*Do Neutron Star Gravitational Waves Carry Superfluid Imprints?**

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Some Essential Facts:

- Can estimate that there are about 2 x 10^8 neutron stars in the galaxy.

- Fermi-temperatures for nucleons is around 10^{12} K.

- Neutron stars probably cool to less than 10^9 K soon after their births.

- Nuclear physics determination of nucleon superfluid transition temperature is also on the order of 10^9 K.

- The overwhelming majority of neutron stars should contain two or more superfluids/superconductors in their cores.

- Glitches are explainable in terms of neutron superfluidity and vortex dynamics.

Possible "Neutron" Star Interiors *

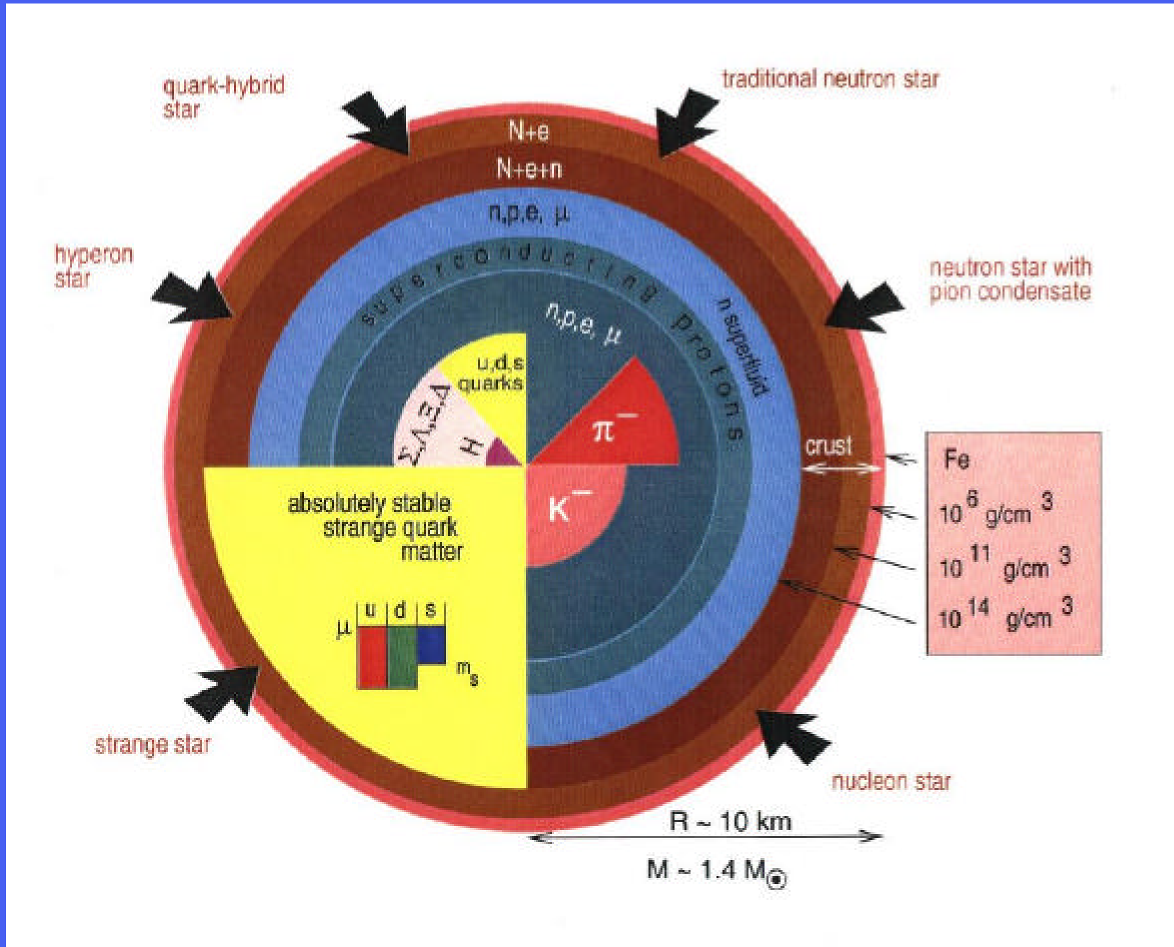
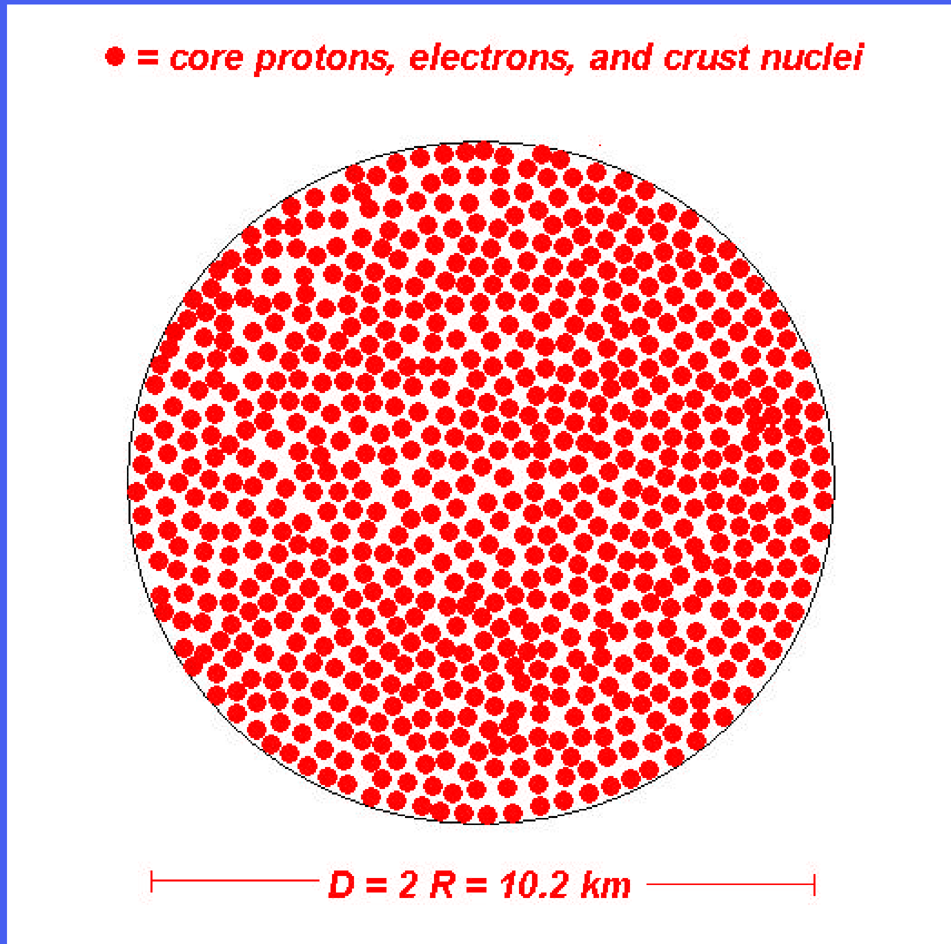


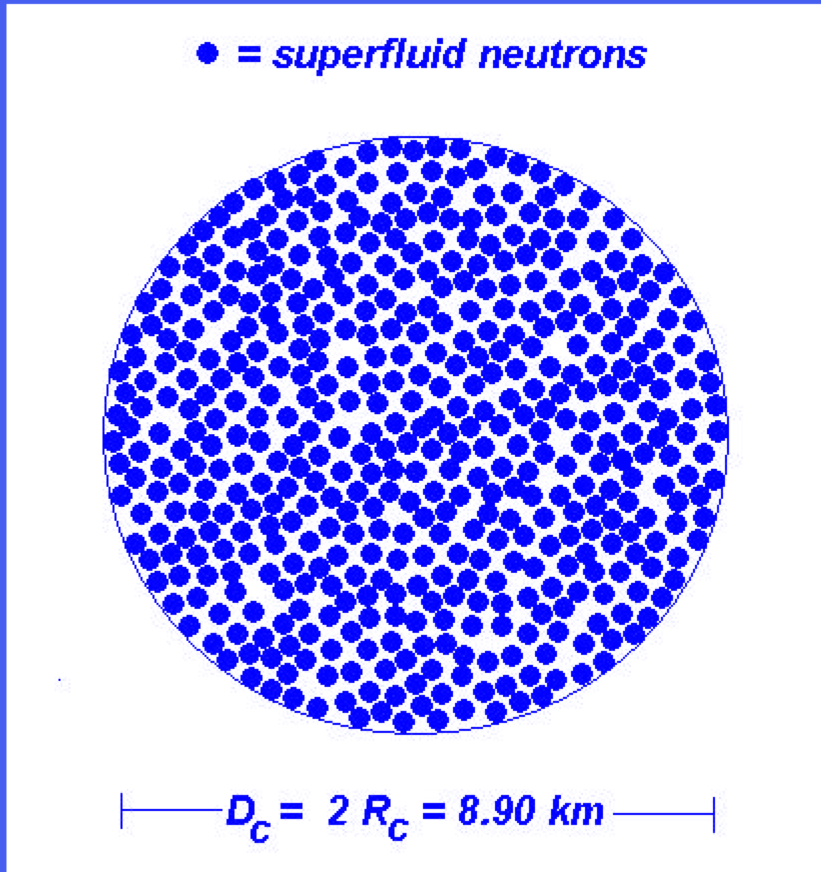
*Figure by F. Weber.

The Simplified Model: Begin With a “Proton” Fluid

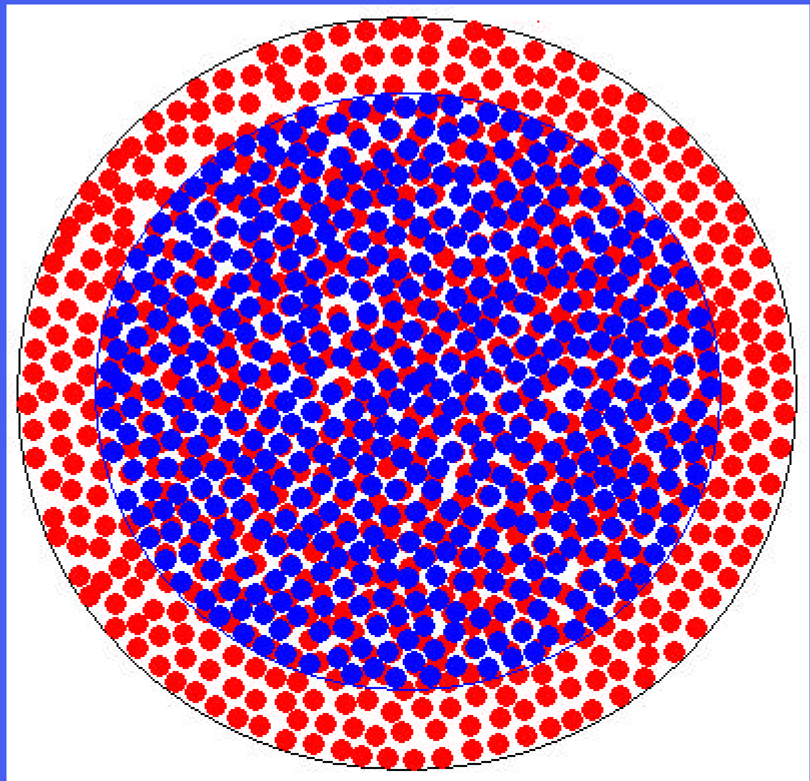
Electromagnetic interaction has the electrons and crust nuclei “locked” to the core protons and electrons (muons, etc) as a single fluid.



Add Some Superfluid Neutrons in the Core



Obtain The Two-fluid Model



The envelope/core interface occurs at about 10^{11} gm/cm^3, with a central proton fraction of about 10%.

The Two-fluid Formalism and Field Equations

- Fundamental Fluid Variables:

  Neutron current: n^μ Proton current: p^μ

- "Master" Function (equation of state):

$$\Lambda = \Lambda(n^2, p^2, x^2) \qquad n^2 = -\, n_\mu n^\mu \qquad p^2 = -\, p_\mu p^\mu \qquad x^2 = -\, n_\mu p^\mu$$

- Field Equations (Carter [1989], Carter and Langlois [1995,1998], Langlois et al [1998]):

Neutrons	***Protons***	
$\nabla_\mu n^\mu = 0$	$\nabla_\mu p^\mu = 0$	$A = -\partial\Lambda/\partial x^2$
$n^\mu \nabla_{[\mu}\, \mu_{\nu]} = 0$	$p^\mu \nabla_{[\mu}\, \chi_{\nu]} = 0$	$B = -2\partial\Lambda/\partial n^2$
$\mu_\nu = B\, n_\nu + A\, p_\nu$	$\chi_\nu = C\, p_\nu + A\, n_\nu$	$C = -2\partial\Lambda/\partial p^2$

The Entrainment Effect

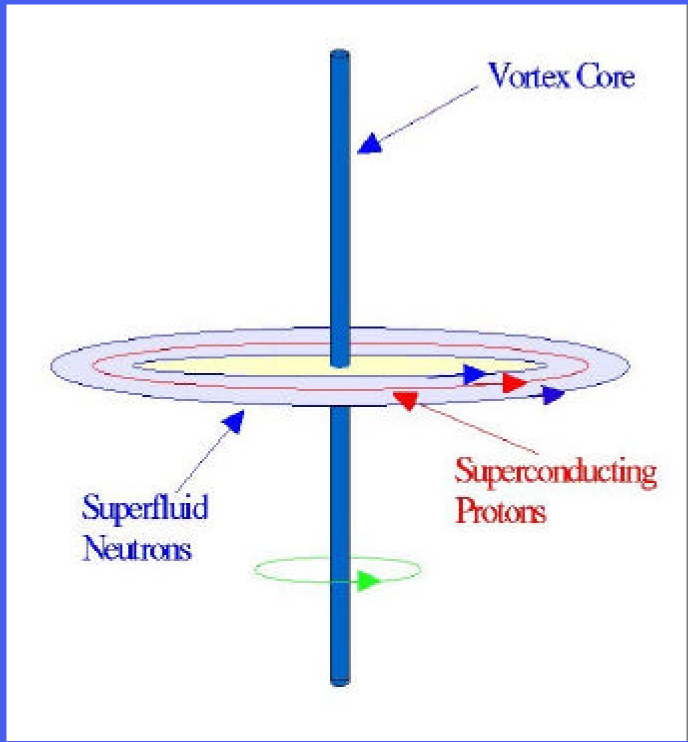
• Although superfluid and superconducting, the nucleons still feel the strong force.

• Bare neutrons and protons are “dressed” by a polarization cloud of both types of nucleons.

• Neutron and proton momenta are each linear combinations of both nucleon velocities.

• When one type of nucleon fluid starts to flow, part of the other fluid is also “pulled” along.

• At the heart of mutual friction: protons cause magnetic field lines to be attached to vortices off of which the electrons scatter dissipatively.



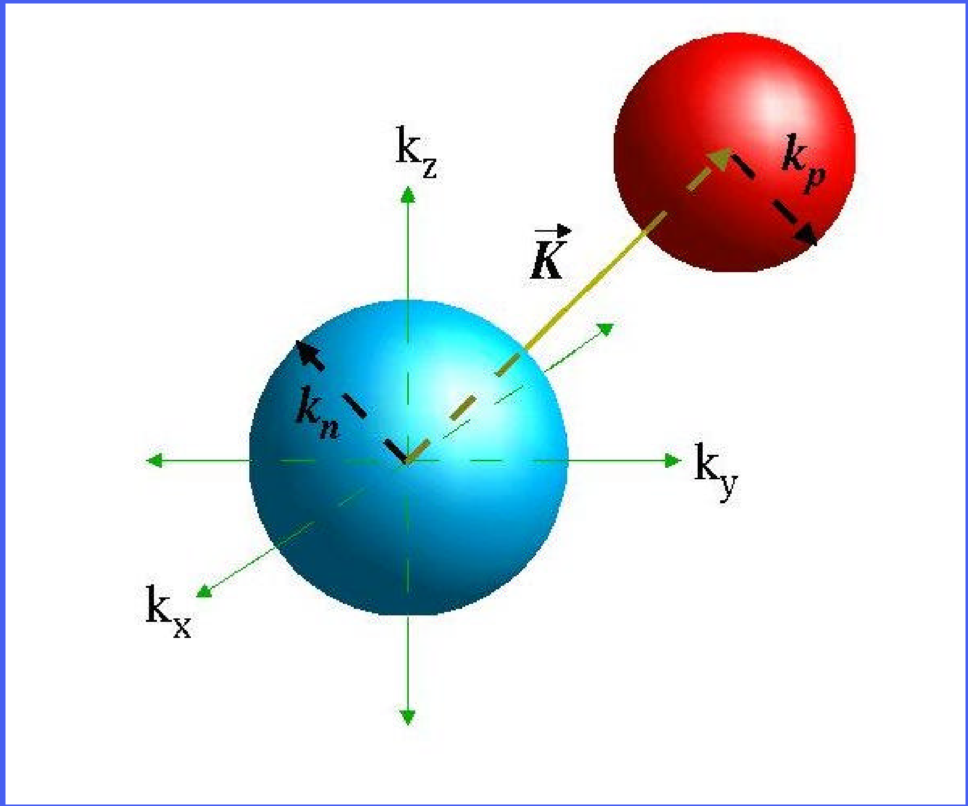
Incorporating the Entrainment Effect

- The averaging procedure is performed in "momentum" space.

- The neutrons and protons fill Fermi spheres of radii k_n and k_p, respectively.

- The proton Fermi sphere is displaced from the origin by $\boldsymbol{K}$.

- k_n, k_p, and K are, in general, functions of n^2, p^2, and x^2.



How to Determine the Master Function

The stress-energy tensor for two-fluid system takes the form:

$$T^{\mu}{}_{\nu} = P\,\delta^{\mu}{}_{\nu} + n^{\mu}\,\mu_{\nu} + p^{\mu}\,\chi_{\nu}$$

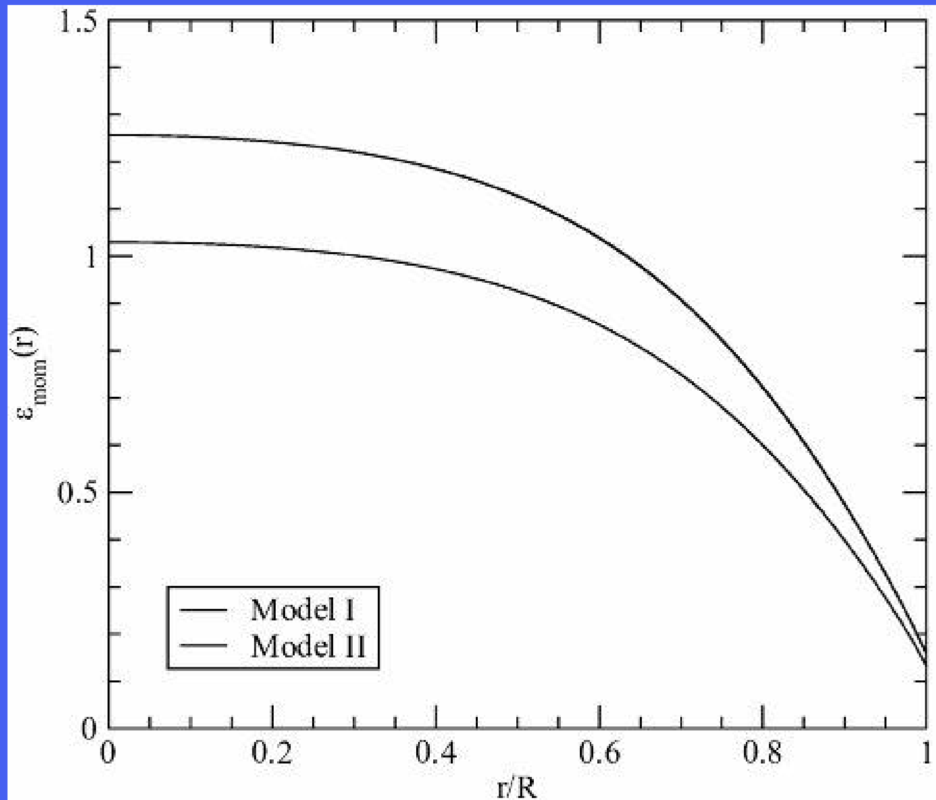
$$P = \Lambda - n^{\mu}\,\mu_{\mu} - p^{\mu}\,\chi_{\mu}$$

Once given the local, average stress-energy $\langle T_{\mu\nu}\rangle$, then the master function is obtained from

$$\Lambda = -\tfrac{1}{2}\langle T\rangle + 3/2\,(p^2\,n^{\mu}\,n^{\nu} + n^2\,p^{\mu}\,p^{\nu} - x^2\,[n^{\mu}\,p^{\nu} + n^{\nu}\,p^{\mu}])\,\langle T_{\mu\nu}\rangle\,/\,(x^4 - n^2\,p^2)$$

Relativistic Mean Field Results for Entrainment

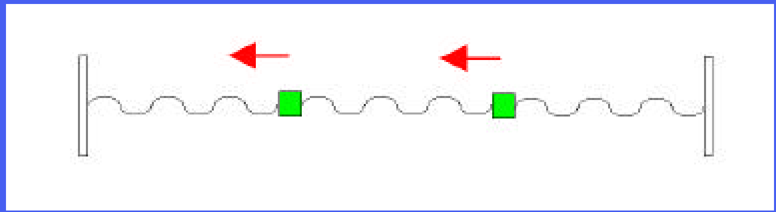
ε_{mom} = - m^2 A / (B C – A^2)

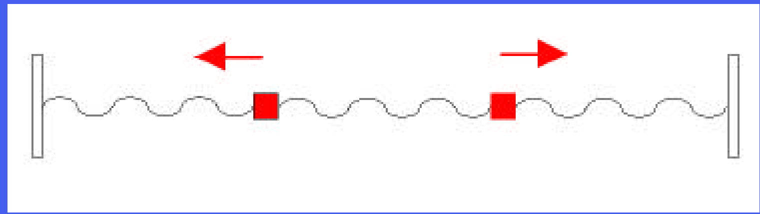


Graph taken from GLC and Joynt (2003). Newtonian calculations show $0.04 < \varepsilon_{mom} < 0.2$.

The Mode Spectrum: Non-rotating Stars

• Two sets of acoustic-like modes (Epstein [1998], Mendell [1991], Lee [1995], etc):





"Ordinary Modes"
~ msec timescales

"Superfluid Modes"
~ sensitive to entrainment

• No g-modes (Lee [1995], Andersson and GLC [2001]).

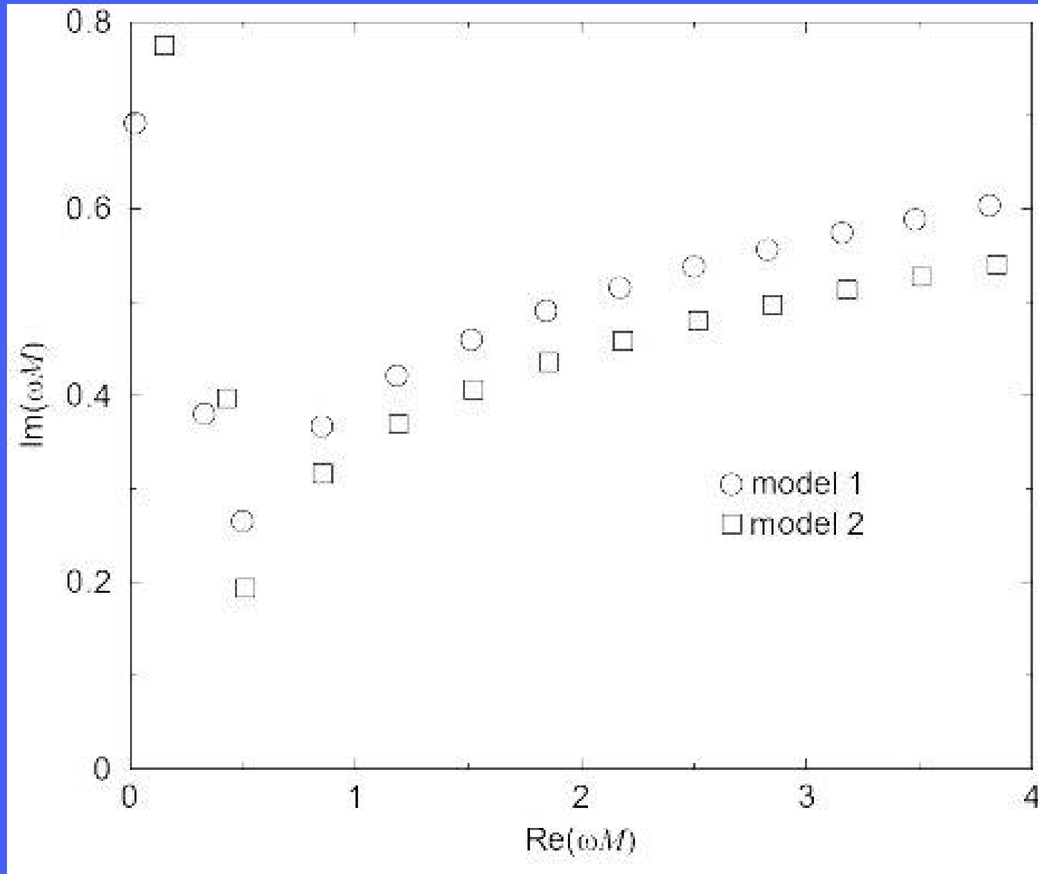
• One set of w-modes (GLC et al [1999]).

• Two sets of spheroidal (polar), and two sets of toroidal (axial) zero-frequency modes (Andersson and GLC [2001], GLC [2002]); degeneracy is broken when the background rotates (Yoshida and Lee [2003] and Prix et al [2003]).

The W-Modes

Model 1: Ordinary fluid.

Model 2: Superfluid

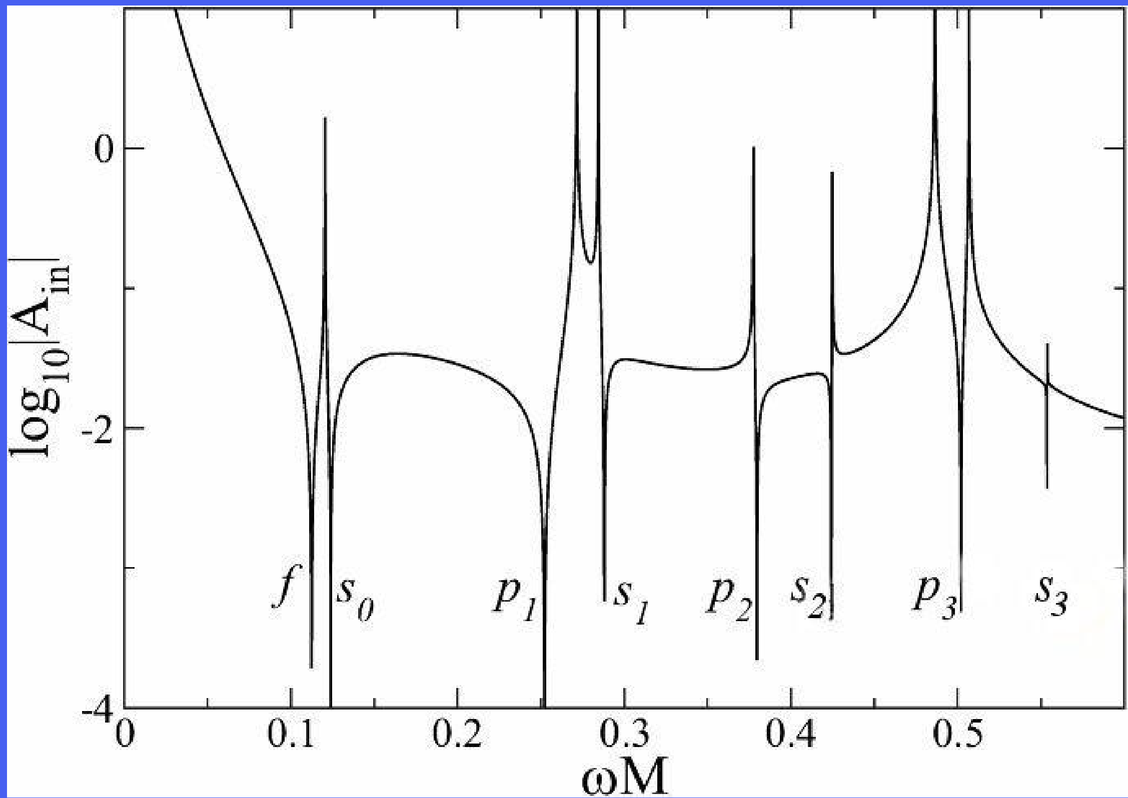


Main point: No doubling of the w-modes.

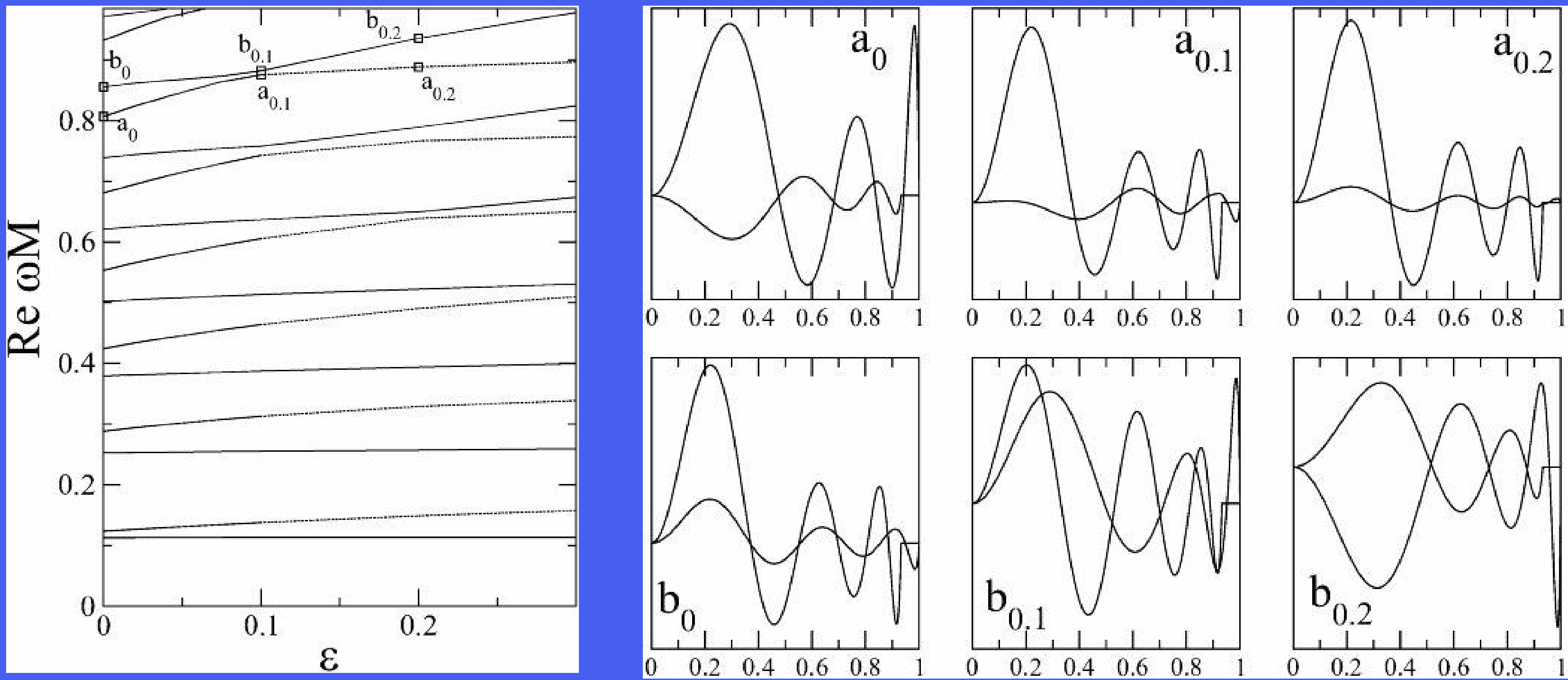
Quasinormal Mode Spectrum

• Plot incoming gravitational wave amplitude vs frequency. Find a splitting of the ordinary fluid modes (GLC et al [1999]; Andersson et al [2002]).

• The s_i modes do lead to gravitational waves (Andersson et al [2002]).



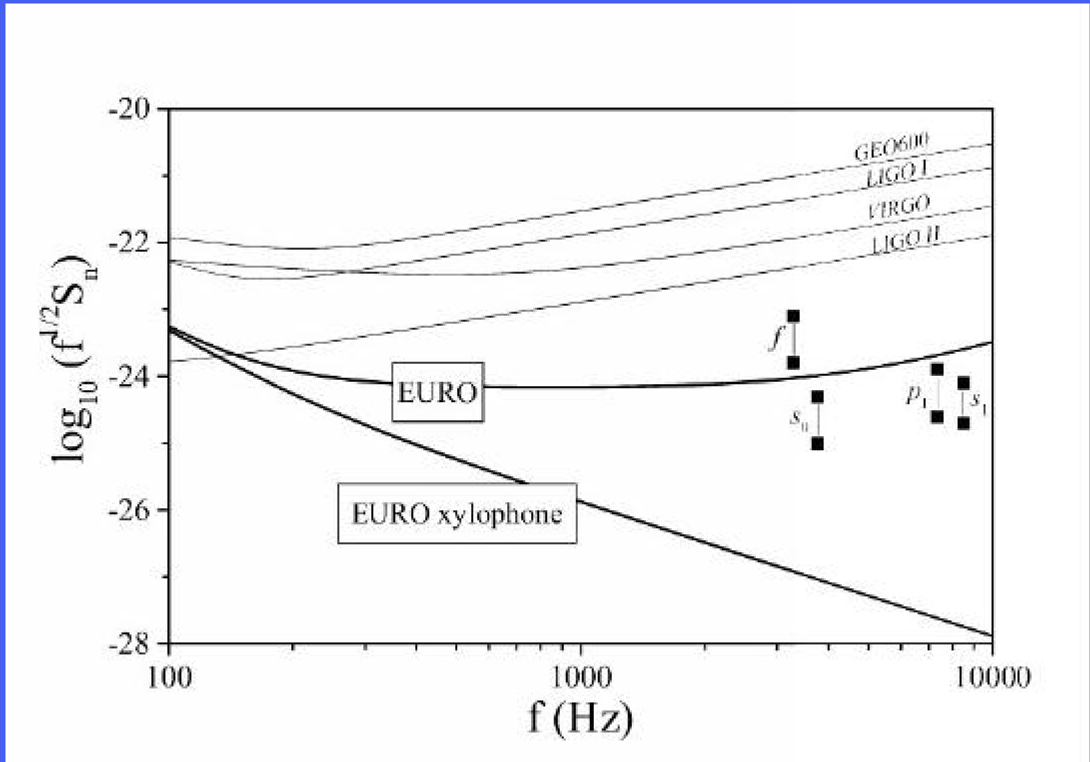
Frequency vs. Entrainment Parameter: Avoided Crossings



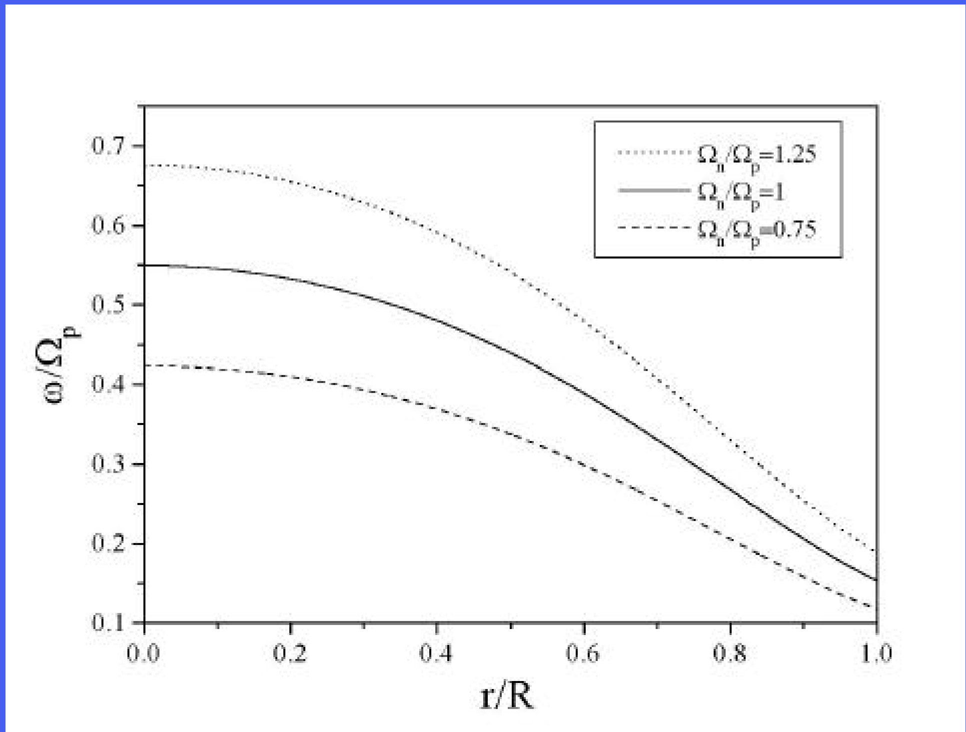
(From Andersson et al [2002]. Also seen for inertial modes by Lee and Yoshida [2003] and Prix et al [2003].)

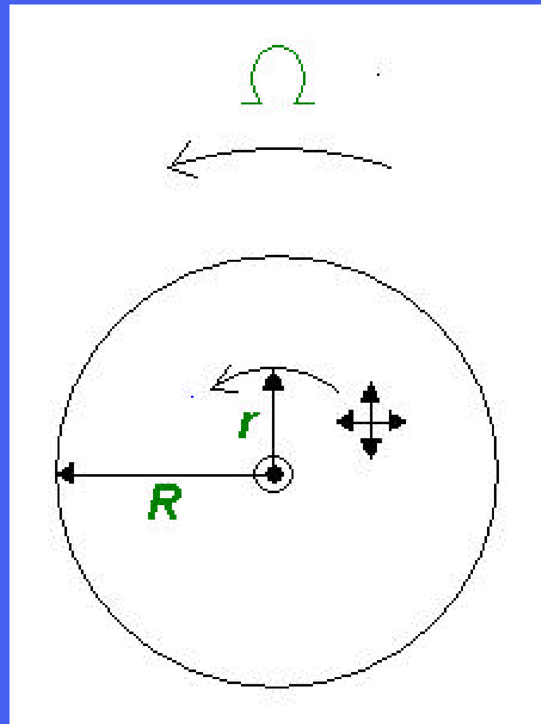
Gravitational Wave Detectibility

Plot of the "noise-as-strain" versus frequency (Andersson and GLC [2001]; Andersson et al [2002]) for gravitational waves produced in (upper) Vela- and (lower) Crab-like glitches.



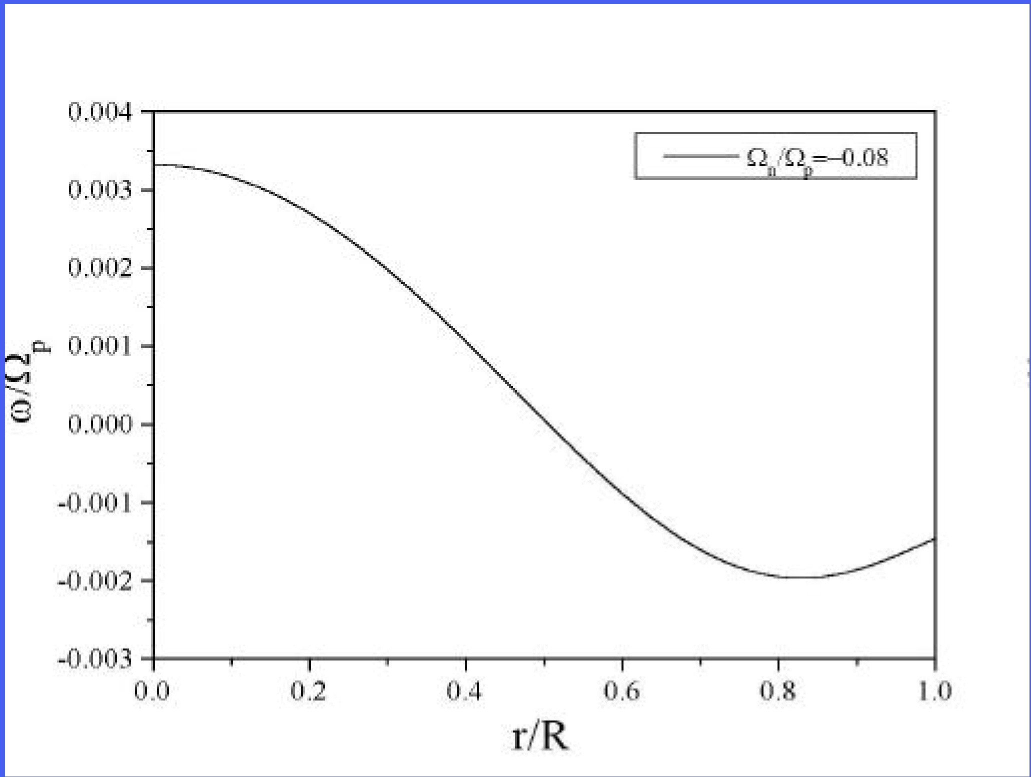
Frame-Dragging wwith a Relative Rotation

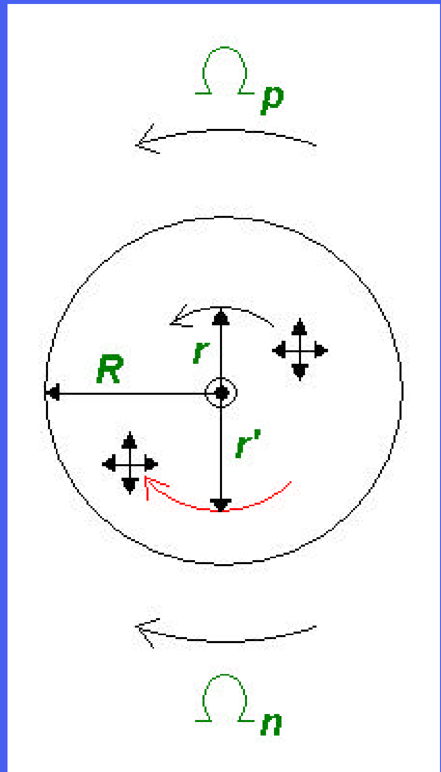




Neutron and proton rotation speed = Ω_n , Ω_p

Pushing the Formalism: "Non-Physical" Rotations



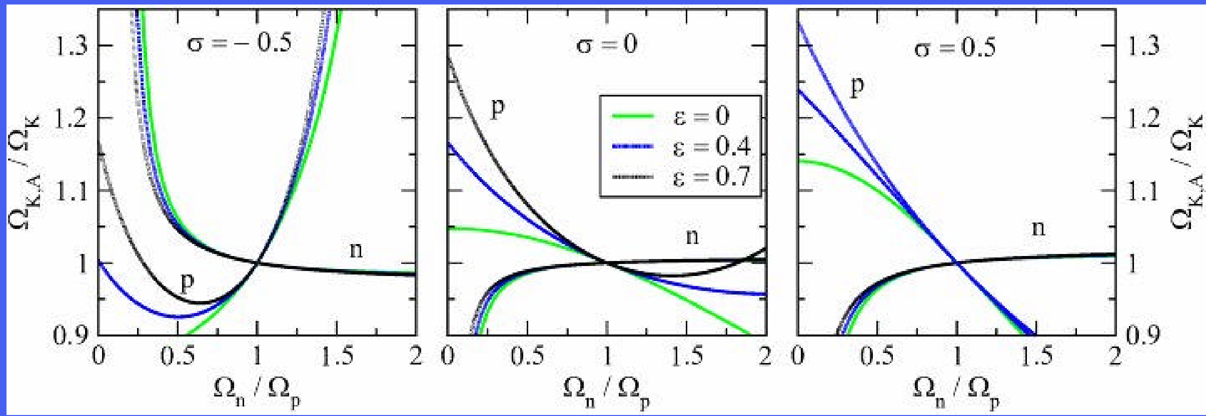


Near the center the proton angular speed dominates, but near the surface the mass of the neutrons dominates.

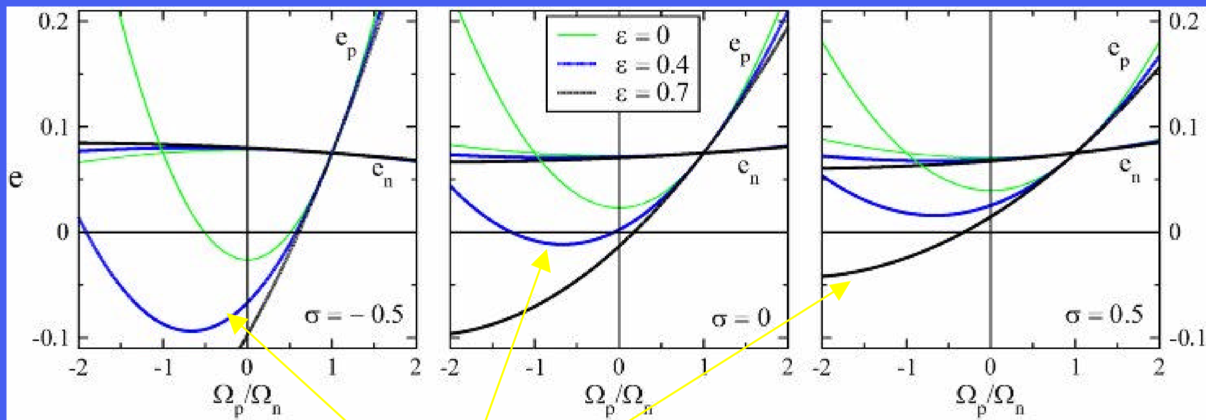
Symmetry Energy Effects

Symmetry energy (here σ) is an additional term in realistic equations of state that vanishes when there are equal numbers of neutrons and protons.

Kepler limit vs relative rotation rate (Newtonian, slow-rotationa).



Ellipticity vs relative rotation rate (Newtonian, slow-rotation).

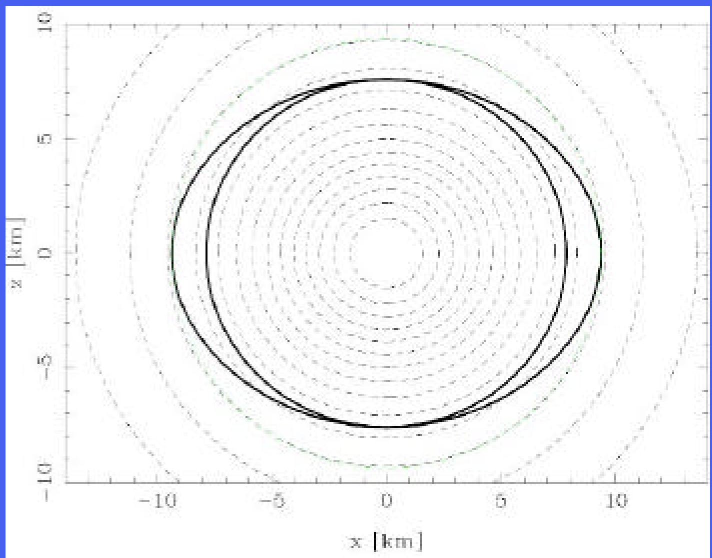


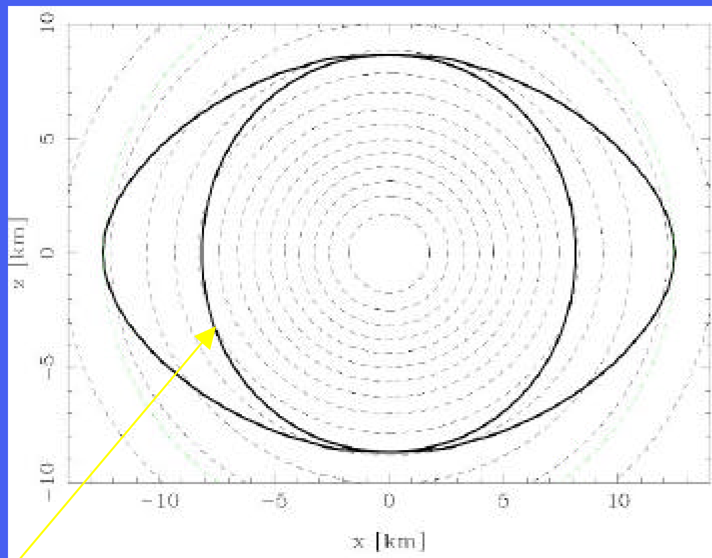
Prolate

Rapidly Rotating Configurations

• Rapidly rotating configurations obtained using LORENE (developed by the Meudon Numerical Relativity group).

• Shown are neutron and proton surfaces through the rotation axis, for different values of the symmetry energy (i.e. zero on the left, and non-zero on the right).





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The Two-Stream Instability

Oh, and on the way to the coliseum … the two-stream instability:

• When two fluids flow through or past each other:

➢ A mode can appear leftward moving with respect to one fluid, but rightward with respect to the other fluid.

➢ Hence, the effective energy of the perturbation can be negative, and thus unbounded from below.

• Known to operate in plasmas (Farley-Bunemann instability); the Kelvin-Helmholtz instability can also be understood this way.

• Shown to be a generic feature of the two-component superfluid (Andersson et al [2003] in PRL); mode calculations reveal unstable modes (Prix et al [2003], now on the archive).

Some Final Questions and Remarks

• What about the loss of g-modes for non-rotating backgrounds?

• Binary evolution before superfluidity gets destroyed? What's the minimal dynamics to use for maximal insight?

• What potential role for the two-stream instability and gravitational wave emission?

• Direct measurement of gravitational waves from superfluid neutron stars could provide a unique probe of the supra-nuclear equation of state (e.g. entrainment and symmetry energy).