Dynamics of differentially rotating neutron stars

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Differential rotation in neutron stars

- Differential rotation can arise in several ways:
  - During the stellar core collapse that creates the NS
  - As the result of a merger with another NS
  - Due to non-linear and magnetic field effects associated with oscillations
  - As the result of accretion

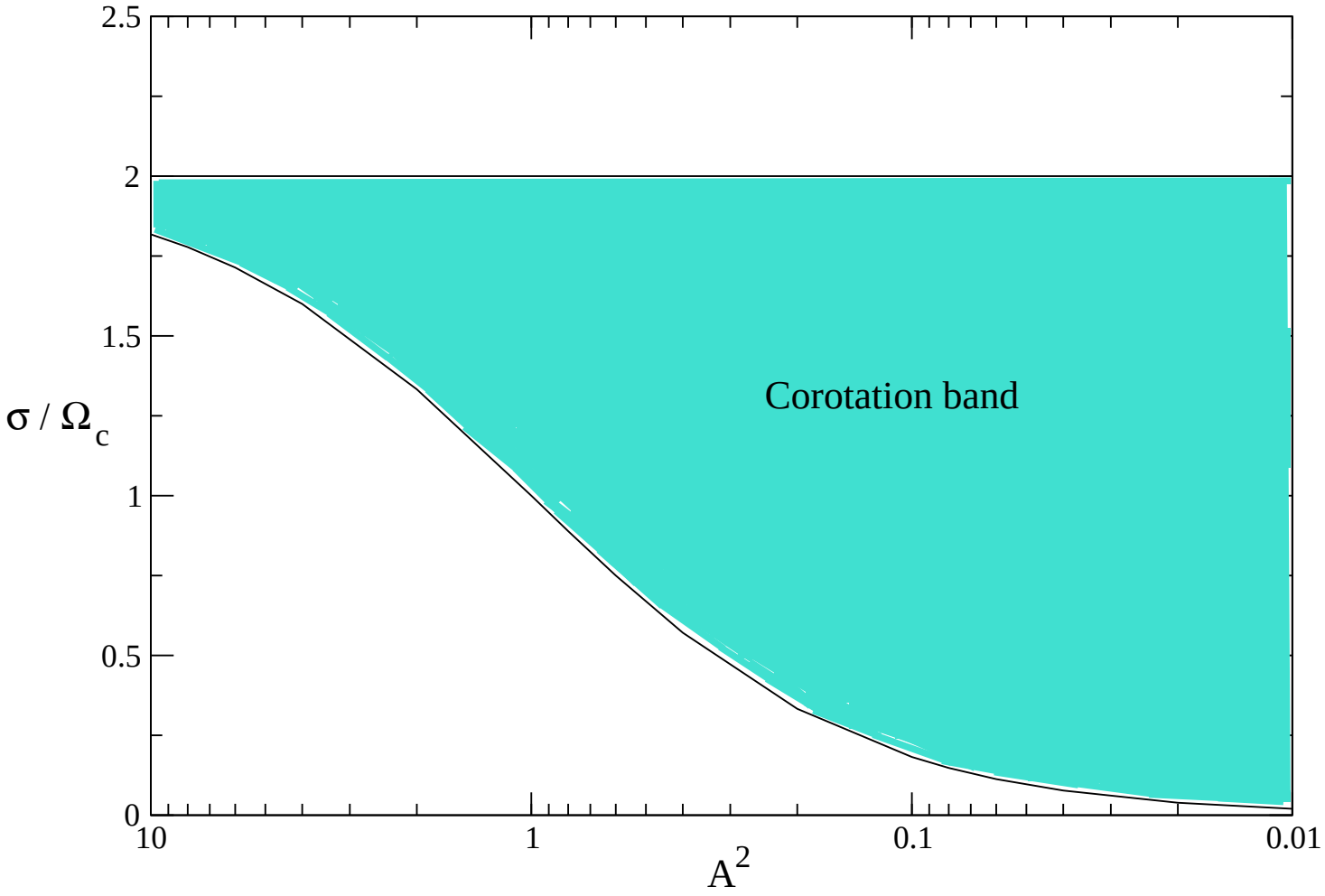
- It will affect the modes of oscillation, introducing
  - Dynamical shear instabilities
  - A continuous spectrum spanning the range of frequencies for which the pattern speed of a mode would equal the angular velocity

Oscillations of a differentially rotating star

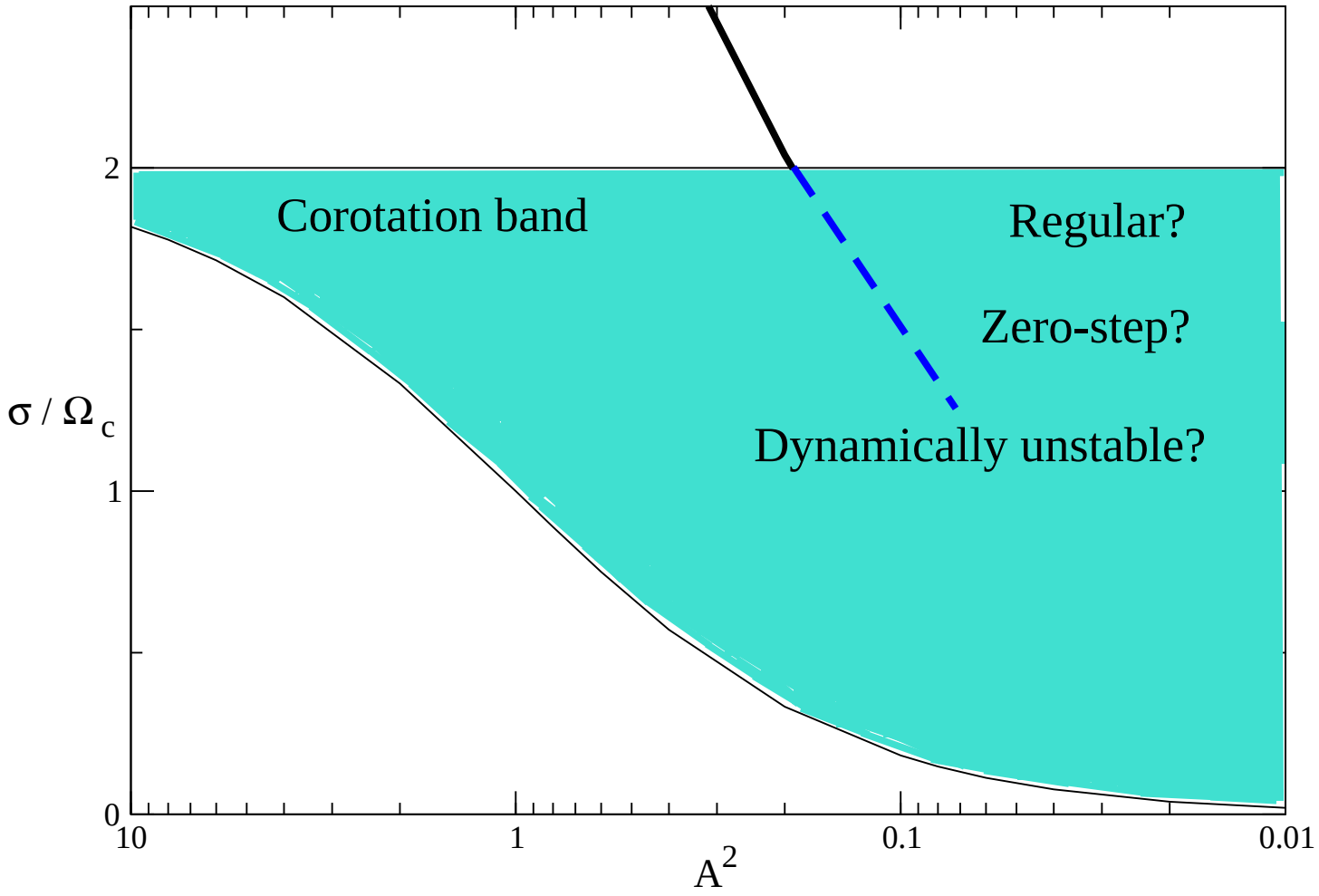
$$L\phi_{,zz} - \bar{\sigma}^2\phi_{,\varpi\varpi} + L\frac{\rho_{,z}}{\rho}\phi_{,z} + \bar{\sigma}^2\phi_{,\varpi}\left[\frac{L_{,\varpi}}{L} - \frac{\rho_{,\varpi}}{\rho} - \frac{1}{\varpi}\right]$$
$$+\phi\left[\bar{\sigma}^2\left(\frac{L}{c^2} + \frac{m^2}{\varpi^2}\right) - \frac{2m\Omega\bar{\sigma}}{\varpi}\left(\frac{\rho_{,\varpi}}{\rho} + \frac{\Omega_{,\varpi}}{\Omega} - \frac{L_{,\varpi}}{L}\right)\right]$$
$$-\frac{\bar{\sigma}^2 L}{c^2}\delta\Phi = 0$$

- Linearized Newtonian perturbations, $\phi = \delta P/\rho$.
- Non-axisymmetric normal modes $\propto \exp(im\varphi - i\sigma t)$.
- P = pressure, ρ = density, c = sound speed, Φ = gravitational potential, $\Omega(\varpi)$ = angular velocity.
- Corotation parameter $\bar{\sigma} = m\Omega - \sigma$. $\bar{\sigma} = 0$ when $\varpi = \varpi_c$.
- Lindblad parameter $L = 2\Omega w_0 - \bar{\sigma}^2$, where vorticity $w_0 = 2\Omega + \varpi d\Omega/d\varpi$. $L = 0$ when $\varpi = \varpi_L$.

J-constant rotation law, $\Omega = \Omega_c A^2/(\varpi^2 + A^2)$



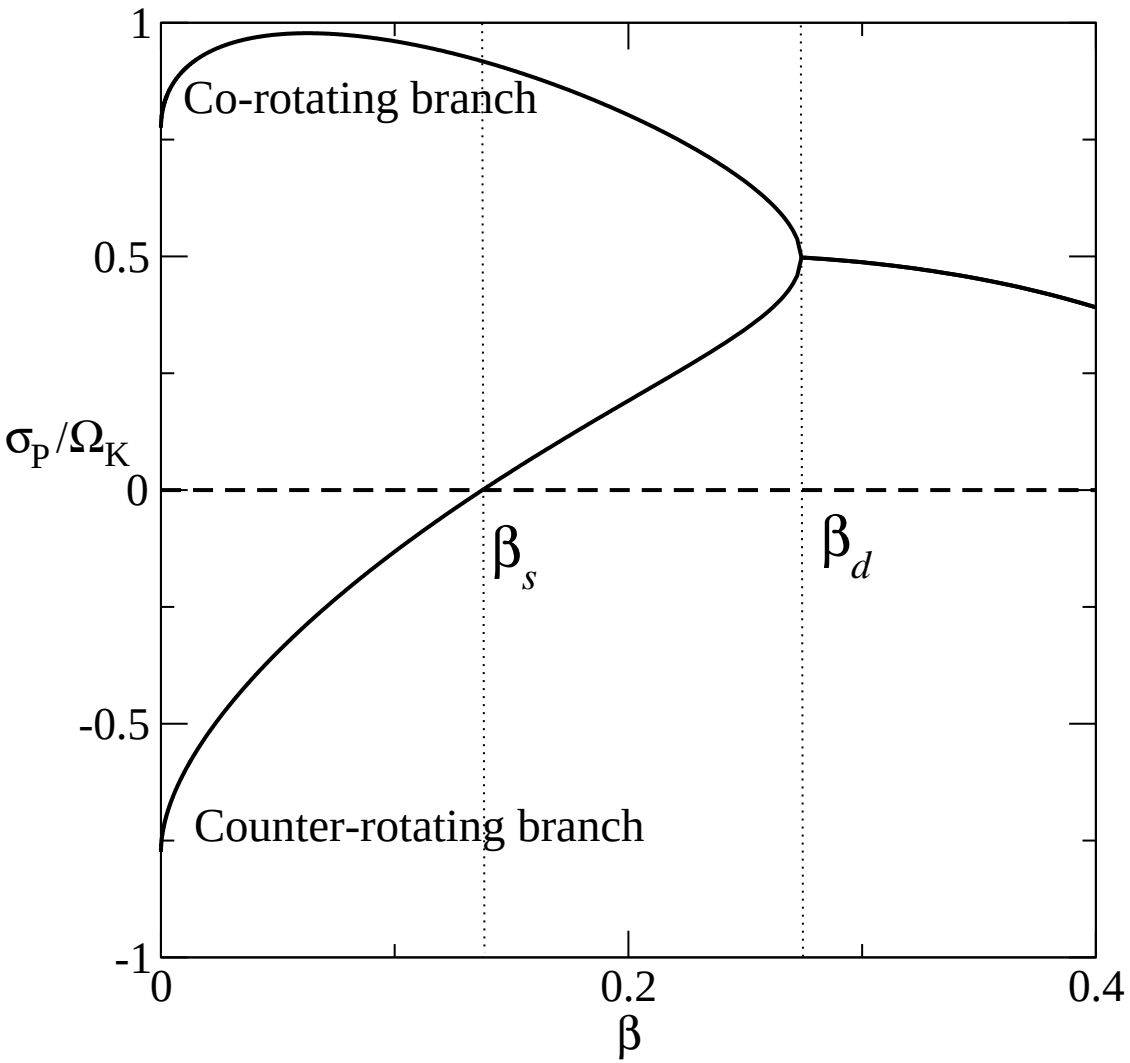
Oscillations in the corotation band (1)



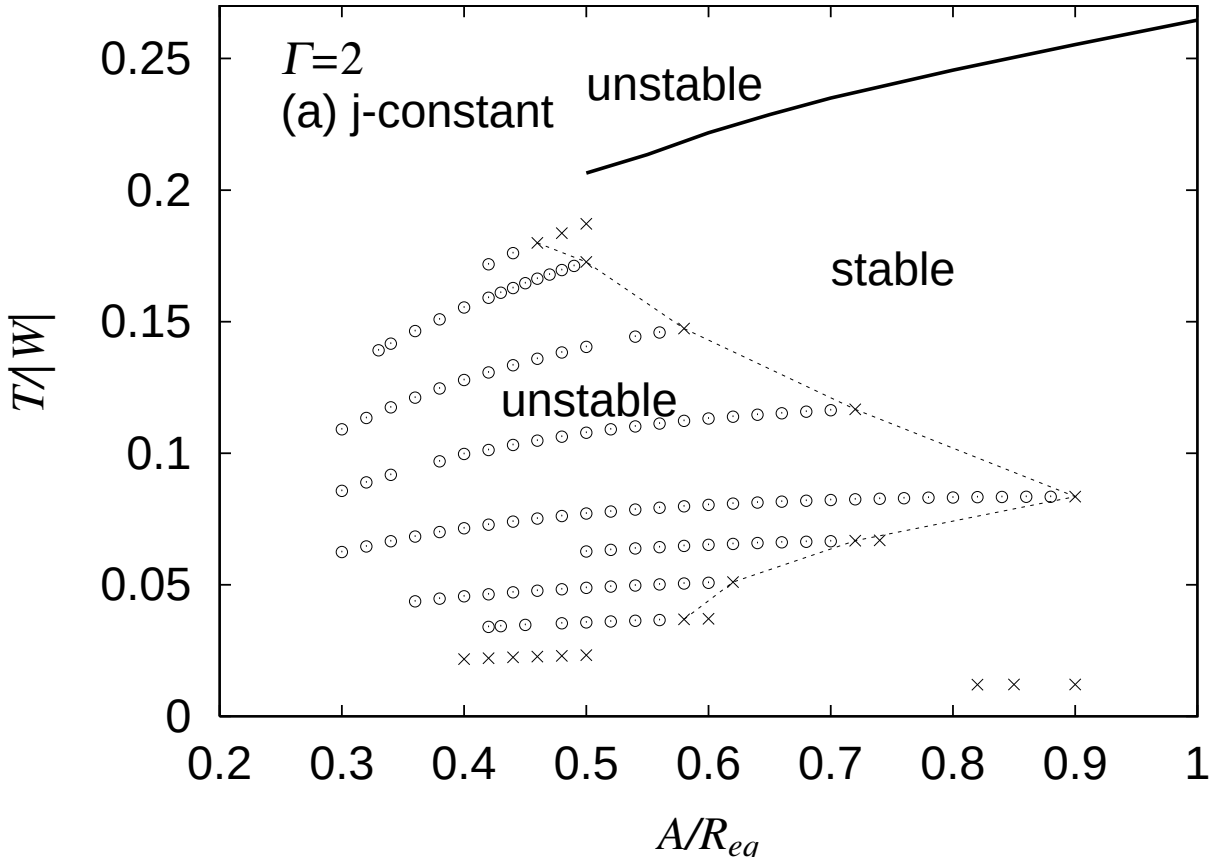
Oscillations in the corotation band (2)

- Regular stable modes:
  - Possible in theory
  - Only one example found for differentially rotating shell
- Continuous spectrum:
  - Individual "eigenfunctions" have singular first derivatives
  - Initial value problem shows that collective perturbation is non-singular, hence physical
  - Behaviour in response to a generic initial perturbation hard to predict analytically
- Zero-step solutions:
  - Frequencies in continuous spectrum where finite step in first derivative vanishes
  - Stand out clearly (in same way as discrete normal modes) in power spectra of numerical time evolutions
- Dynamical shear instabilities
  - Develop in the shell model where modes cross into corotation and degree of differential rotation exceeds a certain threshold
  - Similar instabilities expected in realistic stellar models

The f-modes

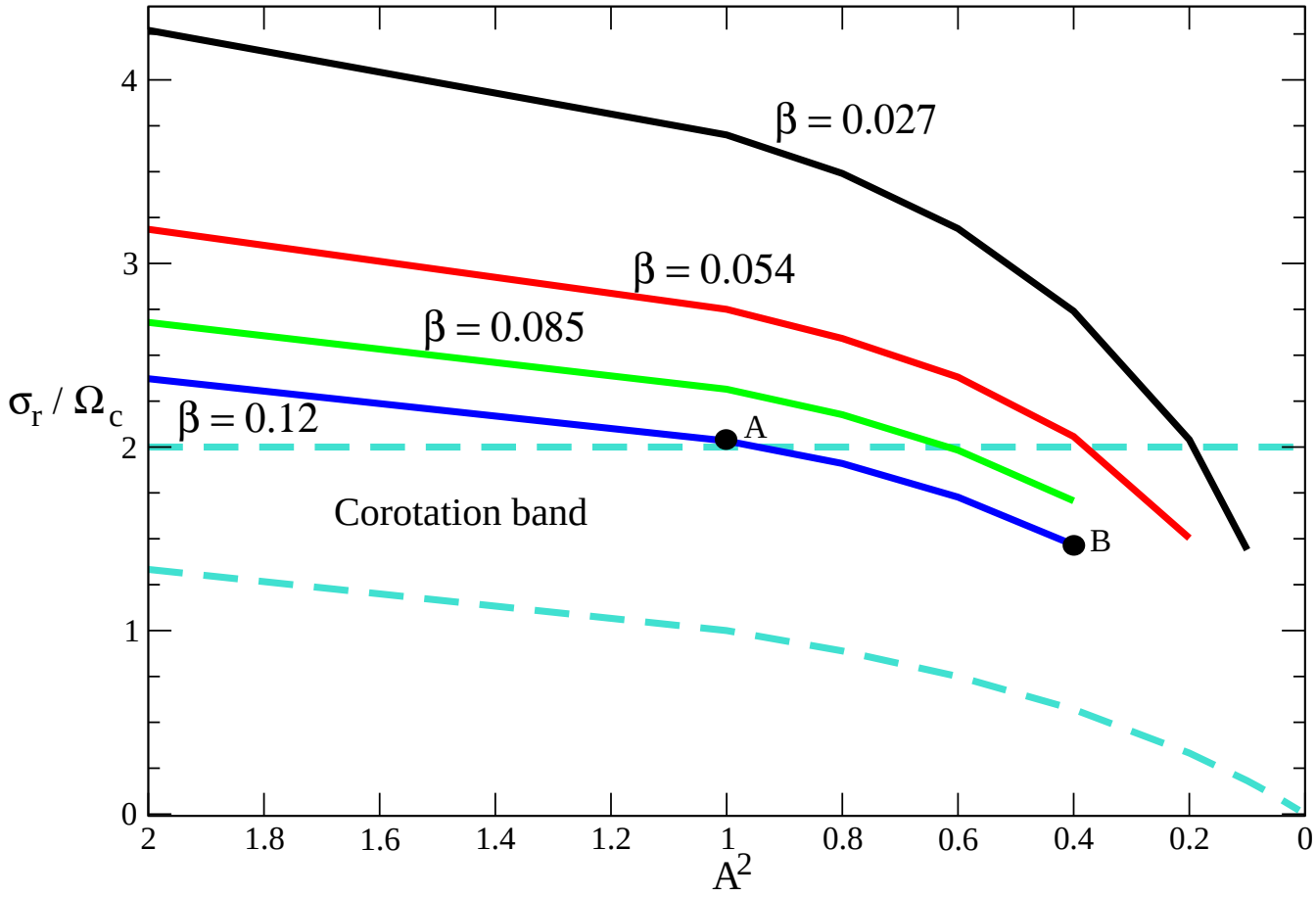

1
0.5
0
-0.5
-1
0
0.2
0.4
β
σP/ΩK
Co-rotating branch
Counter-rotating branch
βs
βd

Low $T/|W|$ dynamical instabilities of the f-mode



Shibata, Karino & Eriguchi, 2003, MNRAS, 343, 619

The f-modes and the corotation band



Conclusions and work in progress

- Dynamical shear instabilities
  - We argue that low $T/|W|$ instabilities arise when the f-mode enters the corotation band at high differential rotation.
  - Testable prediction regarding extent of unstable parameter space.
  - Many issues remain to be resolved.
- Nature of stable modes in the corotation band
- Continuous spectrum
  - Qualitatively identical to that found in our investigation of the differentially rotating shell
  - Dynamical response requires further investigation
  - Effect of viscosity
- Gravitational wave emission
  - Emission from dynamically unstable modes
  - Secular instability conditions