1 - 10$ pc
Density	$\sim 10^7\,\mathrm{M}_\odot \mathrm{pc}^{-3}$
Velocity dispersion	$\sim 100 - 1000\,\mathrm{km\,s}^{-1}$
Relaxation time	$\sim 10^{8-9}$ years

$\times 10^7$

Central Black Hole

Mass	$10^{6-9}\mathrm{M}_\odot$
Size	$R_\mathrm{S} = 2GM_\mathrm{BH}/c^2 = 10^{-7} - 10^{-4}$ pc
Rotation	??

Galactic nucleus simulation

- Goal: simulate a 10^{7-9}-star cluster over 10^{9-10} yrs
- Rich physics (✔: incl. in the code, ✔: to be added, ✘: impossible(?))
  - Self-gravity ✔
  - 2–body relaxation ✔
  - Stellar mass spectrum ✔
  - Anisotropic $\vec{V}$-distribution ✔
  - BH growth ✔
  - Collisions ✔
  - Tidal disruptions ✔
  - Stellar evolution (Giants ✔, remnants ✔)
  - $\star$-captures through emission of grav. radiation ✔
  - Inter. with outer galaxy
  - Others (Binaries ?, Rotation ✘, BH wandering ?, Inter. with disk ?, SF ?...)
- Need for a "fast" algorithm (very large parameter space)
  - Direct N-body too slow $T_{\rm CPU} \propto N^{2-3}$
  - Monte Carlo method: $T_{\rm CPU} \propto N \ln(N)$

    10^{10} yrs evol. with 10^6 part: few CPU-hours/days on PC
  - Continuous methods (direct FP, gas) faster but cannot handle "special events" (collisions, captures...)

MONTE CARLO Code Internals

(Freitag & Benz 2001, A&A 375, 711; 2002, A&A 395, 345)

- Basics
  - Based on Hénon's scheme: Monte Carlo resolution of FP equation
  - Main assumptions: **Spherical symmetry** & **Dynamical equil.**
  - Cluster $\equiv$ **set of particles** with $N_{\rm part} \leq N_*$
    1 particle $\equiv$ **spherical shell** of stars with same E, J, $M_\star$
    Shells create smooth, steady potential
  - R–variable **time steps**: $\delta t(R)^{-1} \propto \left(T_{\rm relax}^{-1} + T_{\rm coll}^{-1}\right)$
  - Particles evolved in pairs; **selection probability**: $P_{\rm selec} \propto \delta t^{-1}$
  - **Orbit sampling**: random position with $\mathrm{d}P/\mathrm{d}R \propto V_{\rm rad}(R)^{-1}$
- Physics
  - **Diffusive 2-body relaxation**: "super-encounter" between neighb. particles with $\theta_{\rm SE} = \sqrt{\langle\theta^2\rangle_{\delta t}} = \sqrt{\delta t/T_{\rm rel}^{(1,2)}}$
  - **Loss cone**: random walk of $\vec{V}$ tip, $N_{\rm steps} = P_{\rm orb}/\delta t$
  - **Collisions**: detection through $P_{\rm coll} = \delta t/T_{\rm coll}^{(1,2)}$. Outcome from 4-D grid of SPH simulations.

Captures by emission of grav. radiation

- Inclusion in MC code: Rough and ready!
  - No-return capture of star if $T_{\rm GW}(e,a) < T_{\rm relax}^{\rm (peri)}$ $(\ll T_{\rm relax})$

$$\begin{aligned} T_{\rm GW} &\simeq \frac{2^{1/2}24}{85}\frac{c^5}{G^3 M_{\rm BH}^2 M_*}(1-e)^{7/2}a^4 \\ &\simeq 3.2\times 10^6\,{\rm yrs}\cdot\left(\frac{M_{\rm BH}}{10^6\,M_\odot}\right)^2\left(\frac{M_*}{1\,M_\odot}\right)^{-1}\left(\frac{R_{\rm p}}{10\,R_{\rm S}}\right)^4\left(\frac{1-e}{10^{-5}}\right)^{-1/2}\end{aligned}$$

  - Instantaneous swallowing

- Monte Carlo simulation provides list of capture events
  Stellar type, M_*, e_0, a_0

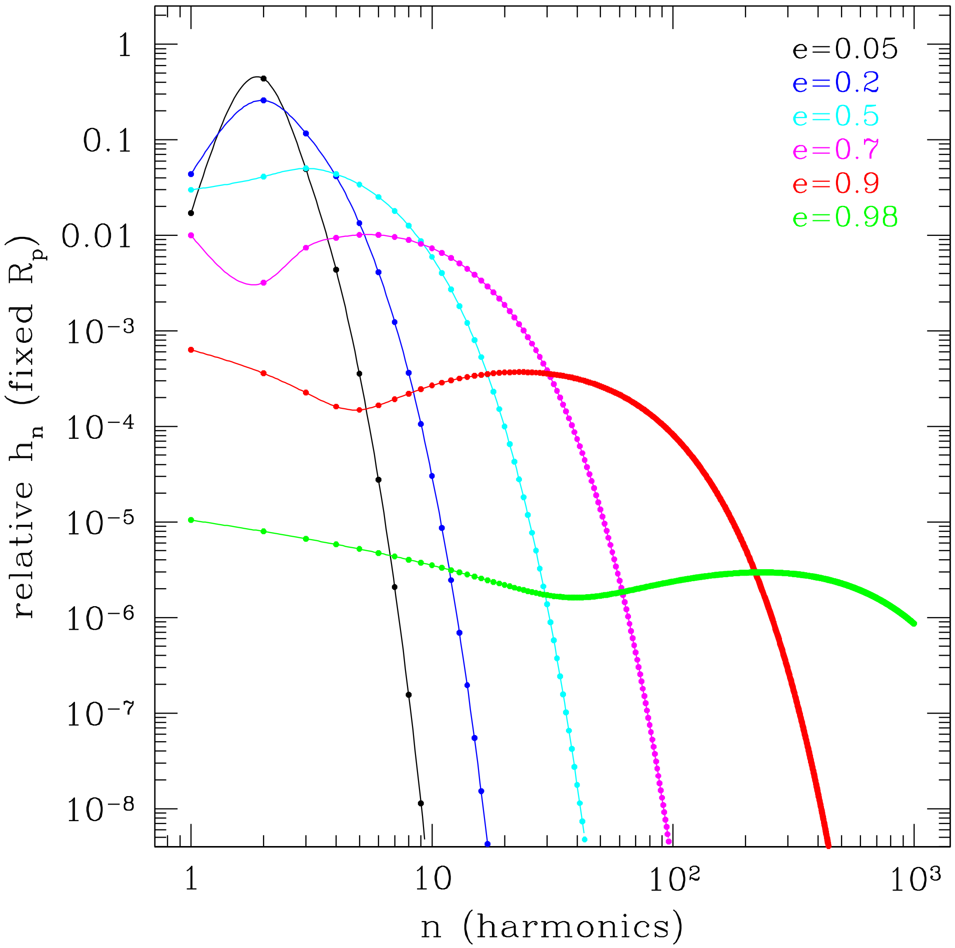
- Orb. evol. for each event computed "off-line" (Glampedakis et al 02)

- Radiation emitted at frequencies $\omega_n = n\sqrt{GM_{\rm BH}a^{-3}},\ n\geq 1$
  with amplitudes

$$h_n = \gamma(n,e)\frac{1}{D}\frac{G^2 M_{\rm BH} M_*}{c^4 a} \simeq 2\times 10^{-27}\gamma\left(\frac{D}{1\,{\rm Mpc}}\right)^{-1}\left(\frac{a}{1\,{\rm pc}}\right)^{-1}\left(\frac{M_{\rm BH}}{10^6\,M_\odot}\right)\left(\frac{M_*}{1\,M_\odot}\right)$$

Amplitude of GW harmonics

Keplerian approximation, quadrupolar component



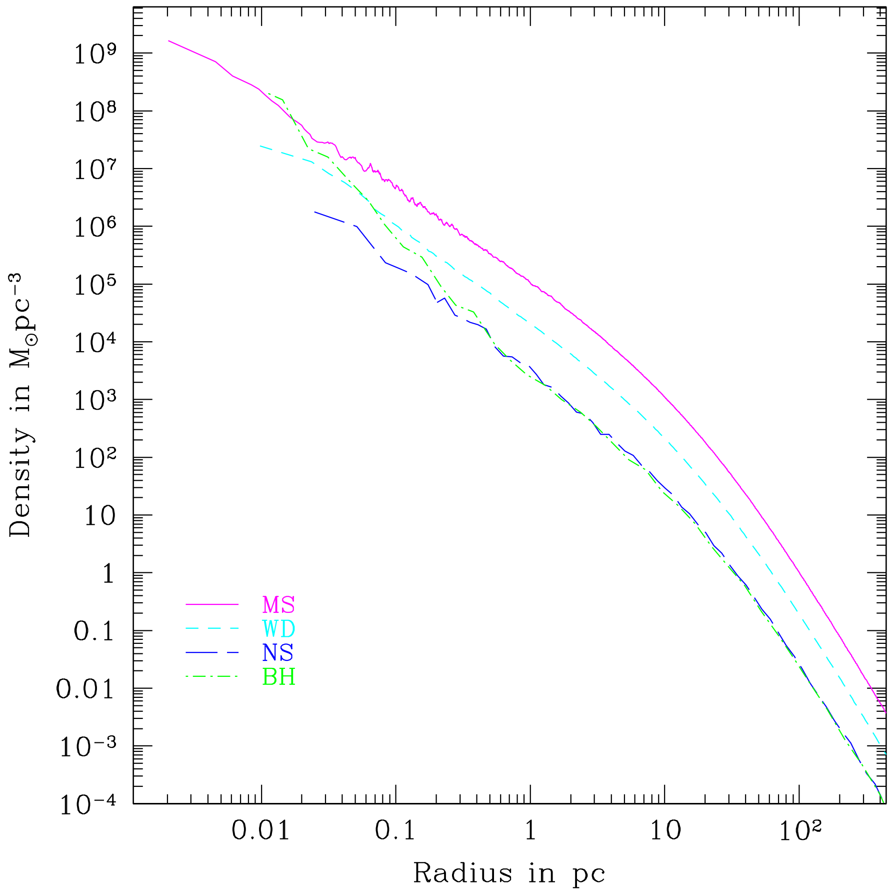
$$h_n = \sqrt{\frac{2}{5}} \frac{G^2 M_{\rm BH} M_*}{c^4 a} \frac{\sqrt{g(n,e)}}{n} \frac{1}{D} \propto \frac{G^2 M_{\rm BH} M_*}{c^4 R_{\rm p}} h(n,e) \frac{1}{D}$$

Cf. Peters & Mathews 63

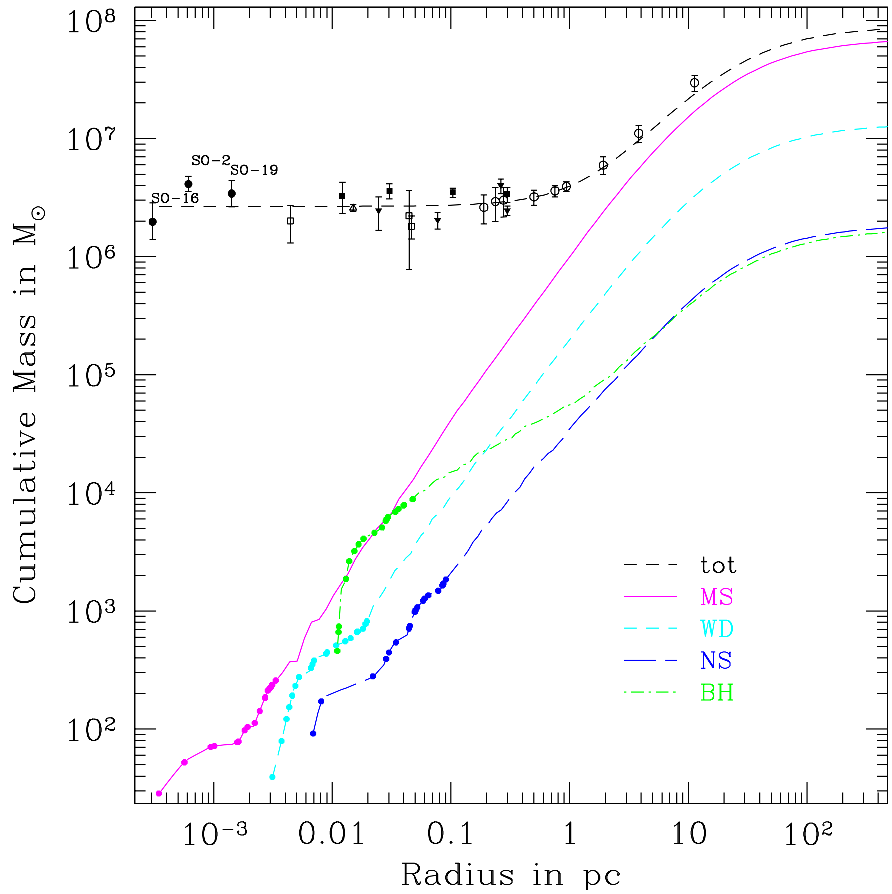
Cluster evolution of Sgr A* model

After 1.32 Gyrs

Density of stars



Enclosed mass

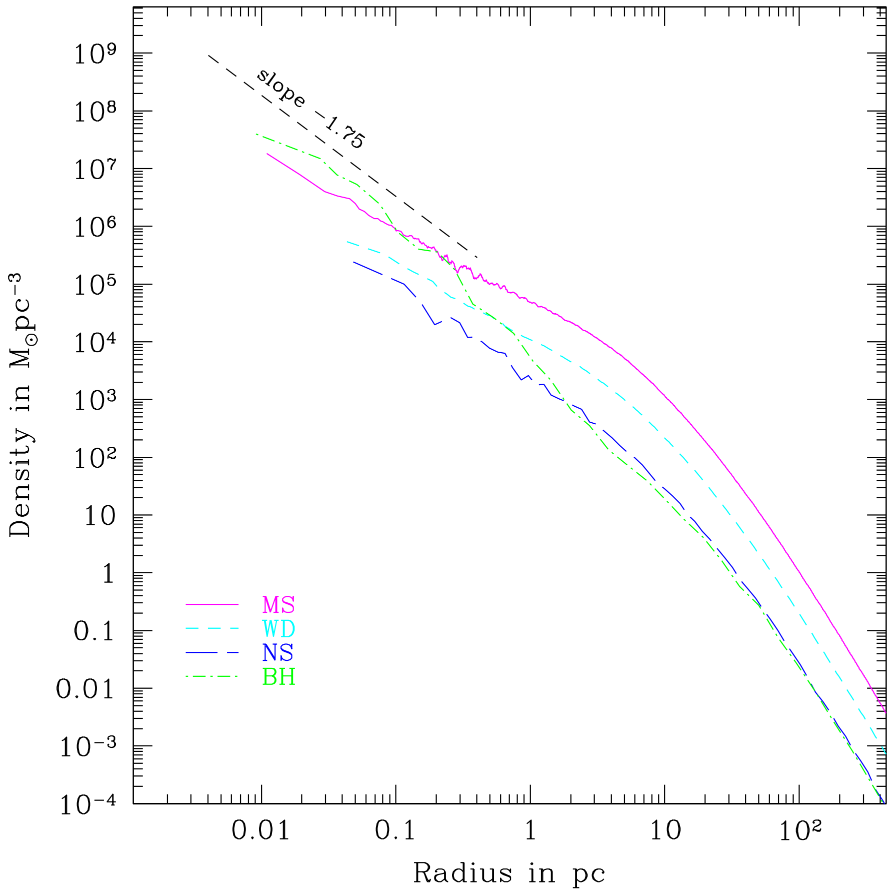


Mass segregation of stellar BHs already important!

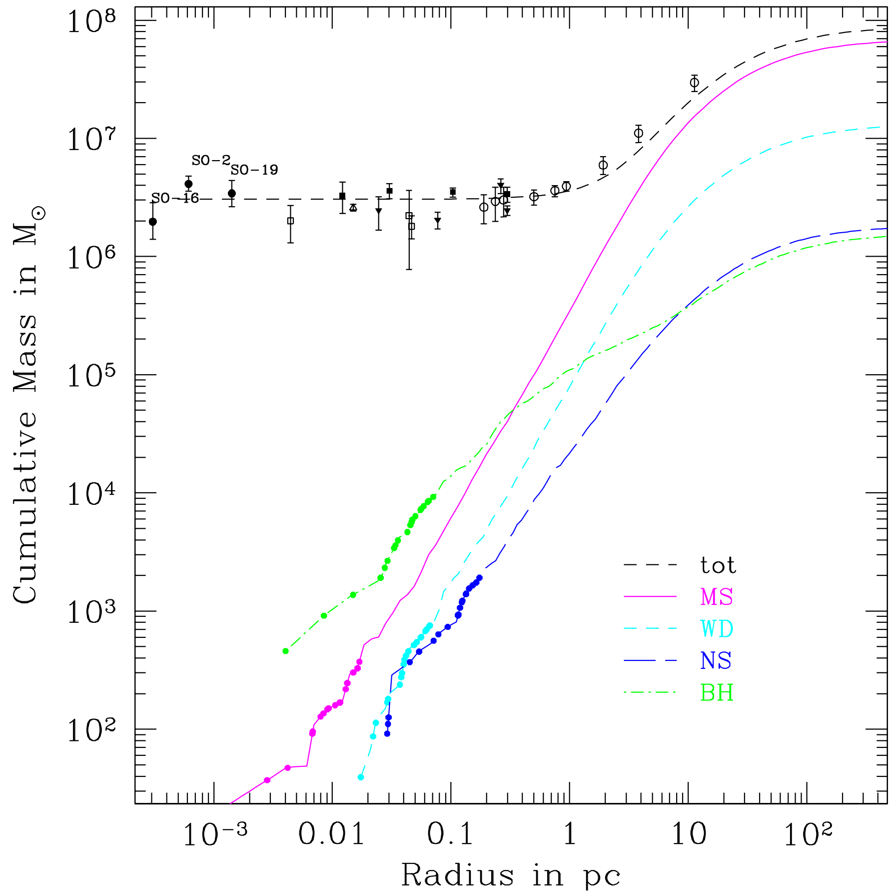
Cluster evolution of Sgr A* model

After 18.2 Gyrs

Density of stars

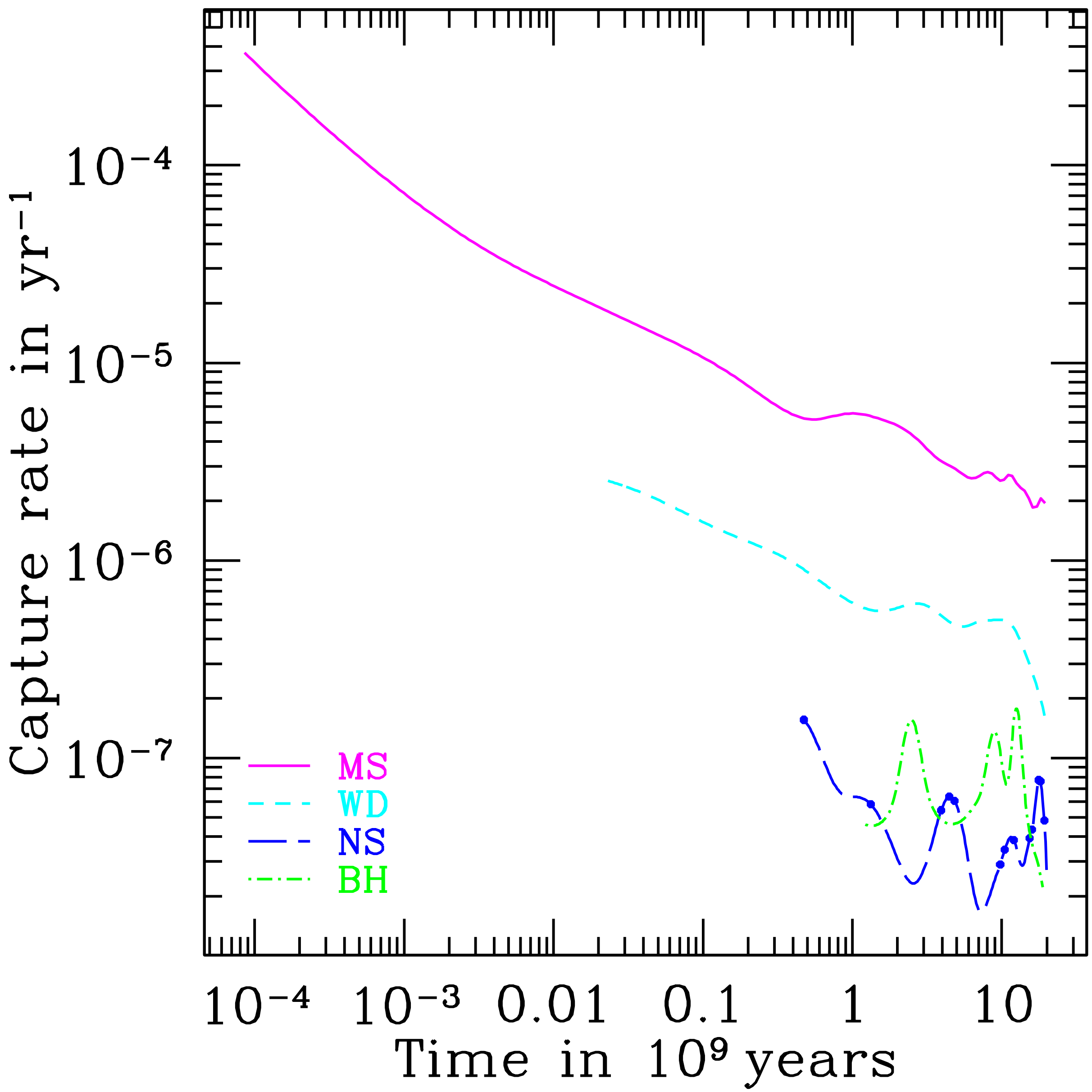


Enclosed mass

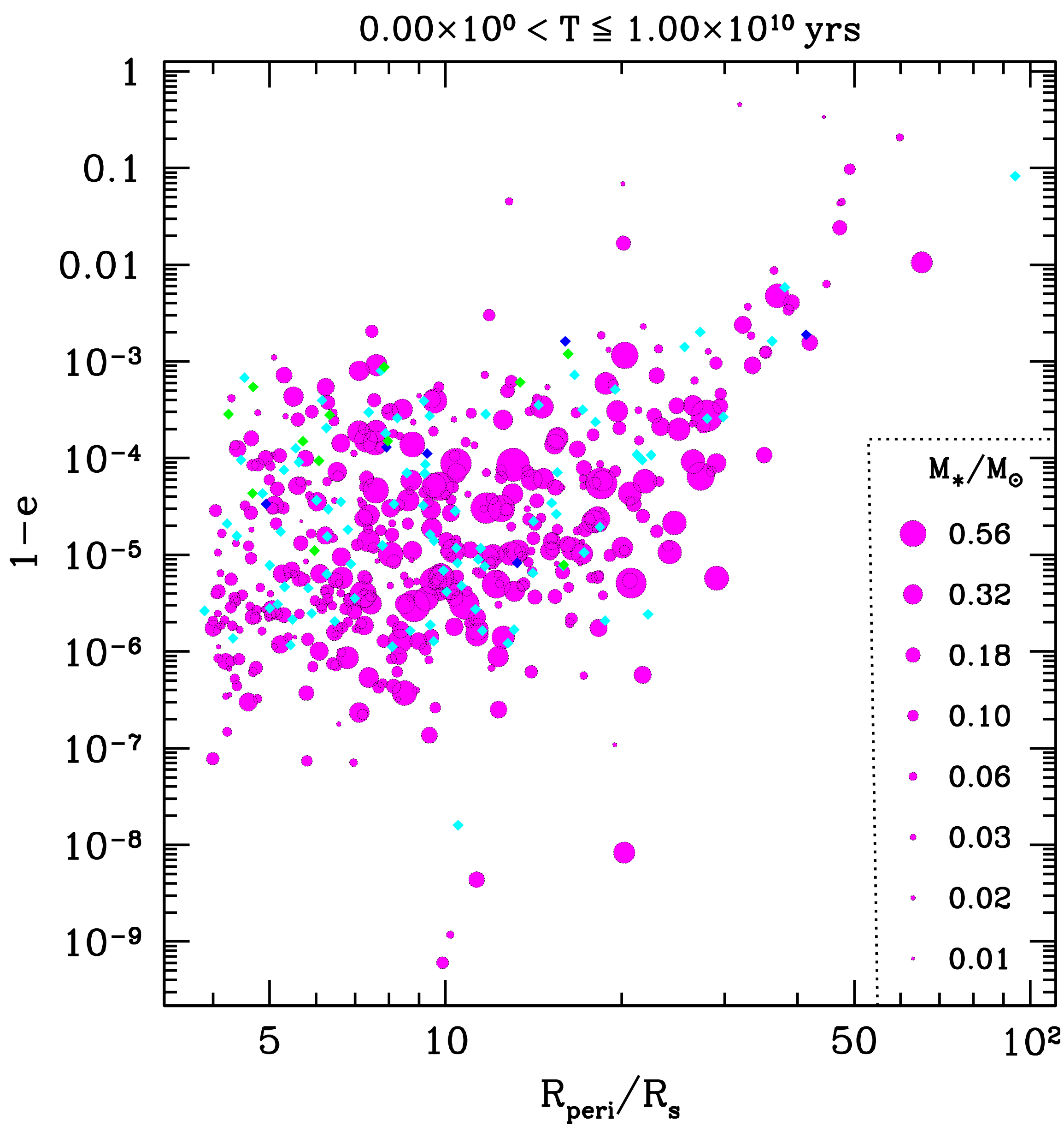


Stellar BHs dominate central region

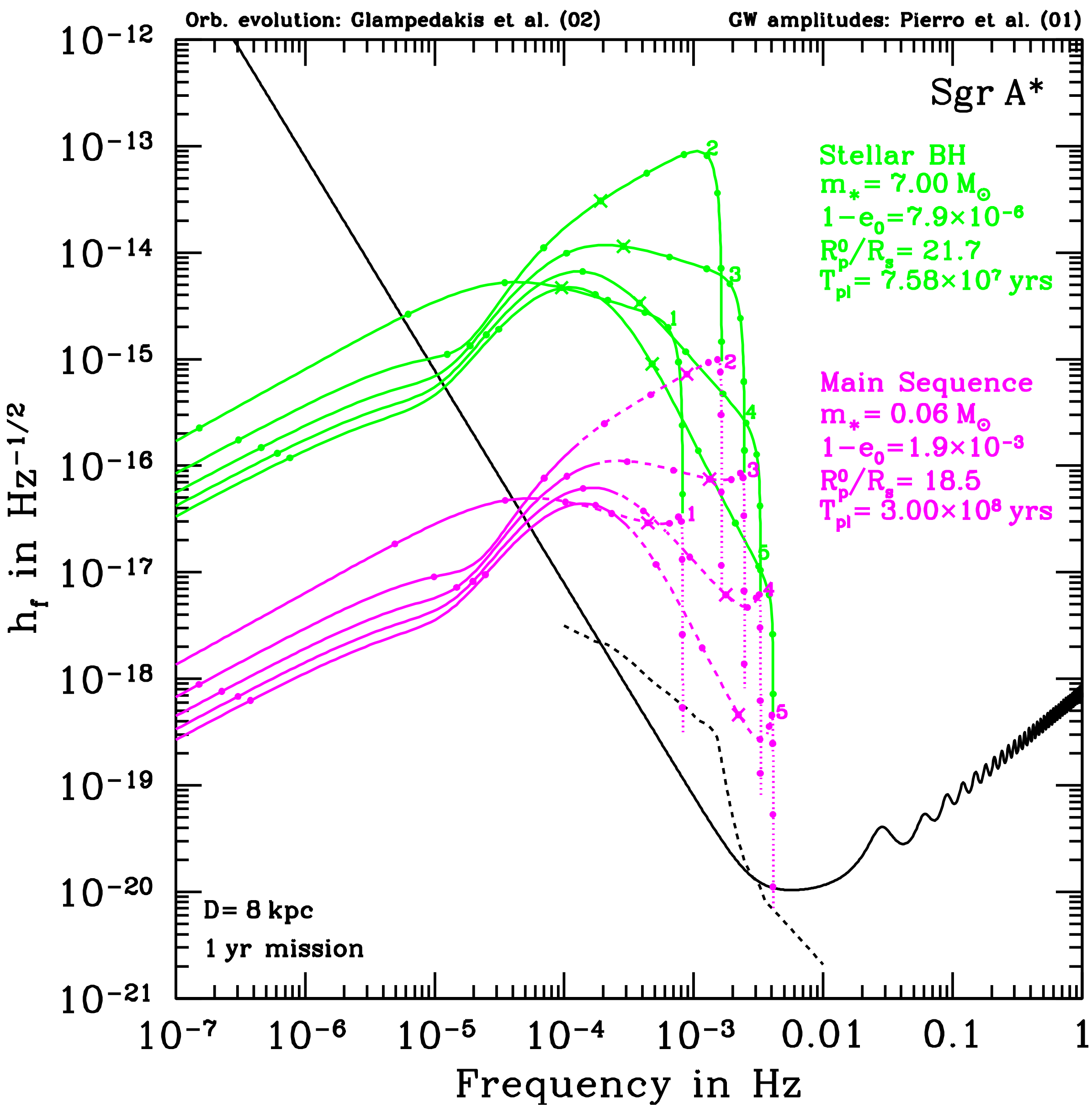
Capture rates in Sgr A* model



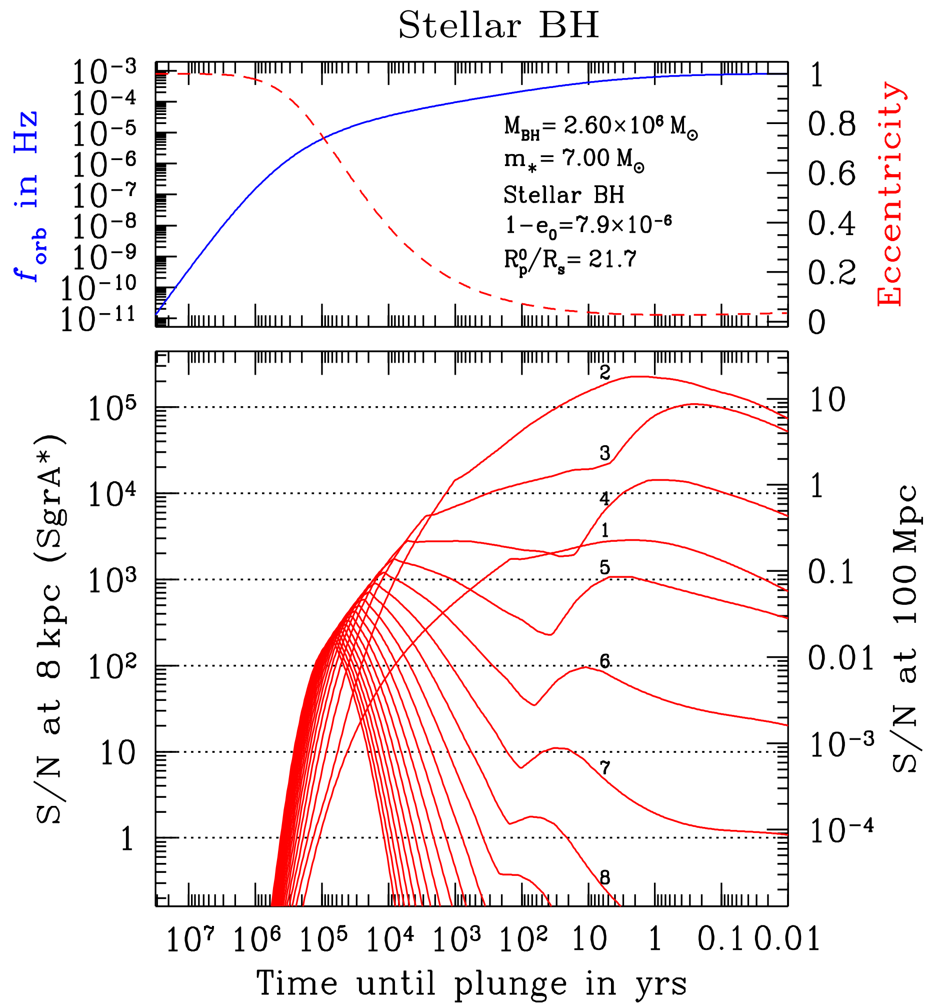
Capture parameters in Sgr A* model

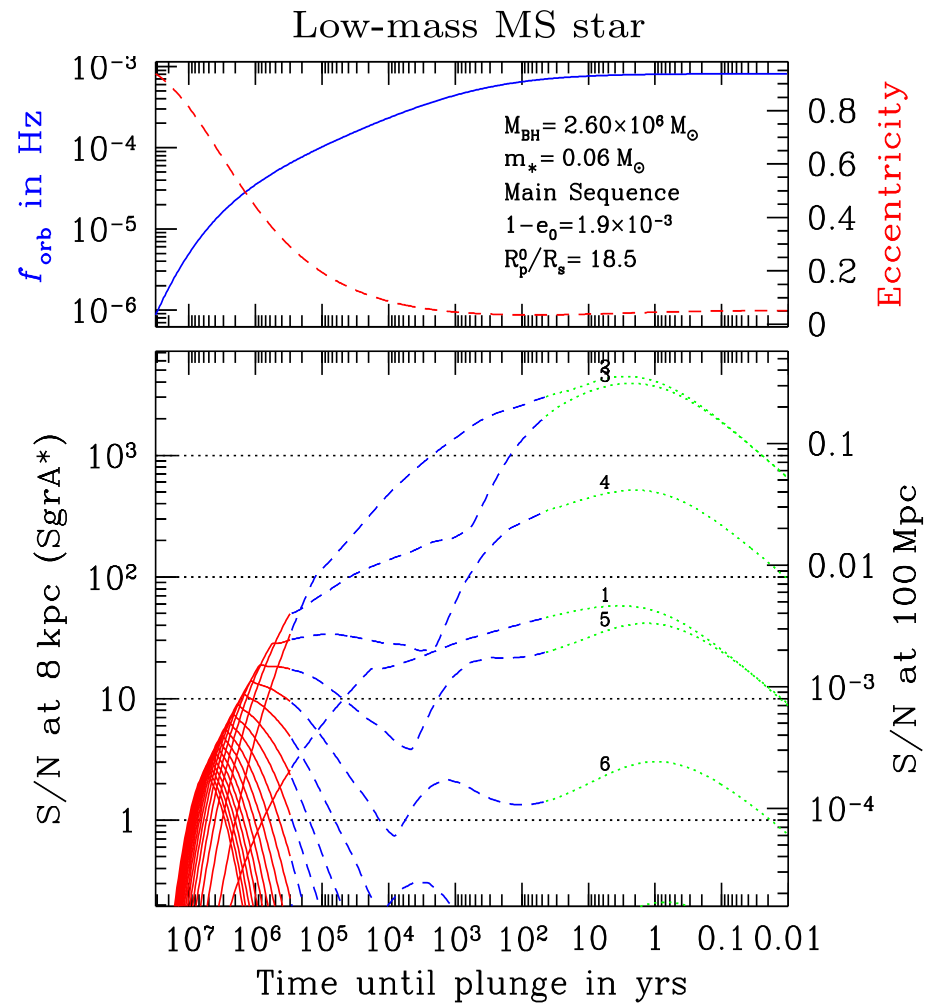


Typical capture events



Typical capture events





$E_{\rm tid} < 0.2E_\star$

$E_{\rm tid} > 0.2E_\star$

$R_{\rm peri} < R_{\rm disrup} \simeq R_*(M_{\rm BH}/M_*)^{1/3}$

Detectable during few 10^5 yrs at Sgr A*

Detectable during $\sim 10\,$yrs at 100 Mpc

Detectable during few 10^6 yrs at Sgr A*

Not detectable at 100 Mpc

Captures: Results of simulations

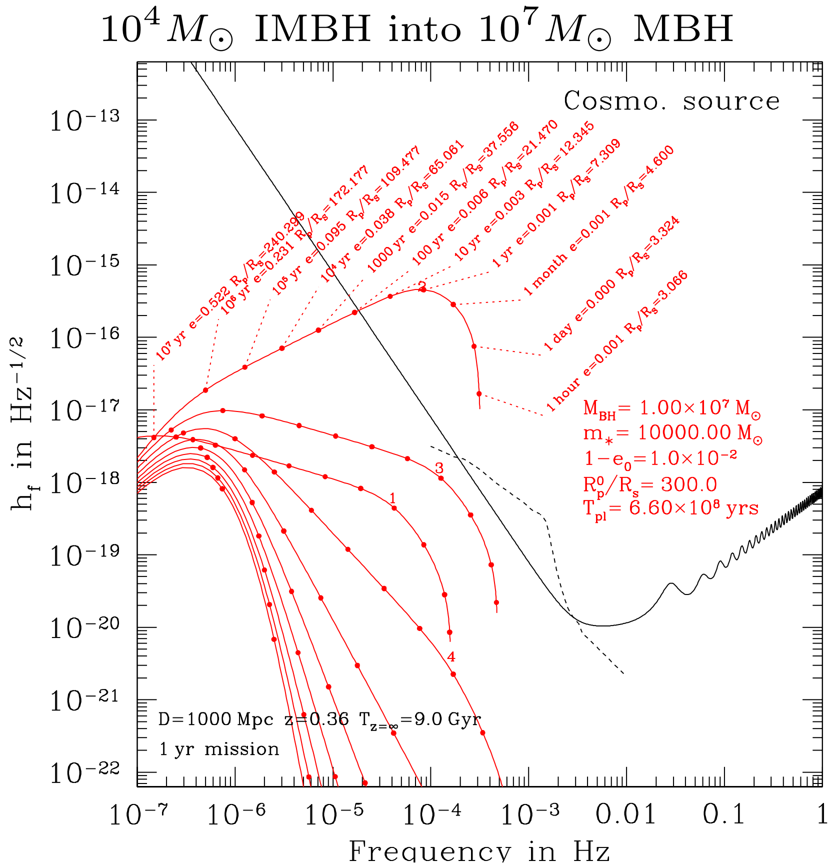
- K models (Freitag 2001, Class. Quant. Grav., 18, 4033)
  Evolved population. Seed BH, $0.5 - 2 \times 10^6$ part.
  - $\dot{N}_{\rm capt} \approx 10^{-7,6,5}\,{\rm yr}^{-1}$ ($*$-BHs) for $N_* = 3.6 \times 10^{7,8,9}$ at $t \simeq 10^{10}$ yrs
    Whether or not *LISA*-detectable
  - $\sim$80, 60, 40 % of stellar BHs swallowed (depends on $T_{\rm relax}$)
- SgrA* models (Freitag 2003, ApJ 583, L21 astro-ph/0211209, astro-ph/0306064)
  Evolved population. $2.6 \times 10^6 M_\odot$ BH. No initial M_* segr. (!), $2 - 6 \times 10^6$ part.

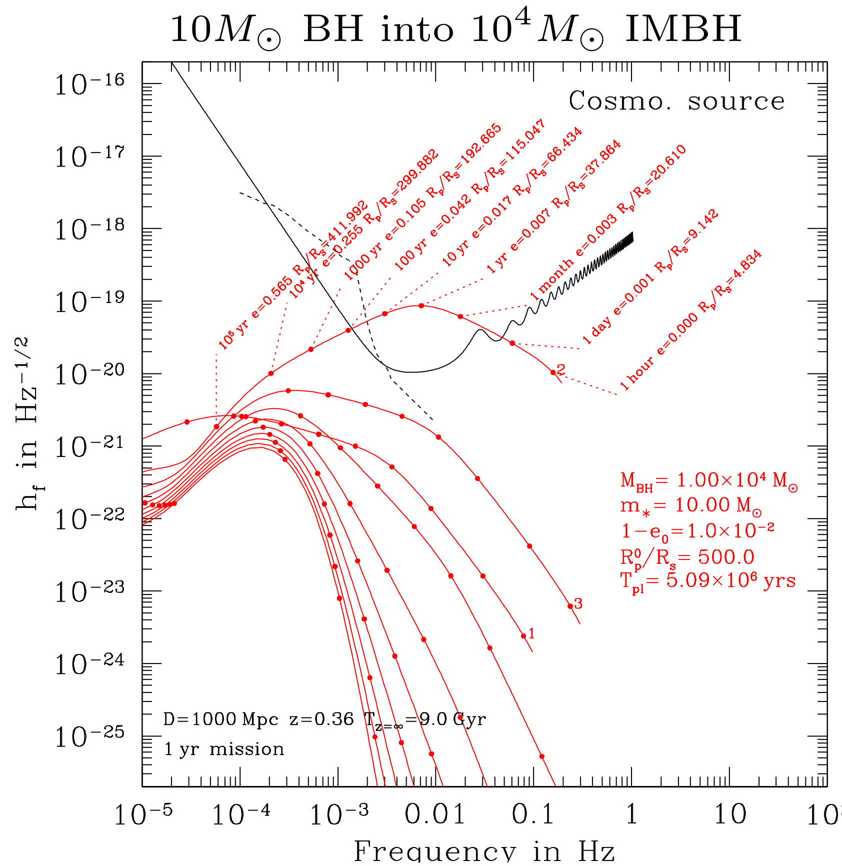
	rates (yr^{-1})	time in *LISA* band (yrs)	# with S/N ≥ 10
MS	few 10^{-6}	$\sim 10^6$	**1–5**
WD	few 10^{-7}	few 10^5	0.3–1
NS, $*$BH	few 10^{-8}–10^{-7}	few 10^4, 10^3	~ 0.01

  - BHs dominate innermost 0.1 pc after few Gyrs, MS stars expelled.
  - MS stars with $M_* \sim 0.05 - 0.1\,M_*$ most resilient
    Role of IMF? Role of tidal interactions?

Beware: low statistics for compact objects

Capture by/of IMBHs??





- IMBHs detectable to 1 Gpc at least!
- A possible scenario: IMBH formed by run-away collisions of stars in dense cluster. Cluster sinks to galactic centre by dynamical friction.

Rate: every few Myrs?

Portegies Zwart & McMillan 02
Rasio, Freitag & Gürkan 03, astro-ph/0304038
Gürkan, Freitag & Rasio 03, astro-ph/0308449
Ebisuzaki et al. 01, Hansen & Milosavljević 03
Portegies Zwart et al. 03

Conclusions and open questions

- Closest source: Low-mass MS stars orbiting Sgr A*
  - $\sim 10^6$ years before plunge $\Rightarrow$ "weak" field
- Other sources: Compact stars in distant galaxies
  - Probably only stellar BHs
  - Last year of inspiral $\Rightarrow$ strong field "geodesy"
  - Capture rates: $\sim 10^{-7} - 10^{-6}\,\mathrm{yr}^{-1}$ for $M_{\mathrm{BH}} \leq 10^7\,M_\odot$
  - Range of $\sim 300\,\mathrm{Mpc} \Rightarrow$ *LISA* detection rate: 0.1–few yr^{-1}
- Many uncertainties...
  - Stellar density, stellar population. How many stellar BHs?
  - Non spherical/stationary potential; Non-relaxational processes
    Nucleus rotation, galaxy mergers, MBH wandering, accretion disk, large angle scat...
  - Number of MBHs with $M_{\mathrm{BH}} \leq 10^7\,M_\odot$ within range?
- IMBHs are very promising sources (if they exist...)