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SPRING SCHOOL ON SUPERSTRING THEORY AND RELATED TOPICS

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Flux compactifications and brane cosmology

Part III

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For many reasons, we think the early Universe underwent a period of accelerating (exponential) expansion of the scale factor:

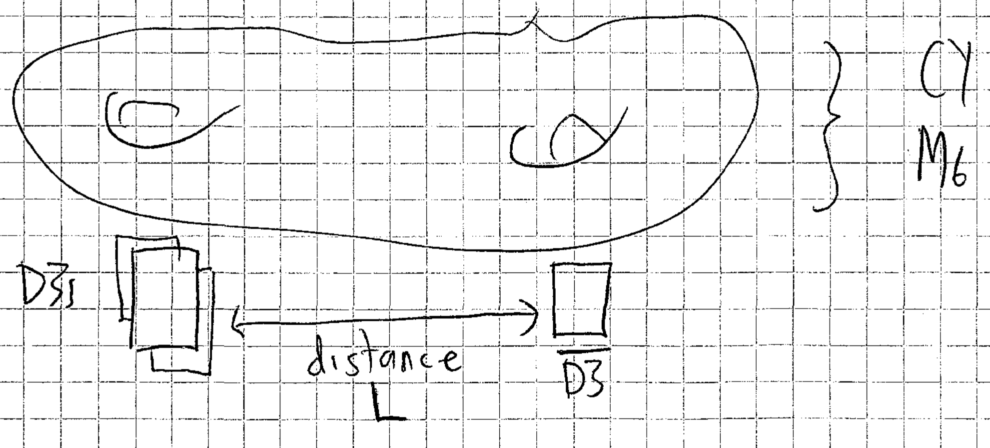
$$a(t) \sim e^{Ht} \qquad H \gg H_{today}$$

$\sim 10^{14}$ GeV in many models

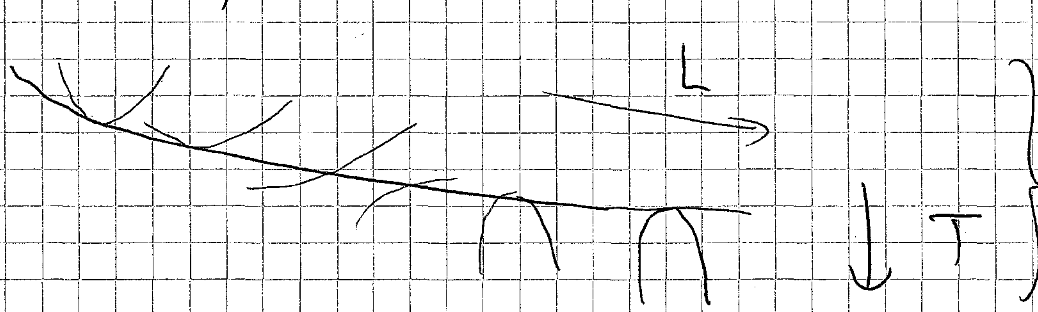
String theory hasn't yet suggested a compelling new way to solve the cosmological problems $\Rightarrow$ natural to ask: can we embed inflation in string theory? At the same time, inflationary models typically rely on finely tuned micro-physics $\Rightarrow$ a stringy embedding could justify them, in principle.

A simple suggestion: "Brane inflation"

E.g.:



- Branes + anti-branes $\perp$ to large CY_3
- Interbrane distance "L" is inflaton
- As $L \to l_s$, tachyon develops $\Rightarrow$ exit inflation by $D3/\overline{D3}$ annihilation:



very similar to Hybrid inflation

This is a nice suggestion because the inflaton is nicely geometrized, and it's dynamics is seemingly isolated from details of M_6 (hence, universal).

PROBLEM:

$$V(L) = 2T_{D3}\left(1 - \frac{1}{2\pi^3}\frac{T_{D3}}{M_{10}^8 L^4}\right)$$

D-$\overline{D}$ tension

Now, "L" isn't a canonically normalized scalar in DBI action, $\phi \equiv \sqrt{T_{D3}}\,L$ is.

$$V(\phi) = 2T_{D3}\left[1 - \frac{1}{2\pi^3}\frac{T_{D3}^3}{M_{10}^8\phi^4}\right]$$

The slow roll parameters include

$$\eta \equiv M_{Pl}^2 \frac{V''}{V}$$

One needs $\eta \ll 1$ to get slow roll inflation.

Now $$M_{PL}^2 = M_{10}^8 \, R^6$$ ← R = radius of M_6

$$\Rightarrow \qquad \eta = O(1) \times \left(\frac{R}{L}\right)^6$$

But $L \lesssim R$ in a compact manifold $\Rightarrow \eta \ll 1$ not attainable!

- More detailed considerations show anisotropic M_6, or special initial conditions, don't help enough...

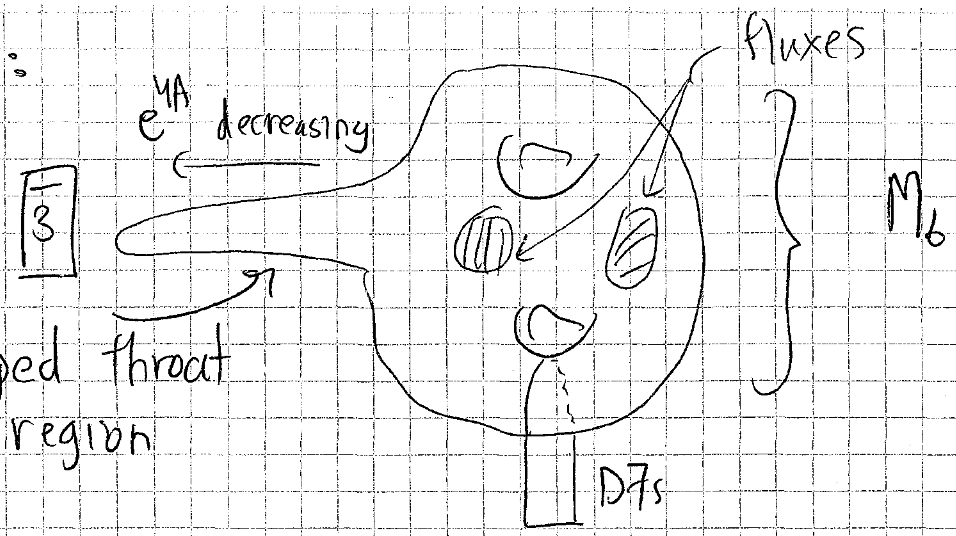
$\boxed{\text{Embedding in IIB compactifications}}$

It is natural to consider the embedding of brane inflation $\in$ IIB flux vacua (D3s, $\overline{\text{D3}}$s, D7s all natural ingredients).

Oth order concern: Must have stabilized moduli in closed string sector. Otherwise $V \to 0$ rapidly as $R \to \infty$ or $g_s \to 0 \Rightarrow$ fast roll to extreme limit of moduli, instead of inflation.

But, we've spent 1st two lectures finding a plausible picture of some IIB vacua w/ moduli stabilized!

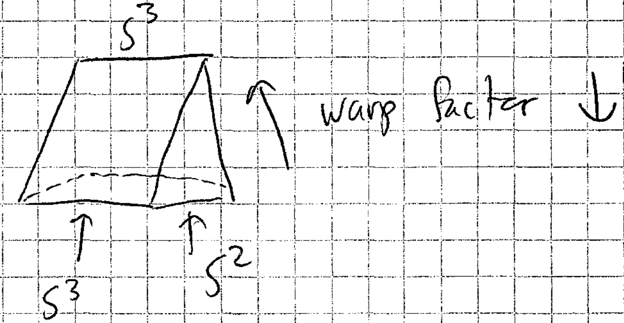
Picture :



A notable feature of some of these models is the warping -- e^{4A} $\mathcal{O}(1)$ in bulk of M_6 but can get very (exponentially) small.

E.g. Consider Klebanov-Strassler sol'n

Non-compact :
"Cone over
$S^3 \times S^2$"



- The warp factor decreases exponentially as one goes towards tip of cone
- The tip is fattened into a "deformed" conifold

$$\sum_{i=1}^{4} z_i^2 = \varepsilon$$

- $\exists$ RR & NS fluxes that "explain" deformation, in a way we'll review.

Can easily embed this into a compact CY.

Conifold $\sum_{i=1}^{4} z_i^2 = 0$ is most generic singularity of M_6.

$\exists$ two natural 3-cycles:

$A = S^3$ @ tip

$B = S^2$ of one $\times$ "radial" (compactified)

Fluxes of K-S sol'n $\Rightarrow$

$$\int_A F_{RR} = M \;,\quad \int_B H_{NS} = -k$$

Result: dual to $SU(N+M) \times SU(N)$ $[N = kM]$

$\mathcal{N}=1$ QFT, coupled to 4d gravity!

- Why is conifold <u>deformed</u>?

$$\int_A \Omega = z \;,\quad \int_B \Omega = \frac{z}{2\pi i} \ln z + \cdots$$

So $$W = \int_{M_6} (F - \tau H) \wedge \Omega$$

$$= -k\tau z + \frac{M}{2\pi i} z \ln z + \cdots$$

<u>Result</u>: z stabilized at $\boxed{z \approx e^{-2\pi k / g_s M}}$

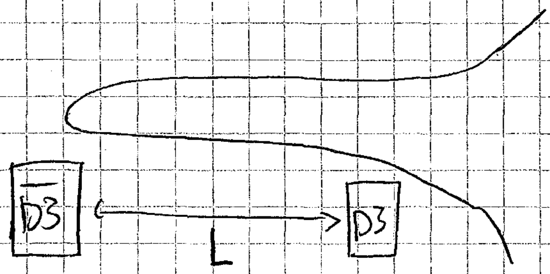
But "z" is size of A-cycle S^3 $\Rightarrow$ a slightly deformed conifold.

The resulting warp factor satisfies

$$e^{A}\Big|_{tip} \approx z^{1/3} \Rightarrow \boxed{e^{A}\Big|_{tip} \approx e^{-2\pi K/3g_s M}}$$

(find by matching to KS solution).

<u>Question</u>: Might the <u>warping</u> help w/inflation?



Scenario: $\overline{D3} \in$ such a geometry feels a <u>force</u>:

model throat as AdS, cutoff in IR

$$ds^2 = h^{-1/2}\left(-dt^2 + d\vec{x}^{\,2}\right) + h^{1/2}\left(dr^2 + \frac{r^2}{R^2}\,\tilde{g}_{ab}\,dy^a\,dy^b\right)$$

$$(F_5)_{rtx^1x^2x^3} = \partial_r h^{-1}$$

$$h(r) = \frac{R^4}{r^4} \quad \left(\equiv e^{-4A}\right)$$

Motion of $D3/\overline{D3}$ derived from DBI:

$$S = -T_3 \int \sqrt{-g}\, d^4x \left(\frac{r^4}{R^4}\right)\sqrt{1 - \frac{R^4}{r^4}\, g^{\mu\nu}\partial_\mu r\, \partial_\nu r}$$

$$\pm\, T_3 \int (C_4)_{tx^1x^2x^3}\; dt\, dx^1\, dx^2\, dx^3$$

- $V(r) = 0$ for D3 ✓ (cancellation above for "+")
- $V(r)$ for $\overline{D3}$ draws <u>quickly</u> to IR (tip)

So the scenario should involve force between $\overline{D3}$ @ tip $r = \underline{\underline{r_0}}$ + D3 at same radial location $\underline{\underline{r_1}}$. Assume $r_0 \ll r_1$.

- Adding D3 @ $r_1 \to$ perturbation to $h(r)$:

$$h(r) \to h(r) + \delta h(r)$$

$$\delta h(r) = \frac{R^4}{N} \frac{1}{r_1^4}$$

So the $\underline{\text{full}}$ potential is now:

$$\boxed{V = 2 T_3 \frac{r_0^4}{R^4} \left\{ 1 - \frac{1}{N} \frac{r_0^4}{r_1^4} \right\}}$$

Notice interaction is $\underline{\underline{\text{very}}}$ weak $\propto r_0^8$ (+ r_0^4 down from tension term) $\Rightarrow$ a beautiful model with low scale, + exponentially small ϵ, η.

$$\left.\begin{aligned} \epsilon &= \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \simeq \frac{8}{N^2} M_P^2 \frac{\phi_0^8}{\phi^{10}} \ll 1 \\ \eta &= M_P^2 \frac{V''}{V} \simeq -\frac{20}{N} M_P^2 \frac{\phi_0^4}{\phi^6} \ll 1 \end{aligned}\right\} \text{easy to get} \ll 1$$

So warping in stabilized flux compactifications allow one to solve "runaway modulus" problem + $\underline{\text{also}}$ get weak enough interbrane forces.

PROBLEM: However, all is not well. We haven't been careful enough in our analysis. Think in 4d EQFT.

The $\mathcal{N}=1$ sugra has

Kähler pot for CY metric

$$\mathcal{K}(\rho,\bar{\rho},\phi,\bar{\phi}) = -3\log\left(\rho+\bar{\rho}-k(\phi,\bar{\phi})\right)$$

describes brane transl. modes

Now the volume in KK ansatz is actually the entire argument of log

$$(\mathrm{Vol})^{2/3} \equiv V = \frac{1}{2}\left(\rho+\bar{\rho}-k(\phi,\bar{\phi})\right)$$

But, in 4d Einstein frame, all of the energies we've been describing scale like inverse powers of V. E.g.

$$T_{\overline{D3}} \propto \frac{1}{V^3}$$

And as branes move $k(\phi,\bar{\phi})$ changes $\Rightarrow$ V changes! E.g. if $k\sim\phi\bar{\phi}$, this $\Rightarrow$ contribution to V during inflation which is $\sim H^2\phi\bar{\phi} \Rightarrow \eta\ \mathcal{O}(1)$.

There are a few attitudes one can take to try to find solutions to this generic issue:

1) Hope to get lucky $\frac{1}{100}$ th of the time.

2) Have inflation occur along $K(\phi, \bar{\phi})$ = <u>constant</u> trajectory -- eg if inflaton is a pseudo Goldstone boson.

3) Consider relativistic branes $\Rightarrow$ $\frac{\dot{\phi}}{\phi^2}$ terms in DBI become important, change the story in an interesting way. } Silverstein + Tong

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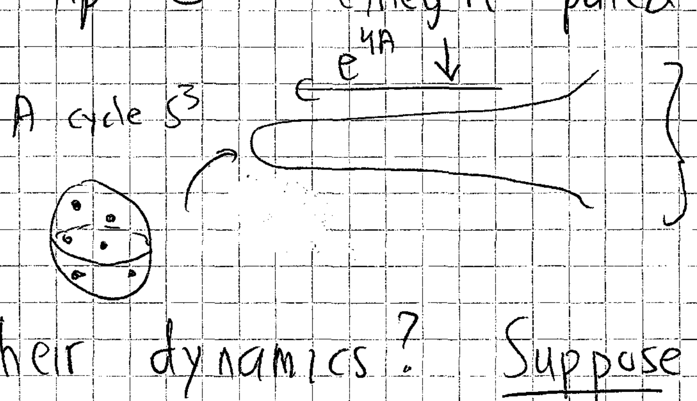
<u>A model with $\overline{D3}$ s only (class 2)</u>

Now, consider just p $\overline{D3}$ branes scattered across the tip S^3 (they're pulled there quickly, recall).



} S^3 has $SO(4)$ symm $\Rightarrow$ maybe flat like PGBs

What is their dynamics? <u>Suppose</u> they were clumped $\rightarrow$ $SU(p)$ gauge theory.

$$dB_6 = \frac{1}{g^2} *_{10} H_3 = -\frac{1}{g_s} dV_4 \wedge F_3$$

Presence of flux deforms DBI potential as in Myers effect :

$$g_s V_{eff}(\Phi) = \sqrt{\det(G_{||})}\left(p - i\frac{4\pi^2}{3} Tr([\Phi^k, \Phi^j]\Phi^l F_{kjl} - \frac{\pi^2}{g_s^2} Tr([\Phi^i, \Phi^j]^2) + \cdots \right)$$

Roughly $F_{kjl} = f\,\epsilon_{kjl}$, $f = \dfrac{2}{b_0^3 \sqrt{g_s^3 M}}$ $\quad b_0 \; O(1)$

Now, $\dfrac{\partial V}{\partial \Phi^i} = 0 \Rightarrow$

$$[[\Phi^i, \Phi^j]\, \Phi^j] - i g_s^2 f \epsilon_{ijk} [\Phi^j, \Phi^k] = 0$$

So constant Φ matrices with

$$[\Phi^i, \Phi^j] = -i g_s^2 f \epsilon_{ijk} \Phi^k$$

solve EOM! So Φ can be any $p \times p$ dim'l matrix rep of $SU(2)$.

- The energetically preferred one is p dim'l irrep
- In it,

$$V_{eff} \sim p\, T_{D3}\left[1 - \frac{8\pi^2}{3 b_0^{12}} \frac{(p^2-1)}{M^2} \right]$$

and the $\overline{D3}$s are a fuzzy $S^2 \subset S^3$ of

$$\boxed{R^2 \simeq \frac{4\pi^2 (p^2-1)}{b_0^8 M^2} R_0^2} \leftarrow \text{radius of A-cycle } S^3$$

Important notes:

1) Can perturbatively roll to p dim'l irrep (the $SU(2)$ matrices aren't most general solns)

2) For p close enough to M, $R \sim R_0$ -- what happens?

Dual analysis using wrapped 5-brane on $S^2 \subset S^3 \Rightarrow$

For $p \gtrsim \frac{M}{12}$, the p $\overline{D3}$s first blow up into an NS 5 on $S^2 \subset S^3$, it then "rolls over" the S^3 & shrinks on other side as $\underline{\underline{(M-p)}}$ D3s!

- $\Delta \left(\int_B H_{NS} \right) = 1 \Rightarrow$ tadpole $\int H \wedge F + N_{D3} - \overline{N}_{\overline{D3}}$ remains constant ...
- From this basic set of facts, can try to make models w/ only $\overline{D3}$ motion on tip (~ Goldstones, naively).

3 promising phases for inflation:
- The p $\overline{D3}$s attracting into clump
- The start of "tachyon condensation"
- For $p \approx \frac{M}{12}$, an intermediate region where NS 5 is near equator of S^3 + can just barely get over it to other side...

Analyzed in hep-th/0403123, doesn't quite work in regime of control.

Other models where a symmetry $\approx SO(4)$ of S^3 is used to try + get slow-roll are being investigated by

Kallosh et al
Tye et al
Iizuka + Trivedi } Just a $\mathbb{Z}_2$; D3 between 2 throats $\Rightarrow$ plateau as it decides which to enter, + slow roll.

References

- Hep-th /0308055 (KKLMMT)

  + previous refs

- For dynamics of $\overline{D3}$s : (in KS throat)

  Hep-th / 0112197 , Hep-th/0403123