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MHD Turbulence in the Heliospheric Plasma: Coherent Structures and Large Scale Correlations

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These are preliminary lecture notes, intended only for distribution to participants.

MHD Turbulence in Heliospheric Plasma: Coherent structures and large scale correlations

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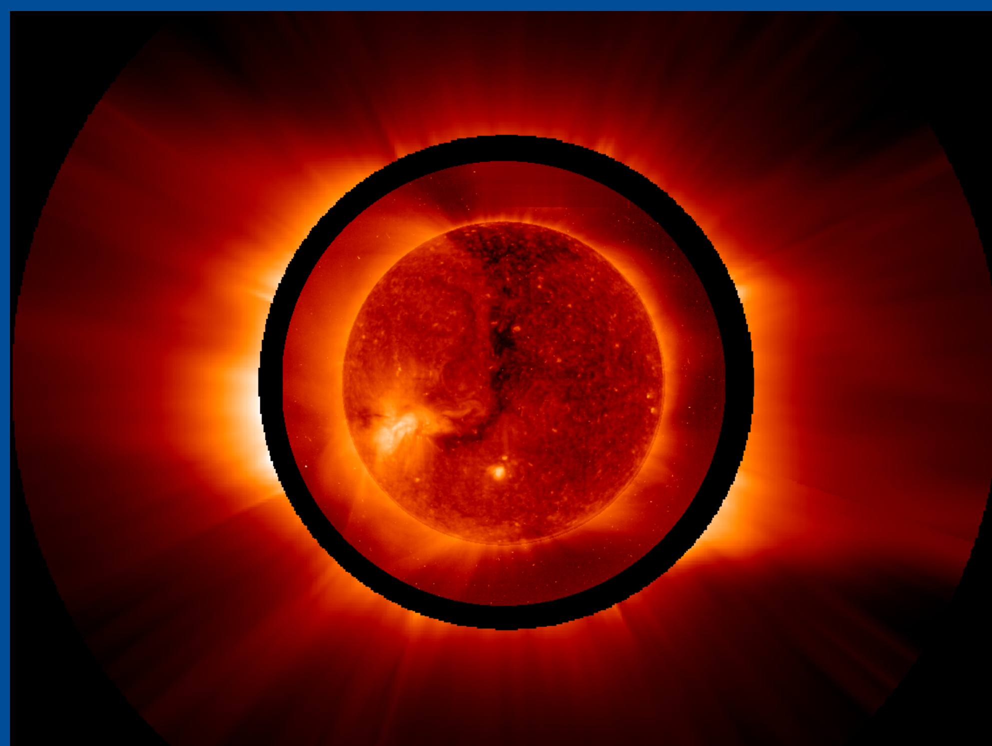
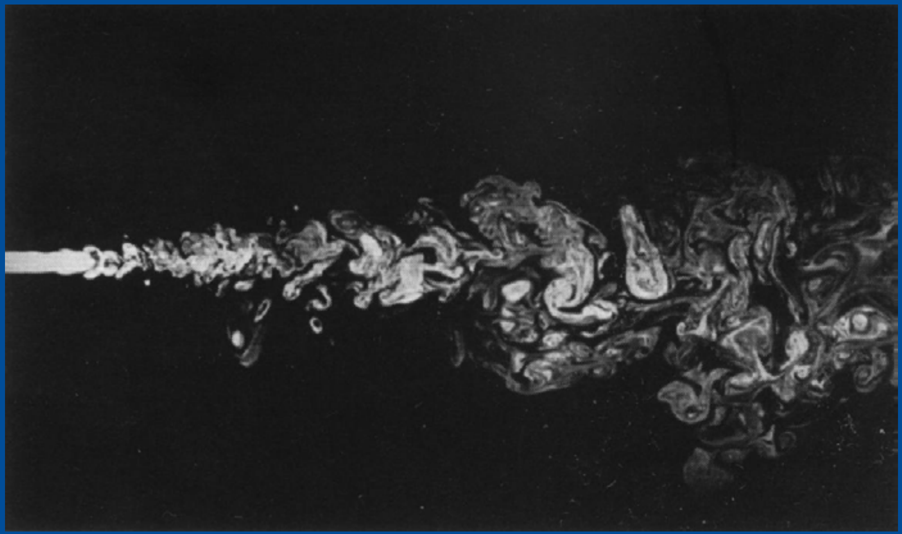
Cosenza - Italy



Main features of turbulent flows



1) Randomness both in space and time.
2) Turbulent "structures" on all scales within the sea of a random background.
3) General unpredictability and instability to very small perturbations.

Turbulence is the result of nonlinear dynamics in DETERMINISTIC (but chaotic) systems

While the details of turbulent motions are extremely sensitive to triggering disturbances, statistical properties are robust quantities

Predictability is reintroduced at a statistical level (via the ergodic theorem and the properties of chaos !).

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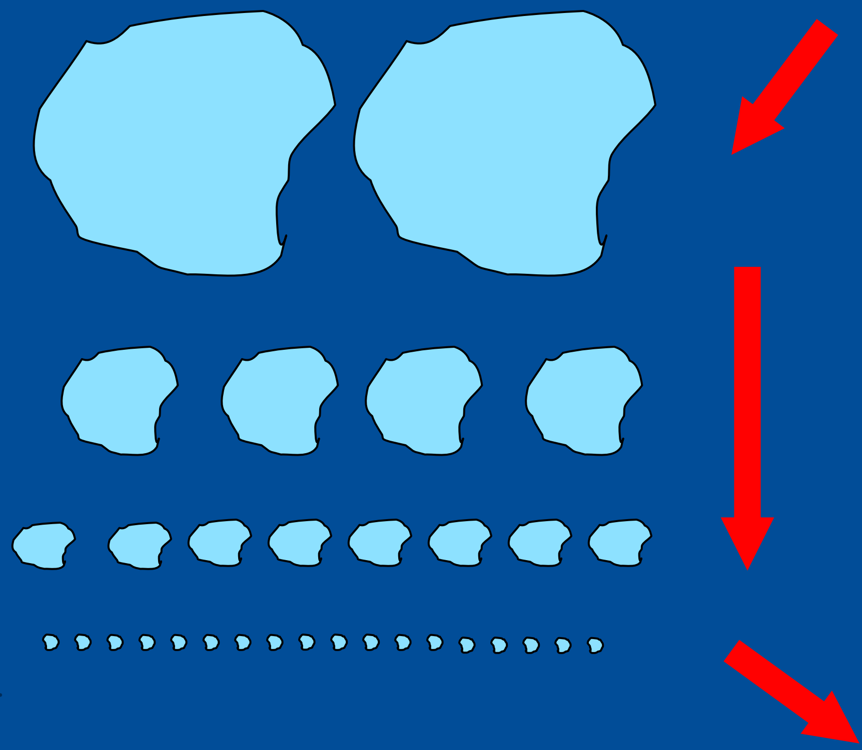
The Richardson's picture

Richardson's phenomenology: break down of eddies at large scales and transfer of energy.



Energy injection range

Inertial range

Energy dissipation range

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Leonardo's view on fluid turbulence

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Leonardo da Vinci (around 1500):

"dove la turbolenza dell'acqua si genera,
dove la turbolenza dell'acqua si mantiene più a lungo,
dove la turbolenza dell'acqua si posa"

"where the turbulence of water is generated,
where the turbulence of water maintains for long,
where the turbulence of water comes to rest"

(Piumati 1894, fo. 74,v - as reported in "Turbulence" by U. Frish)

Modern views on fluid turbulence

Sir Horace Lamb (1932):
"When I die and I go to Heaven, there are two matters on which I hope for enlightment. One is Quantum Electro-dynamics, and the other is the turbulent motion of fluids. And about the former I am really optimistic"

K. L. Sreenivesan (Nature, **344**, 192; 1990):
"One no longer needs to go to Heaven to seek enlightment about Q. E. but, on the Earth turbulence still defies satisfactory description"

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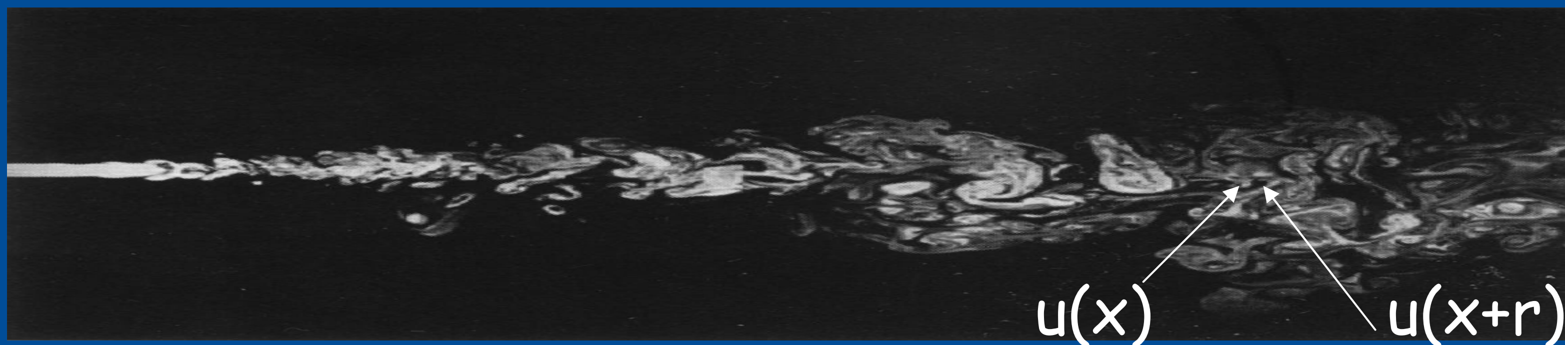
Two-points correlations



Main analysis tools: two-points differences

$$\delta u_r(x) = u(x+r) - u(x)$$

Fluctuations associated to eddies at the scale r .

Statistical homogeneity $\Rightarrow$ the 2-th order moment of two-points differences is related to the energy spectrum

$$\left\langle [\delta u_r(x)]^2 \right\rangle = 2\int_0^\infty E(k)\left[1 - \frac{\sin kr}{kr}\right]dk$$

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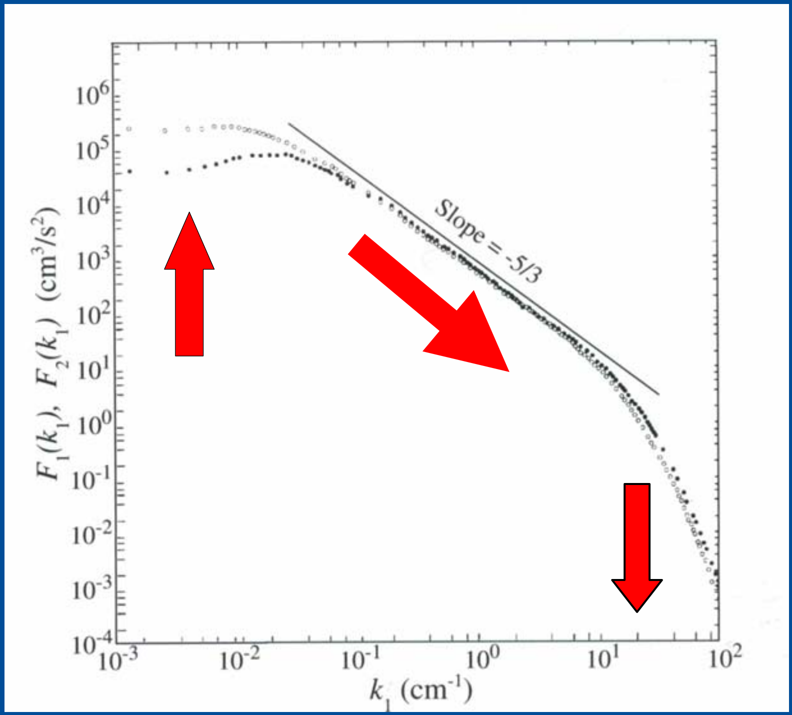
Statistical predictability: universal energy spectrum

Energy input
at large scale L

Energy cascade
in the inertial range
due to non linear
terms

Energy dissipation
at small scale l_d



$$\left\langle \delta u_r^{\,2} \right\rangle^{1/2} \approx r^{1/3}$$

$$E(k) \approx k^{-5/3}$$

A.N. Kolmogorov:
in the inertial range
universal feature

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Solar wind data

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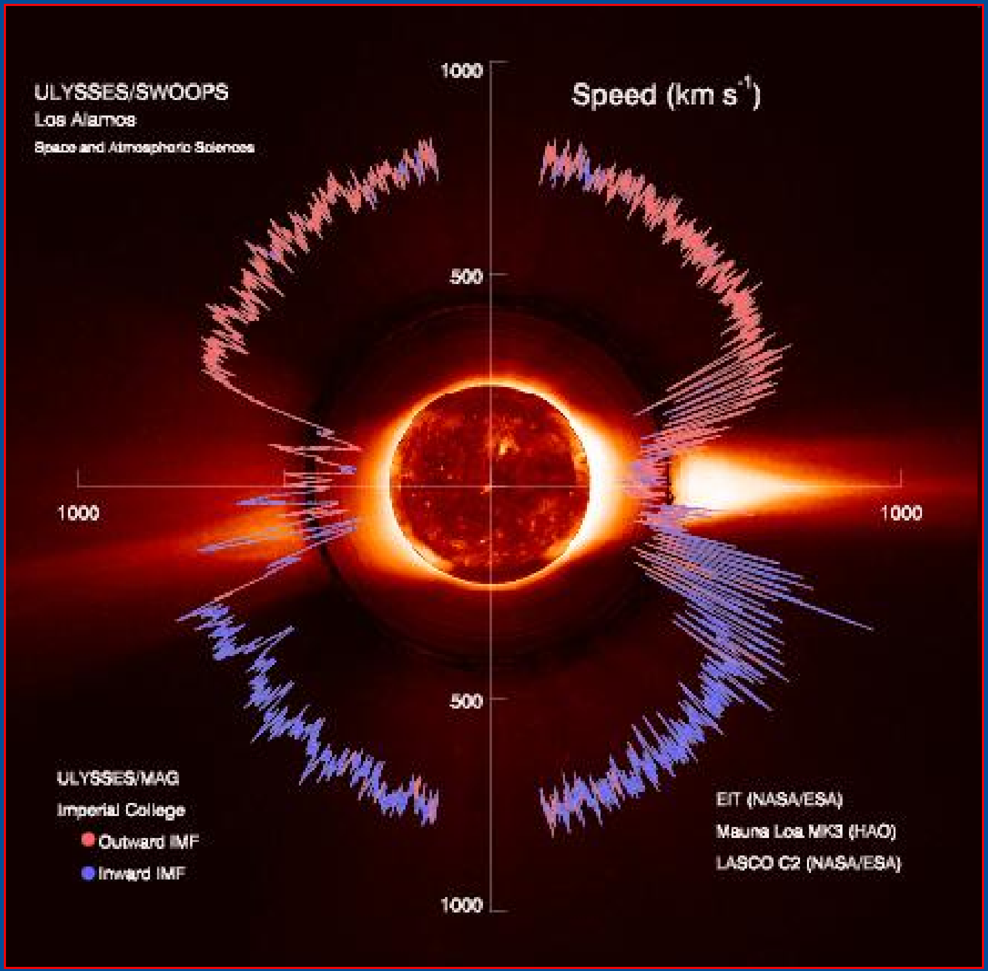
Solar wind: a wind tunnel
Supersonic and superalfvenic flow

$250\ km/s < V_{sw} < 800\ Km/s$

In situ measurements of high amplitude fluctuations of velocity and magnetic fields over 6 decades of frequencies (up to ion cyclotron frequency)

$10^{-6}\ Hz < f < 1\ Hz$



Trieste
2003

Heliospheric current sheet

A current sheet is embedded in the slow wind

In the equatorial region (most widely studied by space experiments):
Bending of the current structure

Stream structure

Fast streams:

v ~ 600 - 800 Km/s
n ~ 1 - 4 cm^{-3}
T ~ 10^4 - 10^5 ^{0}K

Slow streams:

v ~ 250 - 400 Km/s
n ~ 10 - 40 cm^{-3}
T ~ 10^3 - 10^4 ^{0}K

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Power law spectrum

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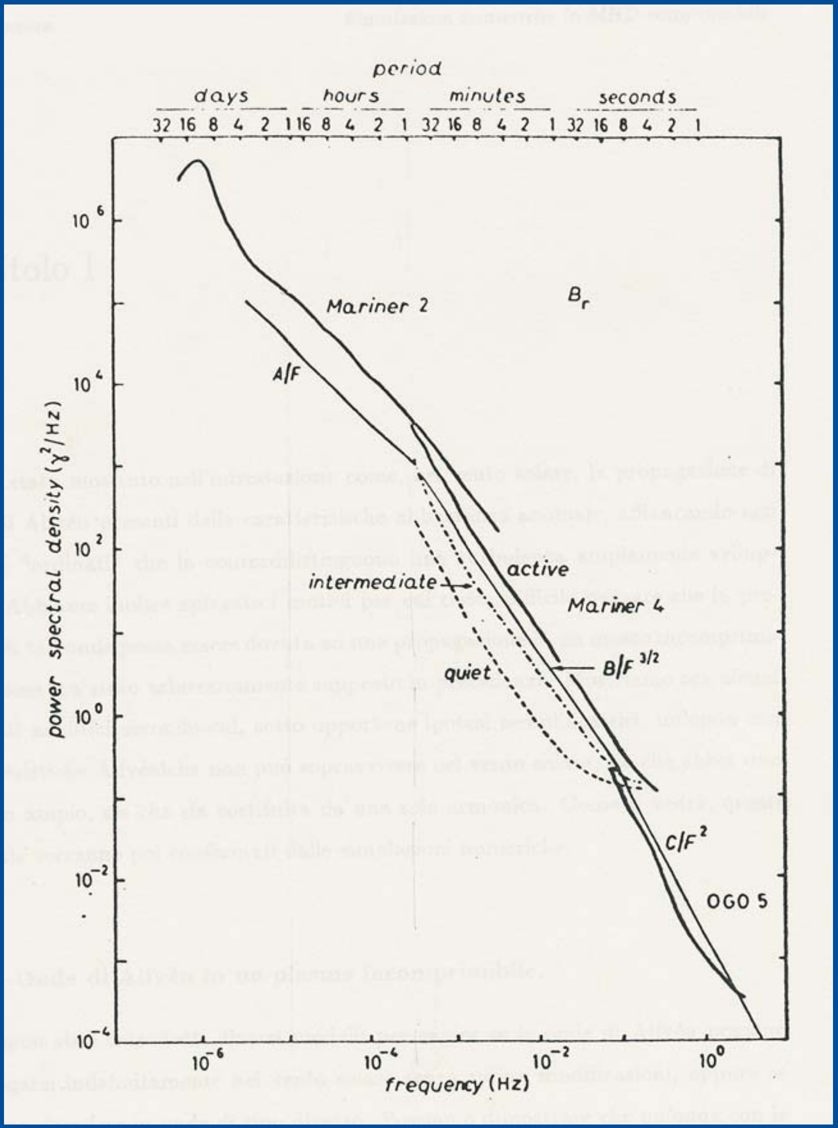
Magnetic and velocity fluctuations display a power law spectrum extending over more than 5 decades (10^{-6} Hz < f < 1 Hz) :

$W \sim f^{-\alpha}$

Signature of NL turbulent energy cascade



From Taylor hypothesis:
$L \sim V_{sw} / f$

Scale range:
400 Km < L < 1 AU

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Alfvenic Correlation

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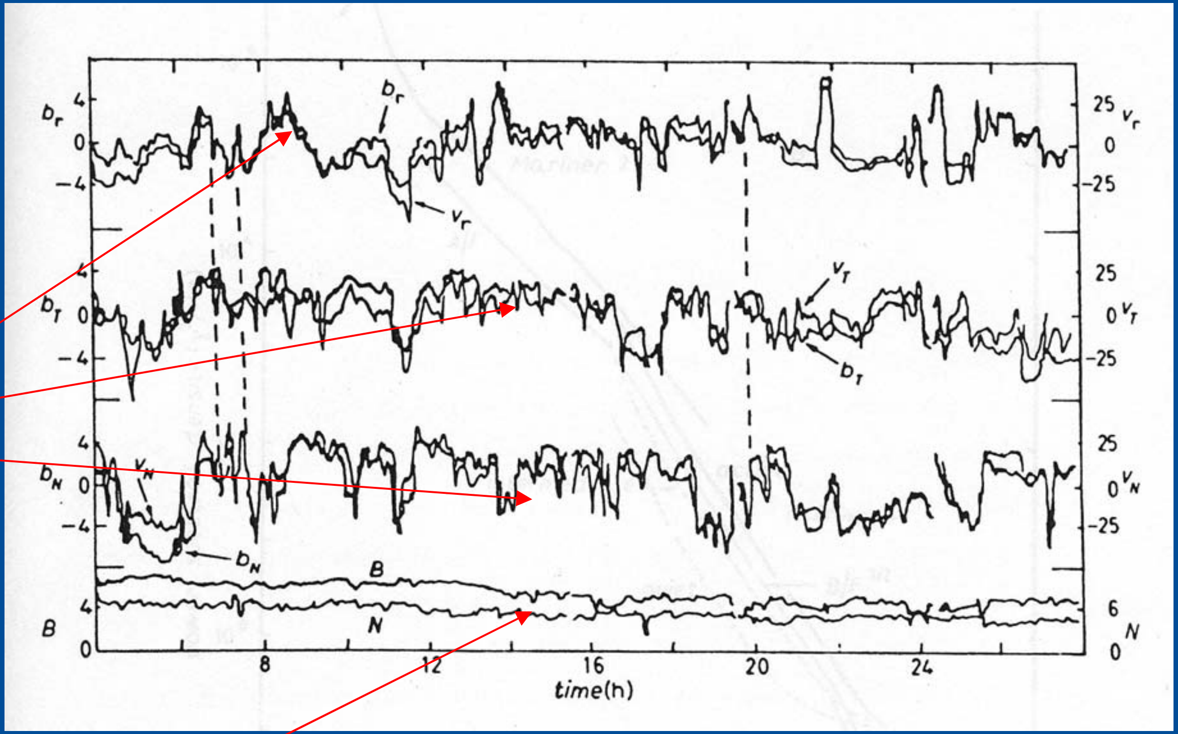
In fast streams:
velocity and magnetic fied fluctuations highly correlated

$\delta \mathbf{v} \sim \sigma\, \delta \mathbf{B} \;/(4\pi\rho)^{1/2}$

σ= +, -

The sign corresponds to
NL Alfven waves
($\delta B / B_0 \sim 1$)
propagating away from the sun.



Low level of density and magnetic field intensity fluctuations $\delta\rho / \rho \sim \delta|B|/B_0 \sim$ few percents
NL effects reduced

In slow streams: (i) almost no $\delta \mathbf{v}$ - $\delta \mathbf{B}$ correlation
(ii) high level of compressive fluctuations $\delta\rho / \rho \sim \delta|B|/B_0 \sim 1$

MHD equations

Elsasser's variables: $$\mathbf{z}^\sigma = \mathbf{v} + \sigma \frac{\mathbf{B}}{4\pi\rho}$$ for $\sigma = +1, -1$

MHD equations: $$\frac{\partial \mathbf{z}^\sigma}{\partial t} + \left(\mathbf{z}^{-\sigma} \cdot \nabla\right)\mathbf{z}^\sigma = -\nabla\left(P + \frac{B^2}{8\pi}\right) + \nu\nabla^2\mathbf{z}^\sigma$$

Nonlinear term

Dissipative term

$$R = \frac{nonlinear}{dissipative} \approx \frac{zL}{\nu}$$

The Reynolds number is
the only parameter of the flow

MHD equations display the same "structure"
as Navier-Stokes equations

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Alfvenic fluctuations

In a statistically homogeneous medium:

$$< \mathbf{B} >= \mathbf{B}_0$$
$$< \mathbf{v} >= 0$$

$$\delta \mathbf{z}^{\sigma} = \delta \mathbf{v} + \sigma \frac{\delta \mathbf{B}}{4\pi\rho}$$

Elsasser's variables fluctuations

for $\sigma = +1, -1$

Ideal MHD NL solutions:

$$P + \frac{B^2}{8\pi} = const.$$

and either

$$\delta \mathbf{z}^{+} = 0$$
$$\delta \mathbf{z}^{-} = \mathbf{F}(\mathbf{r} - \mathbf{v}_A t)$$

$$\longrightarrow \quad \delta \mathbf{v} = -\frac{\delta \mathbf{B}}{4\pi\rho}$$

or

$$\delta \mathbf{z}^{+} = \mathbf{F}(\mathbf{r} + \mathbf{v}_A t)$$
$$\delta \mathbf{z}^{-} = 0$$

$$\longrightarrow \quad \delta \mathbf{v} = \frac{\delta \mathbf{B}}{4\pi\rho}$$

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Ideal MHD quadratic invariants

Energy per mass unit

$$\mathbf{E} = \int \left(\frac{\mathbf{v}^2}{2} + \frac{B^2}{8\pi\rho} \right) dV = \int \left(\frac{z^{+2} + z^{-2}}{4} \right) dV$$

Cross helicity

$$\mathbf{H}_\mathbf{c} = \int (\mathbf{v} \cdot \mathbf{B}) dV$$

Energy and cross helicity conservation is equivalent to pseudo energies conservation

Pseudoenergies:

$$\mathbf{E}^\sigma = \int \frac{z^{\sigma 2}}{2} dV$$

for $\sigma = +1, -1$

In 3D magnetic helicity is also invariant

$$\mathbf{H}_\mathbf{m} = \int (\mathbf{A} \cdot \mathbf{B}) dV$$

In 2D the third invariant is

$$\mathbf{H} = \int |\mathbf{A}|^2 dV$$

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Non linear MHD cascade 1

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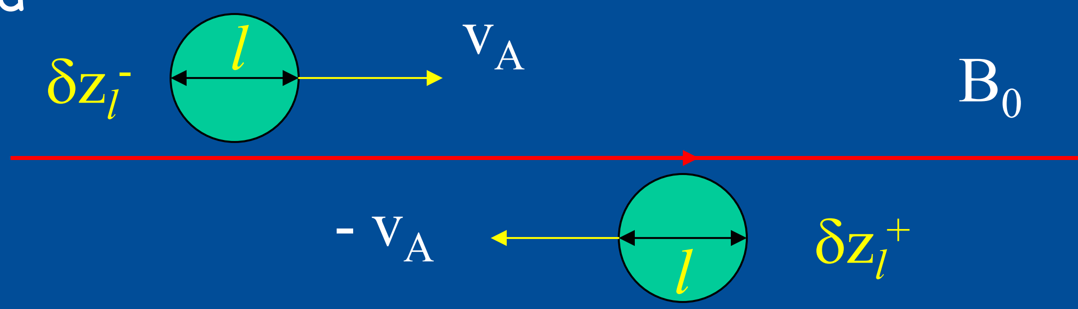
Both pseudoenergies are conserved in the NL cascade along the spectrum

$$\Pi_l^\sigma \sim \frac{\delta z_l^{\sigma 2}}{T_l^\sigma} \sim \varepsilon^\sigma$$

Alfven effect: interacting eddies move in opposite directions in a large scale magnetic field



Interaction stops after a time $\tau_l^A \sim l / v_A$

After N uncorrelated interactions, i.e. after a time $T_l \sim N \tau_l^A$

$$\Delta \mathbf{z}_l^\sigma \sim \sqrt{N}\, \frac{\delta z_l^{-\sigma}\, \delta z_l^\sigma}{l}\, \tau_l^A$$

A significant variation of δz_l^σ requires $\delta z_l^\sigma \sim \Delta z_l^\sigma$ thus

$$N \sim \left(\frac{v_A}{\delta z_l^{-\sigma}} \right)^2 >> 1 \quad \rightarrow \quad T_l^\sigma \sim \left(\frac{v_A}{\delta z_l^{-\sigma}} \right)^2 \frac{l}{v_A}$$

Non linear MHD cascade 2

The energy flow along the spectrum is independent of σ

$$\Pi_l^\sigma \sim \frac{\delta z_l^{\sigma\,2}\,\delta z_l^{-\sigma\,2}}{l v_A} \sim \varepsilon$$

The spectrum is formed when there is a sufficient energy on both modes of propagation, i.e. $\delta z_l^- \sim \delta z_l^+ \sim \delta z_l$ then

$$\delta z_l \sim \varepsilon^{1/4} v_A^{1/4} l^{1/4} \longrightarrow E(k) \propto k^{-3/2}$$

Kraichnan's spectrum

After the spectrum formation pseudoenergies are transferred along the spectrum (and dissipated) at the same rate

$$E^- - E^+ = const \longrightarrow \int \mathbf{v}\cdot\mathbf{B}\, dV = const$$

Cross helicity is conserved while energy is dissipated

In the final state $E^+ = 0$ $\longrightarrow$ $\delta\mathbf{v} = \frac{\delta\mathbf{B}}{\sqrt{4\pi\rho}}$

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Fourier analysis in a cubic box

In a periodic cubic box of size L

$$\mathbf{k} = \frac{2\pi}{L}\mathbf{n}$$

$$z_\alpha^\sigma(\mathbf{r},t) = \sum_{\vec{k}} z_\alpha^\sigma(\mathbf{k},t) e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$z_\alpha^\sigma(\mathbf{k},t) = L^{-3}\int z_\alpha^\sigma(\mathbf{r},t) e^{-i\mathbf{k}\cdot\mathbf{r}} d\mathbf{r}$$

Fourier analysis

Divergenceless condition

3D

$$\mathbf{z}^\sigma(\mathbf{k},t) = \sum_{\alpha=1,2} z_\alpha^\sigma(\mathbf{k},t)\mathbf{e}_\alpha(\mathbf{k})$$

$$\mathbf{k}\cdot\mathbf{e}_\alpha(\mathbf{k}) = 0$$

$$\mathbf{e}_1(\mathbf{k}) = \frac{i\mathbf{k}\times\mathbf{B}_0}{|\mathbf{k}\times\mathbf{B}_0|}; \mathbf{e}_2(\mathbf{k}) = \frac{i\mathbf{k}}{|\mathbf{k}|}\times\mathbf{e}_1(\mathbf{k})$$

2D

$$\mathbf{z}^\sigma(\mathbf{k},t) = z^\sigma(\mathbf{k},t)\mathbf{e}(\mathbf{k})$$

$$\mathbf{k}\cdot\mathbf{e}(\mathbf{k}) = 0$$

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MHD equations for Fourier modes

The evolution of the field for a single wave vector is related to fields of all other wave vectors (convolution term) for which **k = p + q**.

$$\left(\frac{\partial}{\partial t} - i\sigma \mathbf{k}\cdot\mathbf{v}_{\mathbf{A}} + \upsilon k^2\right) z_\alpha^\sigma(\mathbf{k},t) = \sum_{\beta,\gamma=1}^{2} \sum_{\mathbf{p}+\mathbf{q}=\mathbf{k}} M_{\alpha\beta\gamma}(\mathbf{k},\mathbf{p},\mathbf{q}) z_\gamma^\sigma(\mathbf{p},t) z^{-\sigma}(\mathbf{q},t)$$

where

$$M_{\alpha\beta\gamma}(\mathbf{k},\mathbf{p},\mathbf{q}) = \left(i\mathbf{k}\cdot\mathbf{e}_\gamma(\mathbf{q})\right)\left(\mathbf{e}_\alpha{}^{*}(\mathbf{p})\cdot\mathbf{e}_\beta(\mathbf{q})\right)$$

Infinite number of modes involved in nonlinear interactions for inviscid flows

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Direct numerical simulations 1

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Pseudospectral methods:

- Time integration in Fourier space;
- Non linear terms evaluated in physical space;
- From Fourier space to physical space and viceversa Fast Fourier transforms (FFT) are used.

From phenomenological arguments:

$$\frac{k_D}{k_I} \sim \mathrm{Re}^{3/4}$$

High performance computers:

- 2D Re ~ 10^4
- 3D Re ~ 10^3

Direct numerical simulations 2

From Grappin, 1986

Velocity field lines

Magnetic field lines

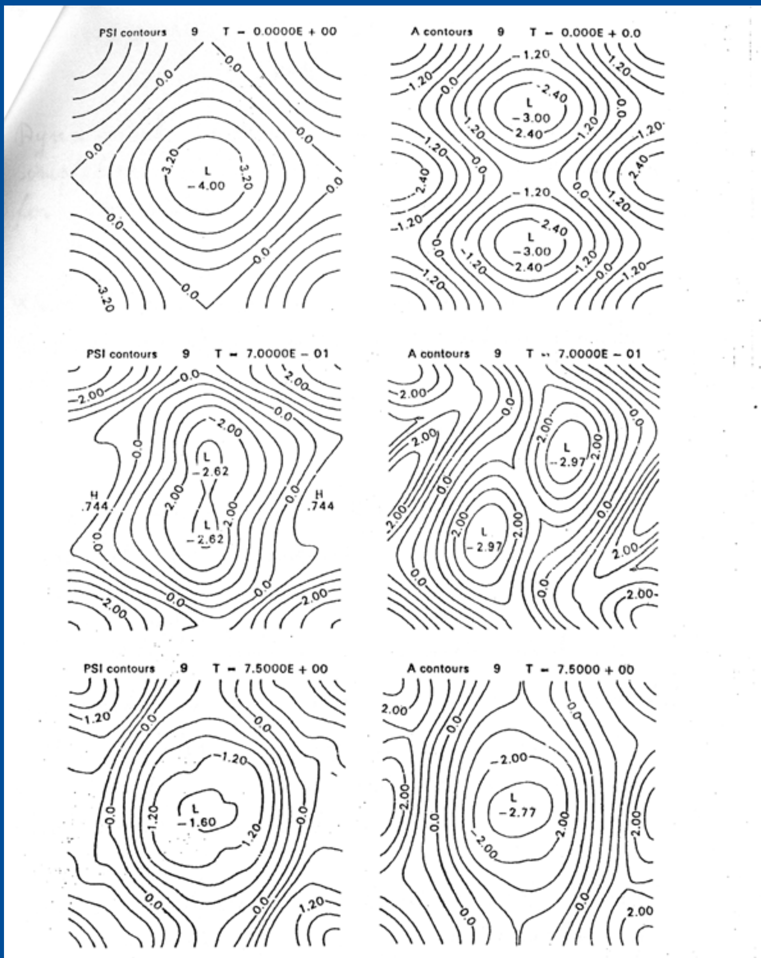

Fig. 3 — Évolution de la corrélation vitesse-champ magnétique d'un écoulement conducteur à nombres de Reynolds cinétique et magnétique de 400, simulée à l'ordinateur. Les courbes de gauche (PSI) sont des lignes de courant : le vecteur vitesse est tangent aux lignes de courant. Les courbes de droite (A) sont des lignes de champ magnétique. A l'instant t = 0 (haut) la corrélation est de 50 %; à l'instant t = 0,7 (milieu) elle est de 52 %; à l'instant t = 7,5 (bas), correspondant à une dizaine de « temps de retournement », la corrélation est passée à 86 %.

Time evolution

Velocity and magnetic field lines are parallel in the final state

Growth of v - B correlation

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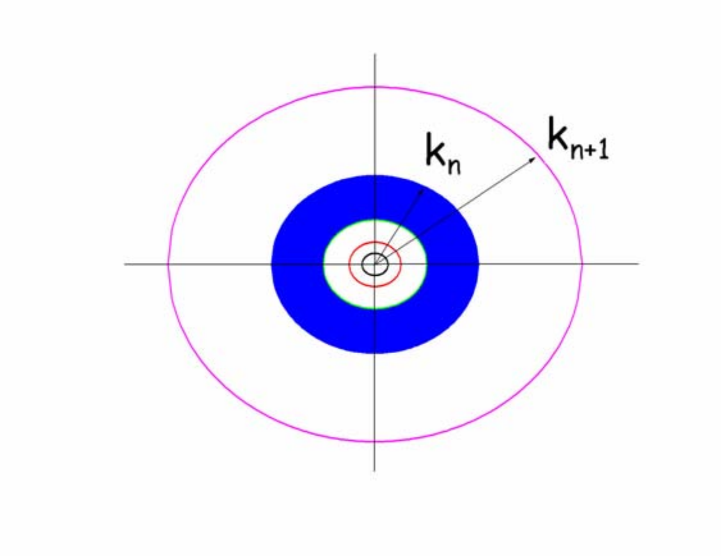
Dynamical (shell) models 1

Shell models are dynamical models representing a simplified version of the spectral equations for turbulence

Step 1
Introduce a logarithmic spacing in the wave vectors space (shells)

$$k_n = k_0 \lambda^n$$
$$n = 1, 2, \ldots, N$$



The intershell ratio in general is set equal to $\lambda = 2$.

We are not interested in the dynamics of each wave vector mode of Fourier expansion, rather in the gross properties of dynamics at small scales.

In this way we can investigate properties of turbulence at very high Reynolds numbers.

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Dynamical (shell) models 2

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Step 2

Assign to each shell ONLY two dynamical variables

$$z_n^{\pm}(t) = u_n(t) \pm b_n(t)$$

Velocity field Magnetic field

The possibility to investigate both spatial and temporal properties of turbulence is ruled out

These fields take into account the averaged effects of velocity modes between k_n and k_{n+1}, that is fluctuations across eddies at the scale $r_n \sim k_n^{-1}$

Step 3

Write quadratic NL equations for these variables where only nearest and next nearest coupling interactions are retained

$$\frac{dz_n^{\sigma}(t)}{dt} = ik_n \sum_{i,j=\pm2,\pm1} M_{i,j} z_{n+i}^{\sigma}(t) z_{n+j}^{-\sigma}(t)$$

Dynamical (shell) models 3

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Step 4

Fix the coupling coefficients M_{ij} imposing the conservation of ideal quadratic invariants (energy, cross helicity, magnetic helicity)

$$\frac{du_n}{dt} + \upsilon k^2 u_n = ik_n\Big[\big(u_{n+1}u_{n+2} - b_{n+1}b_{n+2}\big) - \frac{1}{4}\big(u_{n-1}u_{n+1} - b_{n-1}b_{n+1}\big) - \frac{1}{8}\big(u_{n-2}u_{n-1} - b_{n-2}b_{n-1}\big)\Big]^* + f_n$$

$$\frac{db_n}{dt} + \eta k^2 b_n = ik_n \frac{1}{6}\Big[\big(u_{n+1}b_{n+2} - b_{n+1}u_{n+2}\big) - \big(u_{n-1}b_{n+1} - b_{n-1}u_{n+1}\big) + \big(u_{n-2}b_{n-1} - b_{n-2}u_{n-1}\big)\Big]^* + g_n$$

Forcing terms

Dynamical alignment in shell models

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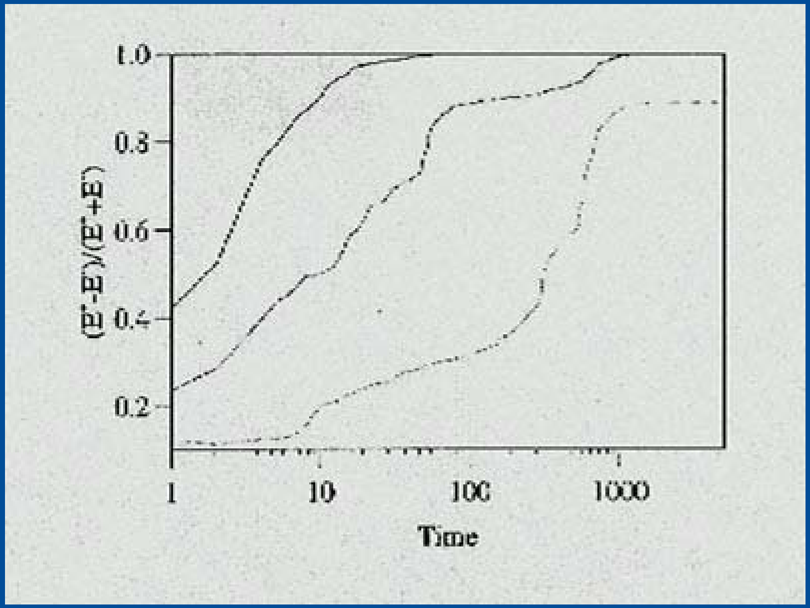
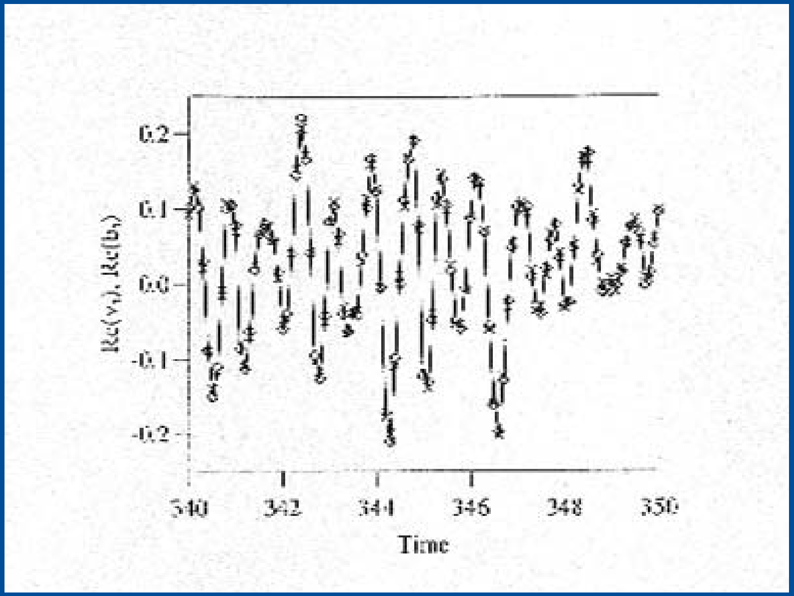
A decay simulation



The cross helicity to energy ratio grows in time

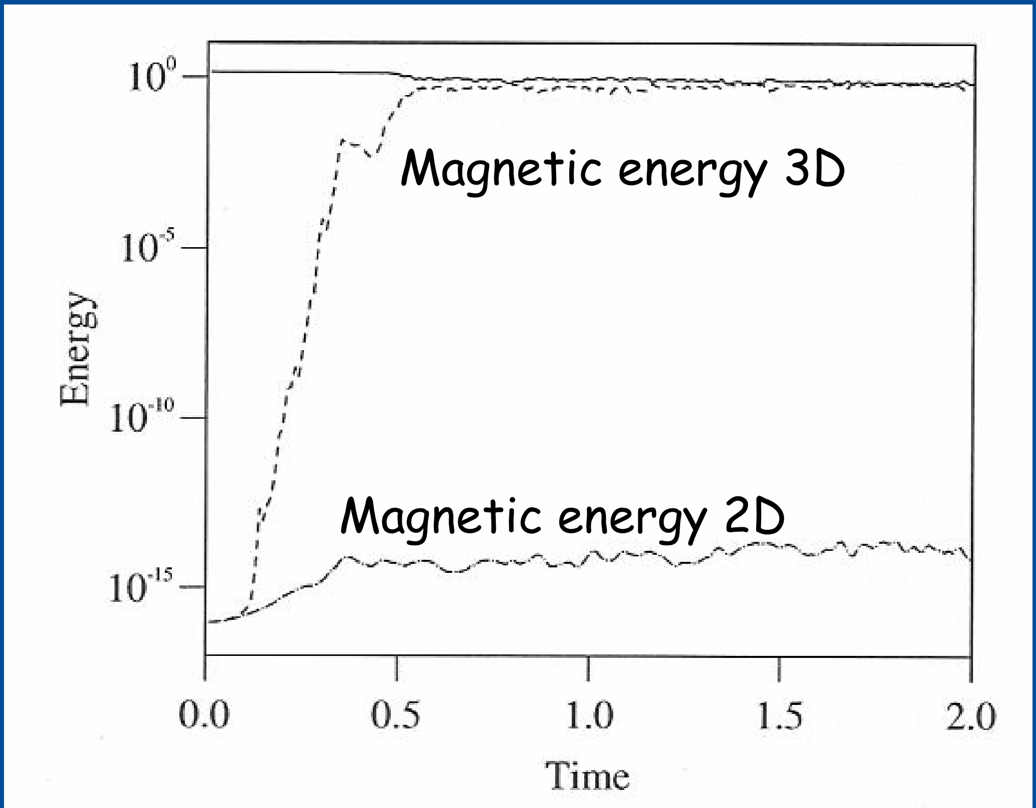
A forced simulation:
Time evolution of mode n = 7, with constant forcing terms.



Velocity (diamonds) and magnetic field (crosses) amplitudes strongly correlated

Dynamo action in shell models

What “turbulent dynamo action” means in the shell model



There exists some “invariant subspaces” which can act like “attractors” for all solutions (stable subspaces).

The fluid subspace is stable (in 2D case) or unstable (in 3D case).

The structure of stable and unstable time-invariant subspaces of real MHD are reproduced in the shell models

Alfvenic correlation: a different scenario 1

Alfenic states, i.e. states of high correlation between velocity and magnetic field fluctuations are spontaneously formed in MHD turbulence

Solar wind turbulence evolves in the reverse way:

- Highly correlated near the sun up to 1 AU
- At larger radial distances from 1 AU to 10 AU:
  - Correlation is progressively lowered
  - Level of compressive $\delta\rho$, $\delta|B|$ fluctuations progressively increased

Possible solution to such paradox:
Solar wind is neither incompressible nor statistically homogeneous

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Alfvenic correlation: a different scenario 2

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Commonly accepted scenario:

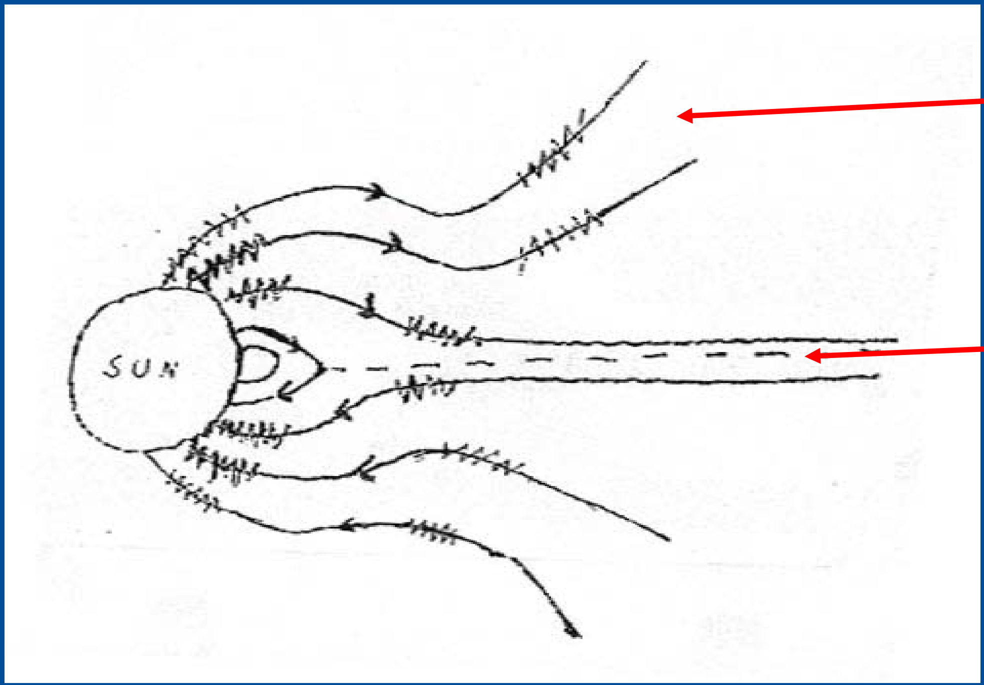
- Main source of fluctuations inside the critical point (solar photosphere)
- Critical point filters out inward propagating waves, only outward propagating Alven waves arrive in the solar wind

Large amplitudes Alfven waves generated at the foot points of open magnetic field lines

- either converge towards the heliospheric current sheet (slow wind)
- or propagate in the fast solar wind

Alfvenic correlation: a different scenario 3

Two interesting physical problems



Fast wind

Slow wind:
current sheet

Large amplitude Alfven waves converging on the current sheet interact with the associated large scale inhomogeneity

Large amplitude Alfven waves propagating in fast wind could be subject to parametric instability

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Turbulence in the heliospheric current sheet 1

Space observations close to the heliospheric current sheet:

- depletion of δv - δB correlation
- higher level of compressive δn, $\delta|B|$ fluctuations
- Kolmogorov - like spectra for v, B, n, |B|

Numerical simulations to study compressive effects (Malara et al., 1996, 1997)

Model:

- A large scale current sheet
- An initial alfvenic perturbation (outward propagating waves on both sides of the current sheet
- An initial uniform $|B|=|B_{eq} + \delta B|$ (no initial ponderomotive force)

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Turbulence in the heliospheric current sheet 2

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Model equations:

$$\frac{\partial \rho}{\partial \tau} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial \mathbf{v}}{\partial \tau} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla(\rho T) + \frac{1}{\rho}(\nabla \times \mathbf{b}) \times \mathbf{b} + \frac{1}{\rho S_\nu}\nabla^2 \mathbf{v}$$

$$\frac{\partial \mathbf{b}}{\partial \tau} = \nabla \times (\mathbf{v} \times \mathbf{b}) + \frac{1}{S_\eta}\nabla^2 \mathbf{b}$$

$$\frac{\partial T}{\partial \tau} + (\mathbf{v} \cdot \nabla)T + (\gamma - 1)T(\nabla \cdot \mathbf{v}) = \frac{\gamma - 1}{\rho}\left[\frac{1}{S_\kappa}\nabla^2 T + \frac{1}{S_\nu}\left(\frac{\partial v_i}{\partial x_j}\frac{\partial v_i}{\partial x_j}\right) + \right.$$
$$\left. + \frac{1}{S_\eta}(\nabla \times \mathbf{b})^2\right]$$

Numerical method:

- 2-1/2 D code
- Pseudospectral: Fourier in the homogeneity direction, Chebshev across the current sheet
- Multidomain: a matching performed to increase resolution inside the current sheet

Turbulence in the heliospheric current sheet 3

Initial conditions:

Magnetic field

$$\mathbf{b}(x,y,0) = A\left\{\varepsilon\cos[\phi(y)]\,\mathbf{e}_x + \sin(\alpha)F(x)\,\mathbf{e}_y + \right.$$
$$\left. + \sqrt{1-\sin^2(\alpha)F^2(x)+\varepsilon^2\sin^2[\phi(y)]}\,\mathbf{e}_z\right\}$$

Velocity field

$$\mathbf{v}(x,y,0) = \sigma(x)\,\frac{\delta\mathbf{b}(x,y,0)}{\sqrt{\rho(x,y,0)}}$$

Density and temperature

$$\rho(x,y,0) = 1 \quad \text{and} \quad T(x,y,0) = T_0$$

Where $\varepsilon = 0.5$,

$$F(x) = \tanh(x)$$

and

$$\phi(y) = 2\sum_{m=1}^{m_{max}} (mk_0)^{-5/3}(\cos mk_0)$$

The equilibrium magnetic field $\mathbf{B}_{eq}$ is obtained by setting $\varepsilon = 0$ and rotates by an angle α in the plane yz

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Turbulence in the heliospheric current sheet 4

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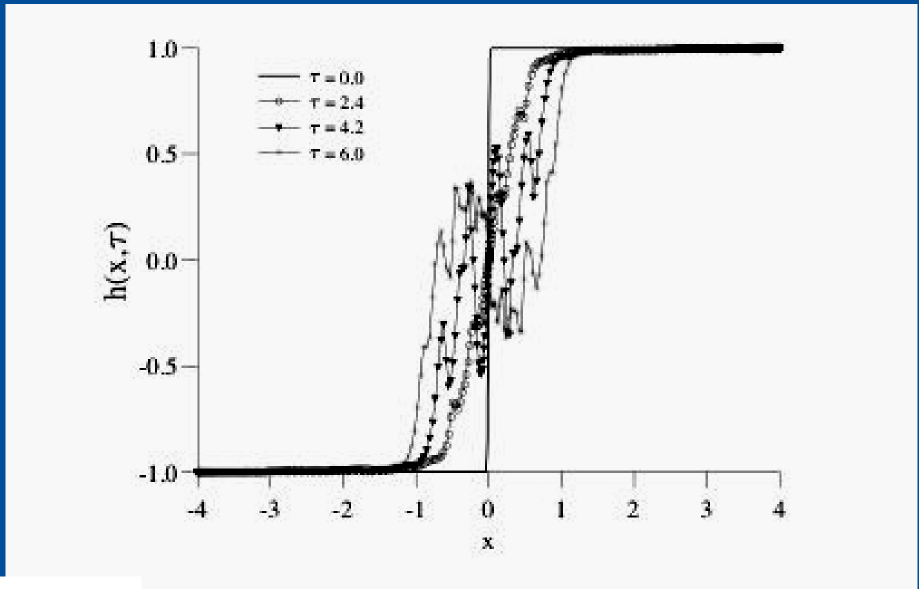
The initial correlation between δ**v** and δ**B** is strongly reduced in the current sheet region



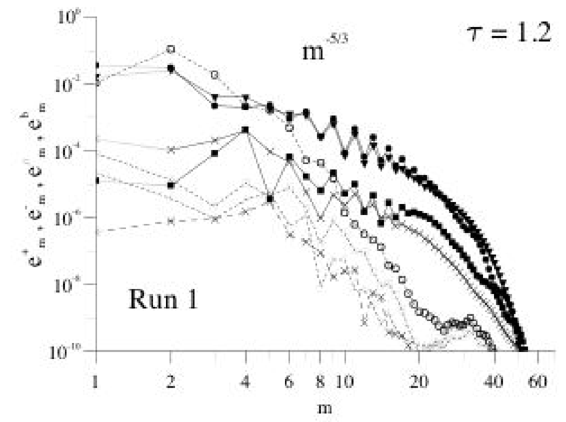


Fig. 3. The Fourier spectra of pseudoenergies fluctuation $e^+_m(x,\tau)$ (circles), $e^-_m(x,\tau)$ (triangles), density fluctuation e^ρ_m (squares) and magnetic field intensity fluctuation e^b_m (crosses) are represented as functions of the wavenumbers at $x = 0.2$ (full lines) and at $x = 3.01$ (dashed lines) at the time $\tau = 1.2$, for $\beta = 0.2$

Compressible fluctuations $\delta\rho$ and $\delta|$**B**$|$ are generated

Inside neutral sheet the spectra for E^+, E^- and $\delta\rho$ are superposed.
The spectral index is 5/3.

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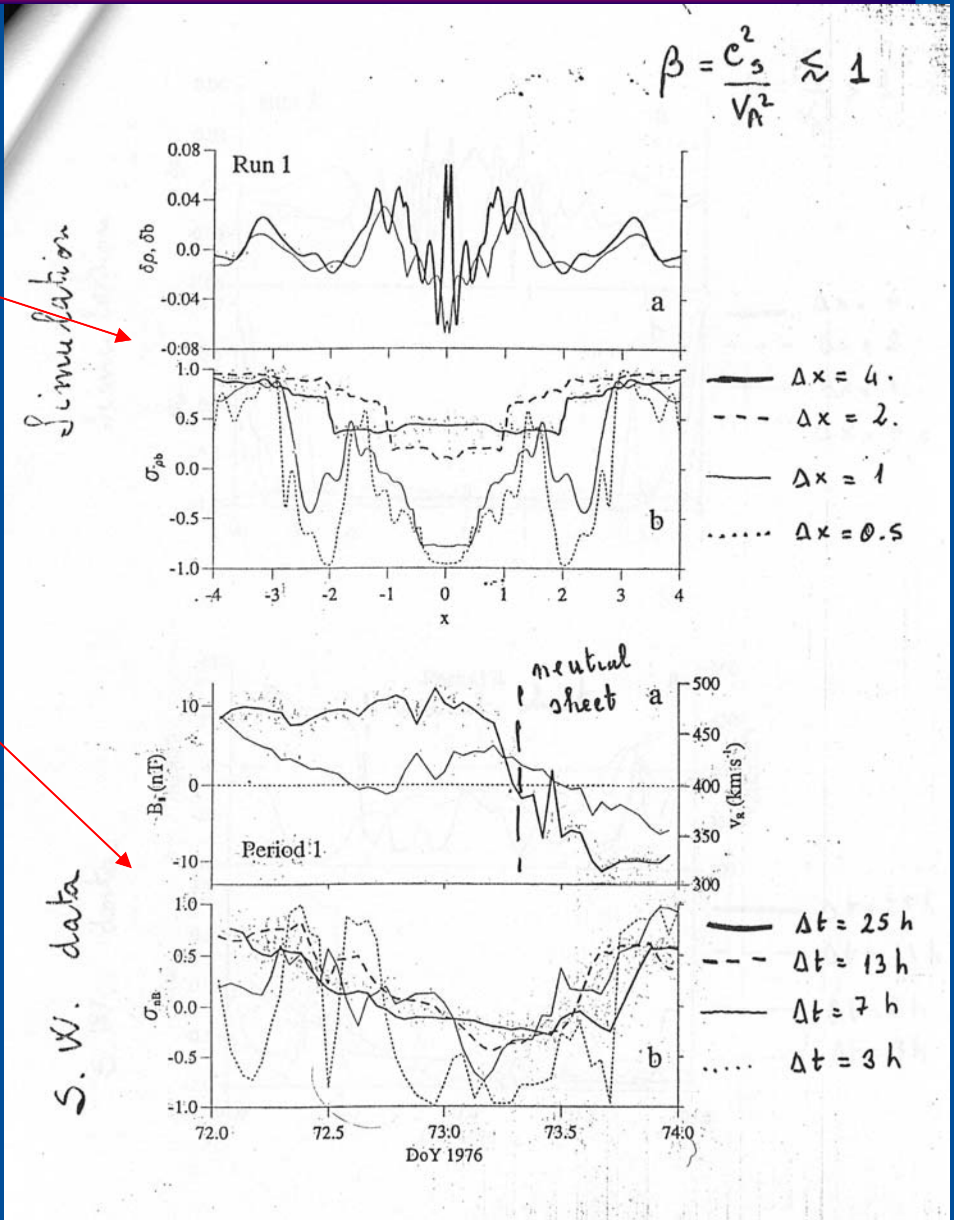
Density magnetic field intensity correlation 1

Simulation vs solar wind data analysis

For $\beta < 1$

$\delta\rho$ and $\delta|B|$:

- anticorrelated at small scales inside current sheet (slow mode fluctuations)
- correlated at large scale outside the current sheet (fast mode fluctuations



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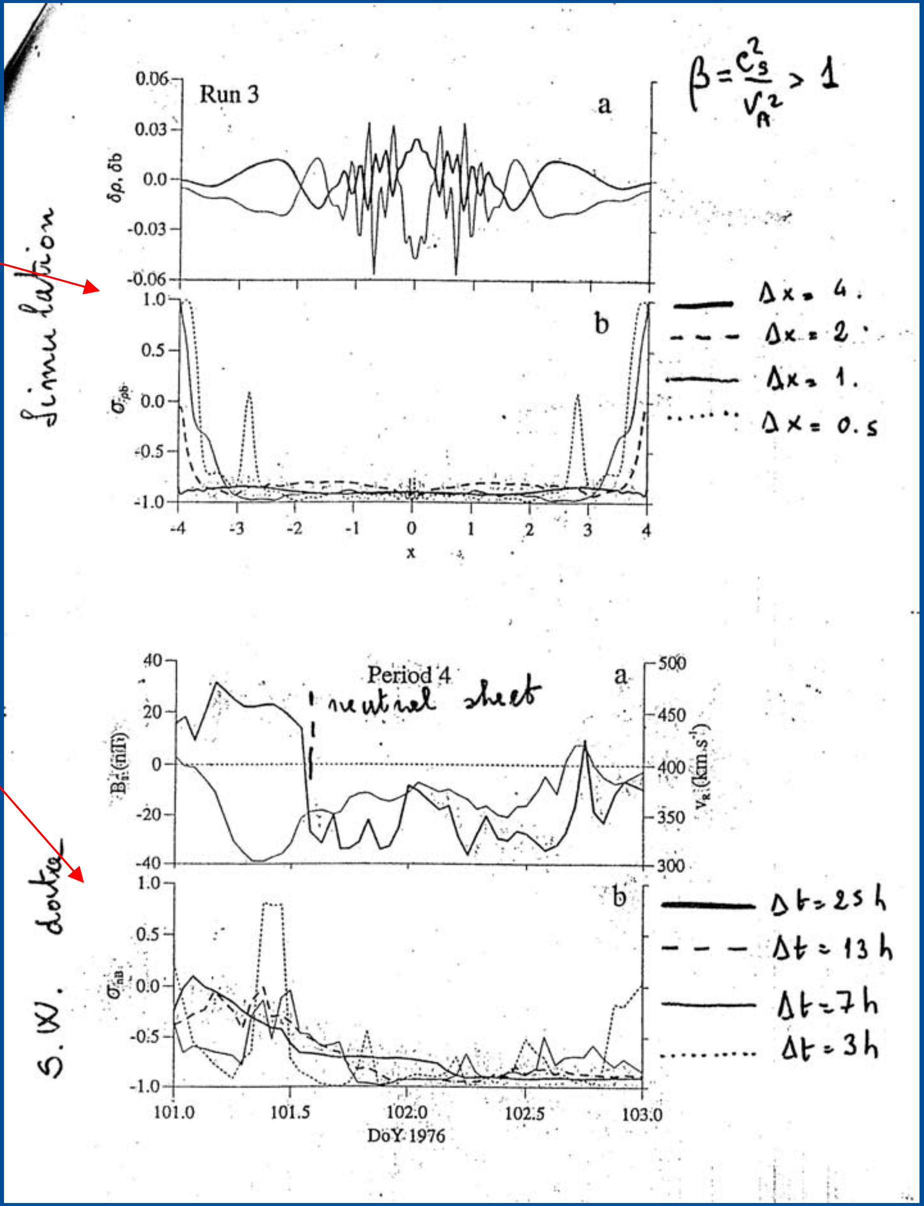
Density magnetic field intensity correlation 2

Simulation vs solar wind data analysis

For $\beta > 1$

$\delta\rho$ and $\delta|B|$:

- anticorrelated at all scales and almost everywhere (slow mode fluctuations)



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Density Temperature fluctuation correlation 1

At small scale $\delta\rho$ - δT correlation mainly positive in the fast streams while it dispays both signs in slow streams (Bavassano et al., 1995):
where does the negaative correlation come from ?

In an ideal fluid entropy S is advected by velocity field like a passive scalar

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right]S = 0$$

S initially uniform
- isoentropic flow
- polytropic equation of state
- only positive correlation are allowed

$$\frac{T}{\rho^{\gamma-1}} = const$$

$$\frac{\delta T}{T} = (\gamma-1)\frac{\delta\rho}{\rho}$$

Small amplitude modes:
both fast and slow modes display $\delta\rho$ - δT positively correlated

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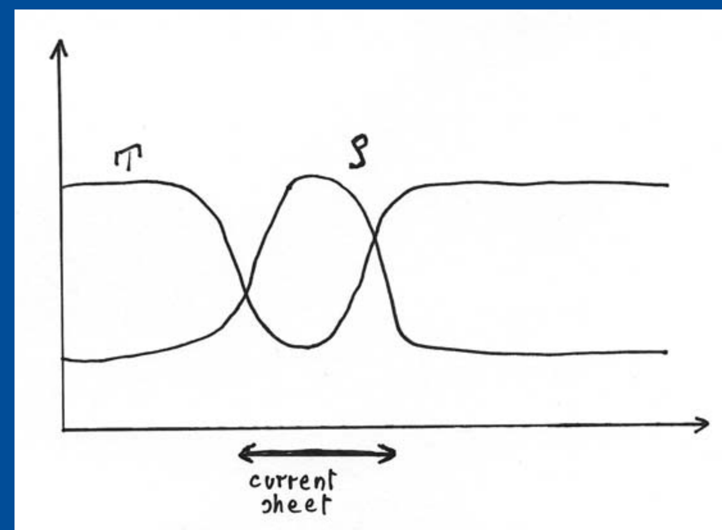
Density Temperature fluctuation correlation 2

Zank and Matthaeus 1991, 1993:
heat conduction term allows for a negative correlation also in a statistically homogeneous configuration (Nearly Incompressible MHD aapproach)

Malara et al., 1998:
Large scale (~1 day) variations of density and temperature furnish a reservoir of entropy fluctuations

$$S \propto \ln\left(\frac{T}{\rho^{\gamma-1}}\right)$$



Plasma turbulence can mix S, moving the S modulation from large to small scales (small amplitude entropy waves display negative $\delta\rho$ - δT correlations)

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Density Temperature fluctuation correlation 3

The sign of $\delta\rho$ - δT correlation results from competition between :

- production of magnetosonic compressive fluctuations $\langle\delta\rho \;\; \delta T\rangle \;\; > \;\; 0$
- entropy cascade $\langle\delta\rho \;\; \delta T\rangle \;\; < \;\; 0$

Numerical model (Malara et al., 1998) :

- Large scale current sheet
- Density and temperature large scale modulation

Initial conditions:

$$\rho(x,y,t=0) = \rho_0 \left\{ 1 + \Delta \left[\frac{1}{\cosh^2(x/a_e)} + p\left(\frac{x}{a_e}\right)^2 \right] \right\}$$

$$T(x,y,t=0) = \rho_0 T_0 / \rho(x,y,t=0)$$

Pressure and magnetic field intensity are constant

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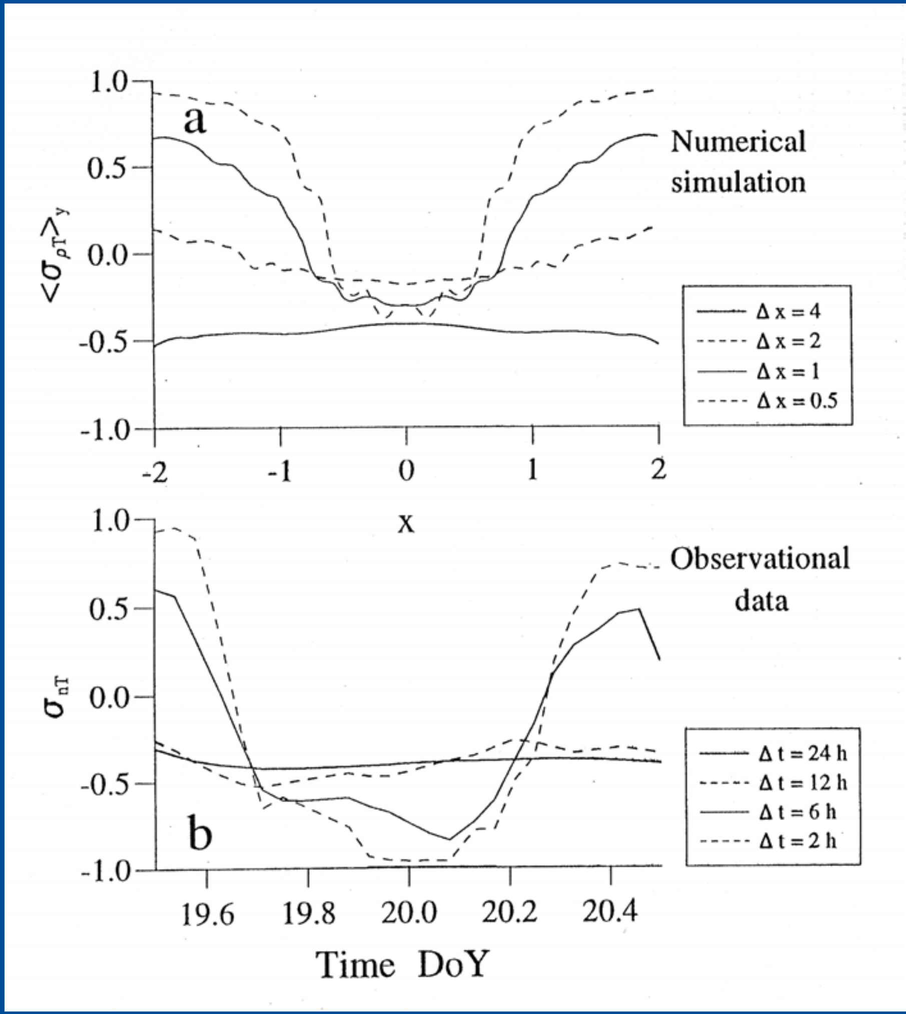
Density Temperature fluctuation correlation 4

At large scales $\langle\delta\rho \;\; \delta T\rangle < 0$ in the whole domain

At small scales

- $\langle\delta\rho \;\; \delta T\rangle < 0$ in the current sheet
- NL interactions more effective inside current sheet where entropy cascade prevails
- $\langle\delta\rho \;\; \delta T\rangle > 0$ outside the current sheet (fast magnetosonic fluctuations propagate outside)



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Parametric Instability of Alfvenic fluctuations 1

Fast speed stream and polar wind much more homogeneous then current sheet region, but E^+/E^- increases with the distance from the sun;
can parametric instability be responsible for this ?

Mother coherent Alfven wave decays in
- an Alfven wave propagating in the opposite direction
- a sound wave

What happens when starting with a broad band initial wave ?

Non linear numerical investigation of evolution of initial conditions

$$\delta\mathbf{B} = B_1\left[\cos(\phi(x))\mathbf{e}_y + \sin(\phi(x))\mathbf{e}_z\right]$$

$$\delta\mathbf{v} = -\frac{v_A}{B_0}\delta\mathbf{B}$$

with

$$\phi(x) = k_0 x + a\sum_{n=-N}^{N} |n|^{-\alpha}\, e^{in(x+\delta)}$$

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Parametric Instability of Alfvenic fluctuations 2

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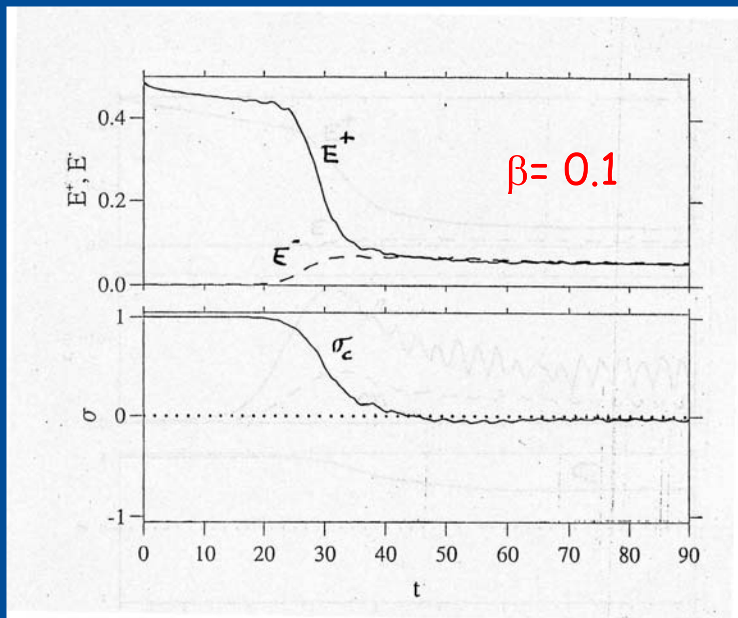
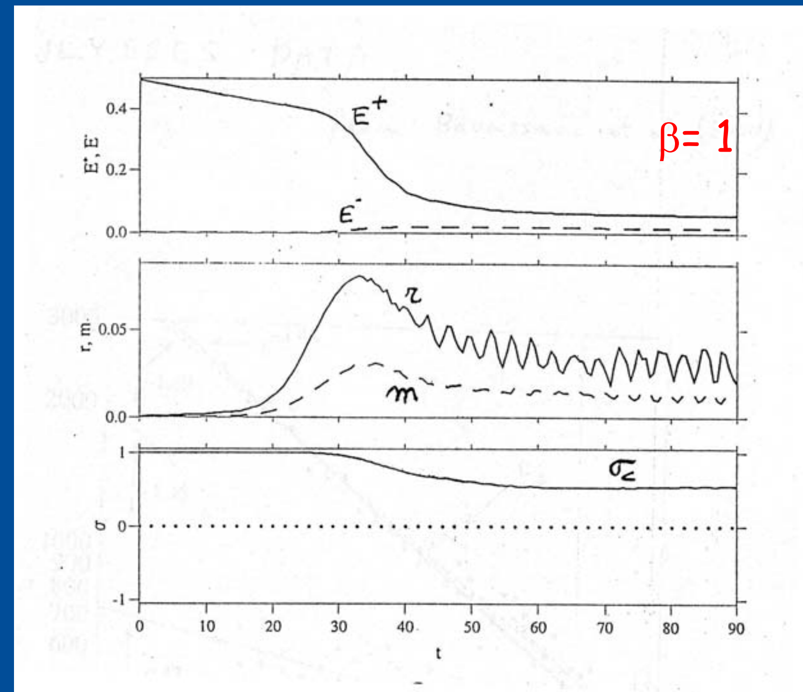
Simulation results:



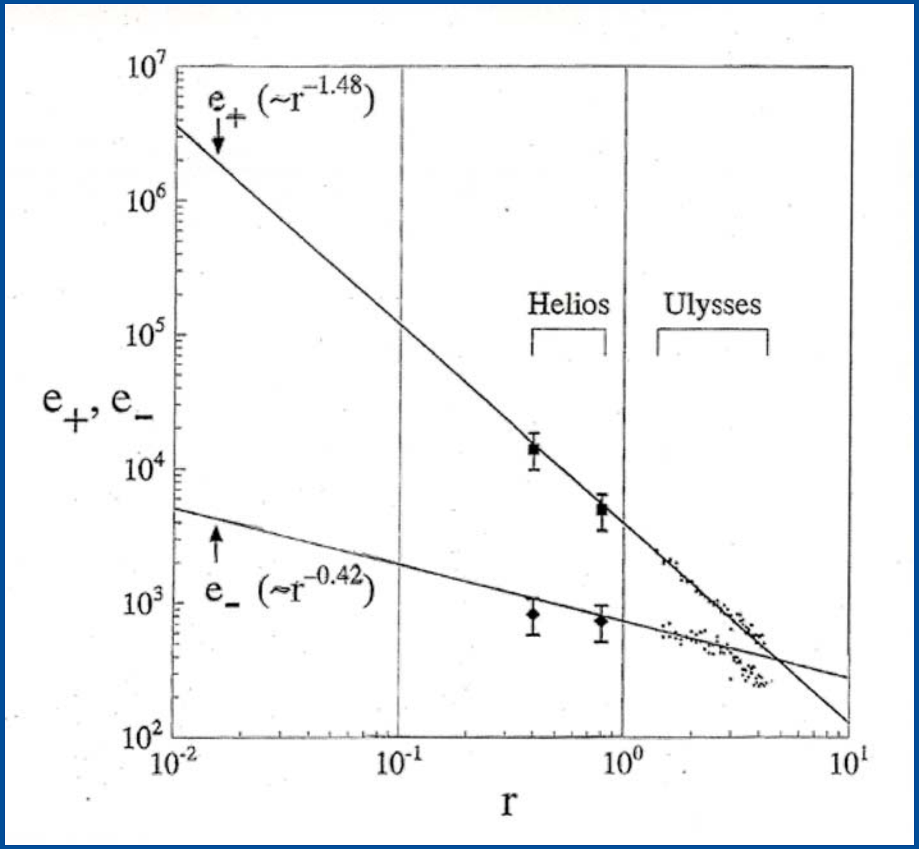


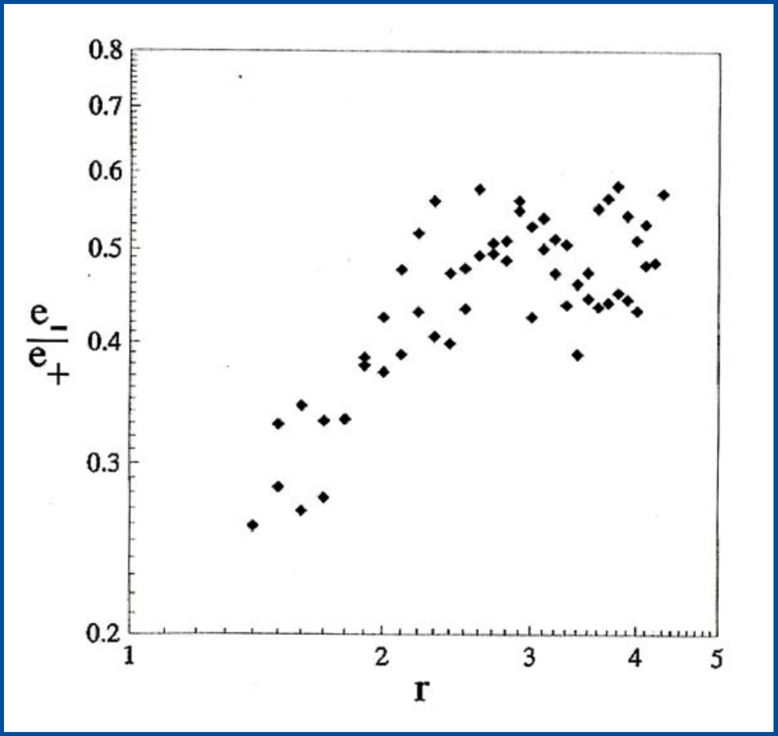
Inward propagating Alfven wave, density and magnetic field intensity fluctuations all grow in time

At instability saturation;

- for small β , E^- grows to the same level as E^+
- for intermediate β , E^- level grows but it remains lower then E^+, the parametric process is unable to completely destroy v - B correlation

Parametric Istability of Alfvenic fluctuations 3





Bavassano et al., 2000

Data analysis: saturation seems to occur at 2.2 AU radial distance at a level $E^-/E^+ \sim 0.5$

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Parametric Instability of Alfvenic fluctuations 4

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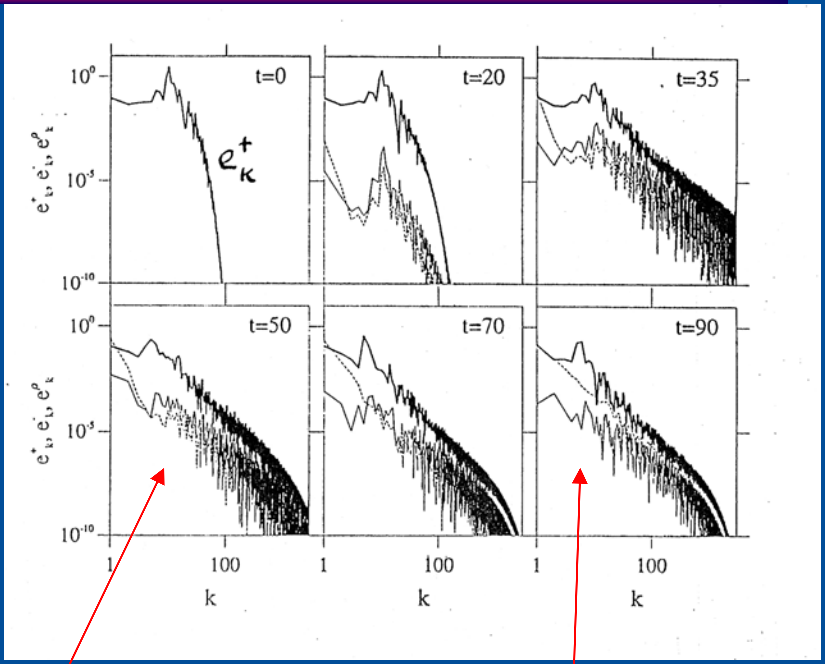
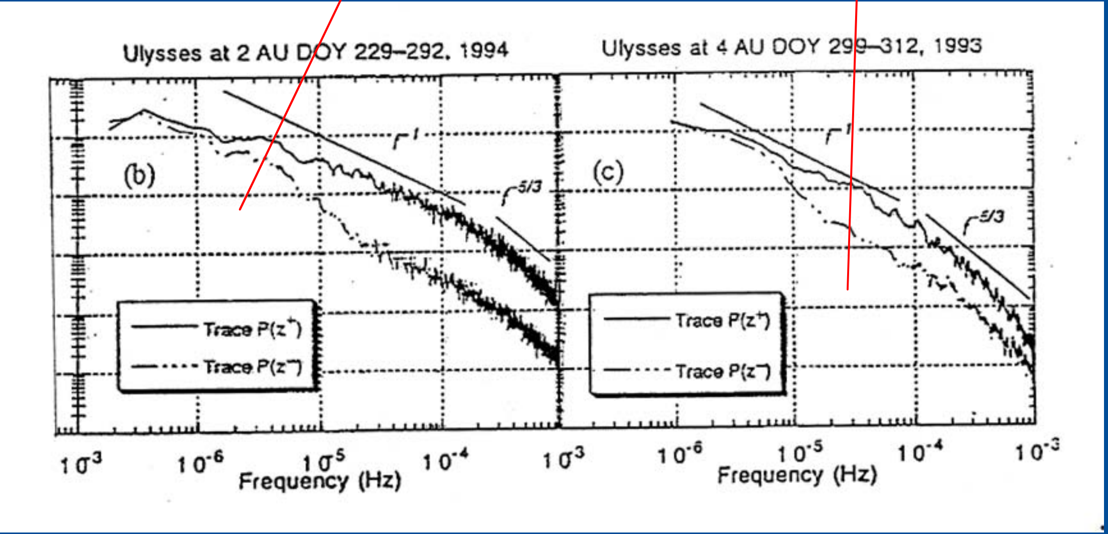
Spectra:

- Both direct and inverse cascades are observed
- At large and small scales E^+ and E^- spectra finally superpose
- At intermediate scales E^+ still dominates
- The spectral index of E^+ progressively decreases and approaches 5/3

--- $E^-(k)$
___ $\rho(k)$





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Conclusions

Analysis of space data has stimulated a lot of theoretical work devoted to understand the oberved properties of MHD turbulence

Numerical simulations are able to explain several observed features of MHD turbulence in solar wind.

In slow wind the interaction of Alfven waves with Heliospheric current sheet produces:

- The depletion of Alfvenic correlation
- The generation of compressive fluctuations
- The spectral features of fluctuations
- The behavior of both r - |B| and r - T fluctuation correlations

In fast wind parametric instability allows to understand

- The time evolution of Alfvenic fluctuations
- The spectral features of both Alfvenic and compressive fluctuations generated

MHD Turbulence in heliosperic plasma: Intermittency and small scale structures

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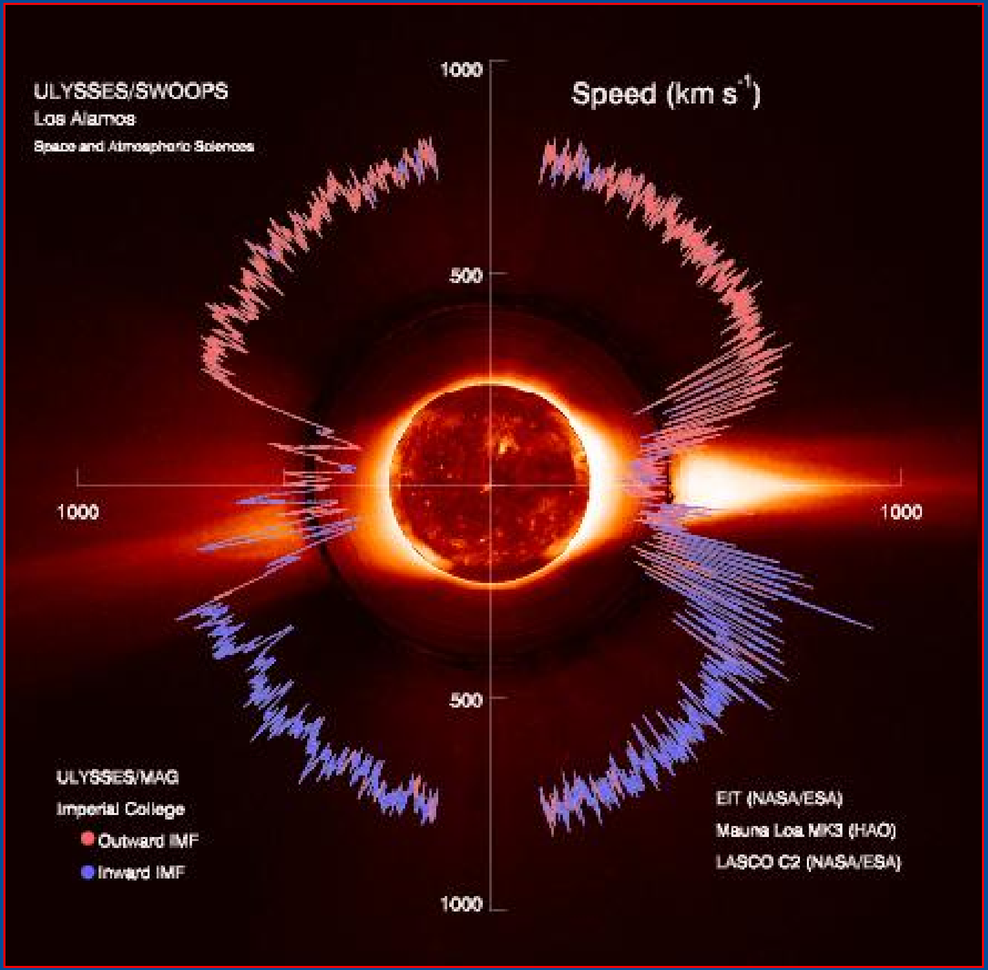
Solar wind: a wind tunnel
Supersonic and superalfvenic flow
250 Km/s < V_{sw} < 800 Km/s

In situ measurements of high amplitude fluctuations of velocity and magnetic fields over 6 decades of frequencies (up to ion cyclotron frequency)

10^{-6} Hz < f < 1 Hz



From Taylor hypothesis: L ~ V_{sw} / f Scale range 400 Km < L < 1 AU

Two-points correlation : higher order moments

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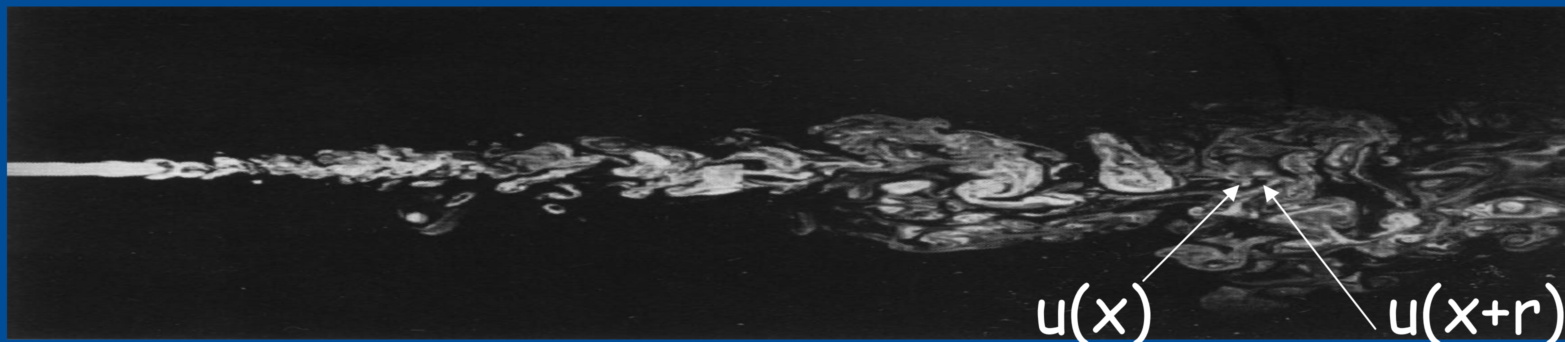

$$\delta u_r(x) = u(x+r) - u(x)$$

Gaussian process: the 2-th order moment is sufficient to fully determine probability density functions (pdf). High-order moments are uniquely defined from the 2-th order (in this sense energy spectra are interesting!)

High-order moments of two-points differences are probes for non-gaussian behaviour of fluctuations

$$S_n(r) = \left\langle [\delta u_r(x)]^n \right\rangle \approx r^{\zeta_n}$$

Kolmogorov's standard turbulence:

$$\zeta_n = n/3 \;\Rightarrow\; E(k) \approx k^{-5/3}$$

Self-similarity of fluctuations

Non linear cascade =
self similar (fractal) process → Pdf for fluctuations should be invariant under a scale change

$$P_r(\delta u_r) = r^{-1/3} F\left(\frac{\delta u_r}{r^{-1/3}}\right)$$

Higher order moments: probes of self similarity

$$S_n(r) = \int_{-\infty}^{\infty} (\delta u_r)^n P(\delta u_r) d(\delta u_r) =$$

$$= r^{n/3} \int_{-\infty}^{\infty} \left(\frac{\delta u_r}{r^{1/3}}\right)^n P\left(\frac{\delta u_r}{r^{1/3}}\right) d\left(\frac{\delta u_r}{r^{1/3}}\right) = A_n r^{n/3}$$

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Self-similarity of fluctuations

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Fluctuations are stochastic variables, high-order moments can be defined in terms of pdfs:

$$S_n(r) = \int_{-\infty}^{\infty} (\delta u_r)^n \, p(\delta u_r) \, d\delta u_r$$

Anomalous scaling exponents → pdfs have also anomalous scalings

$$\delta w_r = \frac{\delta u_r}{\left\langle (\delta u_r)^2 \right\rangle^{1/2}}$$

$$\delta u_r = u(x+r) - u(x)$$

Changing the scale $r \to \lambda r$, it can be shown that, in a pure fractal situation, pdfs at two scales are related

$$pdf(\delta w_{\lambda r}) = pdf(\delta w_r)$$

i.e. the pdfs of normalized fields increments at different scales collapse on the same shape (self-similarity)

Departure from Kolmogorov's scaling

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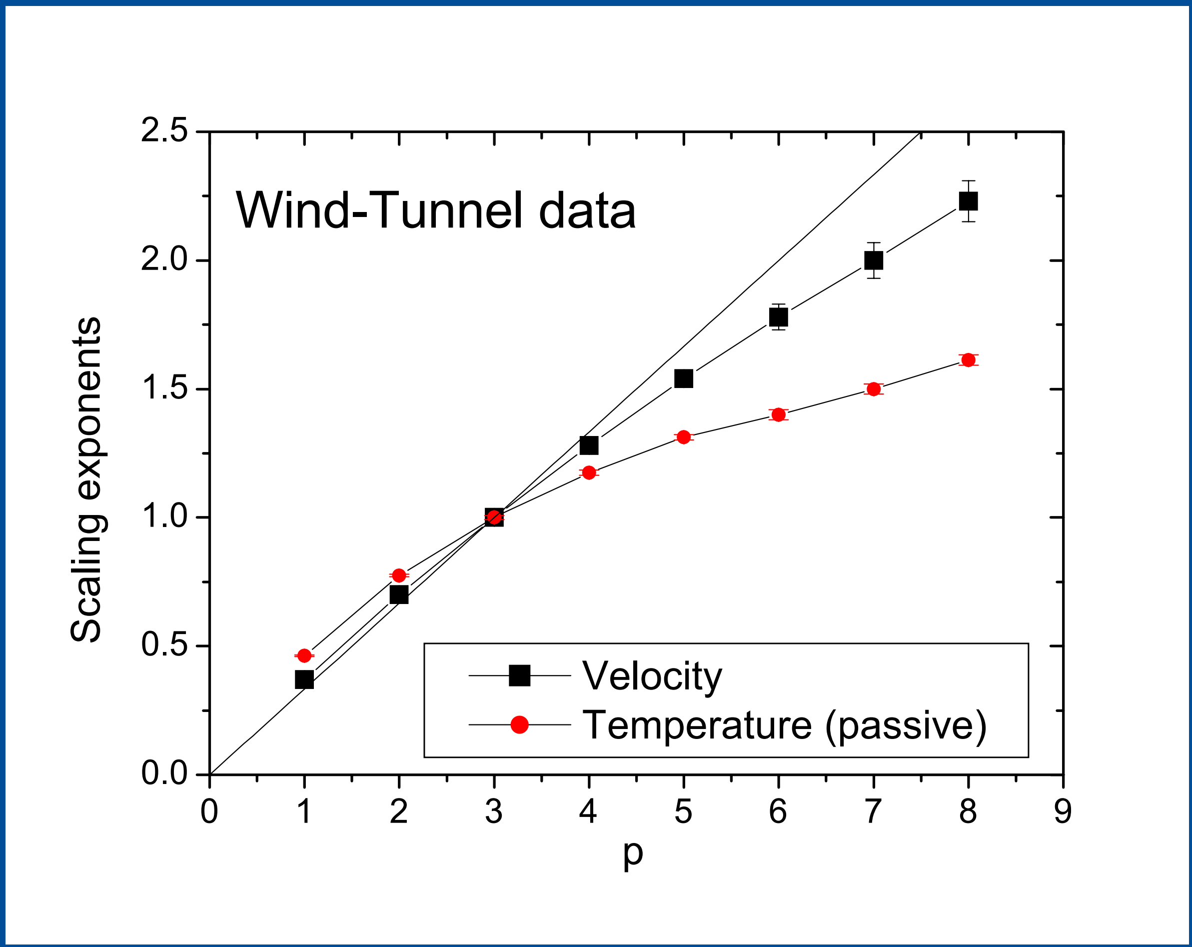
The departure is SIGNIFICANT, and has been attributed to INTERMITTENCY in fully developed turbulence



The temperature field (passive), is different from velocity field.

Intermittency (measured as the difference between the actual scaling exponent and the Kolmogorov's law), is stronger for passive scalar

Laboratory data

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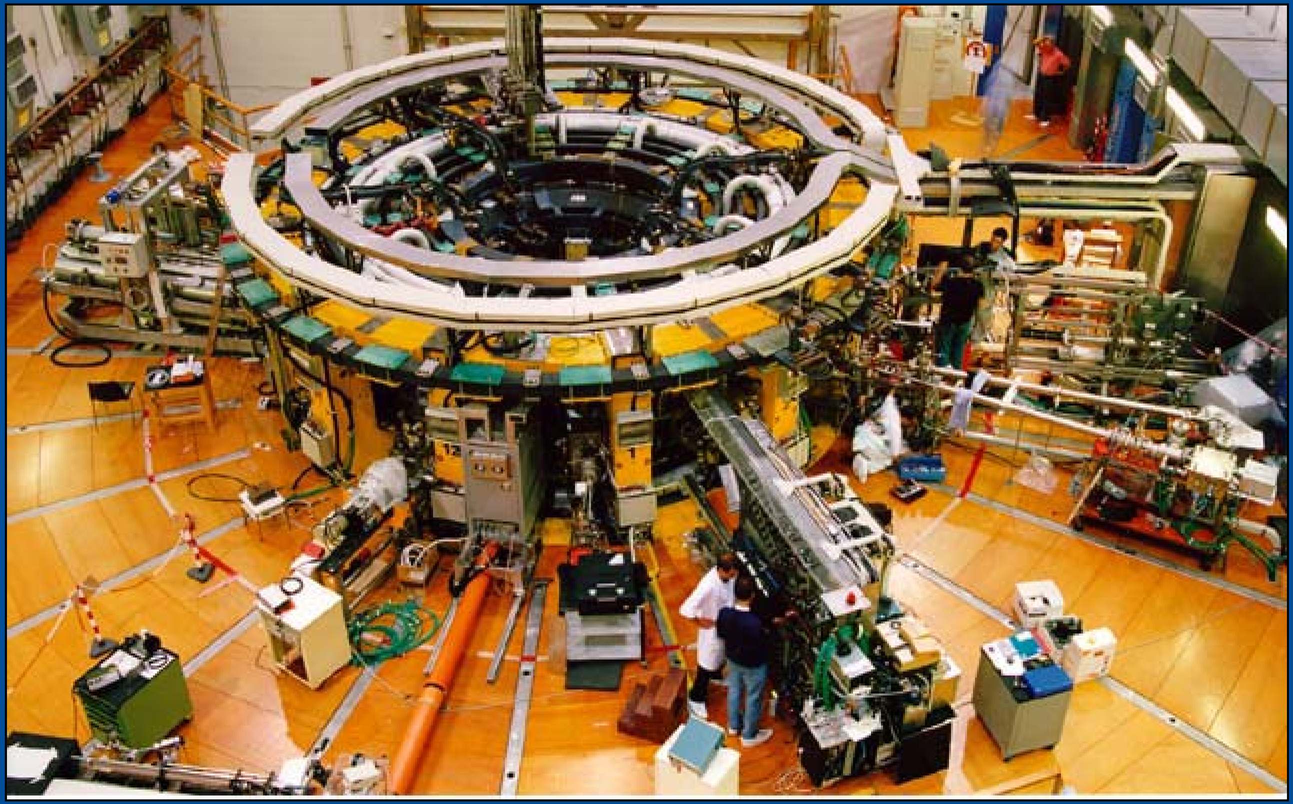
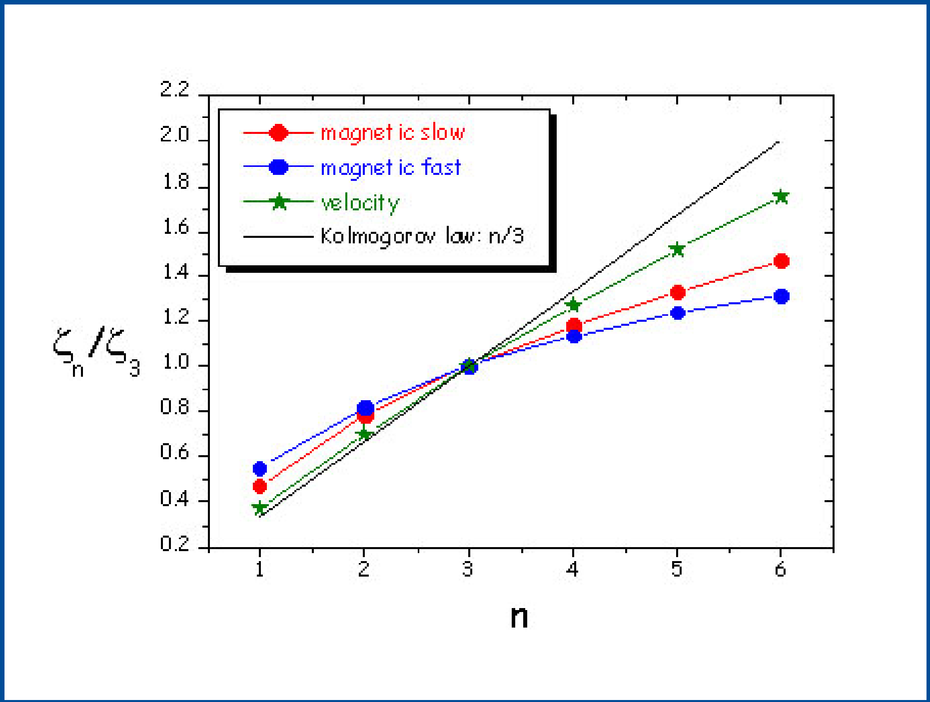
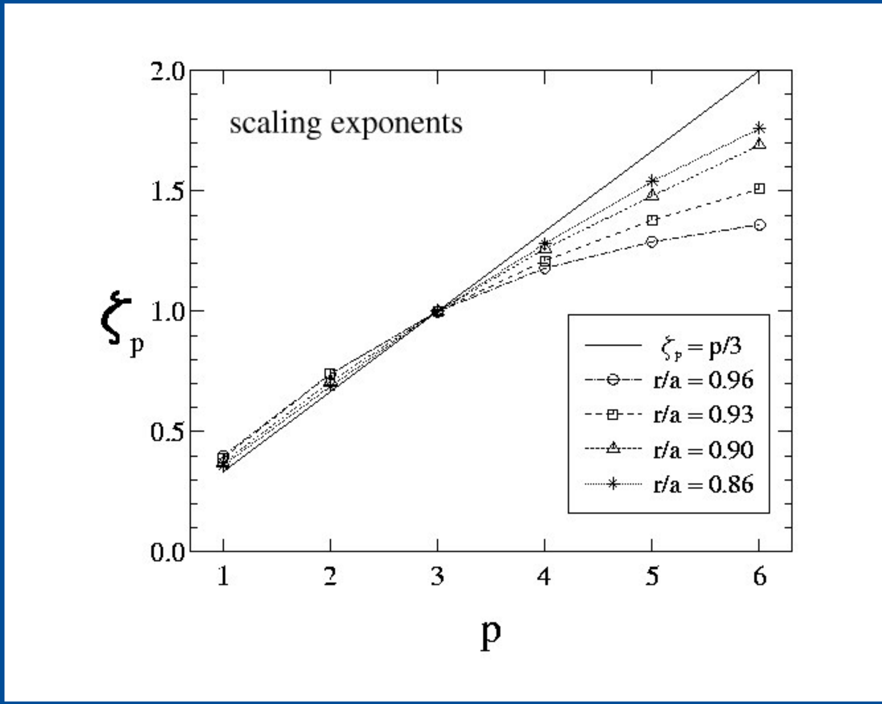


Plasma generated for nuclear fusion, confined in a Reversed Field Pinch configuration (RFX, Padua).
High amplitude fluctuations of magnetic field and floating potential measured at the edge of the device.

Anomalous scalings in plasmas turbulence





Solar wind

Intermittency is stronger for magnetic field than for velocity (magnetic field like a "passive vector" ?)

Laboratory plasma

Intermittency for magnetic turbulence is stronger near the wall

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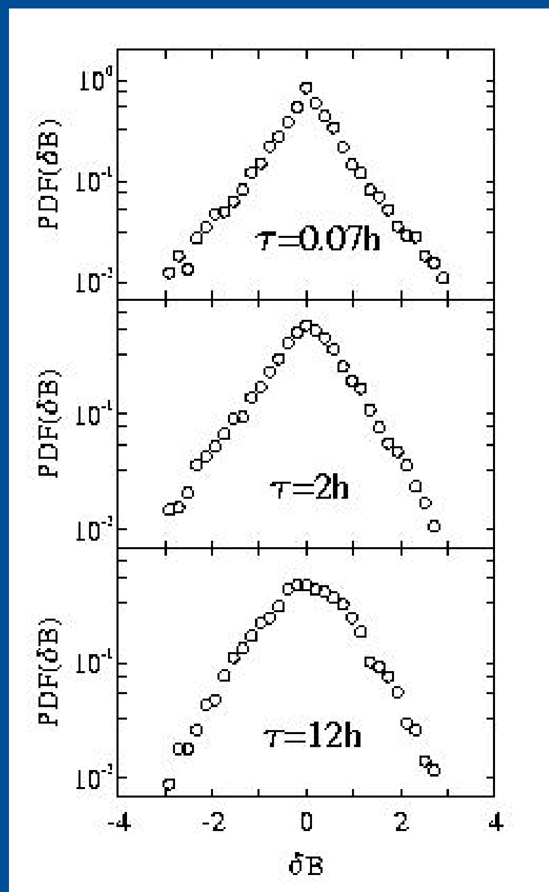
Departure from self-similarity

Anomalous scalings → absence of global self-similarity

Small scales

Inertial range

Large scales



PDFs of normalized fluctuations are not Gaussians

PDFs changes with scale

Note: The role actually played by the energy spectrum E(k) is limited in real turbulence, the phenomenon CANNOT be described taking into account only the 2-th order moment

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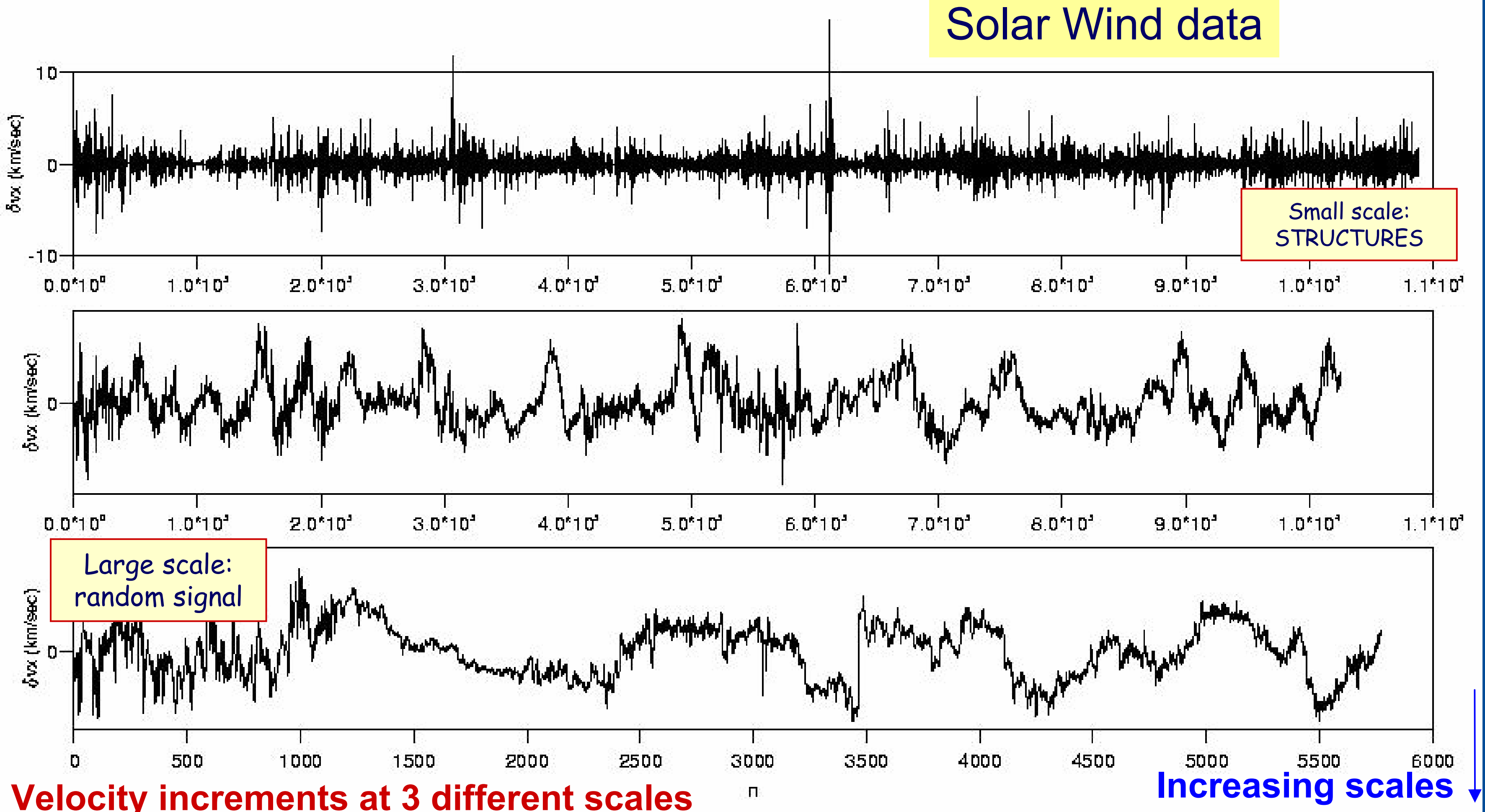
What is "intermittent" ?



Intermittency is due to strong fluctuations confined to small scales. Scaling law depends on position: (MULTIFRACTAL)

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Wavelet analysis of intermittent "structures"

Intermittent "coherent" events within turbulence, on all scales, can be identified through wavelet analysis.

Wavelet transform

$$W(b,a) = C_g^{-1/2} \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} \psi\left(\frac{x-b}{a}\right) f(x)\,dx$$

Haar basis

$$\psi(x) = \begin{cases} 1 & & 0 < x < 1/2 \\ -1 & for & 1/2 < x < 1 \\ 0 & & elsewhere \end{cases}$$

$a = 2^m$ scale dilation $b = 2^m\, i$ position traslation

Signature of intermittency: large isolated values of $|W^m(i)|^2$

Wavelet coefficient classification:

Passive $\left|W^m(i)\right|^2 \le F\left\langle \left|W^m(i)\right|^2 \right\rangle$ Intermittent $\left|W^m(i)\right|^2 > F\left\langle \left|W^m(i)\right|^2 \right\rangle$

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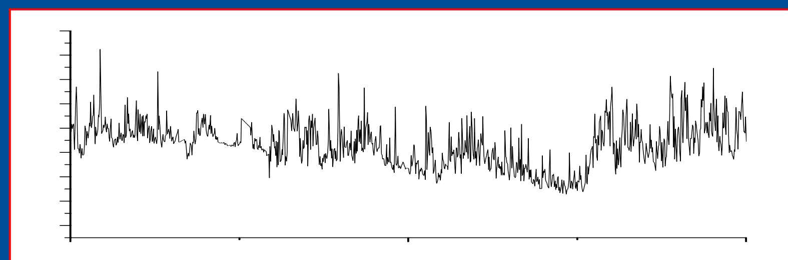
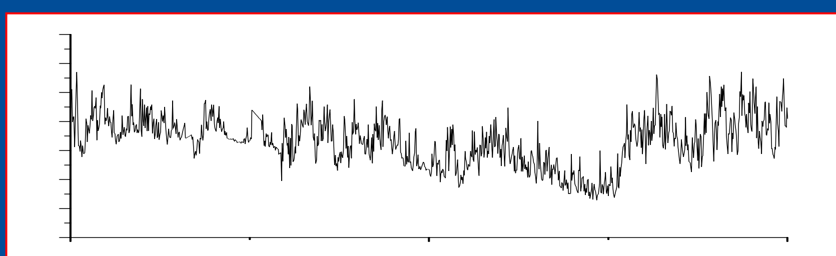
Intermittent “structures” and background

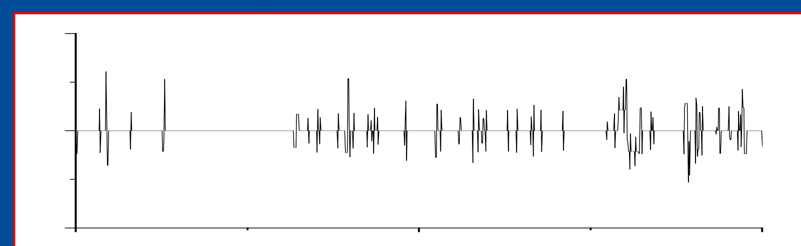
Wavelet coefficients classification (F =5) allows to split the time signal

Complete signal





Gaussian background



Isolated Structures

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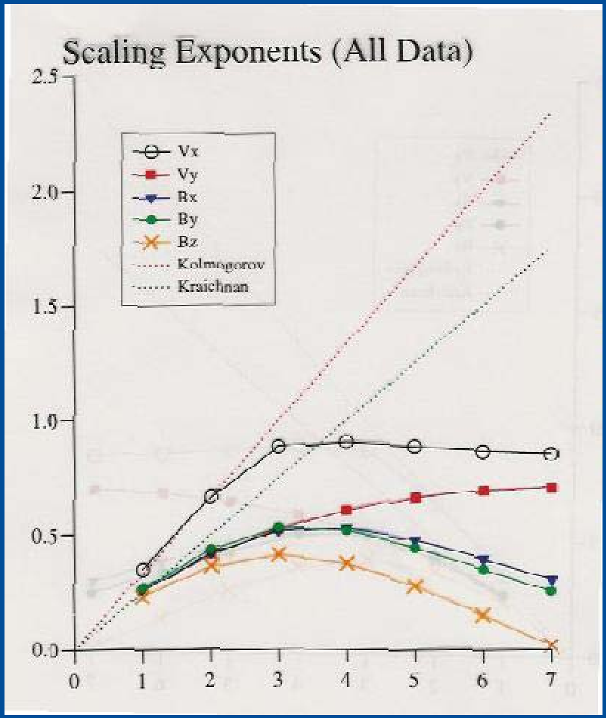
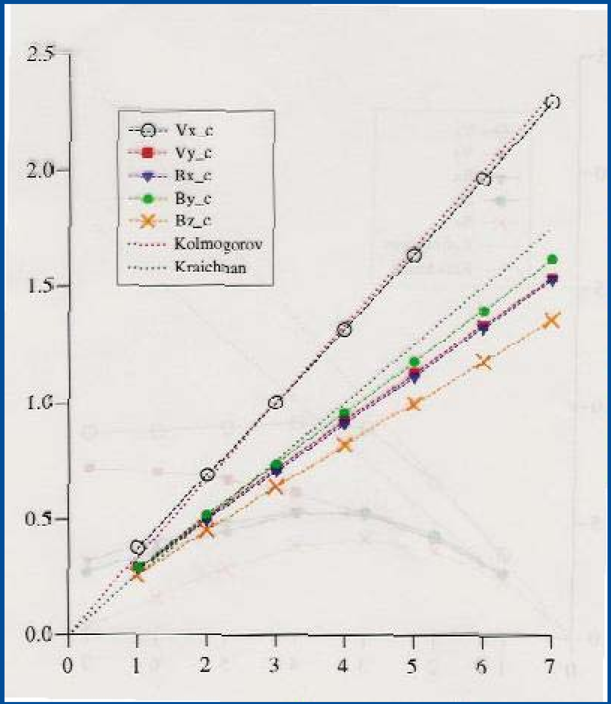
Fractal vs multifractal behavior

After elimination of the intermittent isolated structure from the signal Scaling exponents are linear functions of the order of moment

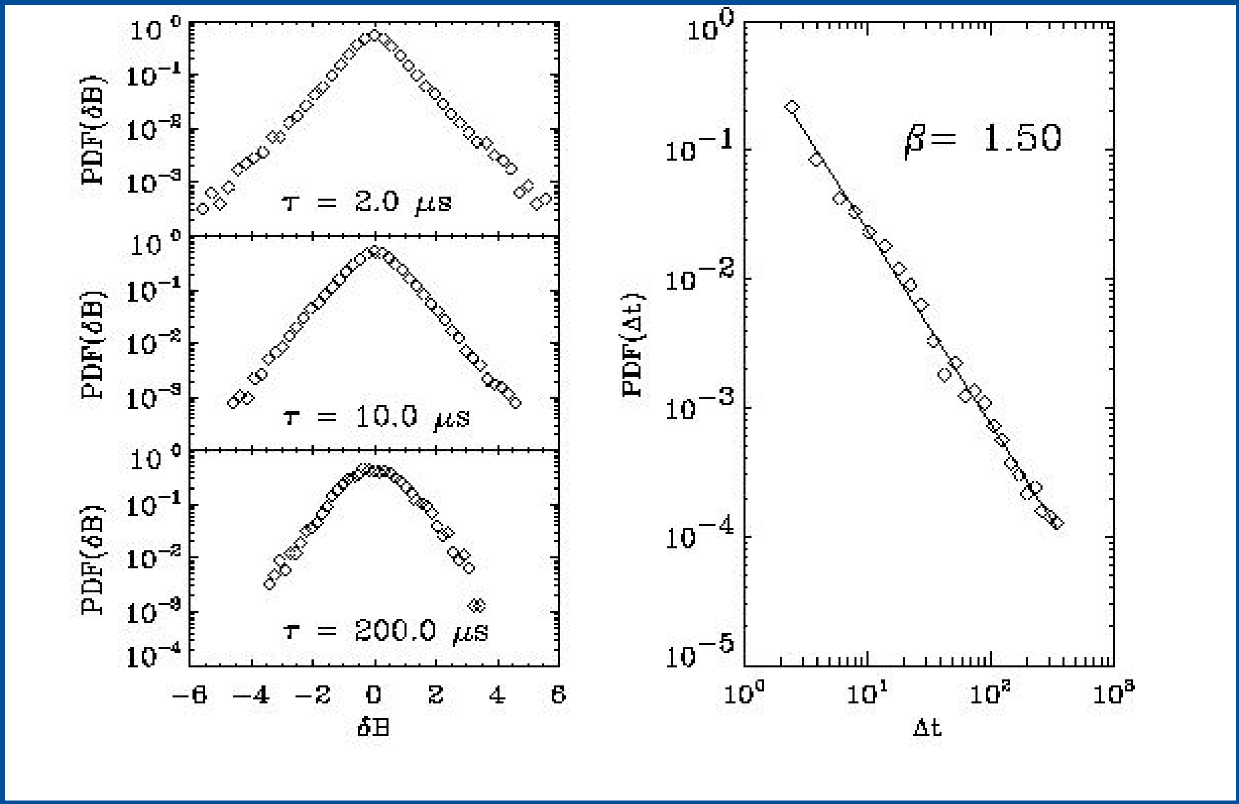


The time signal displays a fractal (selfsimilar) behavior



Waiting times between structures

Laboratory plasma



The times between events are distributed according to a power law

$Pdf(\Delta t) \sim \Delta t^{-\beta}$

The turbulent energy cascade generates intermittent "coherent" events.

The underlying cascade process is NOT POISSONIAN, this means that the intermittent (more energetic) bursts are NOT INDEPENDENT (memory)

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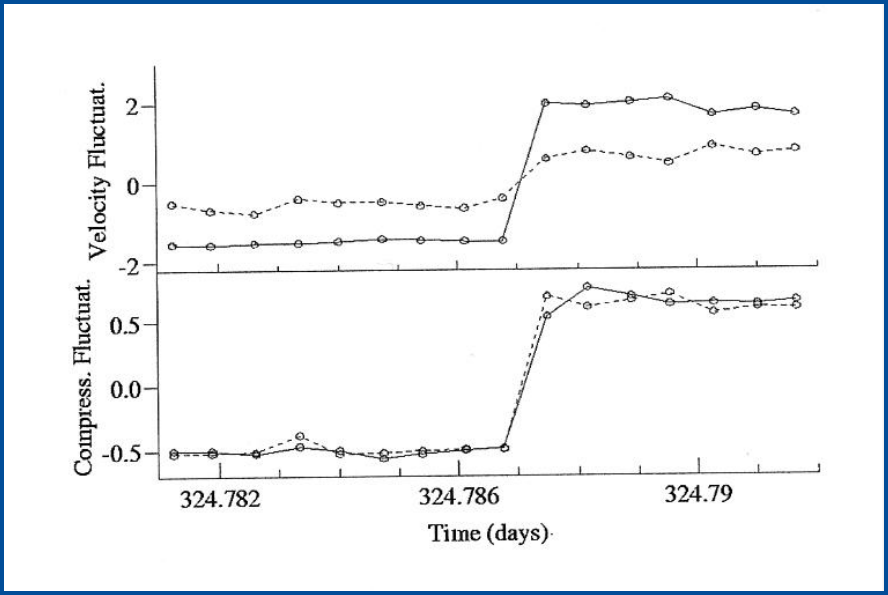
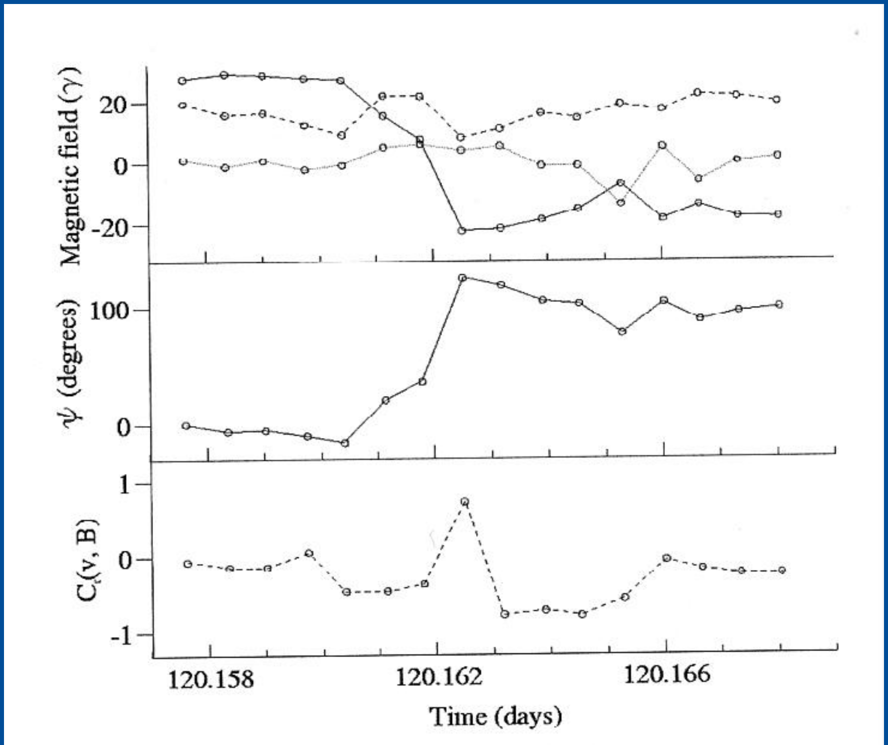
What kind of intermittent structures ?

Minimum variance analysis around isolated structure allows to identify them



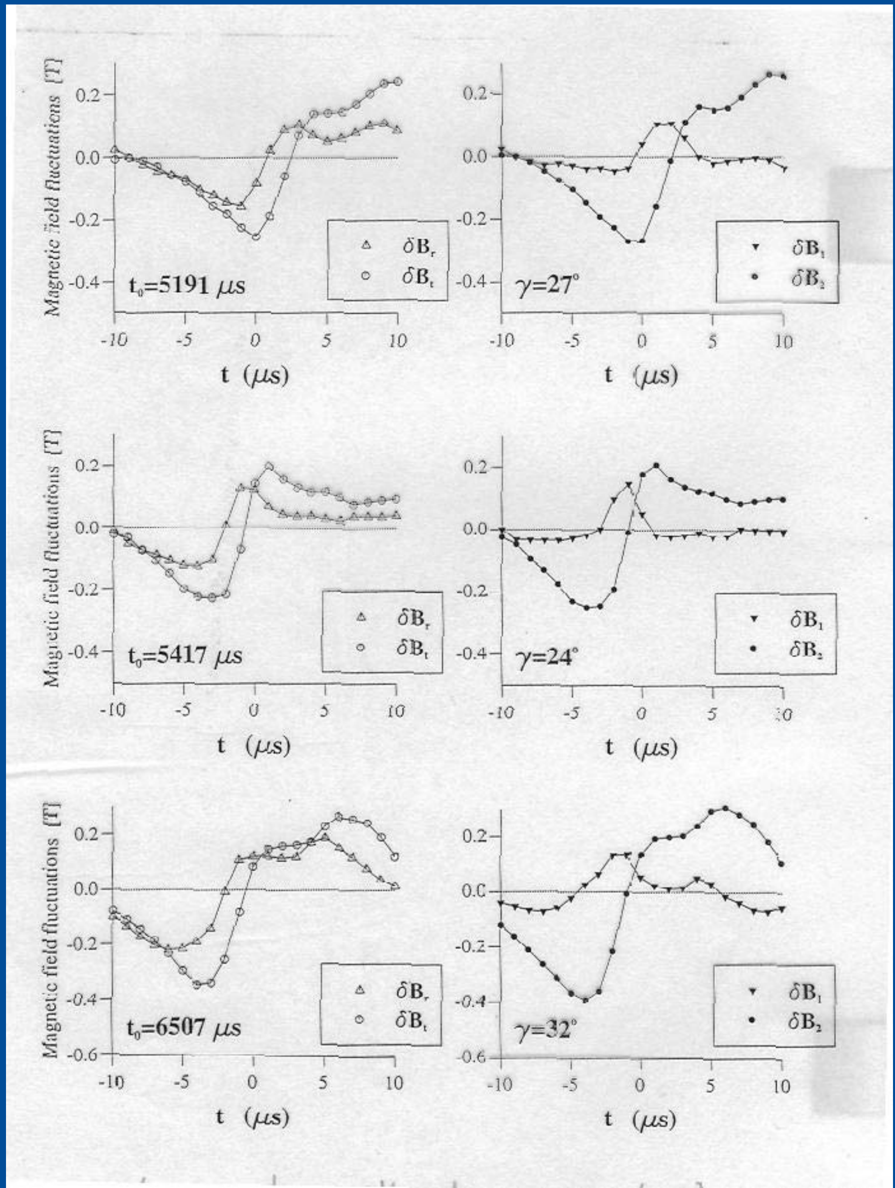
Solar Wind: shock
(compressive structures)



Solar wind: tangential discontinuity (current sheet)

Intermittent structures in laboratory plasmas

RFX edge turbulence:
current sheets



Current sheets are naturally produced as coherent, intermittent structures by nonlinear interactions

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A Statistical approach to Solar Flares

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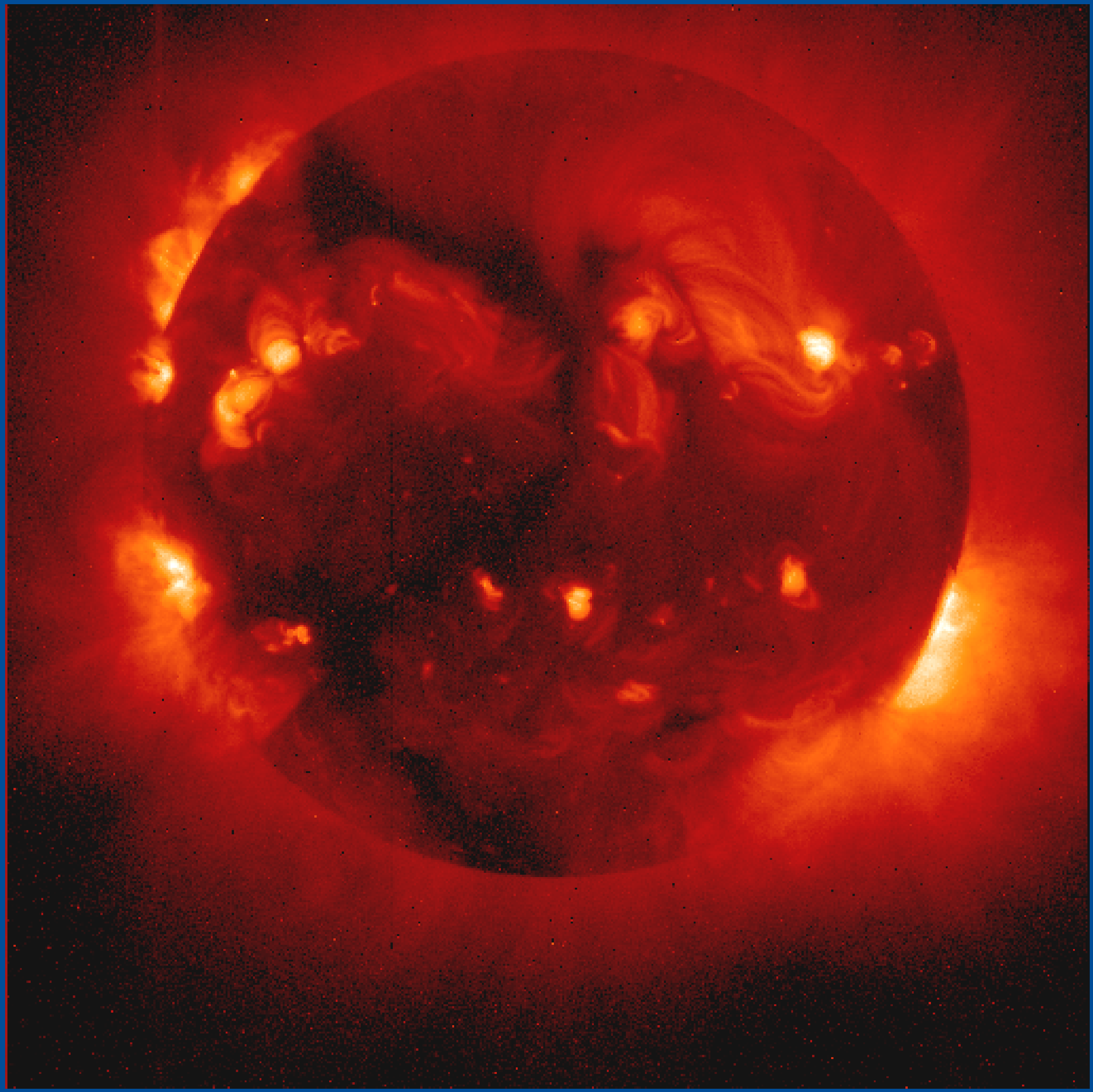
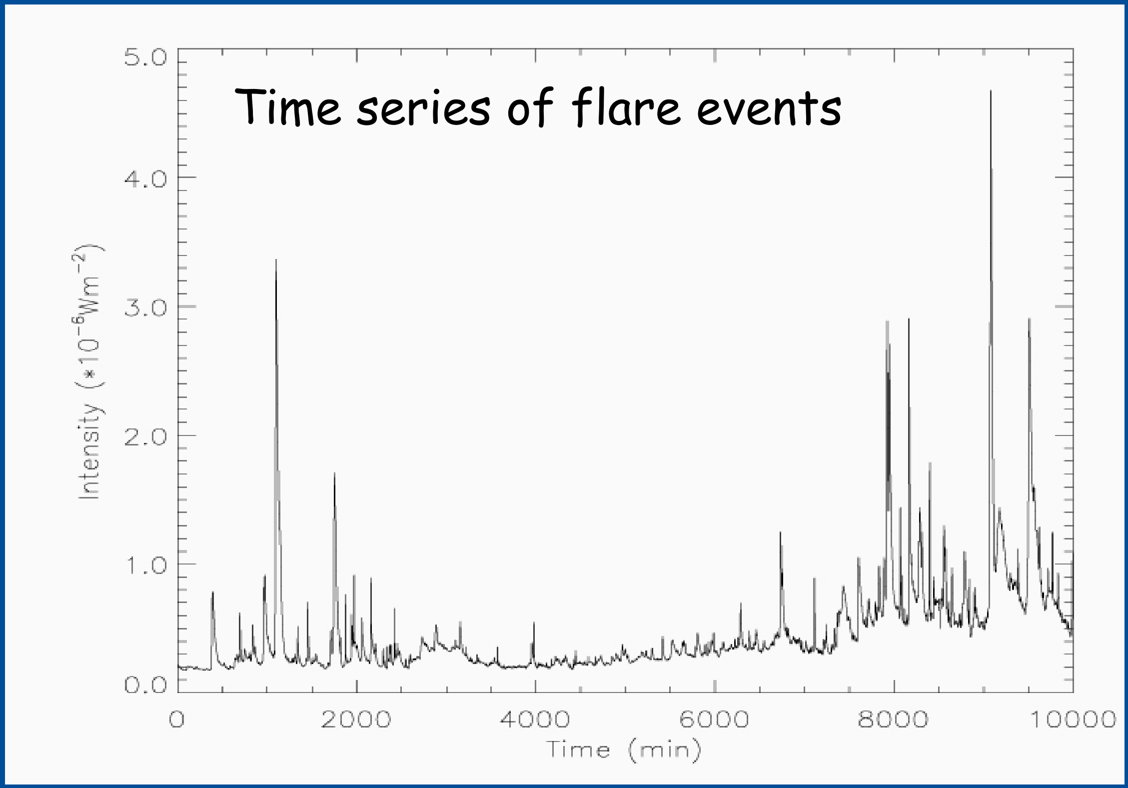


Ratio of EIT full Sun images in Fe XII 195A to Fe IX/X 171A.

Temperature distribution in the Sun's corona:
- dark areas cooler regions
- bright areas hotter regions

Solar flares are impulsive events



Hard X-ray (> 20 keV):

Intermittent spikes

Duration 1-2 s,

E_{max} ~ 10^{27} erg

Numerous smaller spikes down to 10^{24} erg (detection limit)

X-ray corona: superposition of a very large number of flares

Nano flares

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Power laws for statistics of events

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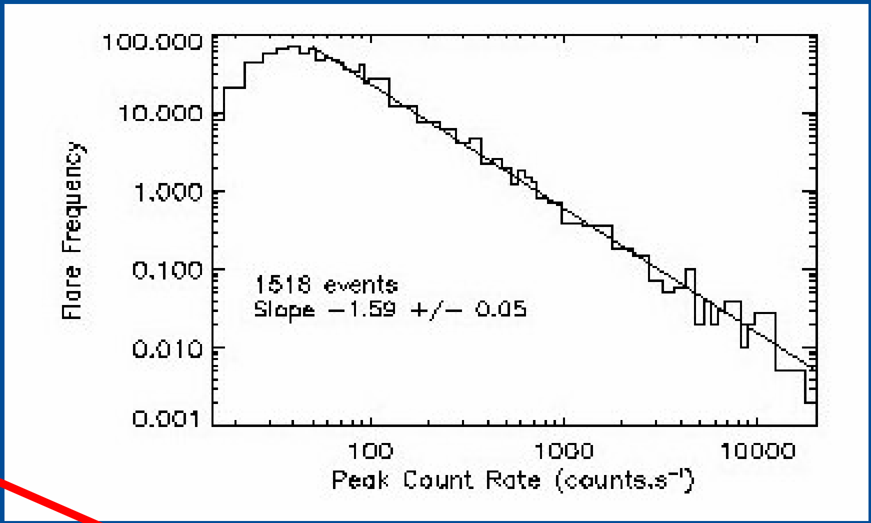
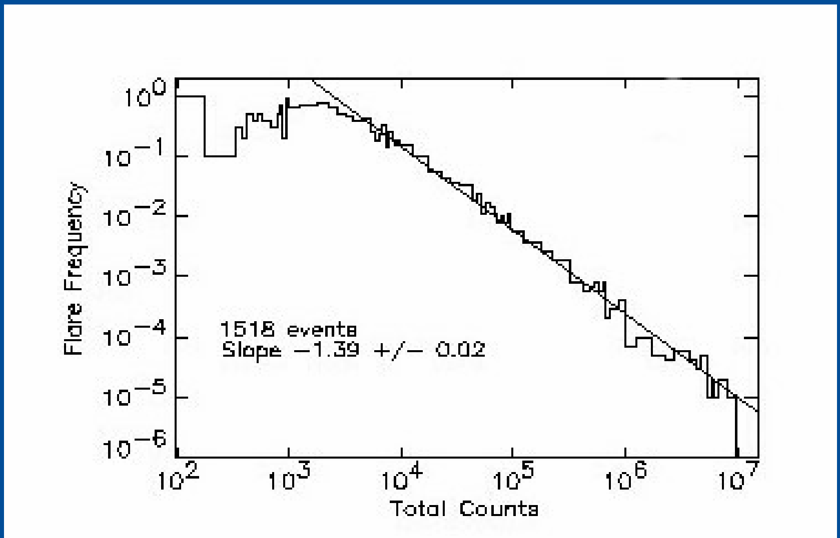
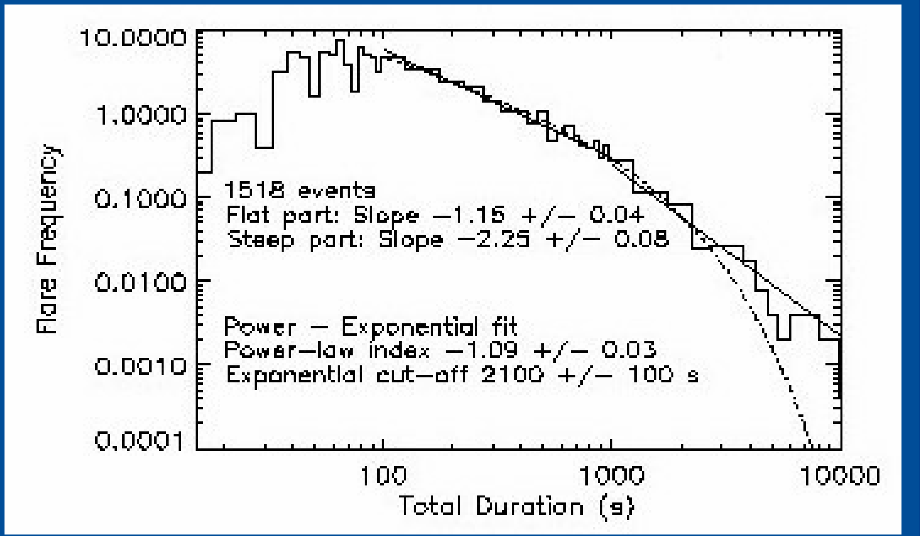
Power peak distribution

Duration distribution

Total energy distribution







Parker's conjecture (1988)

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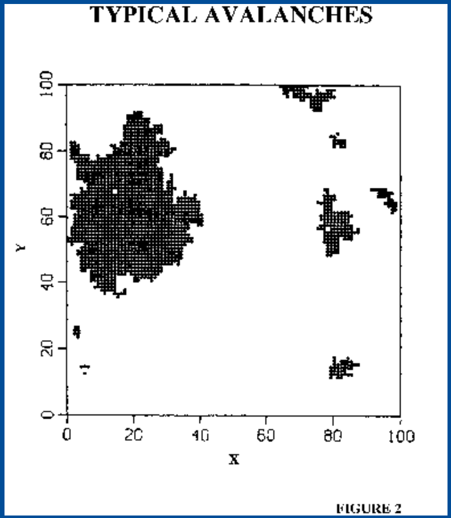
Nanoflares correspond to dissipation of many small current sheets, forming in the bipolar regions as a consequence of the continous shuffling and intermixing of the footpoints of the field in the photospheric convection.

Current sheets: tangential discontinuity which become increasingly severe with the continuing winding and interweaving eventually producing intense magnetic dissipation in association with magnetic reconnection,

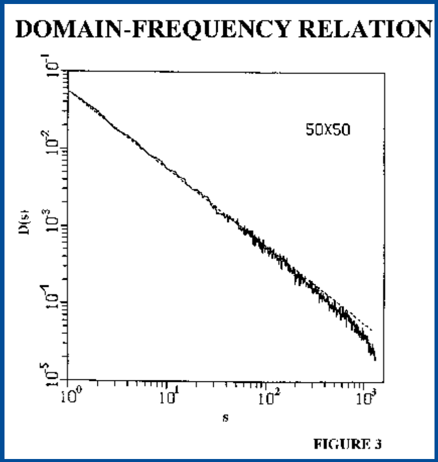
Self-Organized Criticality (P. Bak et al., 1987)

Sand Pile model for Solar Flares:

Fall of grains give rise to an avalanche whose dimension is that of the marginally stable region



Lack of any typical length → Avalanches of all size i.e. FRACTAL PROCESS



Size, lifetimes and number of sand grains in each avalanche are power law distributed.

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Sand Pile Model for Solar Flares (Lu & Hamilton., 1991)

Cellular Automata model for reconnection :

- Vector field B_i on a 3D lattice
- Local slope $dB_i = B_i - \sum_j w_j B_{i+j}$

When $|dB_i|$ > some treshold: instability at position i :

- Field readjusted in the nearby positions so that the grid point i becomes stable
- The readjustment can destabilize nearby points producing an avalanche

Power peak, total energy and duration are power law distributed.

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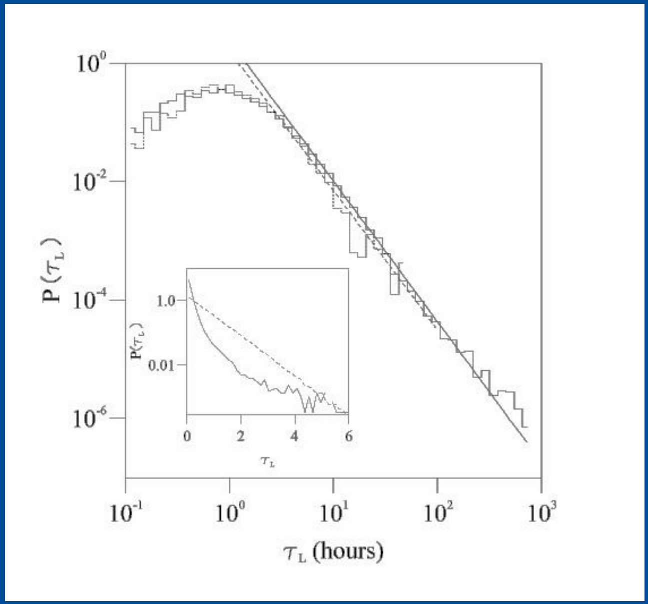
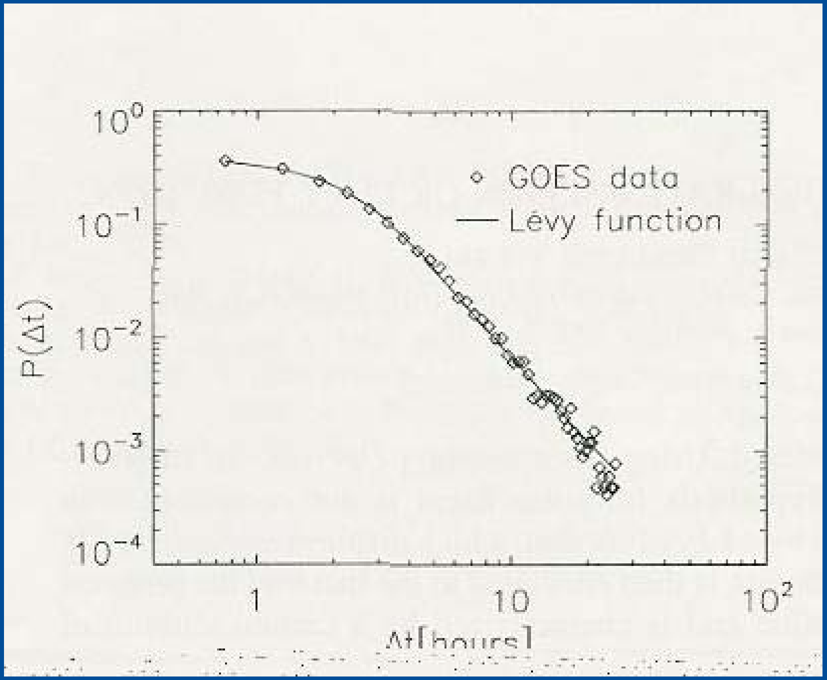
Waiting time distribution



Waiting time distribution of soft X-ray nanoflares displays a power law (Boffetta et al., 1999)

SOC models naturally give rise to an exponential distribution



Waiting time distribution of soft X-ray nanoflares can be fitted with a Levy function (asymptotically power law) (Lepreti et al., 2001)

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Parker's conjecture modified

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Nanoflares correspond to dissipation of many small current sheets, forming in the nonlinear cascade occuring inside coronal magnetic structure as consequence of the power input in the form of Alfven waves due to footpoint motiont.

Current sheets: coherent intermittent small scale structures of MHD turbulence

Turbulence modelling

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Dynamical shell models:

Simulations of the cascade process through the dynamics of nonlinear couplings.

Allow us to obtain the time evolution of fluctuations, to find scaling laws and to describe the chaotic dynamics on the attractor in the phase space.

Geometrical description is neglected but nonlinear interactions are described at best

MHD equations in the wave vectors space

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For
σ = +1, -1

$$\frac{\partial z_\alpha^\sigma(\mathbf{k},t)}{\partial t} = M_{\alpha\beta\gamma}(\mathbf{k})\sum_p z_\beta^\sigma(\mathbf{p},t) z_\gamma^{-\sigma}(\mathbf{k}-\mathbf{p},t)$$

$$M_{\alpha\beta\gamma}(\mathbf{k}) = -ik_\beta\left(\delta_{\alpha\gamma} - \frac{k_\alpha k_\gamma}{k^2}\right)$$

$$z_\alpha^\sigma = u_\alpha + \sigma\frac{B_\alpha}{\sqrt{4\pi\rho}}$$

Infinite wave vectors involved in the sum, but the presence of dissipation introduces a maximum wave vector

$$k_{\max} \approx k_{diss} \approx k_0 R^{3/4}$$

Even in this case we have a large number of variables

$$N \approx (k_{diss}/k_0)^3 \approx R^{9/4}$$

Dynamical (shell) models

1) Introduce a logarithmic spacing of the wave vectors space (shells);

$$k_n = k_0 2^n$$
$$n = 1,2,\ldots,N$$

2) Assign to each shell ONLY two dynamical variables;

$$z_n^{\pm}(t) = u_n(t) \pm b_n(t)$$

3) Write a nonlinear equations with couplings among variables belonging to local shells;

$$\frac{dz_n^{\pm}(t)}{dt} = ik_n \sum_{i,j=\pm 2,\pm 1} M_{i,j} z_{n+i}^{\pm}(t) z_{n+j}^{\mp}(t)$$

4) Finally fix the coupling coefficients imposing the conservation of ideal invariants.

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Properties of MHD shell model

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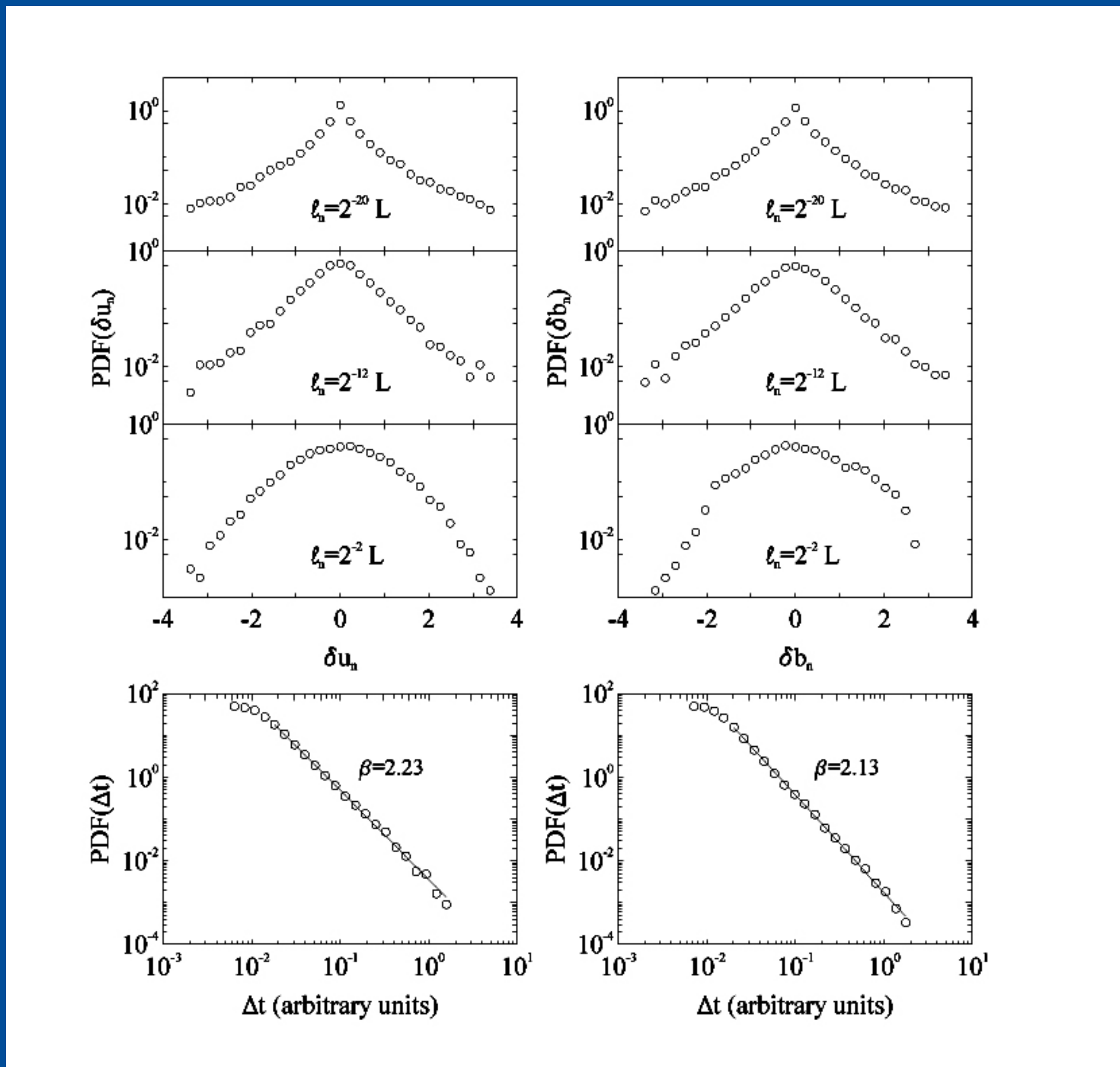

The shell model is able to capture the intermittent behaviour of real turbulence

Chaotic dynamics during the energy cascade generates non poissonian events

Isolated bursts of dissipation in shell model

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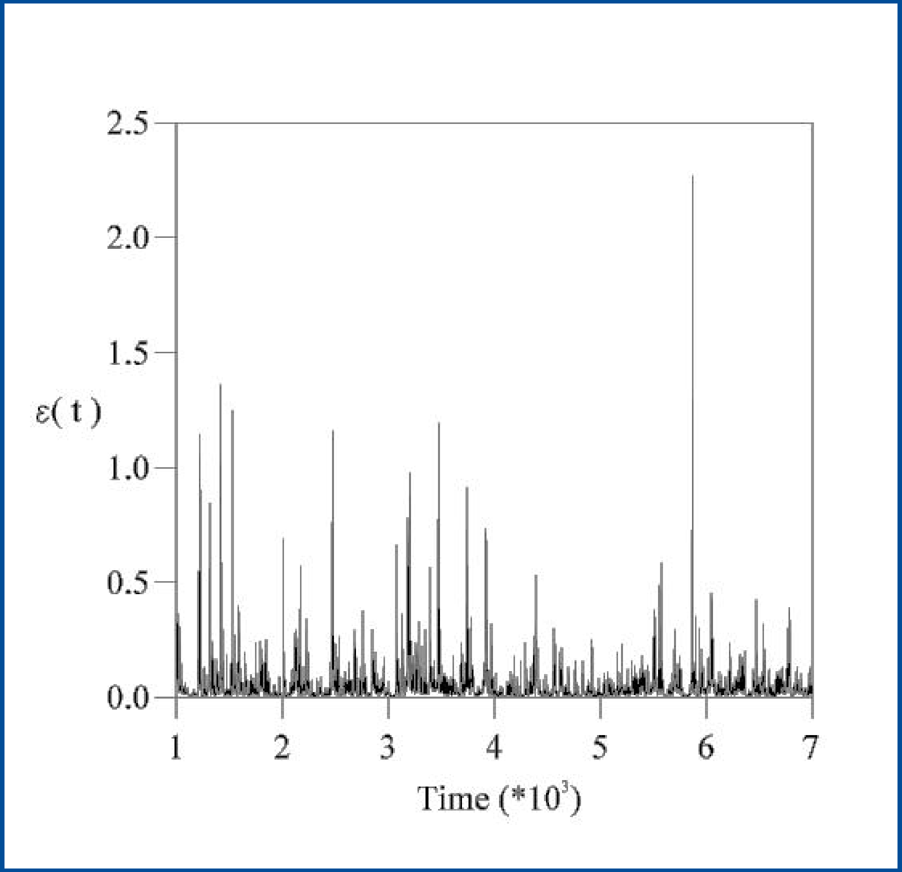
$$\varepsilon(t) = \nu \sum_n k_n^2 |u_n|^2 + \eta \sum_n k_n^2 |b_n|^2$$

The energy dissipation rate is intermittent. Energy is dissipated through impulsive isolated events (bursts).

Bursts can be isolated through a threshold process to make statistics.



Solar Flares: intermittent dissipative events within MHD turbulence?

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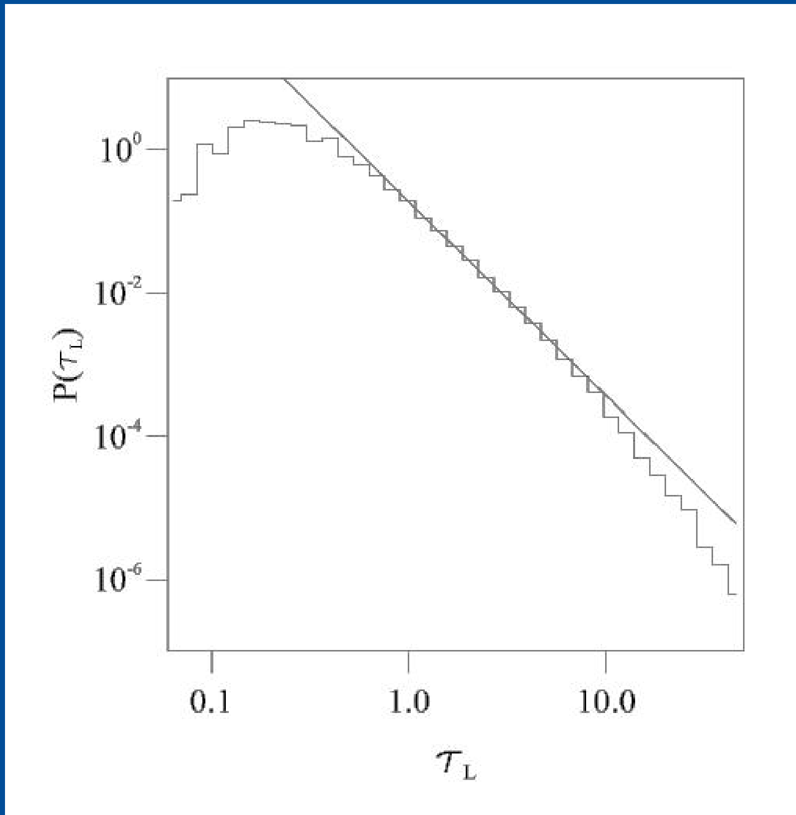

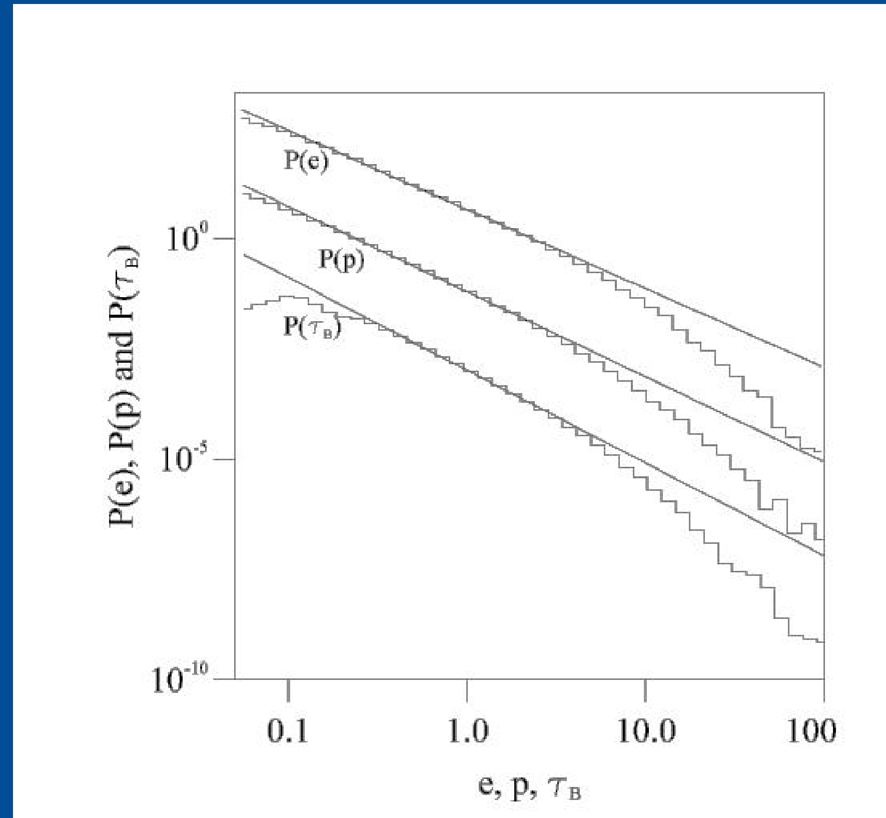

The time between two bursts is τ, we calculate the pdf $p(\tau)$.

1) Total energy of bursts
2) Time duration
3) Energy of peak

In all cases we found power laws.

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MHD Turbulence in Coronal Loops

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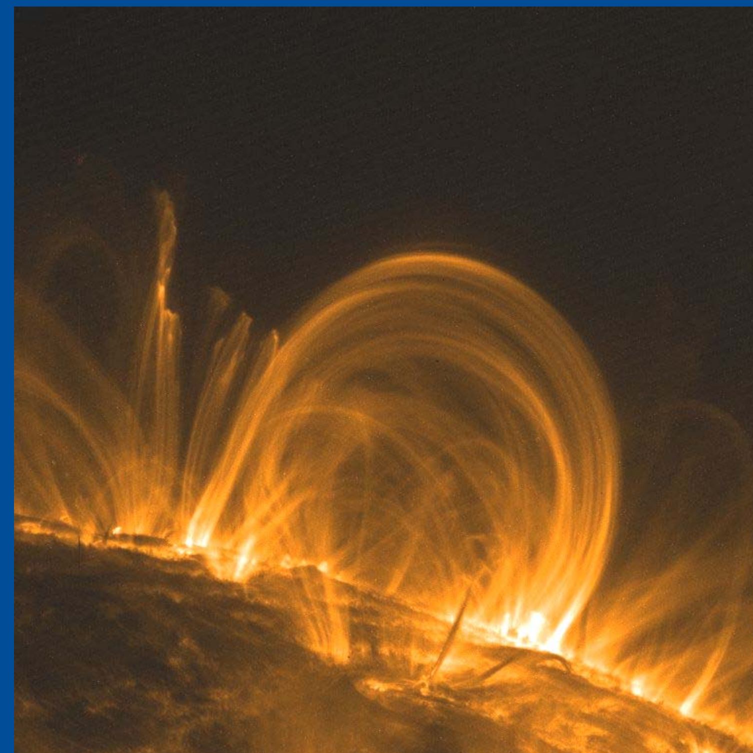
In a coronal loop:



Small beta values

$$\beta \approx 10^{-2} << 1$$

Large aspect ratio

$$R = \frac{L}{L_\perp} >> 1$$

Small perpendicular to parallel magnetic field ratio

$$\frac{B_\perp}{B_0} < \frac{1}{R} << 1$$

Reduced MHD can be used (RMHD):

$$\frac{\partial \mathbf{z}_\perp^{\sigma}}{\partial t} + \left(\mathbf{z}_\perp^{-\sigma} \cdot \nabla_\perp\right)\mathbf{z}_\perp^{\sigma} - \sigma c_A \frac{\partial \mathbf{z}_\perp^{\sigma}}{\partial x} = -\nabla_\perp \left(P + \frac{B^2}{8\pi}\right) + \nu \nabla^2 \mathbf{z}_\perp^{\sigma}$$

- Incompressible MHD in perpendicular variables
- Alfven wave propagation along background magnetic field

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A Hybrid Shell Model

RMHD equations in the wave vector space perpendicular to B_0 :

$$\left(\frac{\partial}{\partial t} - \sigma c_A \frac{\partial}{\partial x}\right) \mathbf{z}_{\perp i}{}^{\sigma}(\mathbf{k},x,t) = \sum_{\mathbf{p}} M_{ilm}(\mathbf{k})\, \mathbf{z}_{\perp l}{}^{-\sigma}(\mathbf{p},x,t)\mathbf{z}_{\perp m}{}^{\sigma}(\mathbf{k}-\mathbf{p},x,t) - \nu k^2 \mathbf{z}_{\perp i}{}^{\sigma}(\mathbf{k},x,t)$$

A shell model in the wave vector space perpendicular to B_0 can be derived:

$$\left(\frac{\partial}{\partial t} - \sigma \frac{\partial}{\partial x}\right) Z_n^\sigma(x,t) = -\chi\, k_n^2 Z_n^\sigma(x,t) + \tag{3}$$
$$ik_n \left(\frac{11}{24} Z_{n+1}^{\sigma} Z_{n+2}^{-\sigma} + \frac{13}{24} Z_{n+1}^{-\sigma} Z_{n+2}^{\sigma} - \frac{19}{48} Z_{n+1}^{\sigma} Z_{n-1}^{-\sigma} - \right.$$
$$\left. \frac{11}{48} Z_{n+1}^{-\sigma} Z_{n-1}^{\sigma} + \frac{19}{96} Z_{n-1}^{\sigma} Z_{n-2}^{-\sigma} - \frac{13}{96} Z_{n+1}^{-\sigma} Z_{n-1}^{\sigma}) \right)^*$$

(Hybrid : the space dependence along B_0 is kept)

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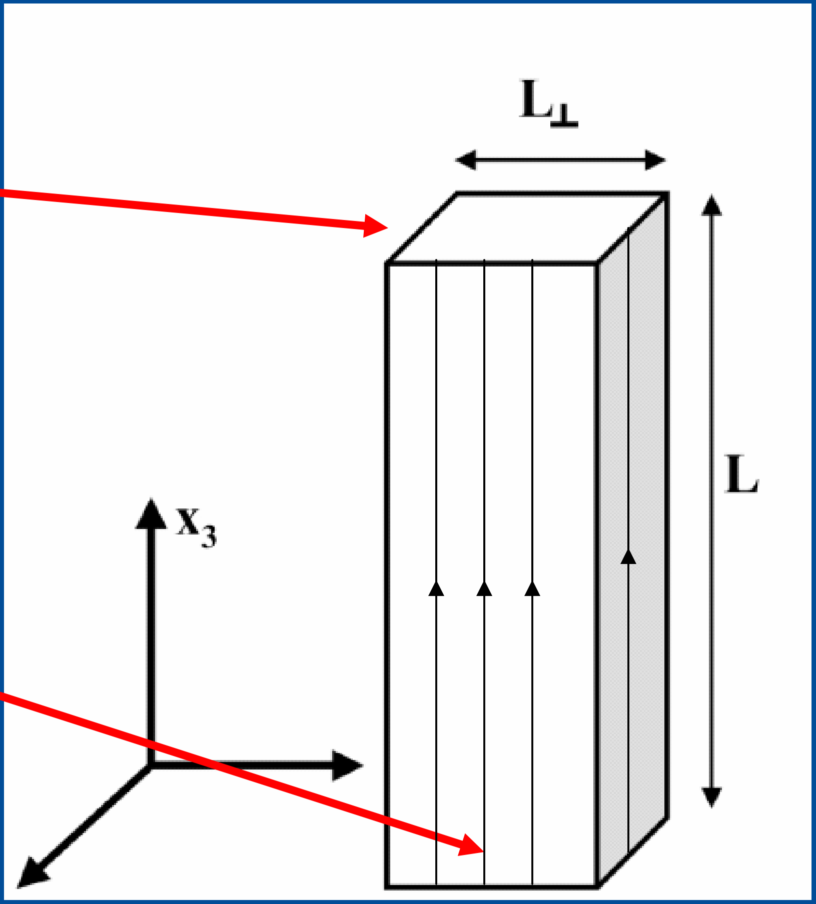
Boundary Conditions

Space dependence along B_0 allows to chose boundary conditions:

Total reflection is imposed at the upper boundary

A random gaussian motion with autocorrelation time t_c = 300 s is imposed at the lower boundary only on the largest scales

The level of velocity fluctuations at lower boundary is of the order of photospheric motions $\delta v \sim 5\ 10^{-4}\ c_A \sim 1$ Km/s



Model parameters: $L \sim 3\ \ 10^4$ Km, $R \sim 6$, $c_A \sim 2\ \ 10^3$ Km/s

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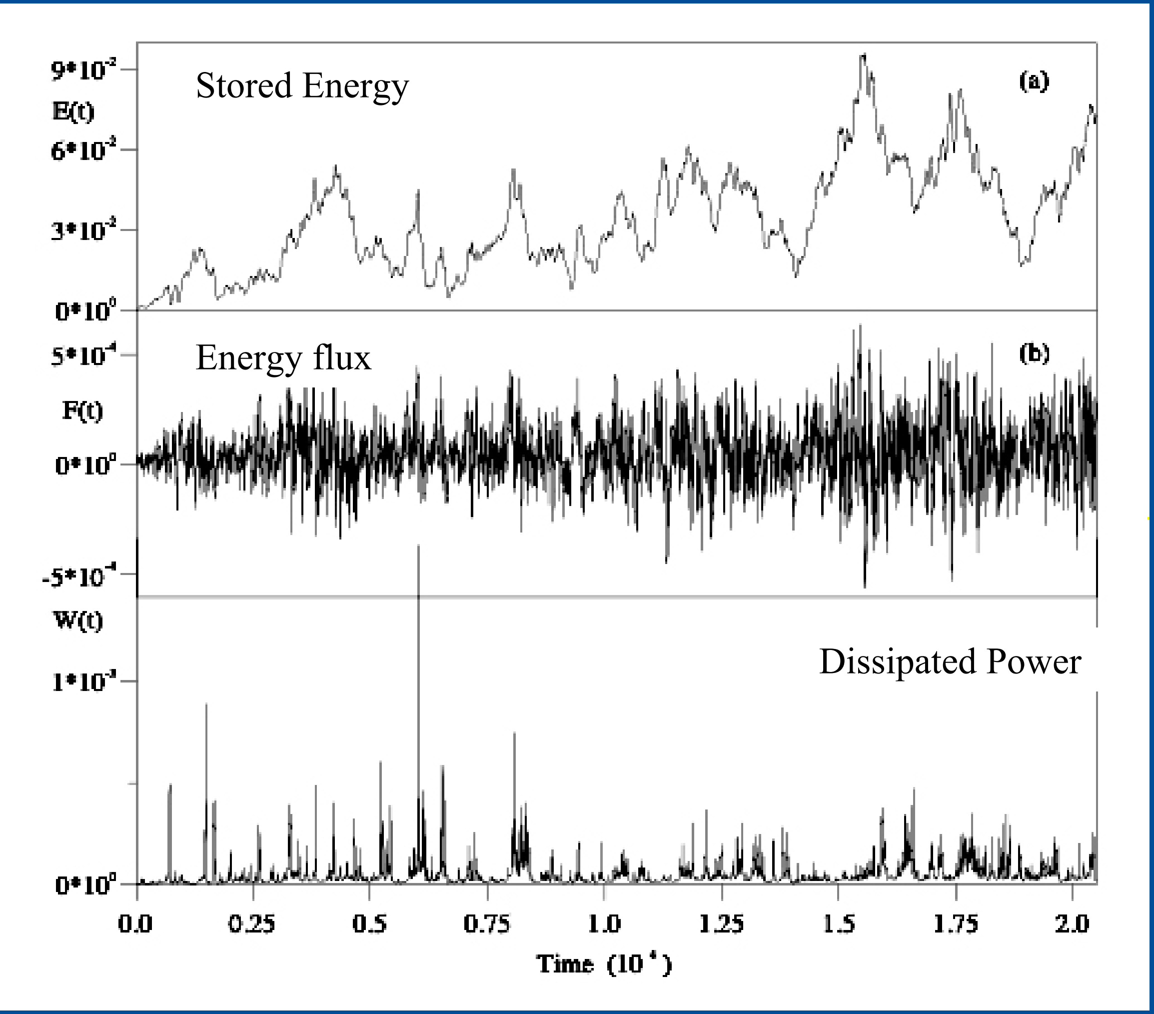
Energy balance

After a transient a statistical equilibrium is reached between incoming flux, outcoming flux and dissipation

The level of fluctuations inside the loop is considerably higher than that imposed at the lower loop boundary

Dissipated power displays a sequence of spikes



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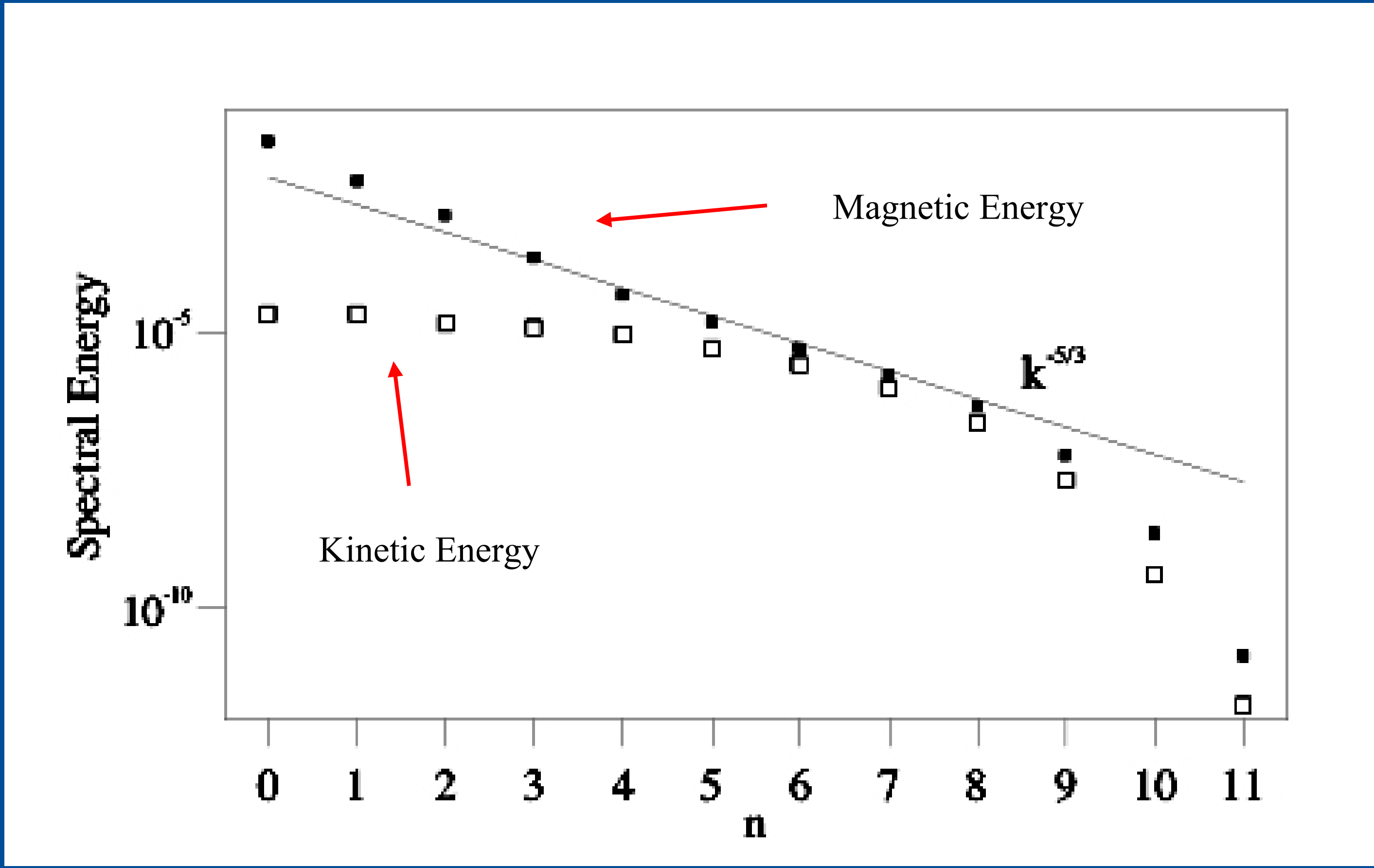
Energy spectra



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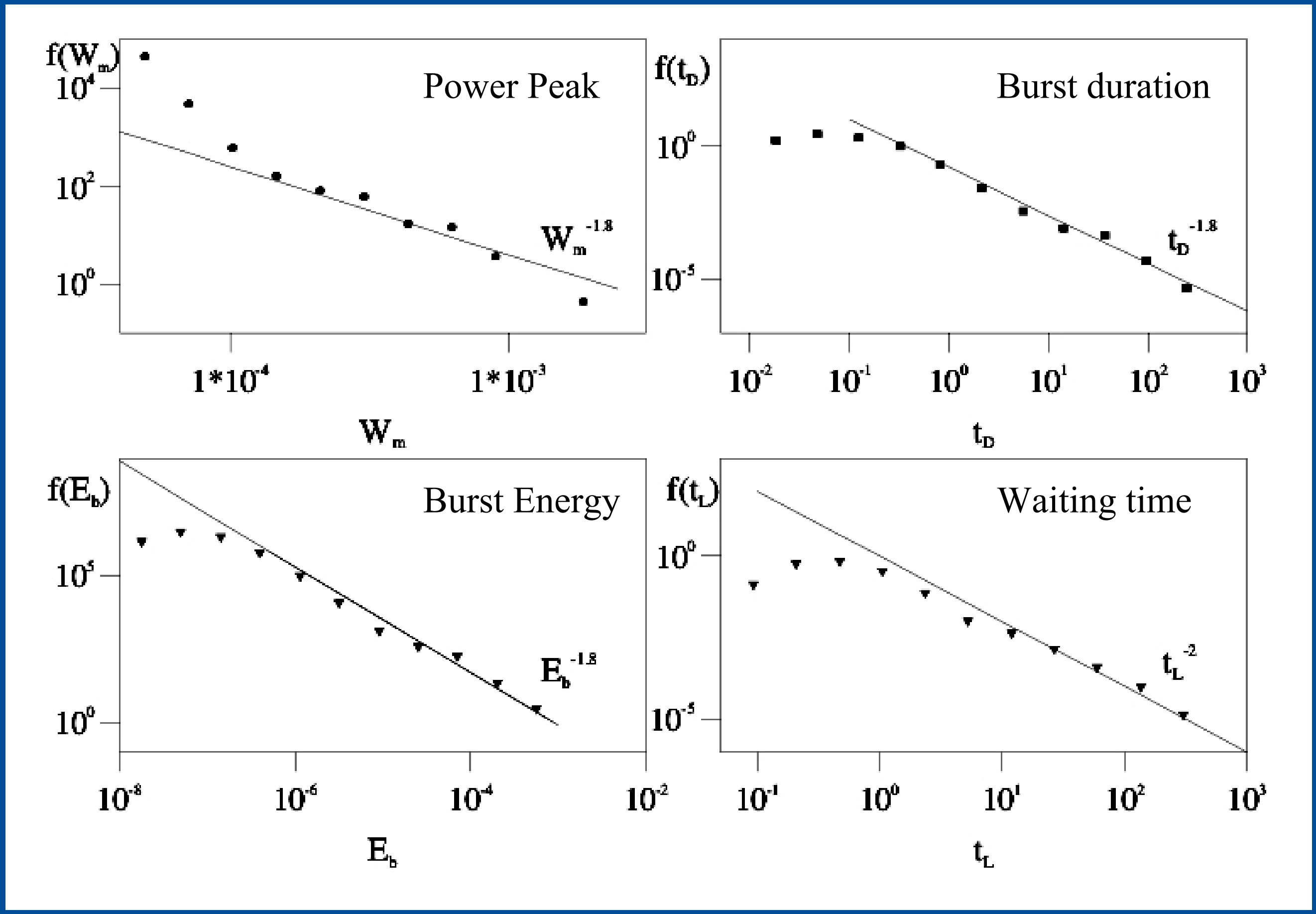
A Kolmogorov spectrum is formed mainly on magnetic energy

Magnetic energy dominates with respect to kinetic energy

Statistical analysis of dissipated power



Power laws are recovered on Power peak, burst duration, burst energy and waiting time distributions

The obtained energy range correspond to nanoflare energy range

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Dissipation mechanism ?

Explicit form of dissipation terms in RMHD:

Viscous term in V_x equation

$$\eta_1\left(\nabla_\perp{}^2 + 4\frac{\partial^2}{\partial z^2}\right)V_x + \eta_3\left(\nabla_\perp{}^2 + 2\frac{\partial^2}{\partial z^2}\right)V_y$$

Viscous term in V_y equation

$$\eta_1\left(\nabla_\perp{}^2 + 4\frac{\partial^2}{\partial z^2}\right)V_y - \eta_3\left(\nabla_\perp{}^2 + 2\frac{\partial^2}{\partial z^2}\right)V_x$$

Resistive term in B_x equation

$$\frac{c^2}{4\pi\sigma_\parallel}\nabla_\perp{}^2 B_x$$

Resistive term in B_y equation

$$\frac{c^2}{4\pi\sigma_\parallel}\nabla_\perp{}^2 B_y$$

Reynolds Numbers

$$R_1 = \frac{VL}{\eta_1} \approx 10^{15}$$

$$R_3 = \frac{VL}{\eta_3} \approx 10^{9}$$

Lundquist Number

$$S = \frac{4\pi\sigma_\parallel}{c^2} L c_A \approx 10^{14}$$

But η_3 term does not contribute to dissipation !

Dissipation lengths much smaller than ion Larmor radius

We need an efficient Kinetic dissipation mechanism !

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Conclusions

Physical understanding of complex nonlinear phenomena occurring in plasmas require the coordinated utilization of different tools: space and laboratory data analysis, simplified dynamical models, numerical simulations.

Dynamical models, in particular, are usefull to describe turbulence, intermittency, anomalous scalings of pdfs, etc.

The intermittent behavior observed in turbulent fluid flows seems to represent a key to explain some burstly phenomena occurring in space, laboratory and solar corona plasmas.

The signature of non linear interactions seems to be a multifractal behavior: SOC models being intrinsecally fractal cannot adequately describe the complexity of turbulent systems. In particular they cannot naturally describe the correlations between the intermittent coherent small scale structures of turbulence.