CANONICAL REDUCTION OF GRAVITY:
FROM GENERAL COVARIANCE TO DIRAC OBSERVABLES
AND
POST-MINKOWSKIAN BACKGROUND-INDEPENDENT GRAVITATIONAL WAVES

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ORTAGGIO, BIJAK (PRAGA)

MASHHOON (COLUMBIA, MISSOURI)

MOTIVATIONS

1) DIRAC-BERGMANN THEORY OF CONSTRAINTS

   ⇒ CANONICAL REDUCTION OF GRAVITY (ANALYTICAL, NO KILLING VECTORS)

   WITH SEPARATION OF GAUGE VARIABLES FROM DIRAC OBSERVABLES

2) UNIFIED SCHEME ENGLOBING BOTH RELATIVITY EXPERIMENTS AND GRAVITATIONAL PHYSICS

3) BACKGROUND-INDEPENDENT QUANTIZATION SCHEME ABLE TO INCORPORATE THE STANDARD MODEL OF ELEMENTARY PARTICLES

   ⇒ COORDINATE-INDEPENDENCE

4) NO POST-NEWTONIAN APPROXIMATION –

   – LORENTZ SIGNATURE AND BRANCH POINT $\sqrt{m^2+\vec{p}^2}$ (HOW TO GET IT BY RESUMMING THE POST-NEWTONIAN EXPANSION ?) AND RELATIVISTIC NON-RIGID ROTATING SYSTEMS

5) SCHEME FOR HAMILTONIAN NUMERICAL GRAVITY IN A COMPLETELY FIXED HAMILTONIAN 3-ORTHOGONAL GAUGE

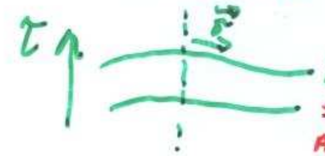
   PERTURBATIVE POST-MINKOWSKIAN APPROXIMATION (DEVELOPMENT IN POWERS OF G) TO SOLVE THE CONSTRAINTS

6) QUANTIZATION OF ATOMIC PHYSICS IN NON-INERTIAL FRAMES IN SPECIAL RELATIVITY

A) ADM CANONICAL METRIC AND TETRAD GRAVITY

NON-COMPACT, GLOBALLY HYPERBOLIC, TOPOLOGICALLY TRIVIAL, ASYMPTOTICALLY FLAT AT SPATIAL INFINITY SPACETIMES OF THE CHRISTODOULOU-KLAINERMANN TYPE   ADM ACTION

3+1 SPLITTING $M^4 \approx R \times \Sigma \quad \Sigma \approx R^3 \mapsto \Sigma_\tau \subset M^4$



SIMULTANEITY AND CAUCHY SURFACES

$z^\mu(\tau, \vec{\sigma})$ EMBEDDING

$\sigma^A = (\tau, \vec{\sigma})$ Σ_τ-ADAPTED 4-COORDINATES (OF RADAR TYPE, BONDI)

$\varepsilon = \pm 1 \quad {}^4\eta_{(\alpha)(\beta)} = \varepsilon\,(+---)$ ${}^4g_{\mu\nu} \mapsto N,\ N^r,\ {}^3g_{rs}$

$${}^4g_{AB}(\tau,\vec{\sigma}) = {}^4E^{(\alpha)}_A(\tau,\vec{\sigma})\ {}^4\eta_{(\alpha)(\beta)}\ {}^4E^{(\beta)}_B(\tau,\vec{\sigma}) = \varepsilon \begin{pmatrix} N^2 - {}^3g_{rs}N^rN^s & -{}^3g_{su}N^u \\ -{}^3g_{ru}N^u & -{}^3g_{rs} \end{pmatrix}(\tau,\vec{\sigma})$$

$${}^4E^A_{(\alpha)}(\tau,\vec{\sigma}) = L^A{}_B\big(\varphi_{(a)}(\tau,\vec{\sigma})\big)\ {}^4\overset{\circ}{E}{}^B_{(\alpha)}(\tau,\vec{\sigma})$$ TETRADS

DYNAMICAL TIMELIKE OBSERVER WITH SPATIAL TRIAD (GYROSCOPES)

COTETRADS ${}^4E^{(\alpha)}_A \mapsto \varphi_{(a)},\ N,\ N_{(a)} = N^r\,{}^3e_{(a)r},\ {}^3e_{(a)r}$

$${}^3g_{rs} = \sum_a {}^3e_{(a)r}\,{}^3e_{(a)s}$$ COTRIAD

BOUNDARY CONDITIONS AT SPATIAL INFINITY (REGGE, TEITELBOIM, BEIG, O'MURCHADHA)

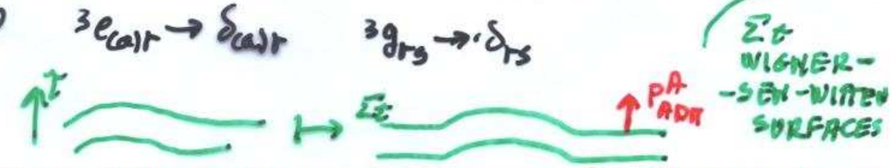
NO SUPERTRANSLATION: SPI GROUP $\mapsto$ ADM POINCARÉ GROUP $P^A_{ADM},\ J^{AB}_{ADM}$ { EXISTENCE ANGULAR MOMENTUM; STANDARD MODEL OF PARTICLES

$N(\tau,\vec{\sigma}) = N_{(as)}(\tau,\vec{\sigma}) + n(\tau,\vec{\sigma}) \qquad N_r(\tau,\vec{\sigma}) = N_{(as)r}(\tau,\vec{\sigma}) + n_r(\tau,\vec{\sigma}) \qquad N_{(as)} \Rightarrow const + \ldots$

ALL FIELDS WITH A SUITABLE DIRECTION-INDEPENDENT LIMIT AT SPATIAL INFINITY

$$n,\ n_r,\ \varphi_{(a)} \to 0 \qquad {}^3e_{(a)r} \to \delta_{(a)r} \qquad {}^3g_{rs} \to \delta_{rs}$$

RESTRICTION ON 3+1 SPLITTINGS



DECOUPLED CENTER OF MASS (POINT PARTICLE CLOCK)   FLAT MINKOWSKI PLANE

$$z^\mu(\tau,\vec{\sigma}) \longrightarrow \delta^\mu_{(\mu)}\left[\, x^{(\mu)}_{(\infty)}(\tau) + \varepsilon^{(\mu)}_r(P_{(\infty)})\,\sigma^r \right]$$ ASYMPTOTIC WIGNER HYPERPLANE

DIRAC ASYMPTOTIC CARTESIAN COORDINATES

ASYMPTOTIC INERTIAL OBSERVERS (FIXED STARS)

ASYMPTOTIC MINKOWSKI 4-METRIC (NOT BIMETRIC THEORY)

$\Rightarrow$ REST-FRAME INSTANT FORM FOR THE UNIVERSE $\begin{cases} \check{p}^F_{ADM} \approx 0 \\ \tau \approx T_{(\infty)} \end{cases}$

SPECIAL RELATIVITY - PARAMETRIZED MINKOWSKI THEORY FOR ISOLATED SYSTEMS

NICE 3+1 SPLITTINGS (ALSO AT SPATIAL INFINITY)

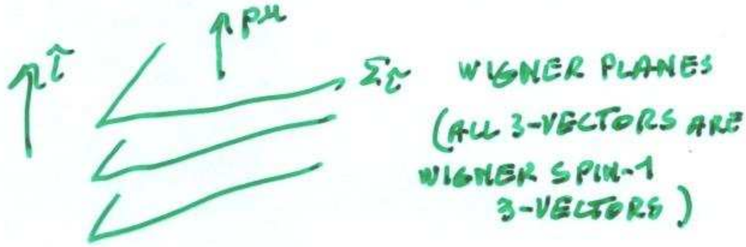
EMBEDDING $z^\mu(\tau, \vec{\sigma})$ $\left[\text{NOT } E^\mu_A = \frac{\partial z^\mu}{\partial \sigma^A}\right]$ AS CONFIGURATION GAUGE VARIABLES

$$ {}^4g_{AB}(\tau, \vec{\sigma}) = \frac{\partial z^\mu(\sigma)}{\partial \sigma^A} \, {}^4\eta_{\mu\nu} \, \frac{\partial z^\nu(\sigma)}{\partial \sigma^B} = {}^4g_{AB}[z] $$

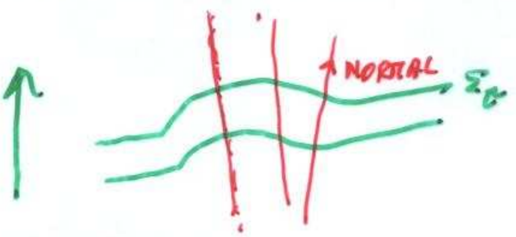
$\mathcal{L}(g[z], \text{matter})$ SPECIAL RELATIVISTIC GENERAL COVARIANCE

$\Rightarrow$ ALL 3+1 SPLITTINGS (AND THEREFORE ALL THE NOTIONS OF SIMULTANEITY) GAUGE EQUIVALENT

REST-FRAME INSTANT FORM



2 ASSOCIATED CONGRUENCES OF TIMELIKE OBSERVERS



$\Downarrow$

INTEGRAL LINES OF ${}^4\check{E}^A_{(\alpha)}$ OF ${}^4E^A_{(\alpha)} = L^A{}_B \, {}^4\check{E}^B_{(\alpha)}$

$\begin{cases} \text{3+1 POINT OF VIEW - NEEDED FOR CAUCHY PROBLEM AND FOR THE CONVENTION ON THE SYNCHRONIZATION OF CLOCKS} \\ \text{1+3 POINT OF VIEW - NEEDED FOR THE LOCAL MEASUREMENTS MADE BY A TIMELIKE OBSERVER ENDOWED WITH A TETRAD} \end{cases}$

14 FIRST CLASS CONSTRAINTS

$n, n_{(a)}, \varphi_{(a)}, {}^3e_{(a)r}$

$\pi^n, \pi^{\vec n}_{(a)}, \pi^{\vec\varphi}_{(a)}, {}^3\pi^r_{(a)}$

$$\left\{\begin{array}{l} \pi^n(\tau,\vec\sigma)\approx 0 \\ \pi^{\vec n}_{(a)}(\tau,\vec\sigma)\approx 0 \\ \pi^{\vec\varphi}_{(a)}(\tau,\vec\sigma)\approx 0 \\ {}^3M_{(a)(b)}(\tau,\vec\sigma) = \left[{}^3e_{(a)r}\,{}^3\pi^r_{(b)} - {}^3e_{(b)r}\,{}^3\pi^r_{(a)}\right](\tau,\vec\sigma)\approx 0 \end{array}\right.$$

PRIMARY CONSTRAINTS

SO(3,1)

SECONDARY CONSTRAINTS

$$\mathcal{H}(\tau,\vec\sigma) = \left[{}^3e\,\epsilon_{(a)(b)(c)}\,{}^3e^r_{(a)}\,{}^3e^s_{(b)}\,{}^3\Omega_{rs(c)} + \frac{1}{8\,{}^3e}\,G_{o(a)(b)(c)(d)}\,{}^3e_{(a)r}\,{}^3\pi^r_{(b)}\,{}^3e_{(c)s}\,{}^3\pi^s_{(d)}\right](\tau,\vec\sigma)\approx 0$$

SUPERHAMILTONIAN CONSTRAINT

$$\mathcal{H}_{(a)}(\tau,\vec\sigma) = -D^{(\omega)}_{(a)(b)r}(\tau,\vec\sigma)\,{}^3\pi^r_{(b)}(\tau,\vec\sigma) = {}^3e^r_{(a)}(\tau,\vec\sigma)\left[{}^3\Theta_r + {}^3\omega_{r(b)}\,{}^3\pi_{(b)}\right](\tau,\vec\sigma)\approx 0$$

OR $\quad {}^3\Theta_r(\tau,\vec\sigma) = \left[{}^3\pi^s_{(a)}\,\partial_r\,{}^3e_{(a)s} - \partial_s\left({}^3e_{(a)r}\,{}^3\pi^s_{(a)}\right)\right](\tau,\vec\sigma)\approx 0$

↳ GENERATES 3-DIFF ON Σ_τ

SUPER MOMENTUM CONSTRAINTS

$\mathcal{H}$ GENERATES HAM GAUGE TRANSF. CONNECTING THE ALLOWED 3+1 SPLITTINGS
NO WHEELER-DE WITT INTERPRETATION

10 ASYMPTOTIC (CONSTANT OF MOTION AND GAUGE INVARIANT) ADM POINCARÉ CHARGES

$$P^A_{ADM} = \check P^A_{ADM} + \int d^3\sigma\,\mathcal{H}^A(\tau,\vec\sigma) \approx \check P^A_{ADM} \qquad J^{AB}_{ADM}\approx \check J^{AB}_{ADM}$$

STRONG (SURFACE INTEGRAL) — WEAK (VOLUME INTEGRAL)

$\check E_{ADM} = c\,\check P^\tau_{ADM}$ DEPENDS ONLY ON ${}^3g_{rs}$ $\Rightarrow$ $\mathcal{H}(\tau,\vec\sigma)\approx 0$ IS AN EQUATION FOR $\boxed{\det {}^3g_{rs}}$ LICHNEROWICZ EQ.

HAMILTONIAN (DE WITT SURFACE TERMS, SPLITTING OF LAPSE AND SHIFT, BOUNDARY CONDITIONS)

$$H_D = \check E_{ADM} + \int d^3\sigma\left[\text{FIRST CLASS CONSTRAINTS}\right]\approx \check E_{ADM}$$

CHOQUET-BRUHAT
MARSDEN, FISHER

THE WEAK ADM ENERGY IS THE HAMILTONIAN OF THE REST-FRAME INSTANT FORM

NO FROZEN REDUCED PHASE SPACE

SCHEME OF GAUGE-FIXINGS

$$H_D = \check{E}_{ADM} + \int d^3\sigma \; \Sigma_\alpha \, \lambda_\alpha(\tau,\vec{\sigma}) \, \underbrace{\Phi_\alpha(\tau,\vec{\sigma})}_{\text{PRIMARY CONSTRAINTS}} + \int d^3\sigma \, [N \, \overbrace{\mathcal{H} + N_r \mathcal{H}^r}^{\text{SECONDARY CONSTRAINTS}}](\tau,\vec{\sigma}) \approx E_{ADM}$$

DIRAC MULTIPLIERS	$\lambda_n(\tau,\vec{\sigma})$	$\lambda_{\vec{n}}(\tau,\vec{\sigma})$	$\lambda_{\vec{\varphi}_{(a)}}(\tau,\vec{\sigma})$	$\lambda_{\alpha(a)}(\tau,\vec{\sigma})$
PRIMARY	$\pi_n(\tau,\vec{\sigma}) \approx 0$	$\pi^r_{\vec{n}}(\tau,\vec{\sigma}) \approx 0$	$\pi_{\varphi(a)}(\tau,\vec{\sigma}) \approx 0$	$\pi_{(a)}(\tau,\vec{\sigma}) \approx 0$
	$\downarrow$	$\downarrow$		
SECONDARY	$\mathcal{H}(\tau,\vec{\sigma}) \approx 0$	$\mathcal{H}_r(\tau,\vec{\sigma}) \approx 0$		
NATURAL GAUGE FIXINGS (SUGANO-KAGRAOKA-KIMURA)	$\chi_{\mathcal{H}}(\tau,\vec{\sigma}) \approx 0$	$\chi_r(\tau,\vec{\sigma}) \approx 0$	$\chi_{\varphi(a)}(\tau,\vec{\sigma}) \approx 0$	$\chi_{\alpha(a)}(\tau,\vec{\sigma}) \approx 0$
	$\downarrow$ SHAPE OF Σ_τ	$\downarrow$ 3-COORD ON Σ_τ	$\downarrow$ ADAPTED COTETRADS	$\downarrow$
	$\partial_\tau \chi_{\mathcal{H}}(\tau,\vec{\sigma}) = \chi_n(\tau,\vec{\sigma}) \approx 0$	$\partial_\tau \chi_r(\tau,\vec{\sigma}) = \chi_{\vec{n}_r}(\tau,\vec{\sigma}) \approx 0$	$\partial_\tau \chi_{\varphi(a)}(\tau,\vec{\sigma}) \approx 0$	$\partial_\tau \chi_{\alpha(a)}(\tau,\vec{\sigma}) \approx 0$
	$\downarrow$ LAPSE (PACKING OF Σ_τ's)	$\downarrow$ SHIFT	$\downarrow$	$\downarrow$
	$\partial_\tau \chi_n(\tau,\vec{\sigma}) \approx 0$	$\partial_\tau \chi_{\vec{n}_r}(\tau,\vec{\sigma}) \approx 0$	$\lambda_{\vec{\varphi}(a)}(\tau,\vec{\sigma})$	$\lambda_{\alpha(a)}(\tau,\vec{\sigma})$
	$\downarrow$	$\downarrow$		
	$\lambda_n(\tau,\vec{\sigma})$	$\lambda_{\vec{n}_r}(\tau,\vec{\sigma})$		

COMPLETELY FIXED HAM. GAUGE $\underset{\text{ON SOLUTIONS OF HAM. EQS (I.E. EINSTEIN EQS)}}{=\!=\!=}$ 4-COORDINATE SYSTEM +
+ 3+1 SPLITTING WITH LEAVES
Σ_τ (IN GENERAL NOT HYPERPLANES)
DEFINED BY AN EMBEDDING
$$z^\mu_\sigma(\tau,\vec{\sigma})$$
$$\parallel$$
CONVENTION FOR THE SYNCHRONIZATION
OF CLOCKS ($\neq$ EINSTEIN'S CONVENTION)

B) GENERAL COVARIANCE, HOLE ARGUMENT, OBJECTIVITY OF SPACETIME POINTS

$^4g_{\mu\nu}$ a) POTENTIAL FOR THE GRAVITATIONAL FIELD

b) CHRONOGEOMETRICAL STRUCTURE $ds^2 = {}^4g_{\mu\nu}(x)\, dx^\mu dx^\nu$

$^4g_{\mu\nu}$ TEACHES RELATIVISTIC CAUSALITY TO THE OTHER FIELDS
LOST IN THEORIES ON BACKGROUND!

IN THE ABSOLUTE MINKOWSKI SPACETIME TIME AND LENGTH MEASUREMENTS DEPEND ON THE CHOICE OF THE SIMULTANEITY SURFACES (CLOCKS AND RODS) BUT ARE NOT DYNAMICAL

TILL NOW - ONLY AXIOMATIC (EHLERS-PIRANI-SCHILD) THEORY OF MEASUREMENT IN G.R. USING TEST OBJECTS

HOLE ARGUMENT (EINSTEIN, STACHEL, ... , LL+PAURI)

EVERYTHING FIXED OUTSIDE

INVARIANCE OF EINSTEIN EQS UNDER ACTIVE DIFF.

$H \mapsto H \quad g \mapsto g' = D_A g \quad g'(D_A x) = g(x)$

$x \mapsto D_A x$

$g'(x) \neq g(x)$ BOTH SOLUTIONS

$\Rightarrow$ DETERMINISM AND OBJECTIVITY OF SPACETIME POINTS DESTROYED

1) WEYL - ONLY COORDINATES INDIVIDUATE THE POINTS OF A MATHEMATICAL MANIFOLD

2) BERGMANN-KOMAR - PASSIVE REINTERPRETATION OF ACTIVE DIFF $x \mapsto x'(x, g)$

DYNAMICAL SYMMETRIES OF EINSTEIN EQS

→ ACTIVE DIFF

→ PULLBACK OF CANONICAL TRANSF.

→ PASSIVE DIF (COORD. TRANSF.)

3) DIRAC - EQUIVALENCE CLASS UNDER HAM. GAUGE TRANSF →

→ DETERMINISTIC DIRAC OBSERVABLES (GAUGE INVARIANT) DO

4) BERGMANN - BERGMANN OBSERVABLES (BO) - SPECIAL COORDINATE-INDEPENDENT DO

5) BERGMANN-KOMAR - INTRINSIC (COORD-INDEPENDENT) COORDINATES FROM EIGENVALUES OF WEYL TENSOR
(SEE ALSO DE WITT)

6) LL-PAURI - GAUGE FIXINGS TO CONSTRAINT DEFINING INTRINSIC RADAR-TYPE COORD.

$\sigma^A - F^{[A]}(DO) \approx 0$

$\Rightarrow$ SPACETIME $\equiv$ GRAVITATIONAL FIELD

EVERY 4-COORDINATE SYSTEM HAS AN ASSOCIATED NON-COMMUTATIVE STRUCTURE

c) SHANMUGADHASAN CANONICAL TRANSFORMATION ADAPTED TO 13 FIRST CLASS CONSTRAINTS (NOT TO THE SUPERHAMILTONIAN CONSTRAINT)

n	$n_{(a)}$	$\varphi_{(a)}$	${}^3e_{(a)r}$
≈ 0	≈ 0	≈ 0	${}^3\pi^r_{(a)}$

$\longmapsto$

n	$n_{(a)}$	$\varphi_{(a)}$	$\alpha_{(a)}$	ξ^r	ϕ	$r_{\bar a}$
≈ 0	≈ 0	≈ 0	≈ 0	≈ 0	π_ϕ	$\pi_{\bar a}$

$\bar a = 1,2$ DO

NOT TENSOR in general

POINT CAN. TRANS F.

X GAUGE VARIABLES (GENERALIZED INERTIAL EFFECTS)

X DO (DETERMINISTIC GENERALIZED TIDAL EFFECTS)

ALL NON-LOCAL – $\alpha_{(a)}, \xi^r, \pi_\phi, r_{\bar a}, \pi_{\bar a}$ NOT KNOWN IN TERMS OF THE ORIGINAL VARIABLES

BUT 1) FROM FINITE GAUGE TRANSF.

$$ {}^3e_{(a)r}(\tau,\vec\sigma) = R_{(a)(b)}\big(\alpha_{(a)}(\tau,\vec\sigma)\big)\,\frac{\partial \xi^s(\tau,\vec\sigma)}{\partial\sigma^r}\;{}^3\hat e_{(b)s}\big[\phi(\tau,\vec\xi(\tau,\vec\sigma)),\, r_{\bar a}(\tau,\vec\xi(\tau,\vec\sigma))\big] $$

2) POINT CAN. TRANSF

$$ {}^3\pi^r_{(a)}(\tau,\vec\sigma) = \sum_s \int d^3\sigma_1\, K^r_{(a)s}(\vec\sigma,\vec\sigma_1,\tau|\alpha_{(a)},\xi^r,\phi,r_{\bar a}]\,\Big(\phi^{-2}e^{-\frac{1}{\sqrt3}\sum_{\bar a}\gamma_{\bar a s} r_{\bar a}}\Big)(\tau,\vec\sigma_1)\cdot $$
$$ \cdot\Big[\frac{\pi_\phi}{6\phi} + \sqrt3\,\sum_{\bar b}\gamma_{\bar b s}\,\pi_{\bar b}\Big](\tau,\vec\sigma_1) + $$
$$ + \int d^3\sigma_1\, F^r_{(a)(b)}(\vec\sigma,\vec\sigma_1,\tau|\alpha_{(a)},\xi^r,\phi,r_{\bar a}]\;\pi^{(\alpha)}_{(b)}(\tau,\vec\sigma_1) + $$
$$ + \sum_u \int d^3\sigma_1\, G^{ru}_{(a)}(\vec\sigma,\vec\sigma_1,\tau|\alpha_{(a)},\xi^r,\phi,r_{\bar a}]\;\pi^{(\xi)}_u(\tau,\vec\sigma_1) \qquad \approx 0 $$

CANONICITY CONDITIONS – $K^r_{(a)s}$ and $F^r_{(a)(b)}$ FROM $G^{ru}_{(a)}$

1) CANONICITY CONDITIONS + 3 ROTATION $(M_{(a)}\approx 0)$ AND 3 SUPERMOMENTUM $(\mathcal{H}_r\approx 0)$ CONSTRAINTS

$\Rightarrow$ SET OF ELLIPTIC PARTIAL DIFFERENTIAL EQs FOR THE KERNEL $G^{ru}_{(a)}$

2) $\phi = (\det {}^3g_{rs})^{1/12}$ CONFORMAL FACTOR OF 3-METRIC – FROM LICHNEROWICZ EQ.

3) π_ϕ – LAST GAUGE VARIABLE (A MOMENTUM; $\xi^r + \pi_\phi$ LORENTZ SIGNATURE)

IT (AND NOT ${}^3K \sim$ YORK TIME) DESCRIBES THE NORMAL DEFORMATIONS

OF Σ_τ  (NOT YORK MAP)

POINT CANONICAL TRANSFORMATION

$$\begin{cases} {}^3e_{(a)r}(\tau,\vec\sigma) = {}^3R_{(a)(b)}(\alpha_{(e)}(\tau,\vec\sigma))\, \dfrac{\partial \xi^s(\tau,\vec\sigma)}{\partial\sigma^r}\; {}^3\hat e_{(b)s}[\phi(\tau,\vec\xi(\tau,\vec\sigma)), r_{\bar a}(\tau,\vec\xi(\tau,\vec\sigma))] \\ \\ {}^3\tilde\pi^r_{(a)}(\tau,\vec\sigma) = \sum_s \int d^3\sigma_1\, K^r_{(a)s}(\vec\sigma,\vec\sigma_1,\tau|\alpha_{(e)},\xi^u,\phi,r_{\bar a}]\, \Big(\phi^{-2} e^{-\frac{1}{\sqrt3}\sum_{\bar a}\gamma_{\bar a s} r_{\bar a}(\tau,\vec\sigma)}\Big)(\tau,\vec\sigma_1) \end{cases}$$

$$\cdot \Big[\frac{\pi_\phi}{6\phi} + \sqrt3 \sum_{\bar b}\gamma_{\bar b s}\pi_{\bar b}\Big](\tau,\vec\sigma_1) +$$

$$+ \int d^3\sigma_1\, F^r_{(a)(b)}(\vec\sigma,\vec\sigma_1,\tau|\alpha_{(e)},\xi^u,\phi,r_{\bar a}]\; \underbrace{\tilde\pi^{(\alpha)}_{(b)}(\tau,\vec\sigma_1)}_{\approx 0} +$$

$$+ \sum_u \int d^3\sigma_1\, G^{ru}_{(a)}(\vec\sigma,\vec\sigma_1,\tau|\alpha_{(e)},\xi^u,\phi,r_{\bar a}]\; \tilde\pi^{\vec\xi}_u(\tau,\vec\sigma_1)$$

CANONICITY

$K^r_{(a)s}$, $F^r_{(a)(b)}$ FROM $G^{ru}_{(a)}$ SOLUTION OF $\quad Q_u = \phi^2\, e^{\frac{1}{\sqrt3}\sum_{\bar a}\gamma_{\bar a u} r_{\bar a}}$

$$\frac{1}{Q^2_{r_1}(\tau,\vec\sigma)}\, \frac{\partial G^{s r_2}_{(a)}(\vec\sigma_1,\vec\sigma,\tau)}{\partial\sigma^{r_1}} + \frac{1}{Q^2_{r_2}(\tau,\vec\sigma)}\, \frac{\partial G^{s r_1}_{(a)}(\vec\sigma_1,\vec\sigma,\tau)}{\partial\sigma^{r_2}} =$$

$$= \Big[\frac{\delta_{(a)r_1}\,\delta^s_{r_2}}{Q_{r_1}(\tau,\vec\sigma)\, Q^2_{r_2}(\tau,\vec\sigma)} + \frac{\delta_{(a)r_2}\,\delta^s_{r_1}}{Q^2_{r_1}(\tau,\vec\sigma)\, Q_{r_2}(\tau,\vec\sigma)}\Big]\, \delta^3(\vec\sigma_1,\vec\sigma)$$

EQUIVALENT TO SOLVE THE 6 ROTATION AND SUPERMOMENTUM CONSTRAINTS IF $G^{ru}_{(a)}$ ALSO SATISFY

1) $\sum_r Q_r(\tau,\vec\sigma)\Big(\delta_{(a)r}K^r_{(b)u} - \delta_{(b)r}K^r_{(a)u}\Big)(\vec\sigma,\vec\sigma_1,\tau) =$

$$= Q_u(\tau,\vec\sigma_1)\Big[Q_a(\tau,\vec\sigma)\,\frac{\partial G^{au}_{(b)}(\vec\sigma,\vec\sigma_1,\tau)}{\partial\sigma_1^u} - Q_b(\tau,\vec\sigma)\,\frac{\partial G^{bu}_{(a)}(\vec\sigma,\vec\sigma_1,\tau)}{\partial\sigma_1^u}\Big] +$$

$$+ \sum_v \frac{\partial Q_u(\tau,\vec\sigma_1)}{\partial\sigma_1^v}\Big[Q_a(\tau,\vec\sigma)\, G^{av}_{(b)}(\vec\sigma,\vec\sigma_1,\tau) - Q_b(\tau,\vec\sigma)\, G^{bv}_{(a)}(\vec\sigma,\vec\sigma_1,\tau)\Big] = 0 \qquad a \not= b$$

2) $\hat D_{(a)(b)r}\, K^r_{(b)u}(\vec\sigma,\vec\sigma_1,\tau) = \Big[\delta_{(a)(b)}\partial_r + \sum_u\big(\delta_{(a)r}\delta_{(b)u} - \delta_{(a)u}\delta_{(b)r}\big)\frac{\partial_u Q_r(\tau,\vec\sigma)}{Q_u(\tau,\vec\sigma)}\Big]\,\cdot$

$$\cdot \Big[\delta^r_{(b)}\delta_{(b)u}\delta^3(\vec\sigma,\vec\sigma_1) - Q_u(\tau,\vec\sigma_1)\frac{\partial G^{ru}_{(b)}(\vec\sigma,\vec\sigma_1,\tau)}{\partial\sigma_1^u} - \sum_v \frac{\partial Q_u(\tau,\vec\sigma_1)}{\partial\sigma_1^v}\, G^{rv}_{(b)}(\vec\sigma,\vec\sigma_1,\tau)\Big] = 0$$

D) COMPLETELY FIXED 3-ORTHOGONAL GAUGE

COMPLETELY FIXED HAMILTONIAN GAUGE $=$

$\underset{\text{ON SOLUTIONS OF EINSTEIN EQS}}{=}$ 4-COORDINATE SYSTEM ON M^4 + A 3+1 SPLITTING Σ_τ $=$

$=$ EXTENDED LABORATORY WITH A SIMULTANEITY NOTION (Σ_τ)

$\varphi_{(a)}(\tau,\vec\sigma) \approx 0$

$\alpha_{(a)}(\tau,\vec\sigma) \approx 0$

$\xi^r(\tau,\vec\sigma) - \sigma^r \approx 0$ WITH $\quad {}^3e_{(a)r} = \delta_{(a)r}\, \phi^2\, e^{\frac{1}{\sqrt3}\sum_{\bar a}\gamma_{\bar ar}r_{\bar a}}$

$$ {}^3g_{rs} = \phi^4 \underbrace{\begin{pmatrix} e^{\frac{2}{\sqrt3}\sum_{\bar a}\gamma_{\bar a1}r_{\bar a}} & 0 & 0 \\ 0 & e^{\frac{2}{\sqrt3}\sum_{\bar a}\gamma_{\bar a2}r_{\bar a}} & 0 \\ 0 & 0 & e^{\frac{2}{\sqrt3}\sum_{\bar a}\gamma_{\bar a3}r_{\bar a}} \end{pmatrix}}_{\det = 1} $$

$\rightarrow$ IMPLIED ELLIPTI PDO FOR $n_r(\tau,\vec\sigma)$

$\pi_\phi(\tau,\vec\sigma) \approx 0$

1) IMPLIES INTEGRAL EQ FOR $n(\tau,\vec\sigma)$

2) LEAVES $r_{\bar a}, \pi_{\bar a}$ 2 CANONICAL PAIRS OF DO

EVEN IF THE SOLUTION $\phi[r_{\bar a},\pi_{\bar a}]$ OF THE (INTEGRO-DIFF) LICHNEROWICZ EQ, RESTRICTED TO THIS GAUGE, $\mathcal{H}(\tau,\vec\sigma)\big|_G = 0$ IS NOT KNOWN

3) DIFFERENT FROM ${}^3K(\tau,\vec\sigma) \approx 0$ (MAXIMAL SLICING CONDITION OF YORK, WITH CONSTANT MEAN EXTRINSIC CURVATURE (CMC) Σ_τ)

IT IS NOT A HARMONIC GAUGE !

Σ_τ NOT CMC

REST-FRAME CONDITIONS $\quad \hat P^r_{ADM}[r_{\bar a},\pi_{\bar a}] \approx 0$

HAMILTONIAN $\quad H_D = \hat E_{ADM} = \int d^3\sigma\, \mathcal{E}_{ADM}[r_{\bar a}, \pi_{\bar a}, \phi(r_{\bar a},\pi_{\bar a})](\tau,\vec\sigma)$

WELL DEFINED ENERGY DENSITY (ON-SHELL IS A FIXATION OF 4-COORDINATES)

$\mathcal{E}_{ADM} \neq T^{\tau\tau}_{LANDAU-LIFSCHITZ}$ (THIS PSEUDOTENSOR IS RELIABLE ONLY IN SPECIAL 4-COORDINATE SYSTEMS)

E) BACKGROUND INDEPENDENT HAMILTONIAN LINEARIZATION

(L., AGRESTI, MARTUCCI, DE PIETRI)

$r_{\bar a}(\tau,\vec\sigma) << 1 \qquad |\partial_u r_{\bar a}(\tau,\vec\sigma)| \sim \frac{1}{L} O(r_{\bar a})$

$|\Pi_{\bar a}(\tau,\vec\sigma)| \sim \frac{K}{L} O(r_{\bar a}) \qquad K = \frac{c^3}{16\pi G}$

$r_{\bar a}, \Pi_{\bar a}$ SLOWLY VARYING OVER WAVELENGTH $L = \frac{\lambda}{2\pi}$

WEAK FIELD APPROXIMATION

RIEMANN $\sim \frac{1}{KL} O(\Pi_{\bar a}) = \frac{1}{L^2} O(r_{\bar a}) \sim \frac{1}{R^2}$ MEAN CURVATURE

1) $a \sim O(r_{\bar a})$ amplitude

2) $\frac{L}{R} = O(r_{\bar a}) \to \frac{\lambda}{2\pi} << R$

a) $\phi = e^{q/2} = (\det {}^3g)^{1/12} \qquad q(\tau,\vec\sigma) = O(r_{\bar a})$

EQ. LICHNEROWICZ $\qquad \Delta q(\tau,\vec\sigma) = \frac{1}{2\sqrt3} \sum_{u\bar a} \gamma_{\bar a u}\, \partial_u^2 r_{\bar a}(\tau,\vec\sigma) + \frac{1}{L^2} O(r_{\bar a}^2)$

$O(r_a)$

$\phi = 1 + \frac12 q + O(r_{\bar a}^2) \qquad q(\tau,\vec\sigma) = \frac{-1}{2\sqrt3} \sum_{u\bar a} \gamma_{\bar a u} \int d^3\sigma_1 \frac{\partial^2_{1u} r_{\bar a}(\tau,\vec\sigma_1)}{4\pi|\vec\sigma - \vec\sigma_1|} + O(r_{\bar a}^2)$

b) ROTATION + SUPERMOMENTUM CONSTRAINT + CANONICITY CONDITIONS

$$ {}^3\pi^r_{(a)}(\tau,\vec\sigma) = \sqrt3 \sum_{\bar a} \gamma_{\bar a r}\, \delta^r_{(a)}\, \Pi_{\bar a}(\tau,\vec\sigma) + $$
$$ + \frac{\sqrt3}{2}\left(1-\delta^r_{(a)}\right) \sum_{\bar a u} \gamma_{\bar a u}\left[1 - 2\left(\delta_{ru} + \delta_{au}\right)\right] \frac{\partial^2}{(\partial\sigma^u)^2} \int_{\sigma^r}^{\infty} d\sigma_1^r \int_{\sigma^a}^{\infty} d\sigma_1^a\, \Pi_{\bar a}(\tau, \sigma_1^r \sigma_1^a \sigma_1^{k\neq r,a}) $$
$$ + O(r_{\bar a}^2) $$

$\int_{-\infty}^{\infty} d\sigma^r\, \Pi_{\bar a}(\tau,\vec\sigma) = 0 \quad r = 1,2,3 \qquad \int_{-\infty}^{\infty} d\sigma^r\, r_{\bar a}(\tau,\vec\sigma) = 0$

c) LAPSE + SHIFT

ALL THE CONSTRAINTS HAVE BEEN SOLVED!

$N(\tau,\vec\sigma) = -\varepsilon + n(\tau,\vec\sigma) = -\varepsilon + O(r_{\bar a}^2)$

$N_r(\tau,\vec\sigma) = n_r(\tau,\vec\sigma) = -\varepsilon\, {}^4g_{\tau r}(\tau,\vec\sigma) = \text{-------}$ $\neq 0 \Rightarrow$ GRAVITOMAGNETISM

NOT HARMONIC GAUGE

d) ${}^4g_{AB} = {}^4\eta_{AB} + {}^4h_{AB}$

${}^4h_{\tau\tau} = 0 + O(r_{\bar a}^2)$

${}^4h_{\tau r} = -\epsilon\, n_r = \dots$

BACKGROUND-INDEPENDENT POST-MINKOWSKIAN SOLUTION

$$
{}^4h_{rs}(\tau,\vec\sigma) = -\frac{2\epsilon}{\sqrt{3}} \sum_{\bar a} \Big[\gamma_{\bar a r}\, r_{\bar a}(\tau,\vec\sigma) - \frac{1}{2}\sum_u \gamma_{\bar a u} \int d^3\sigma_1 \frac{\partial^2_{1u} r_{\bar a}(\tau,\vec\sigma_1)}{4\pi|\vec\sigma-\vec\sigma_1|} \Big]\, \delta_{rs} + O(r_{\bar a}^2)
$$

e) $O(r_{\bar a}^2)$

$$
\begin{aligned}
E_{ADM} = & \frac{12\pi G}{c^3} \int d^3\sigma \sum_{\bar a} \Big[\pi^2_{\bar a}(\tau,\vec\sigma) + \sum_{\substack{\bar a\bar b\\ rs}} \gamma_{\bar a r}\gamma_{\bar b s} \int d^3\sigma_1 d^3\sigma_2 \,\cdot \\
& \cdot \sum_u \zeta^{(0)u}_{(a)r}(\vec\sigma,\vec\sigma_1)\, \zeta^{(0)u}_{(a)s}(\vec\sigma,\vec\sigma_2)\, \pi_{\bar a}(\tau,\vec\sigma_1)\, \pi_{\bar b}(\tau,\vec\sigma_2) \Big] - \\
& - \frac{c^3}{16\pi G} \sum_r \int d^3\sigma \Big[\frac{1}{6} \Big(\sum_{\bar a u} \gamma_{\bar a u} \frac{\partial}{\partial\sigma^r} \int d^3\sigma_1 \frac{\partial^2_{1u} r_{\bar a}(\tau,\vec\sigma_1)}{4\pi|\vec\sigma-\vec\sigma_1|} \Big)^2 - \\
& - \frac{1}{3} \sum_{\bar a} \big(\partial_r r_{\bar a}(\tau,\vec\sigma)\big)^2 + \frac{2}{3} \Big(\sum_{\bar a} \gamma_{\bar a r}\, \partial_r r_{\bar a}(\tau,\vec\sigma) \Big)^2 - \\
& - \frac{1}{3} \sum_{\bar a\bar b u} \gamma_{\bar a r}\, \partial_r r_{\bar a}(\tau,\vec\sigma)\, \gamma_{\bar b u} \frac{\partial}{\partial\sigma^r} \int d^3\sigma_1 \frac{\partial^2_{1u} r_{\bar a}(\tau,\vec\sigma_1)}{4\pi|\vec\sigma-\vec\sigma_1|} \Big] + O(r_{\bar a}^3)
\end{aligned}
$$

$$
P^r_{ADM} = -\int d^3\sigma \sum_{\bar a} \pi_{\bar a}(\tau,\vec\sigma)\, \partial_r r_{\bar a}(\tau,\vec\sigma) \approx 0
$$

REST FRAME CONDITION

$$
J^{rs}_{ADM} = \int d^3\sigma \sum_{\bar a} \pi_{\bar a}(\tau,\vec\sigma) \big(\sigma^r\partial_s - \sigma^s\partial_r\big)\, r_{\bar a}(\tau,\vec\sigma)
$$

f) HAMILTON EQS $\Rightarrow$ $\Box\, r_{\bar a}(\tau,\vec\sigma) = 0$

TT HARMONIC GAUGE POLARIZATIONS $\rightarrow r_{\bar a}(\tau,\vec\sigma)$

NO L.L.

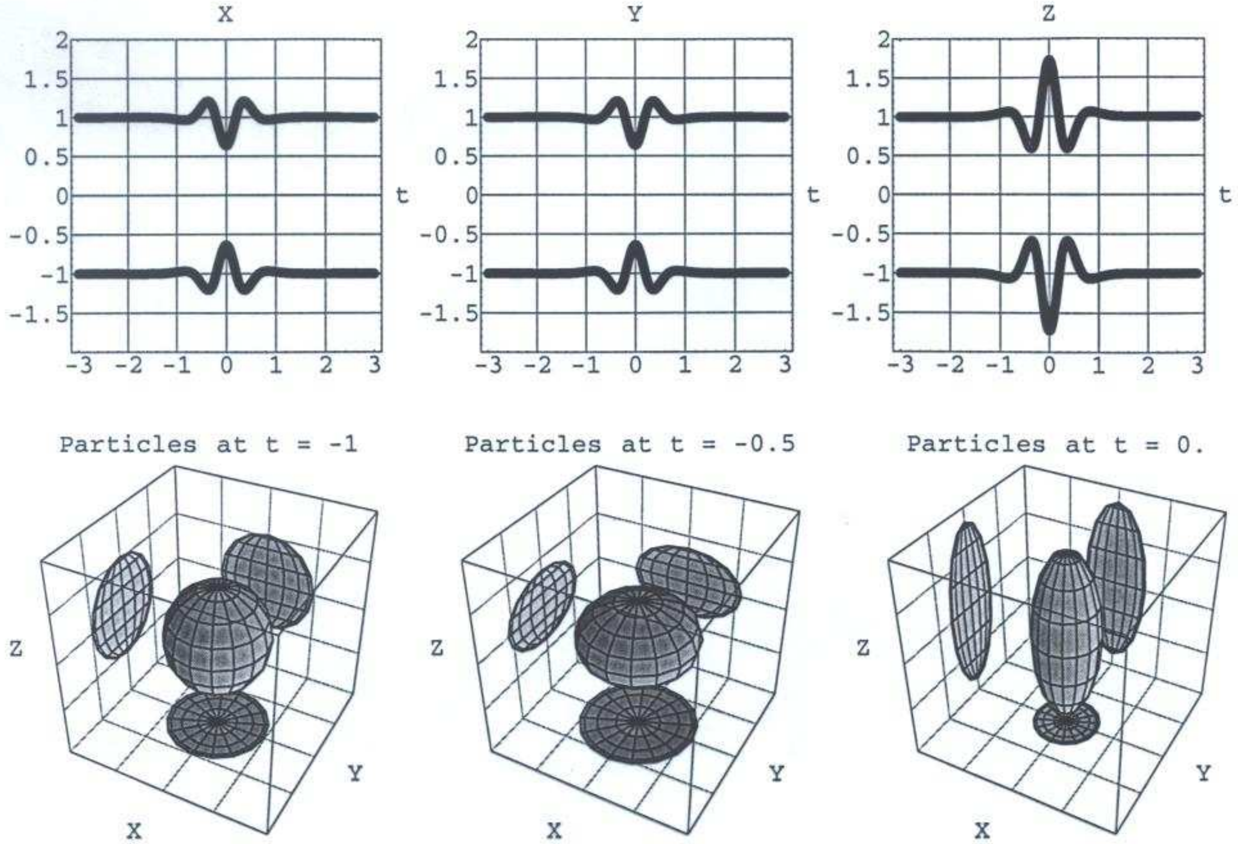
$$
\begin{cases}
r_{\bar a}(\tau,\vec\sigma) = C_{\bar a} \displaystyle\int \frac{d^3k}{(2\pi)^3} \frac{(k_1k_2k_3)^2 e^{-\vec k^2}}{|\vec k|} \Big(e^{-i(|\vec k|\tau - \vec k\cdot\vec\sigma)} + cc \Big) \\
\pi_{\bar a}(\tau,\vec\sigma) = -i \displaystyle\int \frac{d^3k}{(2\pi)^3} \frac{(k_1k_2k_3)^2 e^{-\vec k^2}}{|\vec k|}\, |\vec k| \sum_{\bar b} A^{-1}_{\bar a\bar b}(\vec k)\, C_{\bar b} \Big(e^{-i(|\vec k|\tau - \vec k\cdot\vec\sigma)} - c.c \Big)
\end{cases}
$$

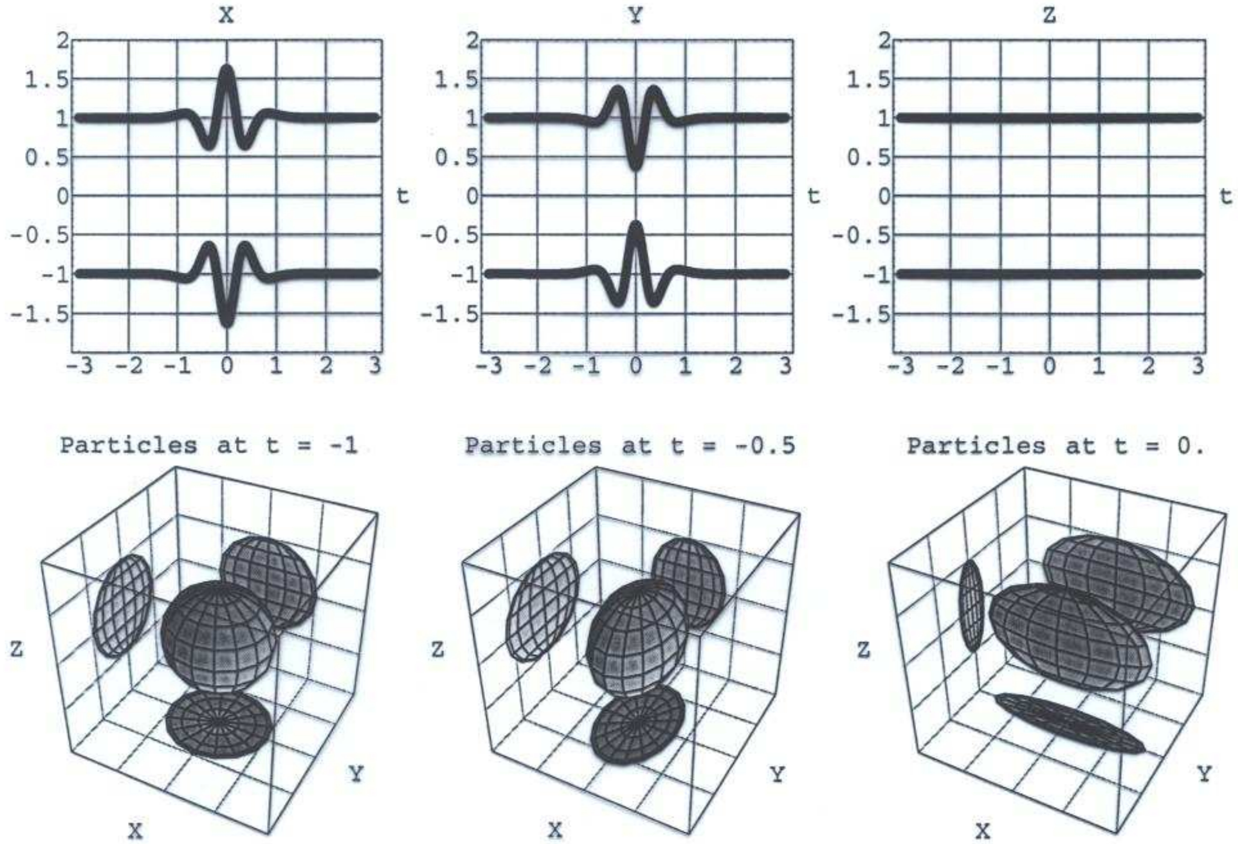
LINEARIZED EINSTEIN EQS SATISFIED

NON TRANSVERSE SOLUTION WITH $\vec P_{ADM} = 0$

g) GEODETIC AND DEVIATION GEODETIC EQS

2 DEFORMATION PATTERNS OF SPHERE OF TEST PARTICLES


X
Y
Z
t
Particles at t = -1
Particles at t = -0.5
Particles at t = 0.


X
Y
Z
t
Particles at t = -1
Particles at t = -0.5
Particles at t = 0.

F) TETRAD GRAVITY + PERFECT FLUID (LL, BARBI, AGRESTI, DE PIETRI)

a) RELATIVISTIC PERFECT FLUIDS (BROWN) WITH EQUATION OF STATE $\boxed{\rho = \rho(n(x), s(x))}$

IN TERMS OF LAGRANGIAN (COMOVING) COORDINATES $\alpha^i(x)$

IDENTIFYING THE FLUX-LINES OF THE 4-VELOCITY FIELD $U^\mu(x)$

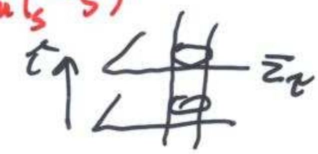
$$S = \int d^4x\, \rho(n(x), s(x)) \qquad n(x) U^\mu(x) = J^\mu(\alpha^i(x)) / \sqrt{{}^4g(x)} =$$

$$\Downarrow \qquad = -\epsilon^{\mu\nu\rho\sigma} \partial_\nu \alpha^1(x) \partial_\rho \alpha^2(x) \partial_\sigma \alpha^3(x)$$

NO CONSTRAINTS
ALL CONSERVATIONS SATISFIED

$$T^{\mu\nu} = (\rho c^2 + P) U^\mu U^\nu - \epsilon P\, {}^4g^{\mu\nu} \qquad P = c^2 \left(n \frac{\partial \rho}{\partial n}\Big|_s - \rho \right)$$

REST-FRAME INSTANT FORM (LL + NOWAK) $\alpha^i(\tau, \vec\sigma)$



ISENTROPIC FLUIDS
$$S = \epsilon c \int d\tau\, d^3\sigma\, \rho(|J(\alpha^i(\tau,\vec\sigma))|)$$

PHASE SPACE $\alpha^i(\tau, \vec\sigma)$, $\Pi_i(\tau, \vec\sigma)$

REST-FRAME HAMILTONIAN $\quad M = \int d^3\sigma\, \mathcal{M}(\tau, \vec\sigma)$

REST-FRAME CONDITION $\quad P^r = \int d^3\sigma\, \mathcal{M}^r(\tau,\vec\sigma) \approx 0 \qquad \mathcal{M}_r = -\Pi_i \partial_r \alpha^i$

M, P^r, J^{AB} POINCARÉ external generators

CLOSED FORM of $\mathcal{M}(\tau, \vec\sigma)$ ONLY FOR

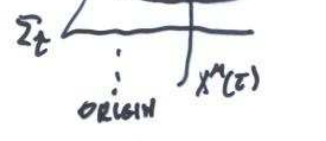
DUST $\quad \rho = \mu n \qquad \mathcal{M} = c \sqrt{\mu^2 c^2 [\det \partial_r \alpha^i]^2 + \Sigma_r \mathcal{M}_r^2}$

PHOTON GAS

OTHERWISE $\mathcal{M} = \mathcal{M}(X)$ WITH X SOLUTION OF TRASCENDENTAL EQ.

DUST $\quad T^{\tau\tau} \approx \mathcal{M} \qquad T^{\tau r} \approx -\epsilon c \mathcal{M}^r \qquad T^{rs} \approx c^2 \frac{\mathcal{M}^r \mathcal{M}^s}{\mathcal{M}}$

DIXON MULTIPOLES WITH RESPECT TO THE CENTER OF MOTION $z^\mu(\tau, \vec\eta(\tau)) = X^\mu(\tau)$



$$T^{AB}(\tau, \vec\sigma) = \sum_0^\infty{}_n \frac{(-1)^n}{n!} q^{r_1 \dots r_n AB}(\tau, \vec\eta(\tau)) \frac{\partial^n \delta^3(\vec\sigma - \vec\eta(\tau))}{\partial\sigma^{r_1} \dots \partial\sigma^{r_n}}$$

(ANALYTIC FUNCTIONS)

MONOPOLES
$$\begin{cases} q^{\tau\tau} = \int d^3\sigma\, \mathcal{M} \\ q^{\tau r} = P^r \approx 0 \\ q^{uv} = c^2 \int \frac{\mathcal{M}^u \mathcal{M}^v}{\mathcal{M}} \end{cases}$$

$q^{r_1 \dots r_n \tau\tau}$, $q^{r_1 \dots r_n \tau r}$, $q^{r_1 \dots r_n uv}$ MASS, MOMENTUM AND STRESS MULTIPOLES

MØLLER CENTER OF ENERGY $\vec\eta(\tau) = \vec R \approx -\frac{\vec K}{M} \qquad q^{r\tau\tau}(\tau, \vec R) = 0$ ZERO MASS DIPOLE

OTHER DESCRIPTION WITH GENERALIZED EULERIAN COORDINATES (LL + ALBA)

$w^i \quad \Pi_{wi} \qquad$ DUST $\quad \mathcal{M} \approx c \sqrt{\mu^2 c^2 + \Sigma_i \Pi^2_{wi}(\tau, \vec\sigma)}$

b) + TETRAD GRAVITY

$$\begin{cases} \mathcal{H} \mapsto \mathcal{H}' = \mathcal{H} - \frac{\epsilon}{c}\mathcal{M} \approx 0 \\ \theta_r \mapsto \theta'_r = \theta_r + \mathcal{M}_r \approx 0 \\ \check{E}_{ADM} \mapsto \check{E}'_{ADM} = \check{E}_{ADM} + \check{E}_{FLUID} \\ \check{P}^r_{ADM} \approx 0 \mapsto \check{P}^r_{ADM} + \check{P}^r_{FLUID} \approx 0 \\ \check{J}^{AB}_{ADM} \mapsto \check{J}^{AB}_{ADM} + \check{J}^{AB}_{FLUID} \end{cases}$$

$\mathcal{M}_r = -\Pi_i \partial_r \alpha^i$

DUST

$$\mathcal{M} = c\sqrt{\mu^2 c^2 [\det \partial_r \alpha^i]^2 + \phi^{-4} \sum_u e^{-\frac{2}{\sqrt{3}}\sum_a \gamma_{au} r_a}\, \mathcal{M}_u^2}$$

$$\check{E}_{FLUID} = \int d^3\sigma\, \mathcal{M}(\tau, \vec{\sigma})$$

SHANMUGADHASAN POINT CAN. TRANSF.

ELLIPTIC EQS FOR $\mathcal{G}^r_{(a)}$

$$\Pi^r_{(a)}(\tau, \vec{\sigma}) = \left(\begin{matrix}\text{AS WITHOUT FLUID BUT WITH} \\ \text{KERNELS DENDING ON } \mathcal{M}, \mathcal{M}_r\end{matrix}\right) + \mathcal{G}^r_{(a)}(\tau, \vec{\sigma} \mid \alpha_{(a)}, \xi^r, \phi, r_a, \mathcal{M}_u]$$

$T^{AB}_{FLUID}(\tau, \vec{\sigma})$ DEPENDS ON $\mathcal{M}, \mathcal{M}_r, {}^3e_{(a)r}, n, n_r$

COMPLETELY FIXED 3-ORTHOGONAL GAUGE

HAMILTONIAN LINEARIZATION $\to$ $\begin{cases}\text{WEAK FIELD + RELATIVISTIC RAREFIED FLUID} \\ \text{NO POST-NEWTONIAN APPROXIMATION}\end{cases}$

$r_a \ll 1$, $\phi = 1 + \frac{q}{2} + O(r_a^2)$, $q = O(r_a)$ $\boxed{\mathcal{M}, \mathcal{M}_r = O(r_a)}$

$$\underset{O(r_a)}{\mathcal{M} = \mathcal{M}_{FLAT}} + \underset{O(r_a^2)}{\mathcal{M}_{(1)}} + O(r_a^3) \qquad T^{AB}_{FLUID} = \underset{O(r_a)}{T^{AB}_{FLAT}} + \underset{O(r_a^2)}{T^{AB}_{(2)}} + O(r_a^3)$$

$\hookrightarrow$ DIXON MULTIPOLES

1) TERMS $O(r_a)$ IN THE LICHNEROWICZ EQ $\longrightarrow$ LINEARIZED $\phi(\tau, \vec{\sigma})$

$\phi = 1 + \frac{1}{2}q$ $\quad q(\tau, \vec{\sigma}) = (\text{AS WITHOUT FLUID}) + \frac{G}{c^4}\int d^3\sigma_1 \frac{\mathcal{M}_{(o)}(\tau, \vec{\sigma}_1)}{|\vec{\sigma} - \vec{\sigma}_1|}$ $\quad \mathcal{M}_{(o)} = c\sqrt{\mu^2 c^2 [\det]^2 + \sum_u \mathcal{M}_u^2}$

2) SOLUTION OF ALL CONSTRAINTS AND CANONICITY CONDITIONS

$$\vec{c}(\vec{\sigma}) = \frac{\vec{\sigma}}{4\pi |\vec{\sigma}|^3}$$

$$\Pi^r_{(a)}(\tau, \vec{\sigma}) = (\text{AS WITHOUT FLUID}) - \sum_s \delta_{s(a)} \int d^3\sigma_1\, c^r(\vec{\sigma} - \vec{\sigma}_1)\, \mathcal{M}_s(\tau, \vec{\sigma}_1) + O(r_a^2)$$

$$\int_{-\infty}^{\infty} d\sigma^r\, \Pi_a(\tau, \vec{\sigma}) = 0 \quad \forall r$$

3) LAPSE AND SHIFTS $\quad n(\tau, \vec{\sigma}) \neq 0$

$c \to \infty$ $\quad n \mapsto \frac{1}{c^2} V_{NEW}$ $\quad n_r \mapsto$ GRAVITOMAGNETIC POTENTIAL $\quad \phi \mapsto 1 + \frac{U}{4c^2}$ $\quad U \neq V_{NEW}$

4) $\check{E}'_{ADM} = \check{E}_{ADM} + \check{E}_{FLUID} + O(r_a^3) \doteq \text{CONSTANT}$

$\check{E}_{FLUID} = \int d^3\sigma\; M_{FLAT}(\tau,\vec{\sigma}) +$ REST ENERGY

$+ \int d^3\sigma_1 d^3\sigma_2 \left[\frac{M_{FLAT}(\tau,\vec{\sigma}_1) M_{FLAT}(\tau,\vec{\sigma}_2)}{|\vec{\sigma}_1 - \vec{\sigma}_2|} - \frac{\mu^2 [\det \partial_r \alpha^i(\tau,\vec{\sigma}_1)]^2}{M_{FLAT}(\tau,\vec{\sigma}_1)} \frac{M_{FLAT}(\tau,\vec{\sigma}_2)}{|\vec{\sigma}_1 - \vec{\sigma}_2|} \right] +$ REL. A-A-A-D NEWTON POTENTIAL

$+ 2\pi \sum_{r\,u} Q_a \int d^3\sigma_1 d^3\sigma_2 d^3\sigma_3\; C^u(\vec{\sigma}_2 - \vec{\sigma}_1)\, C^u(\vec{\sigma}_3 - \vec{\sigma}_2)\, M_r(\tau,\vec{\sigma}_1)\, M_r(\tau,\vec{\sigma}_2) +$ A-A-A-D GRAVITOMAGNETIC POTENTIAL

+ TERMS OF COUPLING OF THE FLUID WITH r_a and π_a TIDAL EFFECTS ON THE FLUID

5) HAMILTON EQS FOR r_a, π_a

$$\Rightarrow \Box r_a(\tau,\vec{\sigma}) = Z_a(\tau,\vec{\sigma}) \doteq (\mathcal{F}_{1a} + \mathcal{F}_{2a})(\tau,\vec{\sigma}) = \left(\alpha_{AB} T^{AB}_{FLAT}\right)(\tau,\vec{\sigma}) + \int d^3\sigma_1\, \beta_{AB}(\vec{\sigma},\vec{\sigma}_1)\, T^{AB}_{FLAT}(\tau,\vec{\sigma}_1) + O(r_a^2)$$

OUTSIDE THE FLUID $\Box r_a \neq 0$ NO MATCHING LIKE IN POST-NEWTONIAN

$r_a(\tau,\vec{\sigma}) = \int G_{RET}\, Z_a$

$$\begin{cases} \int_{-\infty}^{\infty} d\sigma^r\, r_a(\tau,\vec{\sigma}) \doteq \int d\sigma^r\, V\, \mathcal{F}_2 \\ \partial_\tau \int_{-\infty}^{\infty} d\sigma^r\, r_a(\tau,\vec{\sigma}) = \int \mathcal{F}_1 \end{cases}$$

$\downarrow$

6) FLUID HAMILTON EQS — MINKOWSKI EQS + BOUNDARY CONDITIONS

7) DIXON MULTIPOLE EXPANSION IN $Z_a(\tau,\vec{\sigma})$

$\Rightarrow r_a(\tau,\vec{\sigma})$ EVERYWHERE, IN PARTICULAR IN WAVE ZONE

$\Rightarrow {}^4g_{AB}(\tau,\vec{\sigma})$ IN WAVE ZONE KNOWN TERM $+ \frac{1}{c} + \frac{1}{c^2} + \dots$ EXACT RELATIVISTIC FORMULAS

POST-MINKOWSKIAN BACKGROUND INDEPENDENT SOLUTION

$$\frac{d\,\check{E}_{FLUID}}{d\tau} = -\frac{d\,\check{E}_{ADM}}{d\tau}$$

EXACT RELATIVISTIC MULTIPOLAR EXPANSION CONTAINING THE STANDARD TERM IN $P^{\mu\nu\rho\sigma}(\tau)$ OF THE QUADRUPOLE EMISSION FORMULA

$$r_{\tilde a}(\tau,\vec\sigma) = \frac{2\sqrt{3}G}{c^4}\sum_u \gamma_{\tilde a u}\int_{V(\tau-|\vec\sigma-\vec\sigma_1|)} d^3\sigma_1 \, \frac{T^{uu}(\tau-|\vec\sigma-\vec\sigma_1|,\vec\sigma_1)}{|\vec\sigma-\vec\sigma_1|} +$$

$$+ \frac{2\sqrt{3}G}{c^4}\sum_u \gamma_{\tilde a u}\int d^3\sigma_1 \int_{V(\tau-|\vec\sigma-\vec\sigma_1|)} d^3\sigma_2 \, \frac{c^u(\vec\sigma_1-\vec\sigma_2)}{|\vec\sigma-\vec\sigma_1|}\,\frac{\partial T^{su}(\tau-|\vec\sigma-\vec\sigma_1|,\vec\sigma_2)}{\partial\sigma_2^s} +$$

$$+ \frac{\sqrt{3}G}{c^4}\sum_r \gamma_{\tilde a r}\int \frac{d^3\sigma_1}{|\vec\sigma-\vec\sigma_1|}\,\frac{\partial^2}{(\partial\sigma_1^r)^2}\Big[-\int_{V(\tau-|\vec\sigma-\vec\sigma_1|)} d^3\sigma_2 \, \frac{\sum_u T^{uu}(\tau-|\vec\sigma-\vec\sigma_1|,\vec\sigma_2)}{4\pi\,|\vec\sigma_1-\vec\sigma_2|} +$$

$$+ \sum_{u\neq a}[1-2(\delta_{ru}+\delta_{ra})]\int_{-\infty}^{\sigma_1^u} d\sigma_2^u \int_{-\infty}^{\sigma_1^a} d\sigma_2^a \int_{V(\tau-|\vec\sigma-\vec\sigma_1|)} d^3\sigma_3 \, c^u(\vec\sigma_2-\vec\sigma_3)\Big|_{\substack{\sigma_2^k=\sigma_1^k \\ k\neq u,a}} \frac{\partial T^{su}(\tau-|\vec\sigma-\vec\sigma_1|,\vec\sigma_3)}{\partial\sigma_3^s}\Big]$$

$$+ (\text{HOMOGENEOUS SOLUTION}) =$$

$$\underset{\text{MULTIPOLAR EXPANSION}}{=} \frac{\sqrt{3}G}{c^4}\sum_r \gamma_{\tilde a r}\Big\{ \frac{\ddot p^{\,rr\tau\tau}_{\vec R}(\tau-|\vec\sigma-\vec R|)}{|\vec\sigma-\vec R|} - \int d^3\sigma_1 \sum_s \frac{\ddot p^{\,sr\tau\tau}(\tau-|\vec\sigma-\vec\sigma_1|)}{|\vec\sigma-\vec\sigma_1|}\,\frac{\partial c^r(\vec\sigma_1-\vec R)}{\partial R^s} -$$

$$- \frac{1}{2}\int \frac{d^3\sigma_1}{|\vec\sigma-\vec\sigma_1|}\,\frac{\partial^2}{(\partial\sigma_1^r)^2}\Big[\sum_u \frac{\ddot p^{\,uu\tau\tau}(\tau-|\vec\sigma-\vec\sigma_1|)}{4\pi\,|\vec\sigma_1-\vec R|} +$$

$$+ \sum_{u\neq a}[1-2(\delta_{ru}+\delta_{ra})]\sum_s \ddot p^{\,su\tau\tau}(\tau-|\vec\sigma-\vec\sigma_1|)\int_{-\infty}^{\sigma_1^u} d\sigma_2^u \int_{-\infty}^{\sigma_1^a} d\sigma_2^a \, \frac{\partial c^u(\vec\sigma_2-\vec R)}{\partial R^s}\Big|_{\sigma_2^k=\sigma_1^k}\Big] +$$

$$+ \sum_{n=1}^{\infty}\frac{1}{n!}\Big(\sum \frac{\partial^n}{\partial R^{r_1}\dots\partial R^{r_n}}\Big(\frac{q^{r_1\dots r_n rr}_{(\vec R)}(\tau-|\vec\sigma-\vec R|)}{|\vec\sigma-\vec R|} - \dots\Big)\Big)\Big\}$$

$$\Rightarrow \quad {}^3g_{rs}(\tau,\vec\sigma) = \delta_{rs}[---] \qquad {}^4g_{rs} = -\epsilon\, {}^3g_{rs}$$

$${}^4g_{\tau\tau}(\tau,\vec\sigma) = \epsilon - 2n(\tau,\vec\sigma) = \epsilon\Big\{1 - \frac{2G}{c^4}\Big[\frac{2\Pi_R - \mu^2 c^4\int d^3\sigma_1 \frac{\det[\partial_r \alpha^i(\tau,\vec\sigma_1)]^2}{M_{(\alpha)}(\tau,\vec\sigma_1)}}{|\vec\sigma-\vec R|} + \sum_{n=2}^{\infty} ----\Big\}$$

$$\xrightarrow[c\to\infty]{} \epsilon\Big[1 - \underbrace{\frac{2G}{c^2}\int d^3\sigma_1 \frac{\mu \det[\partial_r \alpha^i(\sigma_1)]}{|\vec\sigma-\vec\sigma_1|}}_{\text{NEWTON POTENTIAL}} + ----\Big]$$

$${}^4g_{\tau r}(\tau,\vec\sigma) = -\epsilon\, n_r(\tau,\vec\sigma) = -----$$

↳ DEPENDS ON $\Pi_{\tilde a}(\tau,\vec\sigma)$

$$\frac{d E_{ADM\,(grav)}}{d\tau} = -\frac{d E_{ADM\,(fluid)}}{d\tau} \quad \text{CONTAINS} \quad \dddot p^{\,rs\tau\tau}$$

G) FUTURE DEVELOPMENTS

FINISH RELATIVISTIC QUADRUPOLE EMISSION FORMULA
(USE WILL-WISEMAN or DAMOUR-BLANCHET REGULARIZATION)

⇒ ENOUGH TECHNOLOGY TO FACE THE FOLLOWING PROBLEMS

HAM. EQS. FOR A BALL OF FLUID
- LINEARIZED EQS + BOUNDARY CONDITIONS
- CAUCHY PROBLEM FOR STARS (RENDALL - OPEN)
- MODIFICATION OF KEPLER ROTATION CURVES OF GALAXIES

BARBI, LL

⇓

HAM. NUMERICAL GRAVITY IN A FIXED 3-ORTHOGONAL GAUGE WITH ONLY DO

DE PIETRI, LL

LINEARIZED THEORY
EFFECTIVE REL 2-BODY PROBLEM FOR 2 BALLS OF FLUID (BINARIES)
- DIXON MULTIPOLES
- MØLLER CENTER OF ENERGY AND SPIN FRAME

⇓

GEN. REL. 2-BODY PROBLEM — $L \neq j$ (EINSTEIN-HOFFMAN-INFELD)

⇓

GEN. REL. HARMONIC OSCIL.
- PROTOTYPE DYN. CLOCK
- THEORY OF MEASUREMENT WITH DYN. NOT TEST OBJECT (BEYOND EHLERS-PIRANI-SCHILD, DE WITT)

TG + EM FIELD
DOUBLE RADIATION GAUGES
DO FOR BOTH
GRAV. LENSING

TG + SCALAR FIELDS

HAM. FORMULATION OF NEWMAN-PENROSE (BERGMANN-KOMAR INTRINSIC COORD)
DO = BO
SPECIAL GAUGES

ORTAGGIO
PAURI, LL

TG + FERMIONS

FOLDY-WOUTHUYSEN TRANSF. FOR SPINNING PARTICLES IN GRAV FIELDS VERSUS PAPAPETROU POLE-DIPOLE SYSTEMS

LUNARI, BINI, LL

ALBA, LL

QUANTIZATION OF PARTICLES AND FIELDS IN NON-INERTIAL FRAMES IN SPECIAL REL.
- QUANTIZE ONLY DO NOT GAUGE VARIABLES (INERTIAL EFFECTS)
- ATOMIC PHYSICS IN ACCELERATED FRAMES
- SYNCHRONIZATION OF DISTANT CLOCKS (GPS, ACES, LISA, RADAR-COORD, MODIFICATION OF EINSTEIN CONVENTION)
- INTERPRETATION OF THE EQUIVALENCE PRINCIPLE

SPECIAL RELATIVITY ADMITS MANY MORE NON-INERTIAL FRAMES OF VOID EINSTEIN SPACETIMES WITHOUT MATTER (3-CONFORMALLY FLAT SIMULTANEITY SURFACES ONLY).

QUANTIZATION OF GRAVITY IN THE SPECIAL NP+SHAN GAUGES
QUANTIZE ONLY COORD-INDEPENDENT DO = BO (TIDAL)
NOT GAUGE VARIABLES (INERTIAL)
- CAUSALITY (3+1) RESPECTED
- ONLY 3-METRIC QUANTIZED LIKE IN LOOP QUANTUM GRAVITY

HAMILTONIAN BLACK HOLES

MARSDEN, MONCRIEF, ARMS FISHER
CONE OVER CONE STRUCTURE OF SINGULARITIES FOR $^4g_{\mu\nu}$
- ADD KILLING EQS AS HAM. CONSTRAINTS

⇓

DO BECOME FUNCTIONS OF GAUGE VARIABLES

⇓

ONLY SINGULARITIES + INERTIAL EFFECTS

BINI, LL.

PERTURBATIVE SCHEME FOR HAM. NUMERICAL GRAVITY IN THE COMPLETELY FIXED 3-ORTHOGONAL GAUGE

POST-MINKOWSKIAN APPROXIMATION → FORMAL SERIES IN POWERS OF G.

$$
\begin{cases}
\phi(\tau,\vec{\sigma}) = \phi_{(0)}(\tau,\vec{\sigma}) + \sum_{1}^{\infty}{}_n G^n \phi_{(n)}(\tau,\vec{\sigma}) \\
m(\tau,\vec{\sigma}\,|\,\phi, r_{\bar{a}}, \alpha^i, \Pi_i] = m_{(0)}(\tau,\vec{\sigma}) + \sum_{1}^{\infty}{}_n G^n m_{(n)}(\tau,\vec{\sigma}) \\
G^{ru}_{(a)}(\vec{\sigma},\vec{\sigma}',\tau\,|\,\phi, r_{\bar{a}}] = G^{ru}_{(0)(a)}(\vec{\sigma},\vec{\sigma}',\tau) + \sum_{1}^{\infty}{}_n G^n \, G^{ru}_{(n)(a)}(\vec{\sigma},\vec{\sigma}',\tau) \\
\zeta^{a}_{(b)}(\tau,\vec{\sigma}\,|\,\phi, r_{\bar{a}}, \alpha^i, \Pi_i] = \zeta^{a}_{(0)(b)}(\tau,\vec{\sigma}) + \sum_{1}^{\infty}{}_n G^n \, \zeta^{a}_{(n)(b)}(\tau,\vec{\sigma})
\end{cases}
$$

1) ITERATIVE ELLIPTIC EQS FOR THE KERNELS $\zeta^{ru}_{(n)(a)}$ AND FOR $\zeta^{a}_{(n)(b)}$

(FROM CANONICITY, ROTATION AND SUPERMOMENTUM CONSTRAINTS)

2) INTEGRO-DIFFERENTIAL LICHNEROWICZ EQ → ITERATIVE ELLIPTIC EQS FOR $\phi_{(n)}$

$$\left(\Delta_{(0)} + \frac{1}{8}\,{}^3\tilde{R}_{(0)}(\tau,\vec{\sigma})\right) \phi_{(0)}(\tau,\vec{\sigma}) = 0$$

$$\left(\Delta_{(n)} + \frac{1}{8}\,{}^3\tilde{R}_{(n)}(\tau,\vec{\sigma})\right) \phi_{(n)}(\tau,\vec{\sigma}) = F_{(n)}[n-1, n-2, \dots]$$

$\Downarrow$

3) HAMILTONIAN $\quad \hat{E}_{ADM}[\phi, r_{\bar{a}}, \alpha^i, \Pi_i] = \frac{1}{G} E_{(-1)} + E_{(0)} + \sum_{1}^{\infty}{}_n G^n E_{(n)} \qquad E_{(n)}[r_{\bar{a}}, \Pi_{\bar{a}}, \alpha^i, \Pi_i]$

ITERATIVE HYPERBOLIC HAM. EQS $\qquad \partial_\tau B^{(n)}_{(\tau,\vec{\sigma})} = \{ B(\tau,\vec{\sigma}), E_{(n)} \} \qquad B = r_{\bar{a}}, \Pi_{\bar{a}}, \alpha^i, \Pi_i$

4) ITERATIVE INTEGRAL EQS FOR LAPSE $\qquad n = n_{(0)} + \sum_{1}^{\infty}{}_n G^n n_{(n)}$

5) SHIFT $\quad n_r = n_{(0)r} + \sum_{1}^{\infty}{}_n G^n n_{(n)r} \quad$ FROM $\phi_{(n)}, G^{ru}_{(n)(a)}, \zeta^{a}_{(n)b}, n_{(n)}$

L. LUSANNA - TOWARDS A UNIFIED DESCRIPTION OF THE 4 INTERACTIONS
IN "QFT: A 20TH CENTURY PROFILE" HEP-TH/9907081

L. LUSANNA - THE REST-FRAME INSTANT FORM OF METRIC GRAVITY
GEN. REL. GRAV. 33, 1579 (2001) GR-QC/0101048

L. LUSANNA AND S. RUSSO - A NEW PARAMETRIZATION FOR TETRAD GRAVITY
GEN. REL. GRAV 34, 189 (2002) GR-QC/0102074

R. DE PIETRI, L. LUSANNA, S. RUSSO AND L. MARTUCCI - DIRAC'S OBSERVABLES FOR THE REST-FRAME INSTANT
FORM OF TETRAD GRAVITY IN A COMPLETELY FIXED 3-ORTHOGONAL GAUGE
GEN. REL. GRAV. 34, 877 (2002) GR-QC/0105084

J. AGRESTI, R. DE PIETRI, L. LUSANNA AND L. MARTUCCI - HAMILTONIAN LINEARIZATION OF THE REST-FRAME
INSTANT FORM OF TETRAD GRAVITY IN A COMPLETELY FIXED 3-ORTHOGONAL GAUGE: A RADIATION
GAUGE FOR BACKGROUND-INDEPENDENT GRAVITATIONAL WAVES IN A POST-MINKOWSKIAN
EINSTEIN SPACETIME SUBMITTED TO GEN. REL. GRAV GR-QC/0302084

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POINT-EVENTS: THE ROLE OF GRAVITATIONAL AND GAUGE DEGREES OF FREEDOM
IN GENERAL RELATIVITY GR-QC/0301040

L. LUSANNA - THE N- AND 1-TIME CLASSICAL DESCRIPTIONS OF N-BODY RELATIVISTIC KINEMATICS
AND THE ELECTROMAGNETIC INTERACTION Int. J. MOD. PHYS A12, 645 (1997)

D. ALBA, L. LUSANNA and M. PAURI - CENTERS OF MASS AND ROTATIONAL KINEMATICS FOR THE
RELATIVISTIC N-BODY PROBLEM IN THE REST-FRAME INSTANT FORM
J. MATH. PHYS 43, 1677 (2002) HEP-TH/0102087

- MULTIPOLAR EXPANSIONS FOR CLOSED AND OPEN SYSTEMS OF RELATIVISTIC PARTICLES
SUBMITTED TO J. MATH. PHYS

L. LUSANNA AND M. MATERASSI - A CANONICAL DECOMPOSITION IN COLLECTIVE AND RELATIVE VARIABLES OF A
KLEIN-GORDON FIELD IN THE REST-FRAME INSTANT FORM
INT. J. MOD. PHYS. A15, 2821 (2000) HEP-TH/9904202

L. LUSANNA and D. NOWAK-SZCZEPANIAK - THE REST-FRAME INSTANT FORM OF RELATIVISTIC
PERFECT FLUIDS WITH EQUATION OF STATE $\rho = \rho(n, s)$ AND OF NON-DISSIPATIVE
ELASTIC MATERIALS INT. J. MOD. PHYS A15, 4943 (2000) HEP-TH/0003095

D. ALBA AND L. LUSANNA - GENERALIZED EULERIAN COORDINATES FOR RELATIVISTIC FLUIDS:
HAMILTONIAN REST-FRAME INSTANT FORM, RELATIVE VARIABLES, ROTATIONAL KINEMATICS
TO APPEAR IN INT. J. MOD. PHYS HEP-TH/0209032

- SIMULTANEITY, RADAR 4-COORDINATES AND THE 3+1 POINT OF VIEW ON ACCELERATED
OBSERVERS IN SPECIAL RELATIVITY

H. CRATER AND L. LUSANNA - THE REST-FRAME DARWIN POTENTIAL FROM THE LIENARD-WIECHERT SOLUTION IN THE RADIATION GAUGE ANN. PHYS. (NY) 289, 87 (2001) HEP-TH/0001046

D. ALBA, H. CRATER AND L. LUSANNA - THE SEMICLASSICAL RELATIVISTIC DARWIN POTENTIAL FOR SPINNING PARTICLES IN THE REST-FRAME INSTANT FORM: TWO-BODY BOUND STATES WITH SPIN 1/2 CONSTITUENTS INT. J. MOD. PHYS. A16, 3365 (2001) HEP-TH/0103109