Finding Event Horizons in Numerical Spacetimes

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Outline

- Basic Properties of the Event Horizon
- Methods for Finding the Event Horizon
- Level Set Description of a Surface
- Algorithm
- Tracking the Generators
- Examples of Event Horizons
- Concluding Remarks

Basic Properties of the Event Horizon

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The event horizon is generated by a congruence of null geodesics that once they join onto it stay on it forever.

This implies that the event horizon is (almost everywhere) a smooth null surface. The exceptions are the caustics.

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However, that means that an outgoing null geodesic will converge exponentially towards the event horizon when integrated backwards in time.

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   - Using explicit surface representations (Anninos et. al., 1995, Libson et. al., 1996)

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f(t, x^i) &= r - s(t, \theta) = z - s(t, \rho) = 0 \\
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   - Using level set description $f(t, x^i) = 0$ (Used here).

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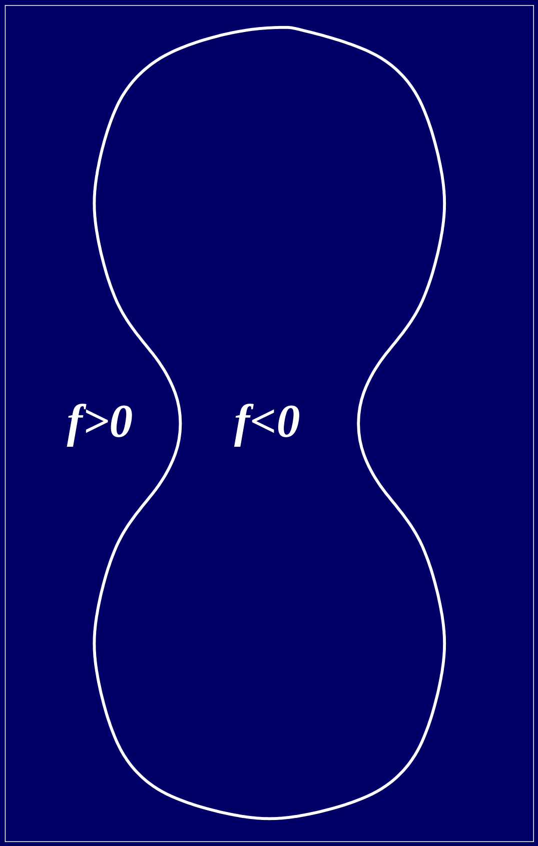

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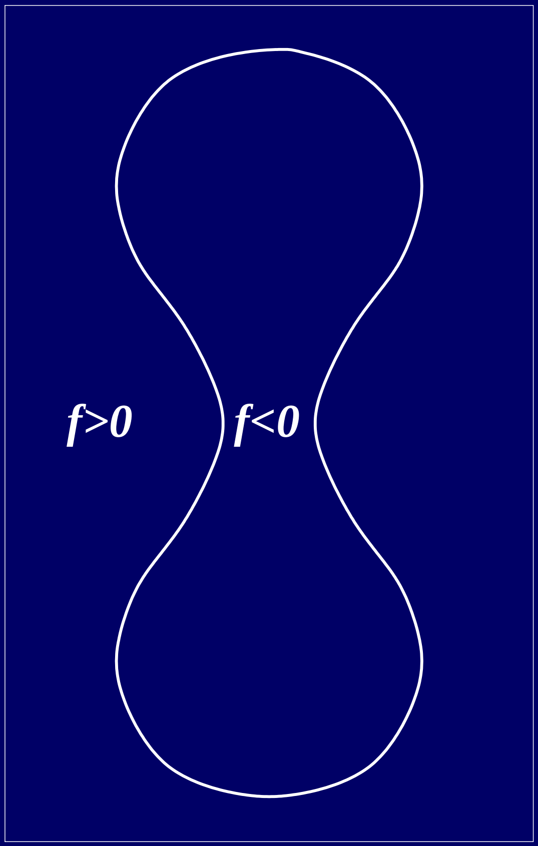

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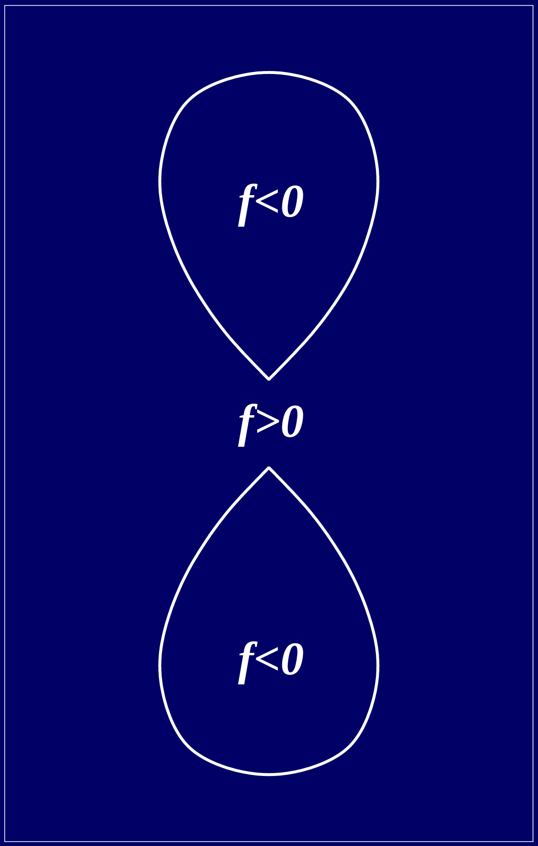

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The requirement that the surface is null, amounts to

$$n_\mu n^\mu = g^{\mu\nu} n_\mu n_\nu = g^{\mu\nu} \partial_\mu f \partial_\nu f = 0.$$

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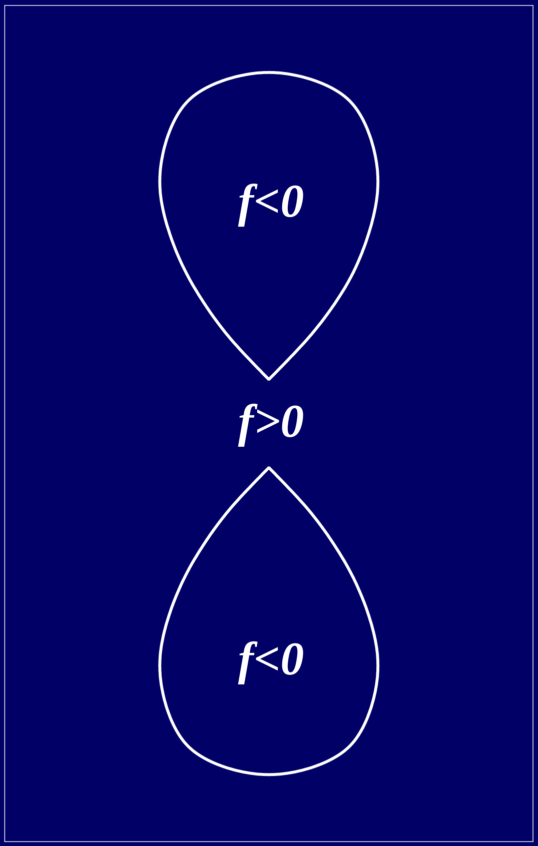

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This is the evolution equation that is integrated backwards in time given an initial guess for the event horizon on the final evolution slice.

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3. Evolve these surfaces backwards in time.

4. When the distance between the surfaces becomes small enough, the event horizon has been located and can be tracked until the initial data slice is reached.

Tracking the Generators

The generators of the event horizon satisfy in general

$$\frac{dx^\mu}{d\lambda} = A(x^\alpha) g^{\mu\nu} \partial_\nu f \,.$$

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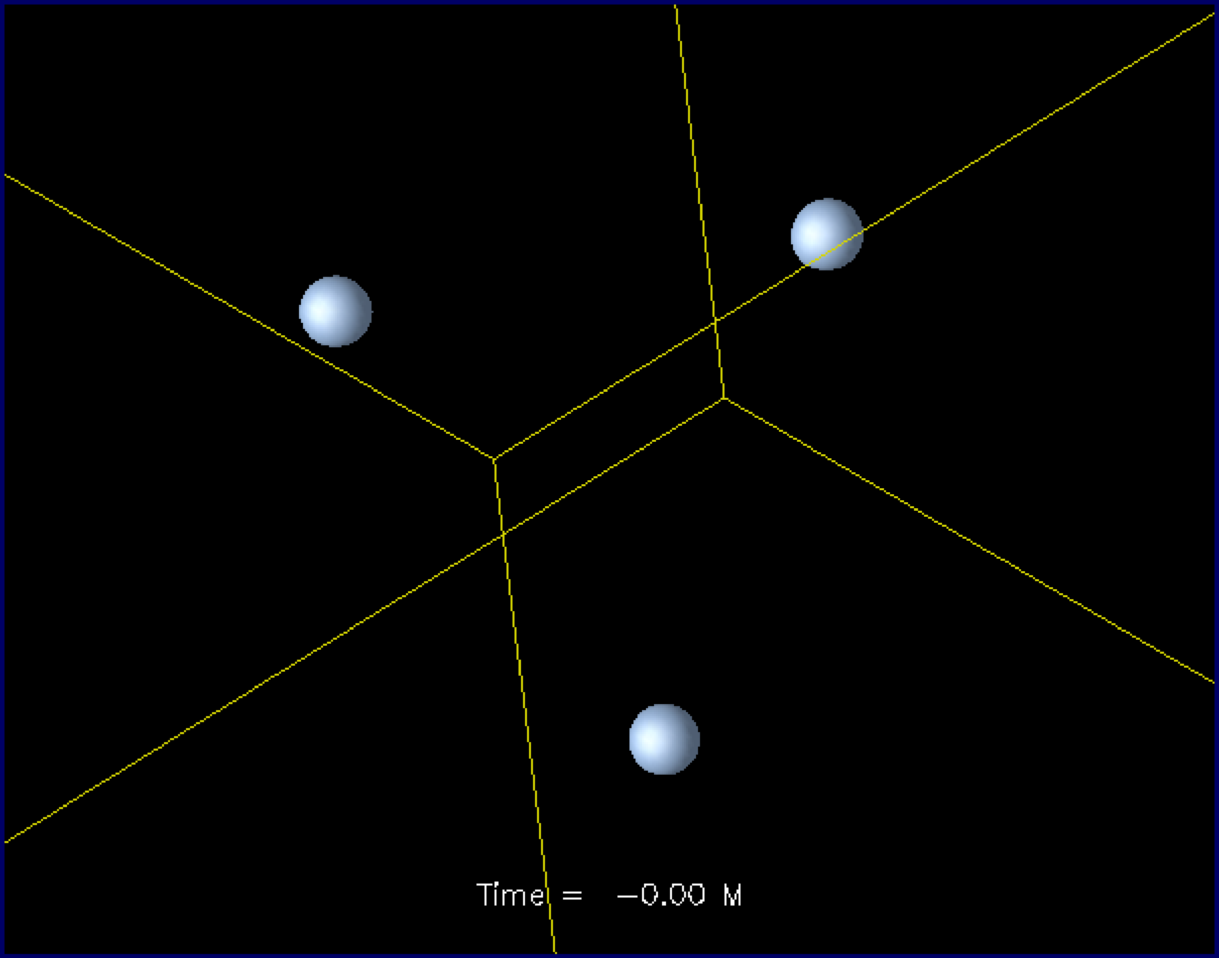
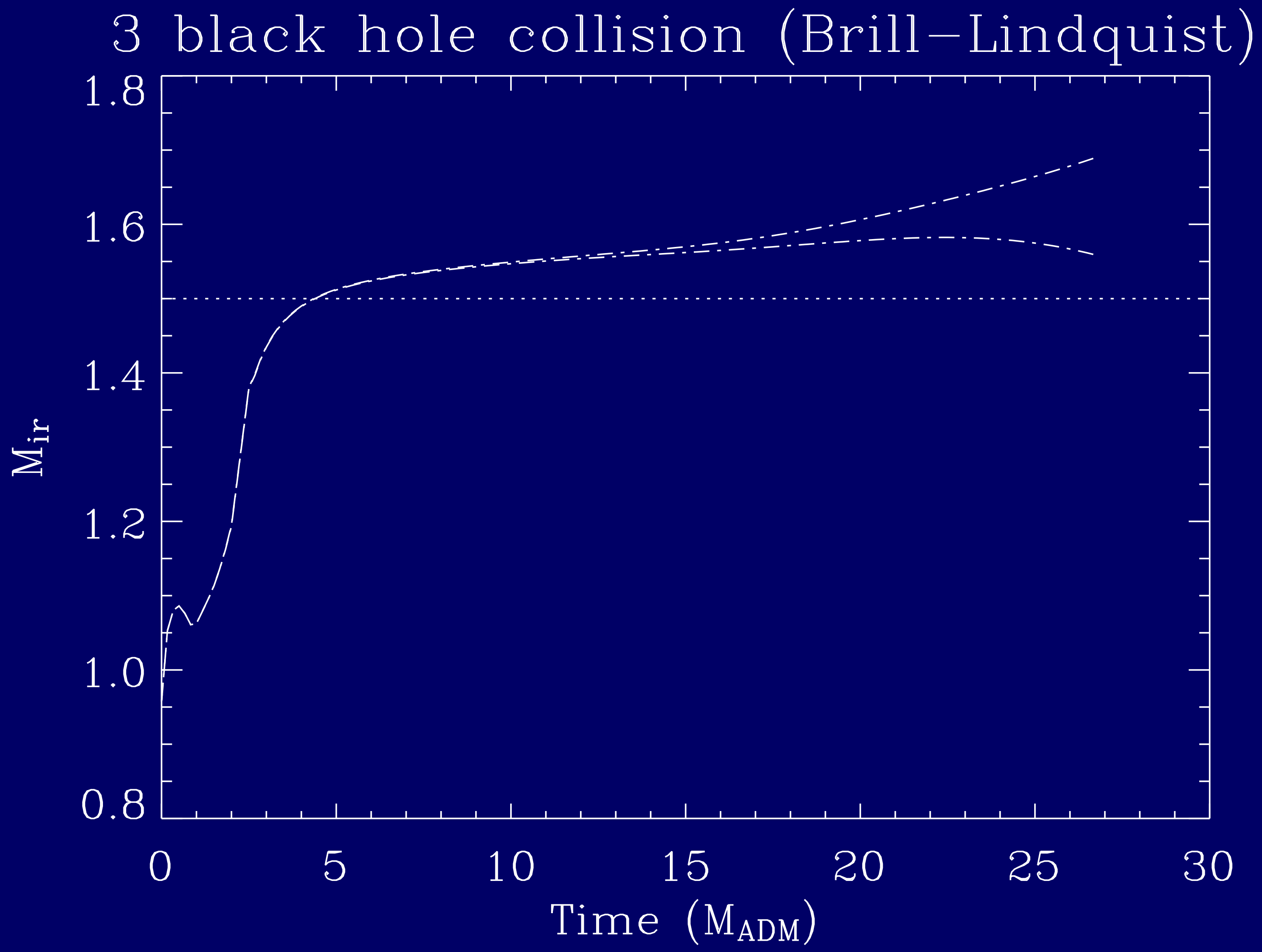
Thus the generators can be tracked without calculating derivatives of the metric. However interpolation to the generator position is necessary.

3 Black Hole Spacetime I



3 Black Hole Spacetime II



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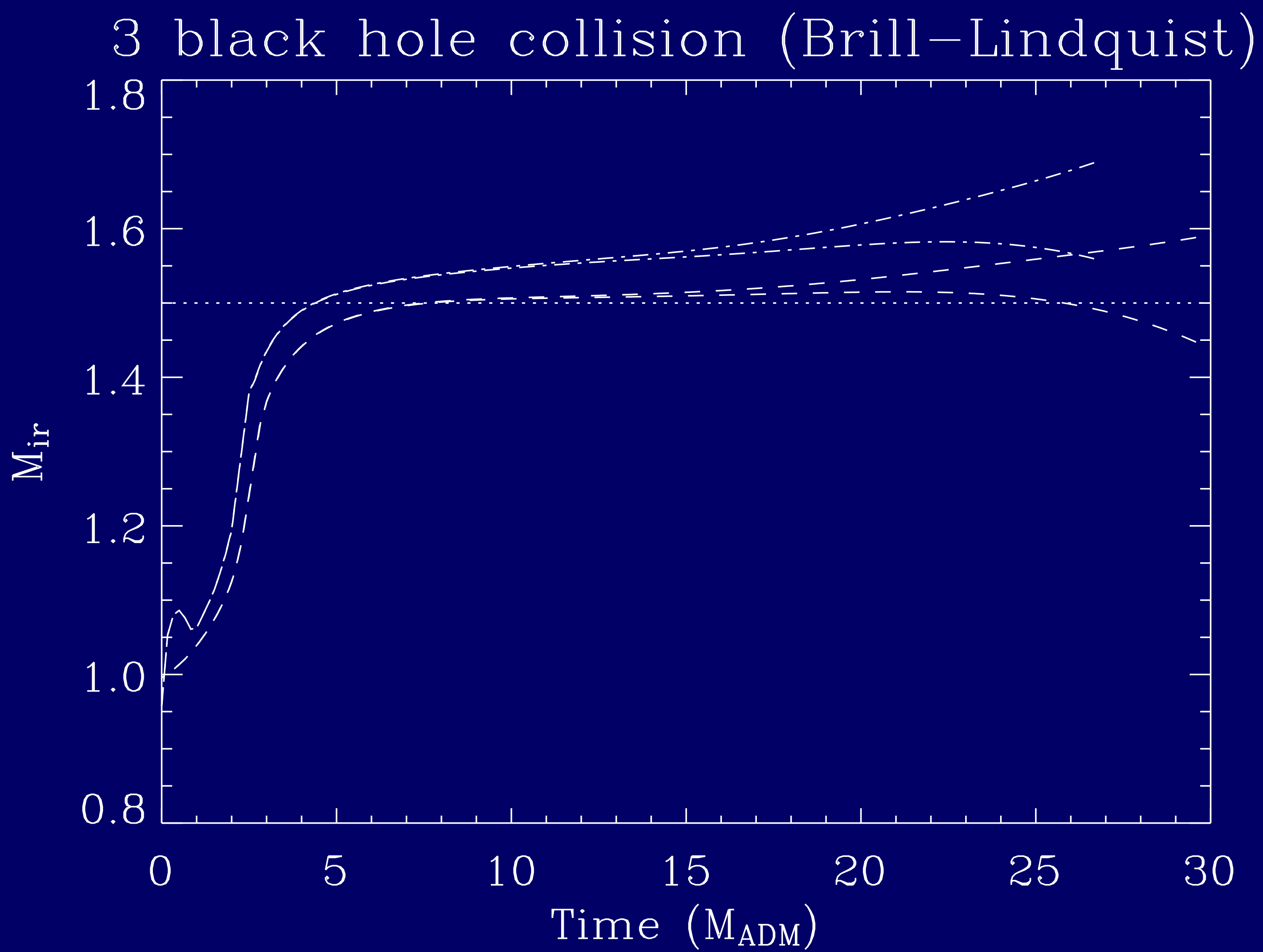

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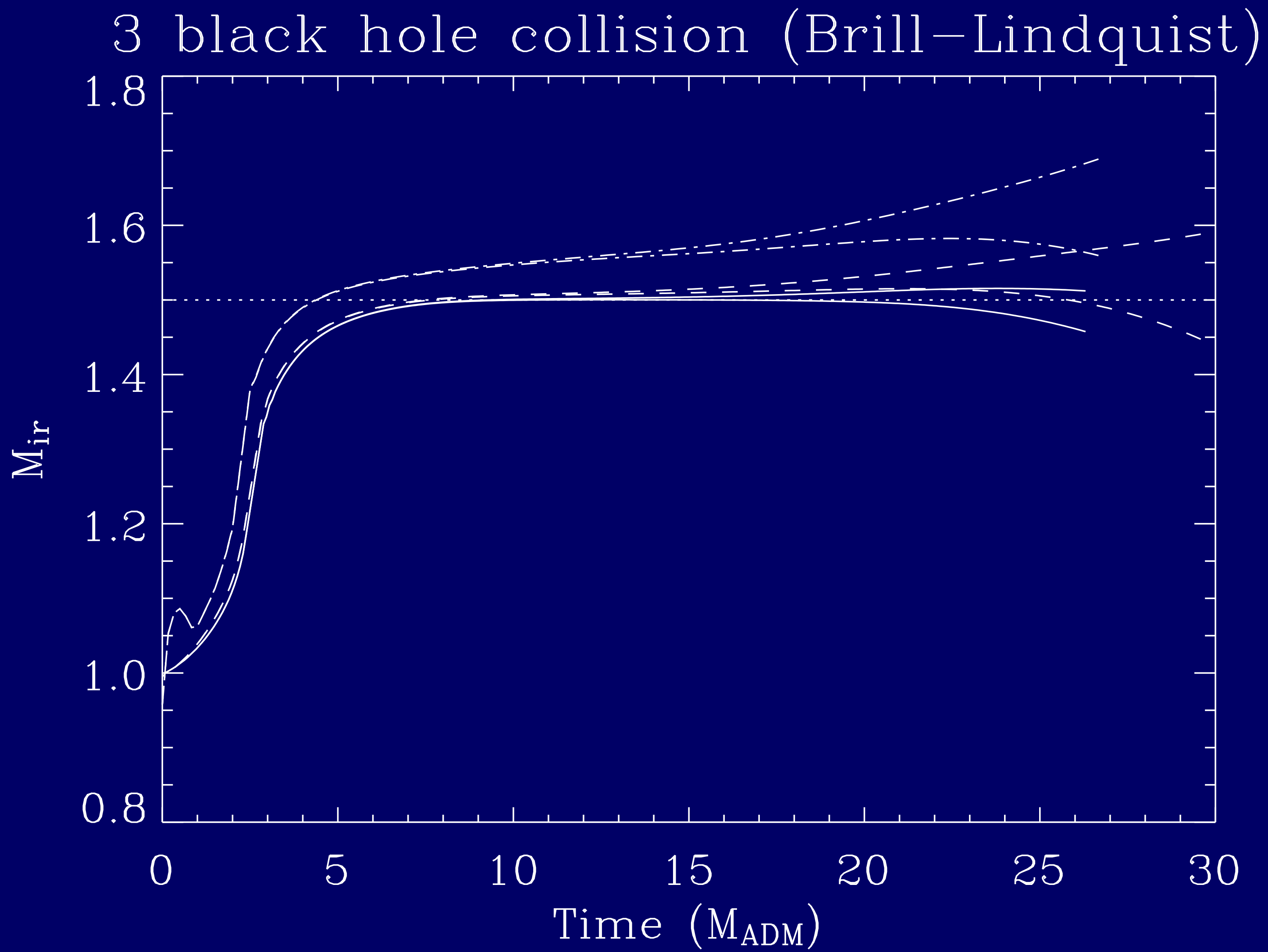
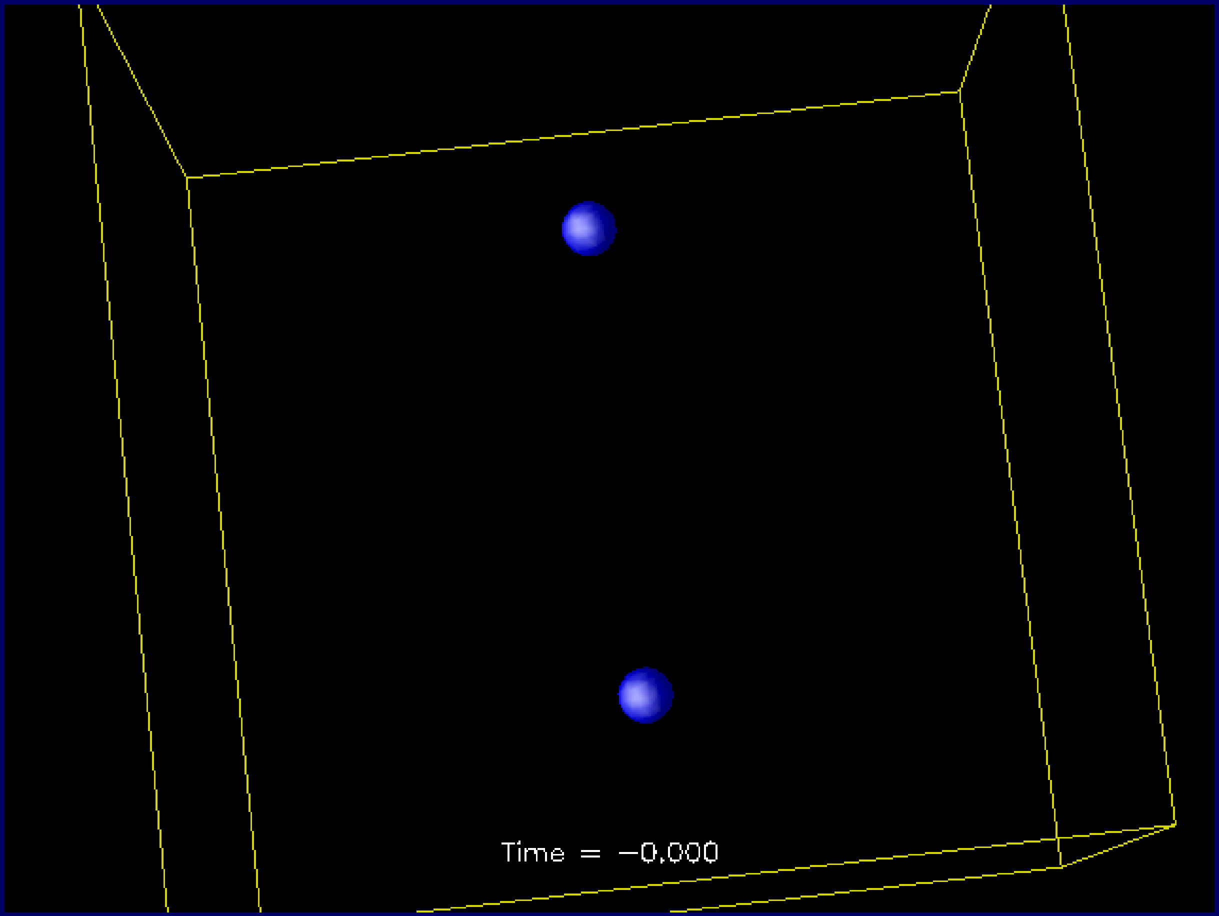
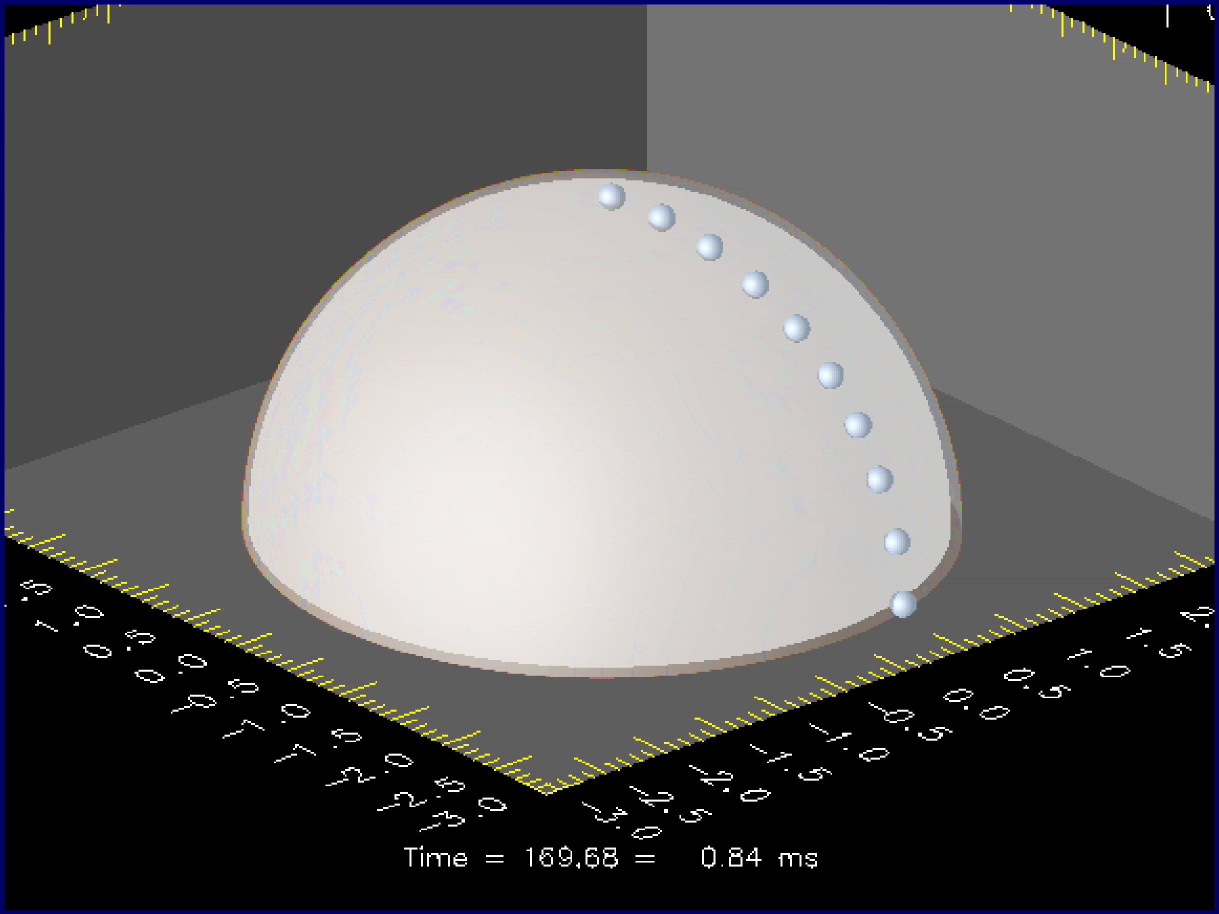


Orbiting Black Holes



Numerical run done by Frank Herrmann.

Collapsing Rotating Neutron Star



Numerical run and visualization done by Ian Hawke.
Further details on the physics in Ian's talk tomorrow.

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- For further details see gr-qc/0305039.