Highly Distorted Black Holes

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+ L. Kidder

1. Test numerical codes

2. Explore generation of harmonics/modes in black-hole spacetimes

3. Probe binary-black-hole problem and data analysis

1. Test of numerical codes

+ good test :
  - solvable with today's resources
  - captures an element of the complexity of the binary black-hole (BBH) problem
  - contains interesting physics

example : wobbling black holes — Laguna
head-on collisions — Seidel

+ single black-hole evolutions last thousands of M
  Gómez et. al (1998)
  E. Seidel's talk
  P. Laguna's talk
  Scheel et al (2002)
  Yo et al (2002)

Ready for dynamic systems

+ distorted black holes test in time-dependent way :
  - formulation of Einstein field equations
  - gauge conditions (α, β^i)
  - boundary conditions
  - wave extraction

Brandt et al (2003), Alcubierre & Brügmann (2001), Gómez (2001), Baker et al (20…
Allen et al (1998) ...

2. Generation of harmonics / modes

What modes are generated if you hit a black hole as hard as you can? — S. Finn

+ Linear theory : quasi-normal frequencies are well known
(K. Kokkotas & B. Schutz Living Reviews)

+ Non-linear generation of modes
mode coupling
transition to the linear regime

+ Allen et al Cauchy framework (1998)
gr-qc/9806014

- non-linear waves extracted through matching to perturbative solutions at $r = 15M$

- time-symmetric I.D.

- small amplitudes

P. Papadopoulos PRD 65 084016 (2002)

+ Characteristic Initial Value Problem; Ingoing null hypersurfaces

+ Initial data

$$\gamma(r,y) = \frac{\lambda}{\sqrt{2\pi w}} e^{-(r-r_c)^2/w^2} Y_{\ell 0}(y)$$

$m=0$ ∴ axisymmetric

+ Exaction of the nonlinear response

$$\gamma(v,r,y) = \sum_{\ell=2}^{\infty} \gamma_\ell(v,r) \; Y_{\ell 0}(y)$$

inverted to get

$$\gamma_\ell(v,r) = 2\pi \int_{-1}^{1} \gamma(v,r,y) \; Y_{\ell 0}(y)\, dy$$

+ Energy in mode coupling from ℓ to ℓ'

$$E_{\ell\ell'} = \varepsilon_{\ell\ell'} \lambda^{s_{\ell\ell'}} M$$

$\ell \to \ell'$	$s_{\ell\ell'}$	$\varepsilon_{\ell\ell'}$	
$3 \to 2$	4.0	2.0×10^{-2}	*
$3 \to 4$	4.0	5.5×10^{-3}	
$3 \to 5$	6.0	1.5×10^{-3}	
$3 \to 6$	4.1	2.3×10^{-2}	*

Y. Zlochower, R. Gómez, S. Husa, L. Lehner, J. Winicour
gr-qc/0306098 (2003)

+ CIVP with outgoing null hypersurfaces
non-axisymmetric ($m = 0$, $m \neq 0$)

+ ingoing pulse at $r = 3M$ $A \leq 0.36$

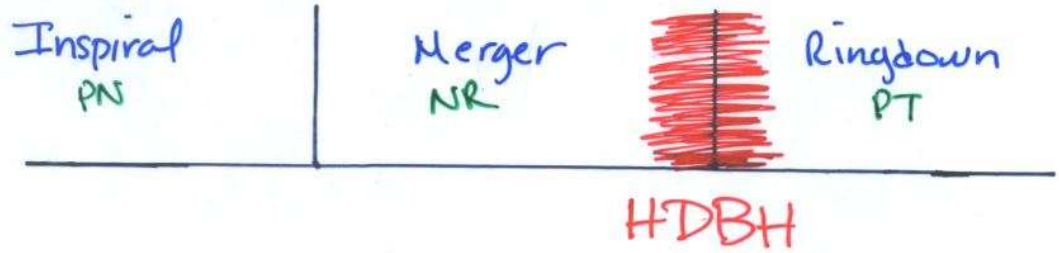
+ compute spectra

- large phase shifts $\leq 15\%$ in 1/2 cycle

- stronger than linear generation of gravitational wave output

- in $m \neq 0$ case, generation of radiation in polarization states not present in linearized approximation

3. Probing the BBH problem + data analysis



+ Information about the end of the merger phase
→ HDBH

+ Baker et al. (2002): early domination of QN frequencies prior to linear regime

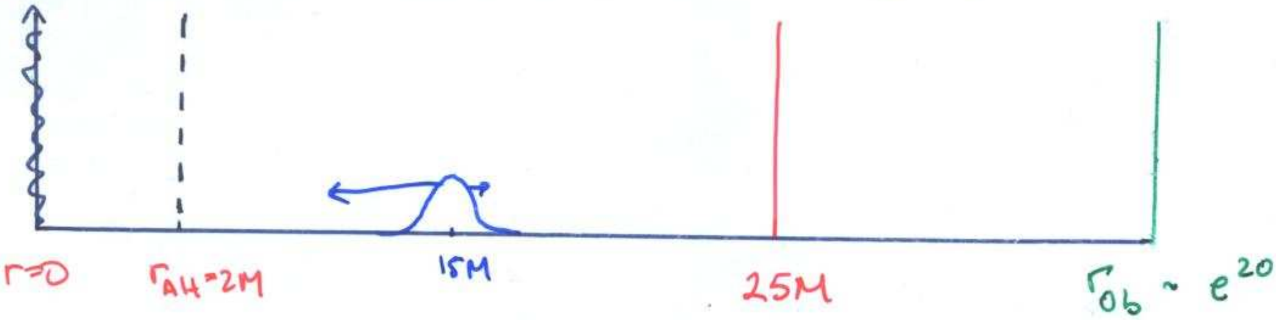
+ Study the transition from non-linear to linear with initial data (A, ω, ℓ, m)

+ search for bulk features ⟶ data analysis

- not have 1000's of templates

- Flanagan + Hughes (1998) suggest bulk features to identify sources in data stream

Distorted a black hole: scattering problem



Our Approach:

+ Cauchy 3+1 code
+ solve the initial data with a pseudo-spectral elliptic solver
+ Evolve black hole and wave with pseudo-spectral code
+ good framework for this problem

Initial Data

+ ingoing Eddington-Finkelstein black hole + pulse

$$\gamma_{ij} = \gamma^0_{ij} + A h_{ij}$$

+ h_{ij} is a gravitational wave (Teukolsky) $\ell = 0, 1, 2, 3, 4 \ldots$ $-\ell \le m \le \ell$

  $A \ll 1 \rightarrow$ linear gravitational wave

+ shape of pulse $F(x) = e^{-(x-x_0)^2/\omega^2}$

* + the constraints are then solved

Evolution Code

+ first-order symmetric hyperbolic form of equations (KST)
  exact, densitized lapse and exact shift

+ pseudo-spectral collocation method
  ⟹ high accuracy
  exponential convergence

  π collocation points / wavelength (PSC) vs. $\sim 20/\lambda$ (FD)

+ trivial excision
+ multi-domain PSC ⟹ "fixed mesh refinement"
  - move outer boundary very far away
  - resolve gravitational wave

(Bartnik + Norton (1999), Bonazzola et al (1999), Gourgoulhon (2002), Grandclément (2002))

+ represent our variables

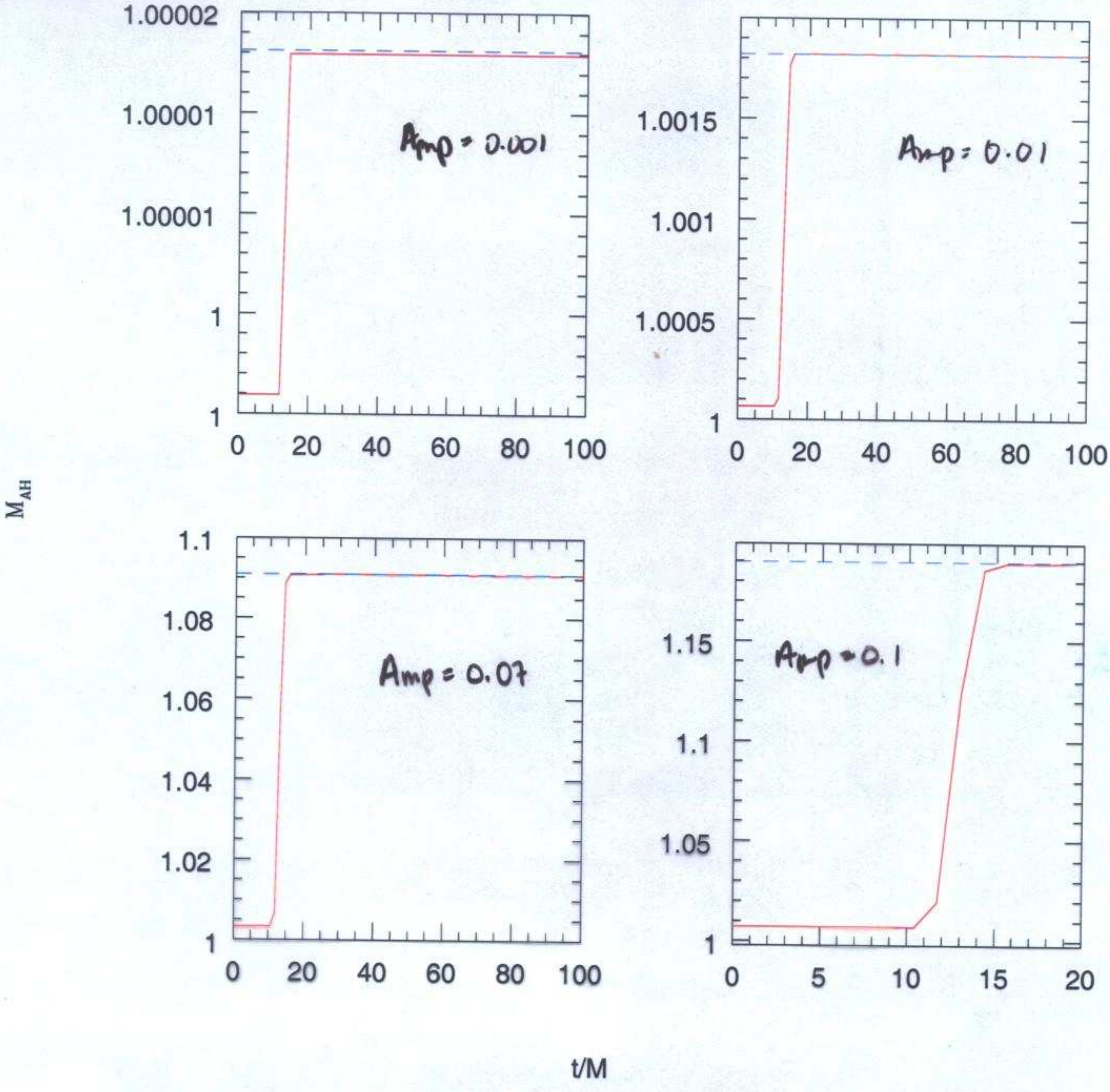
$$g_{xx}(r,\theta,\varphi,t) = \sum_{\ell m} A_{\ell m}(r,t)\, Y_{\ell m}(\theta,\varphi)$$

⟹ mode decomposition into $Y_{\ell m}$ is straight forward

(E. Seidel, use of AH to read QNMs)

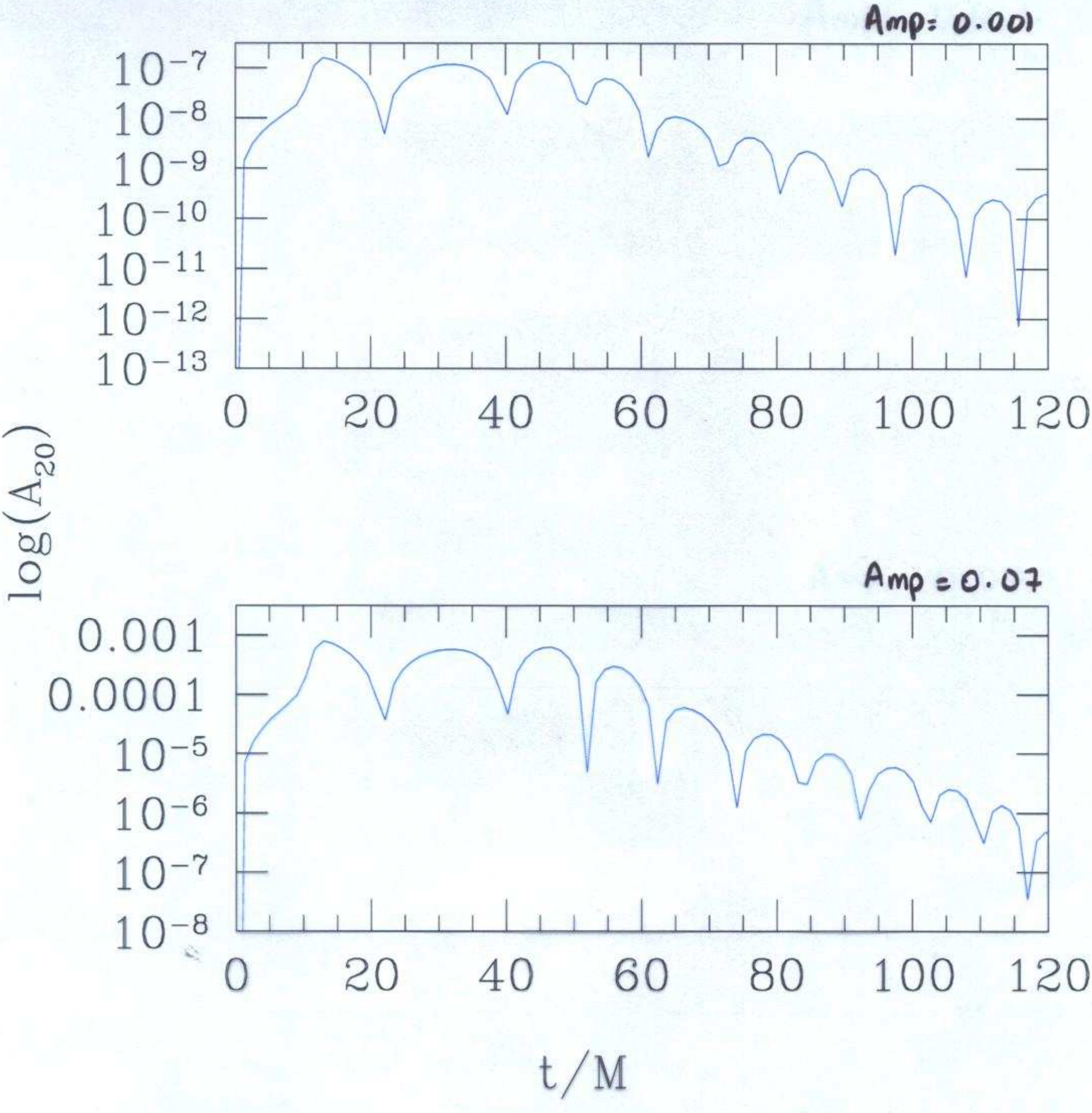
+ pulse starts at 15 M (unlike the 3M of Papadopoulos + Zlochower) → ingoing toward black hole

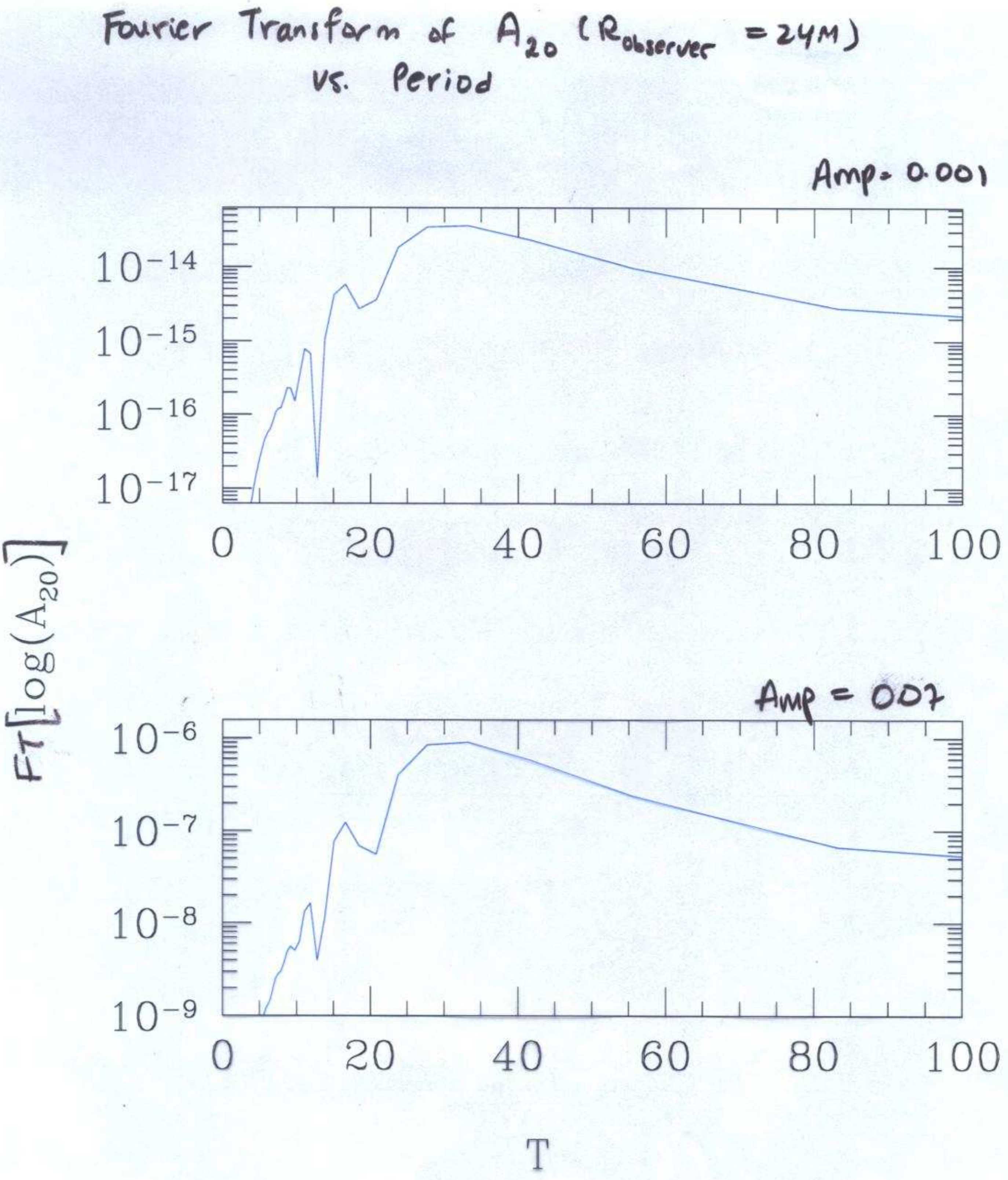
Mass of Apparent Horizon vs. time
compared to ADM Energy of Initial Data



- - - - - E_{ADM} (Initial Data)

Amplitude of spherical Harmonic (A_{20}) $l=2$, $m=0$ versus time at $R_{observer} = 24M$





Fourier Transform of A20 (Robserver = 24M) vs. Period
Amp= 0.001
10−14
10−15
10−16
10−17
0
20
40
60
80
100
FT[log(A20)]
Amp = 007
10−6
10−7
10−8
10−9
0
20
40
60
80
100
T

Our Plan

+ Systematic study (ω, A, ℓ, m) of HDBHs
+ Add highest possible A.
+ Add angular momentum, $\vec{J}$.
    - important for observations
      astrophysical BHs $\vec{J} \neq 0$
    - harder to get clean in-state with $\vec{J} \neq 0$
      ∴ we start pulse far enough away
      to be in Schwartschild or flat space
    - rotation couples modes ∴ use perturbative
      calculation to disentangle linear from
      non-linear couplings.

Open Questions

+ What are the salient features of HDBH
  gravitational waves?
+ How far can we push the linearized regime?
+ How does / when does the transition occur
  to QN ringing in the BBH system?