Searching For Black Hole Binaries In Interferometer Data

Current Status and future prospects

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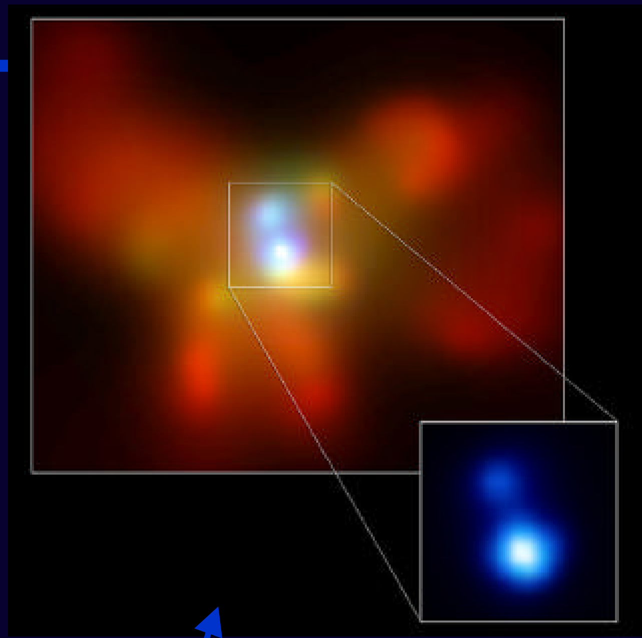
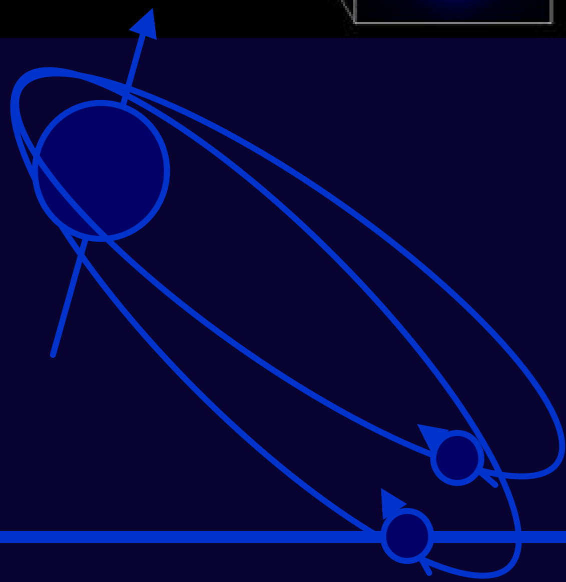
Trieste Conference on Sources of Gravitational Waves

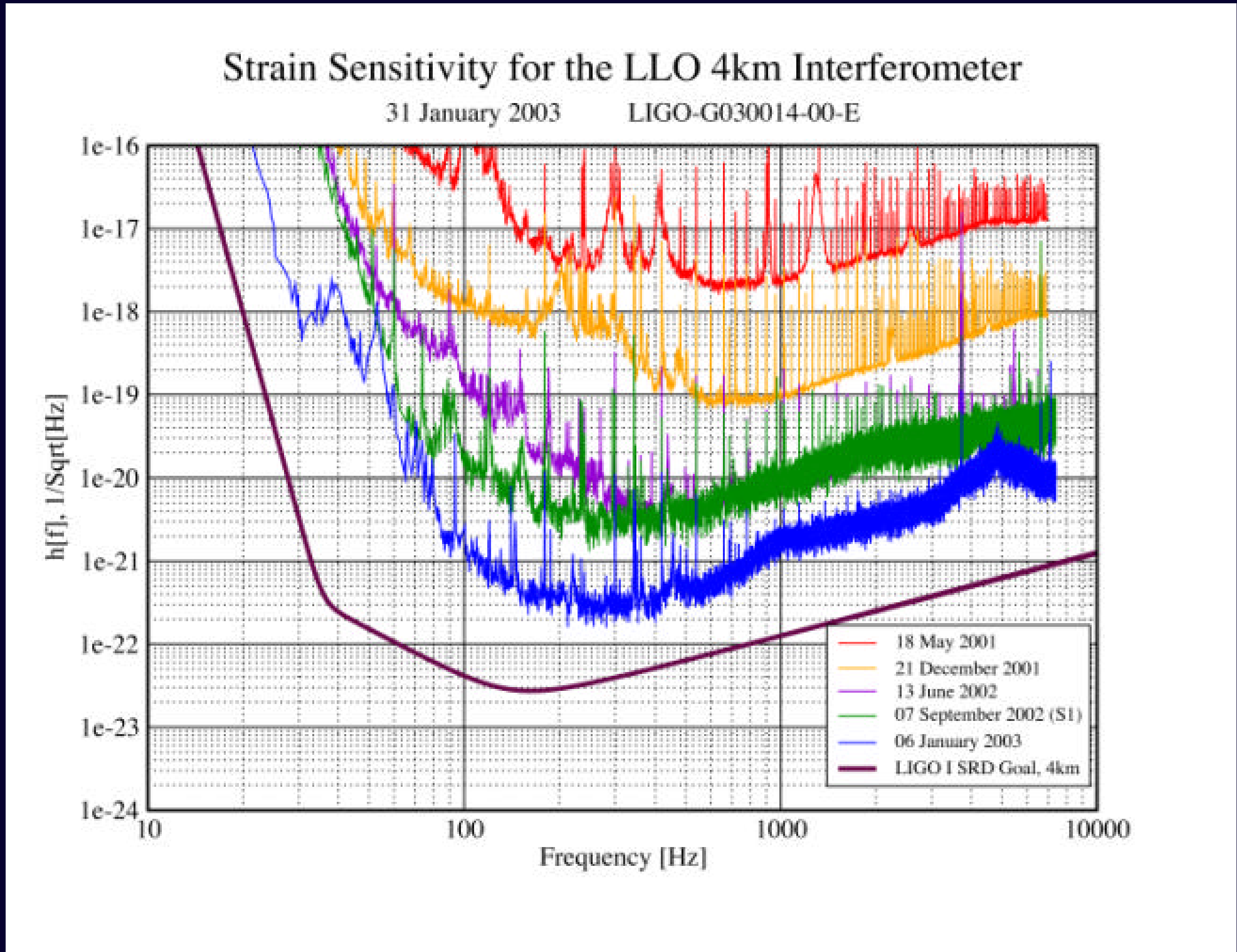
Systems We Want To Be Able Detect



- Binaries consisting of black holes of comparable masses – supermassive black hole binaries or stellar mass black hole binaries
- Small black holes (or neutron stars or white dwarfs) falling into big black holes
- Black holes with or without spins
- Binaries in arbitrary directions in the sky with arbitrary orientation of their orbital planes



What do we have to detect them



Plan of the talk

- Data Analysis
  - What are we up against?
  - Types of GW signals
  - Why is GW data analysis challenging?
- Sources and Waveforms
  - Stellar mass BHs falling into super-massive BHs
  - Super-massive black hole mergers
  - Stellar mass black hole mergers
- Detection schemes
  - parameter space and number of templates
  - search algorithms
    - matched filtering and geometric approach to signal analysis
    - time-frequency analysis
- Testing strong gravity
- Open problems in data analysis

Gravitational Wave Data Analysis
The basics

A good data analysis algorithm can greatly improve detection rates

For every factor of 2 improvement in SNR you get a factor of 8 in detection rate

What are we up against?

- measuring strains that arise from sub-nuclear length changes
  - almost anything can cause a disturbance
- unknown environmental b/g
  - seismic disturbances
  - solar flares and magnetic storms, cosmic rays, ...
- unknown instrumental b/g
  - electronic noise in feedback systems, laser frequency and intensity fluctuations, thermal fluctuations in mirror substrates, thermal vibration of suspension systems, ...
- non-Gaussian and non-stationary backgrounds
  - changing detector configuration
  - stochastic release of strain energy in suspension systems
  - electronic feedback ...

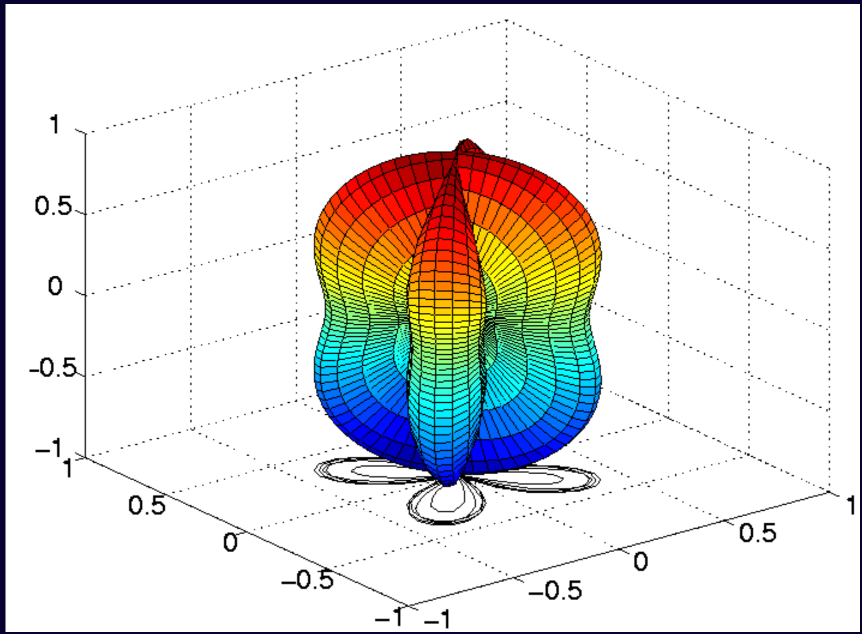
Important to understand detectors before any analysis begins - *Detector Characterization* - a huge effort

Types of gravitational wave signals

- Transients - last for a short duration - detector is stationary
  - Transients with known shape
    - e.g. black hole binaries, QNM
  - Transients with unknown shape
    - e.g. supernovae, NS-BH collision
  - low event rates and small signal strengths
- Stochastic backgrounds
  - population of astronomical sources; primordial stochastic signals; backgrounds from early Universe phase transitions
  - discriminating gravitational wave b/g from each other and from instrumental/environmental b/g
- Continuous waves - last for a duration long enough so that detector motion cannot be neglected
  - Typically very weak amplitude
    - signal power a billion times smaller than noise power
  - long integration times needed
    - several months to a year
  - slowly changing frequency and amplitude
    - system evolves during the observational period

Why GW data analysis is challenging?

- All sky sensitivity
  - Quadrupolar antenna pattern
  - multiple detectors to determine direction to source
- Wide band operation
  - 1 kHz bandwidth at 100 Hz
- Large data rates
  - Hundreds of instrumental and environmental channels
  - up to 10 MB per second from each detector
- Low Event rates
  - Initial interferometers
    - 1/300 years to 1/year
  - Advanced interferometers
    - 2/month to 10/day
- Large number of parameters
  - 2-10-dimensional parameter space - masses, spins, direction, distance, ...

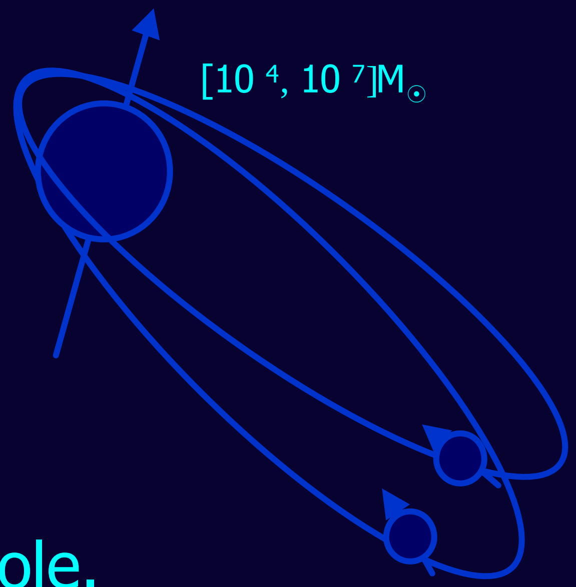


BBH Waveforms

Giving phasing information is not always easy or useful

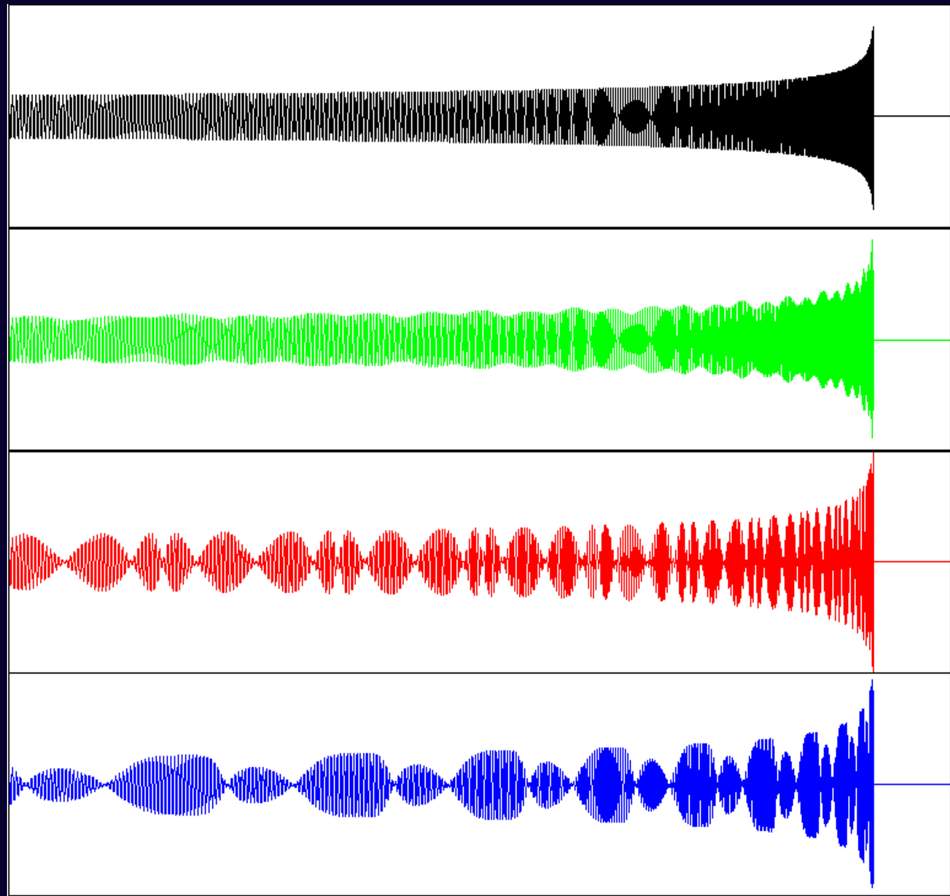
Stellar mass inspiral into a massive hole

- Massive BH with a stellar mass BH companion scattered into tight orbit via 3-body interaction
- Subsequent evolution:
  - Gradual decrease of eccentricity, little change of periholian, relativistic precession
  - Spin modulated chirps or smirches
- Still quite eccentric when plunges into hole.
  - Waveform maps Kerr geometry. Test No-Hair Theorems.
- A few per year at ~ 1 Gpc (Sigurdsson & Rees)
- May be somewhat larger for ~10 $M_\odot$ holes (Phinney)



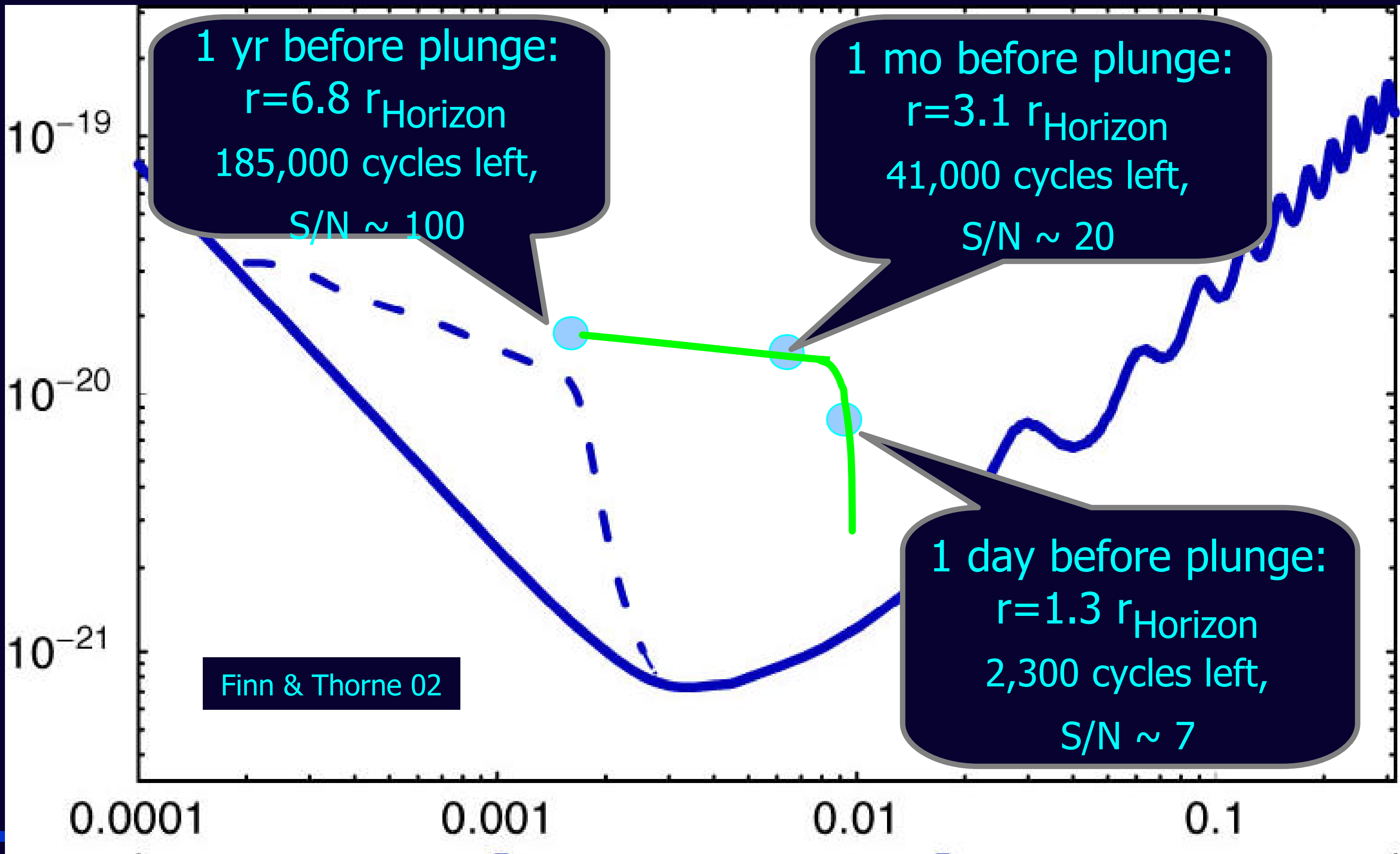
modelling and searching for smirches

- Large parameter space
  - unknown source position and orientation
  - unknown initial directions of orbit and spin angular momenta
- Complicated dynamics
  - spin-orbit and spin-spin couplings
  - eccentric orbit, multipolar source and radiation
  - back-scattering caused by curved background



An Example: Circular, Equatorial orbit - 10 $M_\odot$ + 10^6 $M_\odot$, fast spin

Problems: Visible under SMBHs; Separating them from each other



Including higher order multipoles improves …

Babak and Glampedakis 03

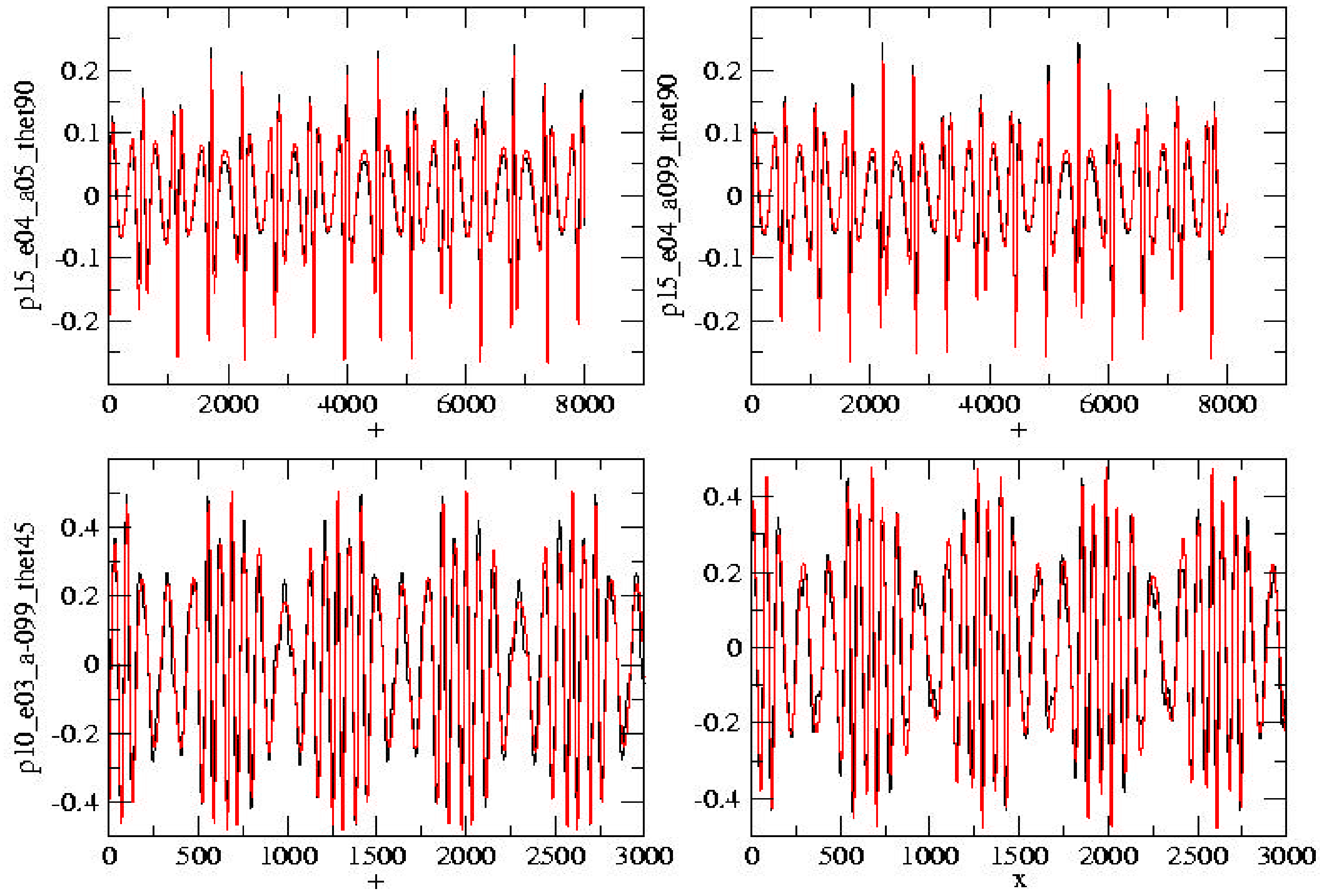
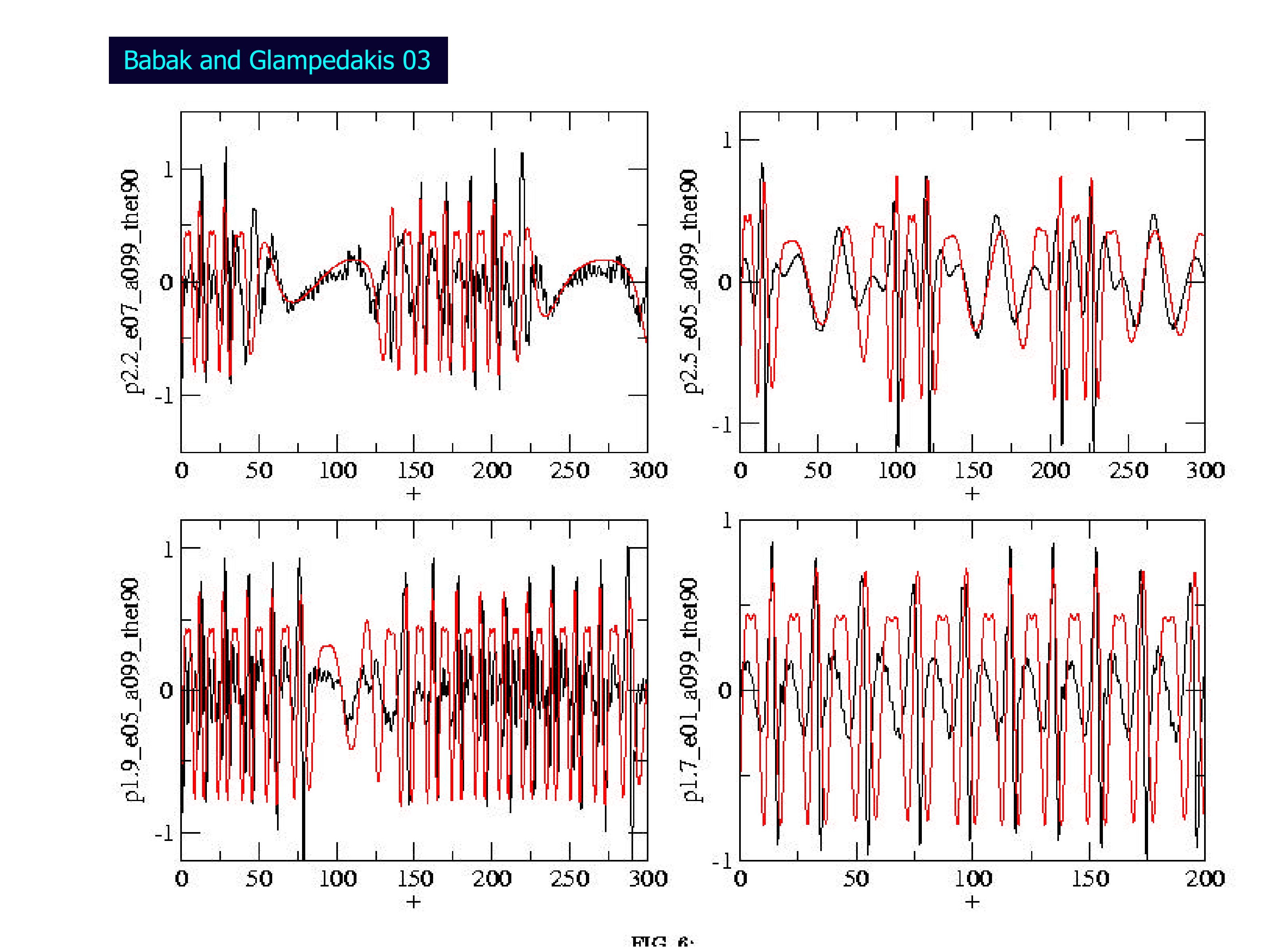
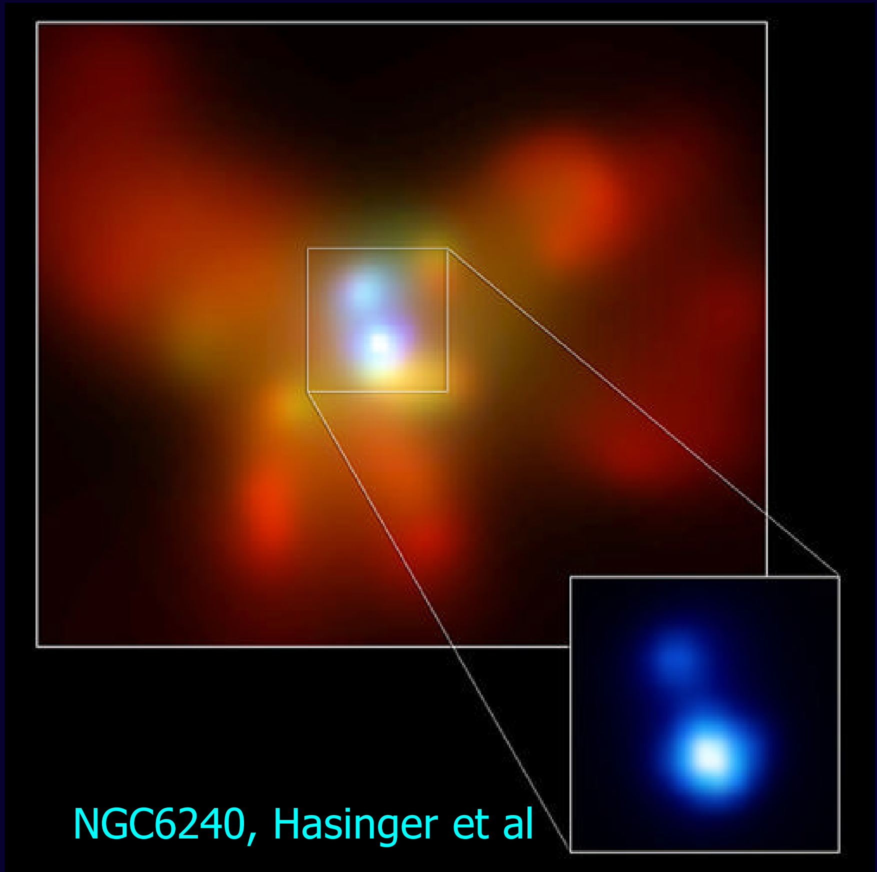


FIG. 4:
Black curve is a numerical waveform, red is a corresponding kludge waveform. Parameters of each waveform you can see on

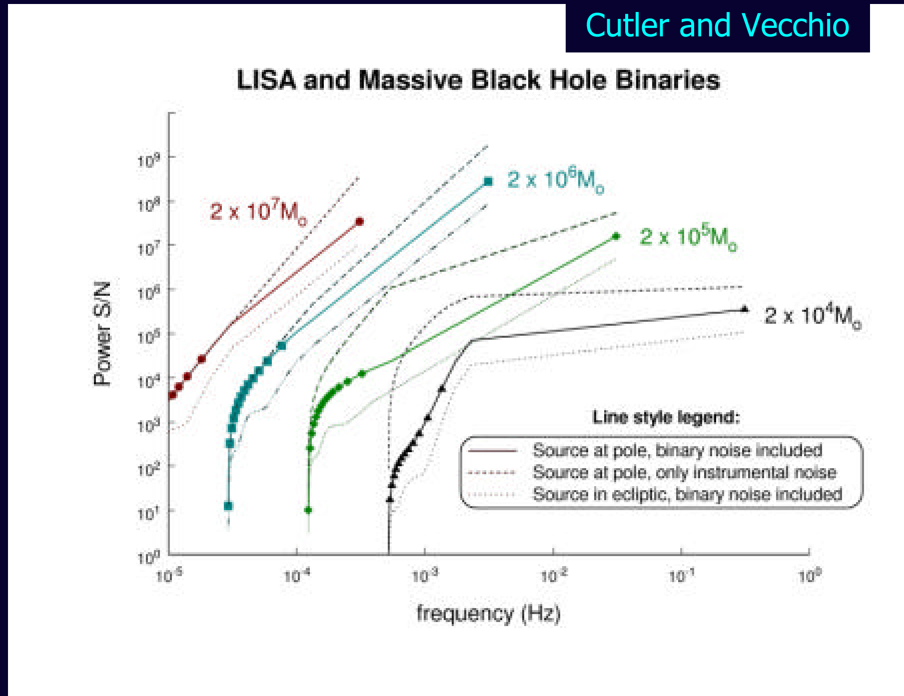
… but not good enough



Merger of supermassive black holes - no templates needed!

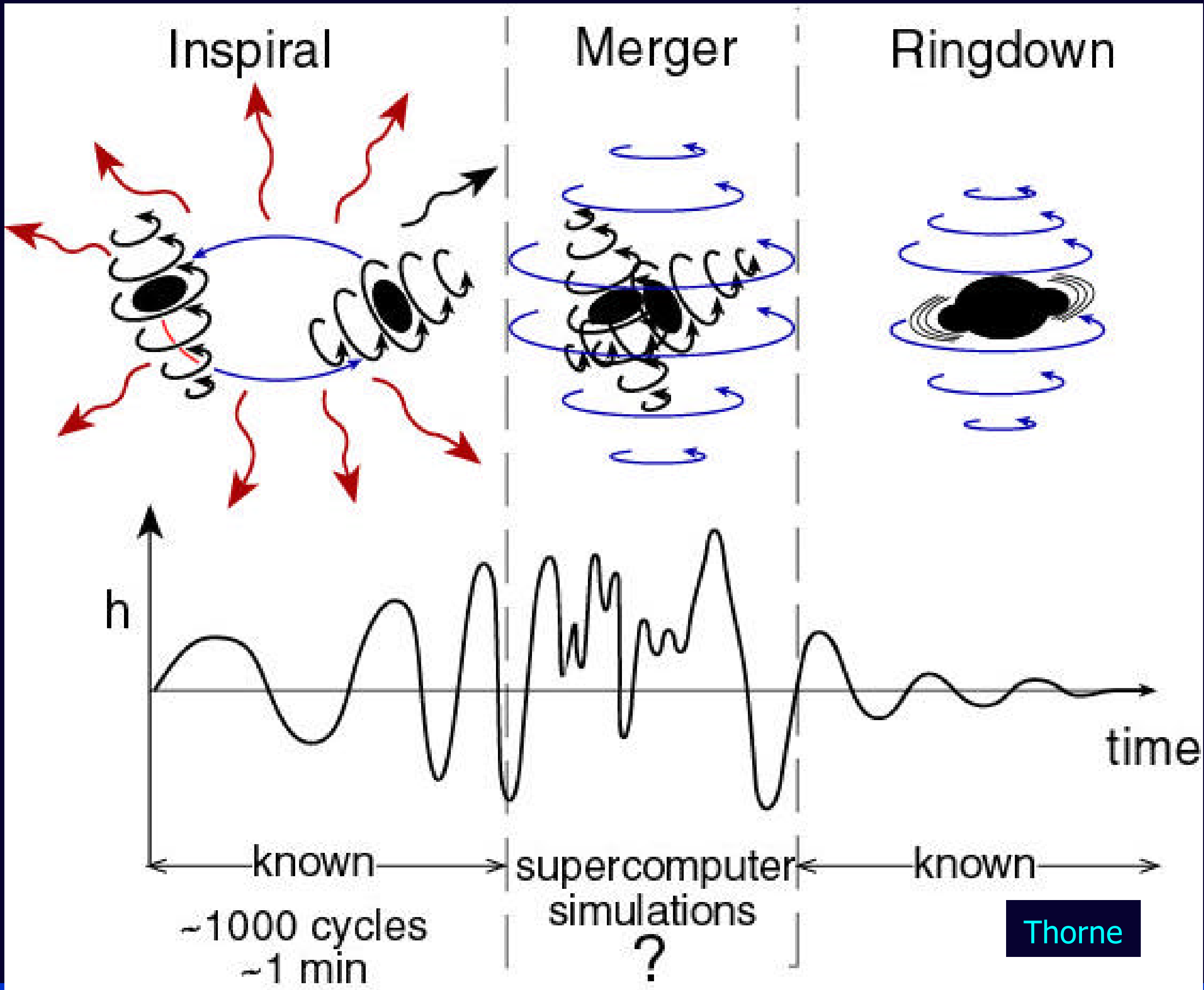


NGC6240, Hasinger et al



The high S/N at early times enables LISA to **predict** the time and position of the coalescence event, allowing the event to be observed simultaneously by other telescopes.

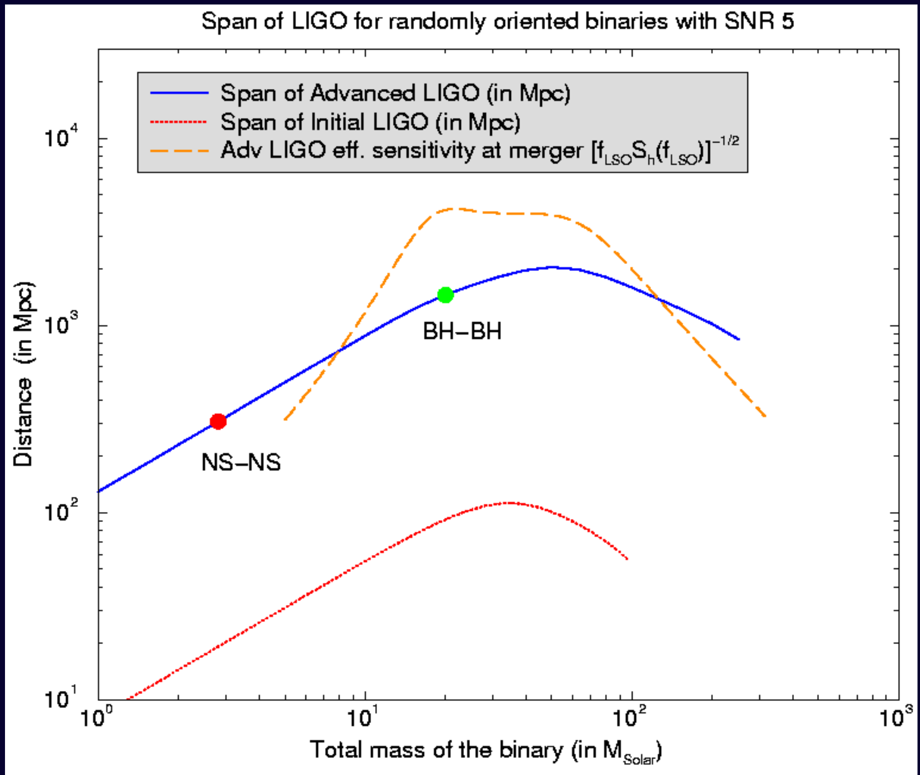
Stellar mass BH-BH Mergers will require accurate templates



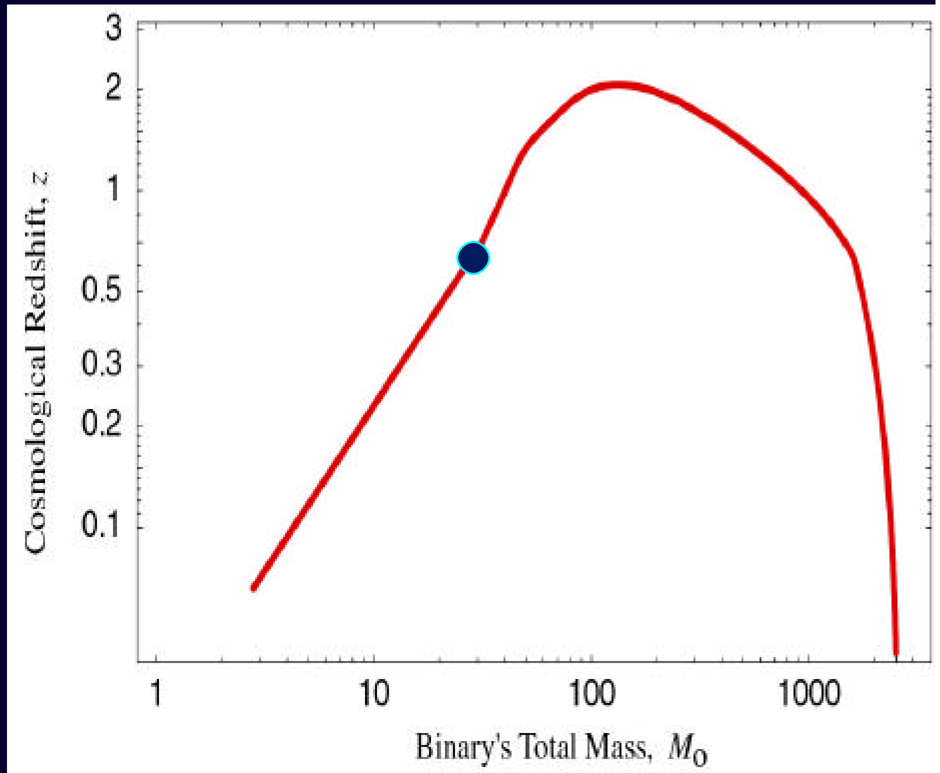
Numerical relativity simulations and analytic insights are are badly needed

BH-BH Mergers in Initial and advanced LIGO

Inspiral Signal Only



Inspiral and Merger



Binary Black Hole Waveforms – Current Status

- Post-Newtonian and post-Minkowskian approximations
  - Energy is known to order O (v^6)
  - Gravitational wave flux is known to order O (v^7) (but still one unknown parameter)
- Improved dynamics by defining new energy and flux functions and their Pade approximants
  - Works extremely well in the test mass limit where we know the exact answer and can compare the improved model with
  - But how can we be sure that this also works in the comparable mass case
- Effective one-body approach
  - An improved Hamiltonian approach in which the two-body problem is mapped on to the problem of a test body moving in an effective potential
  - Can be extended to work beyond the last stable orbit and predict the waveform during the plunge phase until r =3M.
- Phenomenological models to extend beyond the post-Newtonian region
  - A way of unifying different models under a single framework

Post-Newtonian expansions of GW flux and energy

Blanchet, Damour, Iyer, Will & Wiseman 1996; Blanchet 1996

Two quantities determine the GW phasing formula:

1. The gravitational wave flux $\mathcal{F}(v)$

$$\mathcal{F}_{T_n}(v) = \frac{32\eta^2 v^{10}}{5}\left[1 - \left(\frac{1247}{336} + \frac{35\eta}{12}\right)v^2 + 4\pi v^3\right.$$

$$\left. \ldots\,\pi v^5\right],$$

Now known up to 3.5 PN order

But there is one unknown parameter

2. The relativistic energy $E(v)$:

$$E_{T_n}(v) = -\frac{\eta v^2}{2}\left[1 - \left(\frac{9+\eta}{12}\right)v^2 - \left(\frac{81 - 57\eta + \eta^2}{24}\right)v^4\right].$$

Here $v = (\pi m F)^{1/3}$ is post-Newt. expansion parameter m is the total mass and F is GW frequency, $\eta = m_1 m_2/m^2$ is the symmetric mass ratio

Binary Phasing Formula

- Energy balance:

$$\frac{dE(v)}{dt} = - L(v)$$

where L is the GW luminosity, E is the relativistic binding energy

- The phasing of gravitational waves at the dominant post-Newtonian order is twice the orbital phase:

- A phasing formula can be obtained by relativing time-evolution of frequency to energy and luminosity ($f = v^3/\pi M$)

$$\frac{d\Phi_{GW}}{dt} = 2\pi f(t).$$

P-approximants

Damour, Iyer & Sathyaprakash

1. Construct analytically well-behaved new energy and flux functions: Remove branch points in energy; include a linear term to handle log terms in the flux

$$e(v) = \left(\frac{E_{\rm tot}^2 - m_1^2 - m_2^2}{2m_1 m_2}\right)^2 - 1, \qquad f(v;\eta) = \left(1 - \frac{v}{v_{\rm polc}}\right)\mathcal{F}(v;\eta).$$

2. Using Taylor expansions $e_{T_n}(v)$ and $f_{T_n}(v)$ construct Pade approximants $e_{P_n}(v)$ and $f_{P_n}(v)$ which when re-expanded are consistent with PN expansions.

3. Work back and re-define (P-approximants of) energy and flux functions $E_{P_n}(v)$ and $\mathcal{F}_{P_n}(v)$

$$E_{P_n}(v) = \left[1 + 2\eta\left(\sqrt{1 + e_{P_n}(v)} - 1\right)\right]^{1/2} - 1,$$

$$\mathcal{F}_{P_n}(v;\eta) = \left(1 - \frac{v}{v_{\rm polc}}\right)^{-1} f_{P_n}(v;\eta).$$

Cauchy Convergence Table

Compute overlaps (npN,mpN)

Standard post-Newtonian approximants

(1.4,1.4)	*3pN*	*4pN*	*5pN*	*6pN*	*7pN*
3pN		0.63	0.82	0.95	0.92
4pN			0.54	0.60	0.58
5pN				0.88	0.92
6pN					0.99
7pN					

Cauchy Convergence Table

Compute overlaps (npN,mpN)

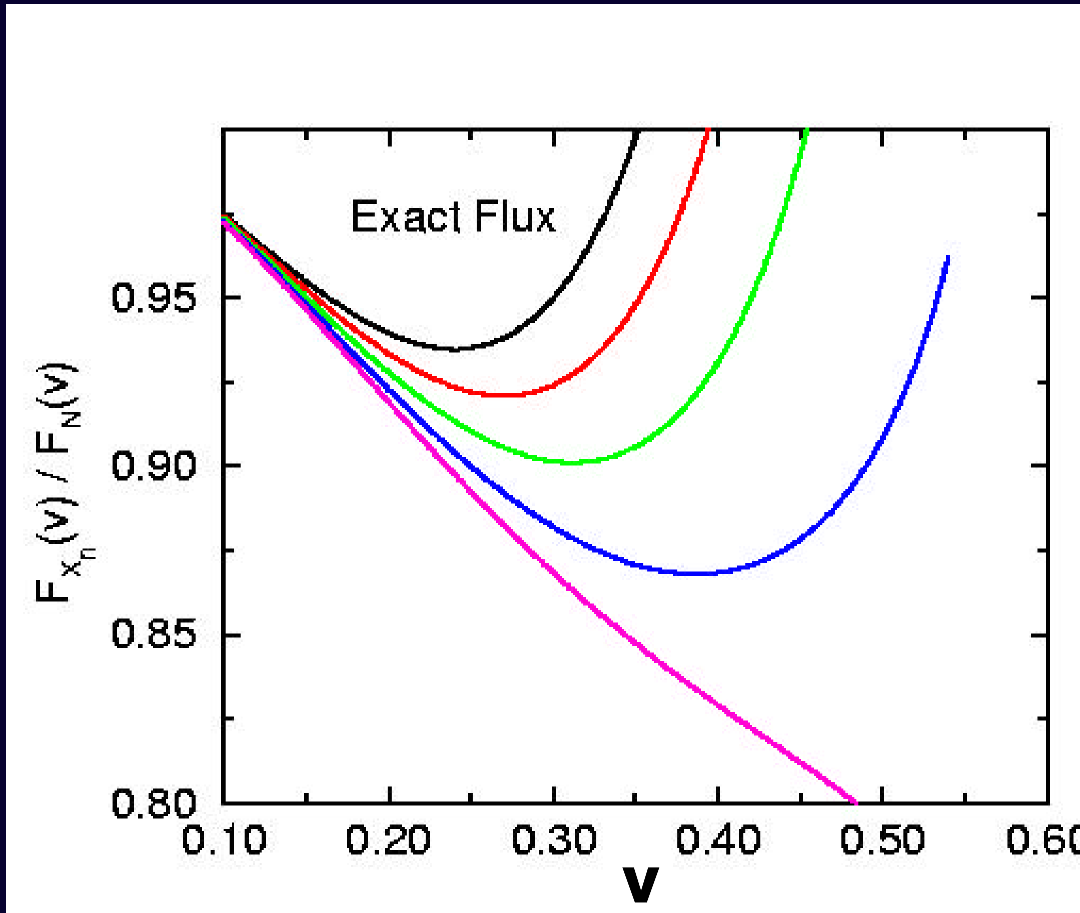
P-approximants

(10,1.4)	*3pN*	*4pN*	*5pN*	*6pN*	*7pN*
3pN		0.41	0.39	0.40	0.40
4pN			0.91	0.99	0.99
5pN				0.94	0.93
6pN					1.00
7pN					

Exact GW Flux - Kerr Case

Shibata 96

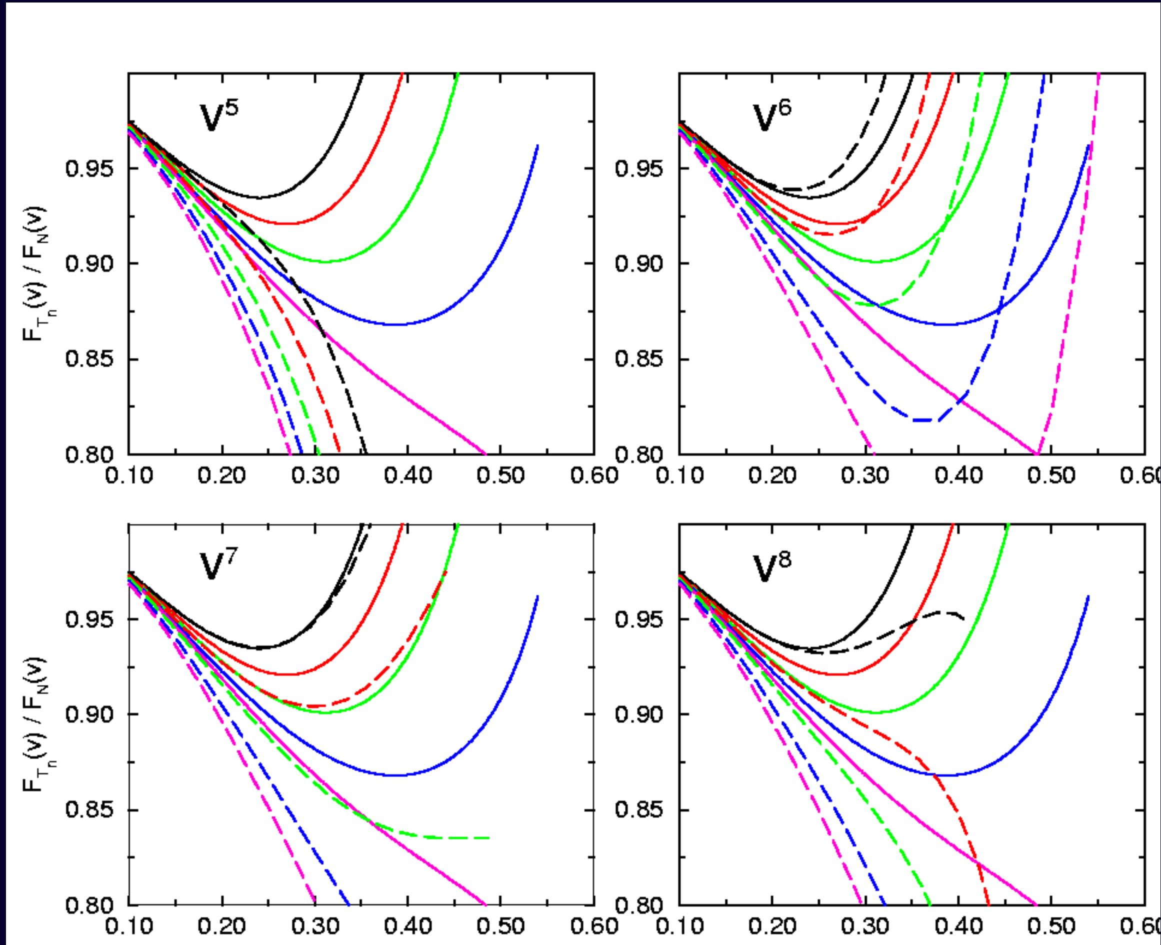
a = 0.0, 0.25, 0.5, 0.75, 0.95



Post-Newtonian Flux - Kerr Case

Tagoshi, Shibata, Tanaka, Sasaki Phys Rev D54, 1429, 1996

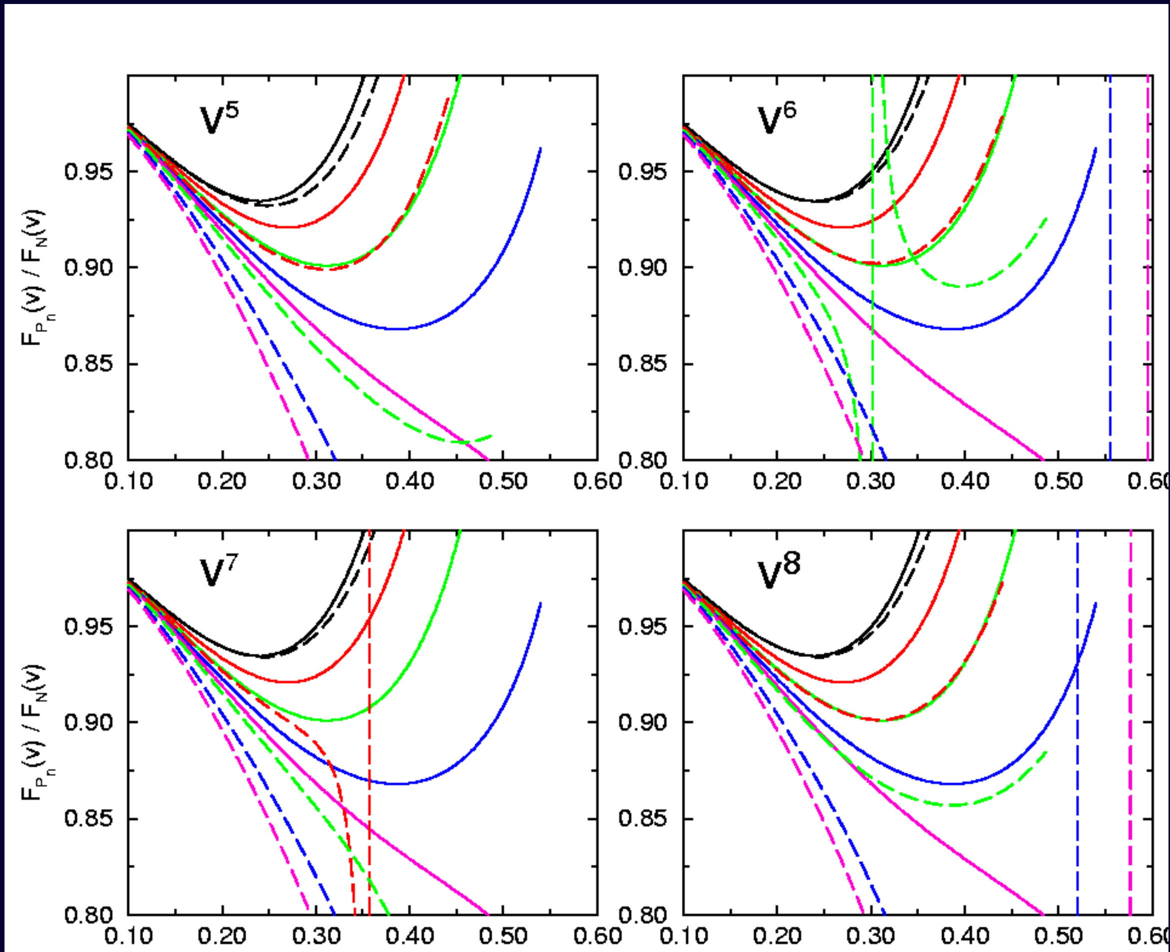
a = 0.0, 0.25, 0.5, 0.75, 0.95



P-approximant flux - Kerr case

Porter and Sathyaprakash 2003

a = 0.0, 0.25, 0.5, 0.75, 0.95



P-approximant flux - Kerr case

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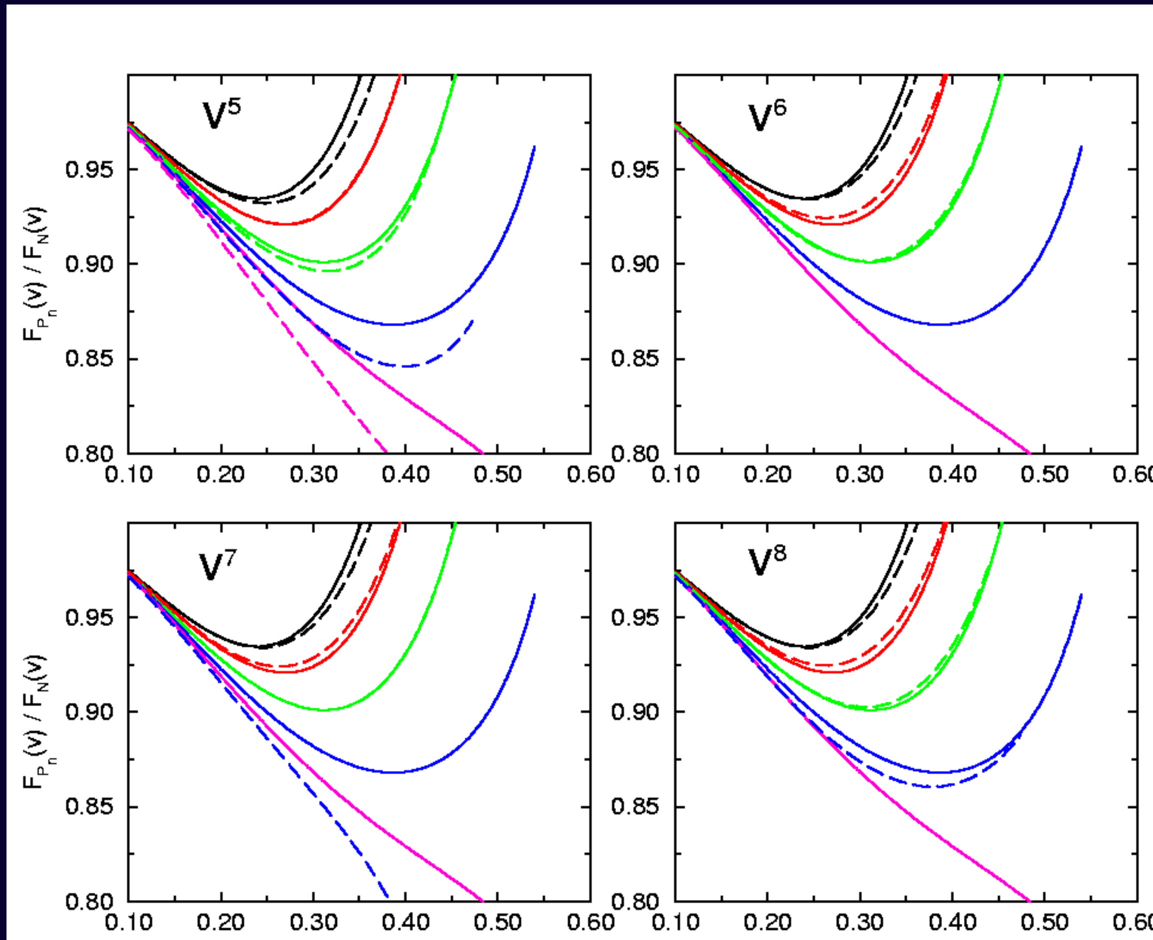


Buonanno-Damour re-summation technique

Buonanno & Damour 1998, 2000

- $\hat{H}(r, p_r, p_\varphi)$ is the Hamiltonian

$$\hat{H}(r, p_r, p_\varphi) = \frac{1}{\eta}\left[1 + 2\eta\left\{-1 + \sqrt{A(r)\left(1 + \frac{p_r^2}{B(r)} + \frac{p_\varphi^2}{r^2}\right)}\right\}\right]^{1/2}$$

- The equations of motion are

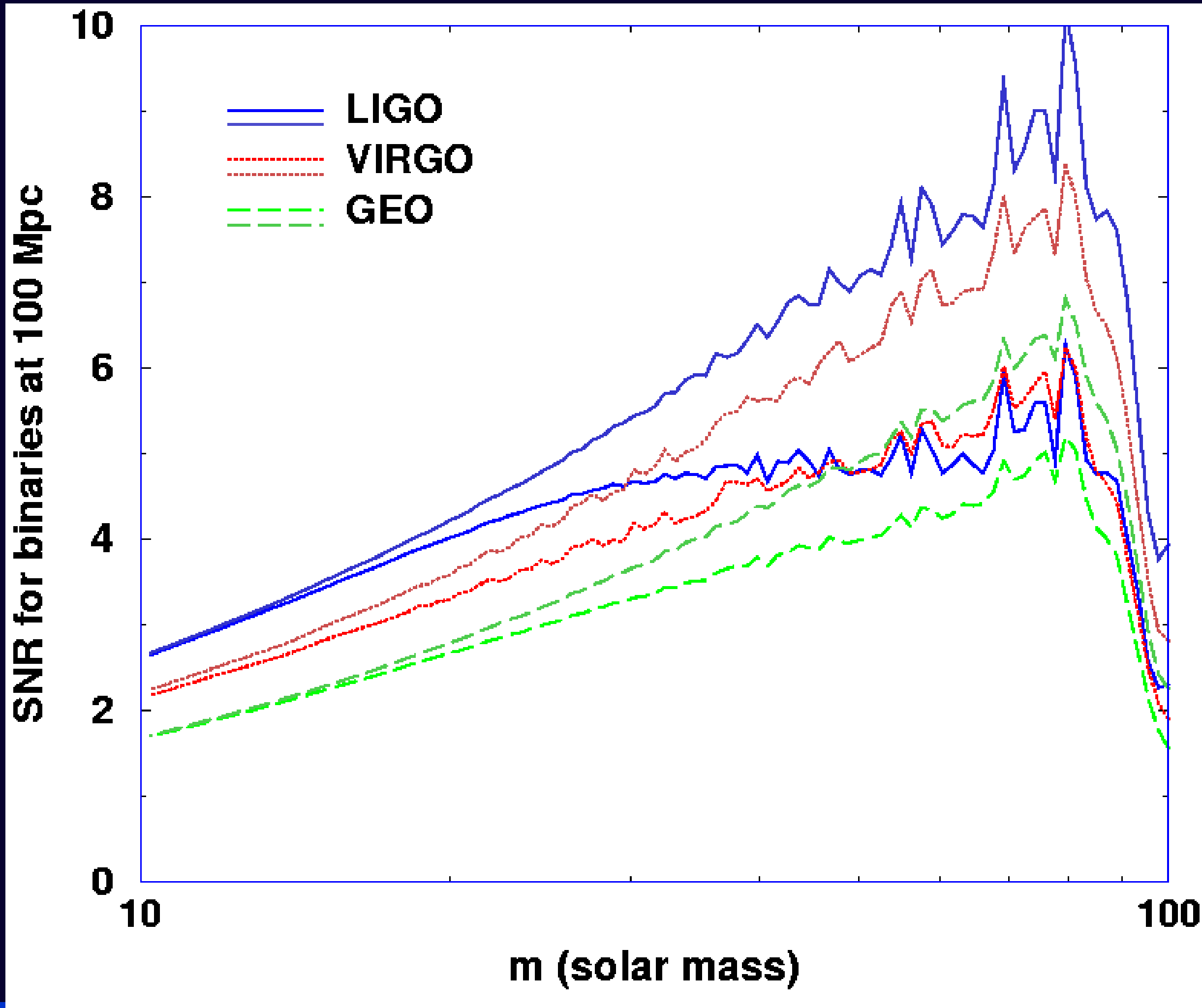
$$\frac{dr}{d\hat{t}} = \frac{\partial \hat{H}(r, p_r, p_\varphi)}{\partial p_r}, \qquad \frac{d\varphi}{d\hat{t}} \equiv \hat{\omega} = \frac{\partial \hat{H}(r, p_r, p_\varphi)}{\partial p_\varphi},$$

$$\frac{dp_r}{d\hat{t}} = -\frac{\partial \hat{H}(r, p_r, p_\varphi)}{\partial r}, \qquad \frac{dp_\varphi}{d\hat{t}} = \hat{\mathcal{F}}_\varphi^{\rm DIS}\hat{\omega}((r, p_r, p_\varphi)).$$

- $A(r)$ and $B(r)$ are functions that occur in the effective metric

Improvement in SNR with plunge

Damour, Iyer and Sathyaprakash 01



Phenomenological Waveforms – detection template family

- Using the stationary phase approximation one can compute the Fourier transform of a binary black hole chirp which has the form

$$\mathbf{h(f) = h_0\ f^{-7/6} \exp [i\, S\, y_k\, f^{(k-5)/3}]}$$

- Where y are the related to the masses and can only take certain values for physical systems
- Buonanno, Chen and Vallisneri (2002) introduced, by hand, amplitude corrections and proposed that y be allowed to take non-physical values and frequencies extended beyond their natural cutoff points at the last stable orbit

$$\mathbf{h(f) = h_0\ (1 + a\ f^{2/3})\ f^{-7/6} \exp [i\, (y_0\, f^{-5/3} + y_3\, f^{-2/3})]}$$

- Such models, though unrealistic, seem to cover all the known families of post-Newtonian and improved models
  - Such DTFs have also been extended to the spinning case where they seem to greatly reduce the number of free parameters required in a search

Summary on Waveforms

- PN theory is now known to a reliably high order in post-Newtonian theory– $O(v^7)$
- Resummed approaches are (1) convergent (in Cauchy sense), (2) robust (wrt variation of parameters), (3) faithful (in parameter estimation) and (4) effectual (in detecting true general relativistic signal)
- EOB approach gives a better evolution up to ISCO most likely reliable for all - including BH-BH - binary inspirals
- Detection template families (DTF) are an efficient way of exploring a larger physical space than what is indicated by various approximations

data analysis for black hole binary searches

How do we choose our test templates used in our searches?

The problem is similar to finding a suitable coordinate system on a sphere that divides into equal regions

Matched filtering - Basics

Given a signal of known shape $h(t)$ what template (or filter) $q(t)$ should one use to maximise the likelihood of detection?

$$C(\tau) = \int_{t_0}^{t_0+T} dt\ h(t)\ q(t+\tau). \qquad (1)$$

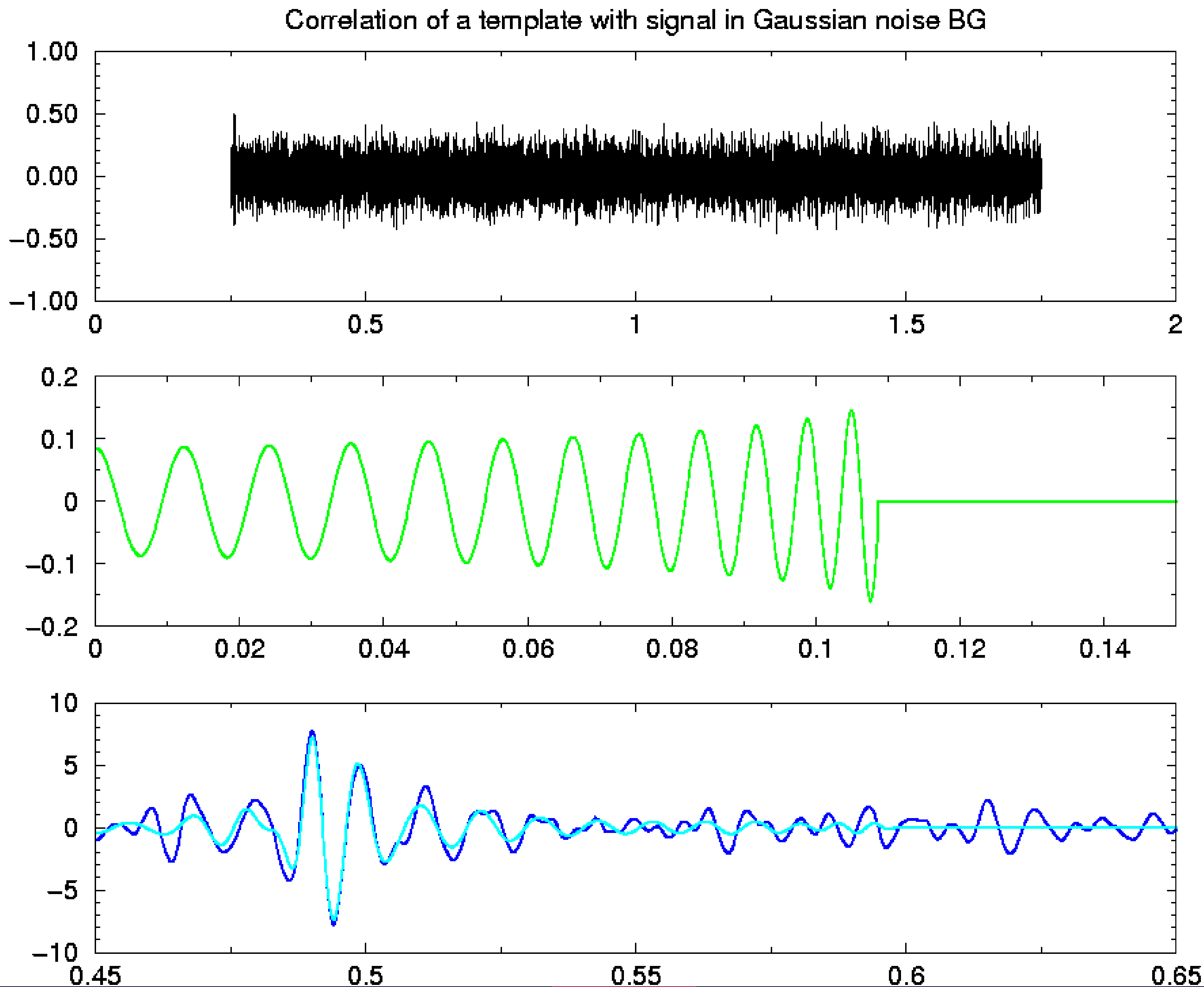
The optimal filter $\tilde{q}(f)$ which maximizes the ratio of the mean $\mu = \langle C \rangle$ to the variance $\sigma = \sqrt{\langle (C-\mu)^2 \rangle}$ of the above statistic (1) is

$$\tilde{q}(f) = \lambda\, \frac{\tilde{h}(f)}{S_h(f)}, \qquad (2)$$

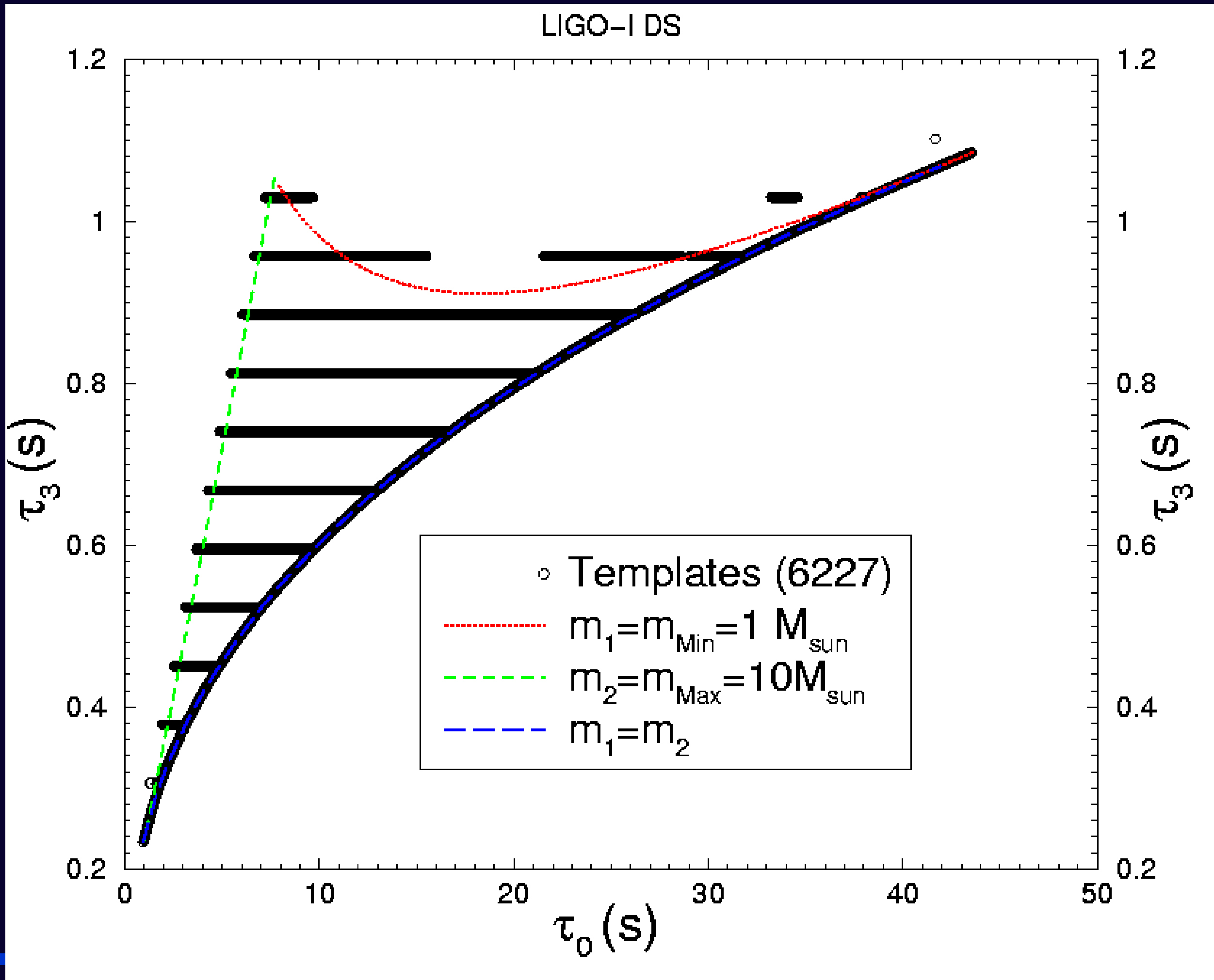
where λ is a normalization constant, $S_h(f)$ is the power spectral density

$$\langle n(f) n(f')^* \rangle = \delta(f - f') S_h(f)$$

that is, the Fourier transform of the auto-correlation function $c(\tau) := \langle n(t+\tau) n(t) \rangle$ of the detector.


Correlation of a template with signal in Gaussian noise BG
1.00
0.50
0.00
−0.50
−1.00
0
0.5
1
1.5
2
0.2
0.1
0
−0.1
−0.2
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
10
5
0
−5
−10
0.45
0.5
0.55
0.6
0.65

Templates to detect NS and BH binaries



Number of independent parameters

Source Parameters

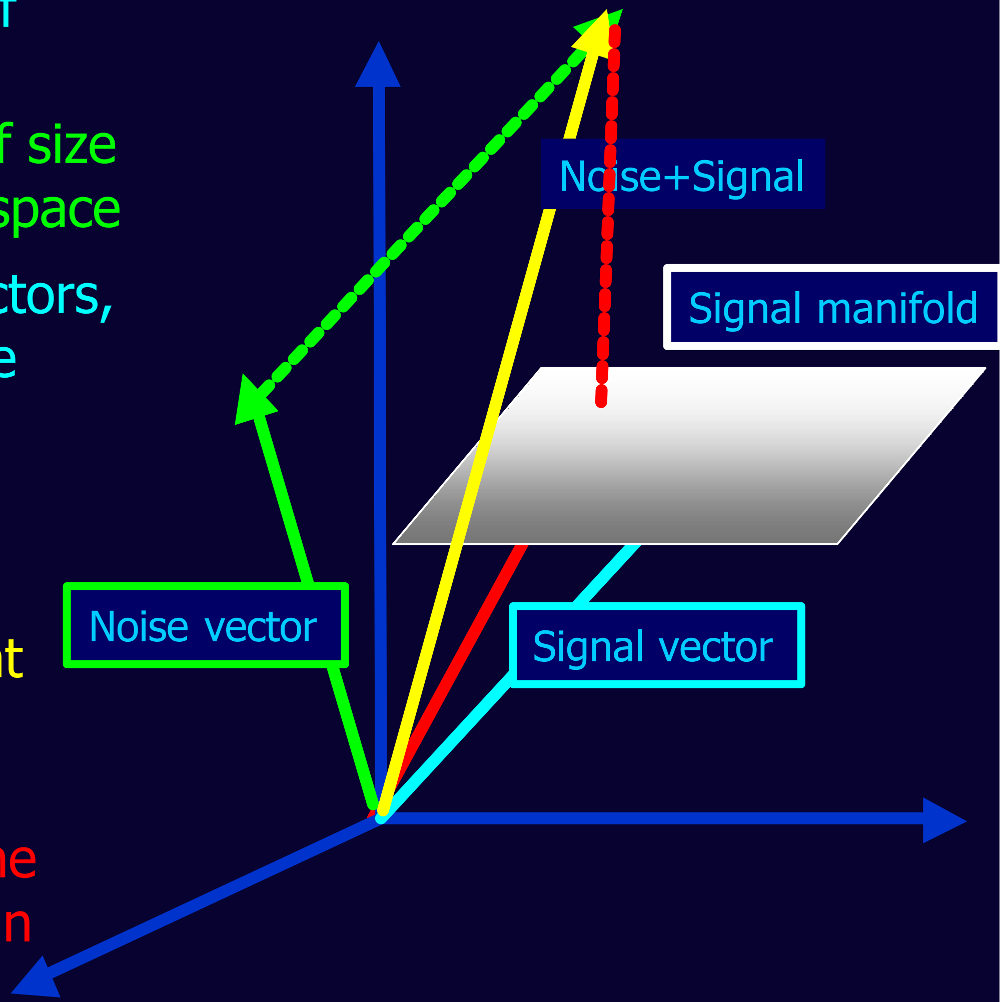
- Source parameters that determine the shape and amplitude of the signal as seen by LISA
  - location of the source (3)
  - masses of the two bodies (2)
  - initial angular momentum (3)
  - initial spins (6)
  - initial eccentricity (1)
  - epoch of merger (1)
  - phase of the signal at merger (1)
- In all 17 parameters
  - but not all are independent

Search Parameters

- Epoch of and phase at merger, distance to the source are not required in a search
- Assume data is de-modulated for different directions on the sky
  - direction cosines not needed
  - a combination of the masses - total chirp time (1)
  - initial eccentricity of the orbit (1)
  - Spin of the MBH (3)
  - opening angle of the orbit when the observation begins (1)
- At most 6 search parameters

Geometrical approach to signal analysis

- An analysis begins with a chunk of data N samples long
- Set of all data chunks $\{x_k\}$ each of size N form an N-dimensional vector space
- Signals from a source are also vectors, but they don't form a vector space
- Set of all signal vectors do form a manifold; signal parameters λ^a serve as a coordinate system
- Noise corrupts the signal and what we measure is signal + noise
- Matched filtering projects the measured (noise + signal) onto the signal manifold – prone to errors in parameter estimation



Scalar product and the metric

- The matched filtering statistic defines a natural scalar product between any two vectors
- The scalar product naturally defines a metric on the signal manifold:

$$g_{ab} = \langle h_{,a} , h_{,b} \rangle$$ - The information matrix

- Correlation between a signal and a nearby template can be expressed in terms of the metric

$$C = 1 - g_{ab}\, d\lambda^a\, d\lambda^b + \ldots$$

- Diagonalize the metric and demand that the distance between templates $d\lambda^a$ are such that C is at least = MM (called minimal match, say 0.95).
- The parameter distance between templates is given by

$$d\lambda^a = [(1 - MM)/g_{aa}]^{1/2}$$

Principal Component Analysis

- given a signal h(t,**p**) compute the information matrix

$$g_{km} = (h_k, h_m)$$

where (a, b) denotes the inner product of vectors a and b defined by matched filtering and a subscript denotes derivative of the signal w.r.t. parameter p^k

- inverse of the information matrix is the covariance matrix

$$G^{km} = [g^{-1}]^{km}$$

- define variance-covariance matrix by: $C^{kk} = G^{kk}$, if k = m

$C^{km} = G^{km} / (G^{kk} G^{mm})^{1/2}$, if k != m

- non-diagonal elements lie in the range [-1,1]
- if $|C^{km}| \sim 1$ means that the parameters are correlated:
- **diagonalize**, principal components are the largest eigenvalues
- number of nearly equal large components gives the effective dimensionality of the parameter space
- Applying this to non-spinning BH binaries automatically shows that there is only 1 ind. param as opp. 4

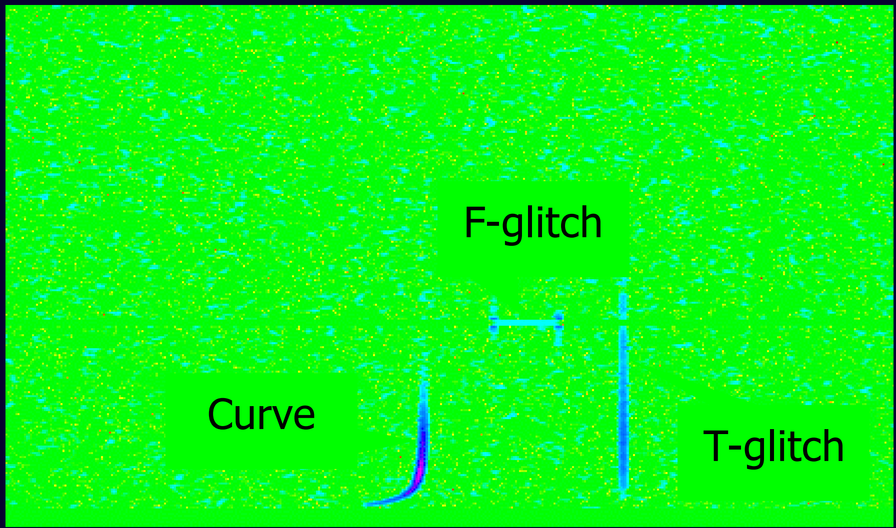
Number of independent parameters in spinning BBH

- Signal Model: (Kidder, Apostolatos et al)

$$h(t) = -A(t) \cos [2\Phi(t) + \phi(t) + \delta\phi(t)]$$

  - A(t, m1, m2, **N, L, S1, S2**) = Amplitude modulation
  - Φ(t , m1, m2, tc, ϕc) = Inspiral phase carrier signal
  - ϕ(t, m1, m2, **N, L, S1, S2**) = Phase modulation
  - $\delta\phi$(t, m1, m2, **N, L, S1, S2**) = Thomas precession
- Principal component analysis suggests that of 12 parameters in the case of smirches only 3 or 4 are independent (Sathyaprakash and Schutz 2003, Barak and Cutler 2003)
- The same analysis for comparable mass black holes suggests that we may require a search in a 6-dimensional space in LIGO/VIRGO/GEO/TAMA data (Sathyaprakash 2003)
- A matched filter search for BH binaries in LIGO/LISA is a difficult task - alternatives needed

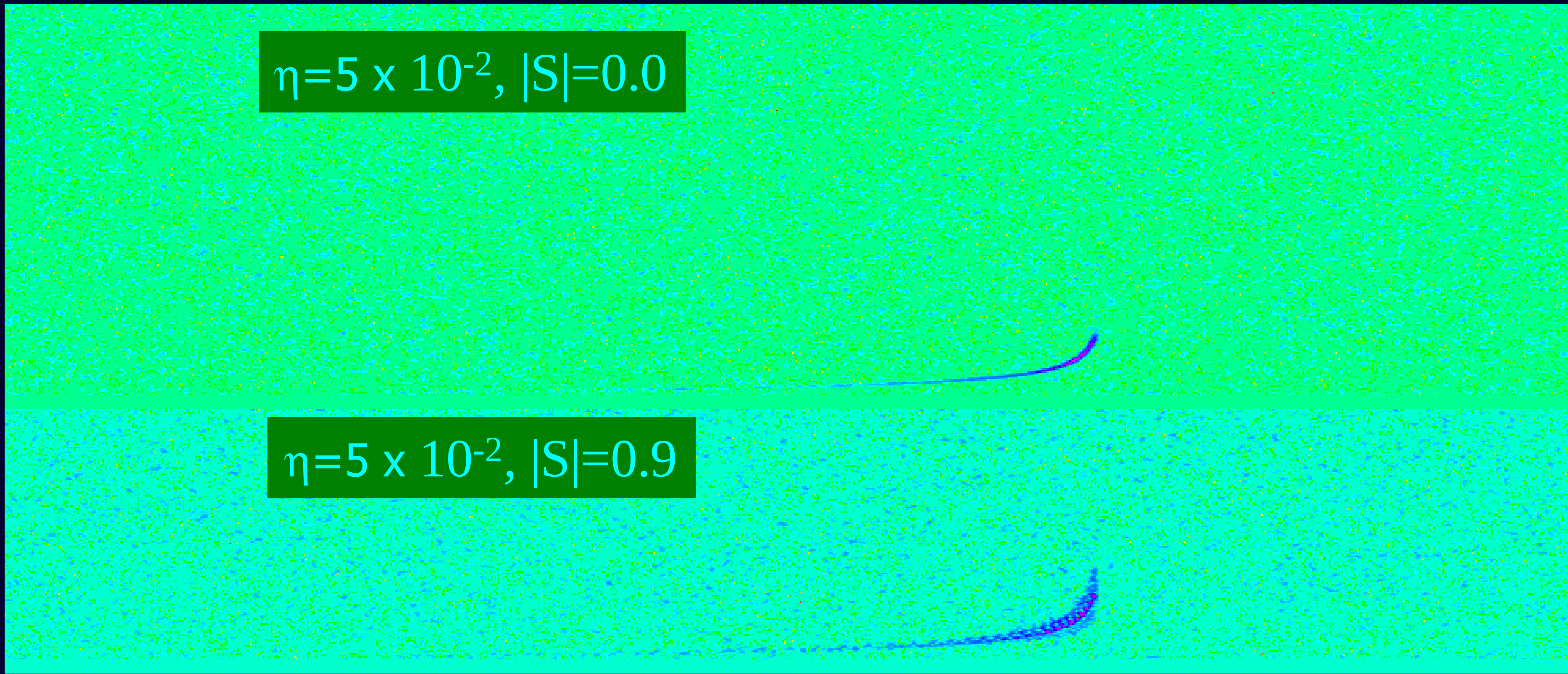
Time-Frequency Analysis - Curves, Blobs and Glitches



- Our inspiral signals are expected to be curves
  - Ignore blobs and glitches
  - Key new (powerful) feature: Use multiple thresholds(s) to pick curves

- Construct spectrograms
  - Short-period Fourier transforms as a function of time;
  - J. Sylvestre used spectrograms to study GW bursts - TF clusters
- Classify features in spectrogram
  - **T-glitch** - a broadband burst lasting for a short duration
  - **F-glitch** - a narrow band signal with a high-Q
  - **Blob** - a homogeneous cluster in the time-frequency plane
  - **Curve** - a filamentary feature in the time-frequency plane

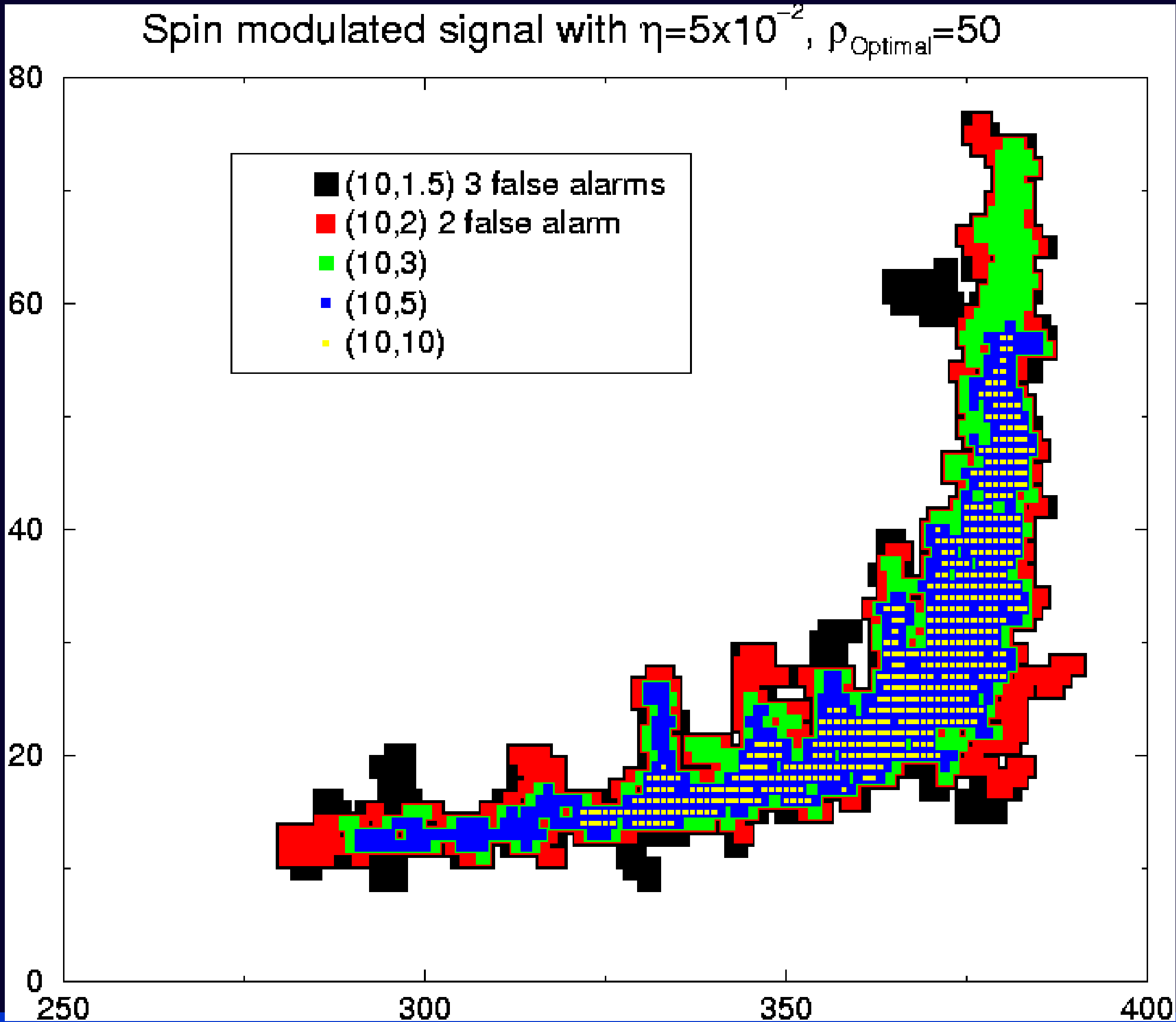
Time-frequency map of spin modulate chirps



- We shall call Spin Modulated Chirps as Smirches
- We clearly see how the spin of the massive black hole smears (or smirches) the waveform.

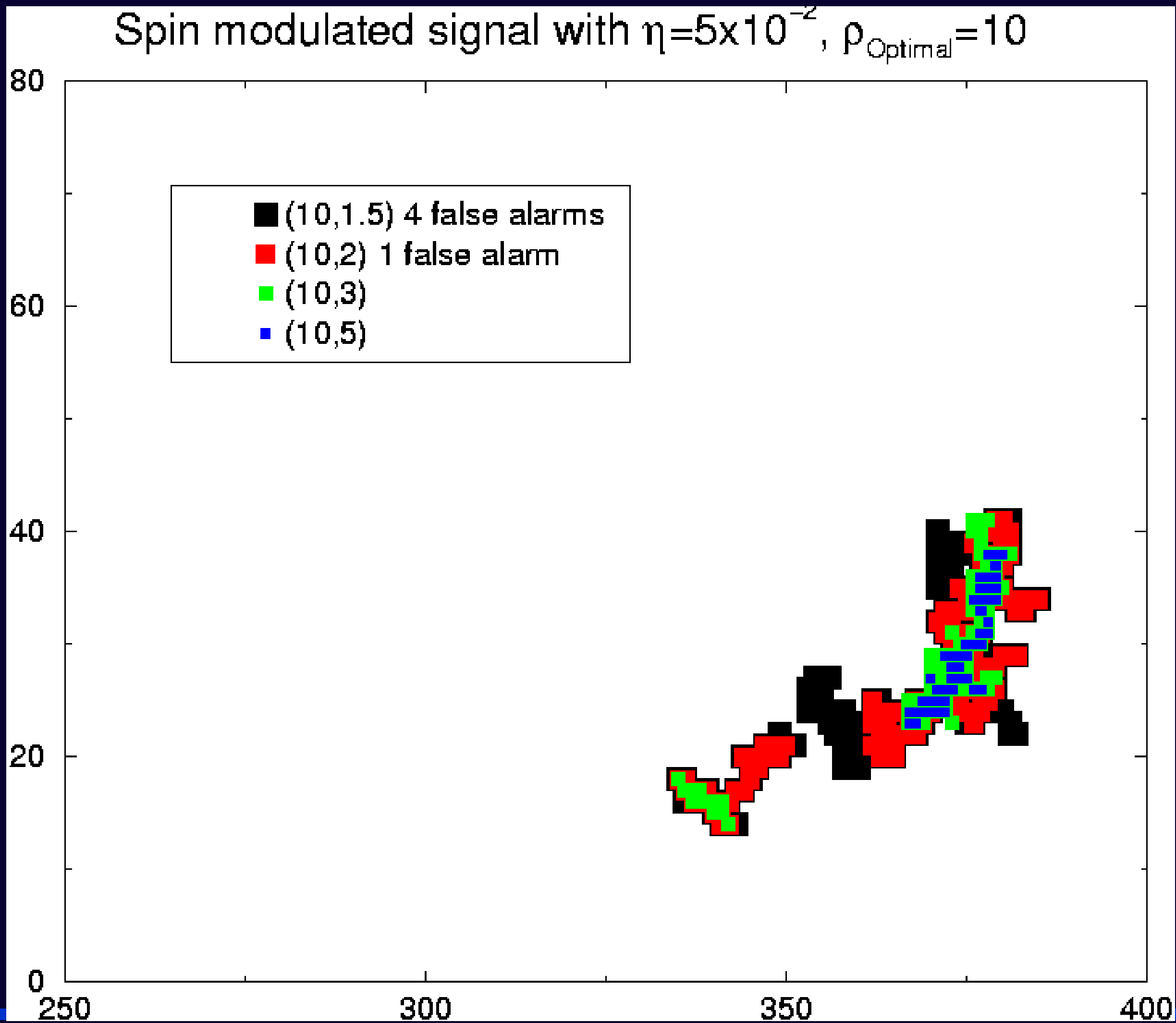
An efficient method to identify Smirches

- Use a first upper threshold to identify peaks in the TF map
- Find pixels attached to the peaks by using a second lower threshold
- Define a threshold pair (u,l) and study TF maps
- This study selects curves that have a fixed number of pixels, but ...



Tracking Smirches with HACR

- We call this two-threshold method Hierarchical Algorithm for Clusters and Ridges (HACR)
- At a fixed upper threshold a lower threshold of u/l ~ 3 maximises the area keeping low false alarm rate
- A single threshold would miss most of the signals as would large values of the ratio u/l.

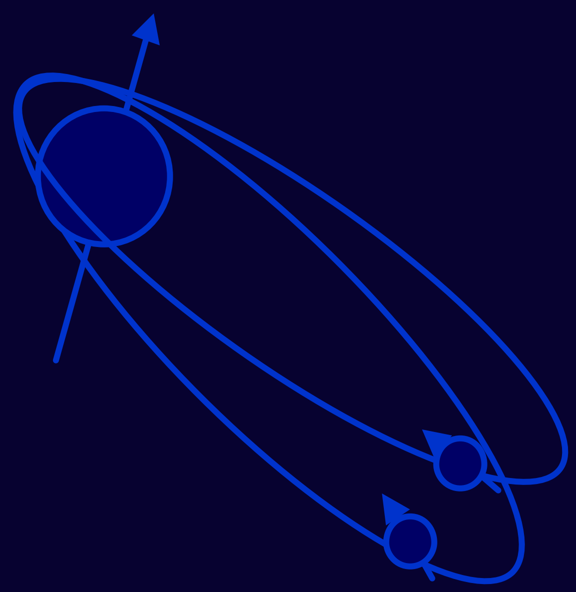


Testing Einstein's Gravity

Gravitational wave observations offer a unique opportunity to test GR in highly relativistic and strongly non-linear regimes

Testing Uniqueness Theorems

- Gravitational wave observations of small black holes falling into large black holes will allow us to measure the multipole moments of a Kerr black hole and compare with what is expected theoretically (Ryan 98)
- In this way one can test whether or not all the multipoles are related to just the spin and the mass of the hole as predicted by general relativity – It is a unique opportunity to test a theorem in geometry by an astronomical observation



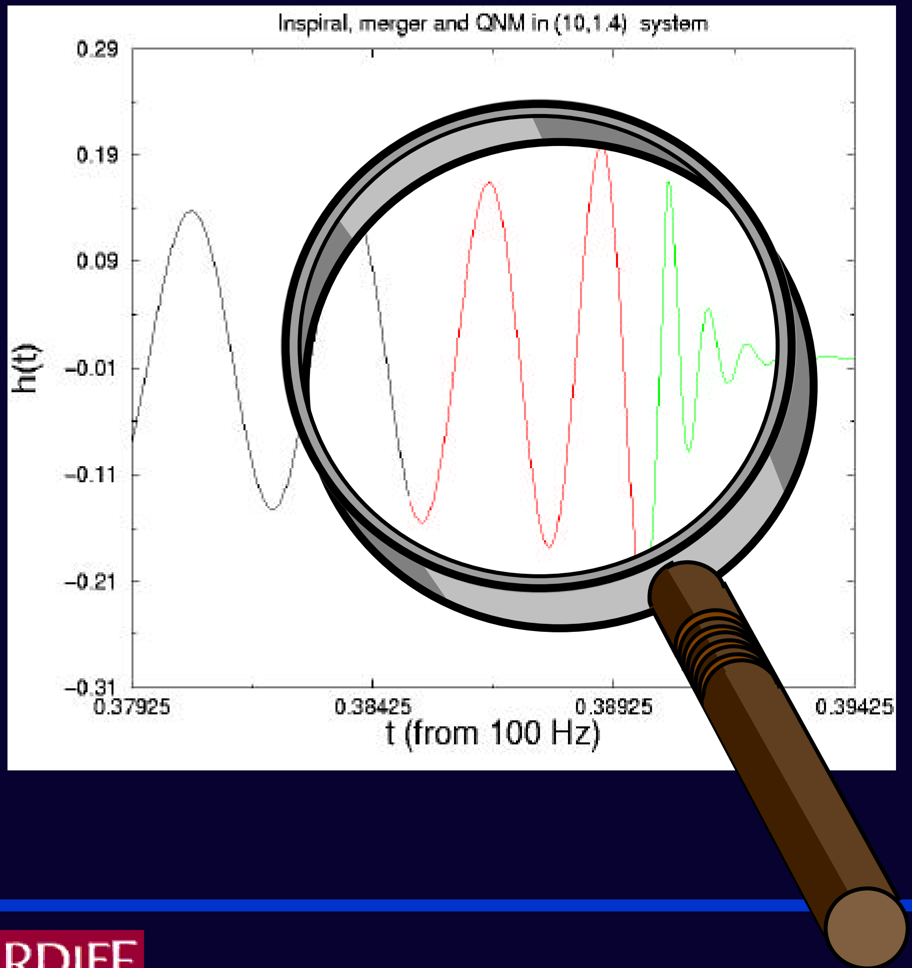
Weighing the Graviton

Cliff Will

- If gravitons are massive then their velocity will depend on their frequency via some dispersion relation
- Black hole binaries emit a chirping signal whose frequency evolution will be modulated as it traverses across from the source to the detector
- By including an additional parameter in matched filtering one could measure the mass of the graviton
  - LIGO, and especially LISA, should improve the current limits on the mass of the graviton by several orders of magnitude

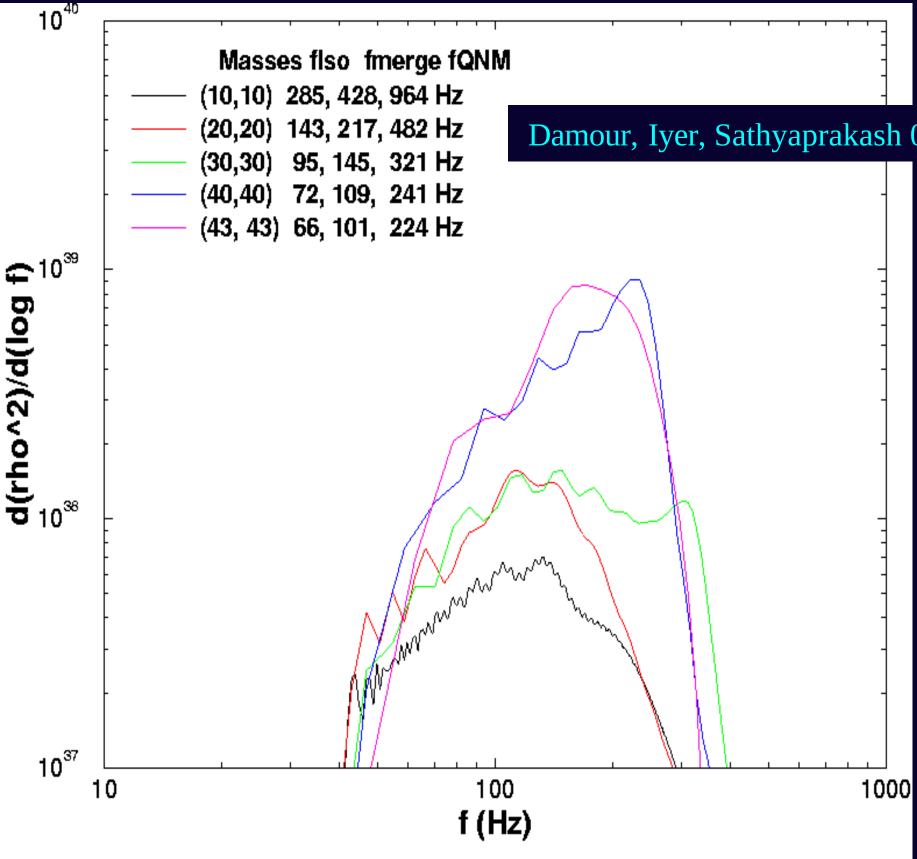
Strong Gravitational Fields

- IFOs act as a magnifying glass revealing different non-linear effects in binaries of different masses
- In the case of stellar mass BH/BH binaries we will observe the merger and quasi-normal mode ringing, gravitational wave tails, precession of the orbital plane, etc.
- Will be able to measure the two independent polarisations of the waves (C Will) and compare the validity of GR with other theories of gravity

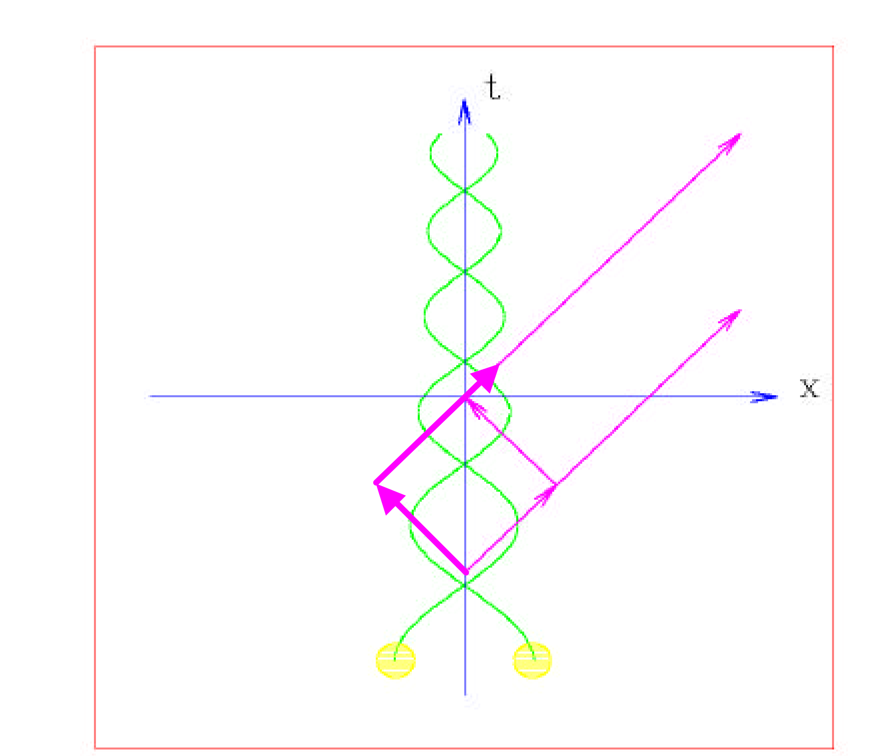


Strong filed tests of general relativity

Merger of BH binaries



Gravitational wave tails



Blanchet and Schaefer 95, Blanchet and Sathyaprakash 96

Open unsolved problems

- **Problem of signal models**
  - Radiation reaction, merger waveform - late time dynamics
- **Problem of template placement**
  - How to choose parameters in a multi-dimensional space?
- **Problem of disentangling signals**
  - How best can foreground signals be resolved from the confusion caused by a background of large population?
- **Problem of non-stationary and non-Gaussian backgrounds**
  - Veto techniques for rejecting instrumental and environmental artifacts
- **Problem of appropriate time-frequency transforms**
  - Are there time-frequency transforms well-suited to signal shapes we encounter?