EQUILIBRIUM AND PULSATIONS OF DIFFERENTIALLY ROTATING NEUTRON STARS

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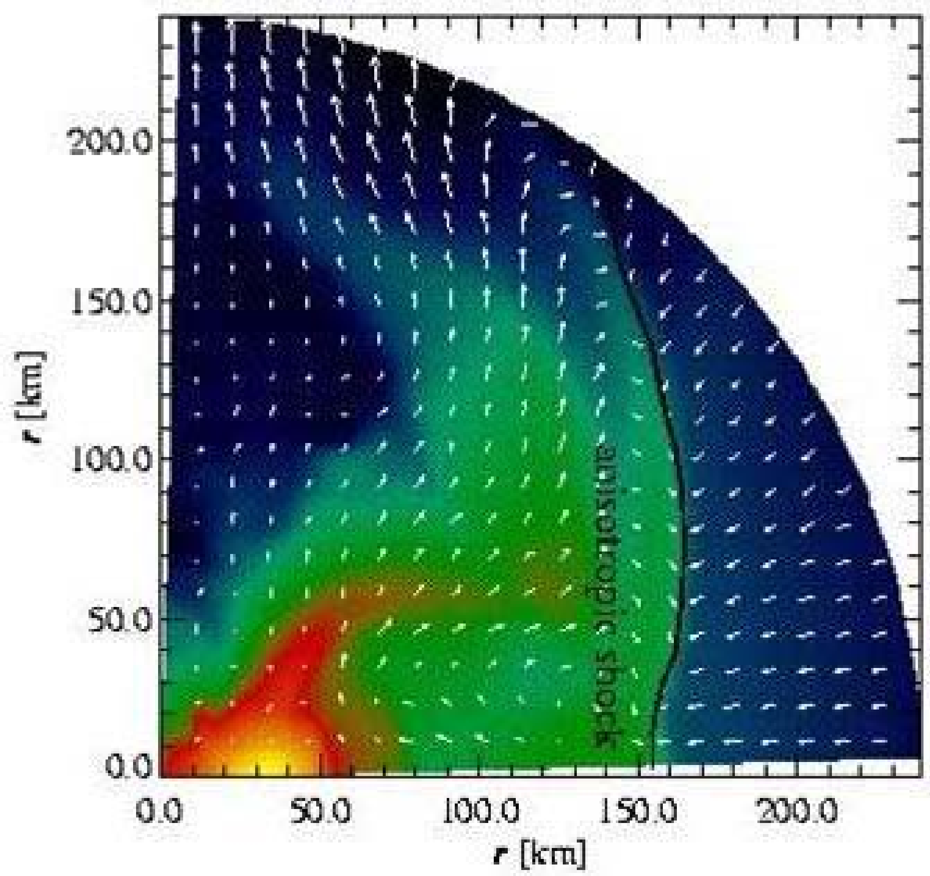
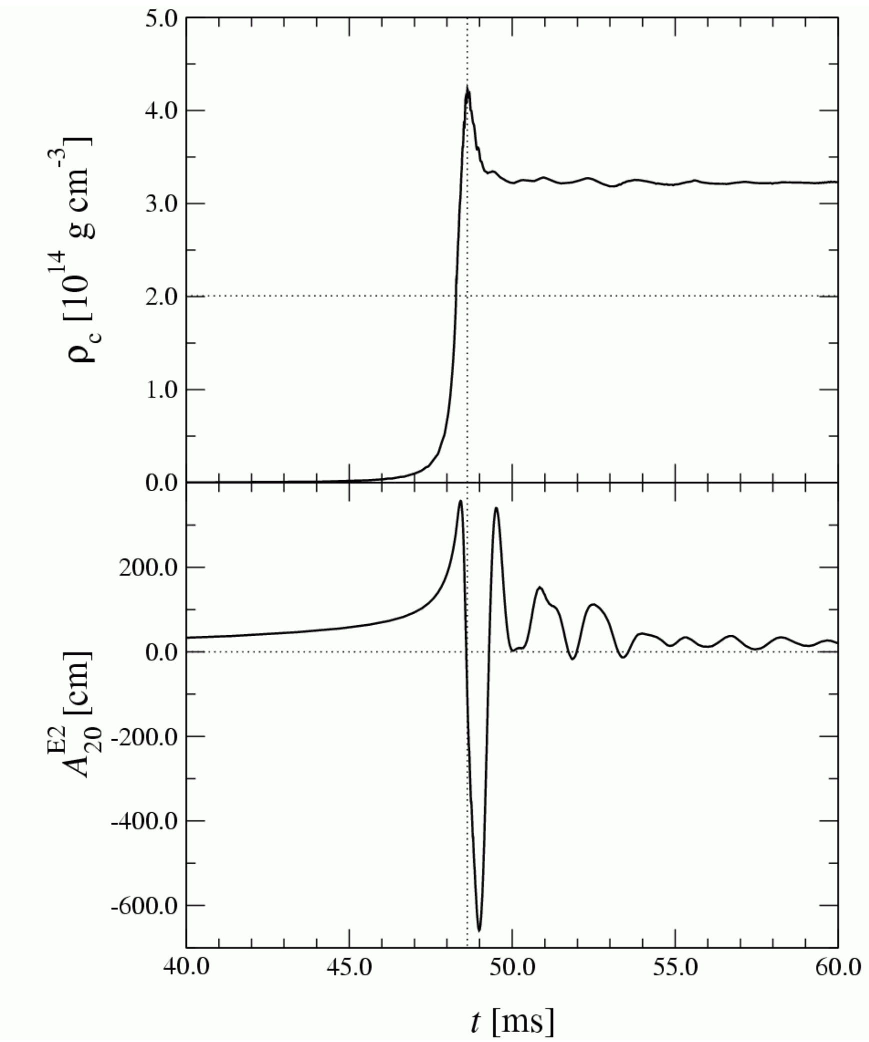
DIFFERENTIALLY ROTATING NEUTRON STARS (I)

Birth of Neutron Stars

Stellar Core Collapse

Accretion-Induced Collapse of WD





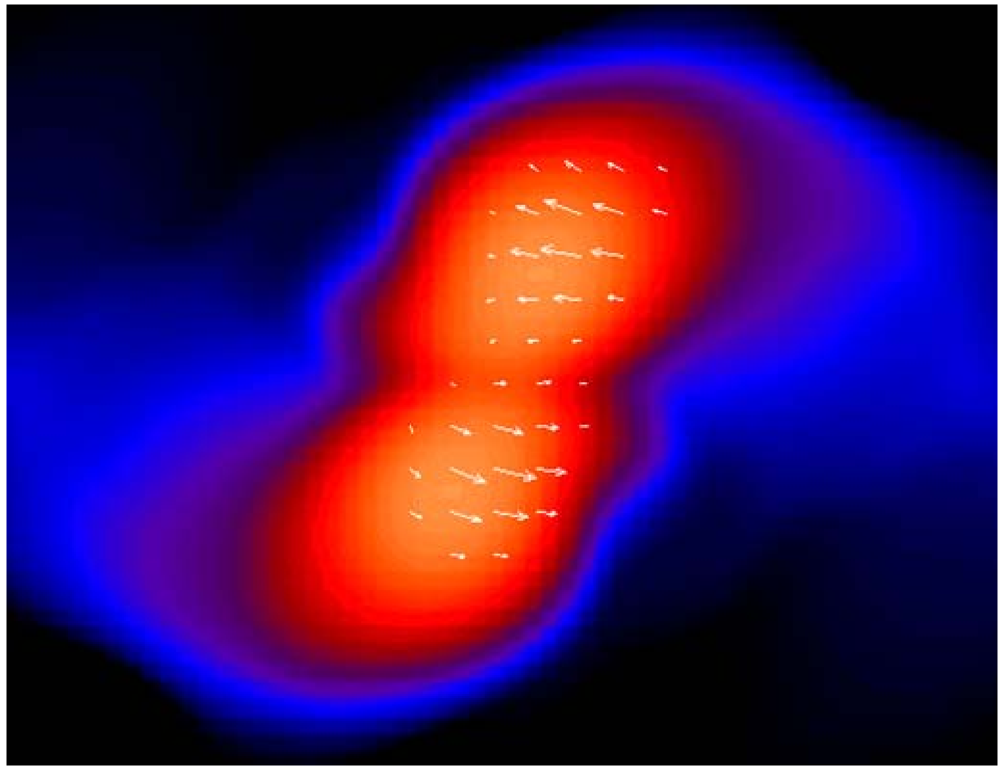
Dimmelmeier, Font & Mueller 2002

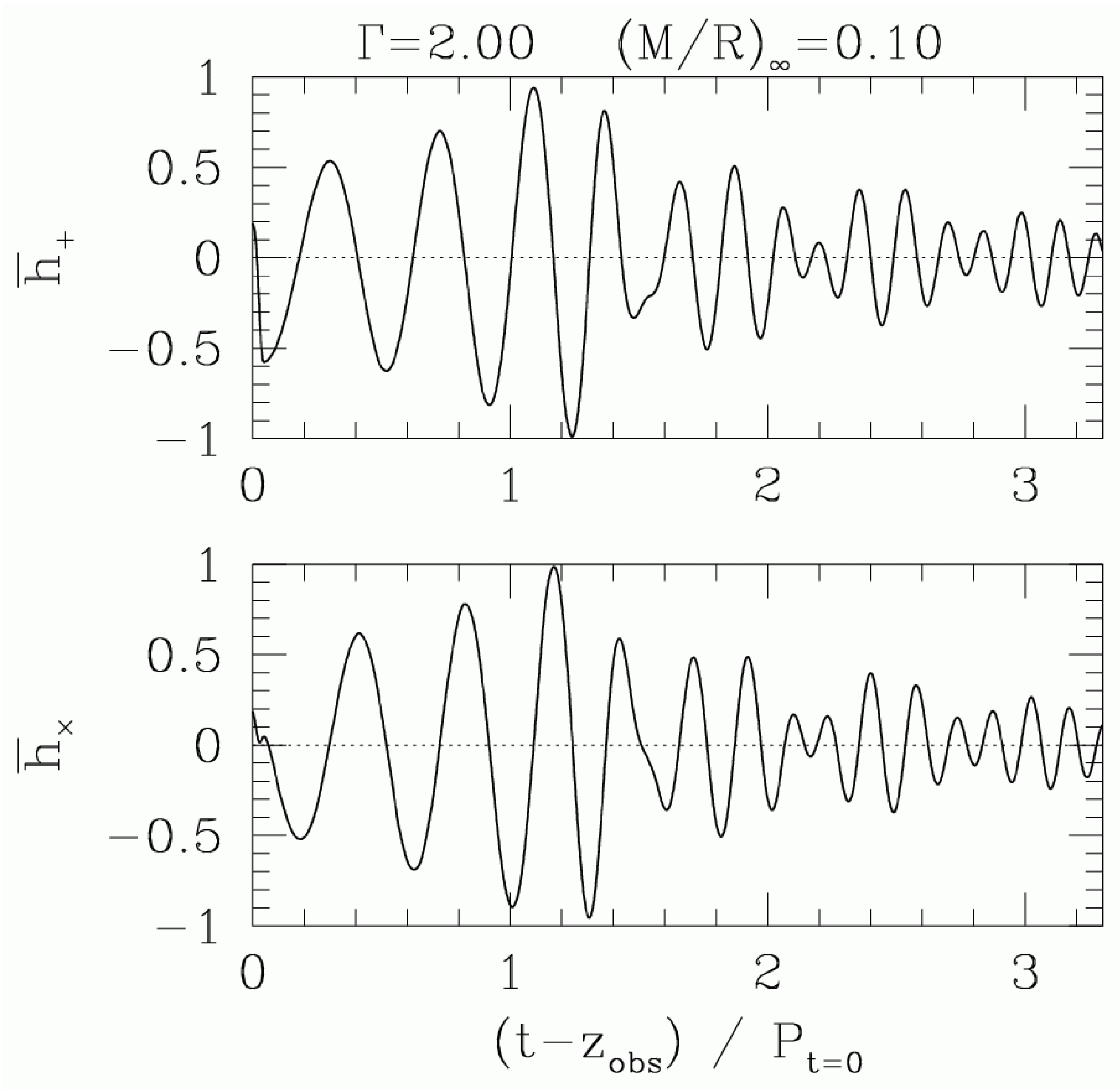
Periodic GW emission due to excitation of l=0 and l=2 modes

Why are the pulsations strongly damped ?

DIFFERENTIALLY ROTATING NEUTRON STARS (II)

Binary Merger





Shibata & Uryu 2002

In many cases, a hypermassive differentially rotating star forms

Part of the signal due to excitation of normal modes

CONSTRUCTION OF INITIAL DATA (I)

Metric:

$$ds^2 = -e^{\gamma+\rho}dt^2 + e^{\gamma-\rho}r^2\sin^2\theta(d\phi-\omega dt)^2 + e^{2\alpha}(dr^2+r^2d\theta^2)$$

Field Equations: in integral form

$$\Delta[\rho e^{\gamma/2}] = S_\rho(r,\mu),$$

$$\left(\Delta + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\mu\frac{\partial}{\partial\mu}\right)\gamma e^{\gamma/2} = S_\gamma(r,\mu),$$

$$\left(\Delta + \frac{2}{r}\frac{\partial}{\partial r} - \frac{2}{r^2}\mu\frac{\partial}{\partial\mu}\right)\omega e^{(\gamma-2\rho)/2} = S_\omega(r,\mu),$$

$$\rho = -\frac{1}{4\pi}e^{-\gamma/2}\int_0^\infty dr'\int_{-1}^{1}d\mu'\int_0^{2\pi}d\phi' r'^2 S_\rho(r',\mu')\frac{1}{|\mathbf{r}-\mathbf{r}'|}.$$

$$r\sin\theta\gamma = \frac{1}{2\pi}e^{-\gamma/2}\int_0^\infty dr'\int_0^{2\pi}d\theta' r'^2\sin\theta' S_\gamma(r',\theta')\log|\mathbf{r}-\mathbf{r}'|,$$

$$r\sin\theta\cos\phi\,\omega = -\frac{1}{4\pi}e^{(2\rho-\gamma)/2}\int_0^\infty dr'\int_0^\pi d\theta'\int_0^{2\pi}d\phi' r'^3\sin^2\theta'\cos\phi' S_\omega(r',\theta')\frac{1}{|\mathbf{r}-\mathbf{r}'|}.$$

RNS code

Stergioulas & Friedman, 1995

$$\begin{aligned}\alpha_{,\mu} = {} & -\nu_{,\mu} - \{(1-\mu^2)(1+rB^{-1}B_{,r})^2 + [\mu-(1-\mu^2)B^{-1}B_{,\mu}]^2\}^{-1} \\ & [\tfrac{1}{2}B^{-1}\{r^2B_{,rr} - [(1-\mu^2)B_{,\mu}]_{,\mu} - 2\mu B_{,\mu}\}[-\mu+(1-\mu^2)B^{-1}B_{,\mu}] \\ & + rB^{-1}B_{,r}[\tfrac{1}{2}\mu + \mu rB^{-1}B_{,r} + \tfrac{1}{2}(1-\mu^2)B^{-1}B_{,\mu}] \\ & + \tfrac{3}{2}B^{-1}B_{,\mu}[-\mu^2+\mu(1-\mu^2)B^{-1}B_{,\mu}] - (1-\mu^2)rB^{-1}B_{,\mu r}(1+rB^{-1}B_{,r}) \\ & - \mu r^2\nu_{,r}^2 - 2(1-\mu^2)r\nu_{,\mu}\nu_{,r} + \mu(1-\mu^2)\nu_{,\mu}^2 - 2(1-\mu^2)r^2B^{-1}B_{,r} \\ & \times \nu_{,\mu}\nu_{,r} + (1-\mu^2)B^{-1}B_{,\mu}[r^2\nu_{,r}^2 - (1-\mu^2)\nu_{,\mu}^2] + (1-\mu^2)B^2e^{-4\nu} \\ & \times [\tfrac{1}{4}\mu r^4\omega_{,r}^2 + \tfrac{1}{2}(1-\mu^2)r^3\omega_{,\mu}\omega_{,r} - \tfrac{1}{4}\mu(1-\mu^2)r^2\omega_{,\mu}^2 + \tfrac{1}{2}(1-\mu^2) \\ & \times r^4B^{-1}B_{,r}\omega_{,\mu}\omega_{,r} - \tfrac{1}{4}(1-\mu^2)r^2B^{-1}B_{,\mu}[r^2\omega_{,r}^2 - (1-\mu^2)\omega_{,\mu}^2]]],\end{aligned}$$

Komatsu, Eriguchi & Hachisu method

Cook, Shapiro & Teukolsky compactified radial coordinate

CONSTRUCTION OF INITIAL DATA (II)

For barotropic EOSs, specific angular momentum measured by proper time of matter is

$$\boxed{j \equiv u^t u_\phi = j(\Omega)}$$

Rotation Law:

$$\boxed{\Omega = \Omega_C - \frac{(\Omega-\omega) r^2 \sin^2\theta\, e^{-2\rho}}{\mathrm{A}^2\left[1-(\Omega-\omega)^2 r^2 \sin^2\theta\, e^{-2\rho}\right]}}$$

Komatsu, Eriguchi, Hachisu

Dimensionless constant:

$$\boxed{\hat{\mathrm{A}} = \mathrm{A}/r_e}$$

Shibata, Baumgarte, Shapiro

Limits:

$$\boxed{\hat{\mathrm{A}}^{-1} \to \begin{cases} 0 & \text{uniform rotation} \\ \infty & j-\text{constant rotation law} \end{cases}}$$

Specific angular momentum conserved during homologous collapse:

$$\boxed{\tilde{j} \equiv u_\phi \left(\frac{\varepsilon + P}{\rho_0}\right)}$$

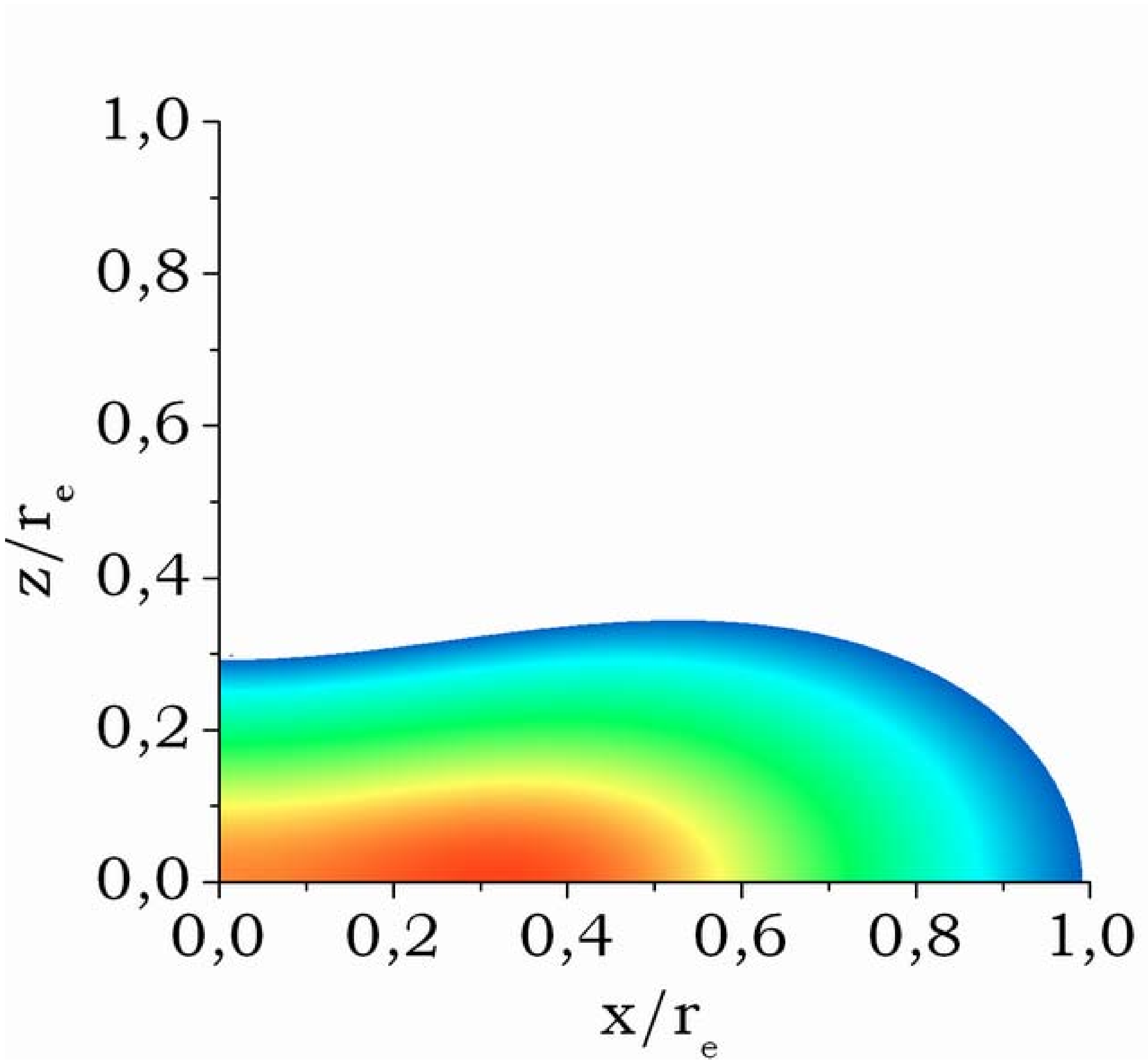
Satisfies Rayleigh criterion for local dynamical stability to axisymmetric perturbations:

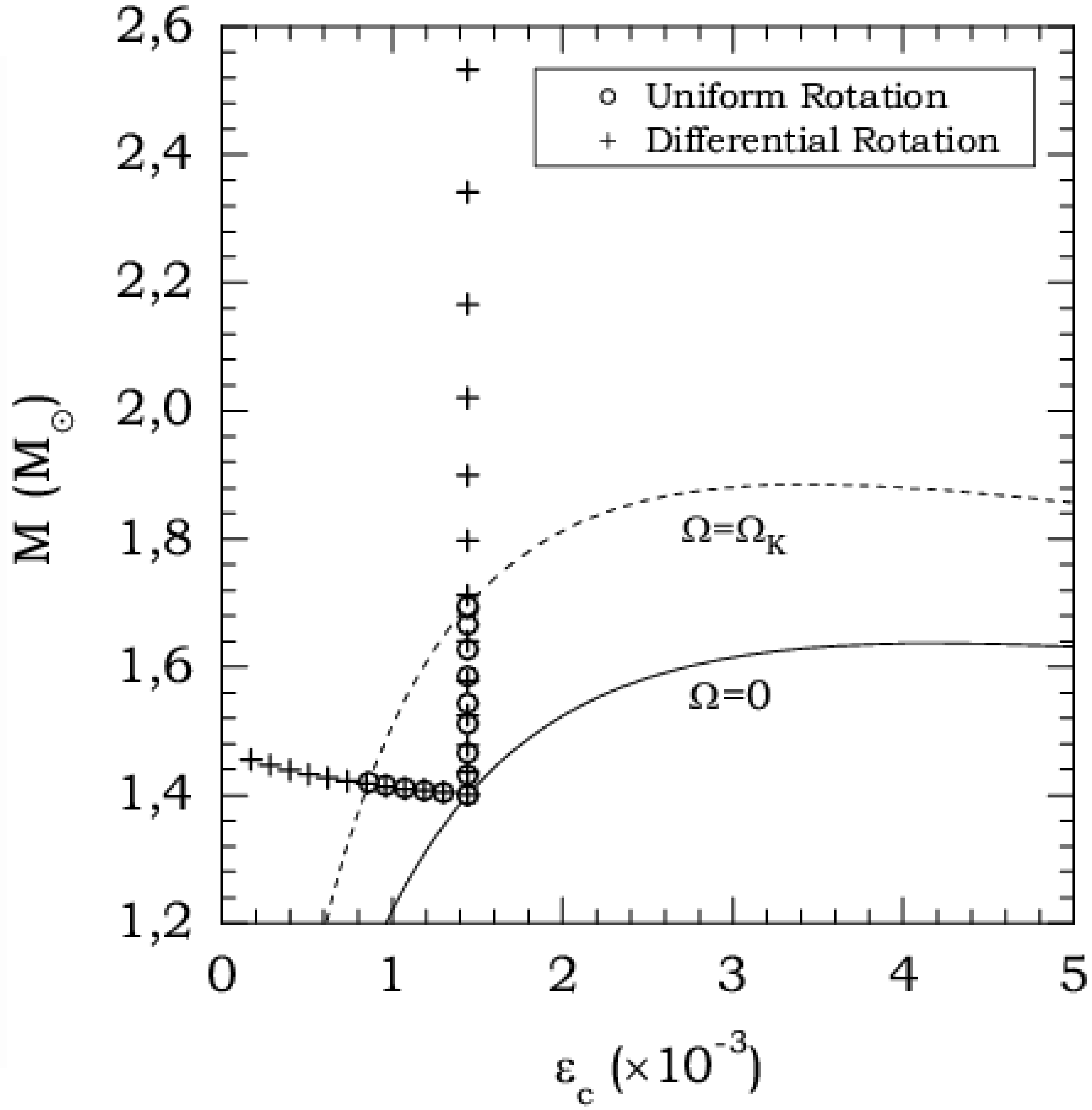
$$\boxed{\frac{d\tilde{j}}{d\Omega} < 0}$$

EQUILIBRIUM MODELS

Choice of $\hat{A}=1.0$ yields $\Omega_e / \Omega_C \sim 1/3$

in agreement with some core collapse and NS merger simulations





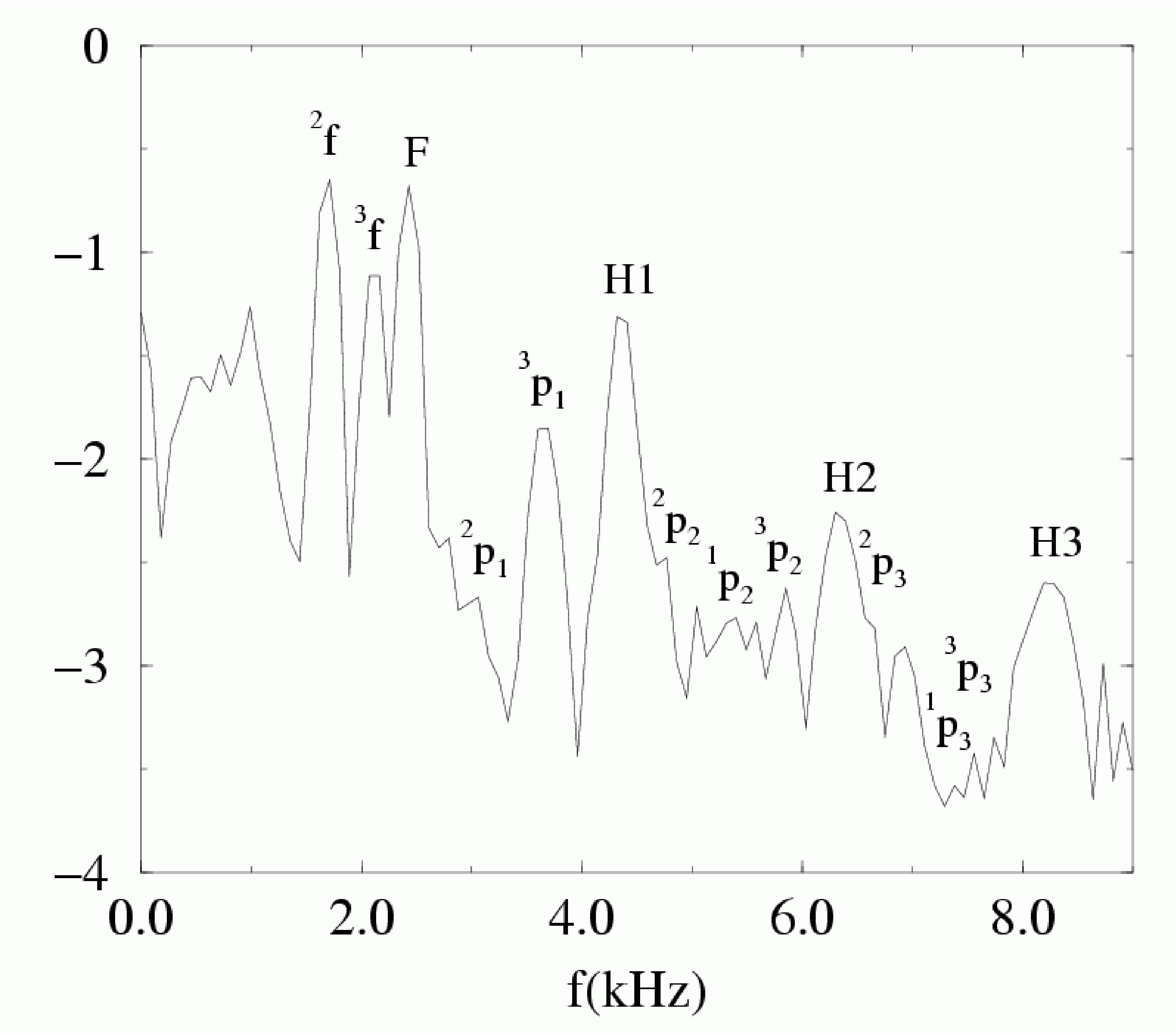
HYDRODYNAMICAL SIMULATIONS

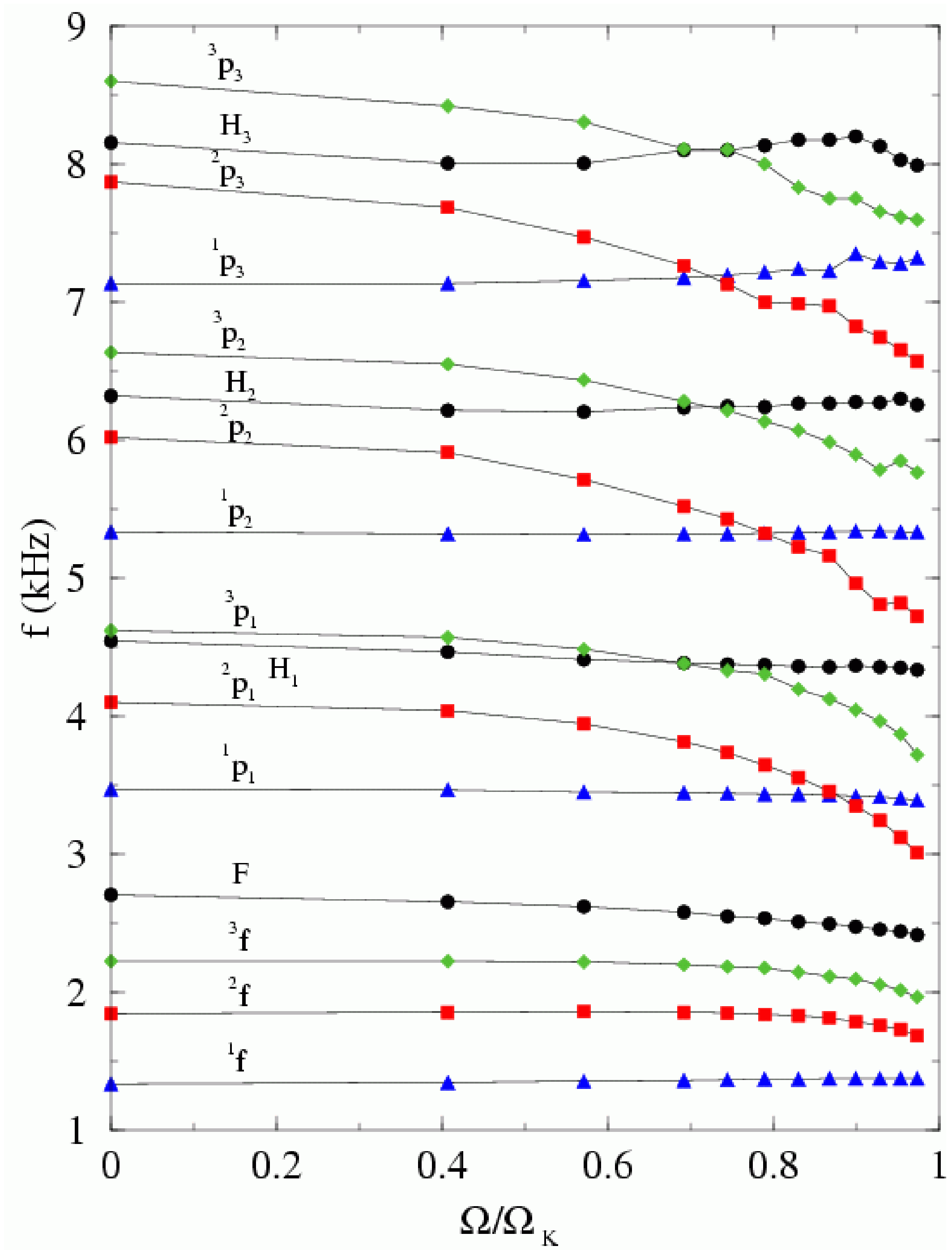
2-D nonlinear evolutions with 3rd order PPM method in Cowling approximation

Font, Stergioulas, Kokkotas (2000)

Axisymmetric mode frequencies for uniform rotation

Font, Dimmelmeier, Gupta, Stergioulas (2001)





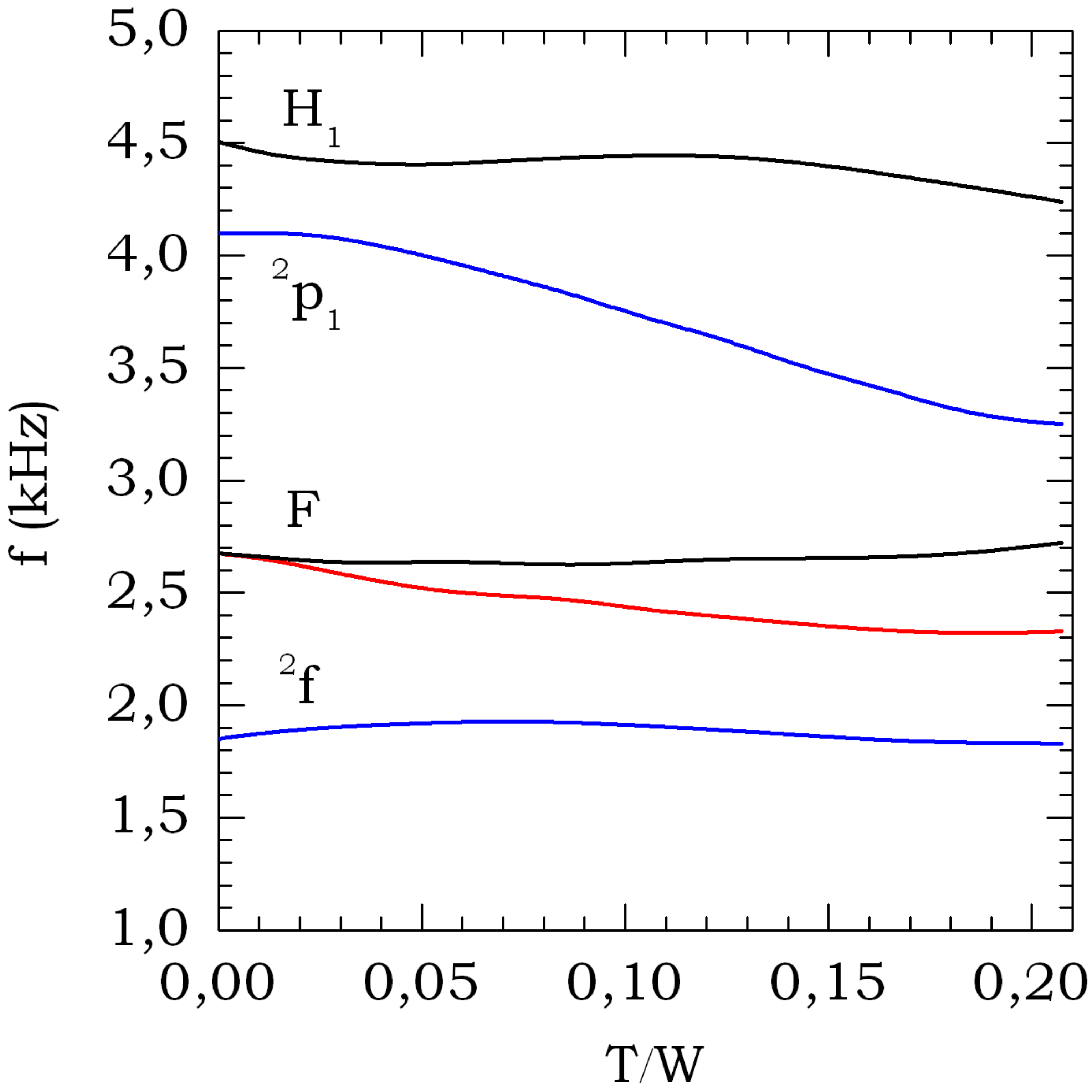
FREQUENCIES FOR DIFFERENTIAL ROTATION

(Stergioulas, Apostolatos, Font 2003)

Fundamental frequencies depend mostly on central density

Differential rotation allows much smaller frequencies

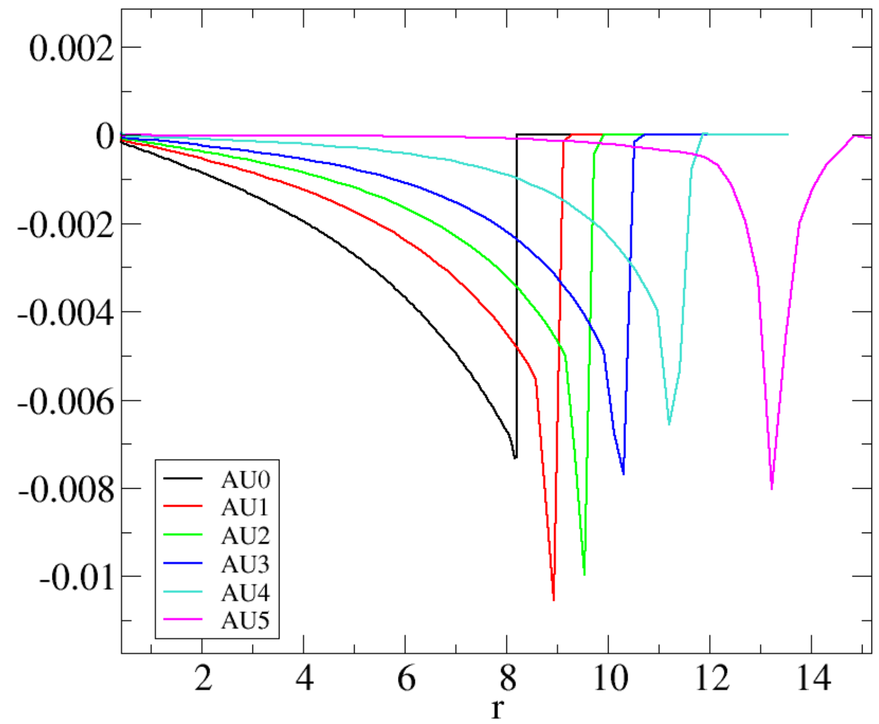
In Cowling approximation fundamental F-mode is split by differential rotation



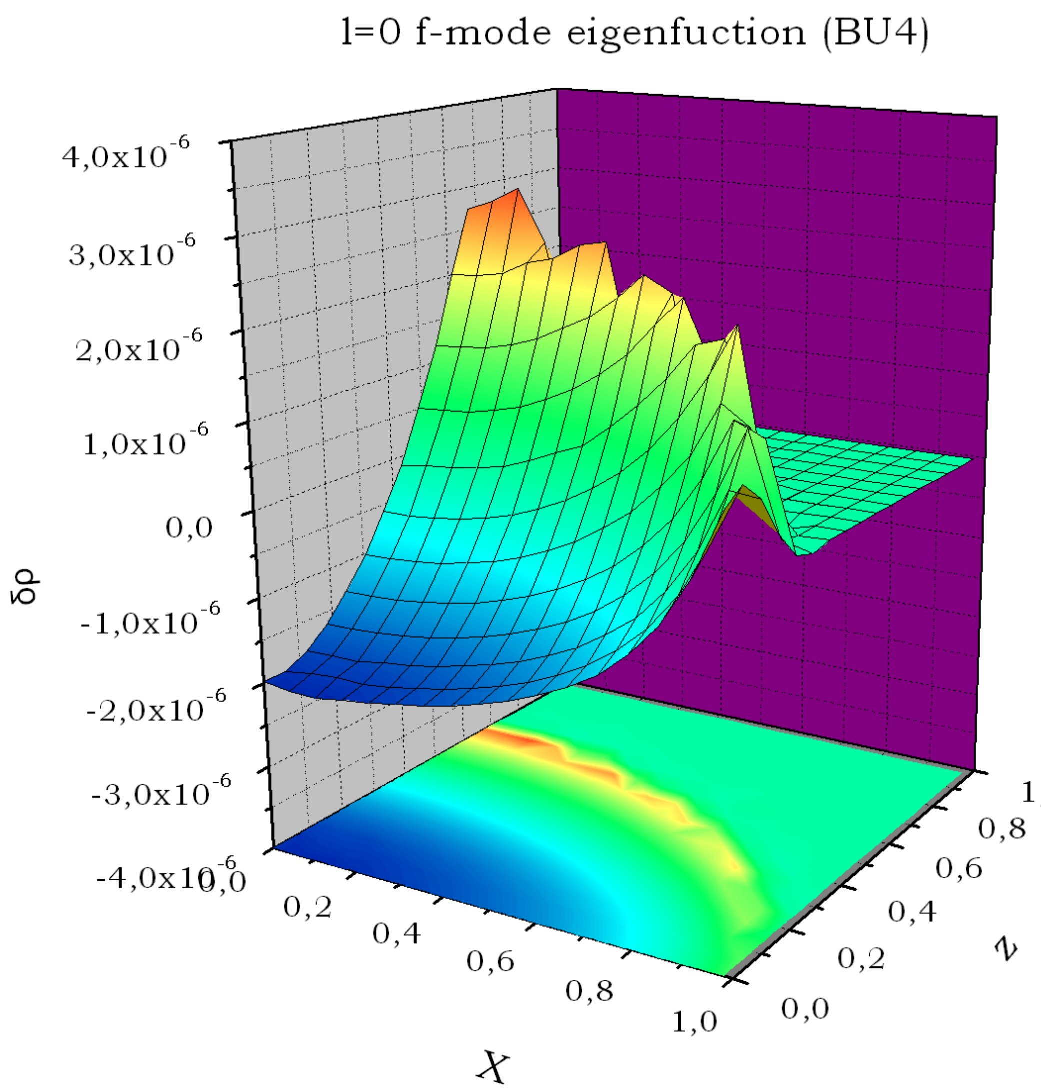
EXTRACTION OF EIGENFUNCTIONS (I)

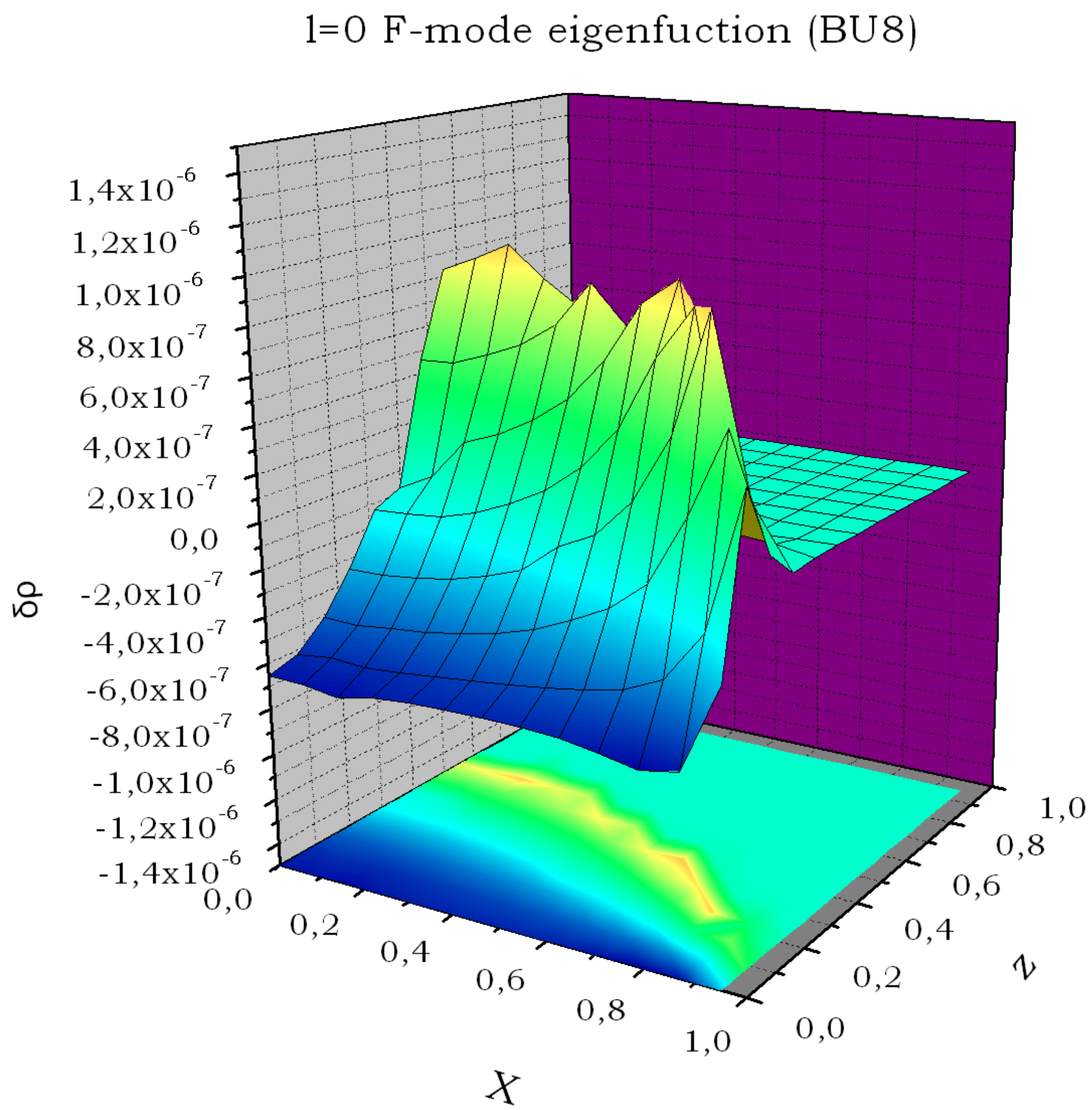
<u>New Fourier Technique</u>

Eigenfunctions are extracted by FFT at all points – magnitude of FFT correlates with shape of eigenfunction



F-mode along x-axis for different rotation rates



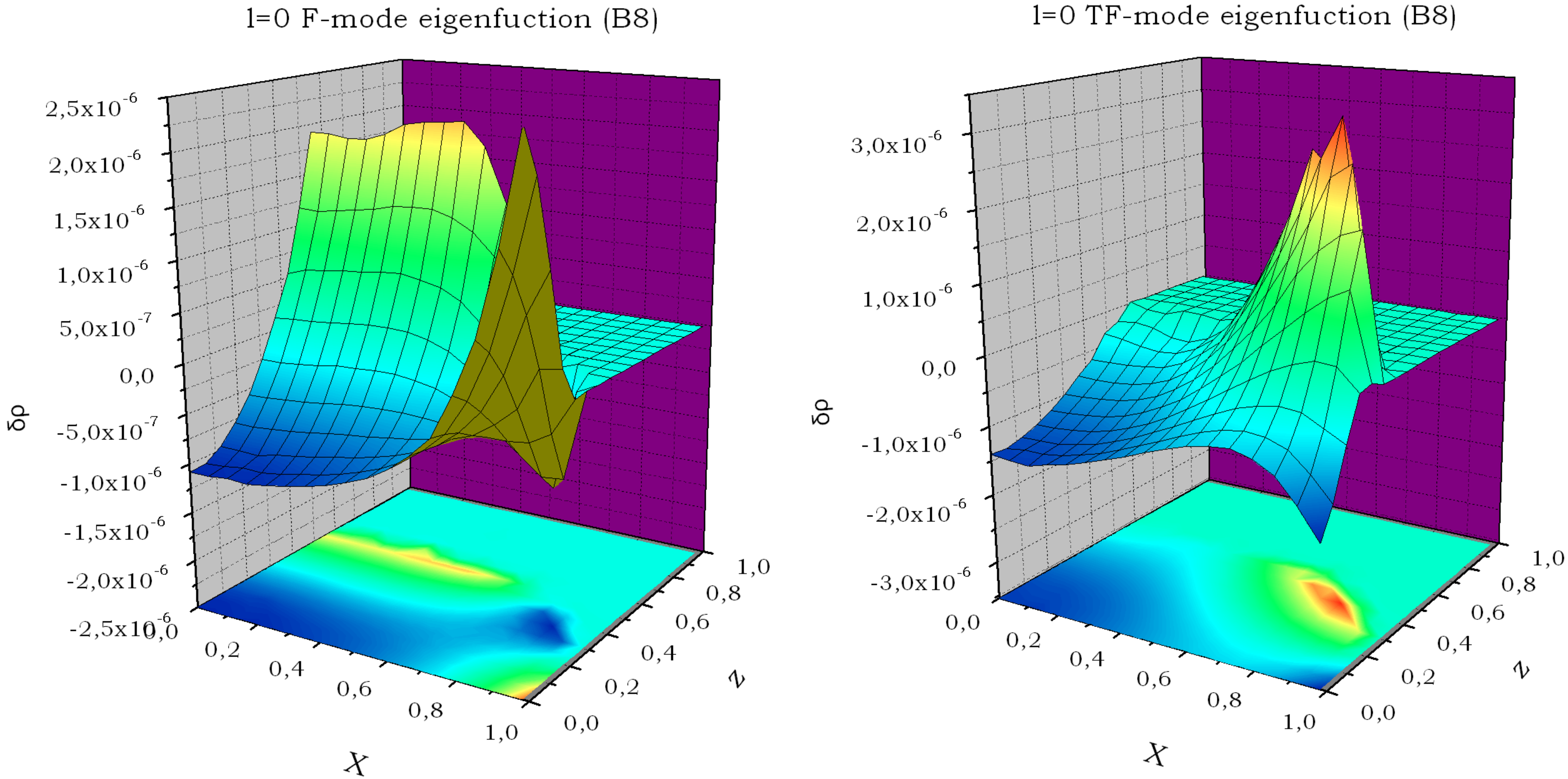


F-mode in xz plane for different rotation rates

EXTRACTION OF EIGENFUNCTIONS (II)

Splitting of Fundamental mode

Comparison of eigenfunctions reveals the very different shape of the two modes

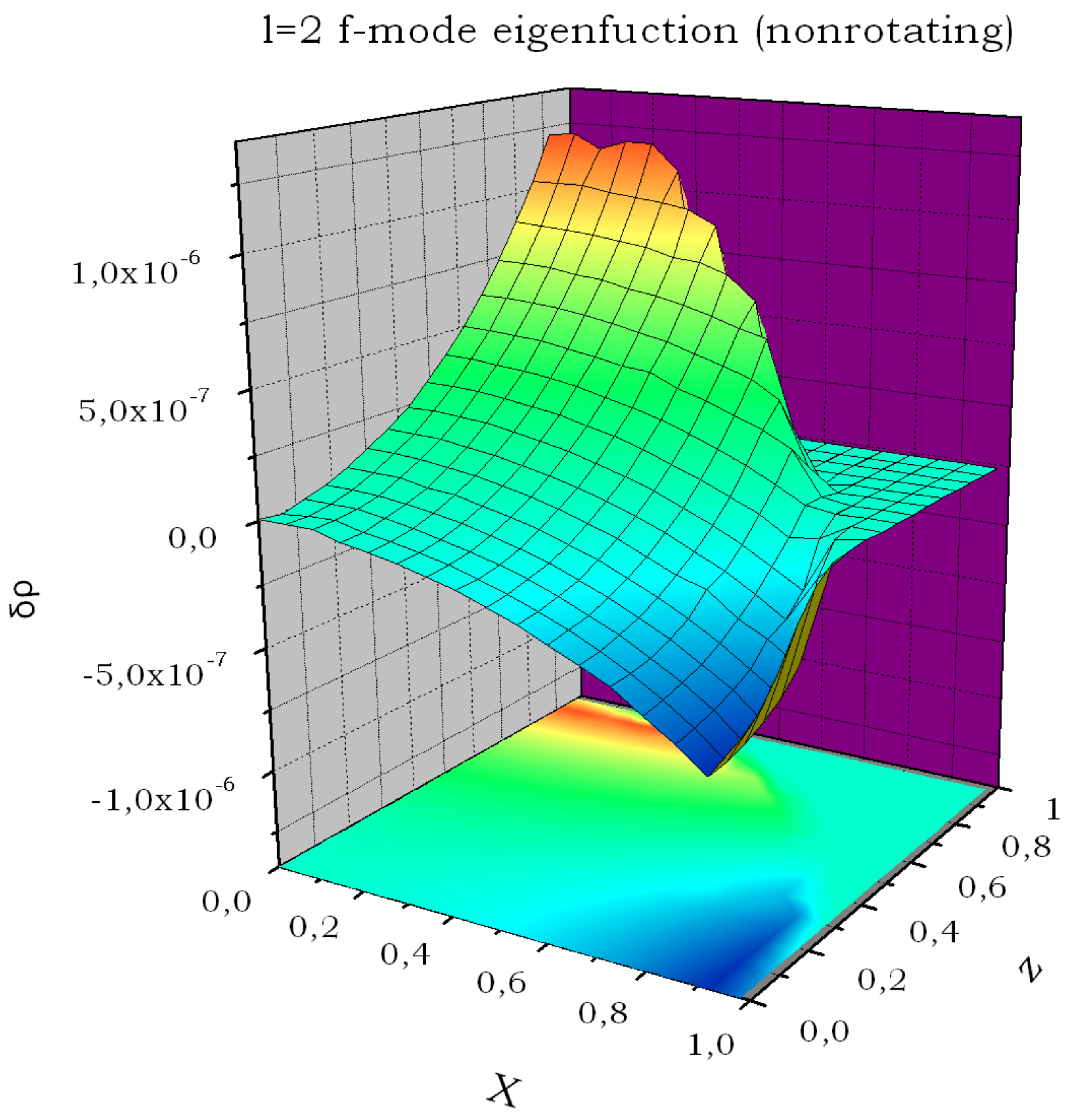


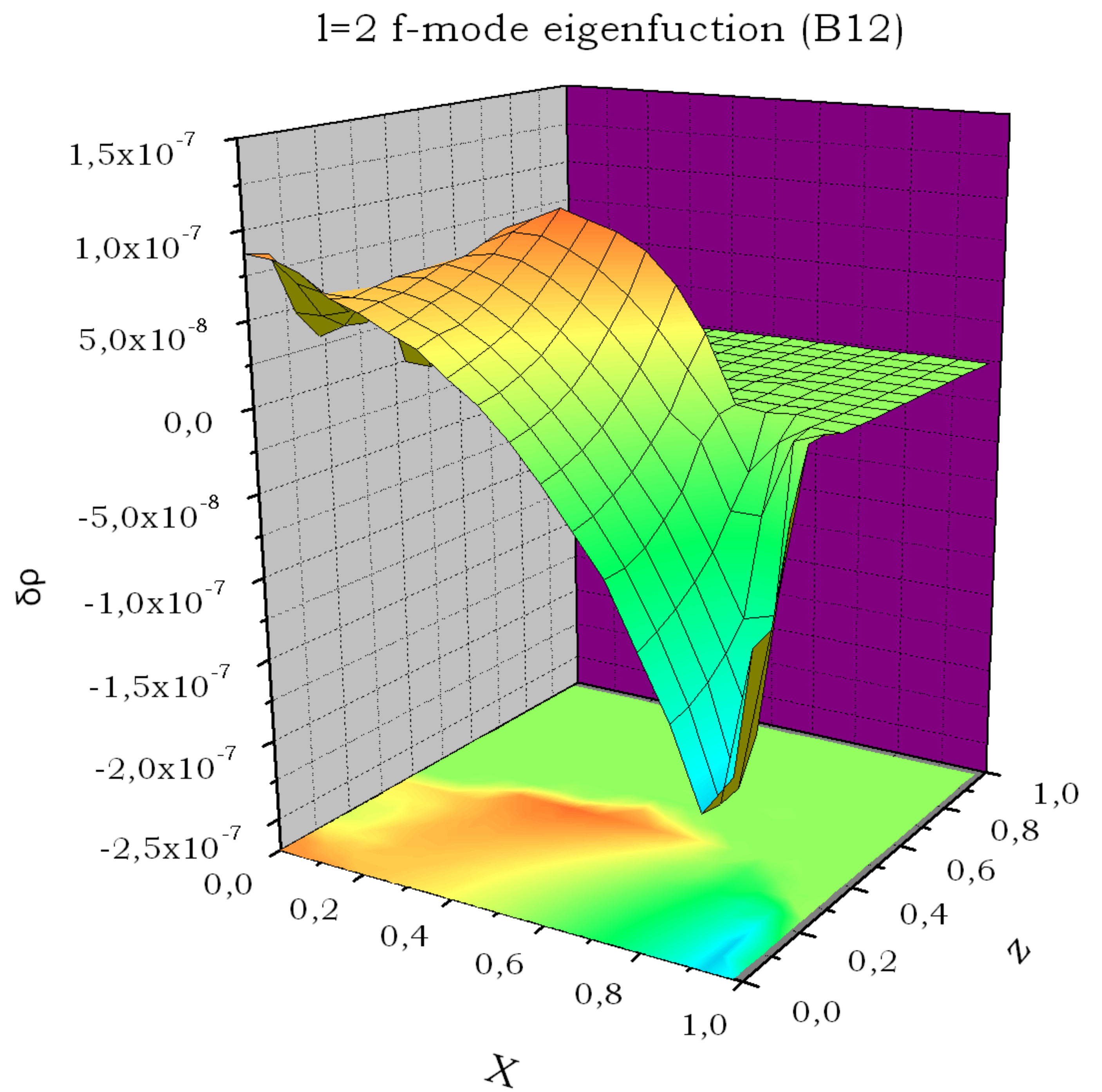
EXTRACTION OF EIGENFUNCTIONS (III)

Effects of rapid rotation on nonradial eigenfunctions

Centrifugal force weakens restoring force near equatorial surface

Nonradial modes become nonzero at center of star





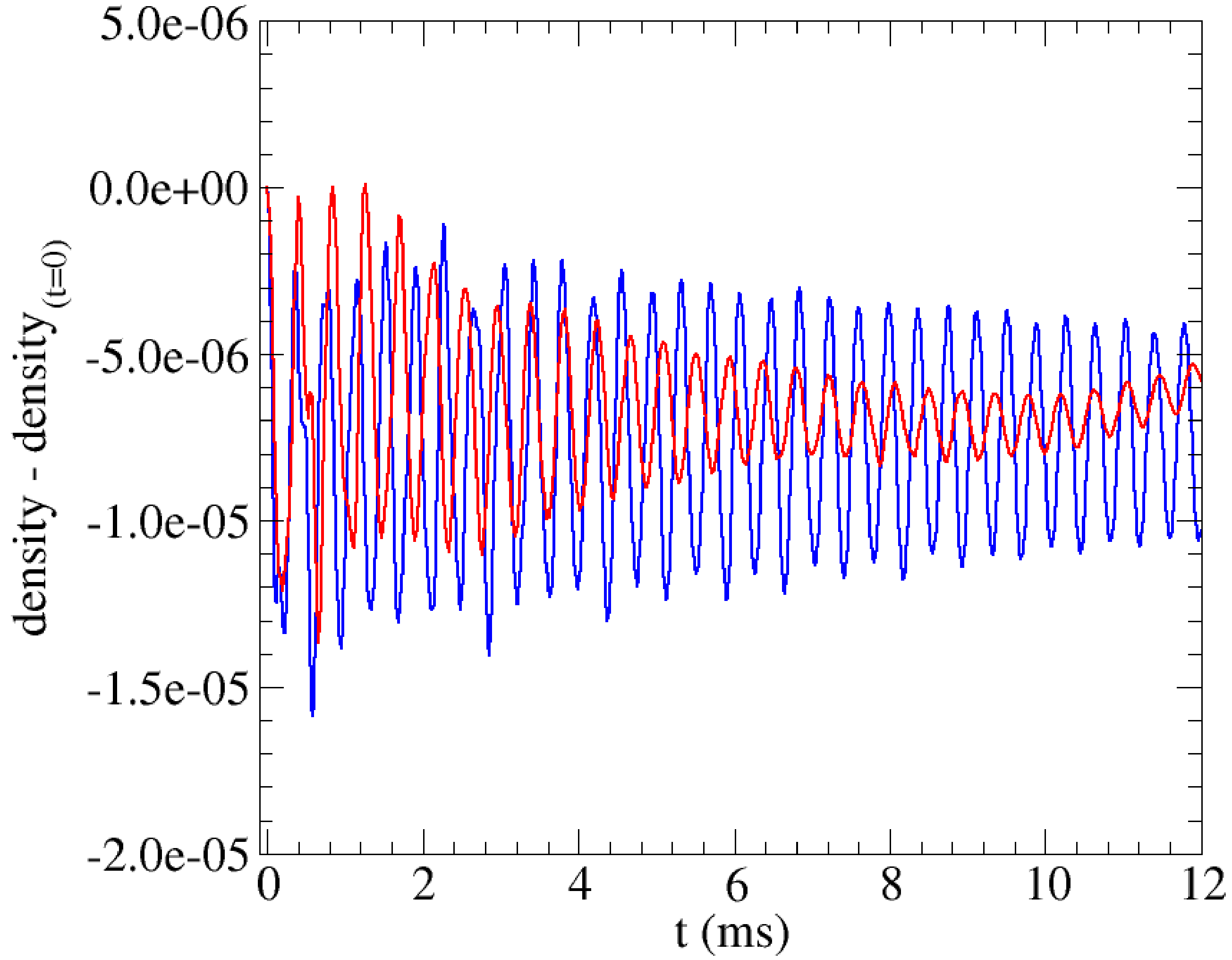
MASS-SHEDDING-INDUCED DAMPING

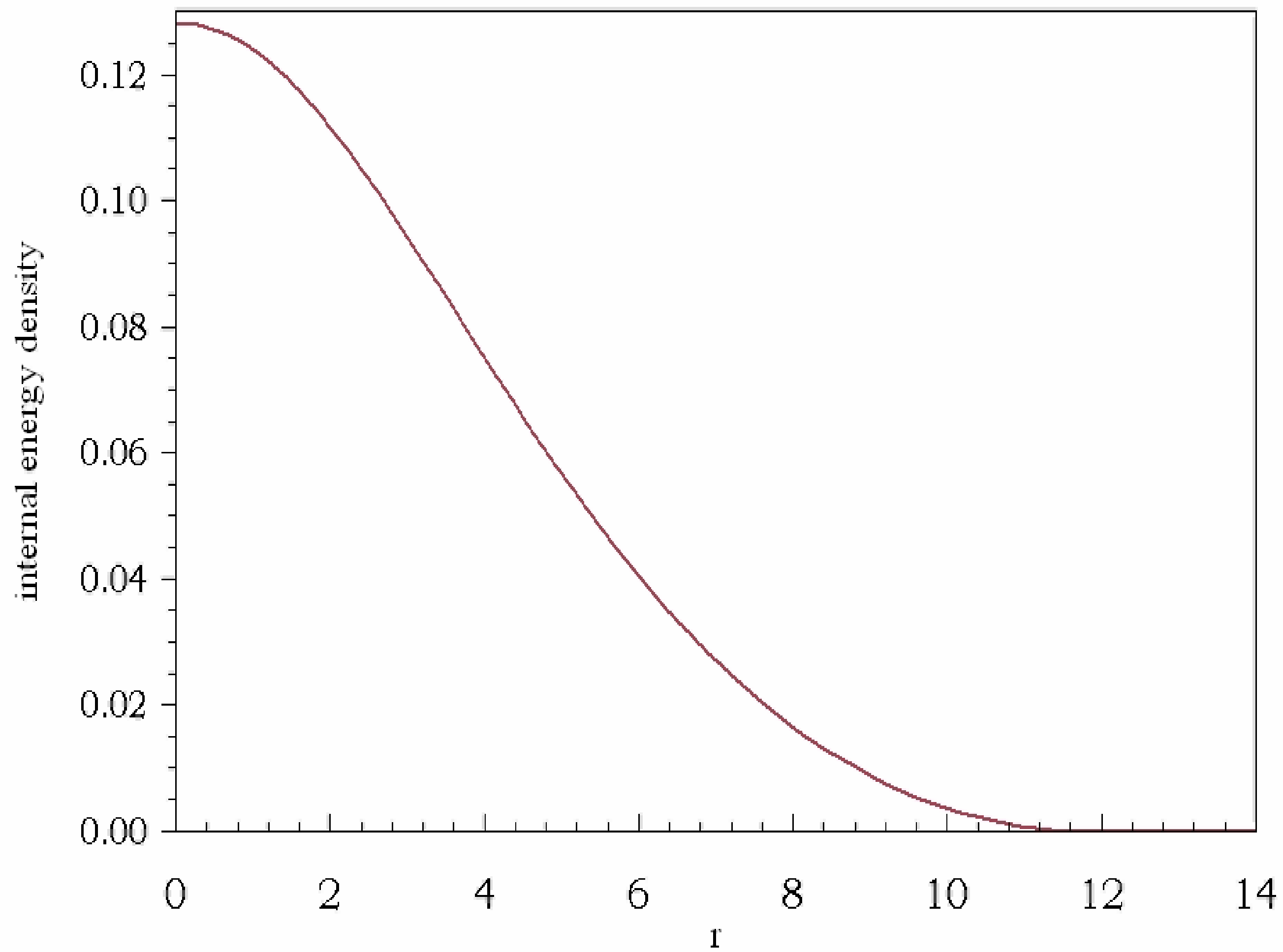
<u>Near Mass-Shedding limit pulsations are strongly damped</u>

Fluid elements near surface are <u>weakly bound</u>

Radial component of pulsations causes <u>mass-shedding in form of shocks</u>

This could enhance strong damping in core-collapse oscillations and set <u>a new saturation amplitude for unstable modes</u>





CONCLUSIONS

- First computation of axisymmetric eigenfunctions for differentially rotating stars
- Splitting of fundamental mode in Cowling approximation
- Rapid rotation causes mass-shedding induced damping of pulsations, setting a saturation amplitude for unstable modes

In progress

- Conformally flat gravity (with H. Dimmelmeier)
- Extensive study of mass-shedding damping rate