

## **AUTUMN COLLEGE ON PLASMA PHYSICS**

13 October - 7 November 2003

# **Results in Hamiltonian Magnetic Field Reconnection**

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Autumn College on Plasma Physics  
Trieste, 2003

# **Results in Hamiltonian magnetic field reconnection**

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# 1 What is Magnetic Reconnection?

Magnetic field line reconnection is one of the most general phenomena in magnetized plasmas and has been widely investigated in astrophysical and space plasmas, in laboratory magnetically confined plasmas and, more recently, in relativistic laser produced plasmas.

## 1.1 Magnetic connections and field topology

The idea of “magnetic field line reconnection” is first based on the the concept of time evolution of magnetic field lines.

The concept of time evolution of magnetic field lines is not a priori obvious as there is in principle no direct way to associate a field line of the vector field  $\vec{B}(\vec{x}, t_1)$  with a field line of  $\vec{B}(\vec{x}, t_2)$ , as field lines in physical space are only defined at fixed time.

Such an association between field lines at different times of a vector field  $\vec{A}(\vec{x}, t)$  is possible, or more precisely is convenient, when we deal with physical systems which obey special topological conservations.

Suppose that the points in the system under consideration move with velocity field  $\vec{v}(\vec{x}, t)$  and that for any two points in the system that at  $t = 0$  are connected by a field line of  $\vec{A}(\vec{x}, 0)$ , at any subsequent time  $t$  there exists a field line of  $\vec{A}(\vec{x}, t)$  that connects these two same points at their new positions, which are given by the trajectories given by the velocity field  $\vec{v}(\vec{x}, t)$ . Then, in this case, we can introduce the concept of field line evolution saying that the new connecting field lines are simply the time evolution of the one we started from at  $t = 0$ .

In incompressible inviscid hydrodynamics the velocity field  $\vec{v}(\vec{x}, t)$  is the velocity  $\vec{u}_f$  of the fluid elements and the connection field lines are those of the fluid vorticity  $\vec{\omega} \equiv \nabla \times \vec{u}_f$ . In this case the connection condition can be expressed in differential form as

$$\frac{d(d\vec{l} \times \vec{\omega})}{dt} = 0, \quad \text{if} \quad d\vec{l} \times \vec{\omega} = 0 \quad \text{at} \quad t = 0, \quad (1)$$

where  $d/dt$  is the Lagrangian time derivative defined by the vector field  $\vec{u}_f$  and expressed in Eulerian variables by  $d/dt \equiv \partial/\partial t + \vec{u}_f \cdot \nabla$  and  $d\vec{l}(t) \equiv \vec{x}_2(t) - \vec{x}_1(t)$  is the difference in position between the two close points  $\vec{x}_2(t)$  and  $\vec{x}_1(t)$  which move with velocities  $\vec{u}_f(\vec{x}_1(t))$  and  $\vec{u}_f(\vec{x}_2(t))$  respectively.

### 1.1.1 Exercise

Prove Eq.(1) using Euler's equation in the form

$$\rho \frac{d\vec{u}_f}{dt} = -\nabla p, \quad (2)$$

with  $\rho$  the constant fluid density and  $p(\vec{x}, t)$  the fluid pressure.

Hint: First apply the operator  $\nabla \times$  to Eq.(2) in Eulerian coordinates and obtain an equation for the fluid vorticity  $\vec{\omega}$  that does not depend on  $p$ .

Then prove that  $d(d\vec{l})/dt = ((d\vec{l}) \cdot \nabla)\vec{u}_f$ .

Finally express the equation for the vorticity in Lagrangian variables and perform the vector product etc...

## 1.2 Magnetohydrodynamics

In ideal magnetohydrodynamics (MHD) the velocity field is the velocity  $\vec{u}$  of the quasineutral plasma elements and the connection field lines are those of the plasma magnetic field  $\vec{B}$ .

Start from Ohm's law

$$\vec{E} + \frac{\vec{u}}{c} \times \vec{B} = \eta \vec{J} + \frac{m_e}{ne^2} \frac{d\vec{J}}{dt} + \frac{1}{nec} \vec{J} \times \vec{B} - \frac{1}{ne} \vec{\nabla} P_e \quad (3)$$

and assume that the r.h.s., which describes the effect of electron resistivity, of electron inertia and of the Hall and of the electron pressure terms, is negligible (or can be reduced to a gradient)

Then Faraday's law combined with Eq.(3) gives

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{u} \times \vec{B}) \quad (4)$$

which has an algebraic structure analogous to that of the vorticity equation in a fluid.

From Eq.(4) we obtain

$$\frac{d(d\vec{l} \times \vec{B})}{dt} = (d\vec{l} \times \vec{B}) (\vec{\nabla} \cdot \vec{u}) - [(d\vec{l} \times \vec{B}) \times \vec{\nabla}] \times \vec{u} \quad (5)$$

i.e., using the continuity equation for the plasma particle density  $n$ ,

$$\frac{d[(d\vec{l} \times \vec{B})/n]}{dt} = -\{[(d\vec{l} \times \vec{B})/n] \times \vec{\nabla}\} \times \vec{u} \quad (6)$$

Equation (4) expresses the condition that within the ideal M.H.D. equations if two plasma points are initially connected by a magnetic field line, they remain connected by a magnetic field line at any subsequent time (regularity properties of the plasma flow have been obviously assumed)

### 1.2.1 Exercises

Prove under the same conditions Alfvén theorem (frozen magnetic flux)

$$\frac{d}{dt} \int_S \vec{B} \cdot d\vec{S} = 0 \quad (7)$$

if the surface  $S$  moves together with the plasma.

Solution

$$\begin{aligned} \frac{d}{dt} \int_S \vec{B} \cdot d\vec{S} &= \int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} + \int_{\partial S} \vec{B} \cdot (\vec{u} \times d\vec{l}) = \\ &= \int_S \left( \frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \times (\vec{B} \times \vec{u}) \right) \cdot d\vec{S}, \end{aligned} \quad (8)$$

etc.....

Prove the conservation of the magnetic helicity  $\vec{A} \cdot \vec{B}$  with the vector potential (integrated inside a closed magnetic flux tube moving with the fluid).

Hint : use

$$\partial_t (\vec{A} \cdot \vec{B}) = \partial_t \vec{A} \cdot \vec{B} + \vec{A} \cdot \partial_t \vec{B} \quad (9)$$

From  $\vec{E} = -\vec{\nabla}\phi - \partial_t \vec{A}/c$  we obtain:

$$\frac{1}{c} \frac{\partial}{\partial t} (\vec{A} \cdot \vec{B}) = -\vec{E} \cdot \vec{B} - \vec{\nabla}\phi \cdot \vec{B} - \vec{A} \cdot (\vec{\nabla} \times \vec{E}). \quad (10)$$

Using  $\vec{\nabla} \cdot (\vec{A} \times \vec{E}) = \vec{E} \cdot (\vec{\nabla} \times \vec{A}) - \vec{A} \cdot (\vec{\nabla} \times \vec{E}) = \vec{E} \cdot \vec{B} - \vec{A} \cdot (\vec{\nabla} \times \vec{E})$  we obtain

$$\frac{1}{c} \frac{\partial}{\partial t} (\vec{A} \cdot \vec{B}) + \vec{\nabla} \cdot [\phi \vec{B} - (\vec{A} \times \vec{E})] = -2\vec{E} \cdot \vec{B} \quad (11)$$

From Ohm's law with vanishing r.h.s. we obtain  $\vec{E} \cdot \vec{B} = 0$ . Then we must compute the contribution of the flux term through the surface  $S$  of a magnetic flux tube occupying the volume  $V$  moving with velocity  $\vec{u}$ . We write  $K_V = \int_V \vec{A} \cdot \vec{B} dV$  and obtain

$$\frac{d}{dt} K_V = \frac{d}{dt} \int_V \vec{A} \cdot \vec{B} dV = \int_V \partial_t (\vec{A} \cdot \vec{B}) dV + \int_S (\vec{A} \cdot \vec{B}) \vec{u} \cdot d\vec{S} = \dots \quad (12)$$

and use Eq. (11) where  $\vec{E}$  is re-expressed in terms of  $\vec{u}$  and  $\vec{B}$  through Ohm's law, recalling that  $\vec{B} \cdot d\vec{S} = 0$  because  $S$  is the surface of a flux tube. ....

### 1.3 Local breaking

The connection theorem (5) is "violated" by the terms on the r.h.s of Eq.(3).

The definition of magnetic field line reconnection requires that this breaking occurs only locally around critical points (or lines.. ) where field lines break and "reconnect" in a different pattern.



This separation between global regions where magnetic connections are preserved and localized regions where they are broken and reorganized is valid only in plasma regimes where the processes leading to the breaking of the connections are *per se* weak but are locally enhanced by the formation at the critical points of small spatial scales (singular perturbations).

#### **1.4 Magnetic connections and energy balance**

Closely related to the topological nature of magnetic reconnection is its interpretation as a process that leads to magnetic energy conversion. This is best understood in the case of ideally stable MHD plasma equilibria with inhomogeneous magnetic fields and current gradients. These equilibria can become unstable when the (infinite number of) constraints arising from the conservation of the magnetic connections between plasma elements are removed and lower magnetic energy states become available to the plasma.

In fact the magnetic energy release and the associated particle acceleration are possibly the features of magnetic reconnection most relevant to astrophysical plasmas whereas in the laboratory plasmas the most important feature is often related to the loss of plasma confinement due to the change of magnetic topology.

This energy release can lead to the interpretation of the nonlinear development of magnetic reconnection as a transition, forbidden within the ideal MHD equations, between two MHD (equilibrium) states with different magnetic energies, the excess energy being eventually dissipated into heat by the effect of the resistivity  $\eta$  (or transported away by accelerated particles).

### **1.5 Dissipationless magnetic reconnection**

This relationship between dissipation and reconnection ceases to be valid in dilute high temperature plasmas. In such collisionless plasma regimes we find that, even maintaining a fluid-like plasma description with a barotropic scalar pressure, magnetic connections are broken not by electron resistivity but by electron inertia.

In these regimes the plasma dynamics is Hamiltonian and the transformation of magnetic energy into plasma kinetic and internal energy is in principle reversible.

Furthermore, contrary to the dissipative case, generalized magnetic connections are preserved by the Hamiltonian plasma dynamics.

In the cold electron limit the new connections are defined by the vector field

$$\vec{B}_e \equiv \vec{B} - (m_e c/e) \nabla \times \vec{u}_e$$

(subscripts  $e$  denote electron quantities) which is proportional to the rotation of the fluid electron canonical momentum and which reduces to  $\vec{B}$  only in the limit of massless electrons.

### 1.5.1 Exercise

Using Ohm's law (3) with on its r.h.s. term only the Hall term leads to the same connection relationship given by Eq. (4) where the electron fluid velocity  $\vec{u}_e$  has been substituted for the plasma velocity  $\vec{u}$ .

Using Ohm's law (3) with on its r.h.s. term only the electron inertia contribution and the Hall term, prove the equivalent of Eq. (4) in terms of the electron fluid velocity  $\vec{u}_e$  and of the field  $\vec{B}_e \equiv \vec{B} - (m_e c/e) \nabla \times \vec{u}_e$ .

## 1.6 Current layers

Current layers are also a generic feature of magnetic reconnection.

*The formation of spatially localized current structures is related to the fact that the magnetic connections in the plasma are broken only locally, around the critical points of the magnetic configuration where the current density accumulates.*

In Hamiltonian plasma regimes the formation of current layers is an even more important feature since, as will be discussed below, their presence decouples the time evolution of the reconnecting magnetic field lines from that of the unbroken field lines of  $\vec{B}_e$ .

In the rest of this lecture I will describe some recent results [2, 3, 4] that have been obtained in the study of the nonlinear evolution of collisionless magnetic field reconnection in a fluid-like two-dimensional (2-D) model where the plasma is embedded in a strong guide magnetic field.

This model is mainly applicable to laboratory plasmas and the configuration I will introduce is constrained by boundary conditions that may be too restrictive for astrophysical plasmas.

This model gives a rather clear picture of the dynamical role played by the generalized connections in the nonlinear process of magnetic field line reconnection and in the magnetic energy redistribution.

## 2 Two-Dimensional Fluid Hamiltonian Model

We refer to high temperature plasma regimes where dissipative effects can be disregarded, but a fluid-like description is still acceptable. For a discussion of the physical requirements of such a regime see e.g. [5].

### 2.1 Magnetic configuration

We consider a 2-D magnetic field configuration uniform along  $z$  with

$$\vec{B} = B_0 \vec{e}_z + \vec{\nabla} \psi \times \vec{e}_z, \quad (13)$$

where  $B_0$  is taken to be constant and  $\psi(x, y, t)$  is the magnetic flux function.

Note that  $\psi(x, y, t) \vec{e}_z$  is the vector potential of the magnetic field in the  $x$ - $y$  plane.

#### 2.1.1 Exercise

Work out the expression that the topological conservation theorems introduced above take in this two-dimensional limit.

Show the role played by the magnetic flux function  $\psi(x, y, t)$  in these new conservation theorems.

## 2.2 Dynamic equations

The plasma dynamics is described by the two-fluid dissipationless “drift-Alfvén” model derived in [5] which includes the effects of electron inertia in Ohm’s law:

$$\partial F / \partial t + [\varphi, F] = e_s^2 [U, \psi], \quad (14)$$

$$\partial U / \partial t + [\varphi, U] = [J, \psi]. \quad (15)$$

Here  $F = \psi + d_e^2 J$  corresponds to the canonical fluid electron momentum and  $J = -\nabla^2 \psi$  is the electron current density along  $z$ ,  $\varphi(x, y, t)$  is the electron stream function, with  $\vec{e}_x \times \nabla \varphi$  the incompressible electron velocity in the  $x$ - $y$  plane, and  $U = \nabla^2 \varphi$  is the plasma fluid vorticity.

The Poisson brackets  $[A, B]$  are defined by

$$[A, B] = \vec{e}_z \cdot \vec{\nabla} A \times \vec{\nabla} B \quad (16)$$

### 2.2.1 Exercise

Show the relationship between the vector field  $\vec{B}_e$  introduced above and the scalar function  $F$ .

The first of the model equations, Eq.(14), describes the electron motion along field lines and is equivalent to the parallel component of Ohm's law, while the second, Eq.(15) originates from the continuity equation and includes parallel electron compressibility.

### 2.2.2 Exercise

Check the approximations used in [5] to derive Eqs.(14,15)

This electron temperature effect becomes important if the sound Larmor radius

$$\varrho_s = (m_i c^2 T_e / e^2 B^2)^{1/2} \quad (17)$$

is comparable to the collisionless electron skin depth

$$d_e = c / \omega_{pe}. \quad (18)$$

The fluid vorticity  $\nabla^2 \varphi$  is related to the ion density variation which is set equal to the electron density variation because of quasineutrality.

### 2.2.3 Exercise

Derive this relationship (see e.g., [5]) in terms of the divergence of the ion polarization drift in the ion continuity equation.

No equilibrium density and temperature gradients effects are included in the present analysis.

The system of Eqs. (14,15) is Hamiltonian with energy

$$H = \int d^2x \left( |\vec{\nabla}\psi|^2 + |\vec{\nabla}\varphi|^2 + d_e^2 J^2 + \varrho_s^2 U^2 \right) / 2. \quad (19)$$

It can be cast in Lagrangian form

$$\partial G_{\pm} / \partial t + [\varphi_{\pm}, G_{\pm}] = 0, \quad (20)$$

for the two Lagrangian invariants

$$G_{\pm} = F \pm d_e \varrho_s U \quad (21)$$

which are advected by the incompressible velocity fields

$$\vec{e}_z \times \vec{\nabla} \varphi_{\pm} \quad (22)$$

where

$$\varphi_{\pm} = \varphi \pm (\varrho_s / d_e) \psi \quad (23)$$

are generalized stream functions and the term  $\pm(\varrho_s / d_e)\psi$  accounts for the thermal electron motion along magnetic field lines.



In the cold electron limit ( $\varrho_s \rightarrow 0$ ) the Lagrangian invariants  $G_{\pm}$  become degenerate and only

$$F = (G_+ + G_-)/2$$

admits a Lagrangian conservative equation advected by the fluid velocity  $\vec{e}_z \times \vec{\nabla}\varphi$ , while the rescaled difference term

$$(G_+ - G_-)/(2d_e\varrho_s) \equiv U$$

obeys Eq.(15).

In 2-D configurations (see exercise above) the advection of Lagrangian invariant quantities is equivalent to the conservation of field line connections. Plasma elements connected by magnetic field lines in the  $x$ - $y$  plane lay on  $\psi = \text{const}$  curves.

If electron inertia is neglected,  $\psi$  is a Lagrangian invariant and magnetic field lines do not break.

For  $d_e \neq 0$  magnetic field lines can break and reconnect but the structure of Eqs.(20) implies that in this Hamiltonian regime the development of magnetic reconnection is constrained by the conservation of the connections given by the field lines of  $G_{\pm}$  (or of  $F$  in the degenerate  $\varrho_s = 0$  case).

### 3 Nonlinear Reconnection Regimes

Eqs.(14,15) can be integrated numerically ([2, 3, 4]) in order to investigate the long term nonlinear evolution of a fast growing (large  $d_e \Delta'$ ) reconnection instability produced by electron inertia in a sheared magnetic equilibrium configuration with a null line

$$\psi_0 = \psi_0(x), \quad \partial\psi_0(x)/\partial x = 0|_{x=0}.$$

Periodic conditions were taken along  $y$  and the configuration parameters were chosen such that only one mode can be linearly unstable, as of interest for laboratory plasmas. The results shown here were obtained with a numerical code that advances the cell averaged values of  $F$  and  $U$  in time using a finite volume technique (i.e., calculating the cell fluxes). A Fourier Transform method is then used to reconstruct the grid points values of  $F$  and  $U$  at the cell corners. Time is advanced using the explicit third order Adams-Bashforts scheme. Typical mesh sizes are  $N_x = 2048$  and  $N_y = 512$ . Random perturbations were imposed on the equilibrium configuration

$$\psi_0(x) = -L/[2 \cosh^2 (x/L)]$$

in a simulation box with

$$L_x = 2L_y = 4\pi L$$

taking

$$d_e = 3/10L$$

and

$$0 < \varrho_s/d_e < 1.5.$$

The accuracy of the integration has been verified by testing the effects of numerical dissipation on the conservation of the energy and of the Lagrangian invariants.

### **3.1 Formation of small spatial scales in the nonlinear phase**

The Lagrangian invariants  $G_{\pm}$  differ from the flux function  $\psi$  by the term  $d_e^2 J \pm d_e \varrho_s U$  which has small coefficients but involves higher spatial derivatives.

As shown by the numerical results in [2], magnetic reconnection proceeds unimpeded in the nonlinear phase because of the development near the  $X$  point of the magnetic island of increasingly small spatial scales that effectively decouple  $\psi$  from  $G_{\pm}$ .

In Hamiltonian regimes the formation of such scales does not stop at some finite resistive scalelength. This corresponds to the formation of increasingly narrow current and vorticity layers.

Because of the conserved  $G_{\pm}$  connections, the spatial localization and structure of these layers depends on the value of  $\varrho_s/d_e$ , as discussed in [2].

### 3.2 Mixing of the Lagrangian invariants and island growth saturation

In the reconnection model adopted, magnetic energy

$$\int d^2x |\vec{\nabla}\psi|^2$$

is transformed, in principle reversibly, into two forms of kinetic energy, one,

$$\int d^2x |\vec{\nabla}\varphi|^2$$

related to the plasma motion in the  $x$ - $y$  plane and one,

$$\int d^2x d_e^2 J^2,$$

to the electron current along  $z$  and, for  $\varrho_s \neq 0$ , into electron parallel compression

$$\int d^2x \varrho_s^2 U^2.$$

The last two energies involve quantities with higher derivatives.

Being the system Hamiltonian, it is not a priori clear whether a reconnection instability can induce a transition between two stationary plasma configurations with different magnetic energies, as is the case for resistive plasma regimes where the excess energy is dissipated into heat.

The first results were obtained in [3], where taking  $\varrho_s/d_e \sim 1$ , it was shown that, in spite of energy conservation, this transition is possible at a “macroscopic” level.

A new coarse-grained stationary magnetic configuration can be reached because, as the instability develops, the released magnetic energy is removed at an increasingly fast rate from the large spatial scales towards the small scales that act a perfect sink. This leads to the saturation of the island growth.

Similarly, the constraints imposed by the conservation of the  $G_{\pm}$  connections cease to matter at a macroscopic level.

The advection of the two Lagrangian invariants  $G_{\pm}$  is determined by the stream functions  $\varphi_{\pm}$ .

The winding, caused by this differential rotation type of advection [3], makes  $G_{\pm}$  increasingly filamented inside the magnetic island, leading to a mixing process similar to that exemplified by the “backer transformation” in statistical mechanics (successive stretching and folding at constant area).

As shown in [3], these filamentary structures of  $G_{\pm}$  do not influence the spatial structure of  $\psi$  which remains regular.

The analogy with the Bernstein-Greene-Kruskal (BGK) solutions of the Vlasov equation obtained in [6] for the nonlinear Landau damping of Langmuir waves was discussed in [3].

The evolution of  $G_+$  for  $\varrho_s/d_e = 1.5$  is shown in Fig.(1) at two successive times (1a,1b) together with the contours of  $J$  (1c) and  $U$  (1d). The contours of  $G_-$  are obtained from those of  $G_+$  by mirror reflection.

### 3.3 Onset of a secondary Kelvin Helmholtz instability: turbulent versus laminar mixing

The advection, and consequently the mixing, of the Lagrangian invariants can be either laminar, as shown in Fig.(1), or turbulent depending on the value of  $\varrho_s/d_e$ .

The transition between these two regimes was shown in [4] to be related to the onset of a secondary Kelvin Helmholtz (K-H) instability driven by the velocity shear of the plasma motions that form because of the development of the reconnection instability.

Whether or not the K-H instability becomes active before the island growth saturates, determines whether a (macroscopically) stationary reconnected configuration is reached and affects the redistribution of the magnetic energy.

In the cold electron limit,  $\varrho_s/d_e = 0$ , the system (14,15) becomes degenerate and the generalized connections are determined by a single Lagrangian invariant  $F$ .

Initially,  $F$  is advected along a hyperbolic pattern given by the stream function  $\varphi$  which has a stagnation point at the  $O$ -point of the magnetic island.

This motion leads to the stretching of the contour lines of  $F$  towards the stagnation point and to the formation of a bar-shaped current layer along the equilibrium null line, which differs from the cross shaped structure found in the initial phase of the reconnection instability for  $\varrho_s/d_e \neq 0$ .

Subsequently,  $F$  contours are advected outwards in the  $x$ -direction. At this stage  $F$  starts to be affected by a K-H instability that causes a full redistribution of  $F$ , as shown at  $t = 103$ .

In this phase the spatial structure of  $F$  is dominated by the twisted filaments of the current density which spread through the central part of the magnetic island.

This is shown in Fig.(2) [ in (2a,2d,2c)  $F$  is shown at increasing times. in (2d)  $J$  and (2e)  $U$  is shown].

The contours of the vorticity  $U$  exhibit a well developed turbulent distribution of monopolar and dipolar vortices, while those of  $\psi$  remain regular although they pulsate in time.

The energy balance shows that part of the released magnetic energy remains in the form of plasma kinetic energy corresponding to the fluid vortices in the magnetic island and that an oscillatory exchange of energy persists (see [7]) between the plasma kinetic energy and the elec-



tron kinetic energy corresponding to the pulsations of the island shape.

This turbulent evolution of the nonlinear reconnection process also occurs in the non degenerate, finite electron temperature, case but as the ratio  $\varrho_s/d_e$  is increased, i.e. as the electron temperature effects become more important, the onset of the K-H instability occurs later during the island growth and its effect on the current layer distribution becomes weaker.

For  $\varrho_s/d_e \sim 1$ , no sign of a secondary instability is detectable during the time the island takes to saturate its growth.

## 4 Conclusions

Mixing plays an important role in collisionless magnetic field line reconnection by easing the constraints imposed by the conservation of energy and of the generalized connections which determine the spatial structure of the current and vorticity layers.

A transition occurs between laminar and turbulent mixing: the control parameter is related to the electron temperature.

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