Binary Pulsar Coalescence Rates and Detection Rates for Gravitational Wave Detectors

Chunglee Kim, Vassiliki Kalogera (Northwestern U.),
and Duncan R. Lorimer (U. of Manchester)

Advanced School and Conference on
Sources of Gravitational Waves
ICTP, Trieste, Italy (Sep. 25, 2003)

Detection of gravitational waves

1. GW detectors

ground-based detectors: AIGO, GEO, LIGO, TAMA, VIRGO

space-based detector : LISA (scheduled in 2012)

2. Candidates of GW detectors

ground-based: “late” inspiral of stellar-mass compact object binaries

LISA: Galactic binary GW foreground, massive BH binaries

3. Strong sources of GW

mainly astrophysical objects: understanding of the physics of those sources is needed.

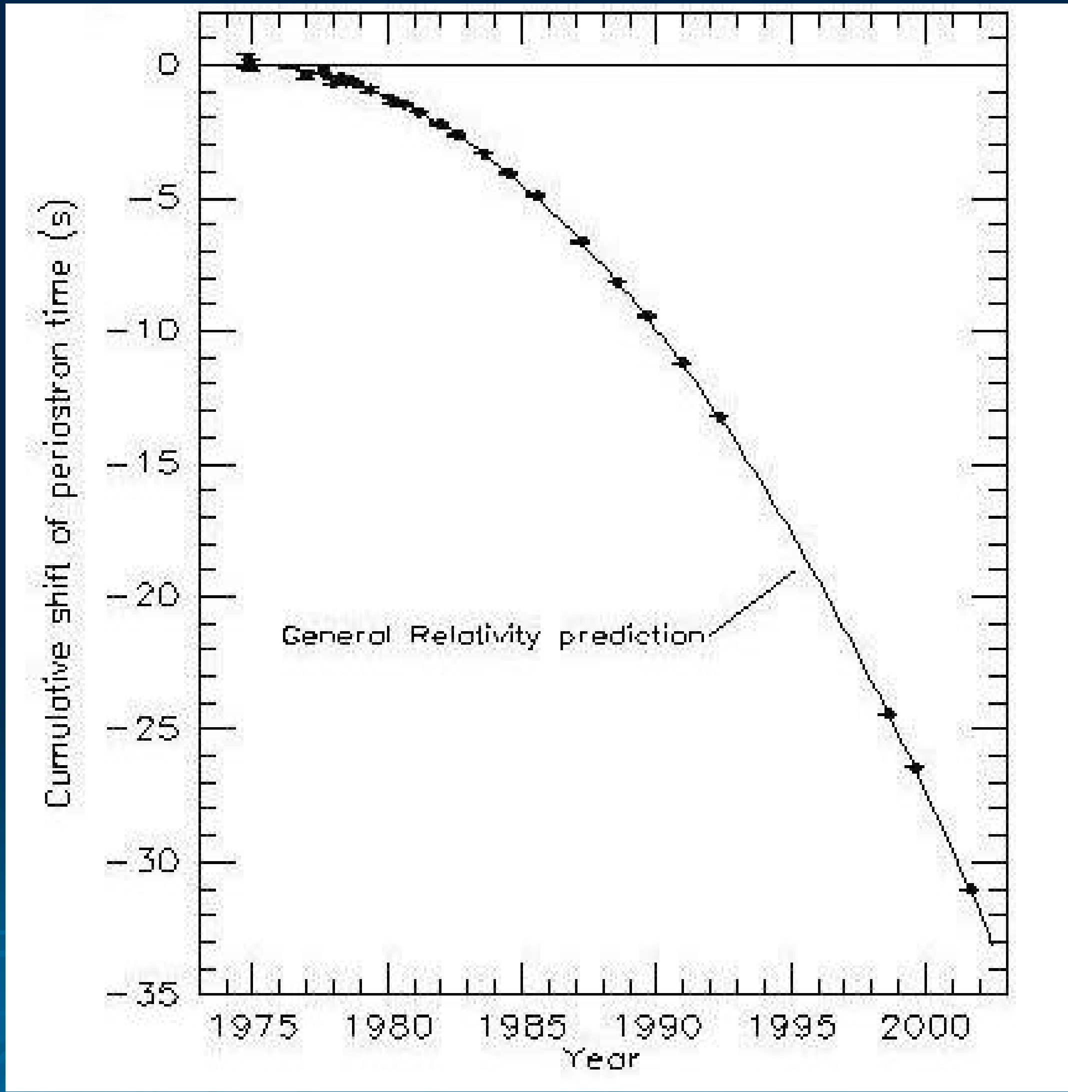
Astrophysics comes in

Binary Compact Object Inspiral:

Indirect evidence of GW emission

- Orbital decay of PSR B1913+16 is consistent with general relativity to within 0.35%

(Hulse and Taylor 1975; Weisberg and Taylor 2002)



Why are pulsar binaries interesting?

Coalescing pulsar binaries: strong candidates of GW detectors

1. Double neutron star (NS-NS) binaries ← ground based

5 systems are known.

2 coalescing systems in the Galactic disk.
(PSR B1913+16 and B1534+12)

2. Neutron star – White dwarf (NS-WD) binaries ← LISA

Currently, more than 40 systems have been detected.

3 coalescing systems are observed.
(PSR J0751+1807, J1757-5322, and J1141-6545)

Estimation of source detection rates

source signal strength → detection volume

instrument sensitivity → detection volume

detection volume → event rate within detection volume

Detectability of the inspiral of pulsar binaries also depends on the frequency of coalescence events.

→ Calculate the coalescence rates of pulsar binaries

Rate calculation: general strategy (Narayan et al.; Phinney 1991)

$$\text{Merger Rate R} = \frac{\text{Number of sources}}{\text{Lifetime of a system}} \times \text{correction factor}$$

- Lifetime of a system = current age of a pulsar + merging time of a system

---

- Number of sources : number of pulsars in a coalescing binary in the galaxy ("scale factor")

- Correction factor : beaming correction for pulsars

Rate calculation: our work (Kim et al. 2003, *ApJ*, **584**, 985)

- Previous studies on the coalescence rate typically have a large uncertainty in the coalescence rate calculation (more than two orders of magnitude) mainly due to the small number of observed samples.

We introduce a new analysis method to give a statistical probability of the coalescence rate.

Small number bias and selection effects for faint pulsars are implicitly included in our method.

Method

1. Model a pulsar population by Monte-Carlo method

- Luminosity distribution

  power-law: $f(L) \propto L^{-p}$, $L_{min} < L$ (L_{min}: cut-off luminosity)

- Spatial distribution

  $$f(R,z) \propto \exp\left(- \frac{R^2}{2R_o^2} - \frac{|Z|}{Z_o} \right)$$

  R_o: radial scale length, z_o: vertical height

2. Pulsar-survey simulation

- consider selection effects of large-scale pulsar surveys
- look for pulsars similar to each of observed pulsars (e.g. PSR B1913+16-like population)

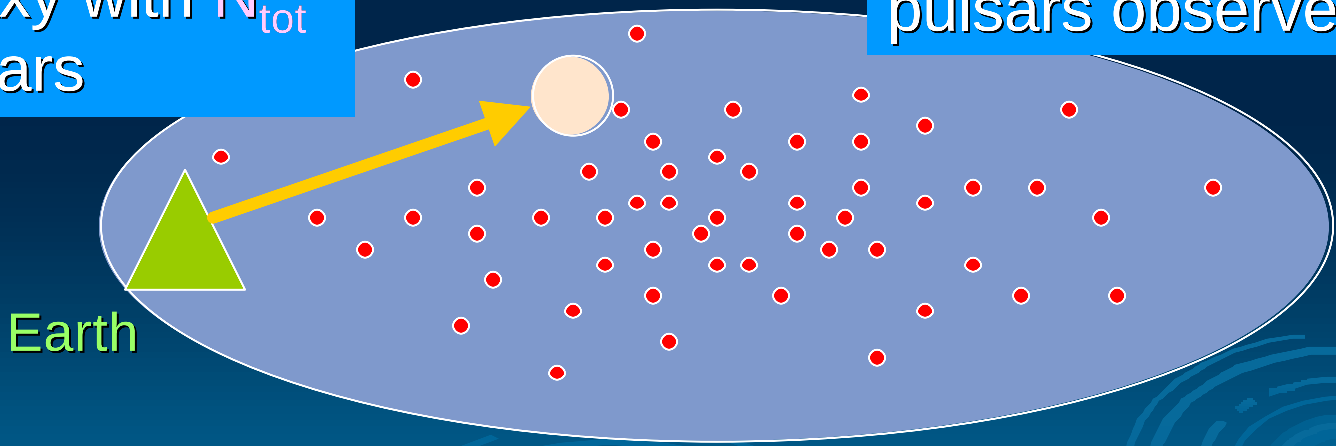
Method

2. Pulsar-survey simulation (cont.)

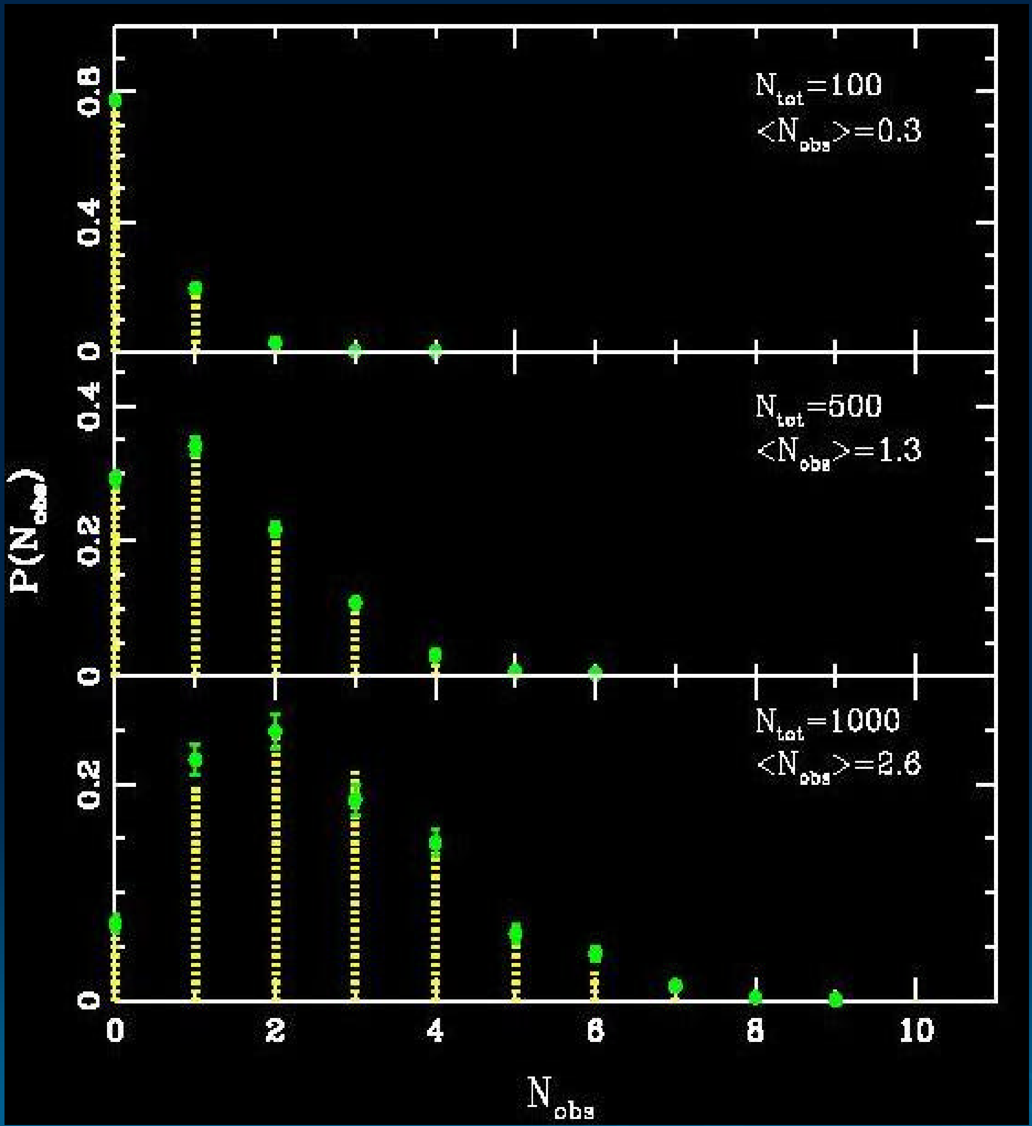
- adapt properties for each observed pulsar (period and pulse width) → calculate how many pulsars similar to the observed one can be detected by pulsar-surveys

fill a model galaxy with N_{tot} pulsars

count the number of pulsars observed (N_{obs})



Statistical Analysis



$P(N_{obs})$ for PSR B1913+16

For a given total number of pulsars, the number detected by pulsar-survey simulation follows a Poisson distribution.

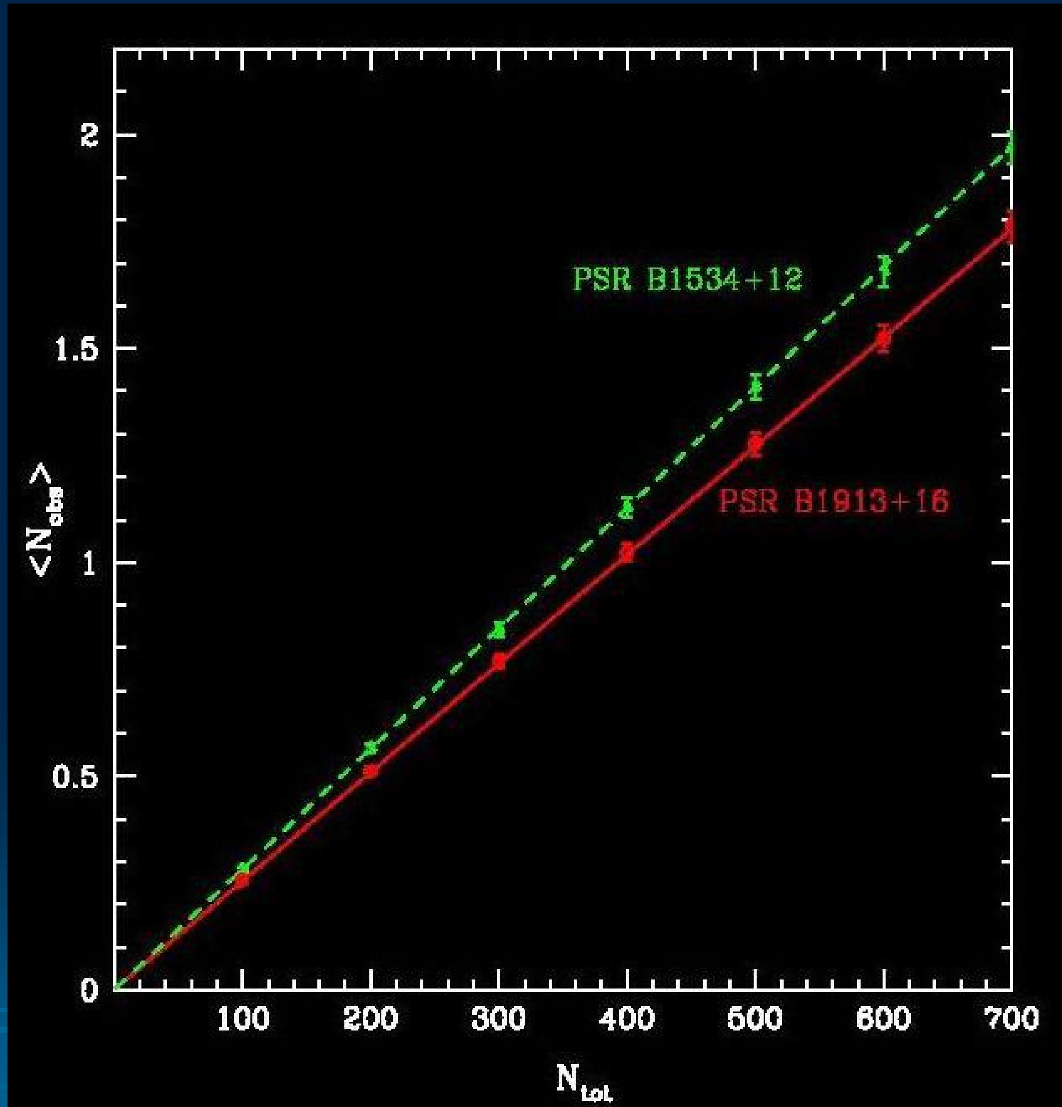
We calculate the best-fit value of $<N_{obs}>$ by fitting the data with the Poisson distribution, $P(N_{obs};<N_{obs}>)$.

Statistical Analysis

$\langle N_{obs} \rangle$ is linearly proportional to the total number of pulsars in a model galaxy (N_{tot}).

$\langle N_{obs} \rangle = \alpha N_{tot}$

where α is a slope.



$\langle N_{obs} \rangle$ as a function of N_{tot} for B1913+16

Statistical Analysis

- We consider each binary system separately by setting $\boxed{N_{obs}=1}$ (small number bias is implicitly included).

Bayes' theorem — Change of variables

$P(1; \langle N_{obs} \rangle) \longrightarrow P(\langle N_{obs} \rangle) \longrightarrow P(N_{tot}) \longrightarrow P(R)$

For an Individual binary i,

$$P_i(R) = C_i^2 R \exp(-C_i R)$$

where $C_i = \left[\dfrac{t_{life}}{N_{tot}\, f_b} \right]_i$

$\longrightarrow$ combine all $P(R)$'s

and calculate $P(R_{tot})$

Results

$P(R_{tot})$

- most probable rate R_{peak}
- statistical confidence levels
- detection rate for GW detector

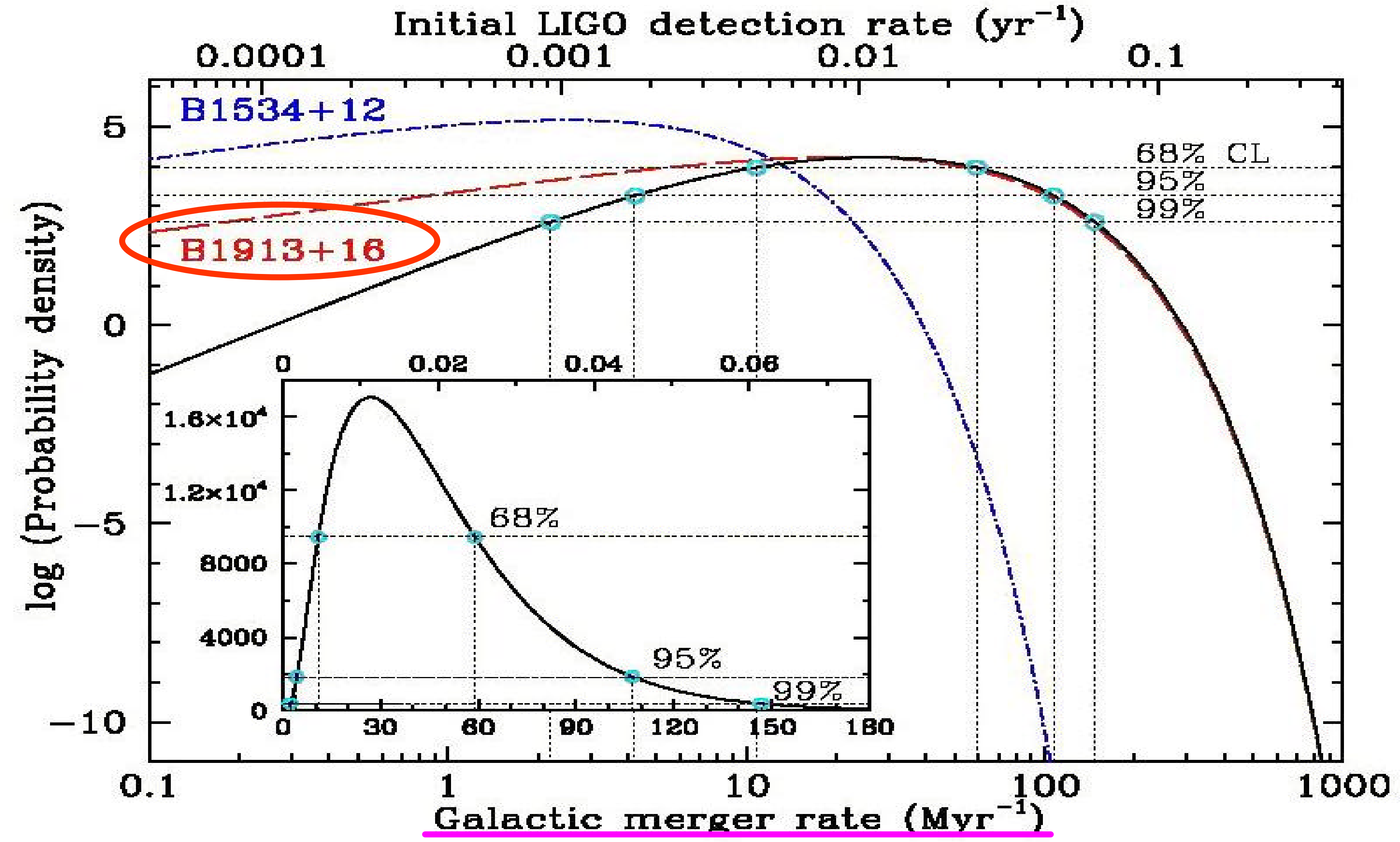
1. Double neutron star (NS-NS) binaries ← ground based

f_{gw}=10-1000 Hz

2 coalescing systems in the Galactic disk

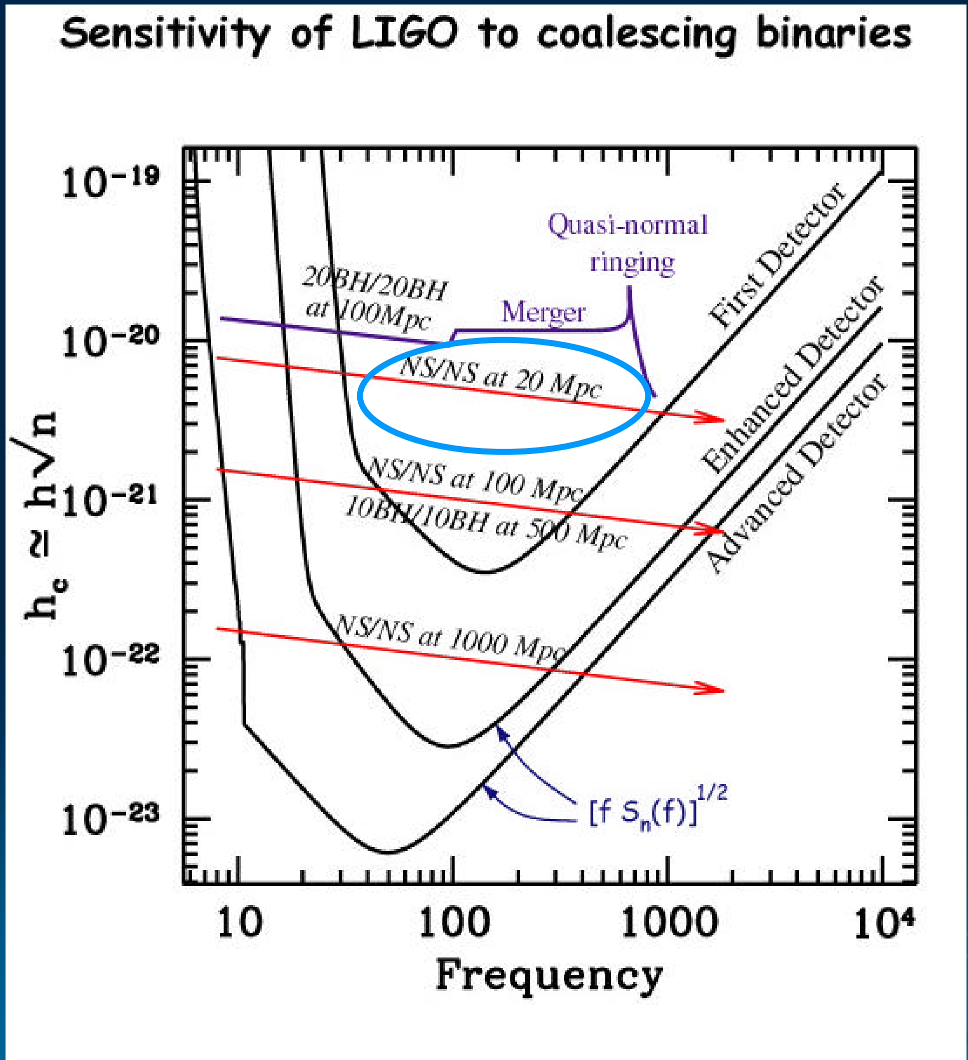
(PSR B1913+16 and B1534+12)

Results: $P(R_{tot})$ of NS-NS binaries



Probability density function of Galactic merger rate of NS-NS binaries and the corresponding detection rate for the initial LIGO

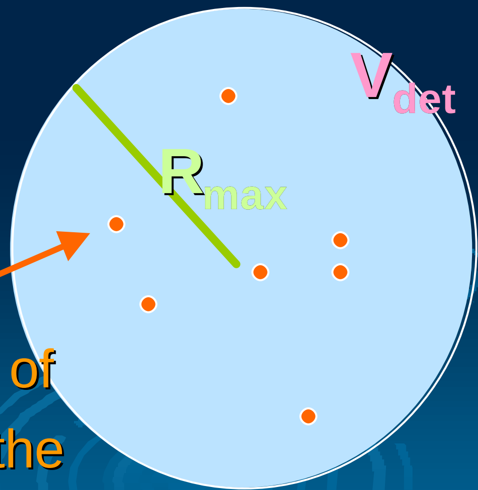
Calculation of detection rate of NS-NS inspirals for LIGO



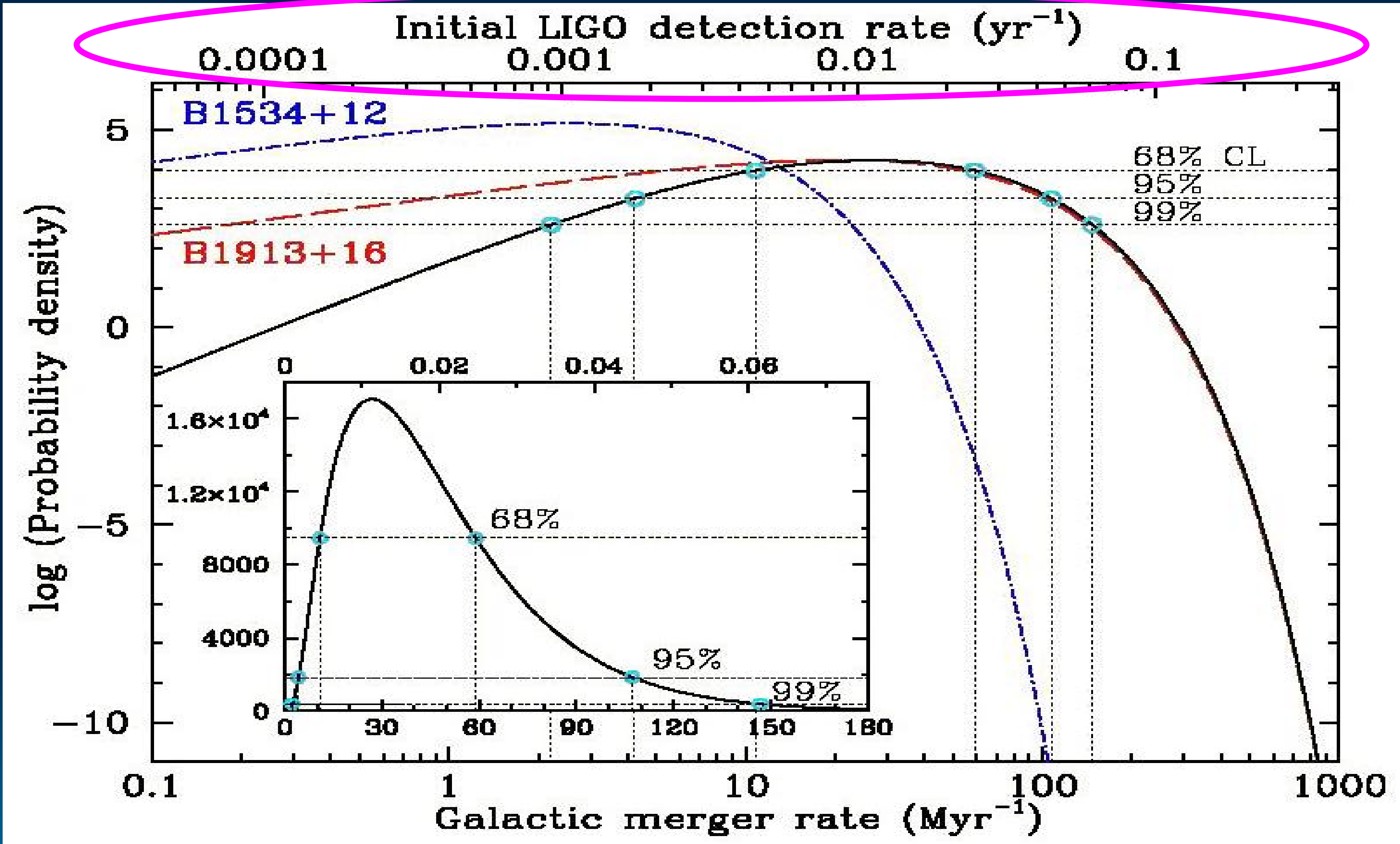
Detection rate = R x ρ_{gal}

R_{max} (ini. LIGO) = 20 Mpc
R_{max} (adv. LIGO) = 350 Mpc
(Finn 2001)

calculate the number density of galaxies within the detection volume



Results: $P(R_{tot})$ of NS-NS binaries



Probability density function of Galactic merger rate of NS-NS binaries and the corresponding detection rate for the initial LIGO

Results: correlation between R_{peak} and model parameters

- Luminosity distribution

power-law: $f(L) \propto L^{-p}$, $L_{min} < L$ (L_{min}: cut-off luminosity)

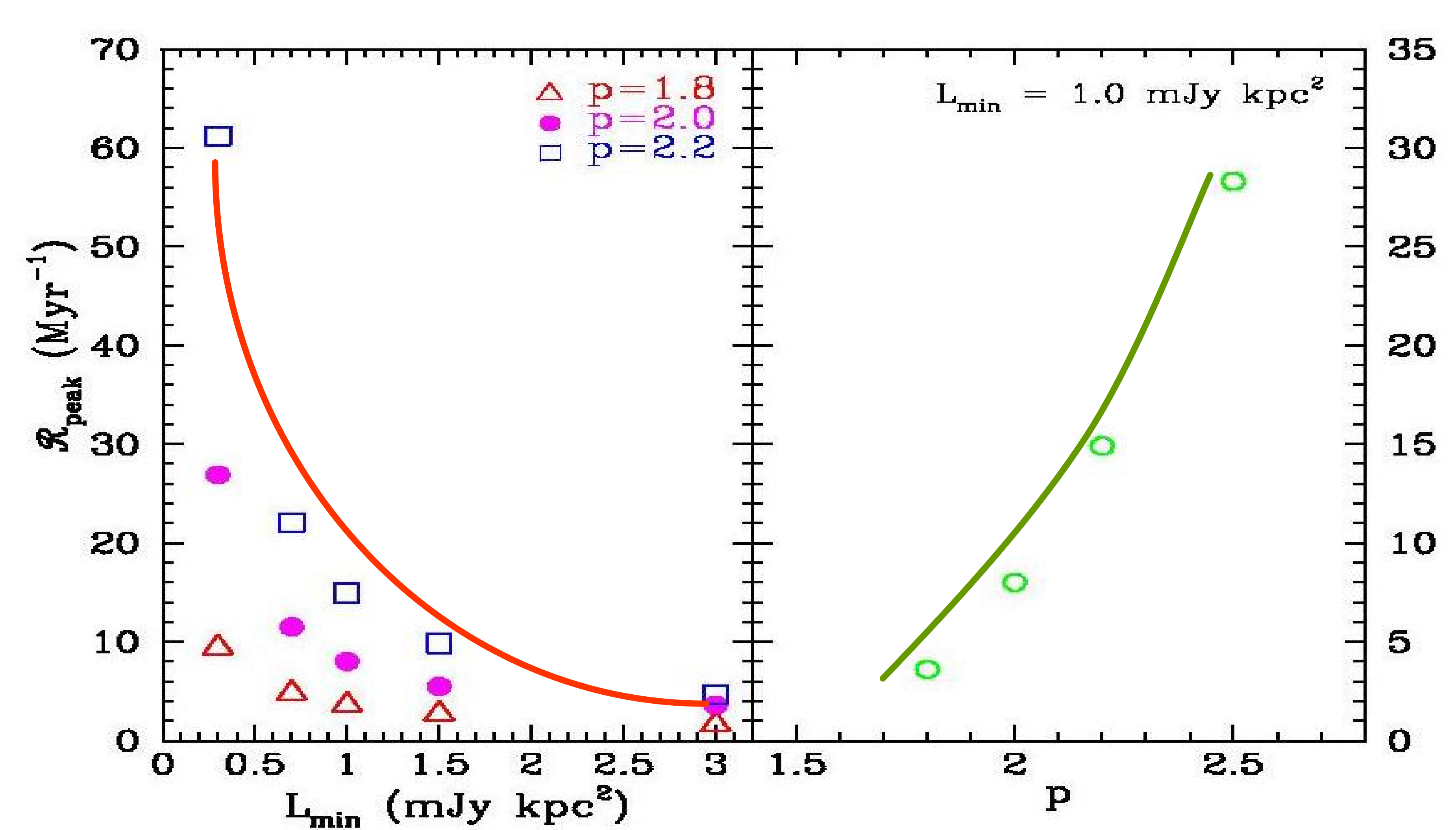
- Spatial distribution

$$f(R,z) \propto \exp\left(-\frac{R^2}{2R_o^2} - \frac{|Z|}{Z_o}\right)$$

R_o: radial scale length, z_o: vertical height

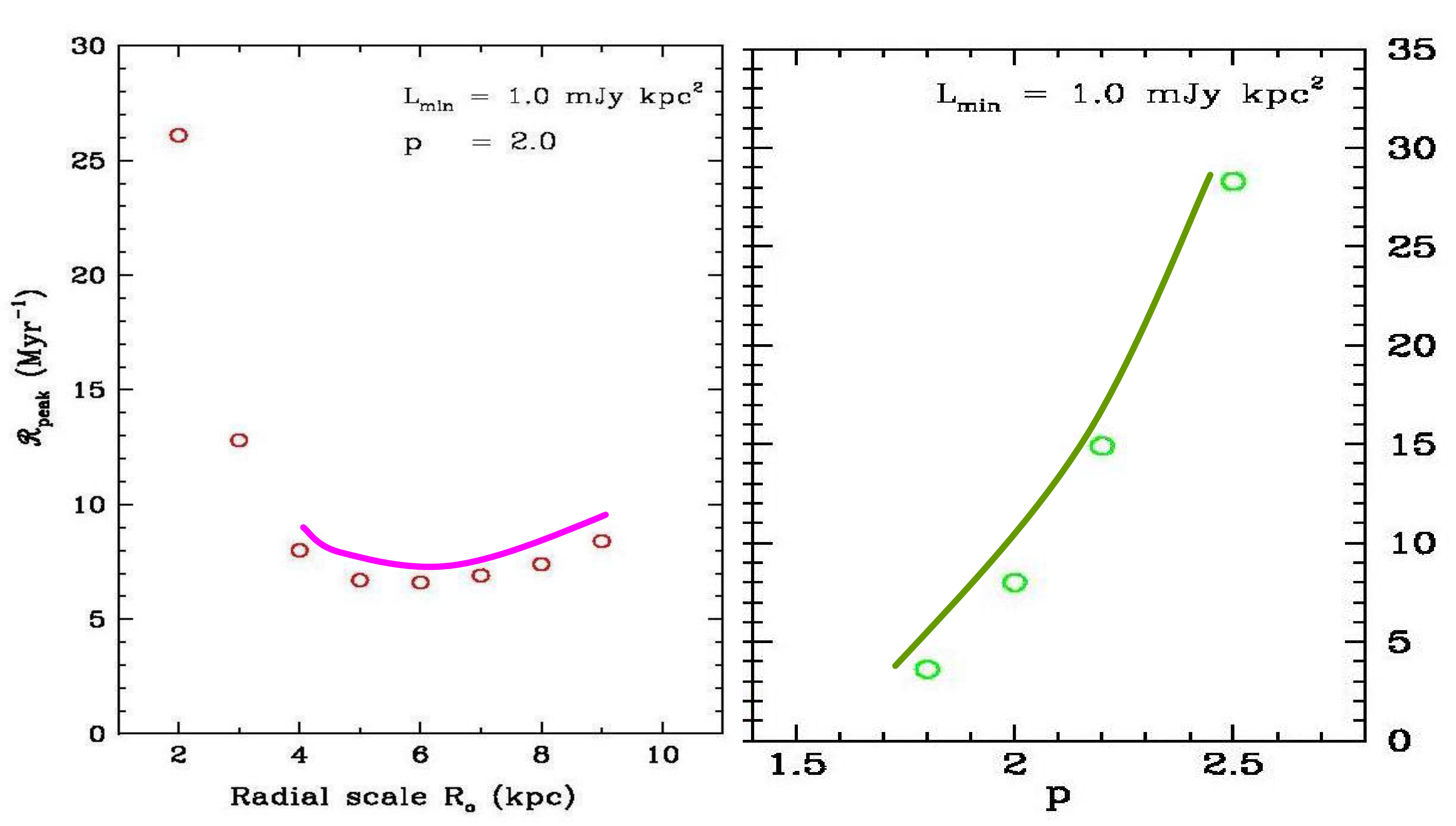
Correlations between the merger rate with parameters of pulsar population models → give constraint to modeling of a pulsar population

Results: R_{peak} vs model parameters I



Correlation between most probable rate R_{peak} and parameters of a pulsar luminosity function (L_{min}: cut-off luminosity, p: power index of L-function)

Results: R_{peak} vs model parameters II



Correlation between most probable rate R_{peak} and parameters of a pulsar luminosity function (L_{min}: cut-off luminosity, p: power index of L-function)

NS-WD binaries

f_{gw}=0.01-100 mHz

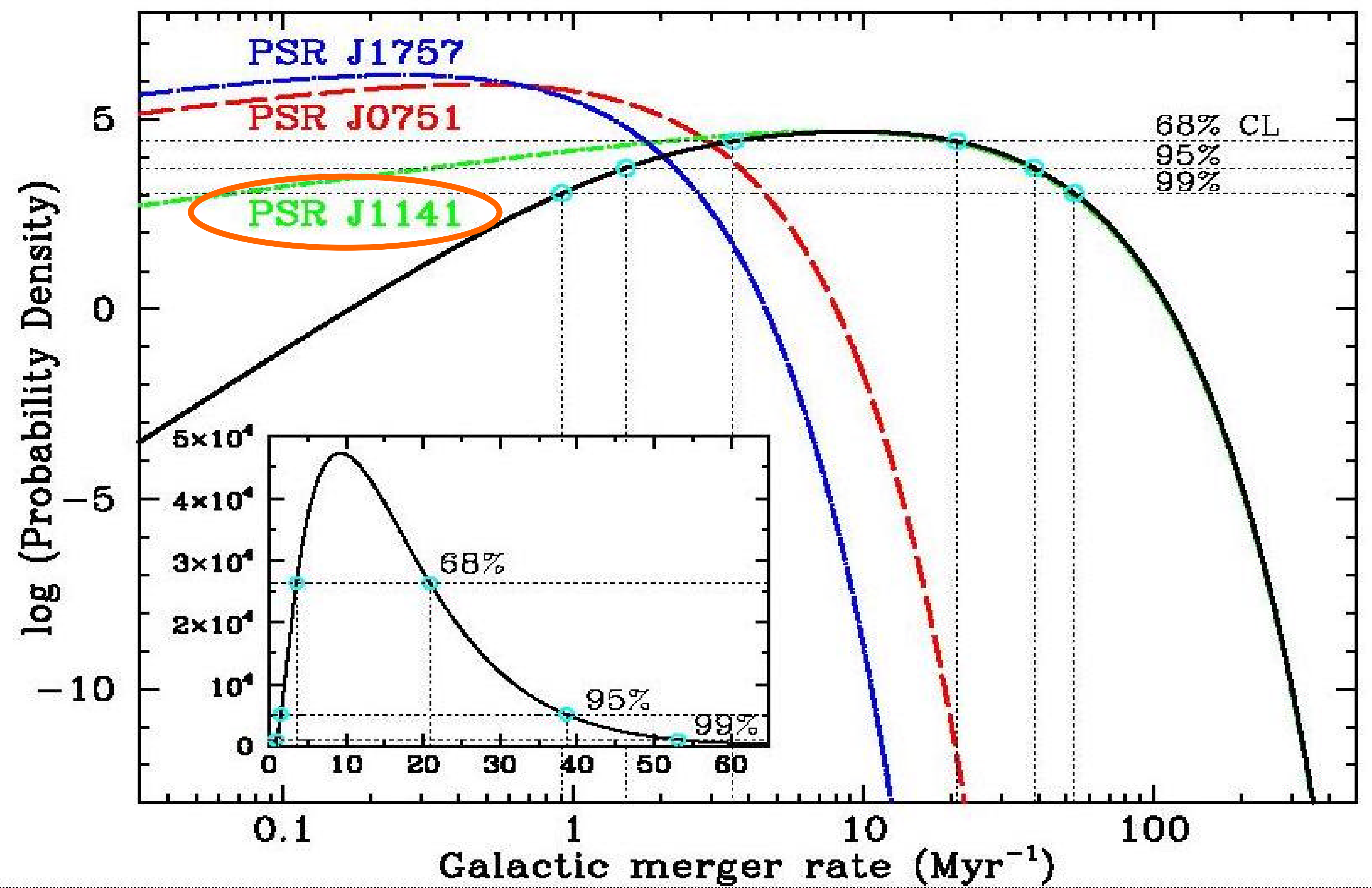
2. Neutron star – White dwarf (NS-WD) binaries ← LISA

Currently, more than 40 systems are detected.

3 coalescing systems are observed.

(PSR J0751+1807, J1757-5322, and J1141-6545)

Results: $P(R_{tot})$ of NS-WD binaries



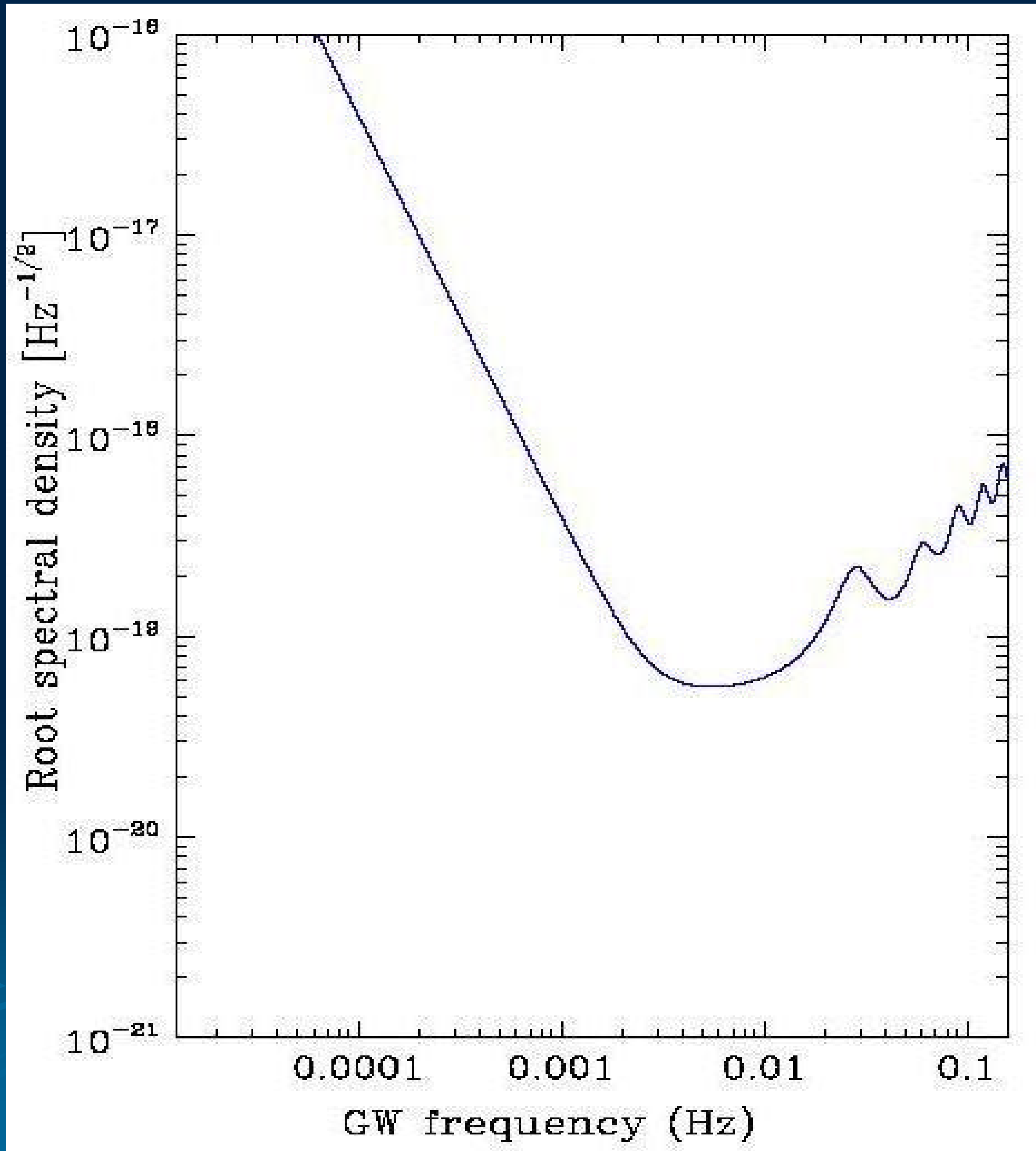
Probability density function of Galactic merger rate for NS-WD binaries

Results: NS-WD binaries

NS-WD binaries contribute to the GW foreground for LISA (cf. WD-WD binaries)

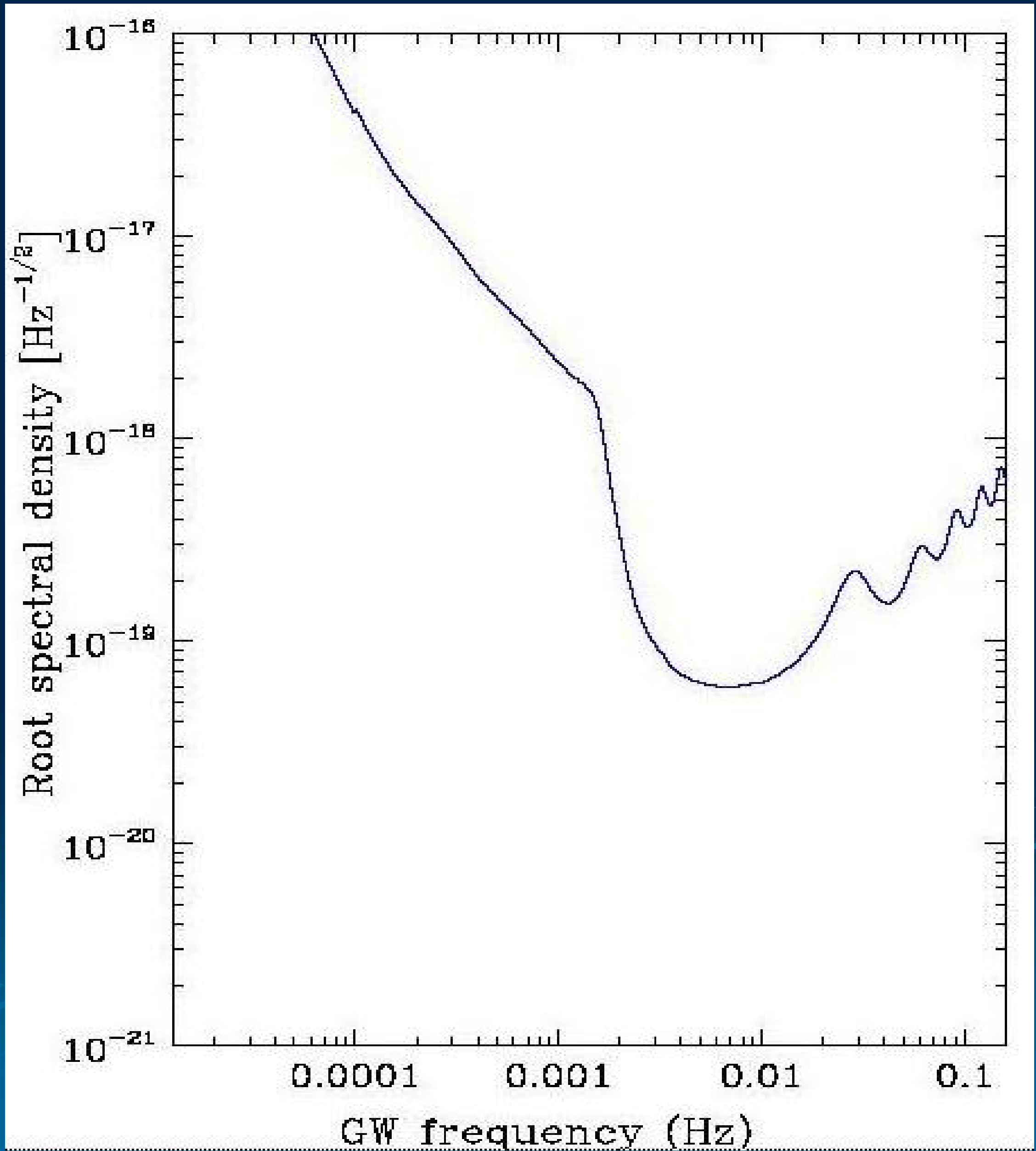
LISA sensitivity curve

http://www.srl.caltech.edu/~shane/sensitivity/



Results: NS-WD binaries

NS-WD binaries contribute to the GW foreground for LISA (cf. WD-WD binaries)

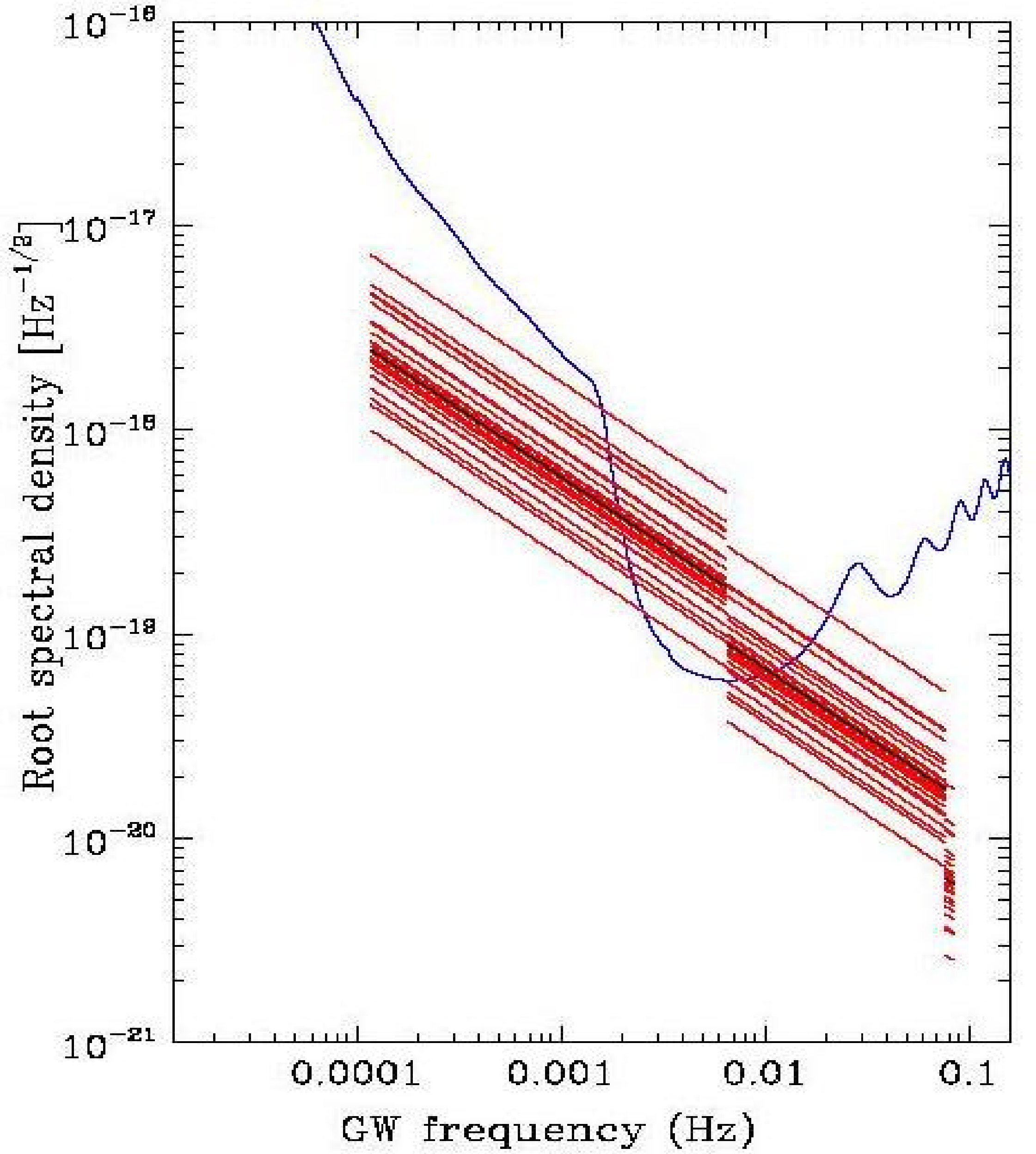


LISA sensitivity curve

http://www.srl.caltech.edu/~shane/sensitivity/

Results: NS-WD binaries

NS-WD binaries contribute to the GW foreground for LISA (cf. WD-WD binaries)



LISA sensitivity curve

http://www.srl.caltech.edu/~shane/sensitivity/

Summary

- Galactic coalescence rates of pulsar binaries

	R_{peak} (Myr^{-1})	R_{det} (ini. LIGO) (yr^{-1})	R_{det} (adv. LIGO) (yr^{-1})
NS-NS	30^{+32}_{-16}	$0.011^{+0.013}_{-0.007}$	61^{+72}_{-36}
	R_{peak} = 2 – 60 per Myr (all models)		
NS-WD	9^{+12}_{-6}		
	R_{peak} = 0.6 – 20 per Myr (all models)		

$$\frac{R_{peak}\,(\text{NS-NS})}{R_{peak}\,(\text{NS-WD})} \sim 3$$

Summary

- Detection rate of NS-NS inspirals

R_{det} (ini. LIGO) = 1 event per 30 – 1000 years…

R_{det} (adv. LIGO) = 3 – 140 events per year

Most optimistic prediction? R_{det} < 300 per year (68% CL) (adv. LIGO)

Future work

- Apply the method to other classes of pulsar binaries (e.g. pulsar binaries in globular clusters)

- Give statistical constraints for binary evolution theory

  → determine a favored parameter space

  based on the rate calculation

  → can be used for the calculation of coalescence

  rates of BH binaries