NEUTRINO TRANSPORT EFFECTS ON PROTO–NEUTRON STAR OSCILLATIONS AND ON THEIR GW EMISSION

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L.G., J.A. Pons, G. Miniutti, *in preparation*

After a SN explosion the stellar remnant,
which is a proto–neutron–star (PNS),
radiates its binding energy ($\sim 0.1\, M_\odot$)
through **neutrino emission**
in some tens of seconds.
A small part of this energy ($\sim 10^{-6} - 10^{-8}\, M_\odot$)
is supposed to be radiated in **GW radiation**,
partially through QNM oscillations.

It whould be very interesting to understand
the characteristics of such GW radiation
and of the QNMs of a PNS.

This problem has been addressed first in

V.Ferrari, G.Miniutti, J.A.Pons,
astro-ph/0210581, **MNRAS** 342 623, (2003)

where the star has been assumed non–rotating,
and the dissipative effects have been neglected.

The star rotation has then been
taken into account in

V.Ferrari, L.Gualtieri, G.Miniutti, J.A.Pons, A.Stavridis,
submitted to **MNRAS**

Today I will talk of our tentative to take into
account $\boxed{\textbf{dissipative effects}}$ in this picture.

Let me briefly recall
the different stages
of the evolution of a PNS:

- **Few tenths of second** after the SN explosion, neutrino diffusion deleptonizes the star, and generates heat.

- After **15–20 seconds**, the PNS has become lepton–poor but it is still hot.
Termally produced neutrinos dominate the emission, cooling the star.

- After $\sim$**50 seconds** the star is much colder and transparent to neutrinos: it has become a neutron star (NS).

While in a **cold NS** the main dissipative effect is (bulk or shear) **viscosity**, in a **hot, newly born PNS** the main dissipative effects are
$\boxed{\textbf{heat and lepton transfer}}$
due to neutrino diffusion.

We have considered an **evolutive model** of *non–rotating* PNS studied in

Pons *et al.*, **ApJ** 513, 780 (1999); **ApJ** 553, 382 (2000).

In this model the structure of the star and the diffusion coefficients are computed at different stages of its evolution from $t_{ev} = 0.2s$ to $t_{ev} = 50s$.

Using this evolutive model as a background,
we want to study
general relativistic perturbations of the star,
by taking into account **dissipative effects**
(but, at the moment, neglecting rotation).

In the framework commonly used
to study dissipative effects on NS oscillations
(see for example Cutler C., Lindblom L., **ApJ** 314, 234 (1987))
one follows the following steps:

- Consider a **perfect fluid** spherically symmetric background

$$
\begin{aligned}
ds^2 &= -e^{\nu}dt^2 + e^{\lambda}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \\
T_{\mu\nu} &= (\rho + p)u_\mu u_\nu + p g_{\mu\nu}
\end{aligned}
$$

  and solve the Einstein equations.

- Consider **perfect fluid** non–radial *perturbations* of that background

$$
\begin{aligned}
ds^2 &= -e^{\nu}(1 + r^l H_0 Y_{lm} e^{i\omega t})dt^2 \\
&\quad -2i\omega r^{l+1} H_1 Y_{lm} e^{i\omega t} dt dr \\
&\quad +e^{\lambda}(1 - r^l H_0 Y_{lm} e^{i\omega t})dr^2 \\
&\quad -r^2(1 - r^l K Y_{lm} e^{i\omega t})(d\theta^2 + \sin^2\theta d\phi^2) \\
\delta u_\mu &= \left(W e^{\lambda/2} \frac{r_{,\mu}}{r} Y_{lm} - V Y_{lm,\mu} \right) i\omega r^l e^{i\omega t}
\end{aligned}
$$

  and solve the perturbed Einstein equations (which in this gauge are the Lindblom–Detweiler (LD) equations).

- The dissipative terms of the stress–enrgy tensor are then computed in terms of such *perfect fluid perturbations*:

$$\delta T_{\mu\nu}^{diss} = \delta q_\mu u_\nu + \delta q_\nu u_\mu + \delta\tau_{\mu\nu} + q_{\mu\nu}\delta\tau \ (q_{\mu\nu} \equiv g_{\mu\nu} + u_\mu u_\nu)$$

with

$$\begin{aligned} \delta q^\mu &= -\kappa \frac{nT^2}{\rho + p} q^{\mu\nu} \left[\delta \left(\frac{\rho + p}{nT} - s \right) \right]_{,\nu} \\ \delta\tau^{\mu\nu} &= -2\eta\delta\sigma^{\mu\nu} \\ \delta\tau &= -\zeta\delta\theta \end{aligned}$$

$\delta\sigma^{\mu\nu}$, $\delta\theta$, shear and expansion of the perturbed fluid motion, can be expressed **in terms of** δu_μ.
κ, η, ζ are the coefficients of thermal conductivity and viscosity, derived from the background model.

- δq_μ, $\delta\tau_{\mu\nu}$, $\delta\tau$ are then used to compute quantities as the **dissipative damping times.**

This approach is useful, but it has some limits:

1. It assumes that the dissipative effects are **much smaller** than the perfect fluid non–radial perturbations. This is surely true in the case of cold NSs, maybe it is less true for hot, newly born PNSs, where dissipative effects can be very relevant.

2. The effects of the dissipative terms on the gravitational perturbations cannot be seen *directly* by using this approach.
   They can only be *inferred*, for example by comparing gravitational damping times with dissipative damping times.
   Furthermore, in this approach there is no way to determine how the frequencies of the modes are affected by dissipative effects.

To avoid these problems, we should **couple consistently** dissipative terms with other perturbations.

The general approach is the following.

- Consider a **dissipative fluid** spherically symmetric background

$$\begin{aligned} ds^2 &= -e^{\nu}dt^2 + e^{\lambda}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \\ T_{\mu\nu} &= (\rho + p)u_\mu u_\nu + p g_{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu \end{aligned}$$

  and solve the Einstein equations.
  Here only heat/lepton transfer terms do appear, because this is the relevant phenomenom in PNSs.

- Consider **dissipative fluid** non–radial perturbations of that background (we restrict ourselves on *polar* perturbations)

$$
\begin{aligned}
ds^2 &= -e^{\nu}(1 + r^l H_0 Y_{lm} e^{\mathrm{i}\omega t})dt^2 \\
&\quad -2\mathrm{i}\omega r^{l+1} H_1 Y_{lm} e^{\mathrm{i}\omega t} dt dr \\
&\quad +e^{\lambda}(1 - r^l H_0 Y_{lm} e^{\mathrm{i}\omega t})dr^2 \\
&\quad -r^2(1 - r^l K Y_{lm} e^{\mathrm{i}\omega t})(d\theta^2 + \sin^2\theta d\phi^2) \\
\delta q_\mu &= \left(0, Q_1 \frac{1}{r} Y_{lm}, Q_2 Y_{,a}\right) r^l e^{\mathrm{i}\omega t} \qquad (a = \theta, \phi) \\
\delta u_\mu &= \left(0, W \frac{e^{\lambda/2}}{r} Y_{lm}, V Y_{lm,a}\right) \mathrm{i}\omega r^l e^{\mathrm{i}\omega t}
\end{aligned}
$$

  and solve the perturbed Einstein equations

$$
\delta G_{\mu\nu} = \delta T_{\mu\nu} = \delta T_{\mu\nu}^{perfect\ \ fluid} + \delta T_{\mu\nu}^{diss} \,.
$$

  In this case we need **new equations** for q_μ and δq_μ, given by the *termodynamics* of the system, that couple with the Einstein equations.

Such coupled system whould give, at the same time, the behaviour of the gravitational perturbations **and** of the dissipative terms.

As a first step we have taken an
extra assumption:
The background heat transfer vector is
less relevant than the **perturbation** of the
heat transfer vector:

$$q^{(0)} \ll \delta q\,.$$

The reason for this assumption is the following:
the background heat transfer appears in

$$G^{(0)}_{\mu\nu} = T^{(0)}_{\mu\nu} = \ldots + q^{(0)}_{\mu} u^{(0)}_{\nu} + u^{(0)}_{\mu} q^{(0)}_{\nu}$$

which yields

$$\dot{M} \sim -q^{(0)}_{r} :$$

heat carried by neutrinos leaves the star,
so its gravitational mass decreases.
This happens with a timescale of **seconds**,
which is the timescale of the star evolution.

The heat transfer perturbation δq_{μ} is **coupled**
with the other perturbations of the star.
If we study the QNMs of the star, with
$\nu \sim 10^2 - 10^3$ Hz and more, the heat transfer
perturbation coupled with them is a δq_{μ}
oscillating with a timescale of **milliseconds**.

So, we can take a $\boxed{\text{perfect fluid background}}$.
This allows us to integrate our equations in the
frequency domain.

The equations for the gravitational and fluid perturbations in the frequency domain

$$\underbrace{H_0,\, H_1,\, K}_{gravitational} \quad \underbrace{W,\, V,\, \delta p}_{fluid}$$

are the LD equations.
By adding the terms arising by

$$\delta T^{diss}_{\mu\nu} = \delta q_\mu u^{(0)}_\nu + \delta q_\nu u^{(0)}_\mu$$

with $\delta q_\mu = \left(0, Q_1 \frac{1}{r} Y_{lm}, Q_2 Y_{,a}\right) r^l e^{\mathrm{i}\omega t}$ we have an undetermined system for $H_0, \ldots, \delta p + Q_1, Q_2$.

We need then some more equations, which come from **thermodynamics**:

- The PNS EOS can be expressed as $\boxed{\rho = \rho(p, s, Y)}$. Defining the **sound speed** $c_s^2 = \left(\frac{\partial \rho}{\partial p}\right)_{s,Y}$, and calling $nT\alpha_1$, $n\mu\alpha_2$ the other thermodynamical derivatives,

  $$d\rho = c_s^{-2} dp + nT\alpha_1 ds + n\mu\alpha_2 dY\,.$$

  The lagrangian perturbations fullfill the same relationship:

  $$\Delta\rho = c_s^{-2}\Delta p + nT\alpha_1 \Delta s + n\mu\alpha_2 \Delta Y\,. \quad (1)$$

  In the adiabatic case, an element of fluid does not exchange heat or matter with its surroundings, so $\Delta s = \Delta Y = 0$ and

  $$\Delta\rho = c_s^{-2}\Delta p\,.$$

- Time component of the stress–energy tensor conservation:

$$\delta\left(u_\mu T^{\mu\nu}_{;\nu}\right) = 0\,.$$

In the adiabatic case $\delta T_{\mu\nu} = \delta T^{perfect\,fluid}_{\mu\nu}$. This yields $\boxed{\Delta s = \Delta Y = 0}$.
This is **not** an *a priori* assumption, but a logical consequence of the choice of $T_{\mu\nu}$.

If we want to describe heat transfer, we have to take

$$\delta T_{\mu\nu} = \delta T^{perfect\,fluid}_{\mu\nu} + \delta q_\mu u_\nu + \delta q_\nu u_\mu\,.$$

The time component of the stress–energy tensor conservation gives then

$$nT\Delta s + n\mu\Delta Y = \frac{1}{\mathrm{i}\omega}e^{\nu/2}u_\alpha\left(\delta q^\alpha u^\beta + \delta q^\beta u^\alpha\right)_{;\beta} \qquad (2)$$

- We need to introduce another quantity, the *neutrino number flux* f^μ, which satisfies the neutrino number density conservation equation

$$(nYu^\mu)_{;\mu} = -f^\mu{}_{;\mu}\,.$$

We assume that, as in q^μ, also in f^μ the first order part dominates: $\boxed{f^{(0)}_\mu \ll \delta f_\mu}$.
Under this hypotesis

$$n\Delta Y = \frac{1}{\mathrm{i}\omega}e^{\nu/2}(\Delta f)^\beta_{;\beta}\,. \qquad (3)$$

- We still need a dynamical equation for δq_μ, δf_μ: the *transport equation.*
  The general relativistic Boltzmann equation for the spherical background of these PNS has been studied in Pons *et al.*, **ApJ** 513, 780 (1999).
  This gives us the background equations that, under our assumptions, can be translated in equations for the perturbations:

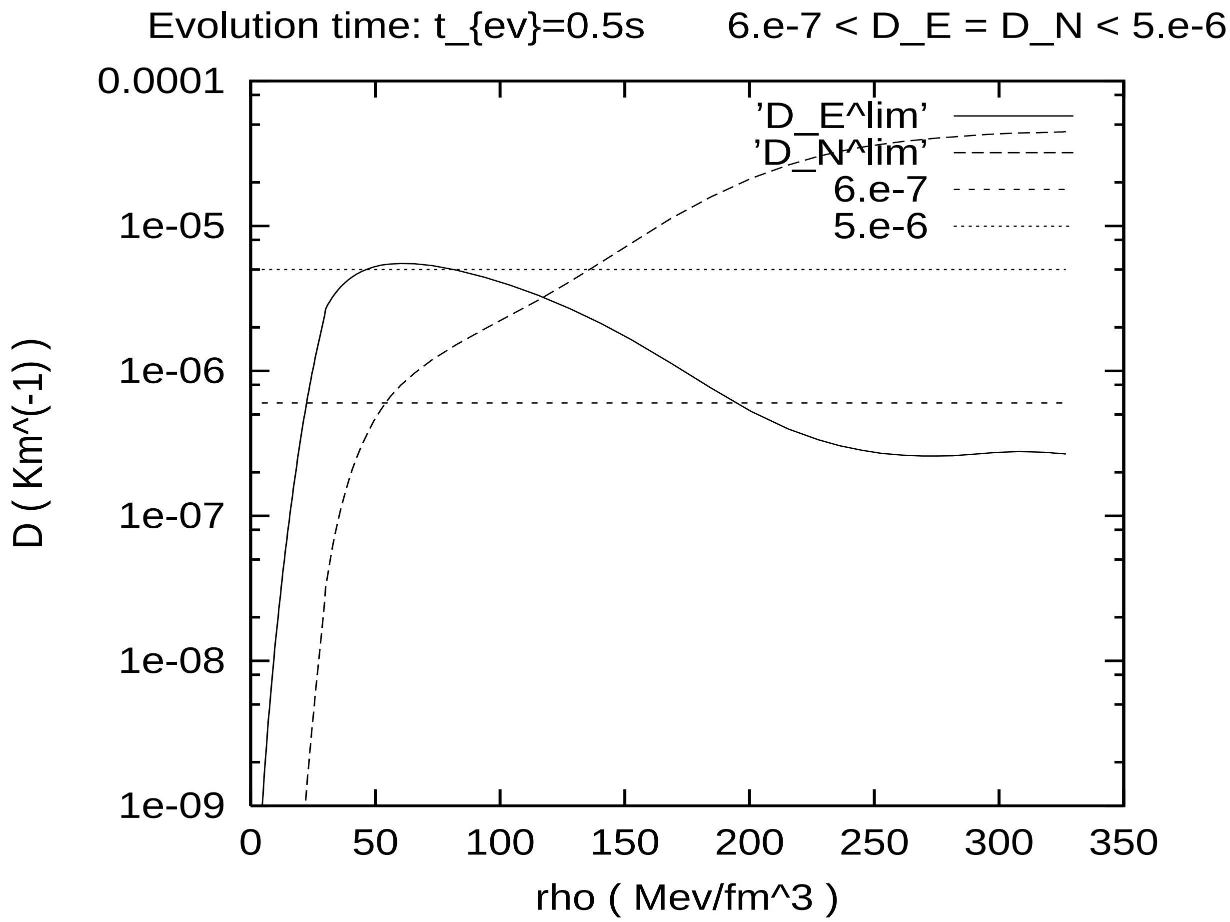
$$
\begin{aligned}
\Delta q_\alpha &= -e^{-\nu/2}\tilde{D}_E \frac{1}{T}\left[\Delta\left(Te^{\nu/2}\right)\right]_{,\alpha} && (4)\\
\Delta f_\alpha &= -e^{-\nu/2}\tilde{D}_N \frac{1}{\mu}\left[\Delta\left(\mu e^{\nu/2}\right)\right]_{,\alpha}\,. && (5)
\end{aligned}
$$

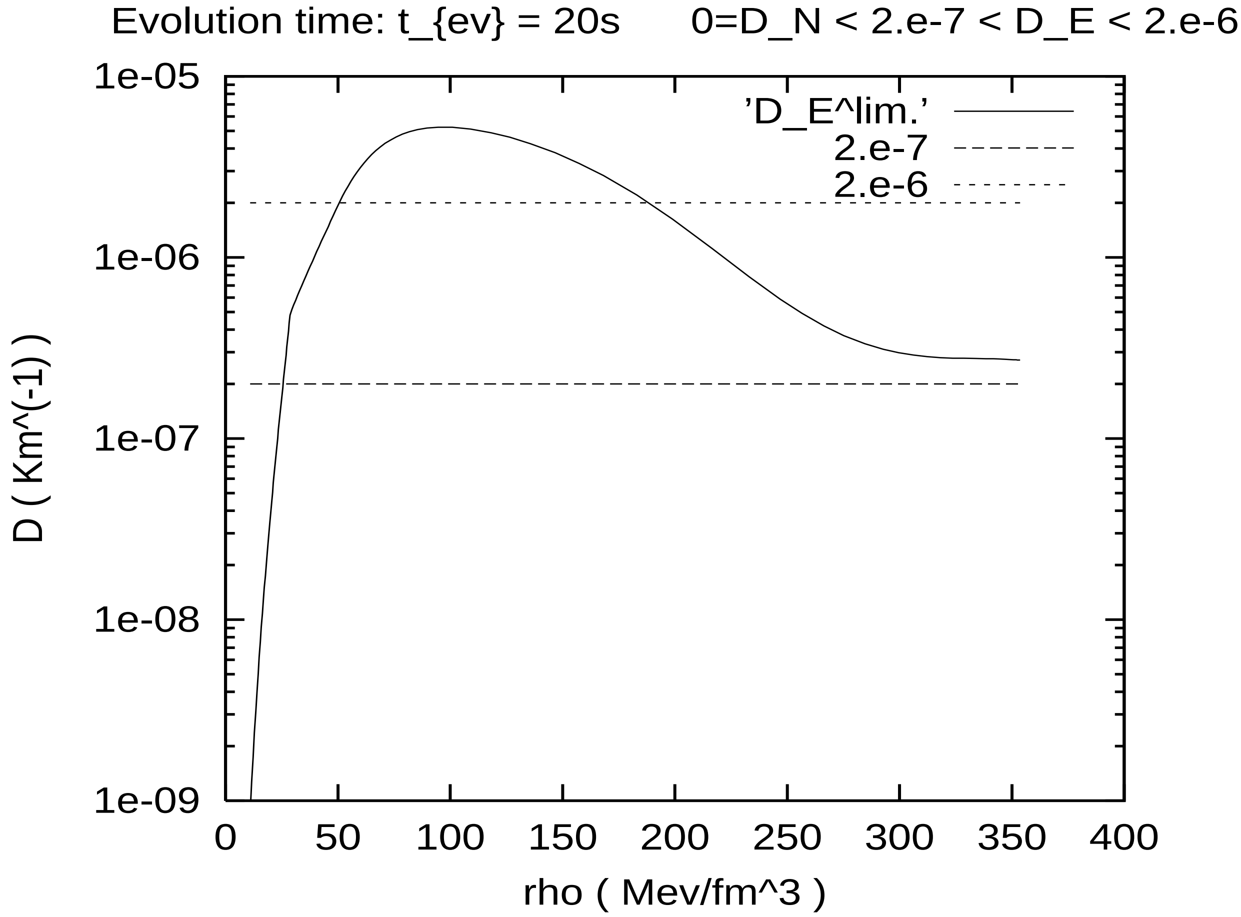
  With these two equations we are able to close the system.

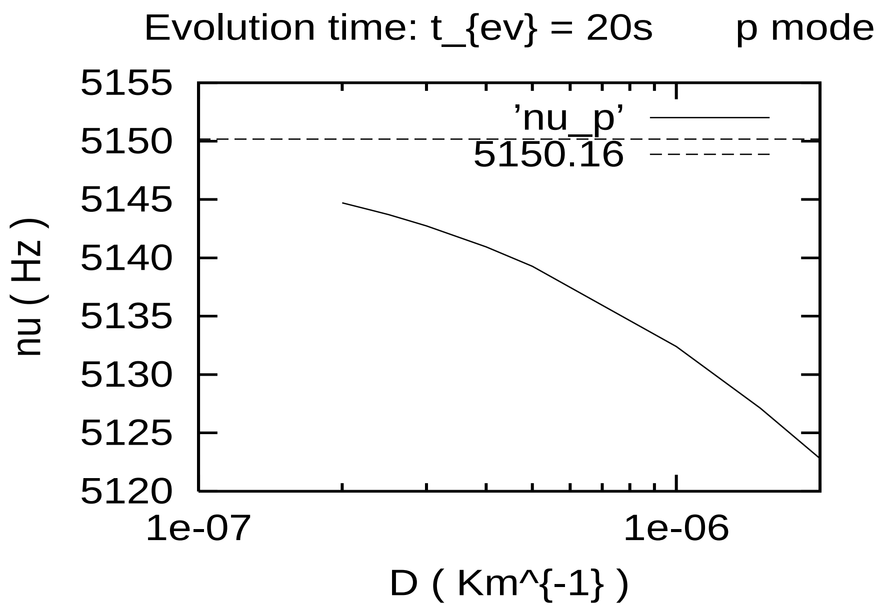
The coefficients $\tilde{D}_E$, $\tilde{D}_N$ are a crucial ingredient: they are given by the microphysics of the system, and are related to neutrino mean free paths by

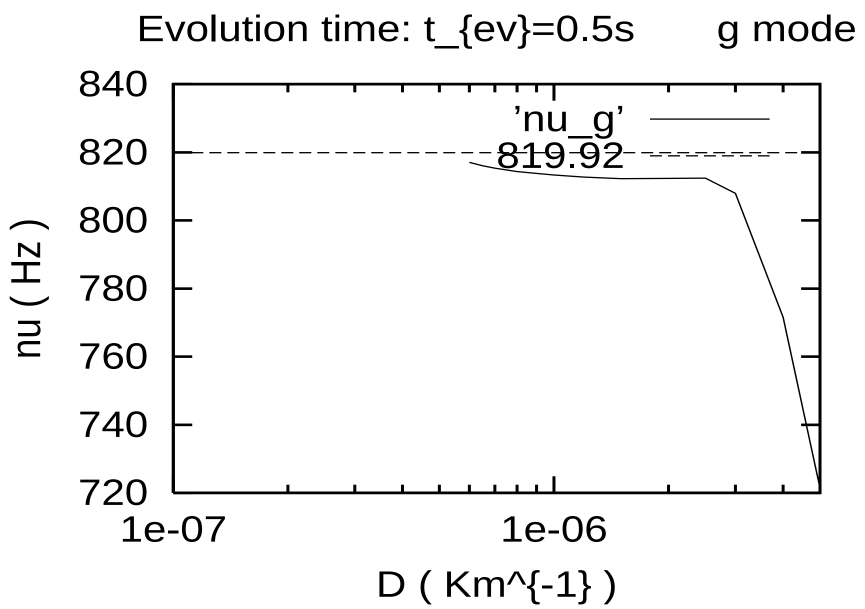
$$
\begin{aligned}
\lambda_e &= \frac{90(\hbar c)^3}{7\pi^2 T^4}\tilde{D}_E\\
\lambda_n &= \frac{6\pi^2(\hbar c)^3}{\mu^4}\tilde{D}_N\,.
\end{aligned}
$$

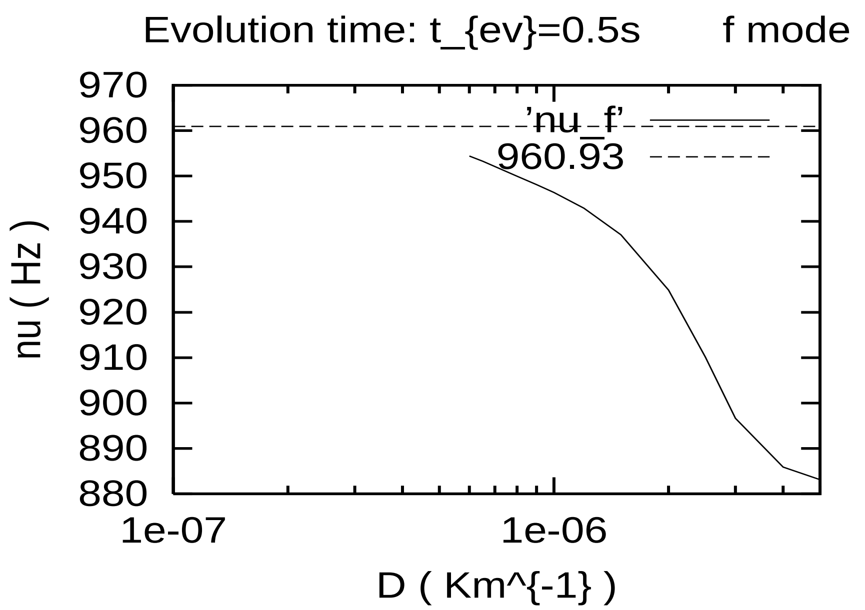
To start with, we have taken $\tilde{D}$ constant throughout the star, and studied the transition between *diffusion approximation* and *semi–transparent* regime: $\boxed{\lambda \lesssim R_{star}}$ which corresponds to $\tilde{D}_E \lesssim \tilde{D}_E^{lim}(r),\ \tilde{D}_N \lesssim \tilde{D}_N^{lim}(r)$.

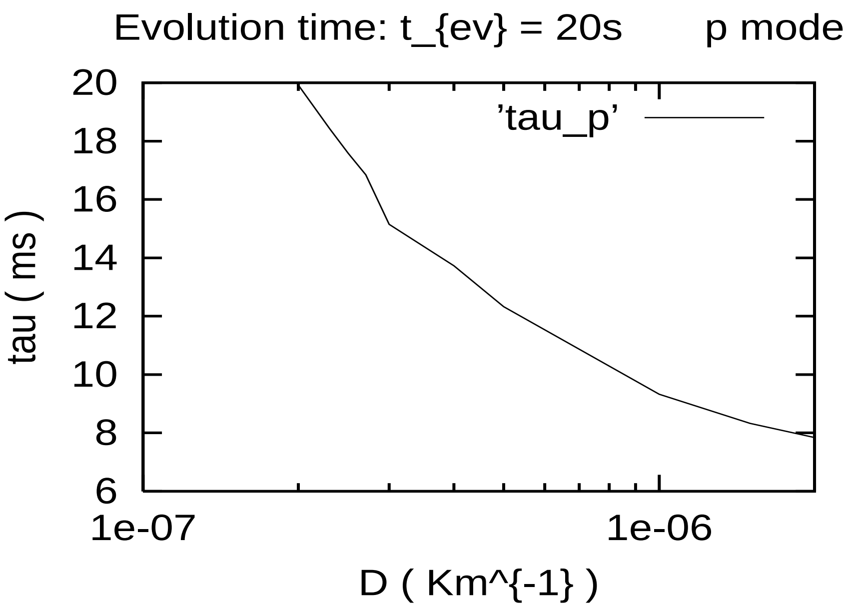

Evolution time: t_{ev}=0.5s 6.e-7 < D_E = D_N < 5.e-6
'D_E^lim'
'D_N^lim'
6.e-7
5.e-6
0.0001
1e-05
1e-06
1e-07
1e-08
1e-09
D (Km^(-1))
0
50
100
150
200
250
300
350
rho (Mev/fm^3)

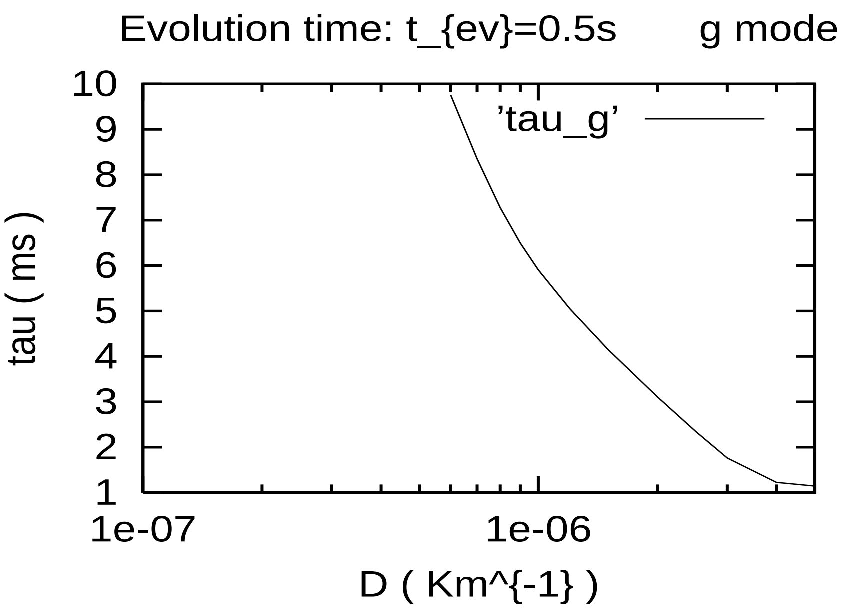

Evolution time: t_{ev} = 20s 0=D_N < 2.e-7 < D_E < 2.e-6
'D_E^lim.'
2.e-7
2.e-6
1e-05
1e-06
1e-07
1e-08
1e-09
D (Km^(-1))
0
50
100
150
200
250
300
350
400
rho (Mev/fm^3)

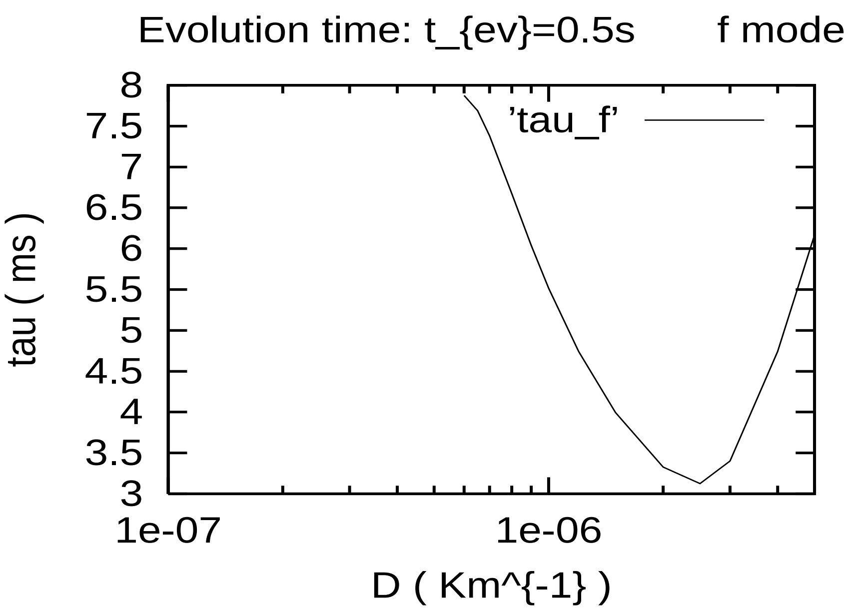

Evolution time: t_{ev} = 20s p mode
5155
5150
5145
5140
5135
5130
5125
5120
nu (Hz)
'nu_p'
5150.16
1e-07
1e-06
D (Km^{-1})


Evolution time: t_{ev}=0.5s g mode
840
820
800
780
760
740
720
nu (Hz)
'nu_g'
819.92
1e-07
1e-06
D (Km^{-1})


Evolution time: t_{ev}=0.5s f mode
970
960
950
940
930
920
910
900
890
880
nu (Hz)
'nu_f'
960.93
1e-07
1e-06
D (Km^{-1})


Evolution time: t_{ev} = 20s p mode
'tau_p'
tau (ms)
D (Km^{-1})


Evolution time: t_{ev}=0.5s g mode
'tau_g'
tau (ms)
D (Km^{-1})


Evolution time: t_{ev}=0.5s f mode
'tau_f'
tau (ms)
D (Km^{-1})

The damping times we find are of the order of magnitude of the dissipation timescale.

Very rough calculation:

the Fourier equation hidden in our system is

$$\frac{\partial \Delta s}{\partial t} \simeq \kappa(\Delta s)'' + \ldots \qquad \left(\kappa \equiv \frac{\tilde{D}}{nTC_p}\right).$$

Its solution goes more or less like

$$\Delta s \sim e^{-\frac{\kappa t}{\lambda^2}}$$

with λ characteristic lenght of the system.

The dissipation timescale is then

$$\tau_{diss} = \frac{\lambda^2}{\kappa}.$$

If we take, for an order–of–magnitude estimate,

$$\lambda \sim R_s\,,$$

we find, for every value of $\tilde{D}$, an estimate for the dissipation timescale.

Such estimate is very close to the gravitational damping time **computed** for the same value of $\tilde{D}$:

$$0.5 < \frac{\tau_{diss}}{\tau_{GW}} < 5\,.$$

So, while in the traditional approach one says *a priori* that the gravitational damping time is equal to τ_{diss}, in our approach this comes out directly from the equations.

Summarizing:

- Both the frequency and the damping time tend, for decreasing $\tilde{D}$, to the adiabatic value.

- In presence of heat transport the frequencies are smaller, as one could expect: even in a simple mechanical oscillator, dissipation has the effect to lower the proper frequency.

- The damping time, computed by taking the imaginary part of the complex frequency of the gravitational waves, changes drastically: in the adiabatic case it is of the order of seconds, while in presence of heat transport (with the values of the constants $\tilde{D}$ we imposed) it is of the order of milliseconds.

The next step will be to take profiles of $\tilde{D}_E$, $\tilde{D}_N$ derived from the microphysics.
Tipically, this means taking *smaller* values of these coefficients.
To do this, we need to integrate our equations in the *time domain*.