Collapse of a Differentially Rotating Supermassive Star

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CONTENTS

1. Introduction

Cosmic Censor Conjecture

Complete gravitational collapse of a body always results in a black hole (BH) rather than naked singularity

(Penrose 1969)

Exception

Collapse of a prolate spheroid with large semimajor axis leads to spindle singularities without apparent horizon

(Nakamura, Shapiro, Teukolsky 1988; Shapiro, Teukolsky 1991)

⬇

Collapse of J/M^2 >1 may show us some violent phenomena in general relativity and in gravitational wave physics

(cf. BH uniquness theorem   Isral 1967; Carter 1971)

Axisymmetric Collapse J/M^2 ∼ 1

- Nakamura (1981)

  Gravitational collapse of a star with radial velocity

  BH formation < $J/M^2 \sim 0.95$ < No apparent horizon

- Stark & Piran (1985)

  Gravitational collapse of n=1 polytropic, uniformly rotating star

  BH formation < $J/M^2 \sim 1.2$ < Flattend disk formation

- Shibata (2000)

  Gravitational collapse of n=1 polytropic, differentially rotating star

  - BH forms when the rest mass exceeds the maximum allowed mass of the equilibrium sequence
  - Shock heating prevents prompt collapse to BH for $J/M^2 \sim 1$

Candidate for Sources of Gravitational Waves

Prevent prompt collapse to BH

- Core bounce and/or shock heating

  Global estimation

  M, J, ... Conserved quantities

$$\frac{M}{R^2} \sim R\Omega^2 \qquad J \sim MR^2\Omega \quad \Rightarrow \quad R_{\rm bounce} \approx M\left(\frac{J}{M^2}\right)^2$$

$J/M^2 > 1$ ... Core bounce

$J/M^2 < 1$ ... Core collapse

Core bounce take place for J/Mˆ2 ~ 1

- Bar Formation

  Global estimation

  M, J, ... Conserved quantities

$$T \sim MR^2\Omega^2 \qquad W \sim M^2/R \quad \Rightarrow \quad \frac{T}{W} \approx \frac{M}{R}\left(\frac{J}{M^2}\right)^2$$

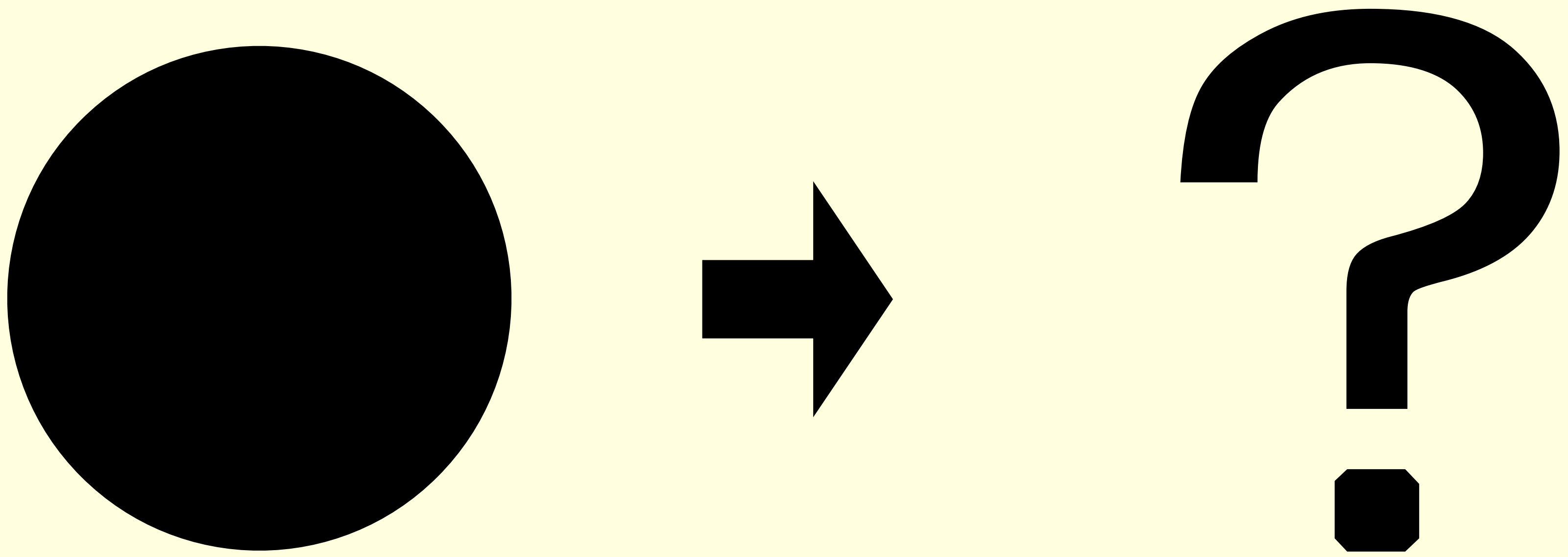
Bar formation take place at R~4M for $J/M^2 \sim 1$

$\Rightarrow$ Raise nonaxisymmetric deformation of the star, which produce gravitational waves

Purpose

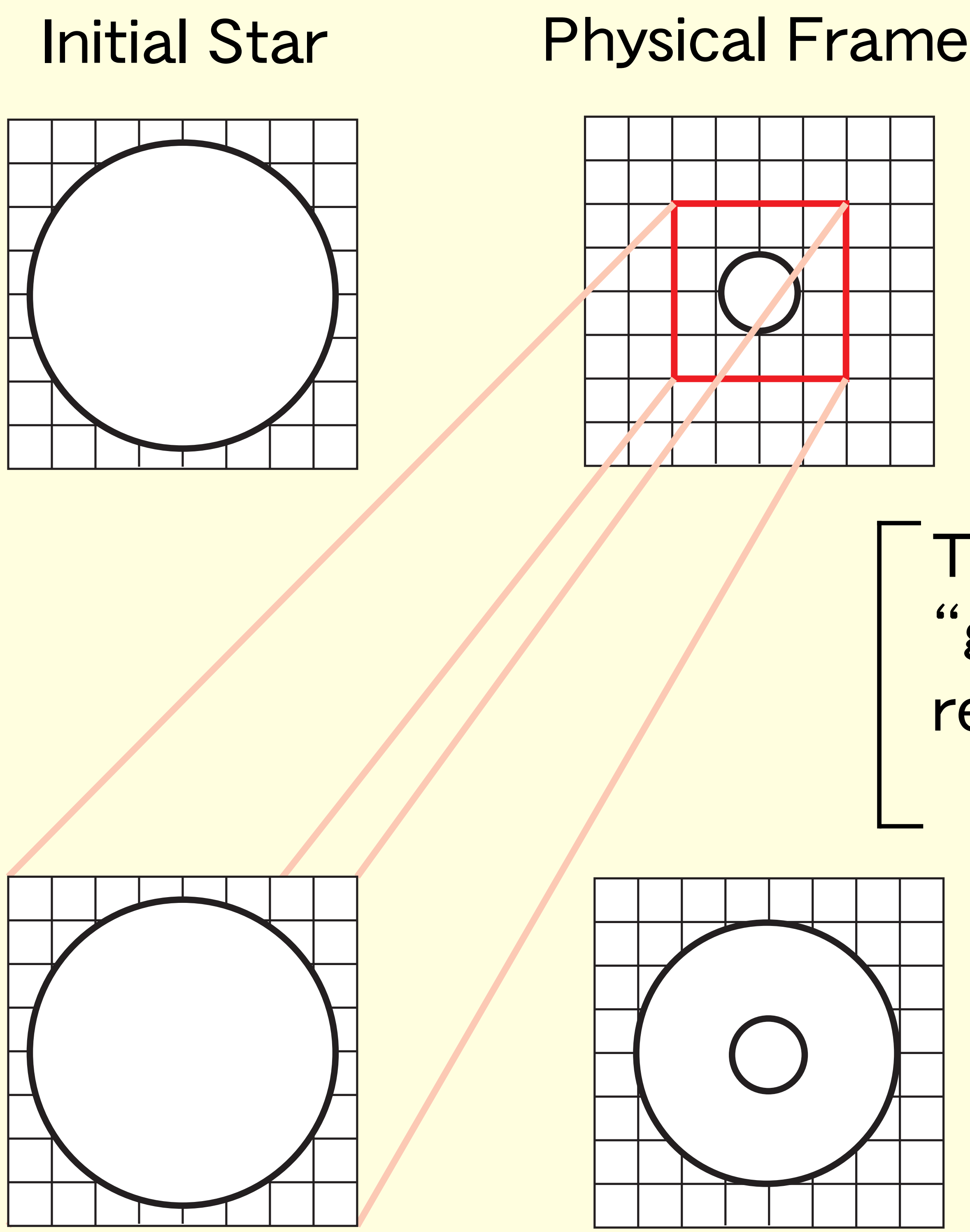
- Verify the Cosmic censor conjecture in the collapse of a differentially rotating supermassive star from equilibrium
- Investigate the final outcome of collapse of a differentially rotating supermassive star
- Verify whether the collapse of a differentially rotating supermassive star becomes a promissing source of gravitational waves

2. 3+1 Relativistic Hydrodynamics in Conformally Flat Spacetime



- Rotating supermassive collapse（R/M～70）
- Conformally flat simulation
- Radiation dominant, n=3 polytropic SMS
- Differential rotation
- Adiabatic collapse ($\tau_{\mathbf{radn}} \gg \tau_{\mathbf{dyn}}$)

Improve the grid resolution



To maintain the grid resolution, we contract our whole grid to one half when the “radius” of the star becomes one half from the previous contraction

This method works approximately “good” when most of the fragments remains in the computational grid
$(\Delta M_{\mathrm{ADM}}/M_{\mathrm{ADM}} \lesssim 10^{-4})$

Satisfactory approximation until the core collapses nonlinearly

Conformally Flat Spacetime

$$ds^2 = (-\alpha^2 + \beta_k \beta^k) dt^2 + 2\beta_k dx^k dt + \psi^4 \delta_{ij} dx^i dx^j$$

α : lapse function β^k : shift vector ψ : conformal factor

- Conformally flat metric with fully relativistic hydrodynamical equations
- Include artificial viscosity to capture shock

➡ 10 elliptic equations to solve gravitational field equations (no need to solve evolution equations)

Advantages

- Satisfactory approximation at the early and intermediate stages of the collapse
- Stable for arbitrary long time, in principle
- Retains all the nonlinear terms necessary to maintain exact dynamics for a spherical star

Disadvantages

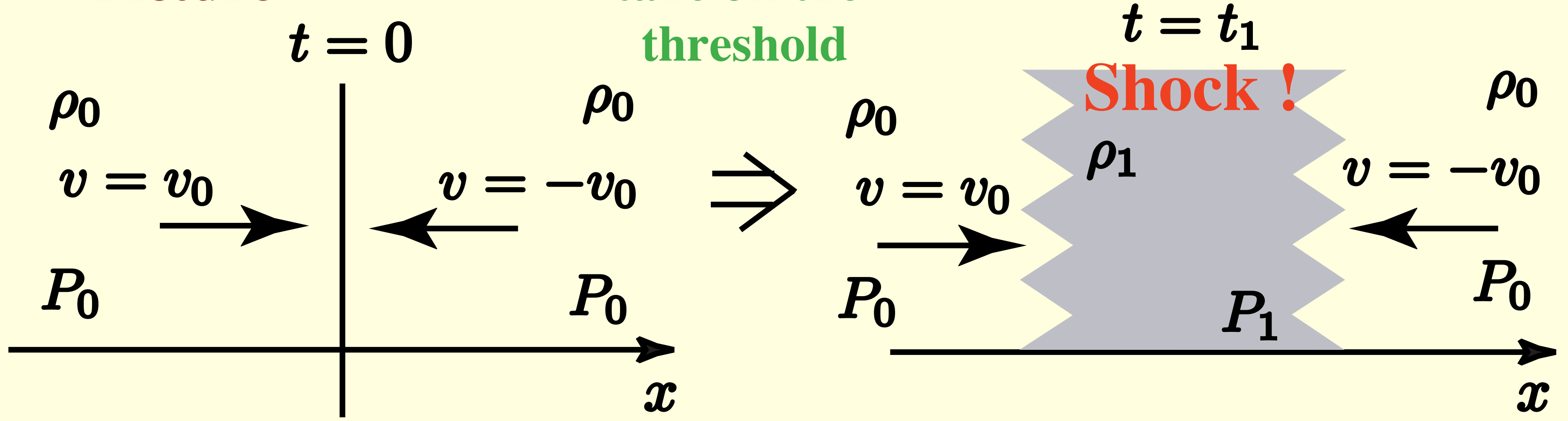
- Dangerous to treat strong gravitation regime
- Impossible to follow BH growth and formation

3. Code Tests

1. 1D Relativistic Wall Shock Problem

Check the validity to treat shock (e.g. Hawley, Smarr, Wilson 1984)

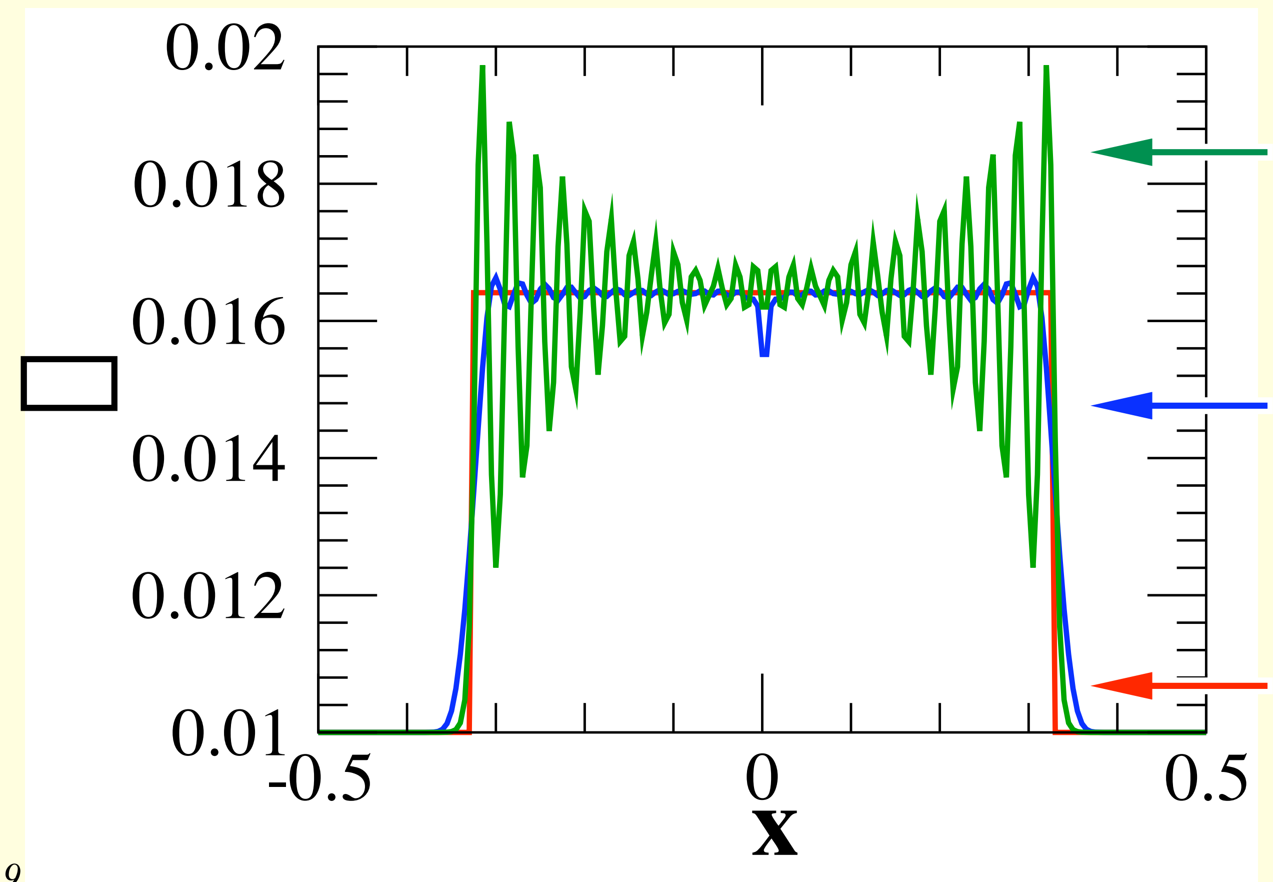
Picture



2 phases of the fluid collide

Comparison t=1.0

Self similar solution
(Analytic solution exists!)



Numerical Results without Artificial Viscosity

Numerical Results with Artificial Viscosity

Analytical Solution

$$\rho_0 = 1 \times 10^{-2}$$
$$v_0 = 0.2$$
$$n = 3$$

Can reproduce the analytic solution with the numerical one about 5% of the maximum density

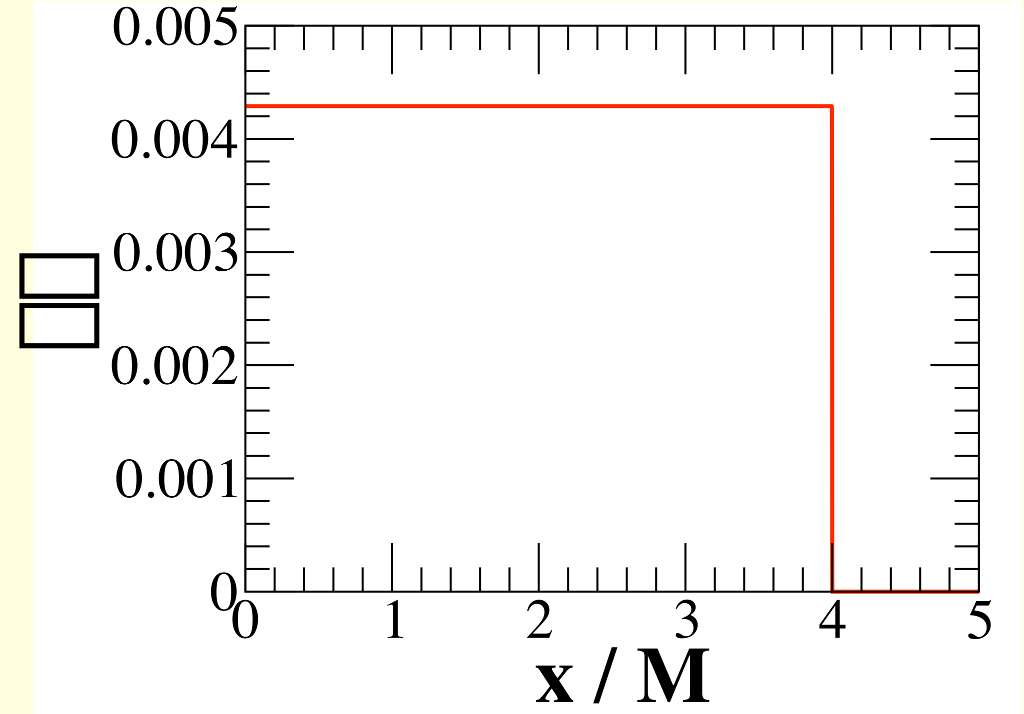
2. Oppenheimer-Snyder Collapse

Conformally flat approximation becomes "exact" for spherical dynamics

⬇

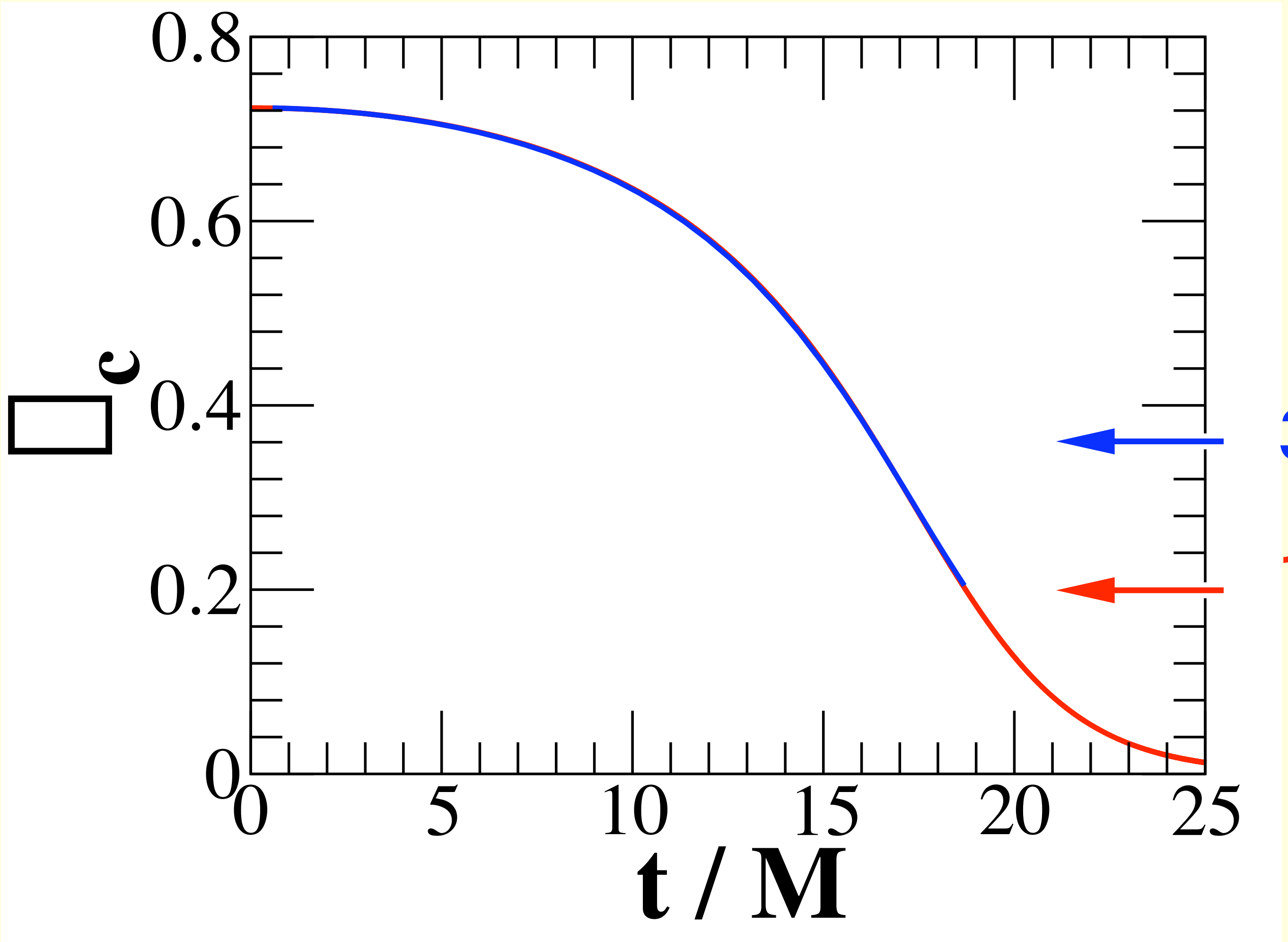
Check the ability of our 3D code to follow spherical dust collapse

Initial Density Profile



Can reproduce the "analytic" 1D solution with the "numerical" 3D within the error of 1% of the central lapse

Central Lapse Function



3D result

1D result

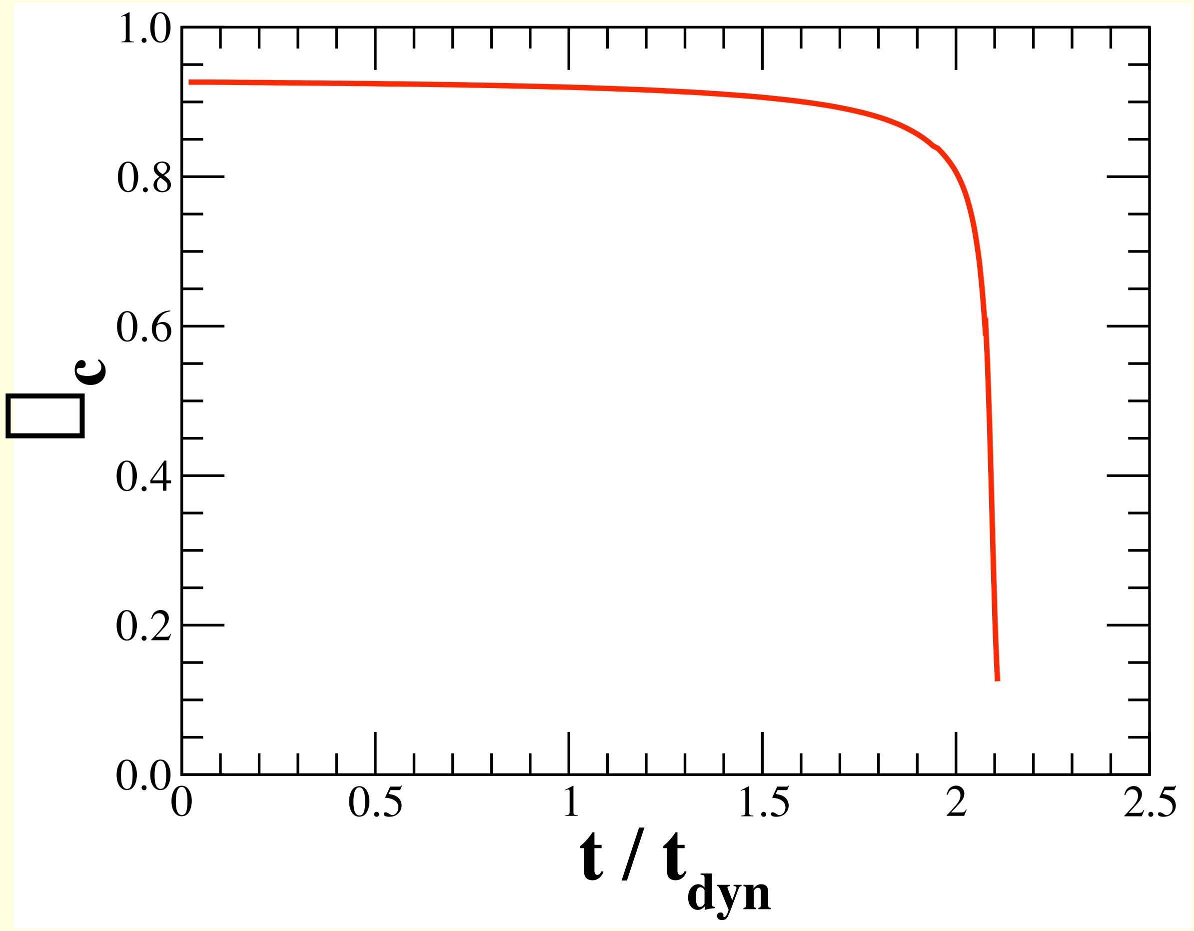
3. Spherical Gravitational Collapse

Check the ability of our code to follow prompt collapse

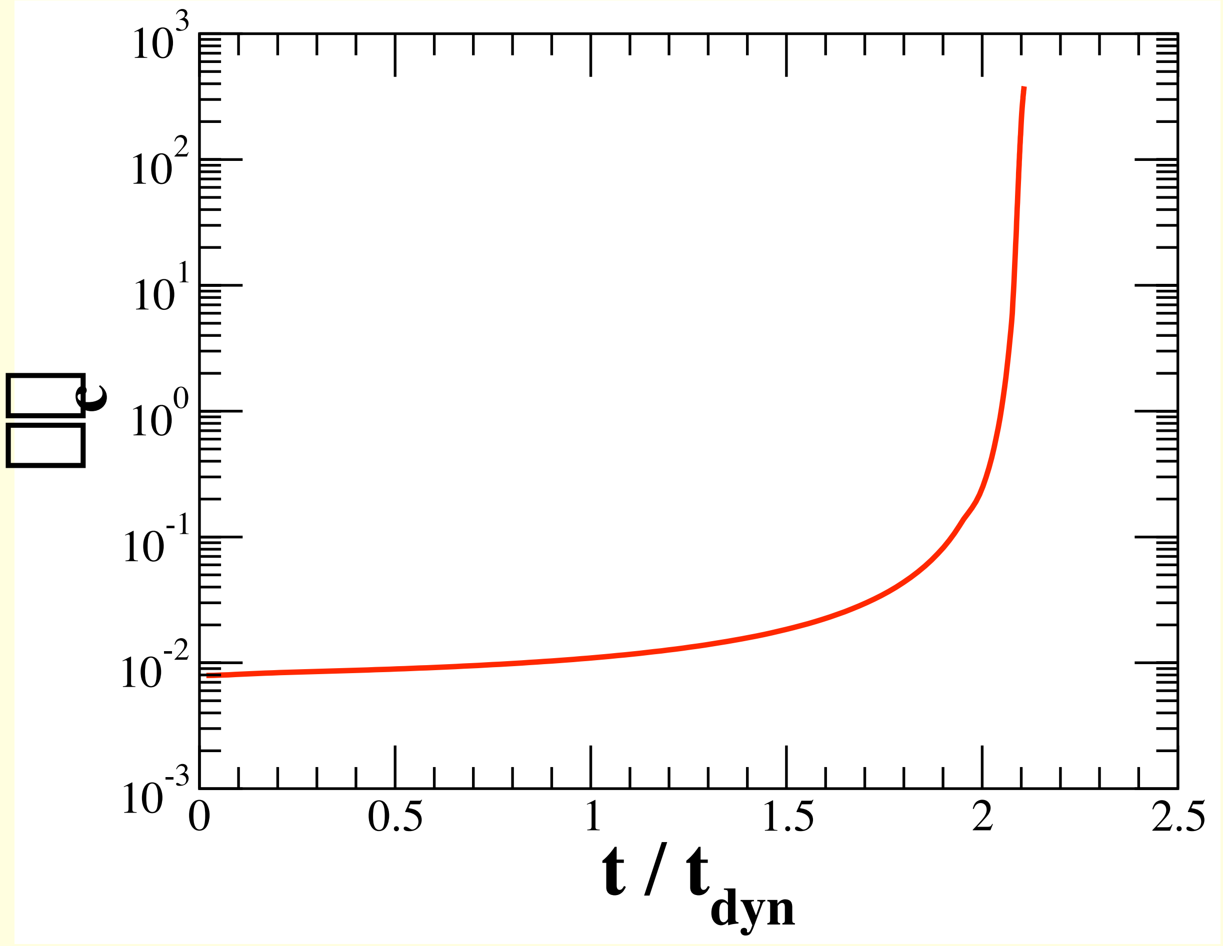
Unstable n=3 Spherical Star

ρ_c	M	R_c/M
3.80×10^{-6}	3.83	65.0

Central Lapse



Central Density



BH is likely to form

➡ Can follow spherical prompt collapse

4. Collapse of a Differentially Rotating Supermassive Star

Initial Condition

Requirements

- Radially Unstable

  Critical Γ in week field, slow rotation (Chandrasekhar & Lebovitz 1968)

$$\Gamma_{\rm crit} = \frac{4}{3} + 2.25\frac{M}{R} - \frac{2}{9}\frac{\Omega^2 I}{|W|}$$

➡ Soft equation of state induces collapse (n=3)

- J/M^2 ~1

  Roche Model (Critical onset of collapse for n=3)

  $J/M^2 \sim 0.9$ R/M $\sim$ 680

  ➡ Differential rotation is necessary to increase J/M^2

- Small Variation of R as Possible

  Naively

$$\frac{M}{R^2} \sim R\Omega^2 \quad \Rightarrow \quad \rho \sim \Omega^2$$

Increasing angular velocity at center increases compaction of the star

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Initial Data Sets

Model	r_{pl}/r_{eq}	T/W	J/M^2	R/M
I	0.60	0.065	0.97	65
II	0.58	0.070	1.01	65
III	0.55	0.076	1.05	65
IV	0.50	0.088	1.14	66

n=3 polytropic equation of state

High degree of differential rotation

$$\Omega_c/\Omega_{eq} \sim 10$$

Computational Resource request

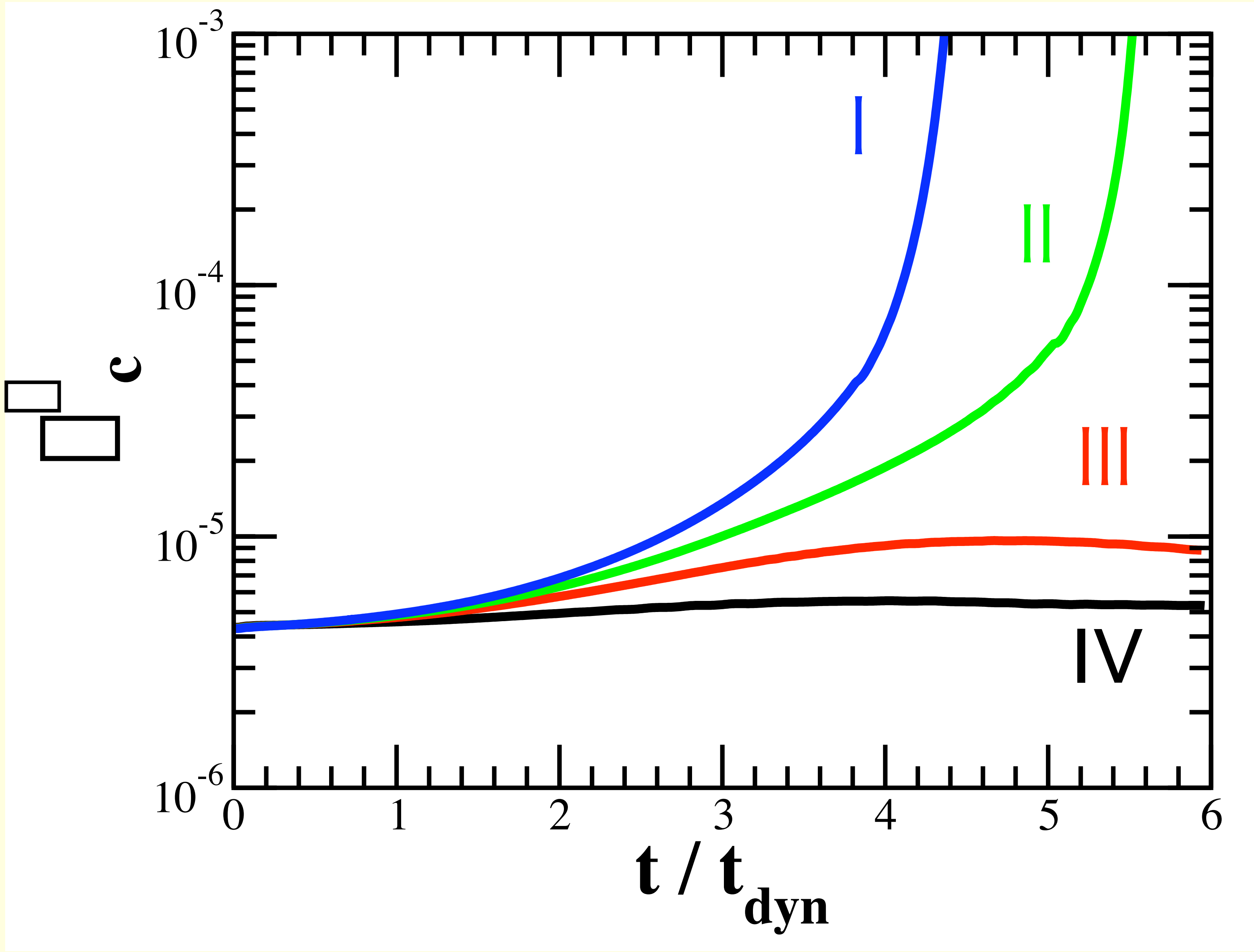
10 elliptic equations with a grid size of (201x201x61)

➡ Request for 150 ~ 300 CPU hours per 1 run

Central Density

We do not know the exact criterion of the radial instability in differentially rotating stars

➡ Evolution is necessary to determine the radial stability of the star



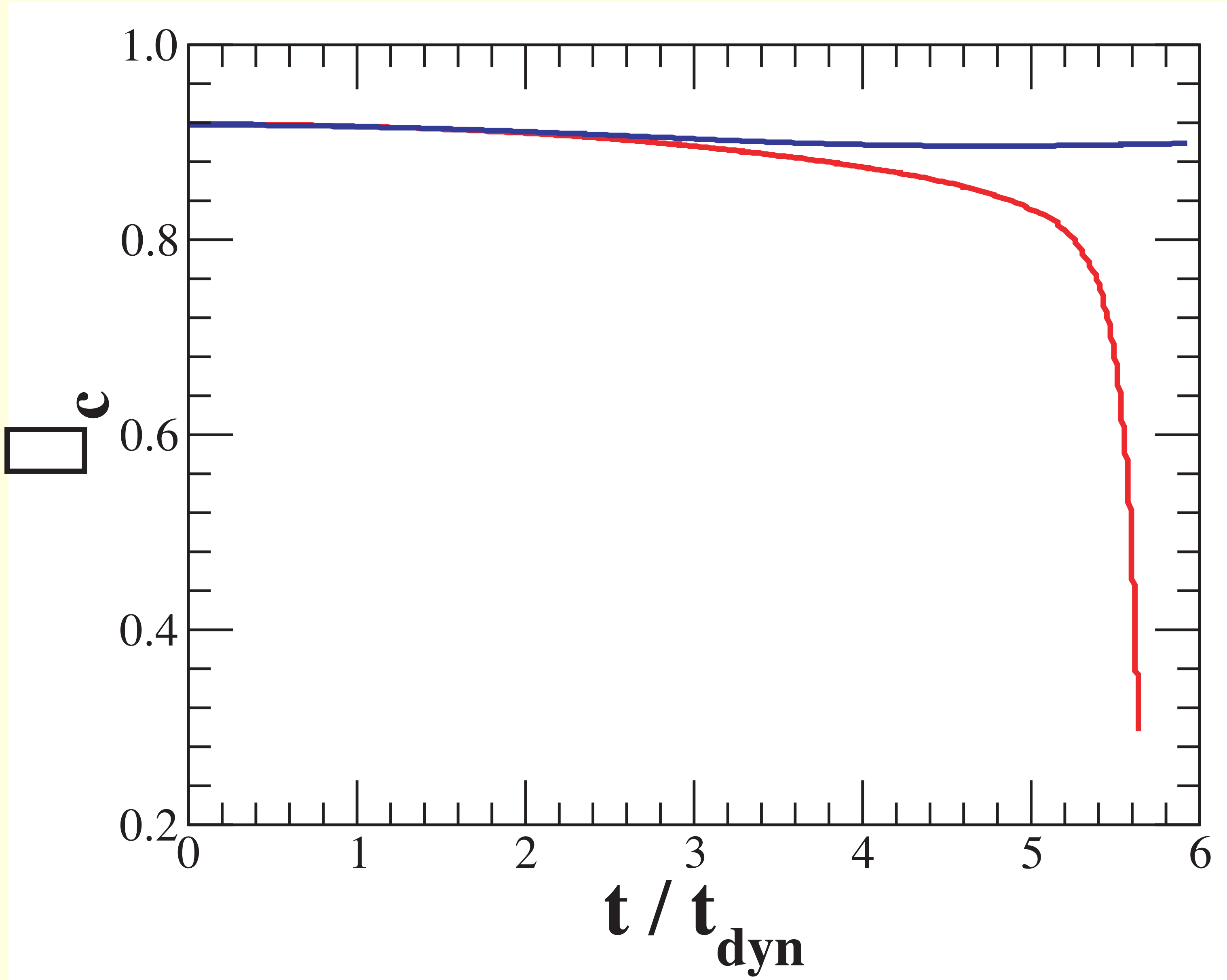
Unstable

Central density has an exponentially growth during the evolution

Stable

Central density remains oscillation around its equilibrium value

Lapse Function



III

Oscillation around the initial equilibrium state

II

Monotonically decreasing

↓

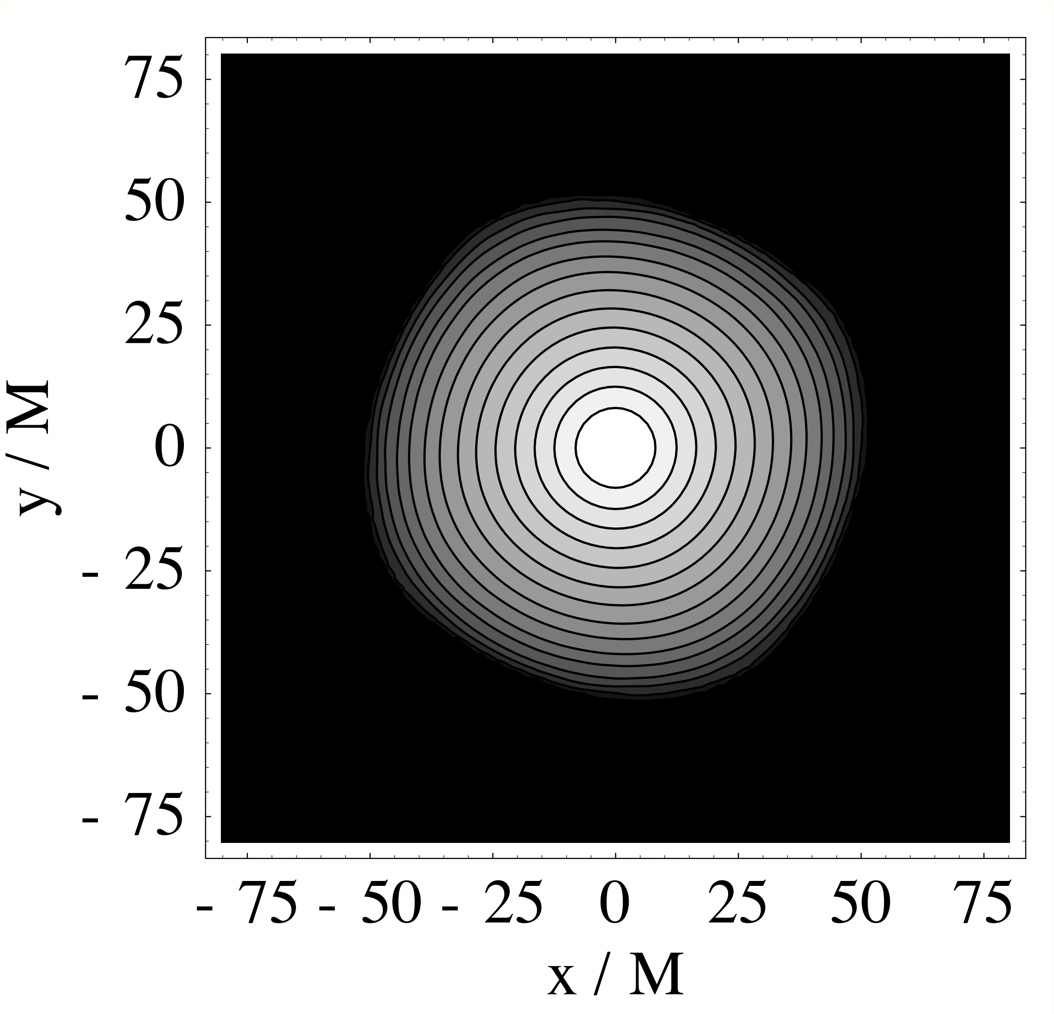
BH is likely to be forming

We terminate the computation due to

1. Lack resolution at the center
2. Extremely slow convergence of the elliptic solver
   (we did not find a code crash)

Final Density Snapshot (Case III Stable)

Density snapshot in the equatorial plane

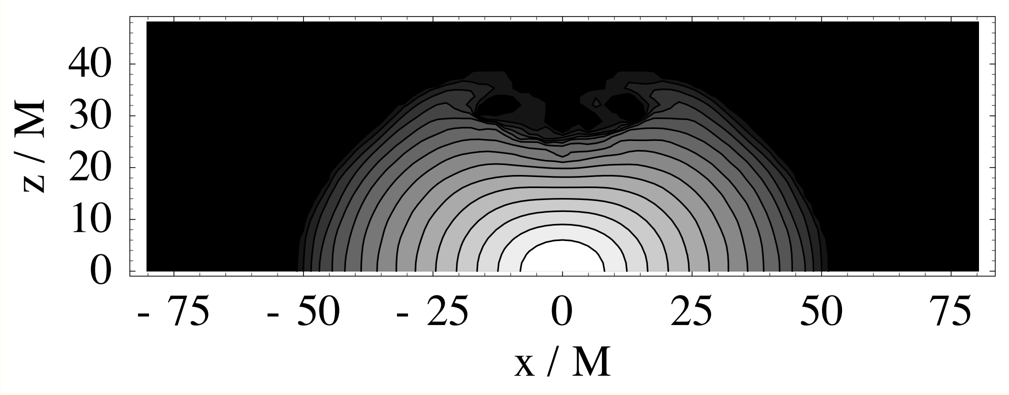


Logarithmic plot in coordinate density

Remains oscillation around the initial equilibrium state

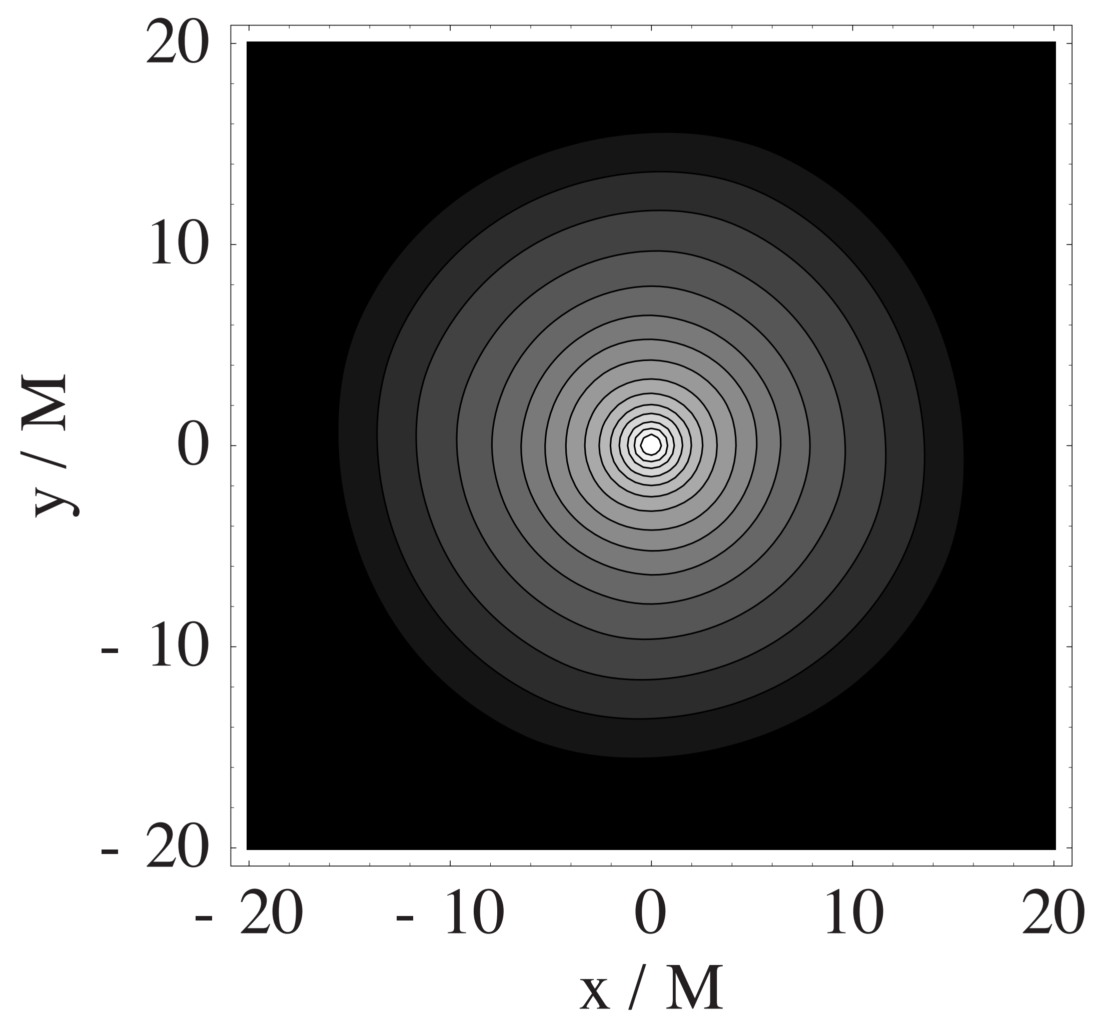
Almost axisymmetric system

Density snapshot in the meridional plane



Final Density Snapshot (Case II Unstable)

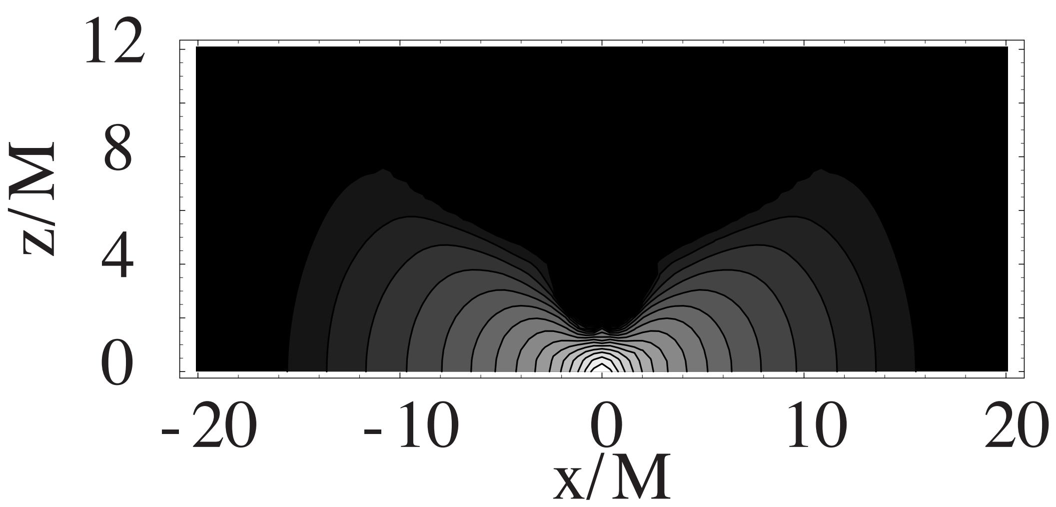
Density snapshot in the equatorial plane



Logarithmic plot in coordinate density

- No evidence for bar structure nor mass ejection from density snapshot
- Almost axisymmetric collapse

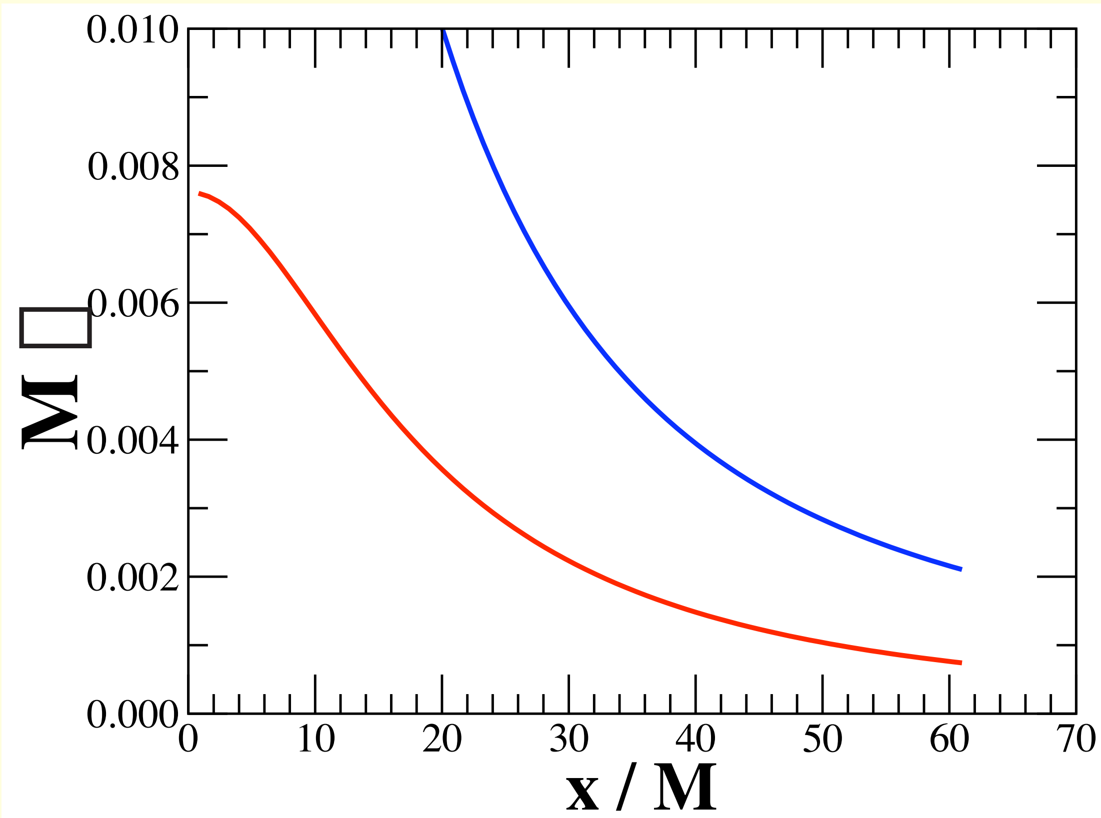
Density snapshot in the meridional plane



↓

Collapse is coherent to form a BH

Angular Velocity



t=0

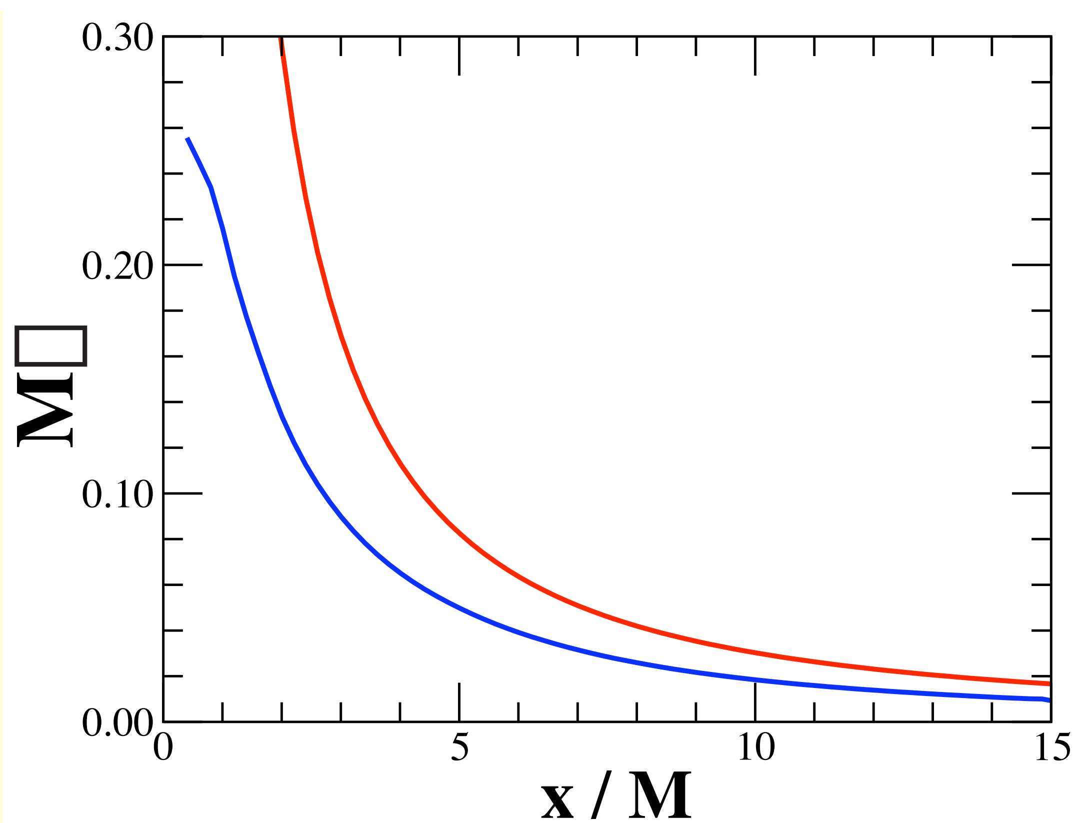
$$\Omega \equiv \left(\frac{m}{\varpi^3}\right)^{1/2}$$

m : cylindrical mass

ϖ : cylindrical radius

Keplarian angular velocity

Angular velocity of the star



termination of integration

- Collapse strengthen the degree of differential rotation
- Rotation at each cylindrical radius grows up to the Keplarian velocity

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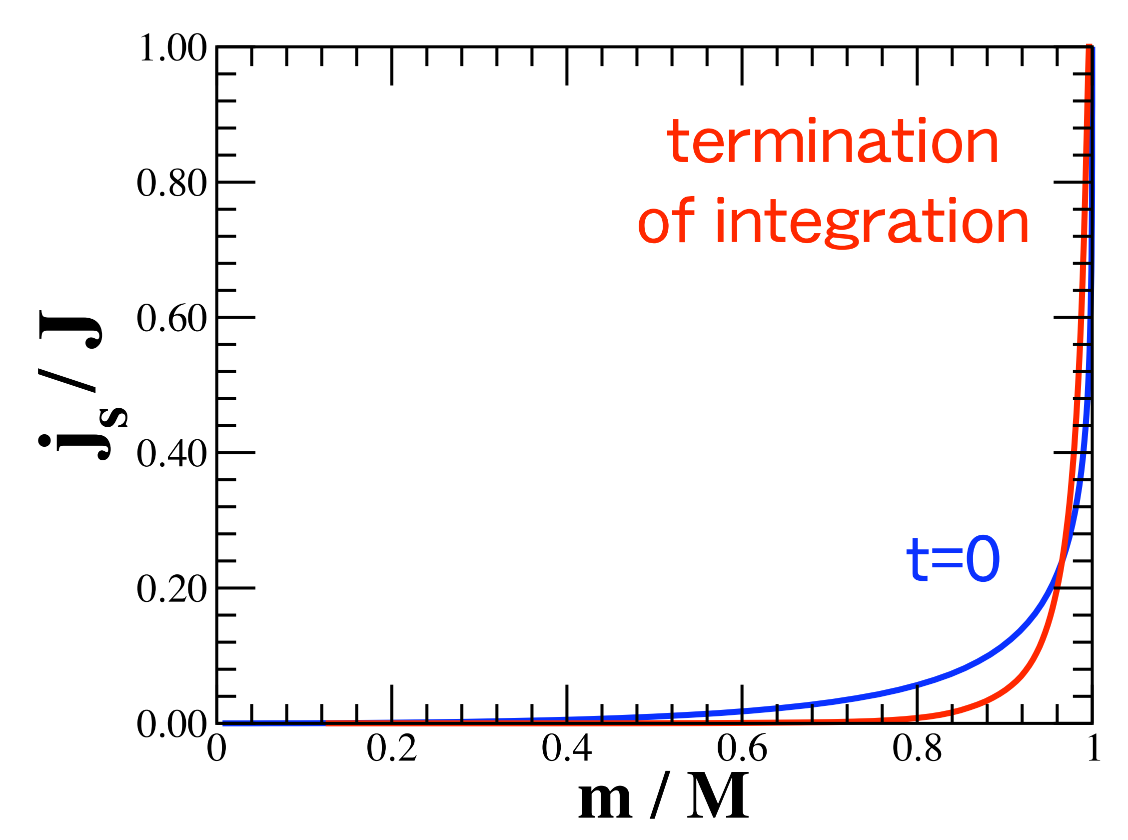
Angular Momentum Distribution

Angular momentum distribution

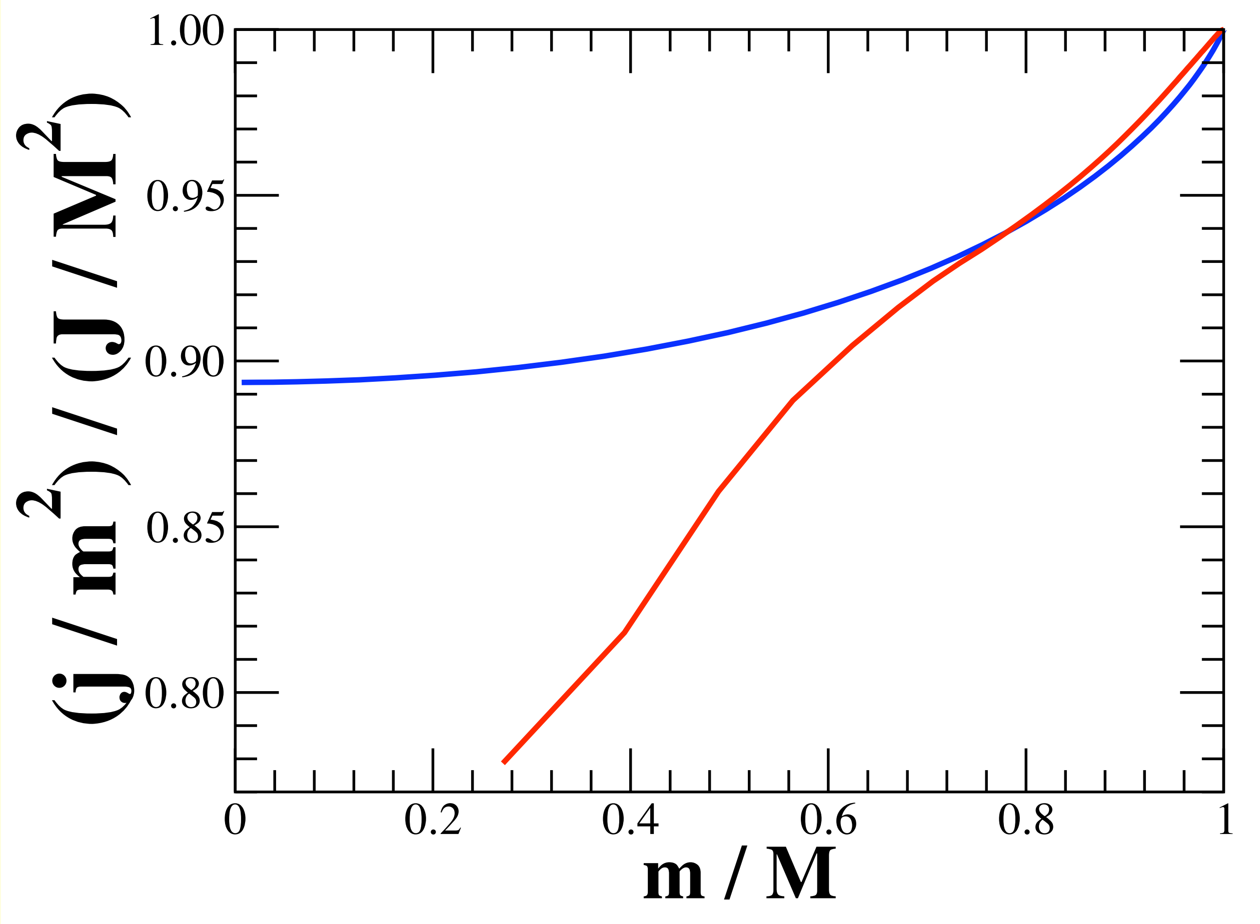


$$j_s = 4\pi \int_0^\infty dz \int_0^\varpi d\varpi \varpi h u_\varphi$$

Gravitational collapse transports a significant amount of mass to the center

$$j = 4\pi \int_0^\infty dz \int_0^\varpi d\varpi \varpi \rho^* h u_\varphi$$

Distribution of j/m^2



BH is likely to form in the condition of $J/M^2 \sim 0.8$

Note that 75% of the rest mass is already inside r < 2M at the termination of our integration

5. Conclusions

We investigate the collapse of a differentially rotating supermassive star by means of hydrodynamical simulations in conformally flat, relativistic gravitation

- For a radially unstable star, angular momentum distributes significantly around the surface of the star and it prevents to form a central core of $J/M^2 \sim 1$
- Collapse of a differentially rotating, relativisticaly unstable supermassive star is coherent and likely leads to the formation of a supermassive BH
- No bar formation from rotating collapse prior to BH formation
- Rotating supermassive star collapse is a promising source of burst gravitational waves, and of quasinormal mode ringing waves (cf. Uniformly rotating SMS collapse Saijo et al. 2002)