Searching for continuous gravitational wave signals

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Sources of Gravitational Waves
Trieste, September 2003

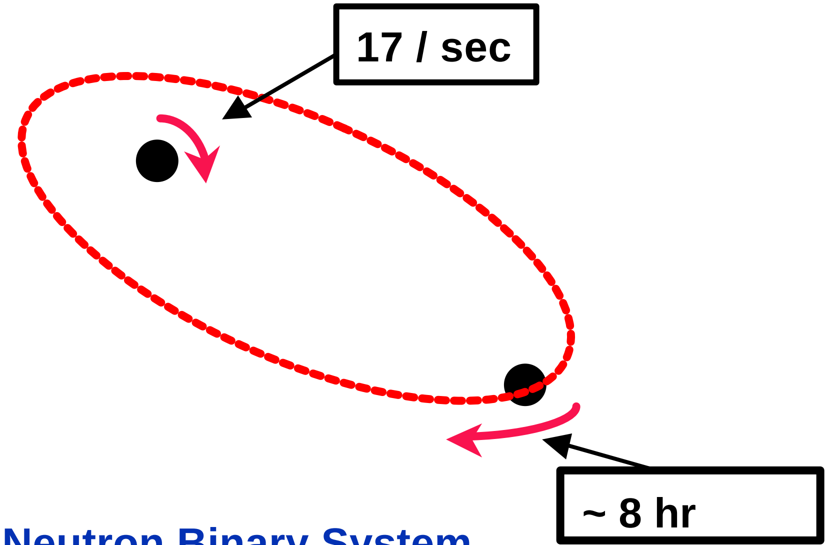
Outline

- Effort for detecting GW
- Gravitational waves from pulsars
- S1 analysis:
  - Frequency domain method
  - Time domain method
- The problem of blind surveys:
  - The incoherent Hough search

Evidence for Gravitational Waves

Neutron Binary System – Hulse & Taylor

PSR 1913 + 16 -- Timing of pulsars

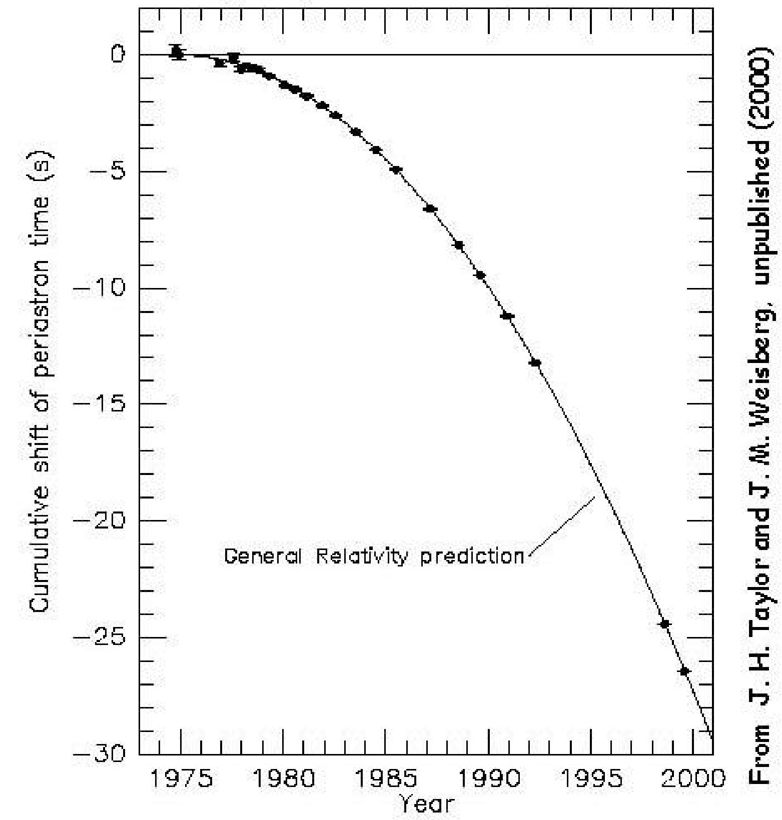


Neutron Binary System
- separated by 10^6 miles
- $m_1 = 1.4 m_\odot$; $m_2 = 1.36 m_\odot$; $e = 0.617$

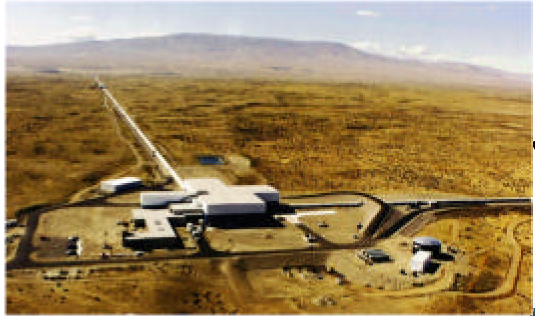
Prediction from general relativity
- spiral in by 3 mm/orbit
- rate of change orbital period

Emission of gravitational waves

Comparison between observations of the binary pulsar PSR1913+16, and the prediction of general relativity based on loss of orbital energy via gravitational waves



An International Network of Interferometers



LIGO Hanford WA (4km & 2km)

TAMA (300m)

LIGO Livingston LA (4km)

GEO (600m)

VIRGO (3km)

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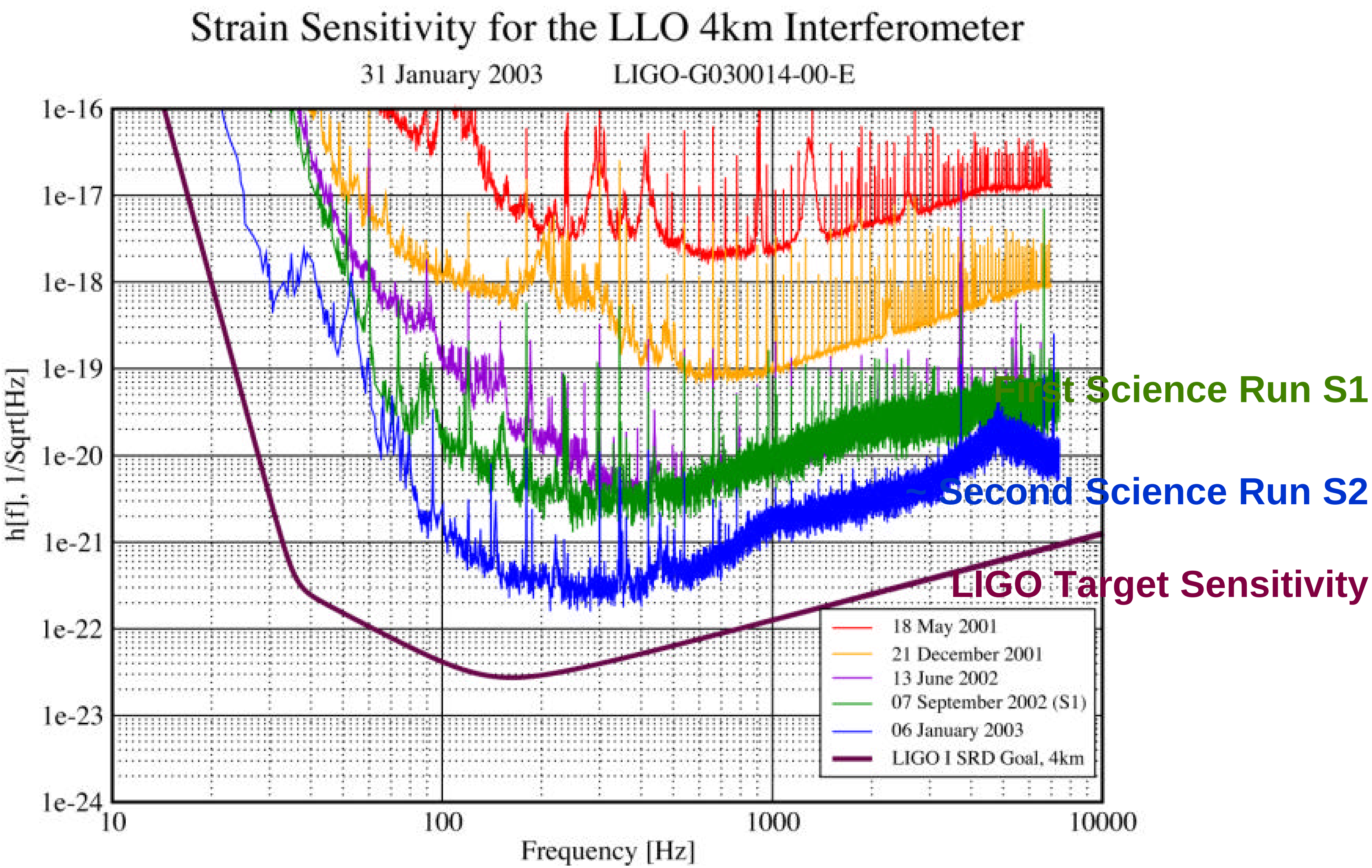
Lock Summaries S1/S2

	S1		S2	
Dates	23/8-9/9/02		14/2-14/4/03	
	Hours		Hours	
Runtime	408	(100%)	1415	(100%)
Single IFO statistics:				
GEO:	400	(98%)		
H1 (4km):	235	(58%)	1040	(74%)
H2 (2km):	298	(73%)	818	(58%)
L1 (4km):	170	(42%)	523	(37%)
Double coincidence:				
L1 && H1 :	116	(28%)	431	(31%)
L1 && H2 :	131	(32%)	351	(25%)
H1 && H2 :	188	(46%)	699	(49%)
Triple coincidence:				
L1, H1, and H2 :	96	(23%)	312	(22%)
Sensitivities:	GEO << H2 < H1 < L1			

LIGO Sensitivity Improvements



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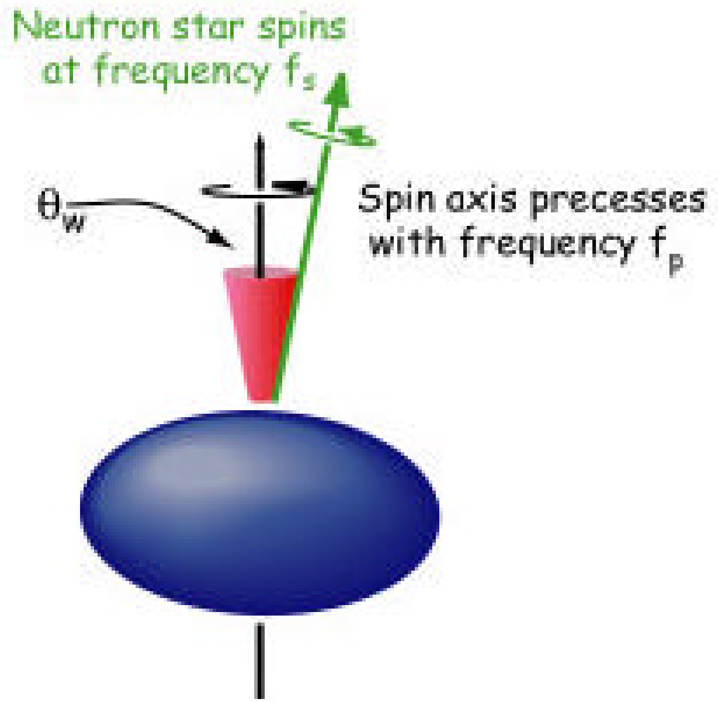
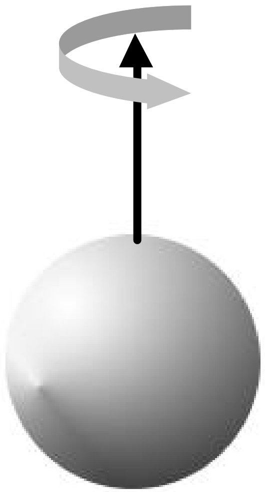
Gravitational waves from pulsars

– Pulsars (spinning neutron stars) are known to exist!

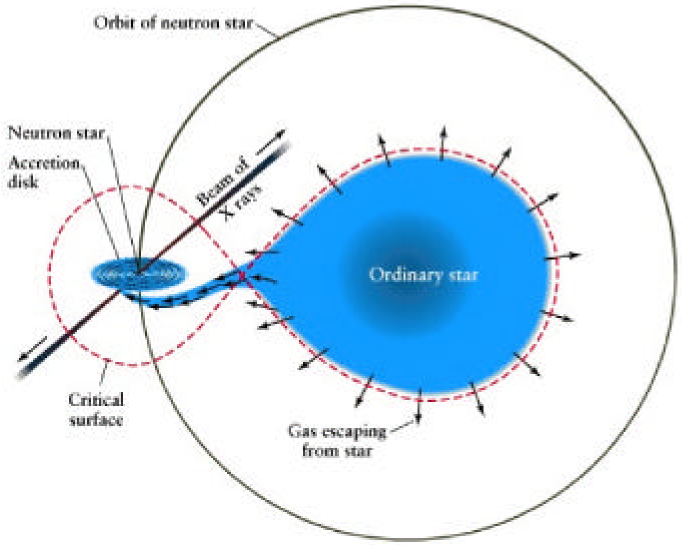
– Emit gravitational waves if they are non-axisymmetric:



Wobbling Neutron Star



Bumpy Neutron Star



Low Mass X-Ray Binaries

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GWs from pulsars: The signal

- The GW signal from a neutron star:

$$h(t) = h_0 \, ?(t) e^{F(t)}$$

A,F amplitude and phase functions

$$h_0 = \frac{8p^2 \mathrm{G}}{\mathrm{c}^4} \frac{I_{zz} f_0^2}{R} e \quad \text{intrinsic amplitude}$$

- (Signal-to-noise)^2 ~ $\displaystyle\int_0^T \frac{h^2(t)}{S_h(f_{gw})} dt$

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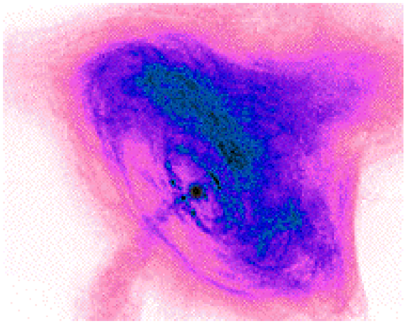
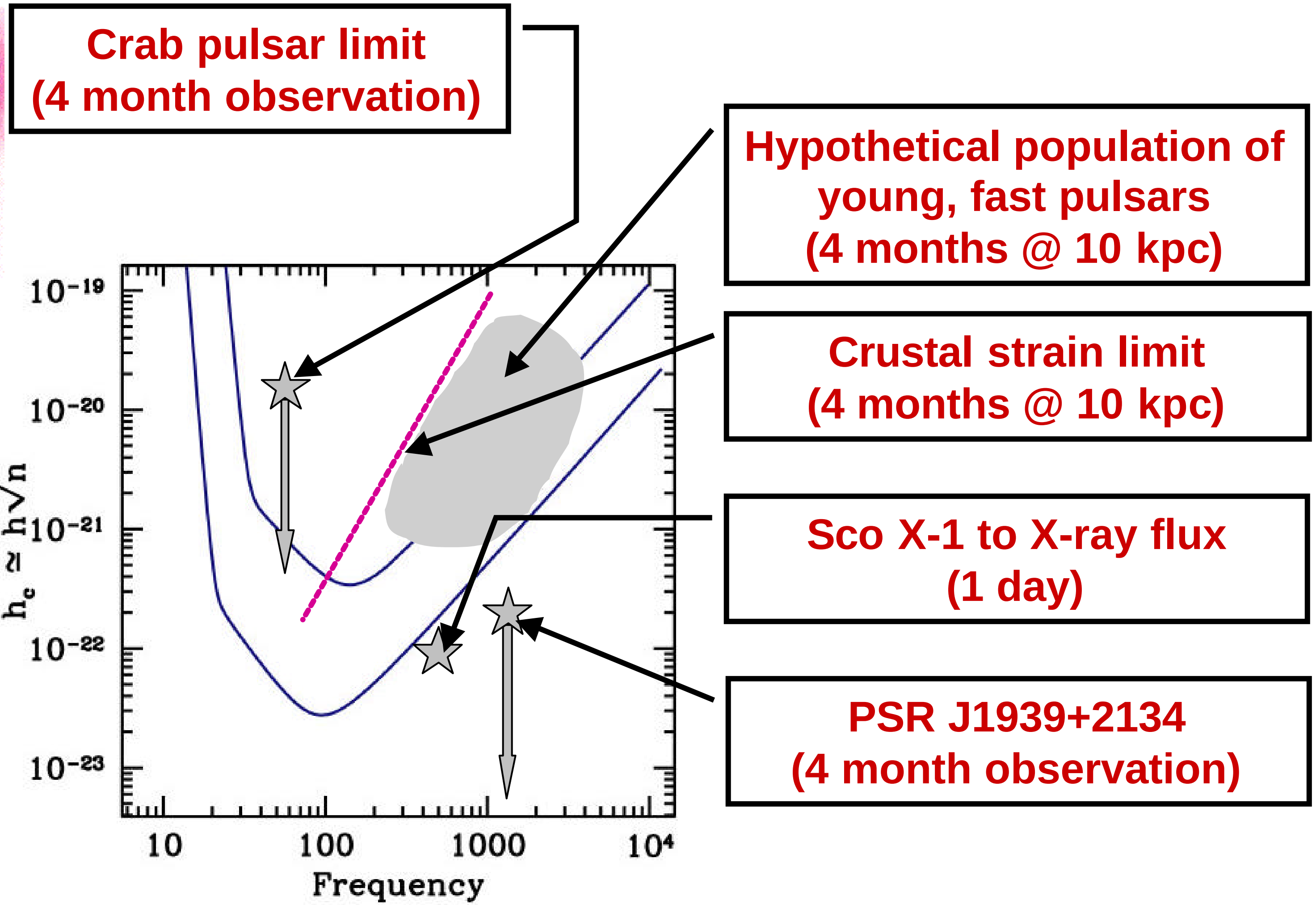
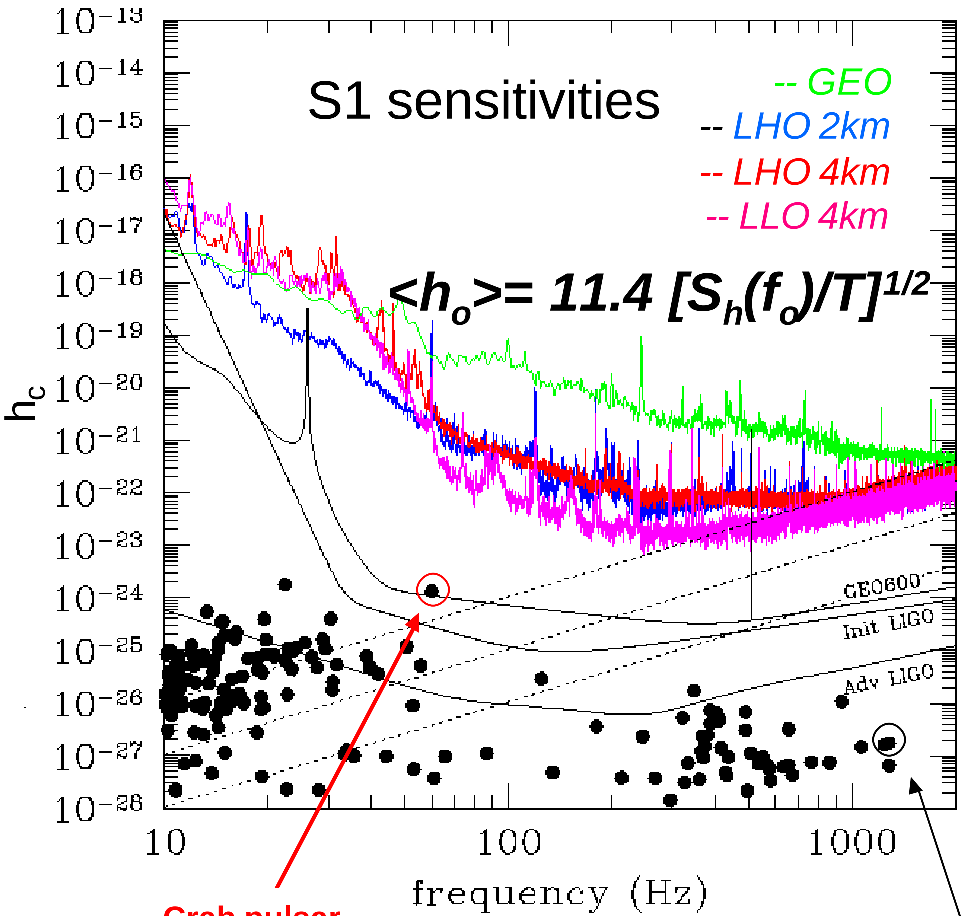
Sensitivity to Pulsars

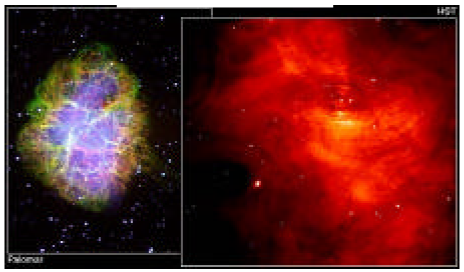




Instrumental sensitivity during S1



Crab pulsar



PSR J1939+2134
P = 0.00155781 s
f_{GW} = *1283.86 Hz*
$\dot{P}$ = 1.0511 10^{-19} s/s
D = 3.6 kpc

- **Detectable amplitudes** with a 1% false alarm rate and 10% false dismissal rate by the interferometers during S1 (colored curves) and at design sensitivities (black curves).
- **Limits** of detectability for rotating NS with equatorial ellipticity ***e* = 10^{-3} , 10^{-4} , 10^{-5} @ 8.5 kpc.**
- **Upper limits** on **$\langle h_o \rangle$** from spin-down measurements of known radio pulsars (filled circles).

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Signal model and parameters

$$h(t) = F_+(t)\,h_+(t) + F_\times(t)\,h_\times(t)$$

$$(a, d),\ f_0,\ f_1, h_0, i, y\ , f_0$$

$$\left.\begin{array}{l} F_+(t, ?\,) \\ F_\times(t, ?\,) \end{array}\right\}$$

beam-pattern functions and depend on the relative orientation of the detector w.r.t. the wave. They depend on *y* and on the amplitude modulation functions a(t) and b(t) that depend on the relative instantaneous position between source and detector.

$$\left.\begin{array}{l} h_+(t) = h_0 \dfrac{1+\cos^2 ?}{2} \cos\ ?\,(t) \\ h_\times(t) = h_0 \cos ?\sin ?\,(t) \end{array}\right\}$$

the two independent wave polarizations.

$$?\ (t) = f_0 + F\,(t)$$

the phase of the received signal depends on the initial phase and on the frequency evolution of the signal. The latter depends on the spin-down parameters and on the Doppler modulation, thus on the frequency of the signal and on the instantaneous relative velocity between source and detector.

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Search for Continuous Waves

S1 Analysis

- **Source:** PSR J1939+2134 (fastest known rotating neutron star) located 3.6 kpc from us.
  - Frequency of source: known
  - Rate of change of frequency (spindown): known
  - Sky coordinates (α, δ) of source: known
  - Amplitude h_0: unknown (though spindown implies $h_0 < 10^{-27}$)
  - Orientation ι: unknown
  - Phase, polarization φ, ψ: unknown
- **S1 Analysis goals:**
  - Search for emission at 1283.86 Hz (twice the pulsar rotation frequency). Set upper limits on strain amplitude h_0.
  - Develop and test an efficient analysis pipeline that can be used for blind searches (**frequency domain method**)
  - Develop and test an analysis pipeline optimized for efficient "known target parameter" searches (**time domain method**)

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Frequency domain method

- Input data: Short Fourier Transforms (SFT) of time series
  - Time baseline: 60 sec.High-pass filtered at 100 Hz & Tukey windowed. Calibrated once per minute
  - Data are studied in a narrow frequency band (0.5 Hz) and $S_h(f)$ is estimated for each SFT.
- Dirichlet Kernel used to combine data from different SFTs (efficiently implements matched filtering).
- Detection statistic used is described in *[Jaranoski, Krolak, Schutz, Phys. Rev. D58(1998) 063001]*
- The ***F*** detection statistic provides the maximum value of the likelihood ratio with respect to the unknown parameters, h_0, $\cos i$, y *and* f_0 given the data and the template parameters that are known

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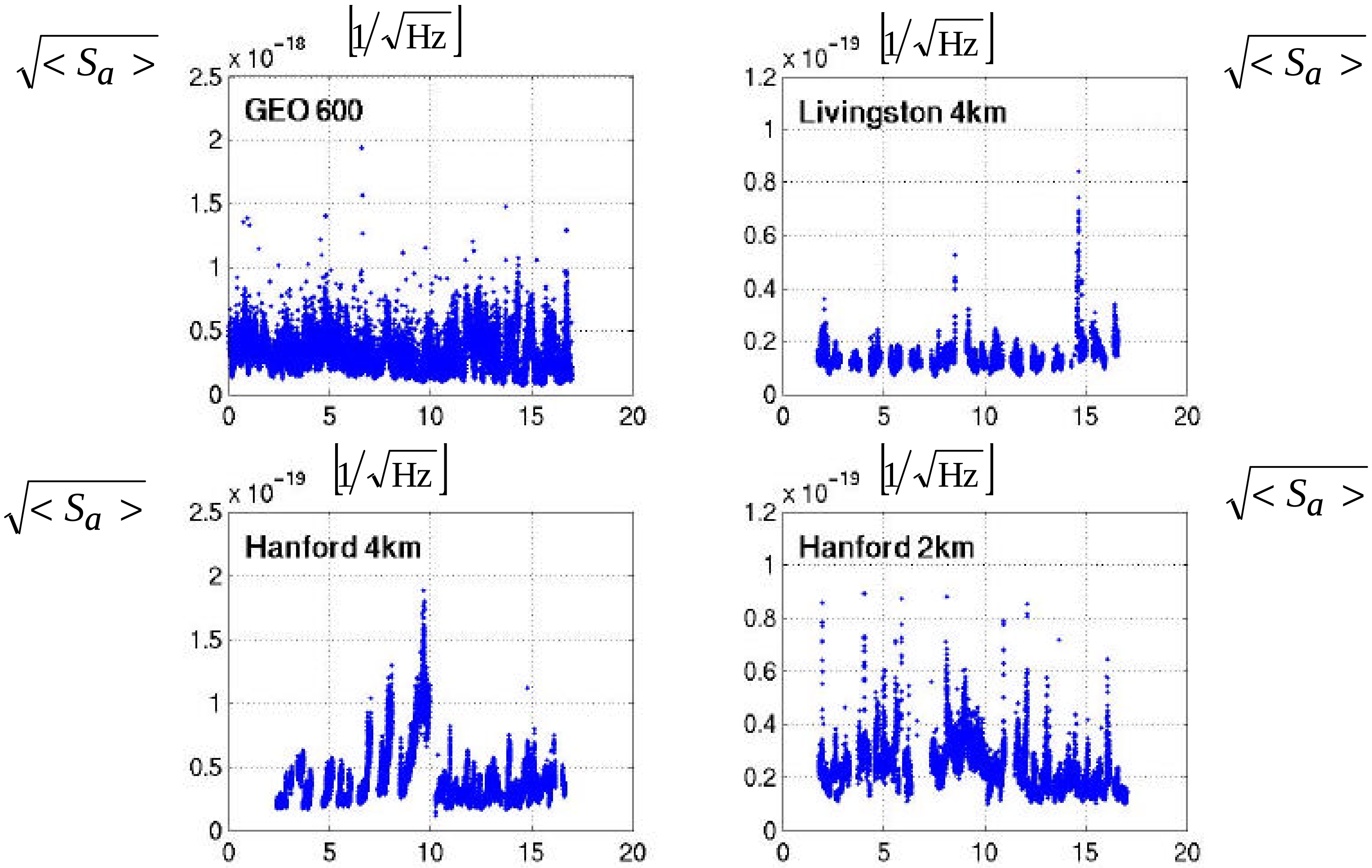
Frequency domain method

- The outcome of a target search is a number ***F**** that represents the optimal detection statistic for this search.
- **2*F**** is a **random variable:** For ***Gaussian*** *stationary noise*, follows a c^2 distribution with 4 degrees of freedom with a non-centrality parameter $l \, \mu (h|h)$. Fixing i, y *and* f_0 , for every h_0, we can obtain a pdf curve: $p(2F|h_0)$
- The **frequentist** approach says the data will contain a signal with amplitude ³ h_0 , with confidence **C**, if in repeated experiments, some fraction of trials C would yield a value of the detection statistics ³ ***F****

$$C(h_0) = \int_{2F*}^{\infty} p(2F \mid h_0) \mathrm{d}(2F)$$

- Use signal injection Monte Carlos to measure Probability Distribution Function (PDF) of *F*

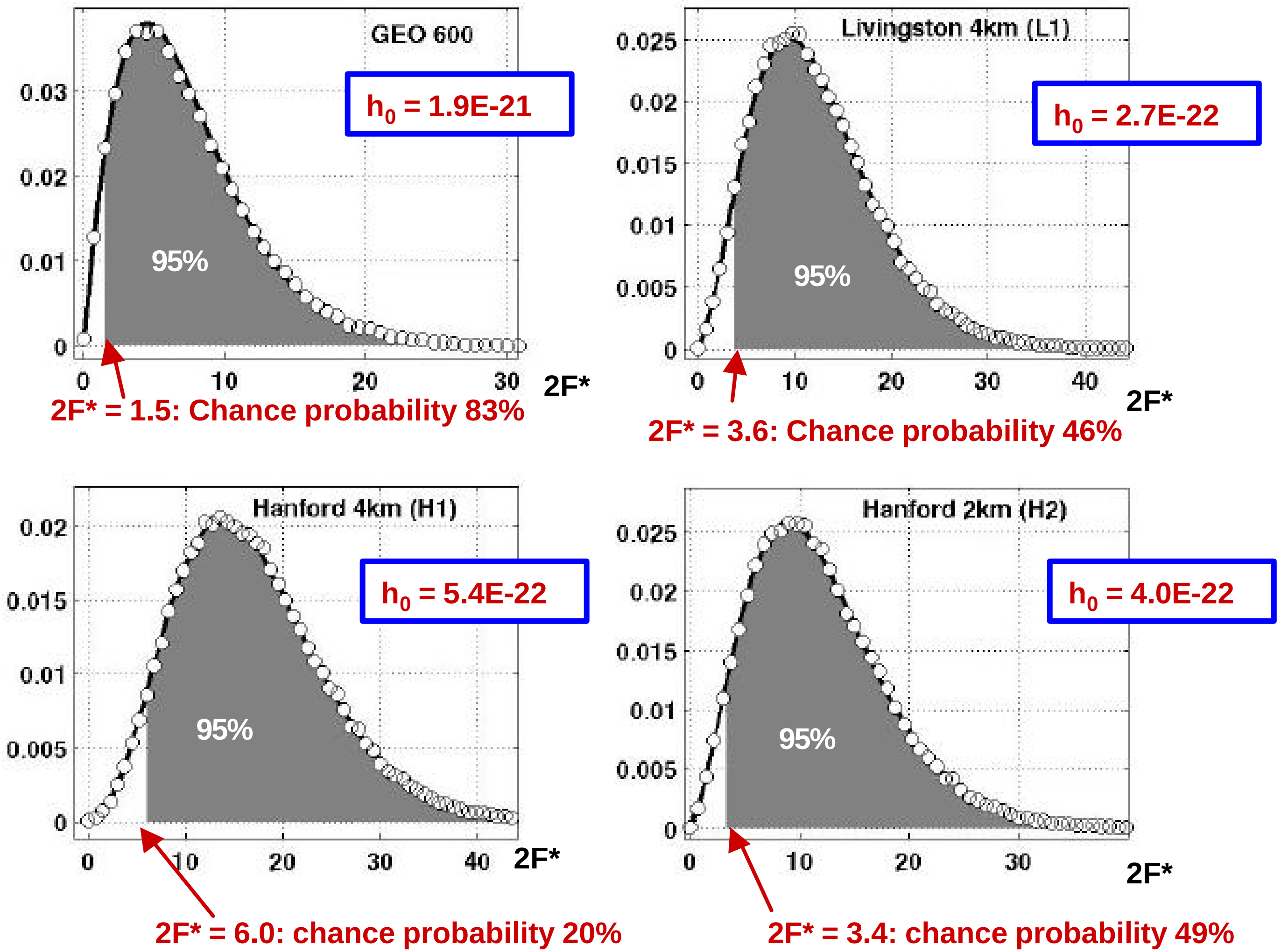
The data: time behaviour (4 Hz band around 1283 Hz)



Measured PDFs for the F statistic

with fake injected *worst-case* signals at nearby frequencies

Note: **hundreds of thousands** of injections were needed to get such nice clean statistics!



Time domain method

- Method developed to handle NS with ~ known complex phase evolution. Computationally cheap.
- Two stages of heterodyning to reduce and filter data:
  - Coarse stage (fixed frequency) 16384 ⇒ 4 samples/sec
  - Fine stage (Doppler & spin-down correction) 240 ⇒ 1 samples/min ⇒ B_k
- Noise variance estimated every minute to account for non-stationarity. ⇒ σ_k
- Standard Bayesian parameter fitting problem, using time-domain model for signal -- a function of the unknown source parameters h_0, ι, ψ and ϕ_0

$$y(t;\mathbf{a}) = \tfrac{1}{4} h_0 F_+(t,\psi)(1+\cos^2\iota)e^{2i\phi_0} - \tfrac{1}{2} i h_0 F_\times(t,\psi)\cos\iota\, e^{2i\phi_0}$$

Time domain: Bayesian approach

- We take a Bayesian approach, and determine the joint posterior distribution of the probability of our unknown parameters, using uniform priors on h_0 ,cos i, y *and* f_0 over their accessible values, i.e.

$$p(\mathbf{a} \mid \{B_k\}) \propto p(\mathbf{a}) \cdot p(\{B_k\} \mid \mathbf{a})$$

posterior prior likelihood

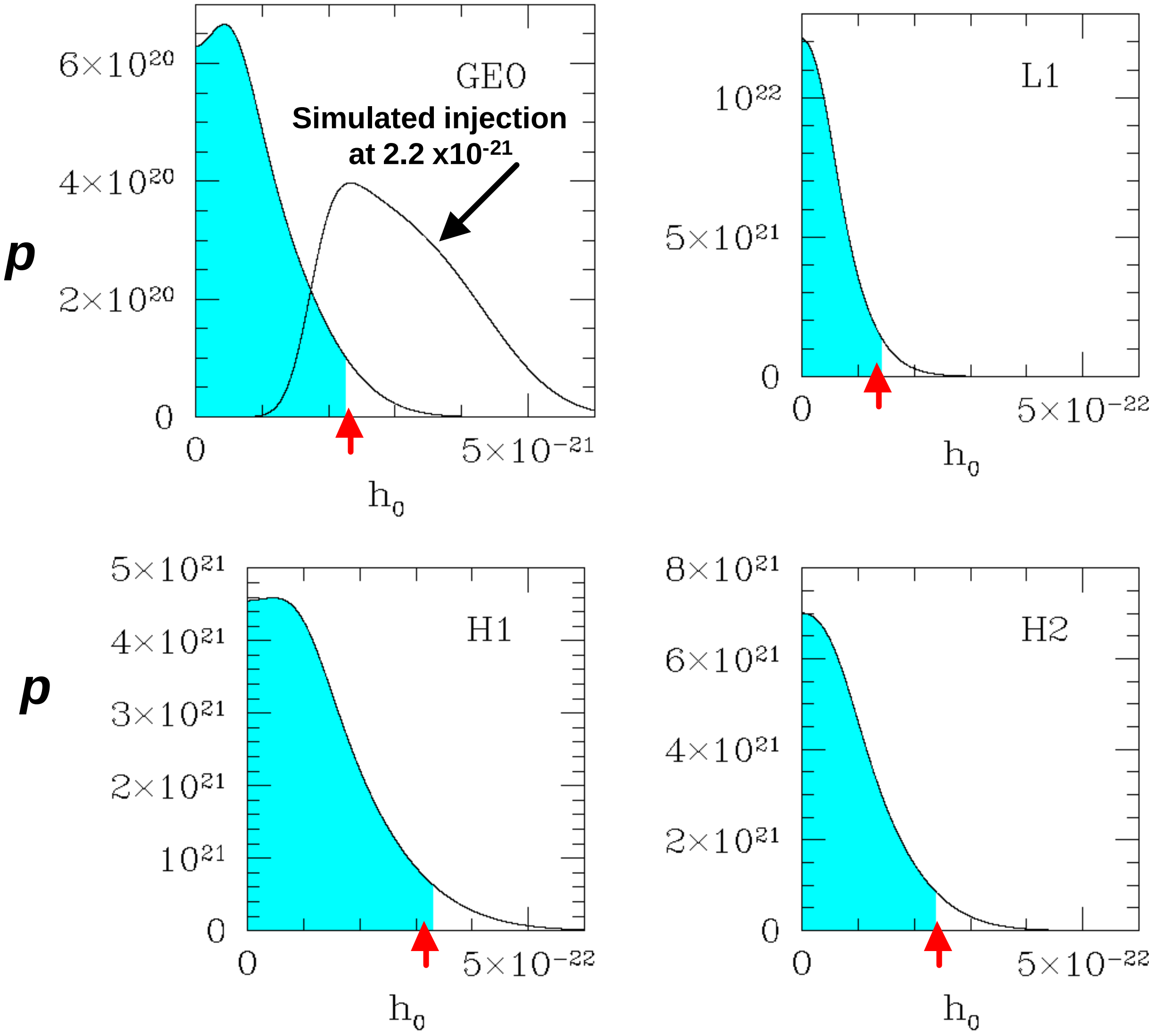
- The likelihood $\propto \exp(-\chi^2/2)$, where $$c^2(\mathbf{a}) = \sum_k \left| \frac{B_k - y(t;\mathbf{a})}{s_k} \right|^2$$
- To get the posterior PDF for h_0, marginalizing with respect to the nuisance parameters cos i, y *and* f_0

$$p(h_0 \mid \{B_k\}) \propto \iiint e^{-c^2/2} \mathrm{d} f_0 \mathrm{d} y \, \mathrm{d}\cos i$$

- The 95% upper credible limit is set by the value h_{95} satisfying

$$0.95 = \int_0^{h_{95}} p(h_0 \mid \{B_k\}) \mathrm{d} h_0$$

Posterior PDFs for CW time domain analyses



shaded area = 95% of total area

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S1 Results

No evidence of continuous wave emission from PSR J1939+2134.

Summary of **95% upper limits for** h_o:

IFO	Frequentist FDS	Bayesian TDS
GEO	$(1.9\pm0.1) \times 10^{-21}$	$(2.2\pm0.1) \times 10^{-21}$
LLO	$(2.7\pm0.3) \times 10^{-22}$	$(1.4\pm0.1) \times 10^{-22}$
LHO-4K	$(5.4\pm0.6) \times 10^{-22}$	$(3.3\pm0.3) \times 10^{-22}$
LHO-2K	$(4.0\pm0.5) \times 10^{-22}$	$(2.4\pm0.2) \times 10^{-22}$

- $h_o<1.4\times10^{-22}$ constrains **ellipticity < 2.7 x 10^{-4}** ($M=1.4\ M_{sun}$, r=10 km, R=3.6 kpc)
- Previous results for this pulsar: **$h_o < 10^{-20}$** (Glasgow, Hough et al., 1983), **$h_o < 1.5 \times 10^{-17}$** (Caltech, Hereld, 1983).

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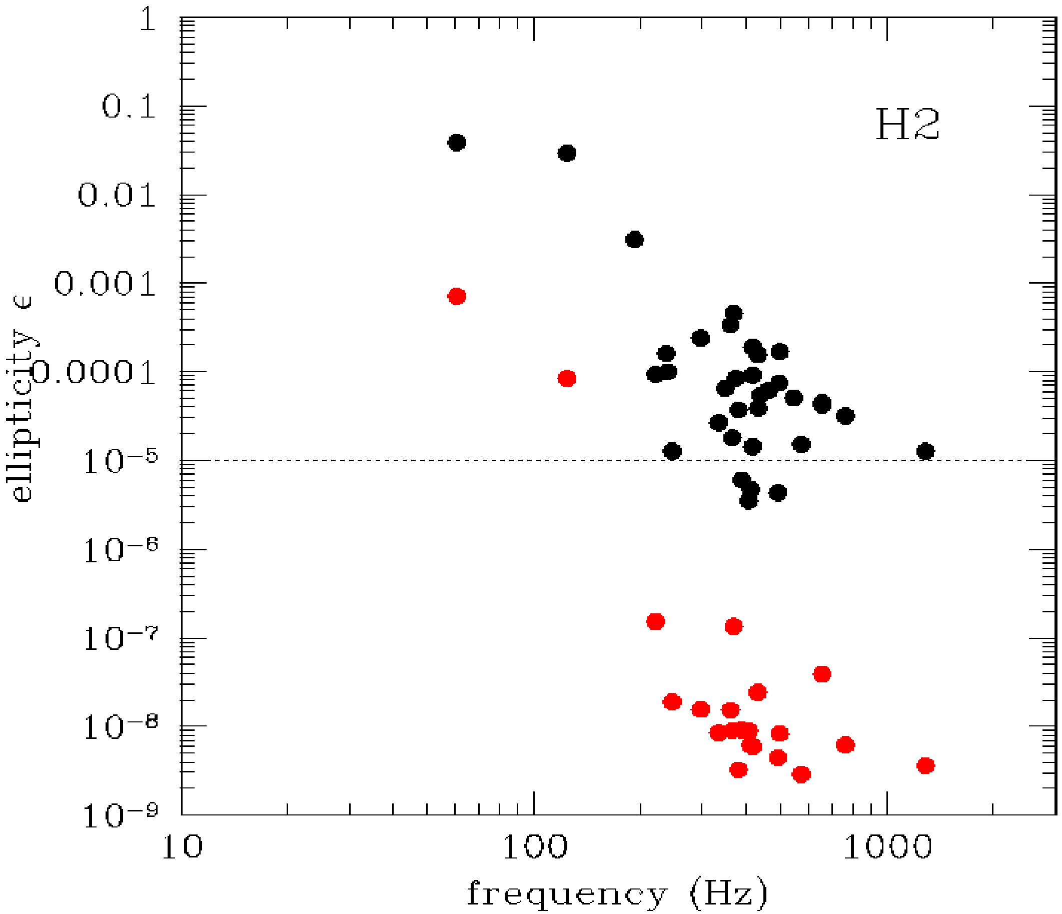
Isolated pulsars with f_{GW} >50 Hz

#	PULSAR	FREQUENCY (HZ)
1	B0021-72C	173.708218966053
2	B0021-72D	186.651669856838
3	B0021-72F	381.15866365655
4	B0021-72G	247.50152509652
5	B0021-72L	230.08774629150
6	B0021-72M	271.98722878893
7	B0021-72N	327.44431861755
8	J0024-7204V	207.90
9	J0030+0451	205.53069927493
10	**B0531+21**	**30.2254370**
11	J0537-6910	62.055096
12	J0711-6830	182.117237666390
13	J1024-0719	193.71568669103
14	B1516+02A	180.06362469727
15	J1629-6902	166.64990604074
16	B1639+36A	96.362234564
17	J1709+23	215.94
18	J1721-2457	285.9893480119
19	J1730-2304	123.110289179797
20	J1744-1134	245.4261237073059
21	J1748-2446C	118.5382530563
22	B1820-30A	183.82343541777
23	B1821-24	327.40566514989
24	J1910-5959B	119.6487328457
25	J1910-5959C	189.48987107019
26	J1910-5959D	110.6771919840
27	J1910-5959E	218.7338575893
28	B1937+21	641.9282611068082
29	J2124-3358	202.793897234988
30	B2127+11D	208.211688010
31	B2127+11E	214.987407906
32	B2127+11F	248.321180760
33	B2127+11H	148.293272565
34	J2322+2057	207.968166802197

Black dots show expected upper limit using H2 S2 data.

Red dots show limits on ellipticity based on spindown.



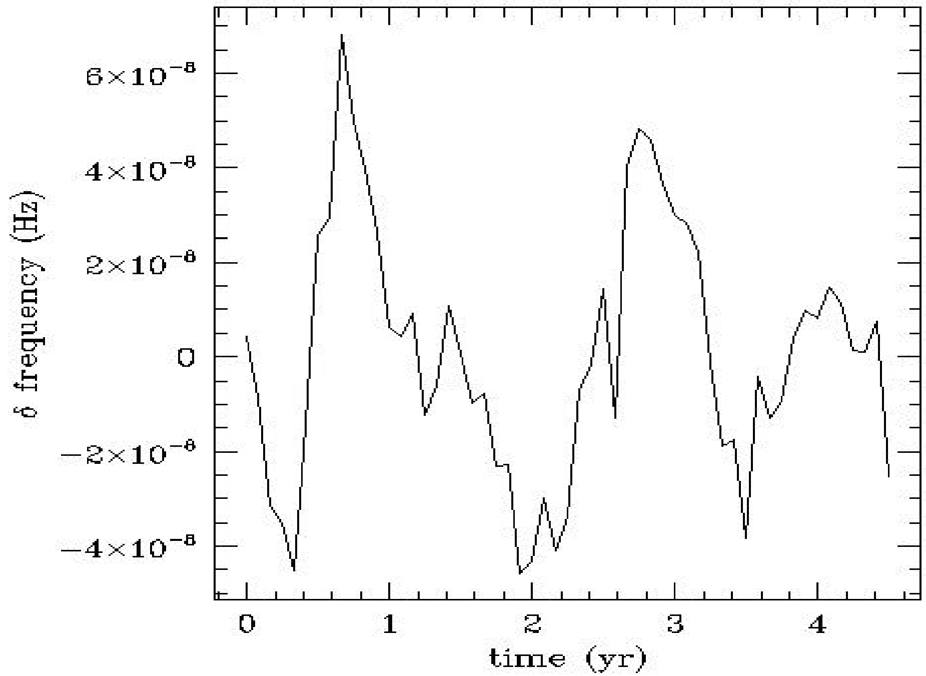
Crab pulsar

Young pulsar (~60 Hz) with large spin-down and timing noise

Frequency residuals will cause large deviations in phase if not taken into account.

Use Jodrell Bank monthly crab ephemeris to recalculate spin down params every month.

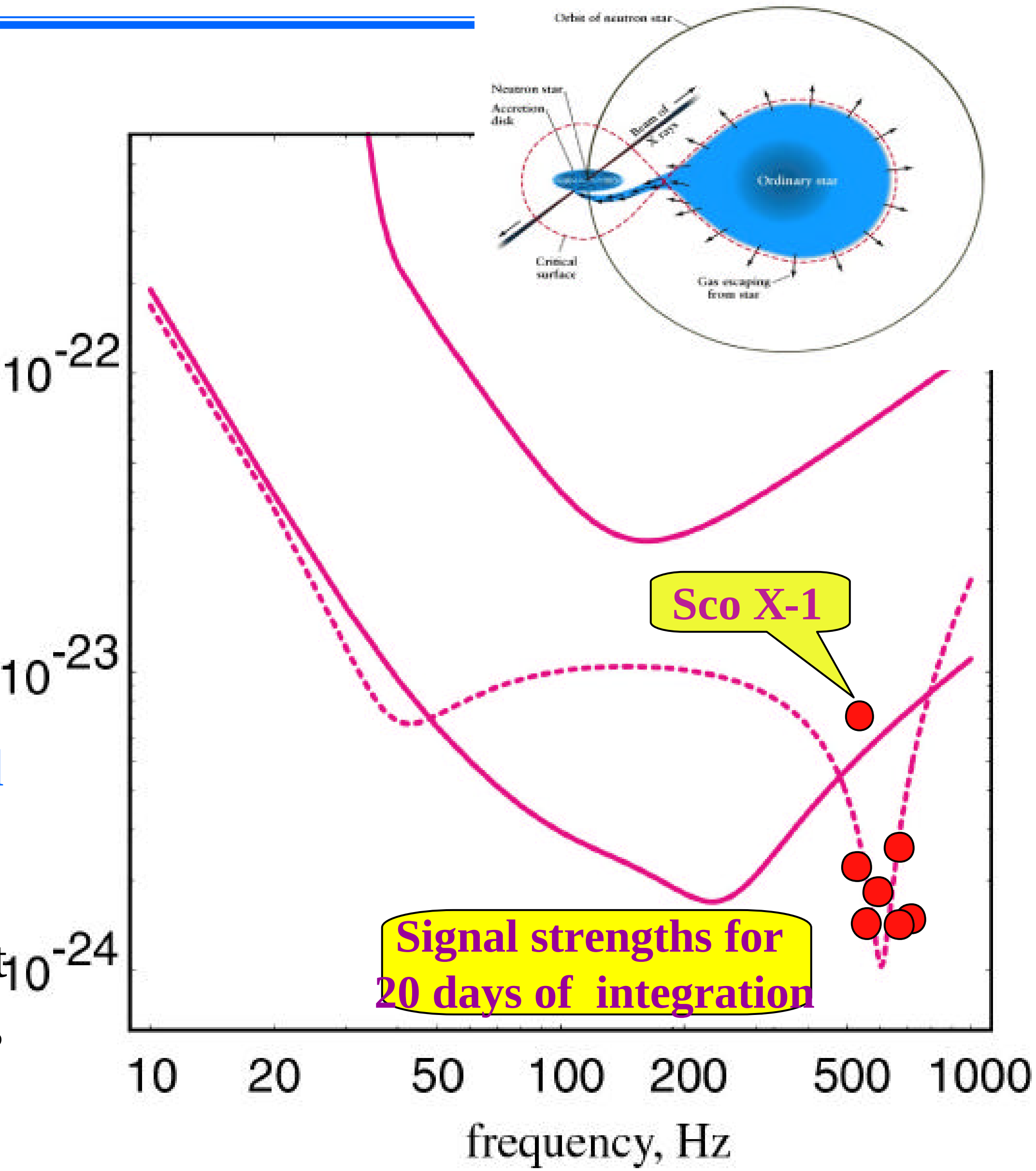
No glitches during S2 but in the event of a glitch we would need to add an extra parameter for jump in GW phase



Time interval (months)	RMS phase deviations (rad)	RMS frequency deviations (Hz)
2	1.3	1.9E-6
4	157.1	9.7E-5
6	2115.7	8.4E-4
8	12286	3.6E-3
10	46229.3	1.0E-2
12	134652.8	2.5E-2

GW pulsars in binary systems

- Physical scenarios:
  - Accretion induced temperature asymmetry (Bildsten, 1998; Ushomirsky, Cutler, Bildsten, 2000; Wagoner, 1984)
  - R-modes (Andersson et al, 1999; Wagoner, 2002)
- LMXB frequencies are clustered (could be detected by advanced LIGO).
- We need to take into account the additional Doppler effect produced by the source motion:
  - 3 parameters for circular orbit
  - 5 parameters for eccentric orbit
  - possible relativistic corrections
  - Frequency unknown a priori



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All-Sky and targeted surveys for unknown pulsars

It is necessary to search for every signal template distinguishable in parameter space. Number of parameter points required for a coherent $T=10^7$s search

[*Brady et al., Phys.Rev.D57 (1998)2101*]:

Class	f (Hz)	t (Yrs)	N_s	Directed	All-sky
Slow-old	<200	$>10^3$	1	**3.7×10^6**	**1.1×10^{10}**
Fast-old	$<10^3$	$>10^3$	1	**1.2×10^8**	**1.3×10^{16}**
Slow-young	<200	>40	3	**8.5×10^{12}**	**1.7×10^{18}**
Fast-young	$<10^3$	>40	3	**1.4×10^{15}**	**8×10^{21}**

Number of templates grows dramatically with the integration time. To search this many parameter space coherently, with the optimum sensitivity that can be achieved by matched filtering, is computationally prohibitive.

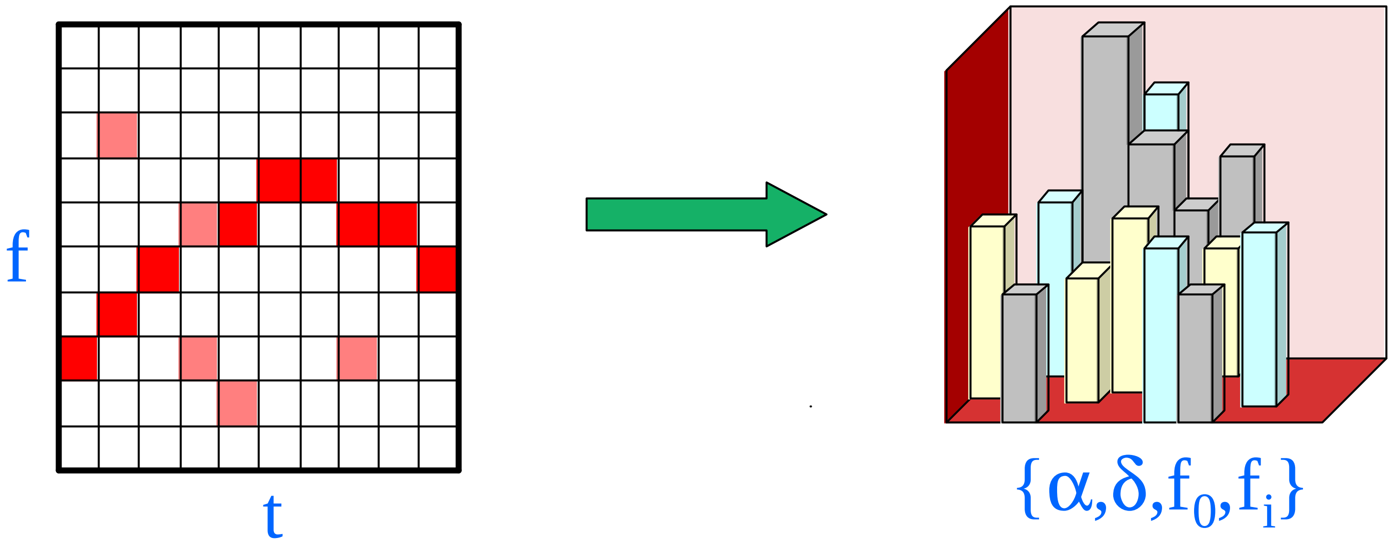
=>It is necessary to explore alternative search strategies

The concept of the Hough transform

– The idea is to perform a search over the total observation time using an incoherent (sub-optimal) method:

We propose to search for evidence of a signal whose frequency is changing over time in precisely the pattern expected for some one of the parameter sets

– The method used is the **Hough transform**



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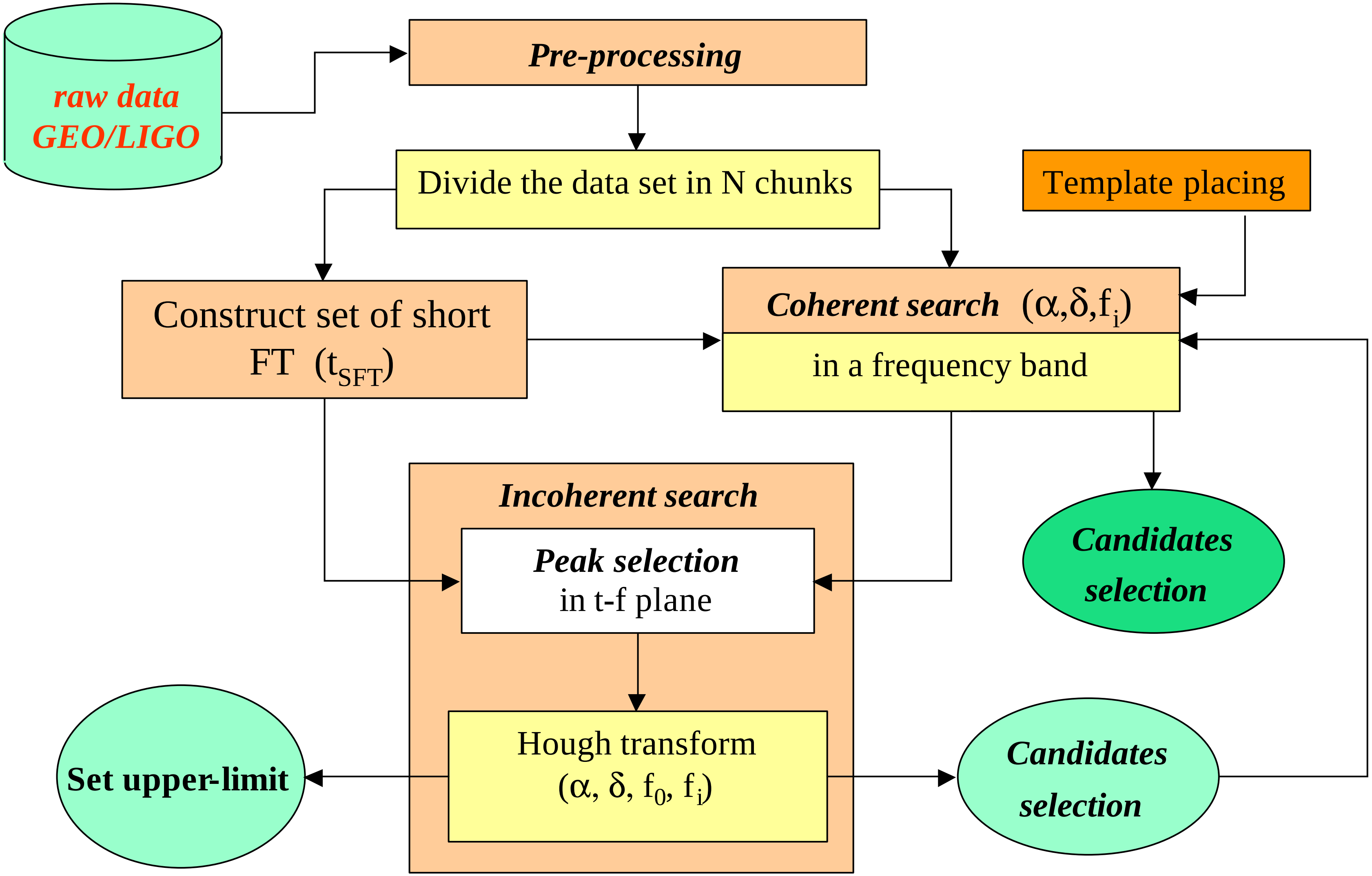
Hierarchical Hough transform strategy



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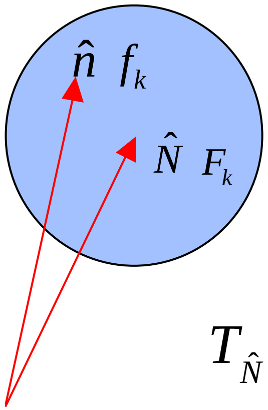
The time-frequency pattern

- SFT data

$$f(t) - f_0(t) = f_0(t)\,\frac{\vec{v}(t)}{c}\cdot\hat{n}$$

- Demodulated data

$$f(t) - F_0(t) = \vec{x}(t)\cdot(\hat{n} - \hat{N})$$



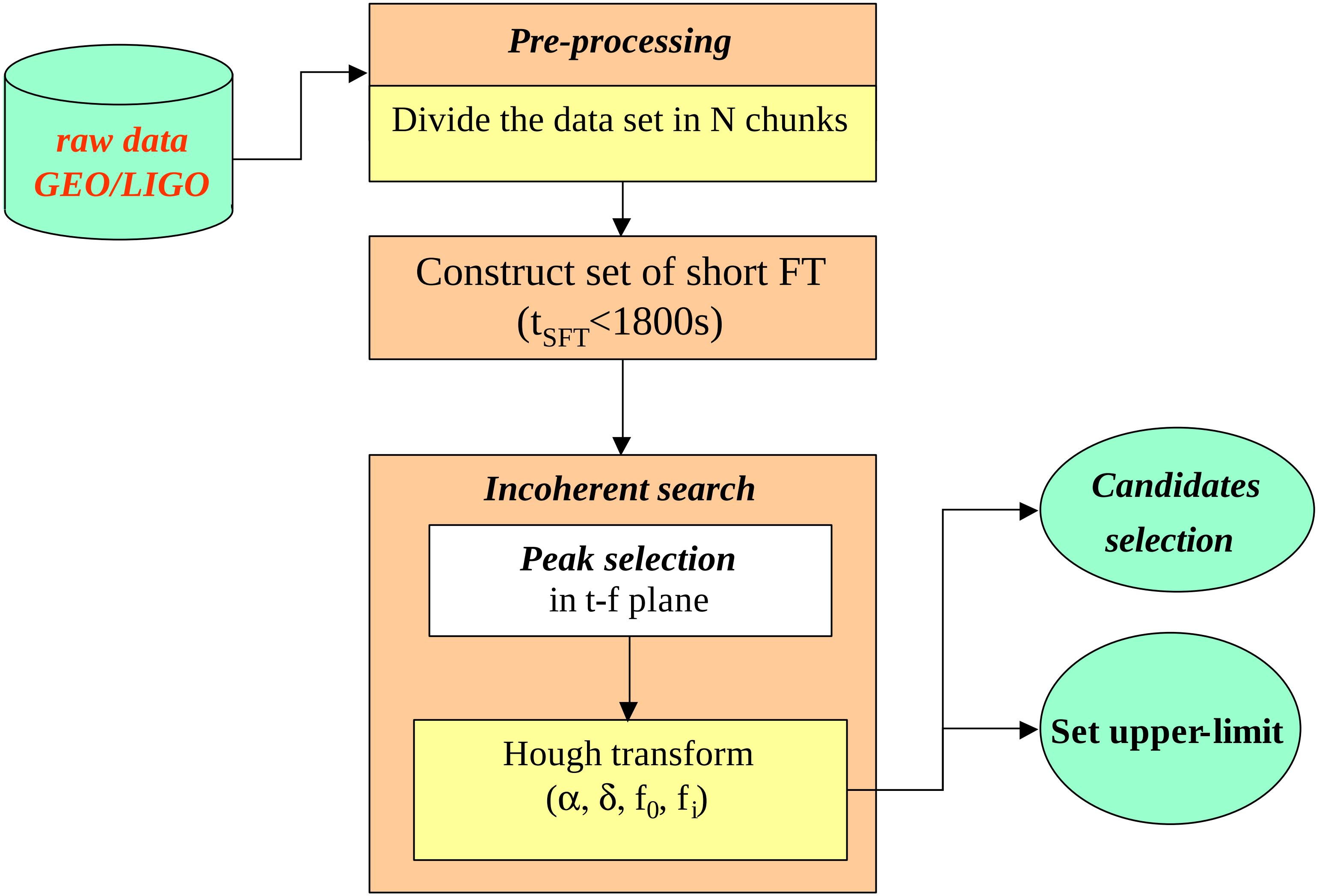
$$F_0(t) \equiv f_0 + \sum_k \Delta f_k \left[\Delta T_{\hat{N}}(t)\right]^k \qquad \Delta f_k \equiv f_k - F_k$$

$$T_{\hat{N}}(t) = t + \frac{\vec{x}(t)\cdot\hat{N}}{c} + \text{☹}$$

Time at the SSB for a given sky position

$$\vec{x}(t) = \left(F_0(t) + \sum_k F_k \left[\Delta T_{\hat{N}}(t)\right]^k\right)\frac{\vec{v}(t)}{c} + \left(\sum_k k F_k \left[\Delta T_{\hat{N}}(t)\right]^{k-1}\right)\frac{\Delta\vec{x}(t)}{c}$$

Incoherent Hough search: Pipeline for S2



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Peak selection in the t-f plane

- Input data: Short Fourier Transforms (SFT) of time series
  (Time baseline: 1800 sec, calibrated)

$$\tilde{x}(f) = \tilde{n}(f) + \tilde{h}(f)$$

- For every SFT, select frequency bins i such

$$\rho_i = \frac{|\tilde{x}(f_i)|^2}{\langle|\tilde{n}(f_i)|^2\rangle} = \frac{|\tilde{x}(f_i)|^2}{S_n(f_i)T_{SFT}}$$

exceeds some threshold ρ_0

➔ time-frequency plane of zeros and ones

- $p(\rho|h, S_n)$ follows a χ^2 distribution with 2 degrees of freedom:

$$\langle\rho_i\rangle = 1 + \frac{|\tilde{h}(f_i)|^2}{S_n(f_i)T_{SFT}} \qquad \sigma_\rho^2 = 1 + \frac{2|\tilde{h}(f_i)|^2}{S_n(f_i)T_{SFT}}$$

- The false alarm and detection probabilities for a threshold ρ_0 are

$$\alpha(\rho_0|S_n) = \int_{\rho_0}^{\infty} p(\rho|0, S_n)\,d\rho = e^{-\rho_0}, \quad \eta(\rho_0|h, S_n) = \int_{\rho_0}^{\infty} p(\rho|h, S_n)\,d\rho$$

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Hough statistics

- After performing the HT using N SFTs, the probability that the pixel $\{\alpha,\delta,f_0,f_i\}$ has a number count n is given by a **binomial distribution:**

$$P(n \mid p, N) = \binom{N}{n} p^n (1-p)^{N-n}$$

$$\langle n \rangle = Np, \quad s_n^2 = Np(1-p)$$

$$p = \begin{cases} a & \text{signal absent} \\ h & \text{signal present} \end{cases}$$

- The Hough false alarm and false dismissal probabilities for a threshold n_0

$$a_H(n_0, a, N) = \sum_{n=n_0}^{N} \binom{N}{n} a^n (1-a)^{N-n}$$

➔ Candidates selection

$$b_H(n_0, h, N) = \sum_{n=0}^{n_0-1} \binom{N}{n} h^n (1-h)^{N-n}$$

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Frequentist Analysis

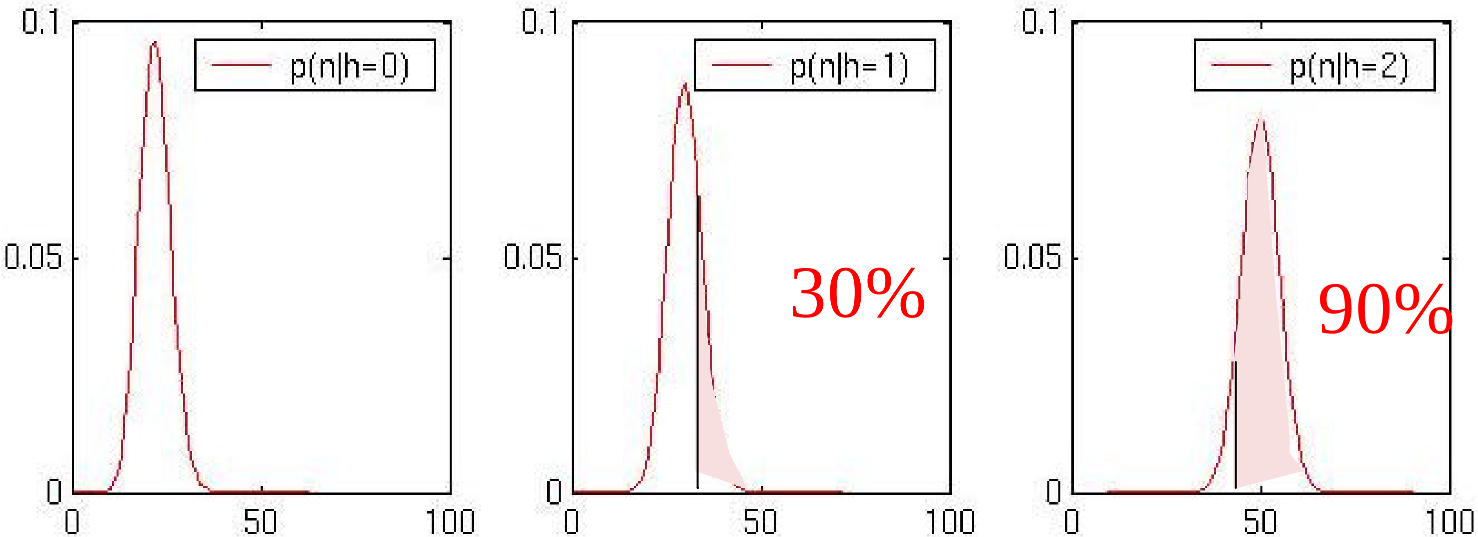
- Perform the Hough transform for a set of points in parameter space $l=\{\alpha,\delta,f_0,f_i\}\in$ **S** , given the data:

$$\textbf{HT: S} \rightarrow \textbf{N}$$
$$\lambda \rightarrow n(l)$$

- Determine the maximum number count n*

$$n^* = \max\, (n(l)) : \lambda \in \mathbf{S}$$

- Determine the probability distribution $p(n|h_0)$ for a range of h_0



Frequentist Analysis

- Perform the Hough transform for a set of points in parameter space $\lambda=\{\alpha,\delta,f_0,f_i\}\in \mathbf{S}$, given the data:

$$\textbf{HT: S} \rightarrow \textbf{N}$$
$$\lambda \rightarrow n(\lambda)$$

- Determine the maximum number count n*

$$n^* = \max\,(n(\lambda)) : \lambda \in \mathbf{S}$$

- Determine the probability distribution $p(n|h_0)$ for a range of h_0
- The 95% frequentist upper limit $h_0^{95\%}$ is the value such that for repeated trials with a signal $h_0 \geq h_0^{95\%}$, we would obtain $n \geq n^*$ more than 95% of the time

$$0.95 = \sum_{n=n^*}^{N} p\left(n|h_0^{95\%}\right)$$

•Compute $p(n|h_0)$ via Monte Carlo signal injection, using $\lambda \in \mathbf{S}$, and $\phi_0 \in [0,2\pi]$, $\psi \in [-\pi/4,\pi/4]$, $\cos\iota \in [-1,1]$.

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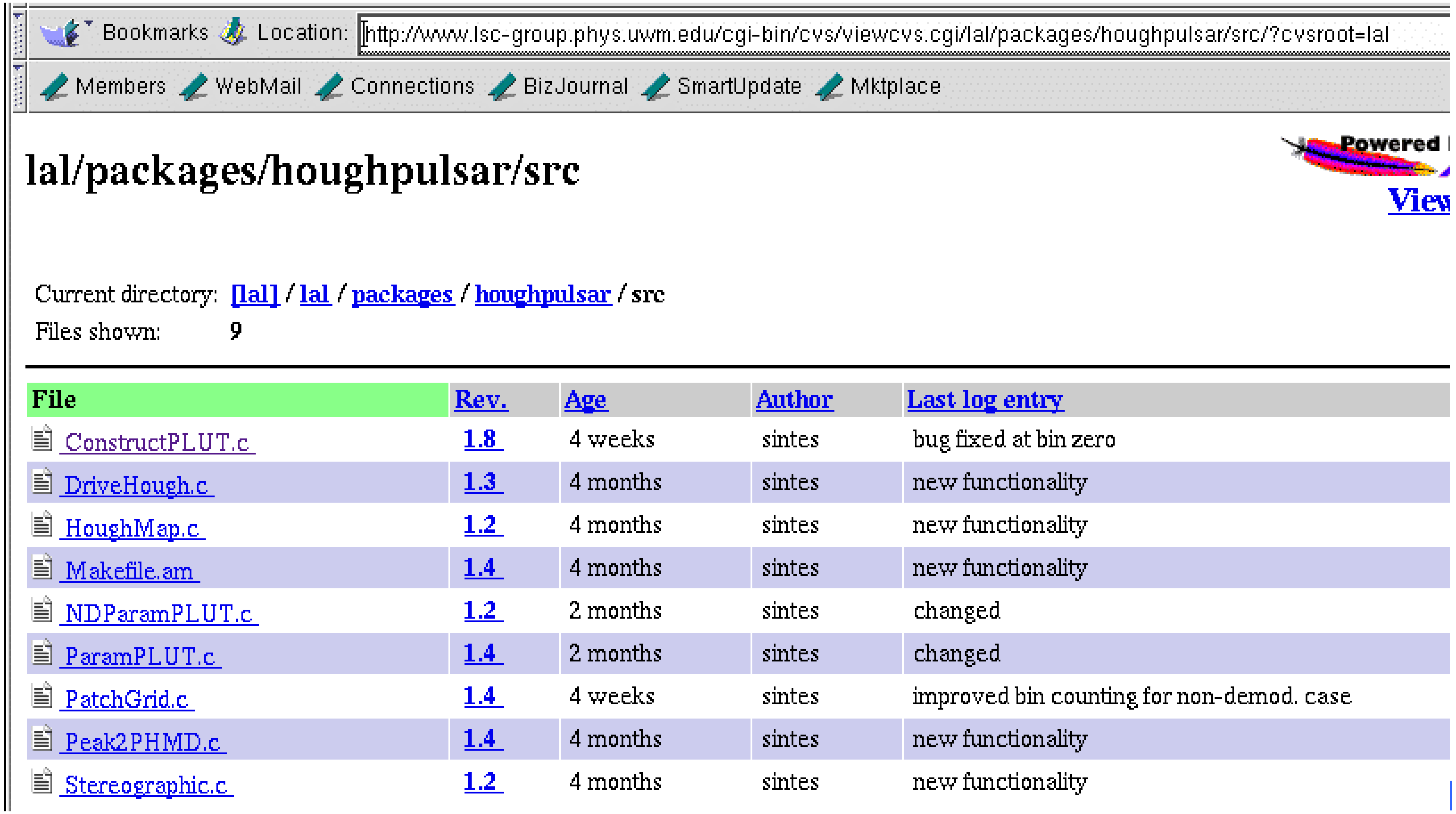
Code Status

- ✓ Stand-alone search code in final phase
- ✓ Test and validation codes (under CVS at AEI)
- ✓ Hough routines are part of the LAL library:



Bookmarks Location: http://www.lsc-group.phys.uwm.edu/cgi-bin/cvs/viewcvs.cgi/lal/packages/houghpulsar/src/?cvsroot=lal

Members WebMail Connections BizJournal SmartUpdate Mktplace

lal/packages/houghpulsar/src

Powered
View

Current directory: [lal] / lal / packages / houghpulsar / src
Files shown: 9

File	Rev.	Age	Author	Last log entry
ConstructPLUT.c	1.8	4 weeks	sintes	bug fixed at bin zero
DriveHough.c	1.3	4 months	sintes	new functionality
HoughMap.c	1.2	4 months	sintes	new functionality
Makefile.am	1.4	4 months	sintes	new functionality
NDParamPLUT.c	1.2	2 months	sintes	changed
ParamPLUT.c	1.4	2 months	sintes	changed
PatchGrid.c	1.4	4 weeks	sintes	improved bin counting for non-demod. case
Peak2PHMD.c	1.4	4 months	sintes	new functionality
Stereographic.c	1.2	4 months	sintes	new functionality

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Code Status II

✓ Auxiliary functions C-LAL compliant
(under CVS at AEI to be incorporated into LAL or LALApps):

- Read SFT data
- Peak Selection (in white & color noise)
- Statistical analysis of the Hough maps
- Compute <v(t)>

➢ Under development:

- Implementation of an automatic handling of noise features. Robust PSD estimator.
- Monte Carlo signal injection analysis code.
- Condor submission job.
- Input search parameter files

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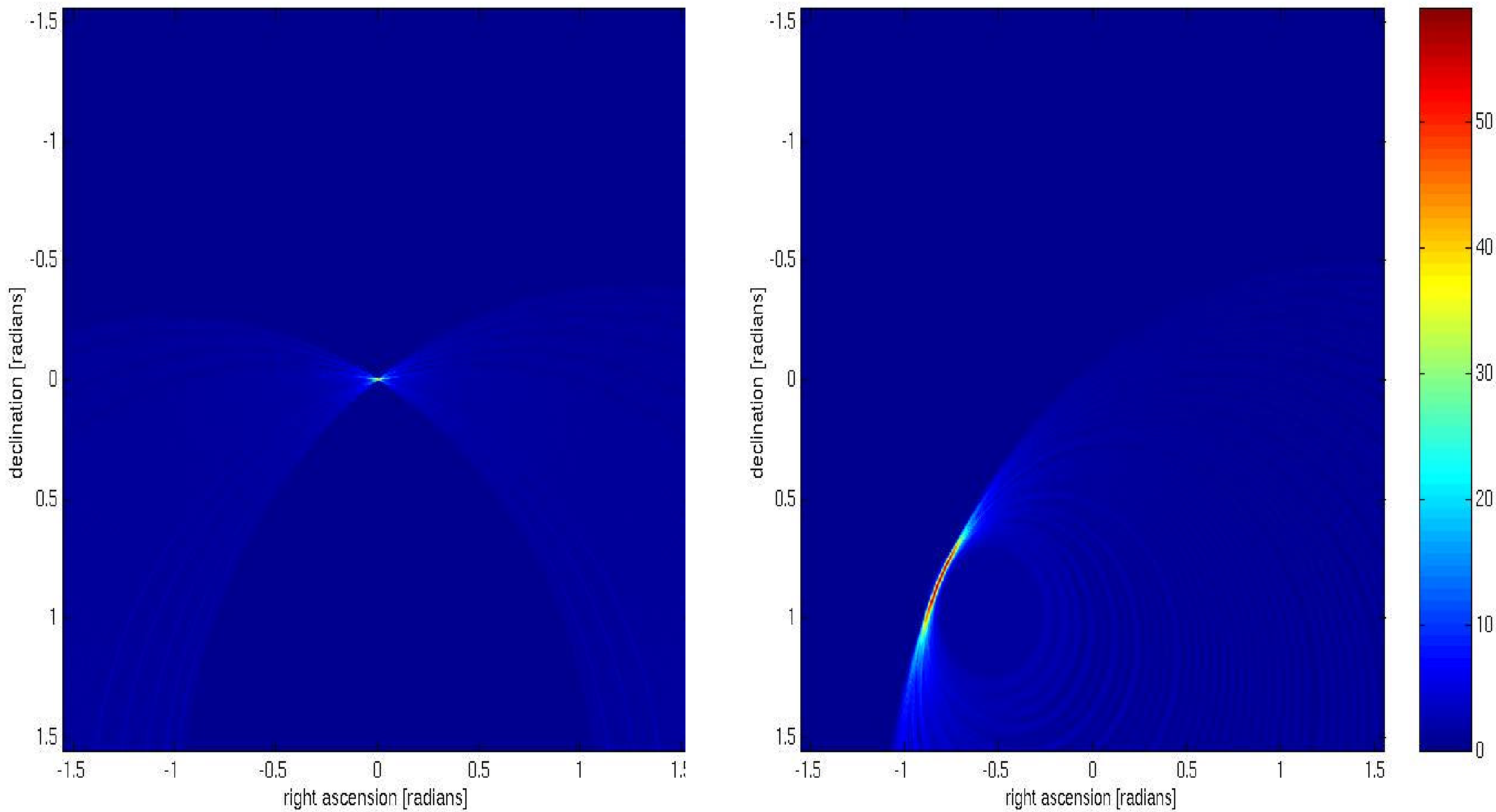
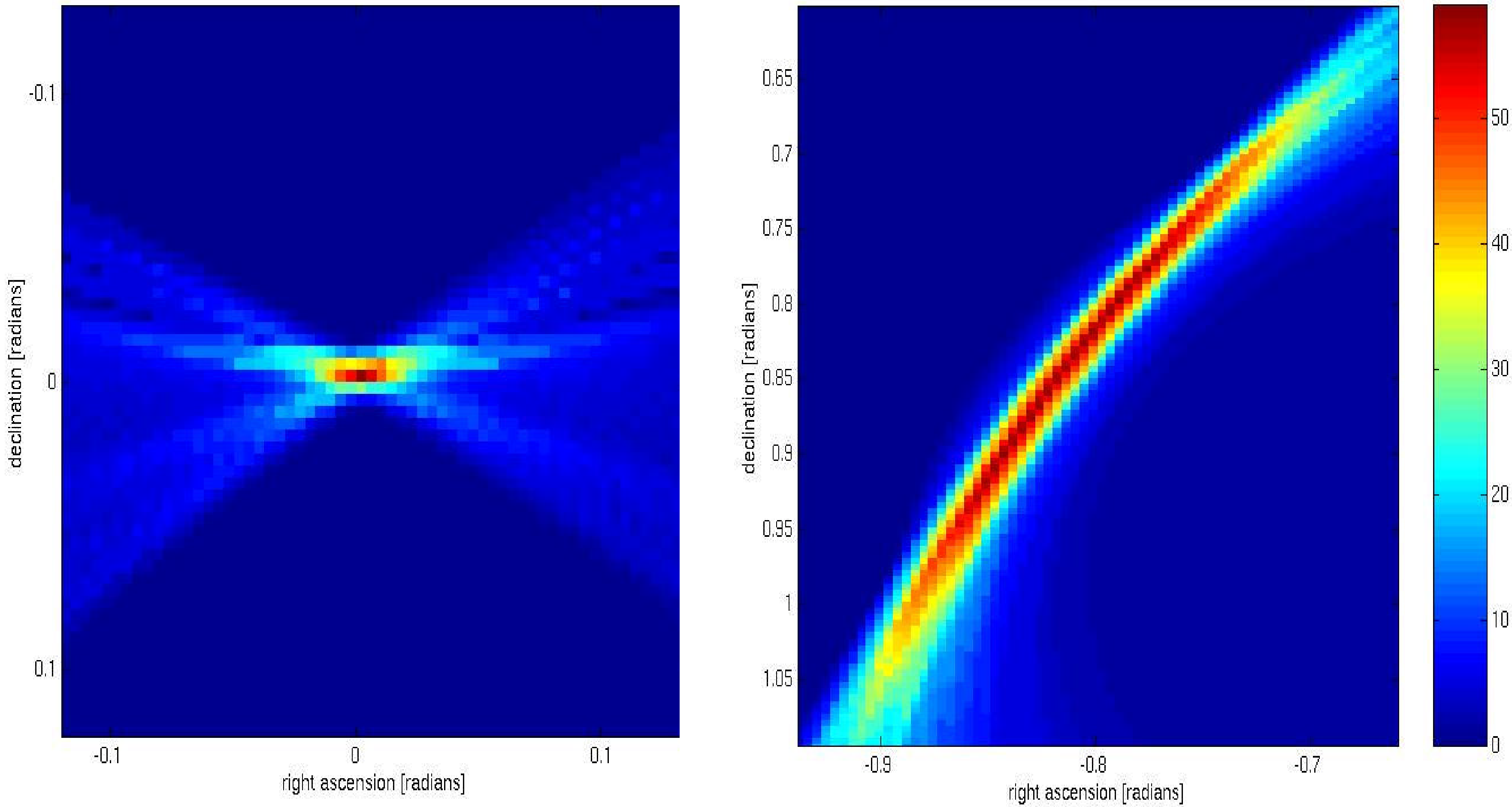
Validation code

-Signal only case- (f_0=500 Hz)



Validation code

-Signal only case- (f_0=500 Hz)



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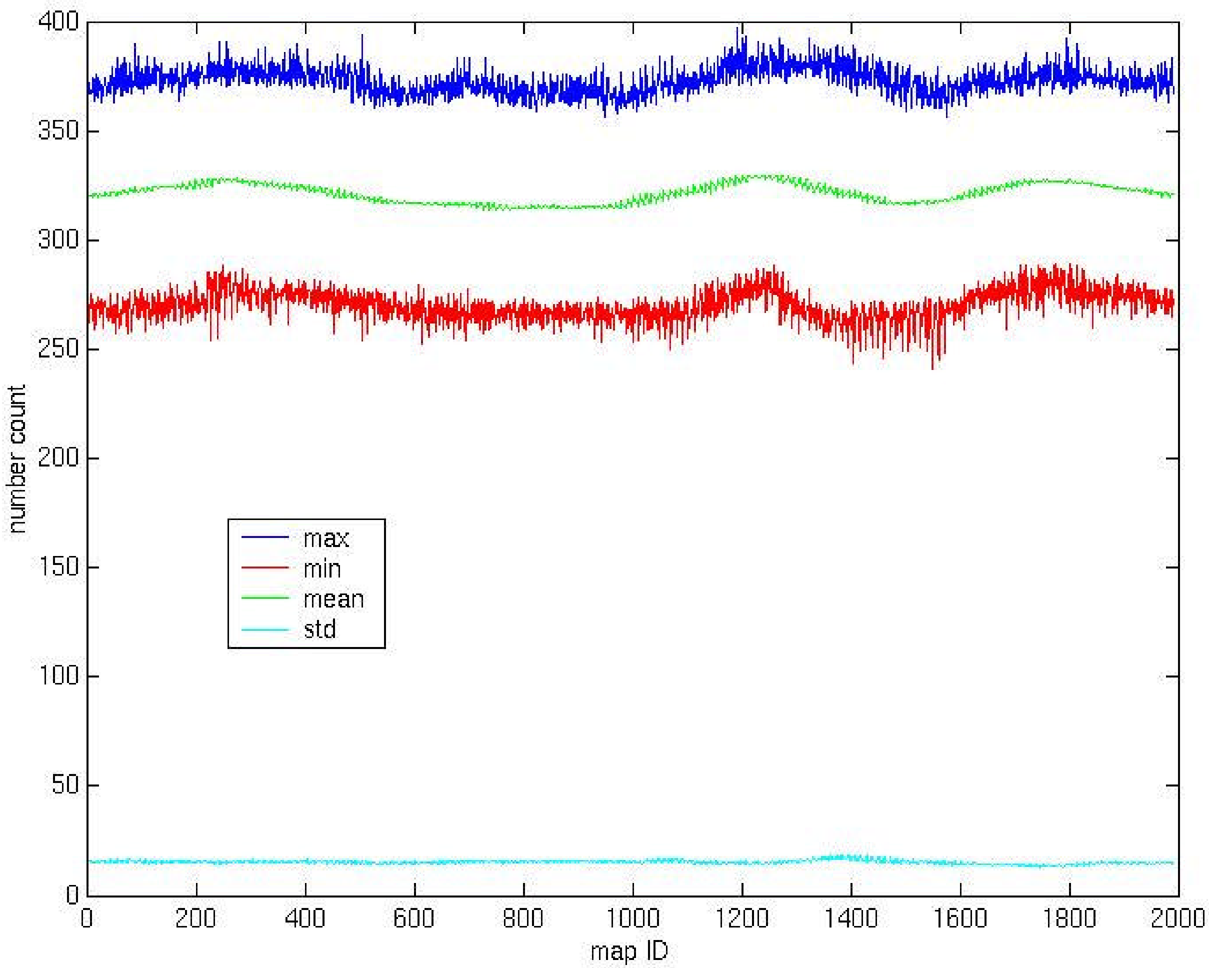
Results on simulated data



Statistics of the Hough maps

f_0~300Hz

t_{obs}=60days

N=1440 SFTs

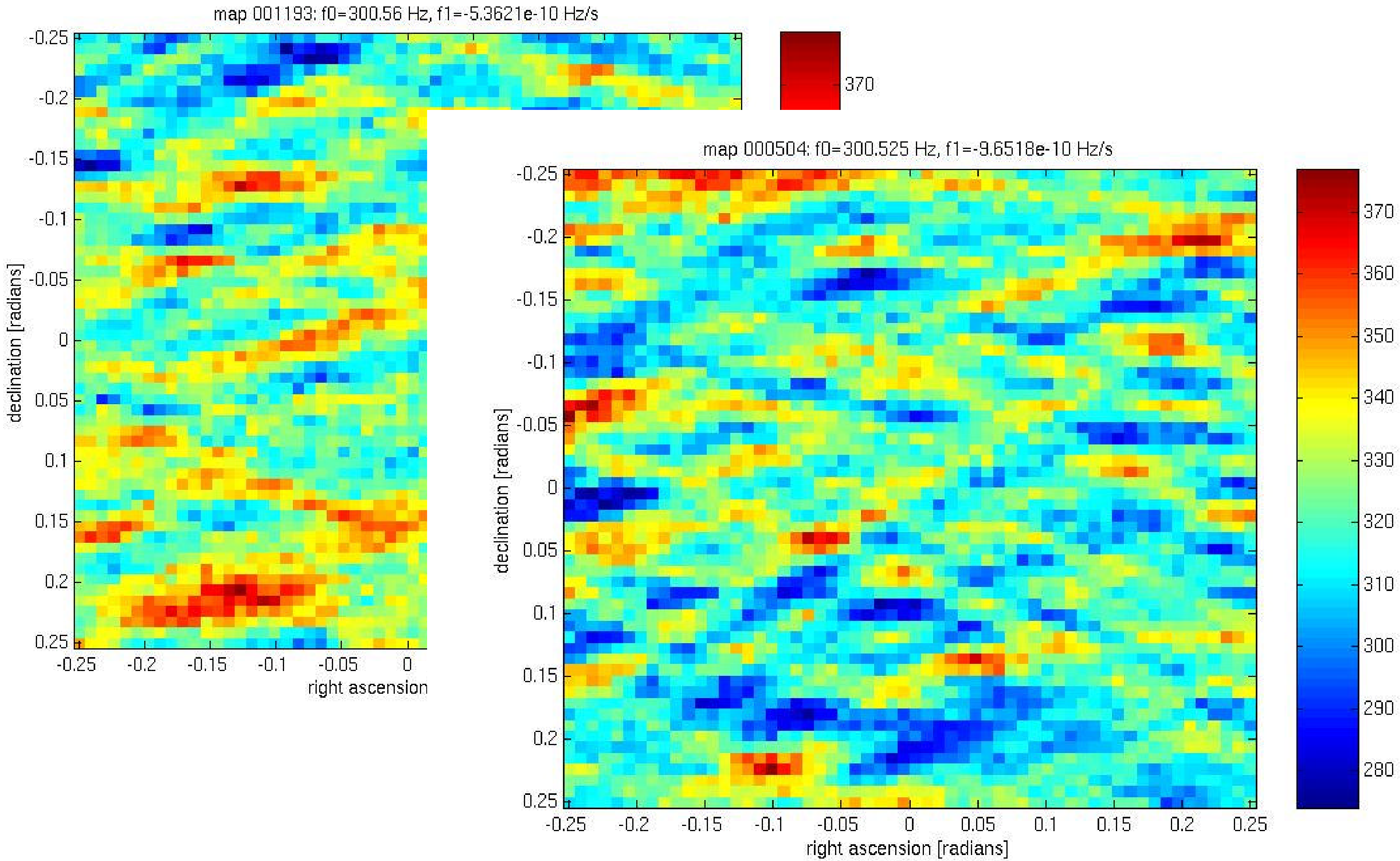
t_{SFT}=1800s

a=0.2231

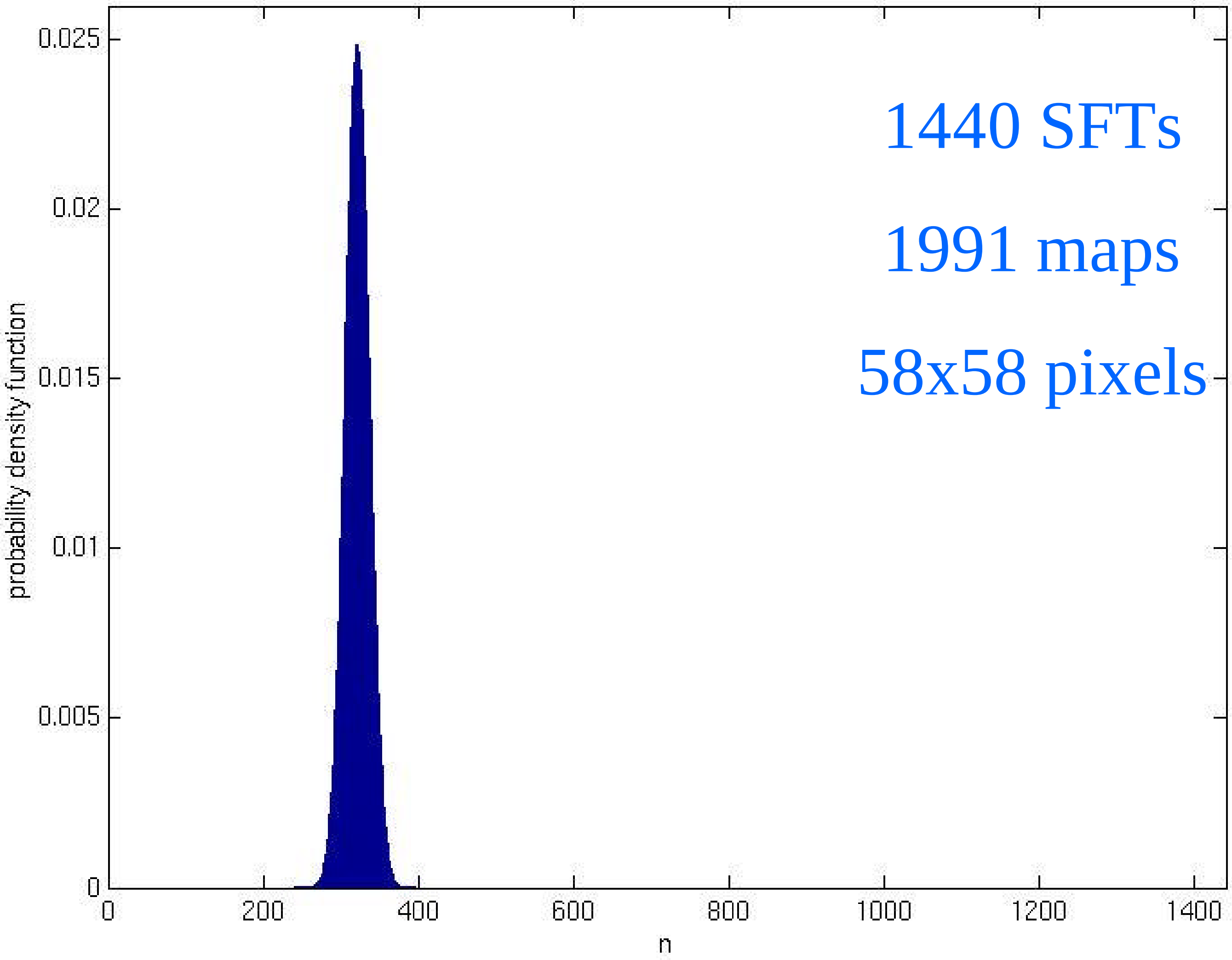
<*n*> = *Na*=321.3074

s_n=Ö(*Na*(1-*a*))=15.7992

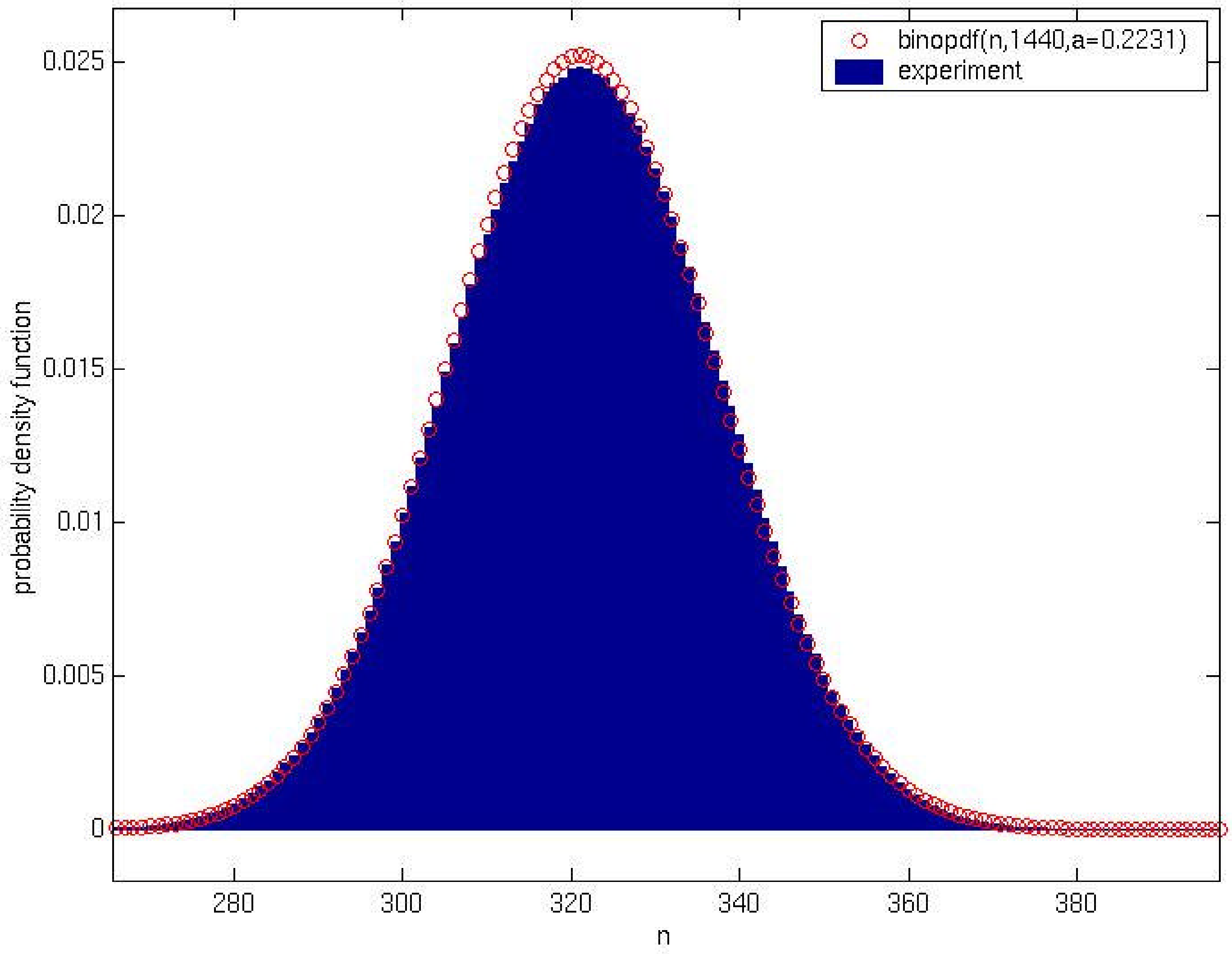
Noise only case



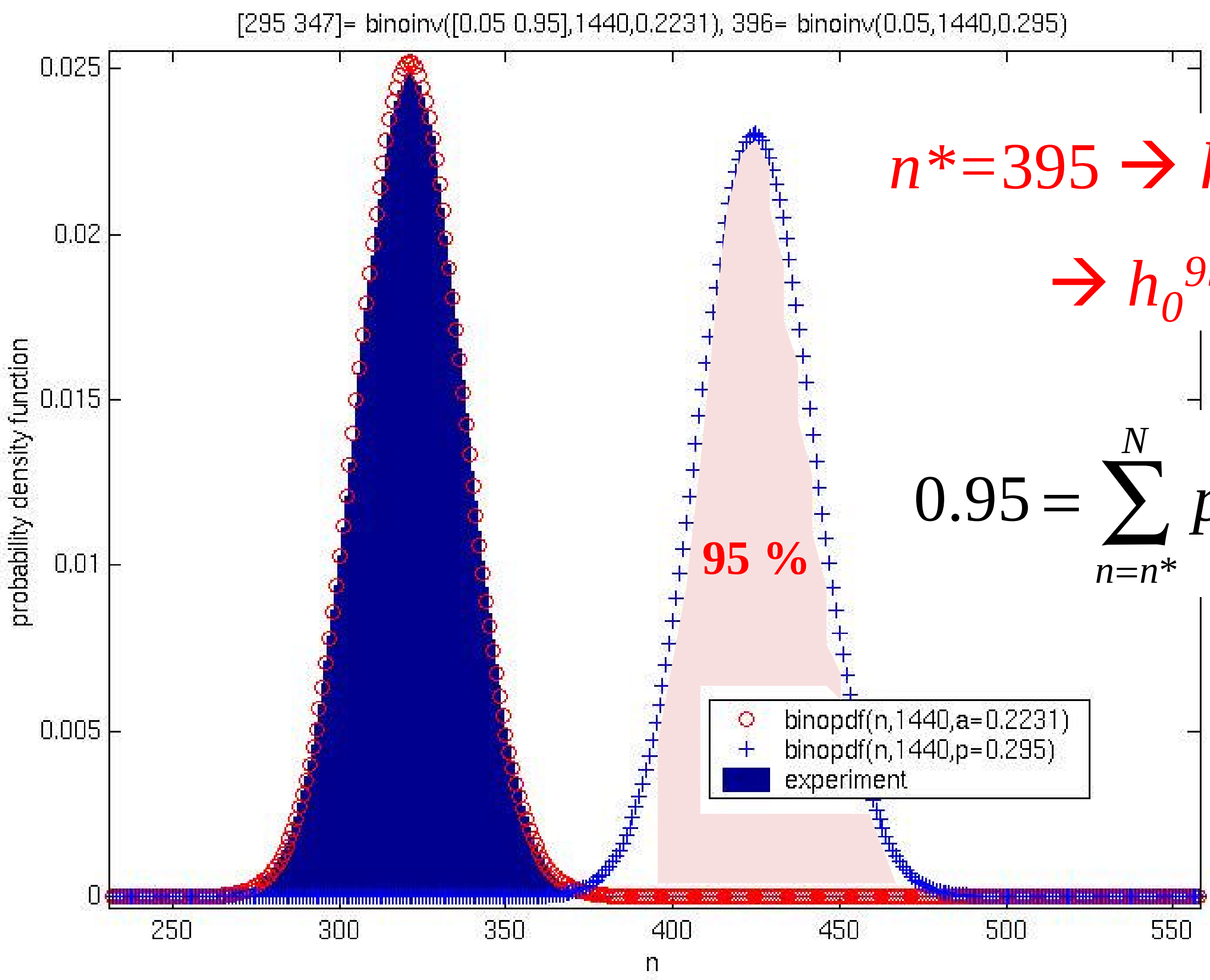
Number count probability distribution



Comparison with a binomial distribution



Set of upper limit.
Frequentist approach.



$n^*=395 \rightarrow h=0.295$

$\rightarrow h_0^{95\%}$

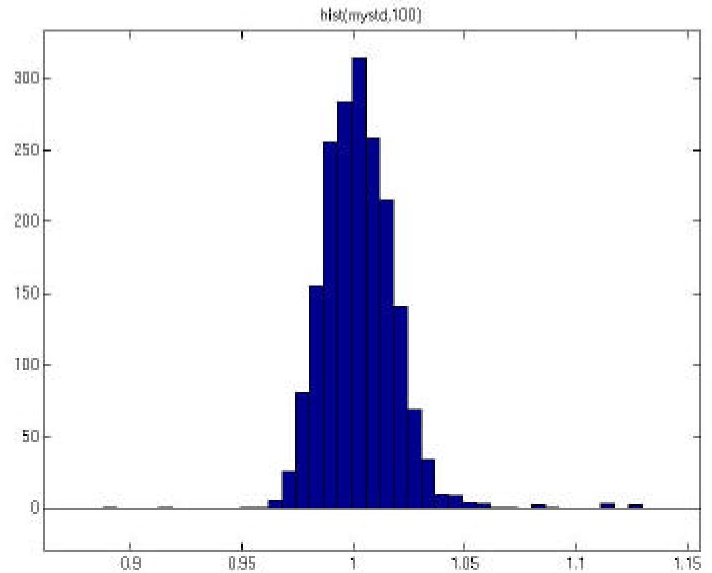
$$0.95 = \sum_{n=n^*}^{N} p\left(n|h_0^{95\%}\right)$$

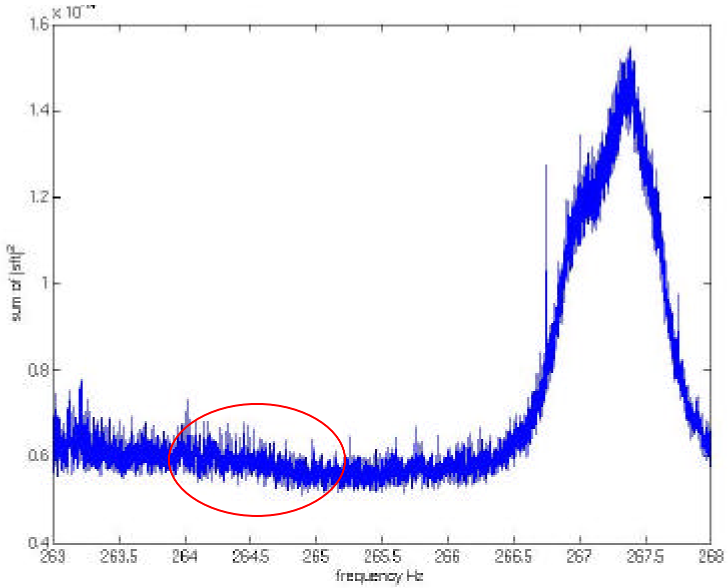
Computational Engine

Searchs offline at:

- Medusa cluster (UWM)
  - 296 single-CPU nodes (1GHz PIII + 512 Mb memory)
  - 58 TB disk space
- Merlin cluster (AEI)
  - 180 dual-CPU nodes (1.6 GHz Athlons + 1 GB memory)
  - 36 TB disk space
- CPUs needed for extensive Monte-Carlo work

H1 Analysis – the SFTs

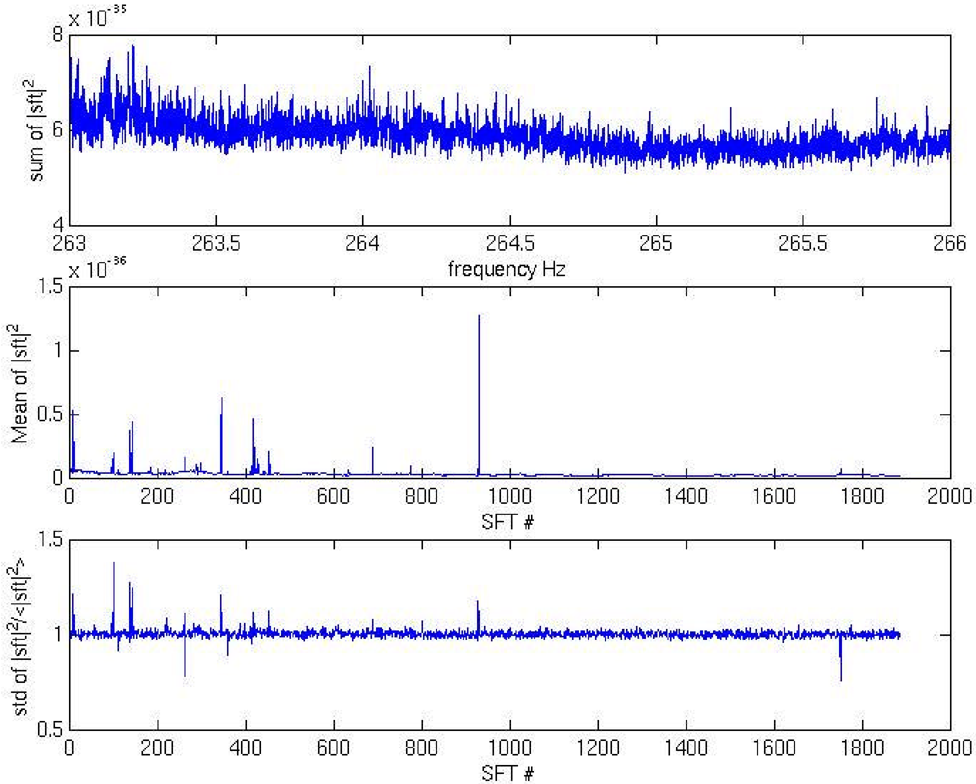




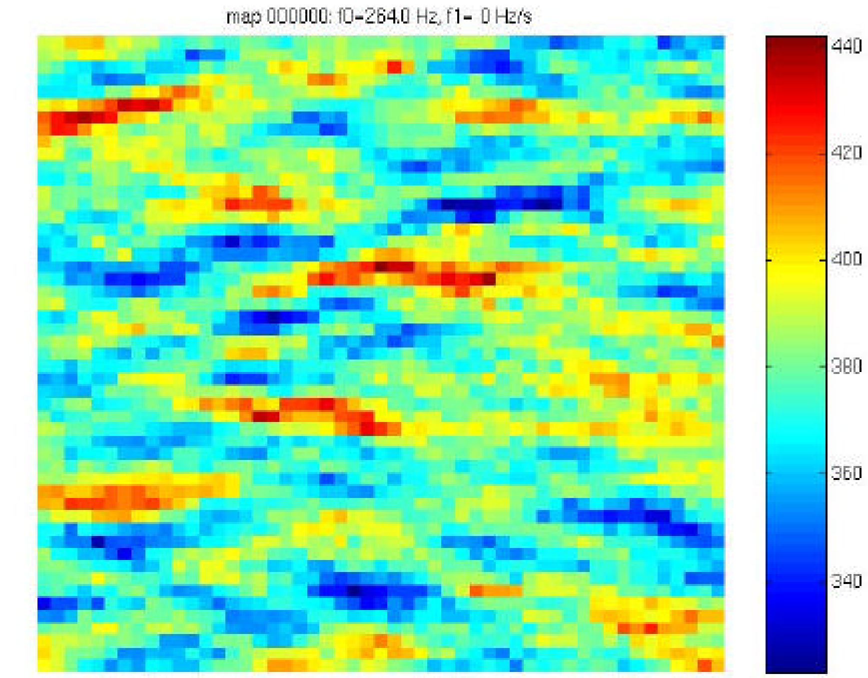
1887 Calibrated SFT’s H1

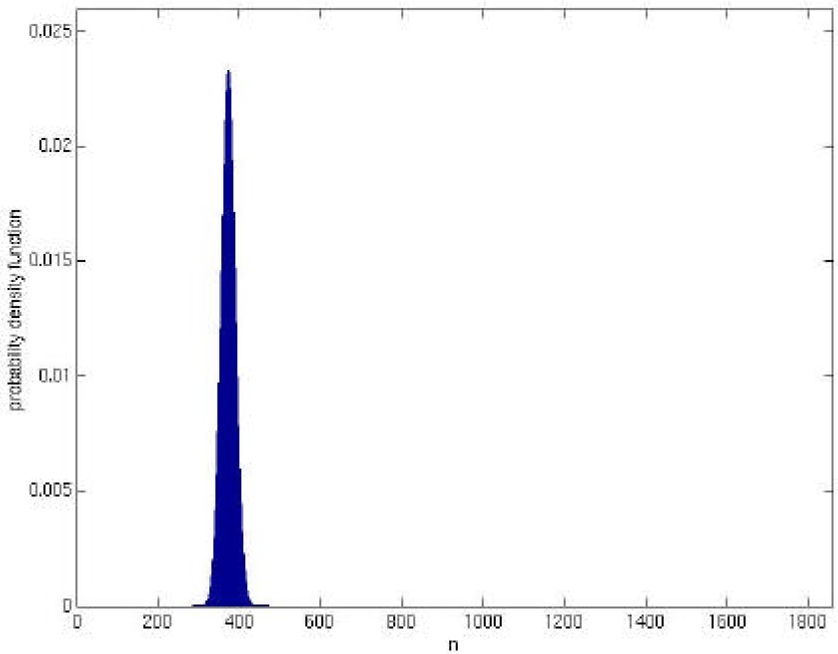
T_{SFT} = 1800s, Band : 263-268 Hz

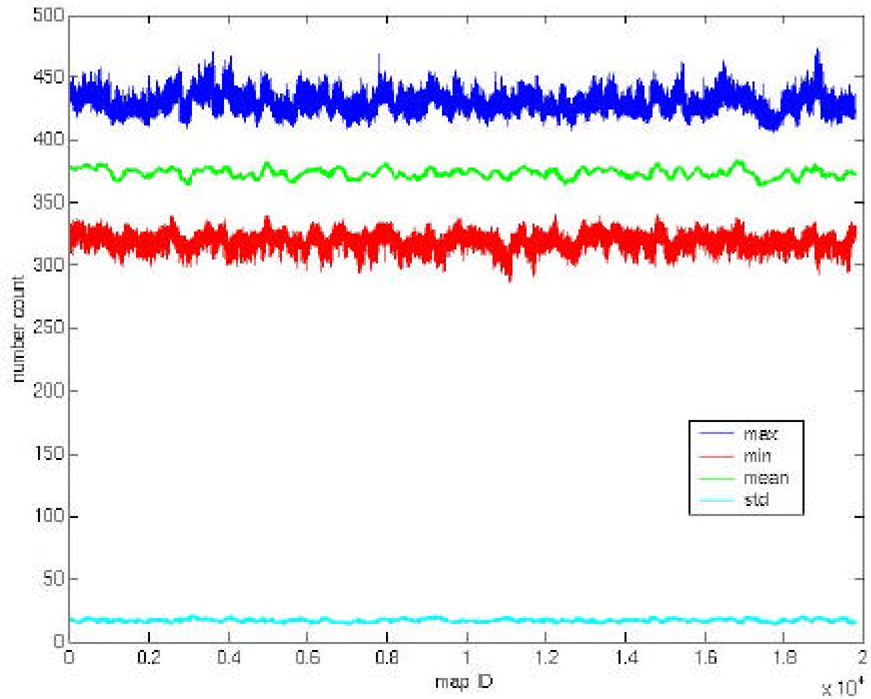
26 SFT, σ≤0.95 or σ≥1.05



H1 Results
264-265 Hz







N=1861 SFTs

a=0.2019

←Mean number count=373.8130

$\langle n \rangle = Na$=375.7294

s_n=Ö(Na(1-a))=17.3168

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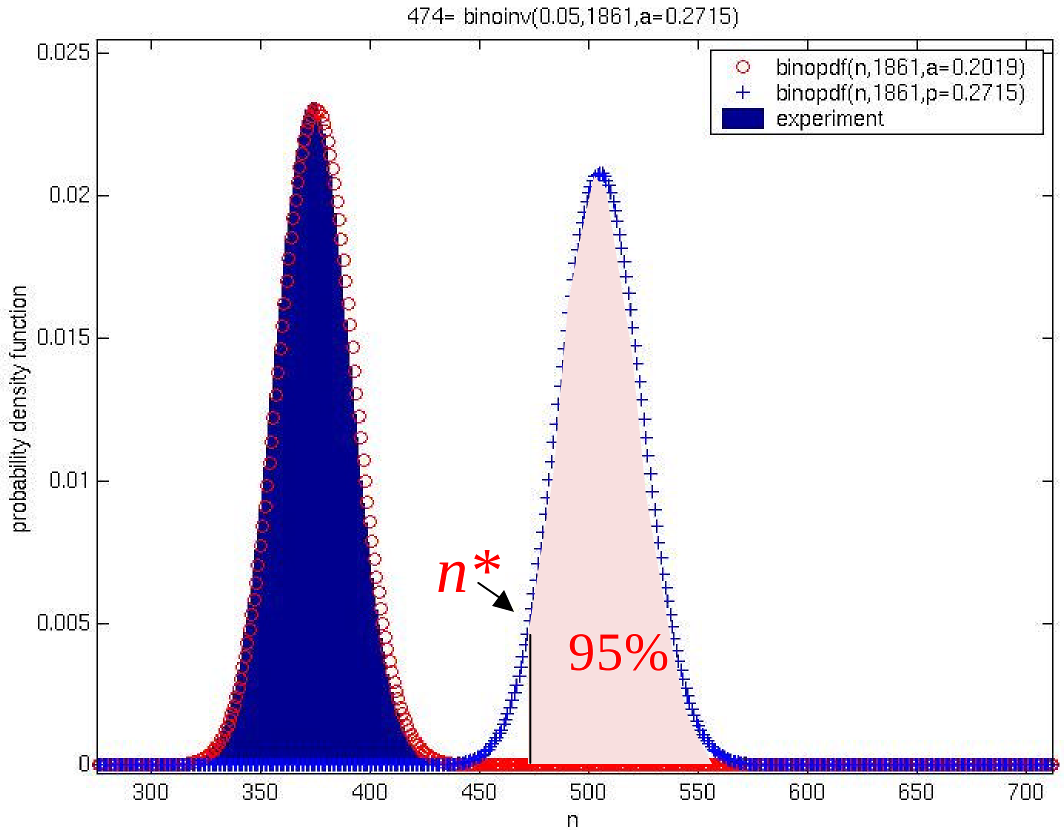
H1 Results 264-265 Hz

"upper limit estimate"



Loudest event n*=473

95 % upper limit

n=473 → *h*=0.2715

$$\frac{|\tilde{h}(f_i)|^2}{S_n(f_i)T_{SFT}} \approx (0.46)^2$$

$$\sqrt{S_n(f_i)} \approx XX \times 10^{-xx} / \sqrt{Hz}$$

$$4|\tilde{h}(f_i)|^2 \approx \frac{3}{50} h_0{}^2 T_{SFT}{}^2$$

→ $h_0^{95\%}$?