Dynamical Instability of Differentially Rotating Polytropes

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Plan of the Talk

- Introduction
- Equilibrium Configurations of Rotating Stars
- Linear Stability Analysis Method
- Dynamical Instability of Differentially Rotating Stars
- Summary and Concluding Remarks

Introduction

Rapidly Rotating Compact Stars

- Neutron stars (NS)
  - Constructed by type II SNe
    - Dense and Compact
    - Rapid rotation
    - High temperature → small viscous effects
  - Limit of rotation?
  - Non-axisymmetric deformation
    - Gravitational Wave (GW)

Rotational instability of fluid

- Rapidly rotating self-gravitating fluid
  - Unstable against non-axsymmetric modes
- Growth of instability ~2 types of timescale
  - Secular instability
  - Dynamical instability
    - Unstable mode will grow rapidly in dynamical timescale
    - Catastrophe of the stellar configuration
      - Binary?
      - Spiral Arms?

Maclaurin Spheroid

- Bar-mode in Rigidly rotating fluid with uniform density (Maclaurin Spheroid)
  - Simple and well-studied
  - Secularly unstable when $T/|W|>0.14$
  - Dynamically unstable when $T/|W|>0.27$
    - $T/|W|$ is a parameter describing the degree of rotation
    - T and W is a rotational kinetic energy and potential, respectively

Realistic Astrophysical Objects

- In simple models, instabilities can appear only in rapidly rotating objects
  - Collapse of massive stellar core (NS)
  - Self induced collapsed object (fizzler)
- Non-uniform density distribution
- Differential rotation driven from contraction
  - Applicability of the limit ?
  - Universal? (Ostriker & Bodenhiemer 1973)
    - $T / |W| > 0.14$ for secular instability?
    - $T / |W| > 0.27$ for dynamical instability?

⇨ Linear Stability Analysis

Linear Analysis

Linear Stability Analysis

- Equilibrium configurations
  - Axisymmetric
  - Newtonian gravity
  - Polytropic EOS: $p = K\rho^{1+1/N}$
  - Differential rotation
- Perturbed equations
  - Adiabatic perturbations
  - Expansion:

$$\delta f(r,\theta,\varphi,t) = \sum_m \exp[-i(\sigma t - m\varphi)] f_m(r,\theta)$$

Equilibrium model

- Basic eq. $$\frac{1}{\rho}\nabla p = -\nabla\phi + r\sin\theta\,\Omega^2\mathbf{e}_R$$
- Poisson eq. $$\Delta\phi = 4\pi G\rho$$
- EOS $$p = K\rho^{1+1/N}$$
- Rotation law $$\Omega = \frac{\Omega_c A^2}{R^2 + A^2} \quad \text{or} \quad \Omega = \frac{\Omega_c A}{\sqrt{R^2 + A^2}}$$

Perturbed Equations

- Perturbed eq. of continuity

$$\frac{1}{\rho_0}\frac{\partial \delta\rho}{\partial t}+\frac{\partial \delta v}{\partial r}+\frac{2}{r}\delta u+\frac{1}{\rho_0}\frac{\partial \rho_0}{\partial r}\delta u+\frac{1}{r}\frac{\partial \delta v}{\partial \theta}+\frac{1}{r\rho_0}\frac{\partial \rho_0}{\partial \theta}\delta v$$
$$+\frac{1}{r}\cot\theta\delta v+\Omega\frac{1}{\rho_0}\frac{\partial \delta\rho_0}{\partial \varphi}+\frac{1}{r\sin\theta}\frac{\partial \delta w}{\partial \varphi}=0$$

- Perturbed eqs. of motions

$$\frac{\partial \delta u}{\partial t}+K\left(1+\frac{1}{N}\right)\frac{\partial}{\partial r}\left(\rho_0^{\frac{1}{N-1}}\delta\rho\right)+\frac{\partial \delta\phi}{\partial r}+\Omega\frac{\partial \delta u}{\partial \varphi}-2\Omega\sin\theta\delta w=0$$

$$\frac{\partial \delta v}{\partial t}+K\left(1+\frac{1}{N}\right)\frac{1}{r}\frac{\partial}{\partial \theta}\left(\rho_0^{\frac{1}{N-1}}\delta\rho\right)+\frac{1}{r}\frac{\partial \delta\phi}{\partial \theta}+\Omega\frac{\partial \delta v}{\partial \varphi}-2\Omega\cos\theta\delta w=0$$

$$\frac{\partial \delta w}{\partial t}+K\left(1+\frac{1}{N}\right)\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}\left(\rho_0^{\frac{1}{N-1}}\delta\rho\right)+\frac{1}{r\sin\theta}\frac{\partial \delta\phi}{\partial \varphi}$$
$$+2\Omega\sin\theta\delta u+2\Omega\cos\theta\delta v+r\sin\theta\delta u\frac{\partial \Omega}{\partial r}+\sin\theta\delta v\frac{\partial \Omega}{\partial \theta}+\Omega\frac{\partial \delta w}{\partial \varphi}=0$$

- Boudary condition

$$\frac{\partial \delta\rho_0}{\partial r}\delta u+\frac{1}{r}\frac{\partial \rho_0}{\partial \theta}\delta v+\left(m\Omega-\sigma\right)\delta\rho=0$$

Linear Stability Analysis (continued)

- Under these conditions
  - Construct the system of perturbed fluid eqs.
  - Solve the system iteratively
  - Obtain the eigenvalue of the mode
- The eigenvalue corresponds to the eigenfrequency of the obtained mode

Dynamical Instability

Realistic Astrophysical Objects

- In simple models, $T/|W| = 0.27$ is the limit of stability
- Recent non-linear simulations
  - Examples of instability occur even

    if $T/|W| < 0.27$, when we consider strong dif. rot.

    eg. Pickett *et al.* 1996, Centrella *et al.* 2001,

    Liu & Lindblom 2001, Shibata *et al.* 2002,etc.

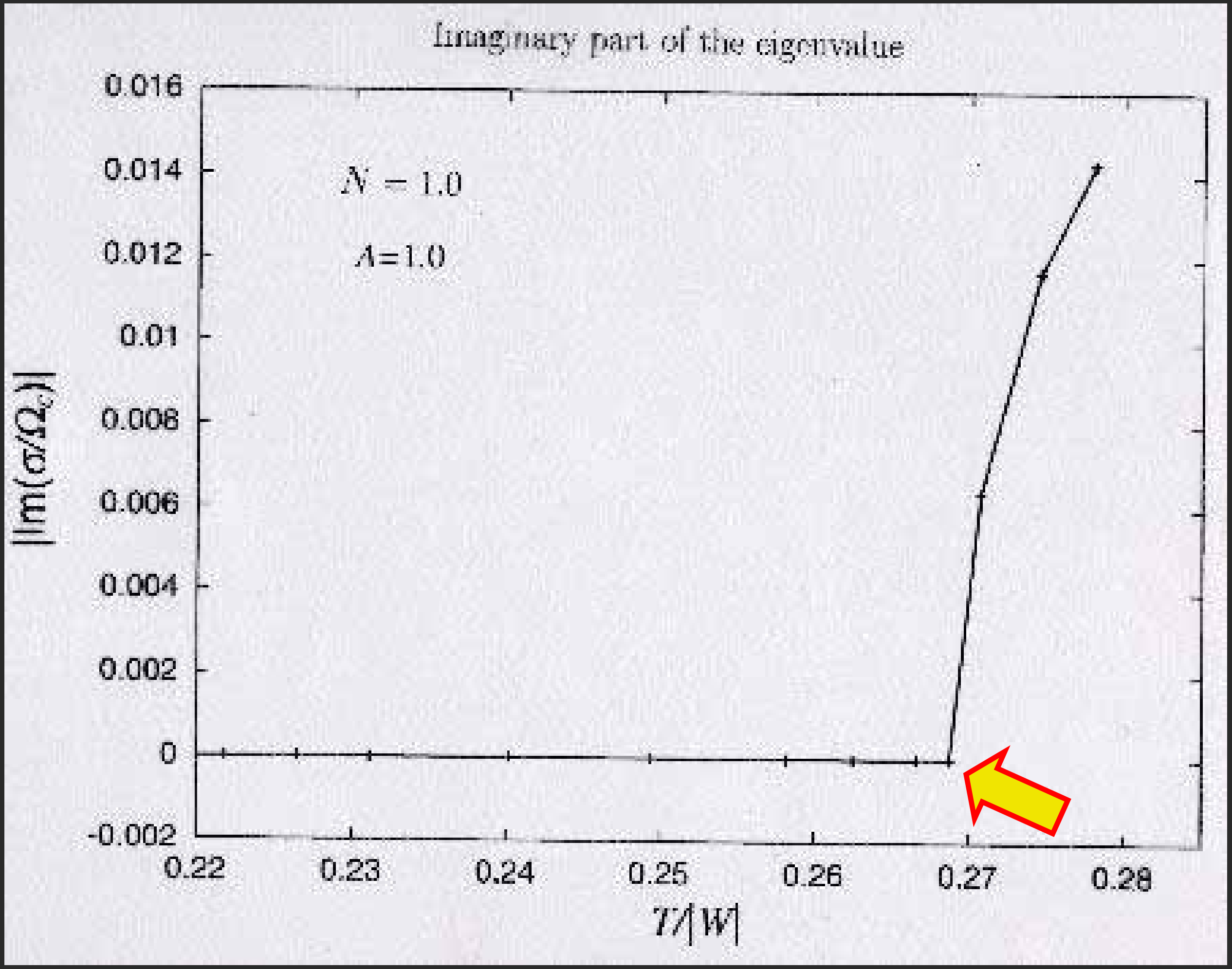
⟹ Linear Stability Analysis

Numerical Computations of Dynamical Instabilities

- Obtain the eigenvalues of modes numerically by the linear analysis method
- When the rot. rate gets higher, the first point where the eigenvalue has a finite imaginary part corresponds to the critical limit of dynamical instability

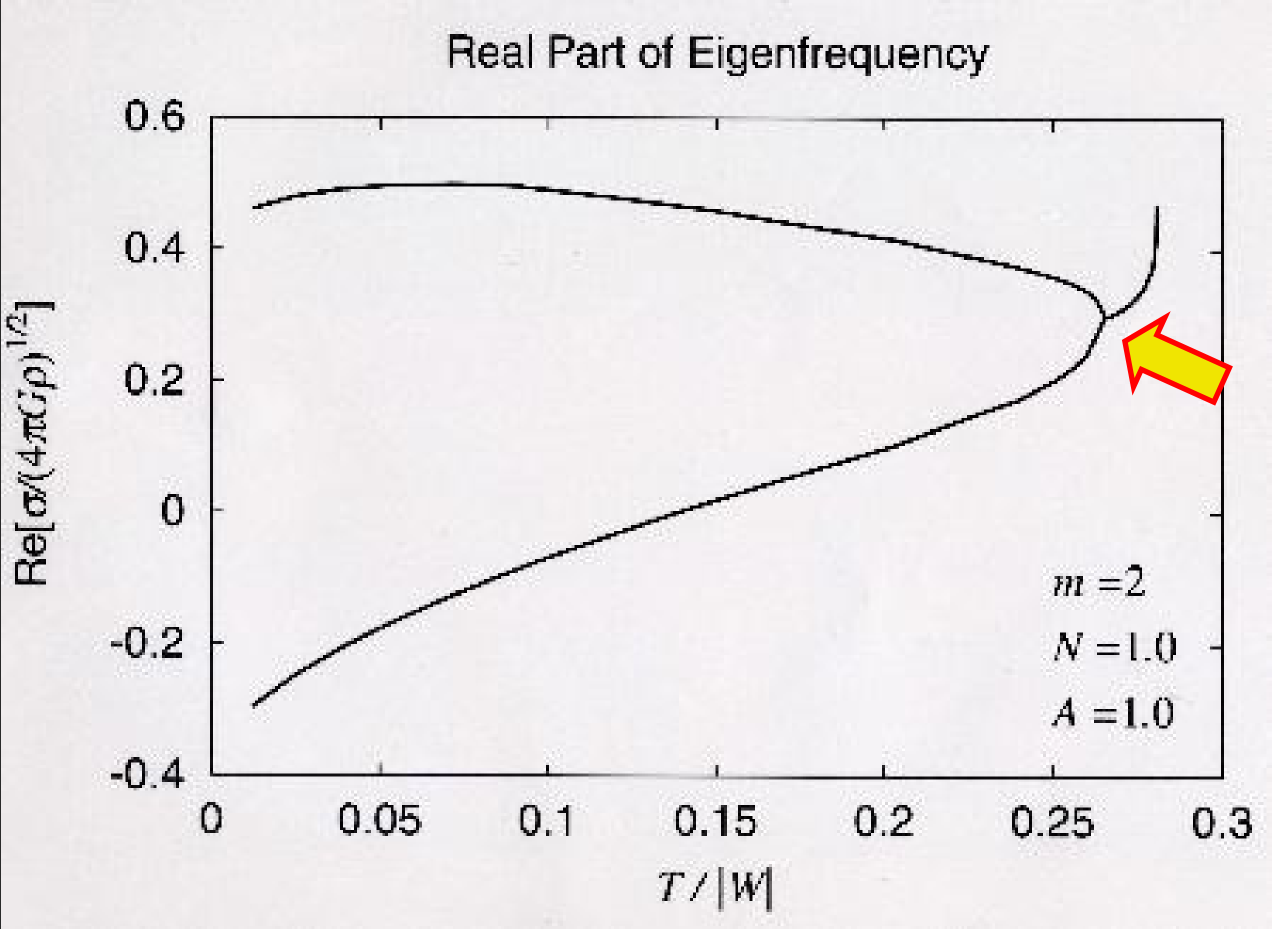
$$\delta f(r,\theta,\varphi,t) = \sum_m \exp[-i(\sigma t - m\varphi)] f_m(r,\theta)$$

Eigenvalue (imaginary part)



- The dynamical instability sets in at the point where the eigenvalue has complex part

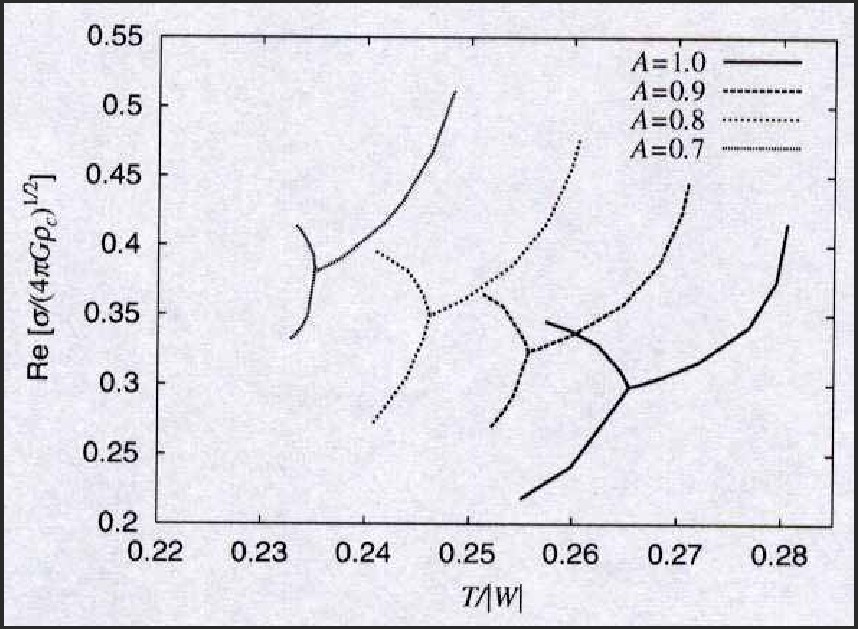
Eigenvalue (real part)

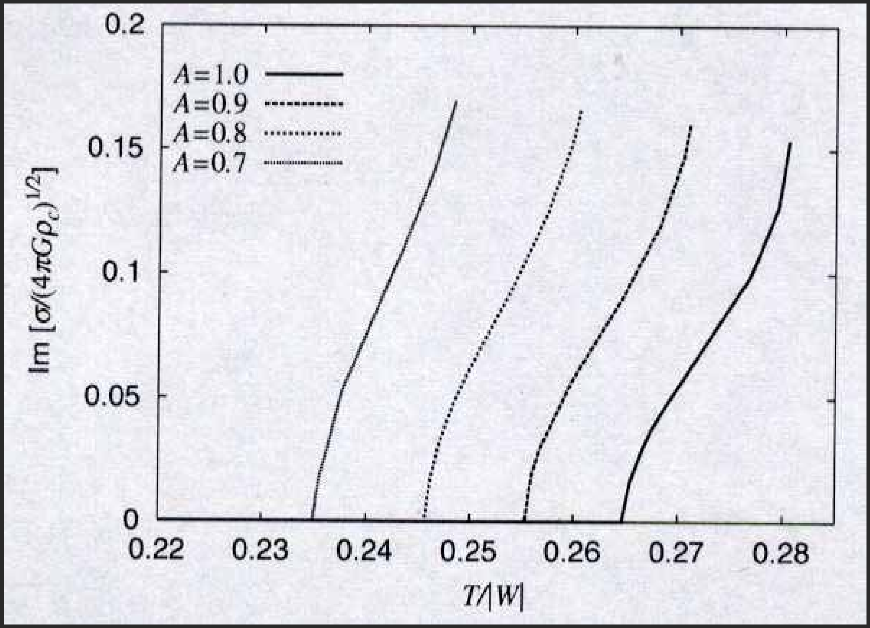


- The dynamical instability sets in at the point where two modes merge

Numerical Result

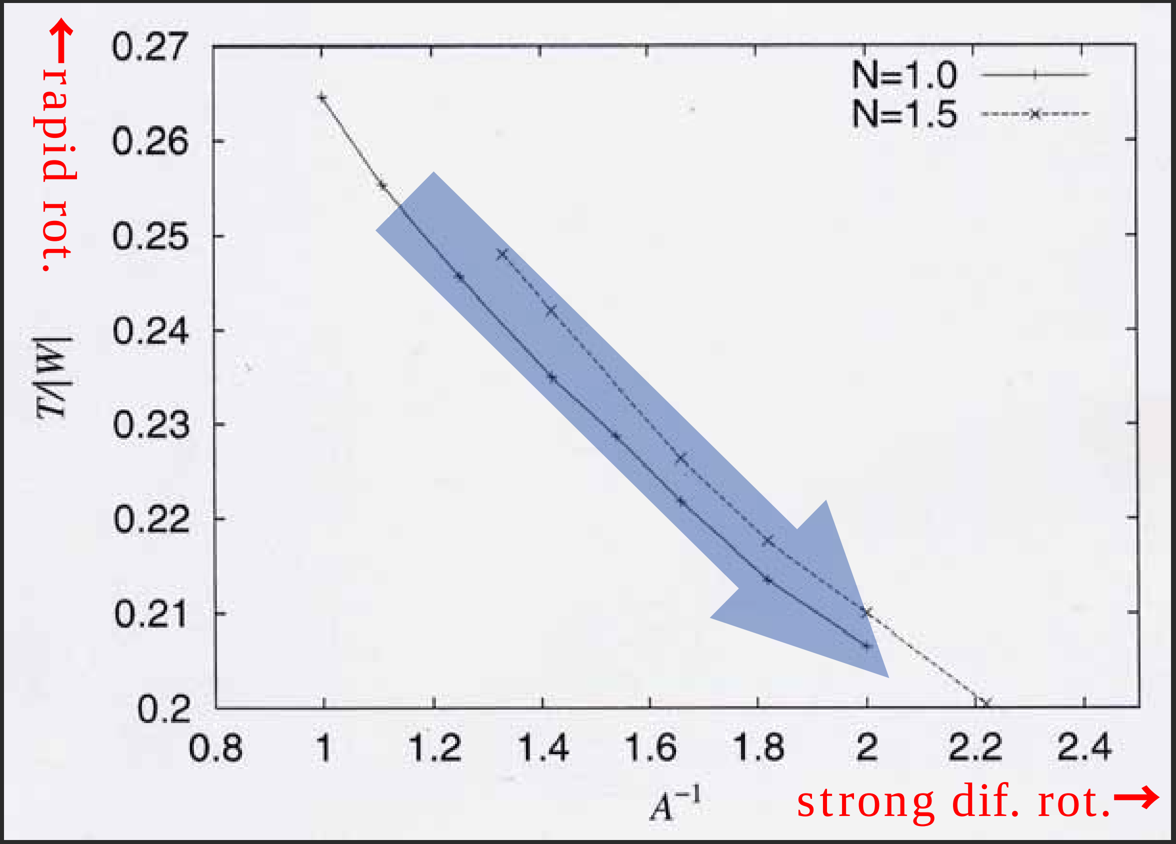
(Karino & Eriguchi 2003)





- Shifts of critical limits of dynamical instability as the degrees of differential rotation

Numerical Result
(Karino & Eriguchi 2003)



- Critical limits of dynamical instability, $(T/|W|)_{crit}$, as the functions of degrees of differential rotation

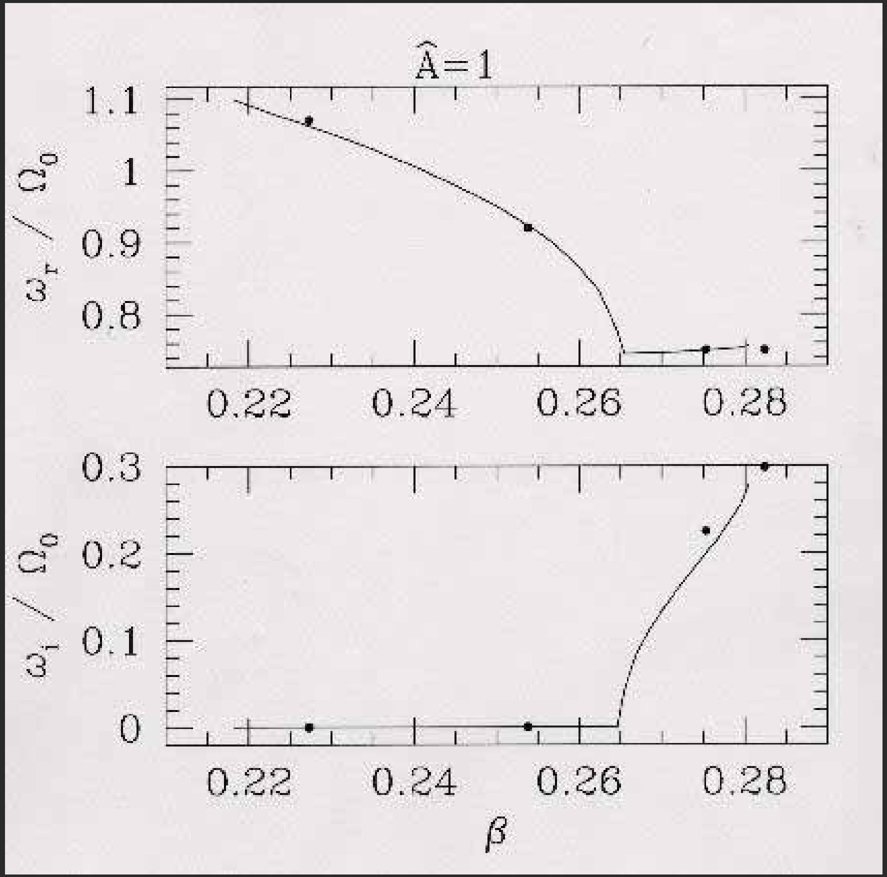
Numerical Results (continued)

- The critical limit of instability tends to decrease when we consider strong differential rotations
- This tendency depends only weakly on the stellar EOS

In differentially rotating stars, dynamical instabilities may occur more easily than ordinary cases

Result of non-linear simulation

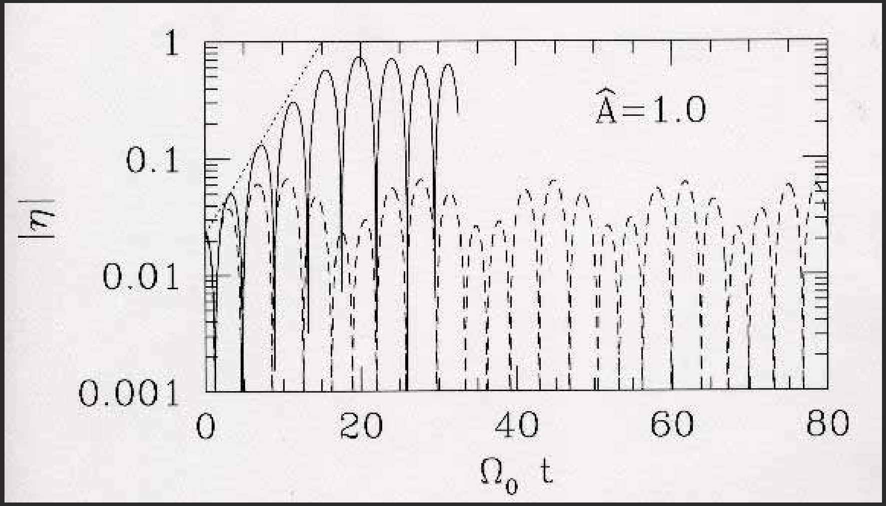
- The results obtained by linear method match with results of non-linear simulations

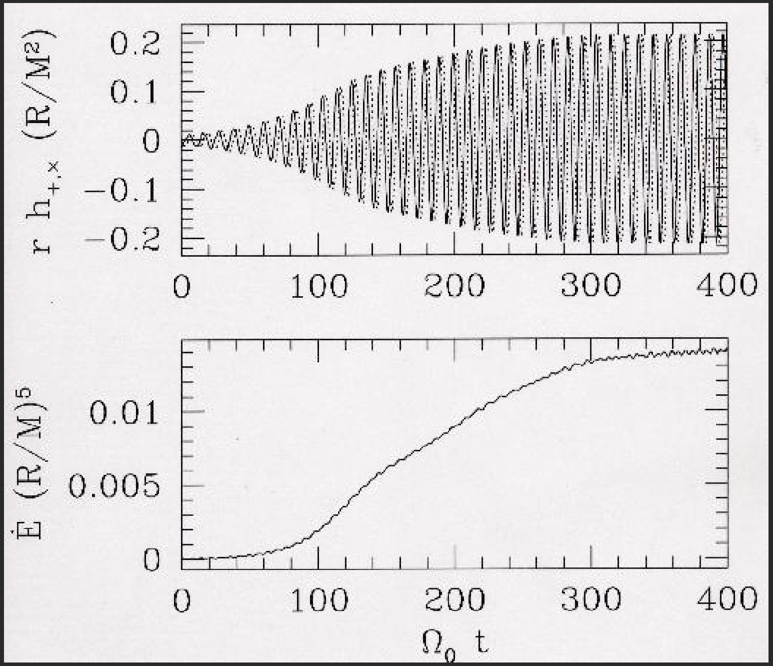


Shibata, Karino & Eriguchi (2002)

GW

- Deformation of the star by the non-linear growth of bar-mode





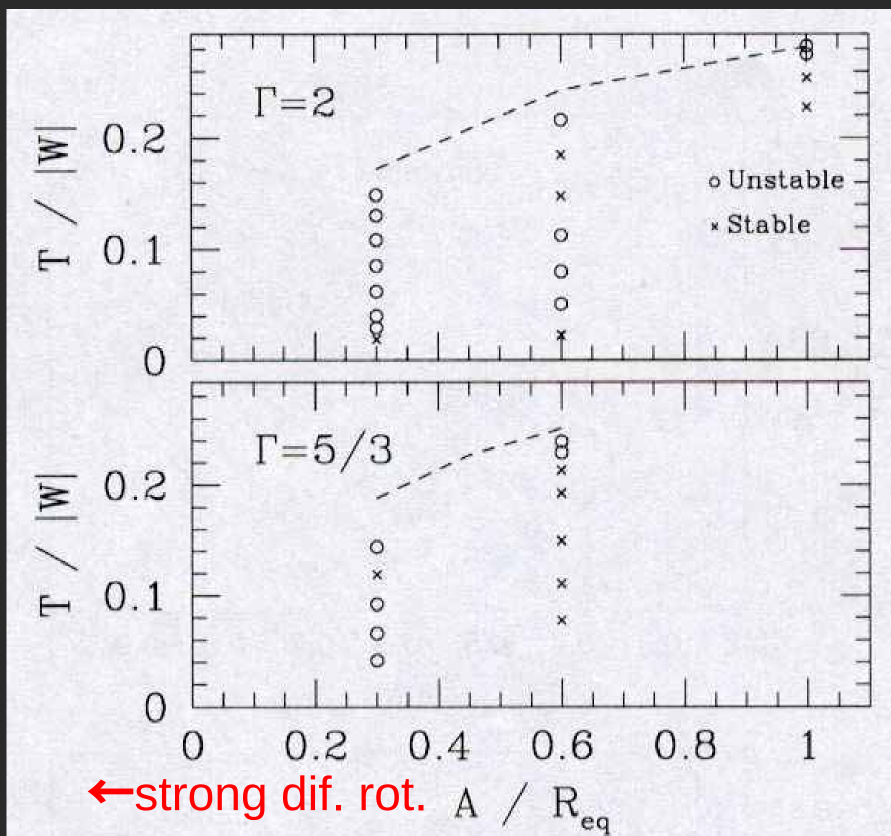
- The wave form is quasi-periodic
  - Effective amplitude

Summary (of this talk)

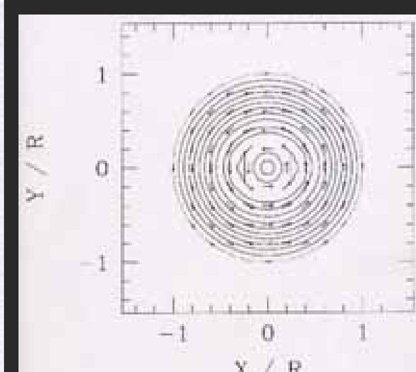
- Maclaurin Spheroid
    - At $T/|W|>0.14$, bar-mode will be unstable secularly
    - At $T/|W|>0.27$, bar-mode will be unstable dynamically
  - Differential Rotation?
- Linear stability analysis
  - Obtain the eigenvalues of modes
- Numerical results
  - Critical limits of dynamical instabilities depend on the effects of differential rotations

Unknown Instability?

- Recently new (?) instability has been found in slowly ($T/|W| \sim 0.1$) and differentially rotating stellar models

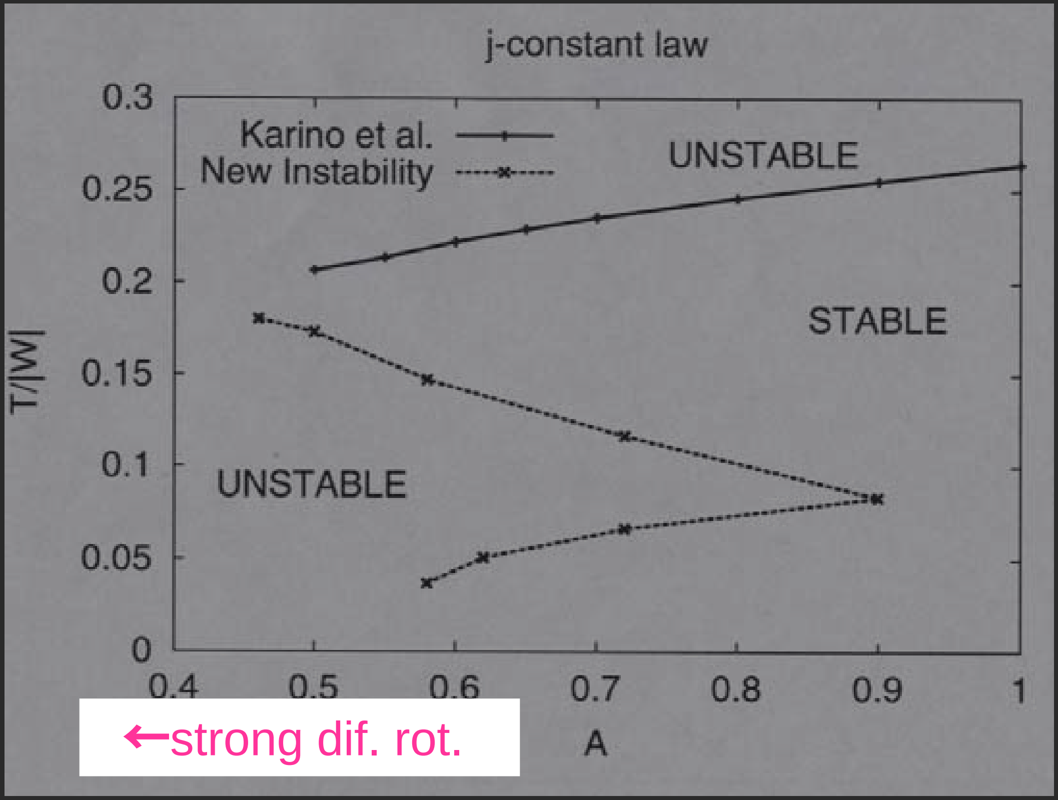


Shibata, Karino & Eriguchi 2003



Unknown Instability?

- Such a new (?) instability can be found by linear method
- The parameter space can be divided into stable and unstable regions



Feature of the Instability?

- This instability appears at $T/|W| \sim 0.05$, and disappears at $T/|W| \sim 0.2$
- The growth rate is small
- Stars with stiff EOS are more unstable