Asymptotic black hole quasinormal modes and the area quantum

Emanuele Berti

Aristotle University of Thessaloniki
berti@astro.auth.gr

In collaboration with:
Kostas Kokkotas, Vitor Cardoso, Hisashi Onozawa

Suggested background reading:
Jacob Bekenstein, "Black holes: Classical properties, thermodynamics, and heuristic quantization", **gr-qc/9808028**

Up-to-date review on recent developments:
V. Cardoso, J. P. S. Lemos, S. Yoshida, **gr-qc/0309112**

Quasinormal modes

Definition: oscillations of a (classical) black hole having
*ingoing waves at the horizon, *outgoing waves at infinity

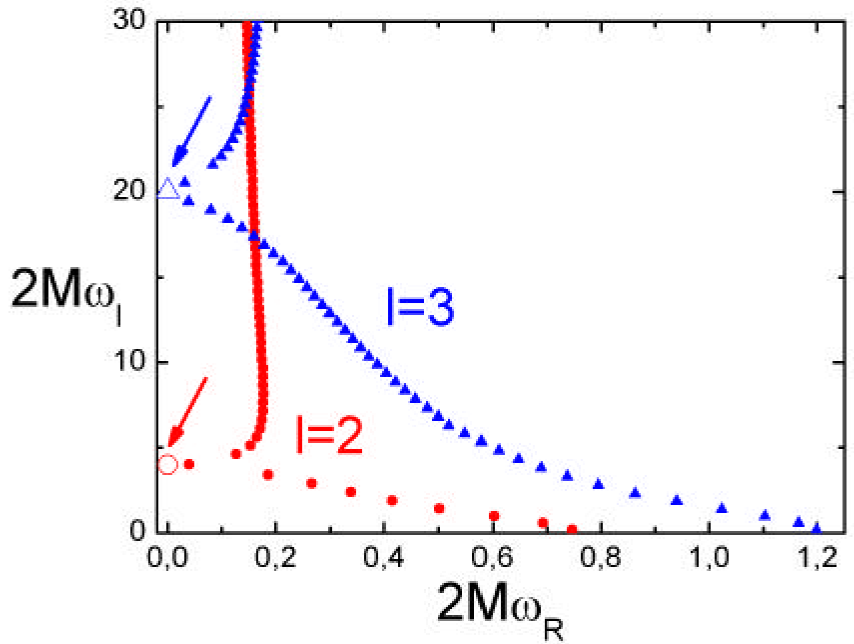
For Schwarzschild black holes and a given perturbing field (s=0,1,2)
this eigenvalue problem has a discrete, l-dependent spectrum

Remarkable numerical discovery:
(Nollert, confirmed by Andersson)
for large n (infinite damping limit)
the frequency of gravitational
(and scalar) perturbations is

•finite

•l-independent

$$\boxed{2Mw = 0.0874247 + \frac{i}{2}\left(n + \frac{1}{2}\right)}$$



Black hole thermodynamics: area is entropy

* Early work on **classical** black holes
(Christodoulou-Ruffini, Bardeen-Carter-Hawking)
showed that black holes obey laws resembling those of thermodynamics,
in which their horizon area plays the role of entropy

** Bekenstein's proposal (1972): area **is** entropy !
For two years this was a one-man view: classically black holes do not radiate,
so the only temperature we can associate to them is T=0

*** Hawking (1974): discovery of black hole radiation
Quantum mechanically black holes do radiate, they have finite temperature T, and

$S=A/4$ (Bekenstein-Hawking entropy)

This formula relates **classical and quantum** properties of black holes!
It has since been derived by many candidate theories of quantum gravity

Black hole area spectrum

Bekenstein (1970's): in any theory of quantum gravity

black hole area must be quantized

Bekenstein, Mukhanov:

if we want to interpret black hole entropy from a **statistical mechanics** point of view, then the area spectrum should be of the form

$$A = g \; l_{Planck}^2 \times n$$

where n is an integer

is undetermined if we don't know how to quantize gravity

Since S=A/4, statistical mechanics suggests it should be of the form

$$g = 4 \ln k$$

Area quantum from QNM's?
Hod's conjecture

Observation: the infinite-damping limit for Schwarzschild QNM frequencies can be written (within the accuracy of the numerics) as

$$w_R = T \ln 3$$

Hod's conjecture (1998): the black hole is the "quantum gravity atom".

Apply Bohr's correspondence principle:

QNM's having large damping do not radiate (the black hole is stable)

Suppose the minimum possible variation in black hole mass is

$$\Delta M = \; w_R$$

Using the mass-area relationship, $A=16\ \ M^2$, this fixes the constant

The conjecture

- is compatible with a statistical mechanics interpretation of black hole entropy
- fixes k=3 ("philosophically favoured": k=2, compatible with Wheeler's "it from bit")

Some new exciting developments

1) Dreyer: Hod's conjecture can be used to fix an ambiguity (the Barbero-Immirzi parameter) appearing in Loop Quantum Gravity
Why is this exciting?
Loop Quantum Gravity becomes predictive!
Puzzling consequence: LQG may be a theory without fermions

2) Motl: **analytical** proof that asymptotic QNM frequencies are indeed given by **Tln3**. This is not just a numerical coincidence!

3) Motl & Neitzke, Kodama-Ishibashi, Birmingham: the "ln3" result applies to even-spin perturbations of (non-rotating) black holes in **any dimension**

4) Birmingham-Carlip-Chen: Hod's conjecture can be "made rigorous" for **2+1 dimensional (BTZ)** black holes
(identifying the mass and angular momentum quanta with the QNM frequency leads to the correct quantum behaviour of the asymptotic symmetry algebra)

All of these developments support Hod's conjecture!

How general is Hod's conjecture?

It seems to apply to non-rotating black holes in any dimension
(as long as the spacetime is asymptotically flat).

1) What about black holes that are **rotating and/or charged?**

A (simplistic?) argument would suggest the asymptotic QNM frequency

$$w = (T \ln 3 + m\Omega) + i2pTn$$

This simple formula is wrong! The situation is more complicated

2) What about black holes in **non asymptotically flat (de Sitter, anti-de Sitter) spacetimes?**

Can they be quantized using Hod's conjecture?
My view: probably not, especially for AdS (the QNM spectrum is different in nature, and the potential has a barrier at infinity)

Asymptotic spectrum of charged black holes

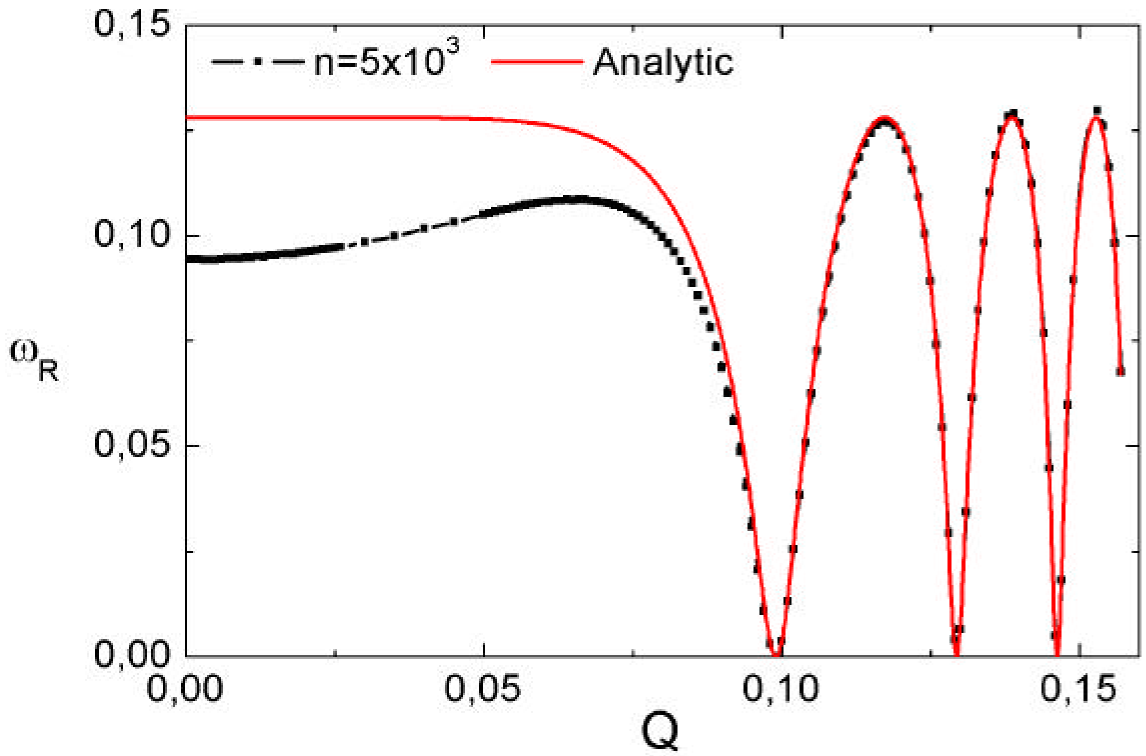
The structure of the asymptotic RN spectrum is now clear.

Numerical calculations (Berti & Kokkotas) and
analytical derivations (Motl & Neitzke, Andersson & Howls)
lead to an implicit formula for QNMs:

$$\exp(bw)+2+3\exp(-\quad w)=0$$

The agreement
with the numerics
is excellent at large Q,
but
the analytic formula
does not reproduce the
Schwarzschild limit
(gives ln5
instead of ln3)

Why?



Asymptotic spectrum of rotating black holes

Rotation splits the modes (Zeeman-like splitting), and the problem involves the solution of a radial and an angular equation coupled together.
Puzzling behaviour! (Berti, Cardoso, Kokkotas & Onozawa)
Hod first suggested

$$w = (\quad + m\Omega) + i2pTn$$

* This only applies to **m>0** modes, and it is likely to be a deep result:

1) It involves the critical frequency for superradiance
2) It implies that the area of rotating black holes is an adiabatic invariant

* For **m=0**, modes **oscillate** as they do in the Reissner-Nordström case.
* For **m<0**, modes seem to tend to a roughly constant value

$$w \approx -mv$$

Kerr black holes: open questions

Numerics indicate that there are
three different behaviours depending on whether
$m>0$, $m=0$, $m<0$

* Which behaviour (if any) is relevant for quantization?

* Why is the Schwarzschild limit discontinuous?
(This happens for charged black holes as well…)

* Can the result be derived analytically?
(Problem: work out the asymptotic behaviour of the angular part)

* What is the physical meaning of ν ?

New twists (almost) every day…

1) Inclusion of the cosmological constant

The QNM spectrum for black holes in **de Sitter** space oscillates as the damping increases, but the **maxima** seem to behave like Schwarzschild modes: they first have a minimum, then asymptote a constant

The QNM spectrum for black holes in **anti-de Sitter** tends to infinity, but the QNM **spacing** tends to a (probably universal) constant

2) Puzzle in the application to Loop Quantum Gravity: **no fermions** allowed!

Among the proposed explanations to include fermions, one is very suggestive: Loop Quantization not of GR, but of a class of supergravity theories (**QNM's favour supersymmetry?**)

3) Oppenheim: push forward Hod's conjecture

If the black hole really plays for quantum gravity the role played by the hydrogen atom for quantum mechanics, then one should be able to find a trace of **multi-level transitions** in the classical black hole spectrum..