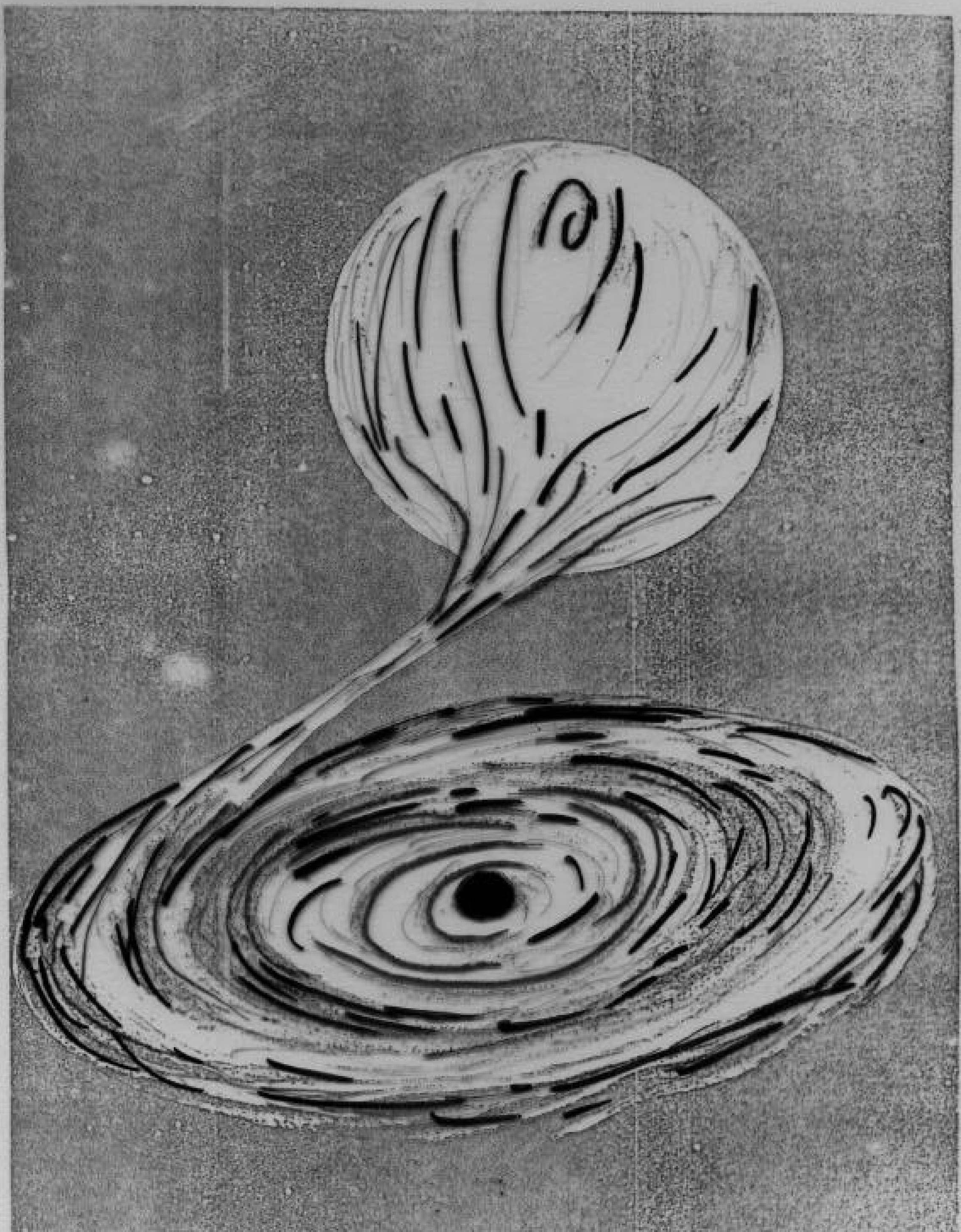
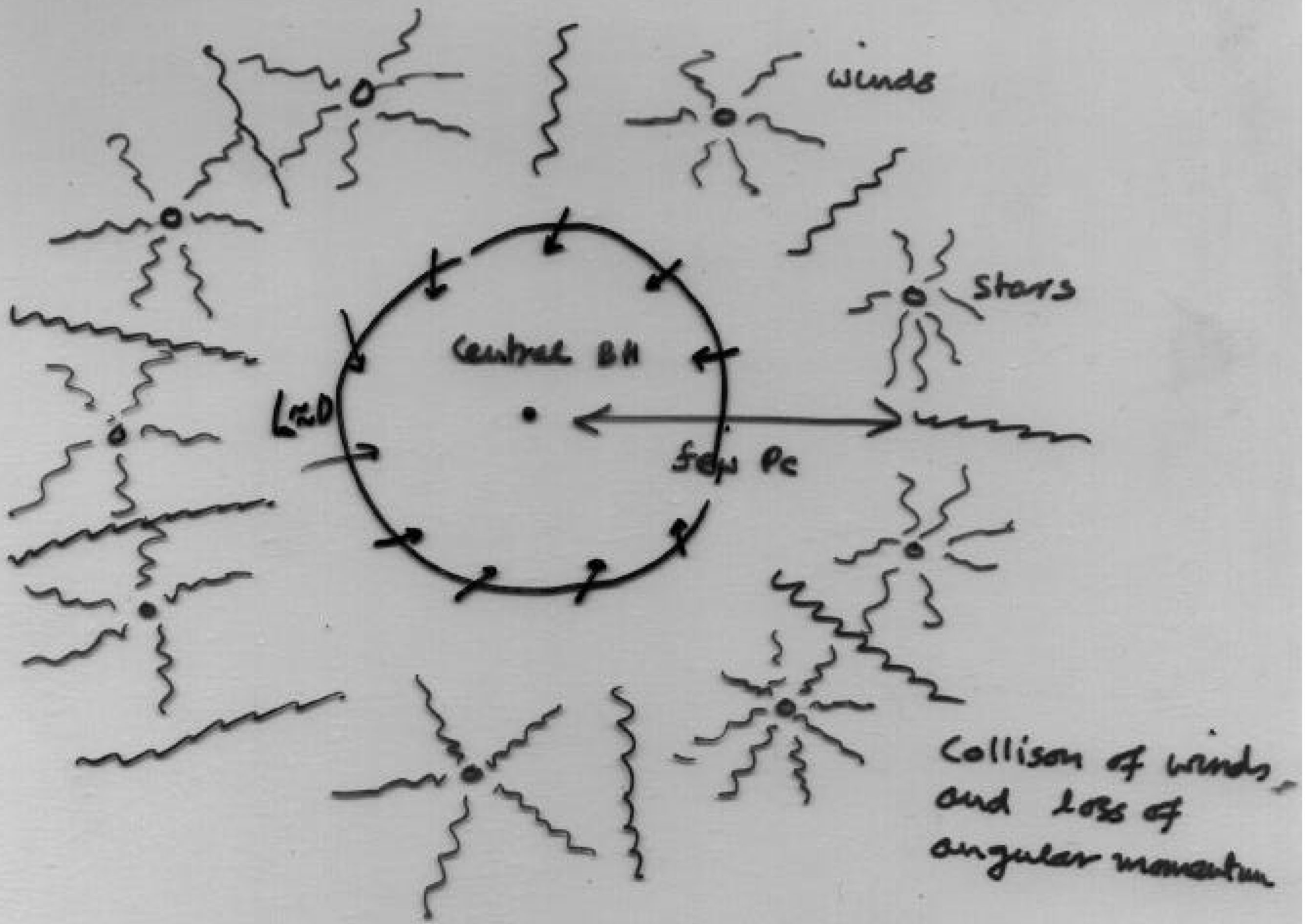


Cartoon diagram of a Stellar mass black hole $(3\text{-}10 M_{\odot})$ accretion from a companion star.

Others are super-massive $(10^{6\text{-}9} M_{\odot})$



AGNS:

1. Outer boundary is <u>Sub-Keplerian</u>
2. Inner boundary is Soft $[v_r = c \text{ @ } r = \frac{2GM}{c^2}]$
3. Disk need not be viscous driven
4. Disk need not be Keplerian
5. Disk must be transonic (Subsonic to Supersonic)
6. No obvious sink of angular momentum

$$ds^2 = -\frac{r^2\Delta}{A}dt^2 + \frac{A}{r^2}(d\varphi - \omega dt)^2 + \frac{r^2}{\Delta}dr^2 + dz^2$$

$$A = r^4 + r^2a^2 + 2ra^2$$

$$\Delta = r^2 - 2r + a^2$$

$$\omega = \frac{2ar}{A}$$

Specific binding energy:

$$u_t = \left[\frac{\Delta}{(1-V^2)(1-\Omega l)(g_{\varphi\varphi} + l\, g_{t\varphi})}\right]^{1/2}$$

where,

$$l = -\frac{u_\varphi}{u_t}, \quad \Omega = \frac{u^\varphi}{u^t}$$

V = radial velocity in the rotating frame

$$\boxed{@ \quad \begin{matrix}\Delta = 0 \\ V = 1\end{matrix}}$$

Fundamentals of Advective DISKS (1)

At the Horizon:

Chakrabarti, 1990

radial velocity ϑ_{r_0} = velocity of light

(on Neutron Star surface $\vartheta_r = 0$)

But

Sound Speed

$$a_s \leq \frac{1}{\sqrt{3}} c$$

$\therefore$ Mach# $= \frac{\vartheta_r}{a_s} > 1$

$\therefore$ Flow must Be supersonic on the horizon! $M > 1$

But in Shakura-Sunyaev Disk

flow is Subsonic

$M < 1$

$\therefore$ Flow must pass through a Sonic point $M = \vartheta/a = 1$

$\therefore$ Flow must be Sub Keplerian

Chakrabarti 1989

Centrifugal Pressure Supported SHOCKS !

$\boxed{\text{CENBOL}}$

(Generally: Centrifugal Pressure Supported Boundary Layer)

Force due to gravity $\sim -\frac{1}{r^2}$

Centrifugal force $\sim +\frac{l^2}{r^3}$

$\therefore$ matter "feels" centrifugal barrier and slows down.

<u>BUT</u> Accelerates again to satisfy inner boundary condition

SO, BLACKS ALSO HAVE BOUNDARY LAYERS !!

Oscn. frequency of QPO is determined by cooling time scale $\equiv$ infall time scale in post shock region

Comparison of transport of angular momentum by Gravity waves and accretion process:-

Rate of loss of ang. mom. due to gravity wave,

$$R_{GW} = \frac{dL}{dt}\bigg|_{GW} = \frac{1}{\Omega}\frac{dE}{dt}$$

Where, $\Omega = \sqrt{GM_1/r^3}$

& $\frac{dE}{dt} = 3\times 10^{33}\left(\frac{\mu}{M_\odot}\right)^2\left(\frac{M_{tot}}{M_\odot}\right)^{4/3}\left(\frac{P}{1hr}\right)^{-10/3}$ ergs/sec.

$\mu = M_1 M_2 (M_1+M_2)$, $M_{tot} = M_1 + M_2$

Rate of gain of angular momentum due to Bondi accretion on Secondary:

$$R_{disk} = \frac{dL}{dt}\bigg|_{disk} = \dot{M}_2\left[l_{Kep}(x) - l_{disk}(x)\right]$$

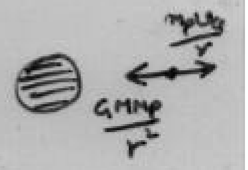
where, $\dot{M}_2 = \frac{4\pi\bar{\lambda}\rho(GM_2)^2}{(v_{rel}^2 + a^2)^{3/2}}$, $\bar{\lambda} \approx 1/2$

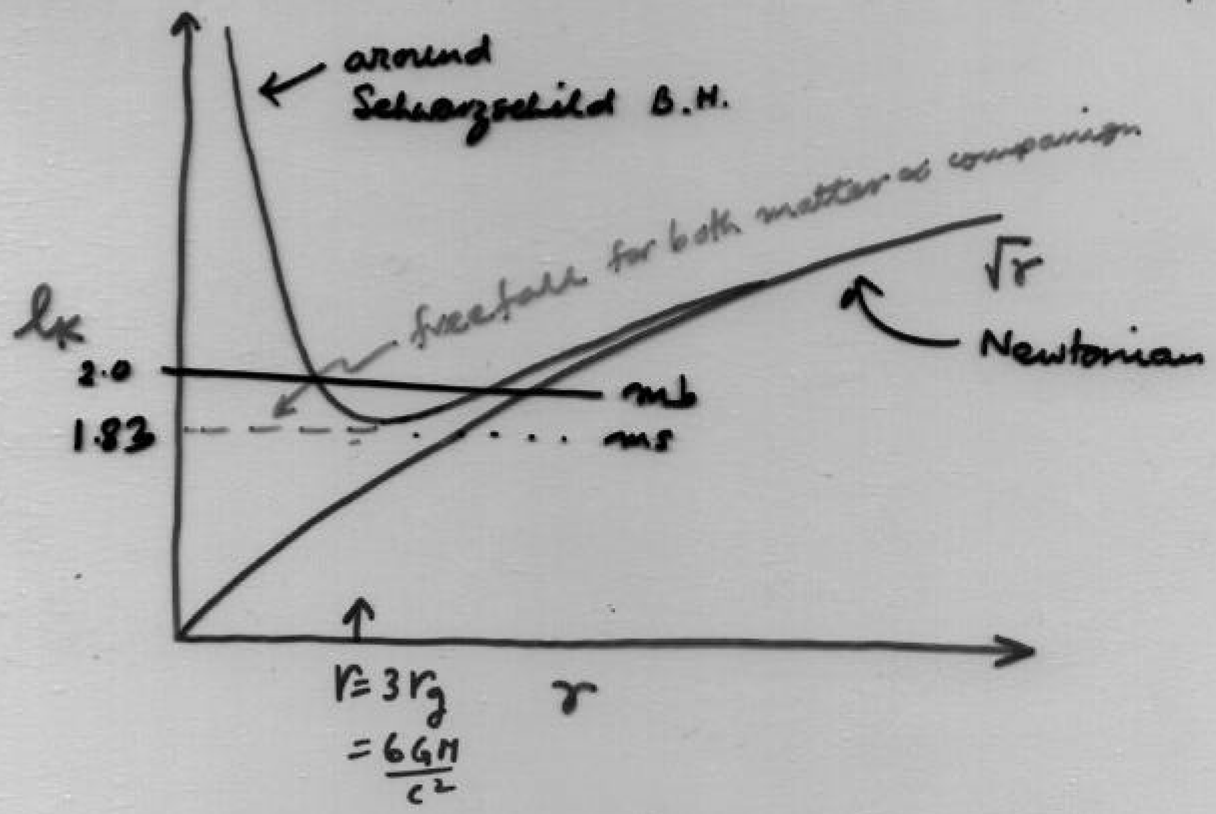
∴ Relative importance $R = \frac{R_{disk}}{R_{GW}} = 1.5\times 10^{-7}\frac{\rho_{10}}{T_{10}^{3/2}}x^4 M_8^2$

in the limit $M_2 \ll M_1$, $L_{disk} \ll L_{Kep}$

At $x = 10$, $M_8 = 10$; $R \simeq 0.015$

The effect is significant!





<u>Keplerian distribution</u> $\left[\frac{GMm_p}{r^2} \approx \frac{m_p v_\phi^2}{r} \approx \frac{m_p l_K^2}{r^3} \right]$

In a Keplerian Disk Angular momentum is transported by viscous torque:

$$\dot{M}(l_1 - l_2) = 4\pi r^2 h f_\phi$$

viscous stress

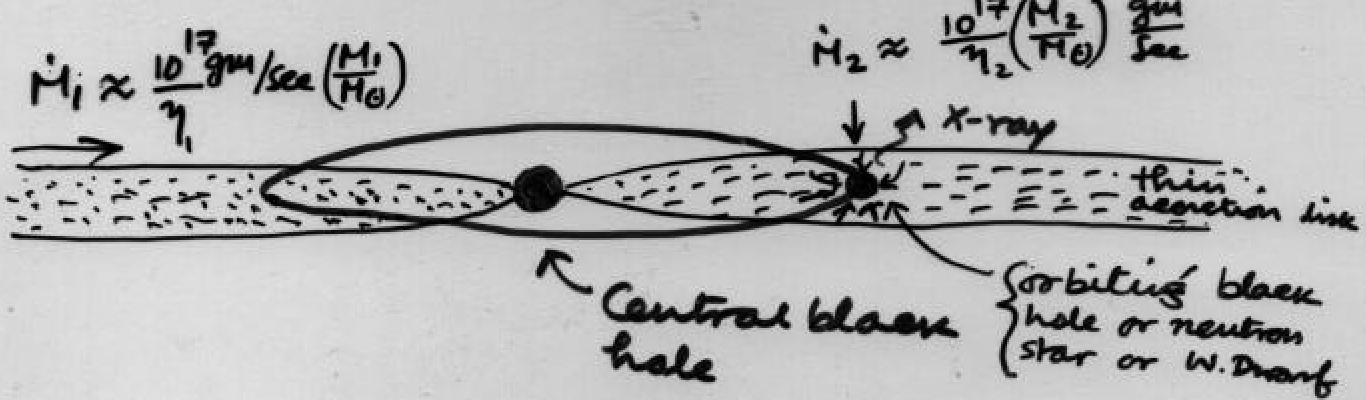
The rate of loss of energy $\frac{dE}{dt}$ in a binary system consisting of two point masses M_1 & M_2 having orbital period P (in hours) is given by ($e=0$)

$$\frac{dE}{dt} = 3\times10^{33}\left(\frac{\mu}{M_\odot}\right)^2\left(\frac{M}{M_\odot}\right)^{4/3}\left(\frac{P}{1\,hr}\right)^{-10/3}\ \frac{ergs}{sec}$$

$$\mu = \frac{M_1 \cdot M_2}{M_1+M_2}\ , \quad M = M_1+M_2$$

The orbital angular momentum loss rate would be:

$$\frac{dL}{dt} = \frac{1}{\Omega_K}\frac{dE}{dt} \qquad \text{[Circular orbit]}$$



in general $\dot{M}_2 << \dot{M}_1$

The rate of gain of angular momentum is given by:

$$\frac{dL_+}{dt} = \alpha \dot{M}_2 \left(\frac{4GM_1}{c}\right)$$

accretion rate onto secondary

normalization constant [angular momentum at inner edge = lmb]

efficiency of angular momentum transport

equating: $\boxed{\frac{dL_+}{dt} = \frac{dL_-}{dt}}$

we obtain, (CHAKRABARTI, 1992)

$$r_{eq} = 1563 \frac{M_1^{1/21}}{[\alpha \dot{M}_2]^{2/7}} \left(\frac{\mu}{M_\odot}\right)^{4/7} \left(\frac{M}{M_\odot}\right)^{8/21} \left(\frac{10^8 M_\odot}{M_1}\right)$$

in units of Schwarzschild Radius of the primary

Example: $M_1 \approx 10^8 M_\odot$, $M_2 = 1 M_\odot$
$\alpha \sim 0.05$, $\eta = 0.057$
(for Schwarzschild B.H)

$$\boxed{r_{eq} = 23\, r_g}$$

Steady source of Gravitational Waves?

frequency of the G.W. $\Rightarrow$ $$f = \frac{1}{\pi}\left(\frac{GM_1}{r_{eq}^3}\right)^{1/2}$$

$$\therefore\ f = 3.7\times10^{-6}\left(\frac{10^8 M_\odot}{M_1}\right)^{-1/2} \frac{(\alpha\dot{m}_2)^{3/7}}{M_1^{1/14}} \left(\frac{M_\odot}{\mu}\right)^{6/7}\left(\frac{M_\odot}{M}\right)^{4/7} \mathrm{Hz}$$

The amplitude of metric perturbation:-

$$|A| = 5.765\times10^{-19}\left(\frac{\mu}{1M_\odot}\right)\left(\frac{M_1}{10^6 M_\odot}\right)^{2/3} f^{2/3}\left(\frac{D}{1\mathrm{Mpc}}\right)^{-1}$$

Comparison of Transport of angular momentum by Gravity waves and the accretion process

Rate of loss of angular momentum due to gravity wave,

$$R_G = \left.\frac{dL}{dt}\right|_{GW} = \frac{1}{\Omega}\frac{dE}{dt}$$

where, $\Omega = \sqrt{\frac{GM_1}{r^3}}$

and $\frac{dE}{dt} = 3\times10^{33}\left(\frac{\mu}{M_\odot}\right)^2\left(\frac{M_{tot}}{M_\odot}\right)^{4/3}\left(\frac{P}{1hr}\right)^{-10/3}$ ergs/sec.

$$\mu = \frac{M_1 M_2}{M_1+M_2}, \quad M_{tot} = M_1 + M_2$$

Rate of Gain of angular momentum due to Bondi accretion on the secondary

$$R_{disk} = \left.\frac{dL}{dt}\right|_{disk} = \dot{M}_2\left[l_{kep}(x) - l_{disk}(x)\right]$$

where, $\dot{M}_2 = \frac{4\pi\bar{\lambda}\rho(GM_2)^2}{(v_{rel}^2 + a^2)^{3/2}}$

Relative importance is

$$R = \frac{R_{disk}}{R_{AW}} = 1.5\times10^{-7} \frac{\rho_{10}}{T_{10}^{3/2}} x^4 m_8^2$$

when $M_2 << M_1$, $l_{disk} << l_{kep}$

Example: at $x=10$
$M_8 = 10$
$R \sim 0.015$!

Or, equivalently write in terms of the accretion rate on massive black hole :-

$$R = 8\times10^{-6} \frac{\dot{m}_1 x^{2.5}}{T_8^{3/2}}$$ ← No M_1, M_2 dependence explicitly

$\dot{m}_1$ = accretion rate (subkeplerian) on the primary

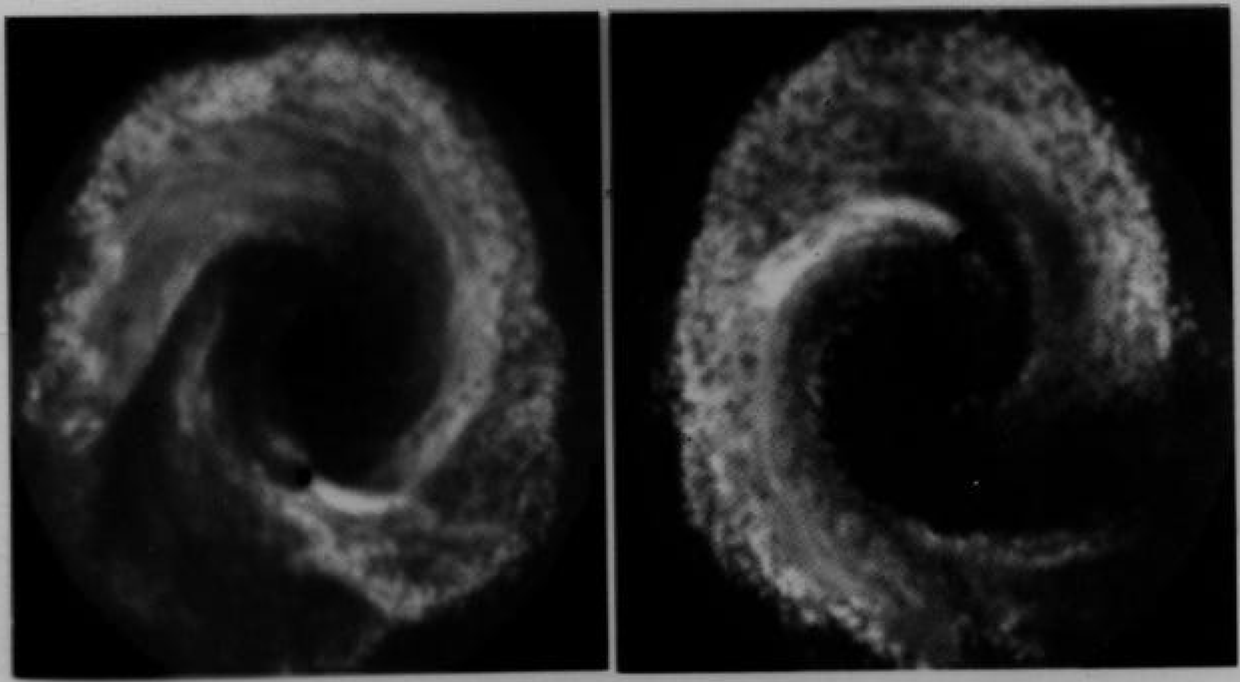
In hard state: $\dot{m}_1 \sim 1$, $x \sim 10$, $T_8 \sim 4$

$\Rightarrow R_{HS} = 3\times10^{-4}$

Independent of Mass of the Central black hole

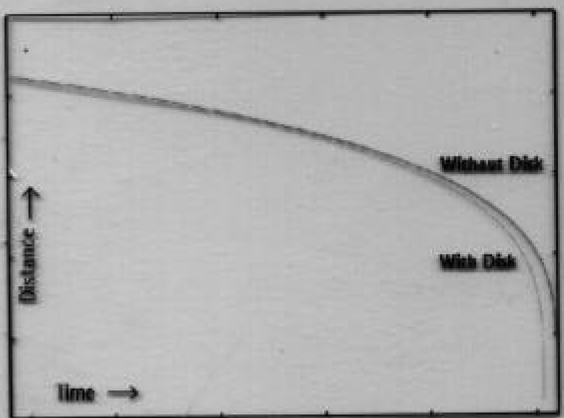
Gravitational Waves From a Binary System in Presence of an Advective Disk

Exchange of angular momentum between the disk and the companion changes the GW emission (Chakrabarti, 1992, 1996)



Molteni, Gerardi and Chakrabarti, 1995

Quicker infall



Chirp Effect

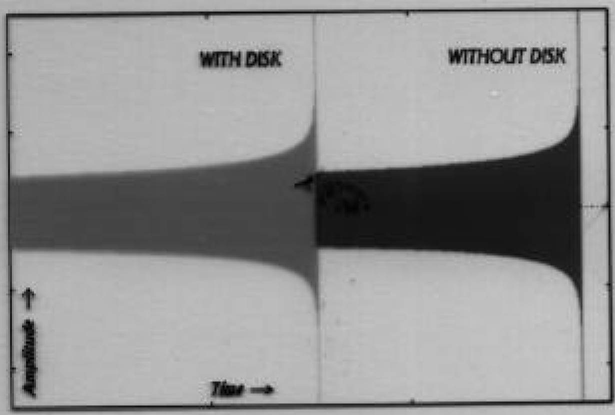
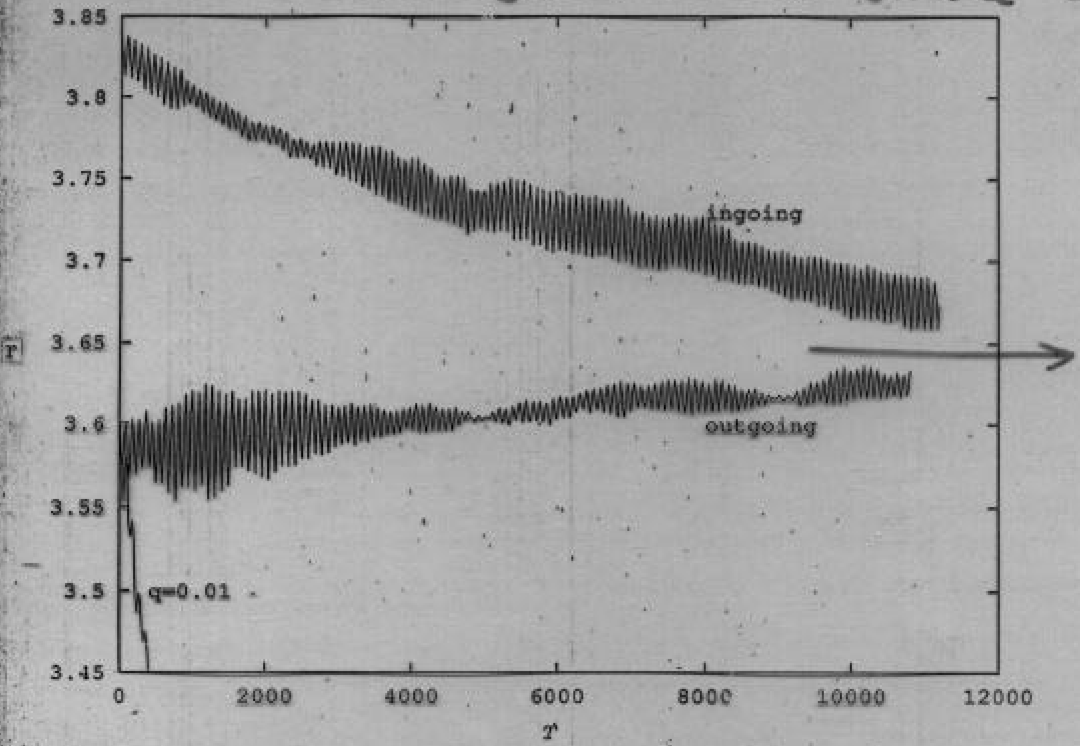


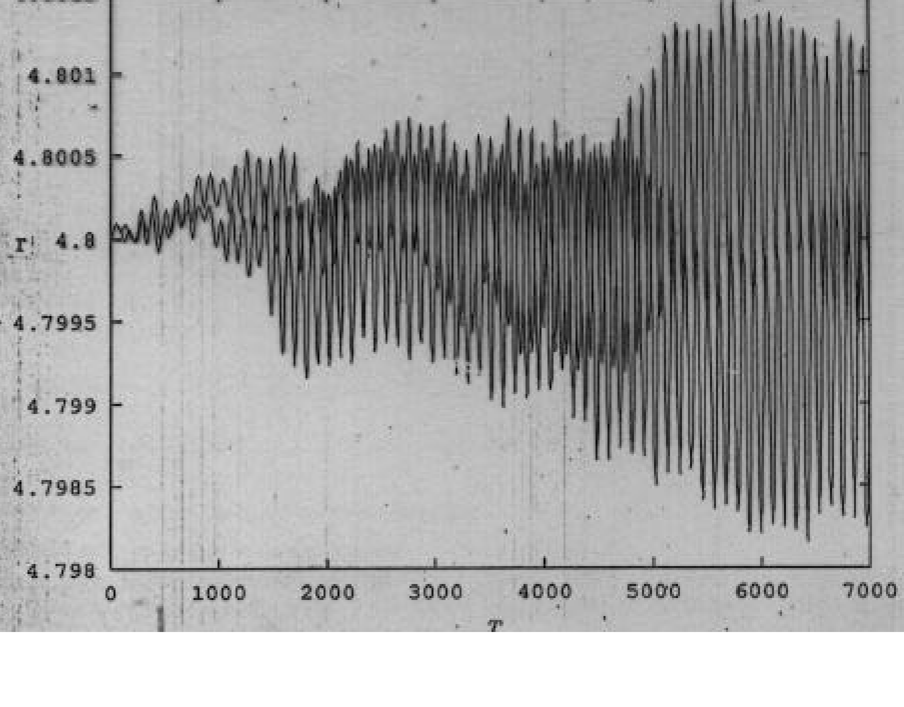
Fig 2

Convergence towards Stable Soln



Aug 1994

Fig 3



back to r_s through gravitational radiation. When it is pushed closer to the black hole at $r = r_s - \epsilon$, it gains *more* angular momentum than it loses, and therefore returns back to r_s. This argument does not hold for $r_{eq} = r_u$. Any small perturbation removes the secondary from this position.

4. STEADY SOURCE OF GRAVITATIONAL WAVES?

Being on a stationary orbit, the secondary will be emitting steady gravitational waves with a frequency

$$f = \frac{1}{\pi}\left(\frac{GM_1}{r_{eq}^3}\right)^{1/2} . \tag{8}$$

(eq. [8]) is only an upper limit of the observed PFR (because of projection effects), so with $M_2 = 700\ M_\odot$ may also be acceptable, which yields $h = 9.5 \times 10^{-18}$ and $T_{PFR} = 3.6$ yr, very close to the observed period. In the above cases, we assume M_2 to be critically accreting; the result is similar when rates computed from other considerations (such as provided by Bondi-Hoyle-Lyttleton) are used.

5. TESTING THE AGN PARADIGM AND CONCLUDING REMARKS

It is well known that when the accretion around a massive black hole takes place in the presence of very low viscosity and

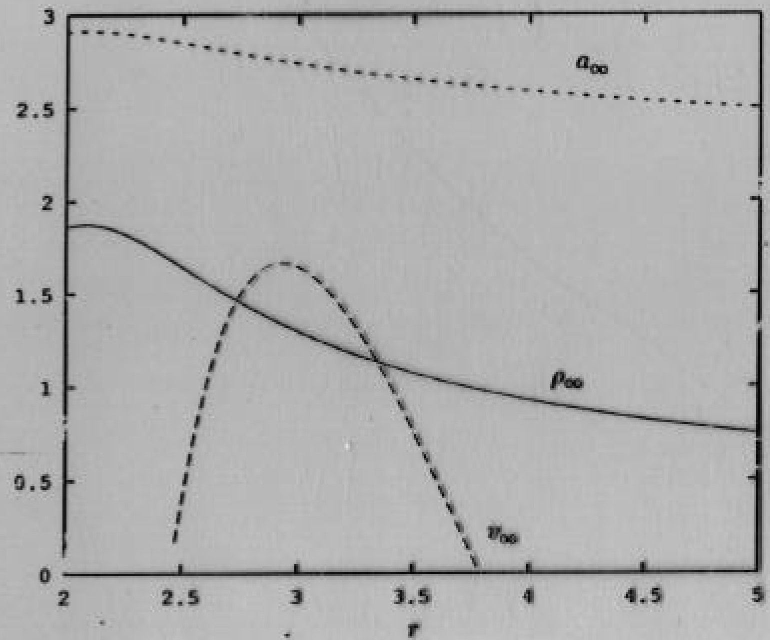


FIG. 2a

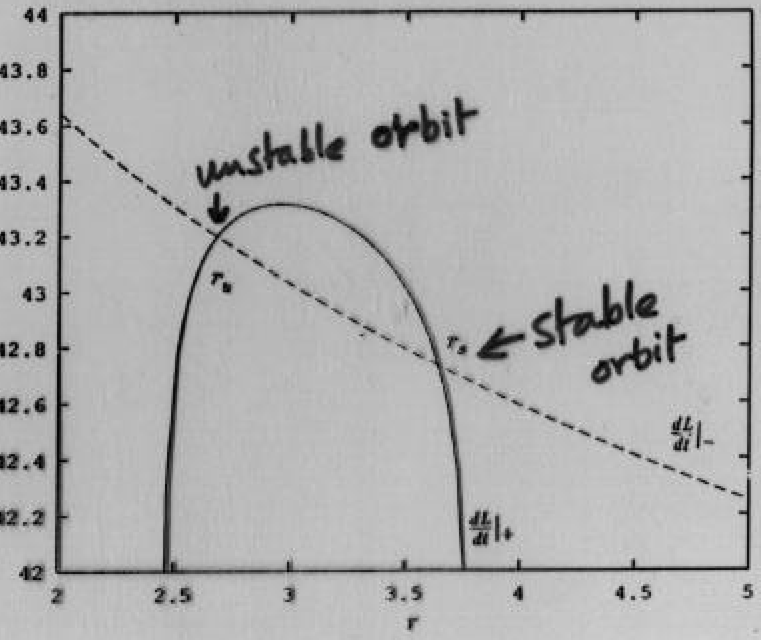
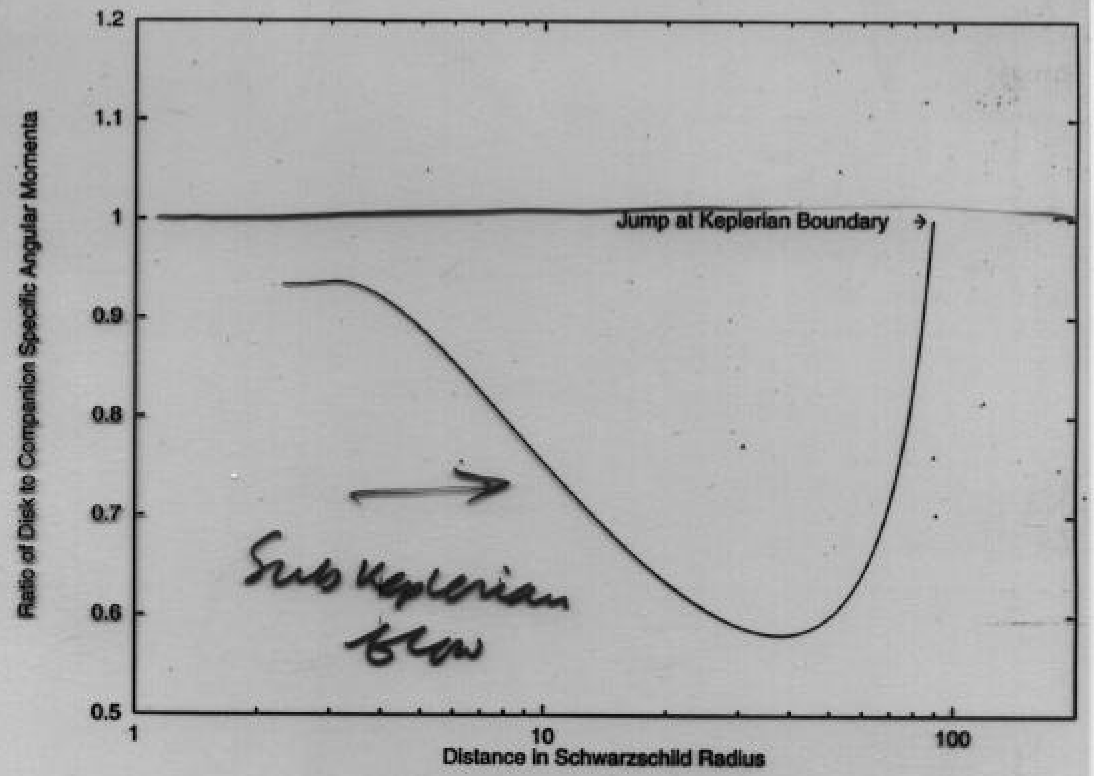
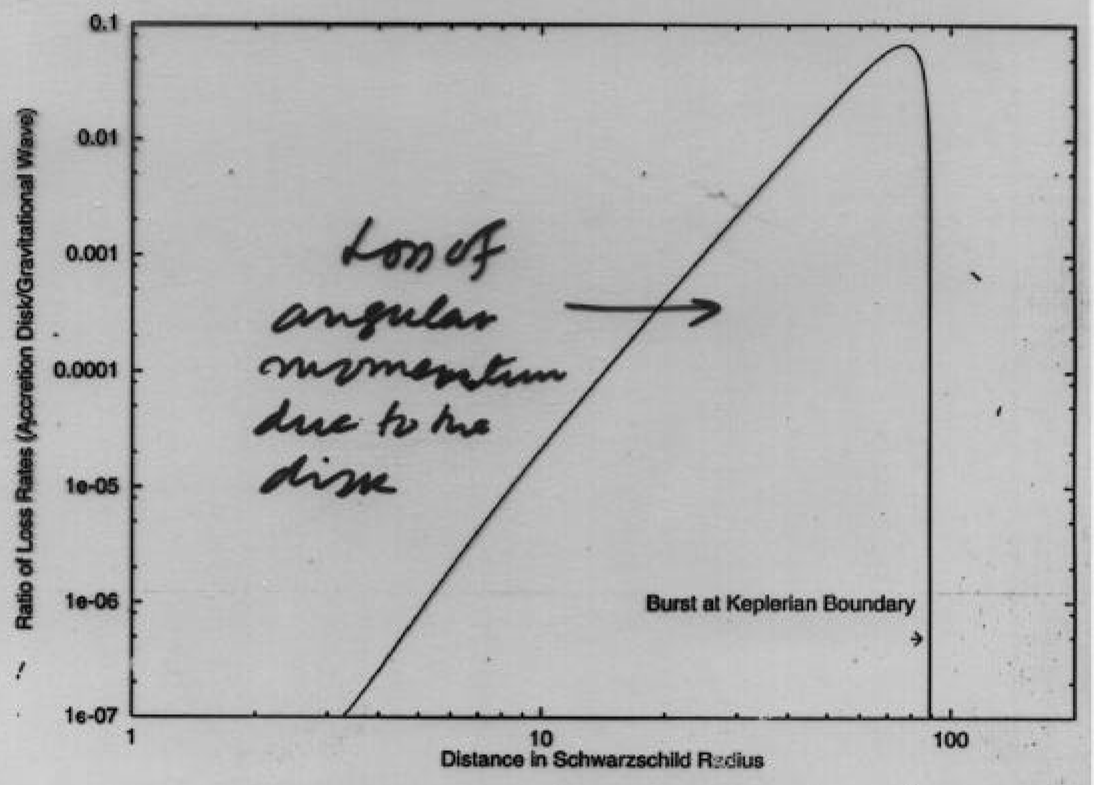


FIG. 2b

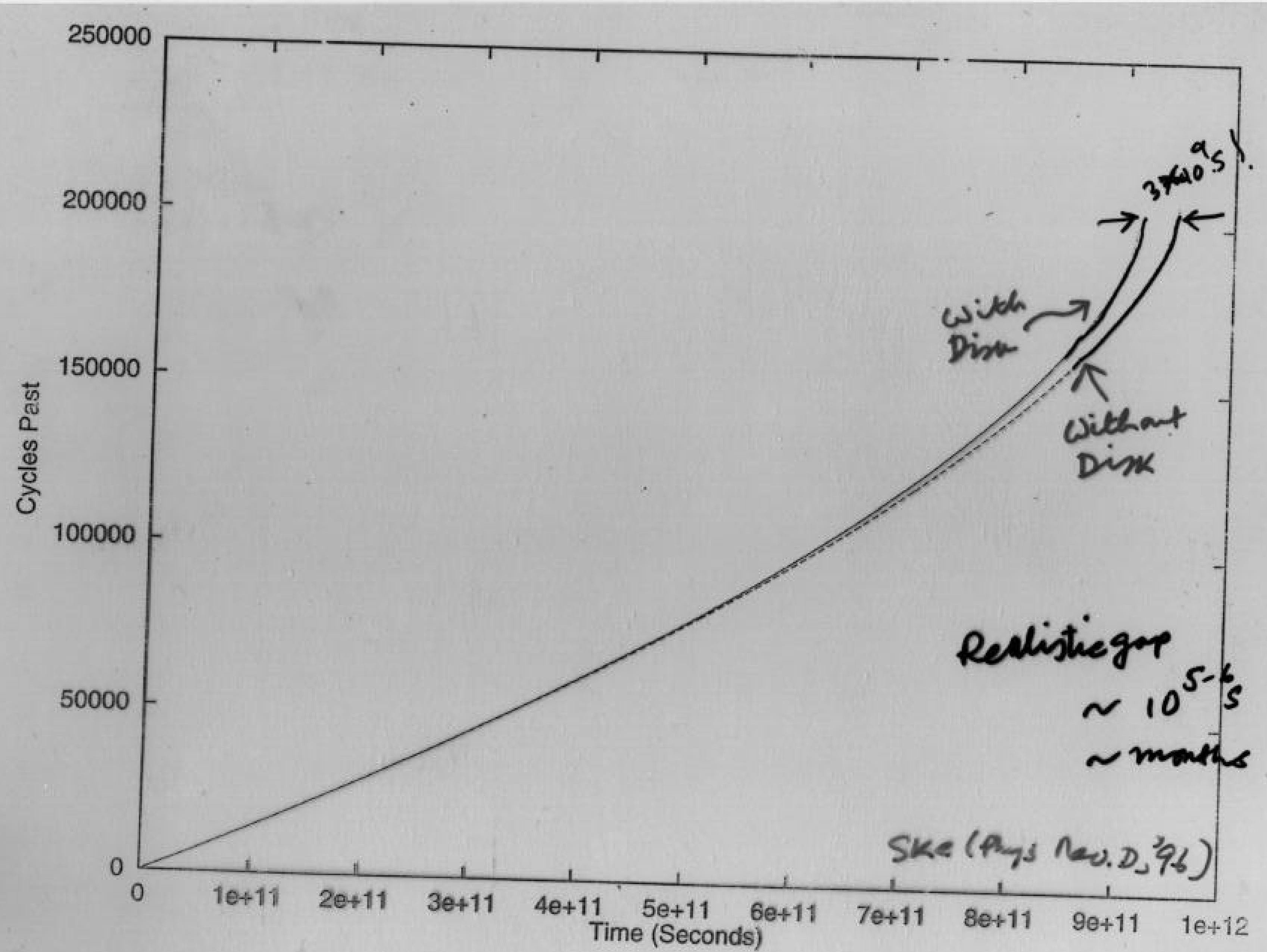
FIG. 2.—(*a*) Variation of (*a*) density $\rho_\infty \times 10^9$ g cm^{-3} (*solid*) and sound speed $a_\infty \times 10^{-9}$ cm s^{-1} (*short dashed*) inside the disk and the relative velocity $v_\infty \times 10^{-8}$ cm s^{-1} (*long dashed*) in the vicinity of the primary black hole, and (*b*) rate of gain dL_+/dt (*solid*) and loss dL_-/dt (*dashed*) of angular momentum by the secondary black hole. Panel (*b*) is drawn in logarithmic scale. At two locations r_s and r_u, balance can take place; however, only r_s is stable. The parameters are $E = 0.01c^2$ ergs g^{-1}, $l = 3.74GM_1/c$ cm^2 s^{-1}, $M_1 = 2 \times 10^9\ M_\odot$, and $M_2 = M_\odot$.

CHAKRABARTI, ApJ, 1993



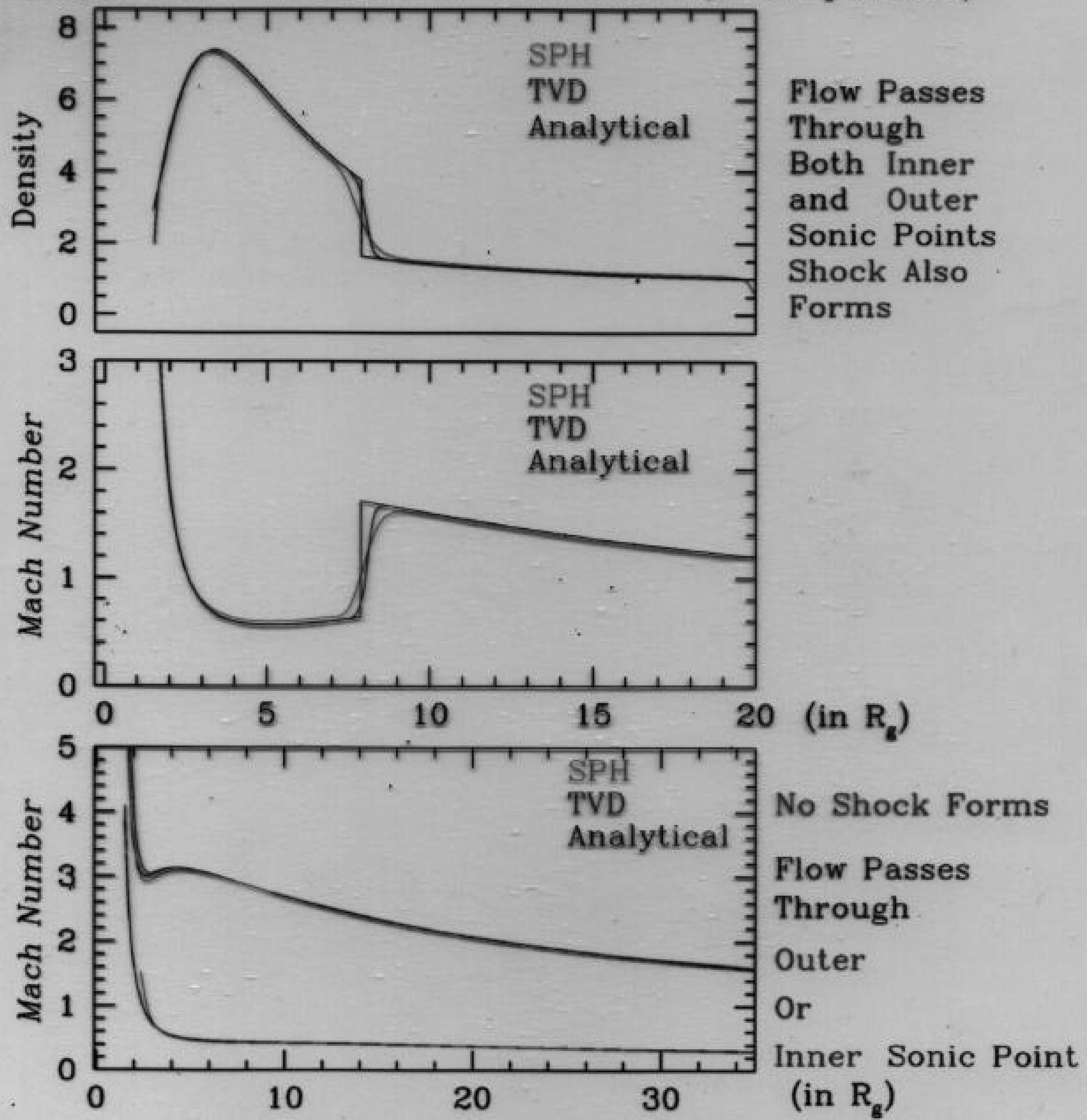


SKC (Phys. Rev. D, '96)



Cycles Past
250000
200000
150000
100000
50000
0
0
1e+11
2e+11
3e+11
4e+11
5e+11
6e+11
7e+11
8e+11
9e+11
1e+12
Time (Seconds)
With Disk
Without Disk
Realistic gap
~ 10^{5-6} s
~ months

Comparison With Numerical Simulations (MRC, ApJ, 1996)



Note: Smoothed Particle Hydrodynamics (SPH) and Total Variation Diminishing (TVD) schemes confirm very well the validity of the theoretical stationary solutions. These theoretical solutions constitute very important test problems against which accuracy of any numerical code (applicable to flows in curved space) could be verified.

Disk model dependence :=

For ADAF $\Rightarrow$ $\underline{R \sim 0}$

Narayan & co. Since the soln is valid for $\dot{m} \simeq 0$

For SLIM disk $\Rightarrow$ $\underline{R \sim 0}$ even if $\dot{m} \sim 100-1000$

Abramowicz & co.

Since the black hole and disk always move with same velocity

In soft state $\ell \sim \ell_{Kep}$

So, $R_{SS} \simeq 0$

$\therefore$ Spectral dependency is present

From observation in UV - Xray

$\Rightarrow$ get L, M_1, $\dot{M}$

From GW measurement

$\Rightarrow$ get M_2

---

frequency of interest:

$T \sim \frac{2GM}{c^3} \sim 10^{-5}\left(\frac{M}{M_\odot}\right) s$

$\nu \sim 10^5 \left(\frac{M_\odot}{M}\right) Hz$

For SMBH $M \sim 10^8 M_\odot$ $\nu \sim 10^{-3} Hz$

for IMB $M \sim 1000 M_\odot$ $\nu \sim 10^2 Hz$

So the wide range covered