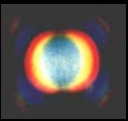


Insertion Device Radiation

Helmut Wiedemann

ICTP, April 2004



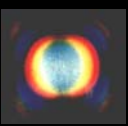
Insertion Device Radiation

Insertion devices do not change the shape of the storage ring!

$$\begin{matrix} \uparrow & \oplus \\ \otimes & \downarrow \end{matrix} B_y \begin{matrix} \uparrow \\ \downarrow \end{matrix} \begin{matrix} \oplus \\ \otimes \end{matrix} 0, z \begin{matrix} \oplus \\ \otimes \end{matrix} dz \begin{matrix} \oplus \\ \otimes \end{matrix} 0$$

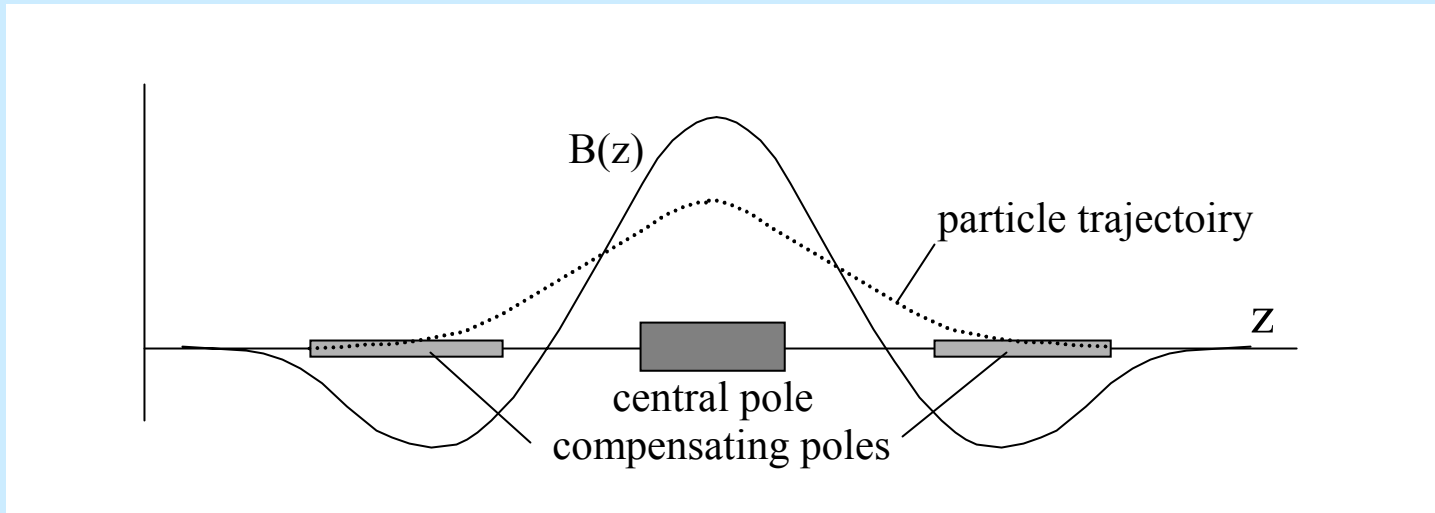
- Wavelength shifter
- Wiggler magnet
- Undulators
- Super bends

Purpose: **harden radiation**
increase intensity
high brightness monochromatic radiation
elliptically polarized radiation

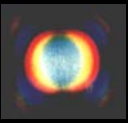


Insertion Device Radiation

wave length shifter



$$B_y = 0, z \ll d, z \gg d$$



Insertion Device Radiation

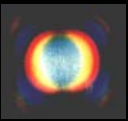
Super bends:

replace conventional bending magnets with super conducting bending magnets causing the same deflection angle.

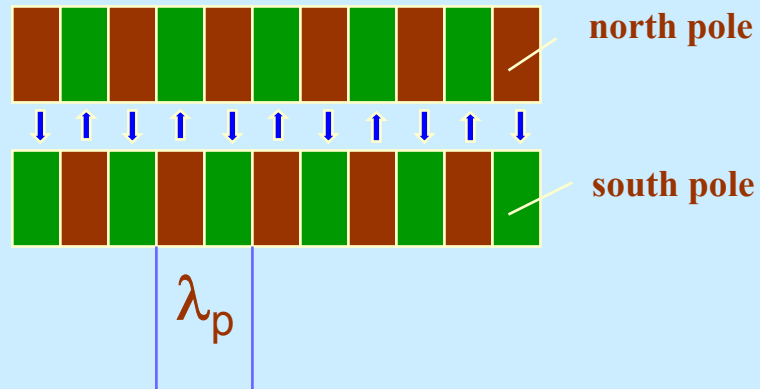
conventional bending magnet



superbend



Periodically deflecting magnets:

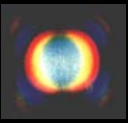


Wiggler magnets, strong field

Undulators, weak field

**Wiggler magnets produce ordinary, broad band
synchrotron radiation;**

Intensity increased by factor N_p (# of poles)



Insertion Device Radiation

deflection angle per half-pole

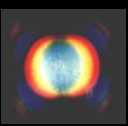
$$B_y(z) = B_0 \cos k_p z$$

$$d\theta = \frac{dz}{\rho} = \frac{eB}{cp} dz \quad \Leftrightarrow \quad \theta = \int \frac{dz}{\rho} = \frac{eB}{cp} \int_0^{\lambda_p/4} \cos k_p z dz = \frac{eB_0 \lambda_p}{2\pi c p}$$

$$\theta = \frac{eB_0 \lambda_p}{2\pi c p} = \frac{K}{\gamma}$$

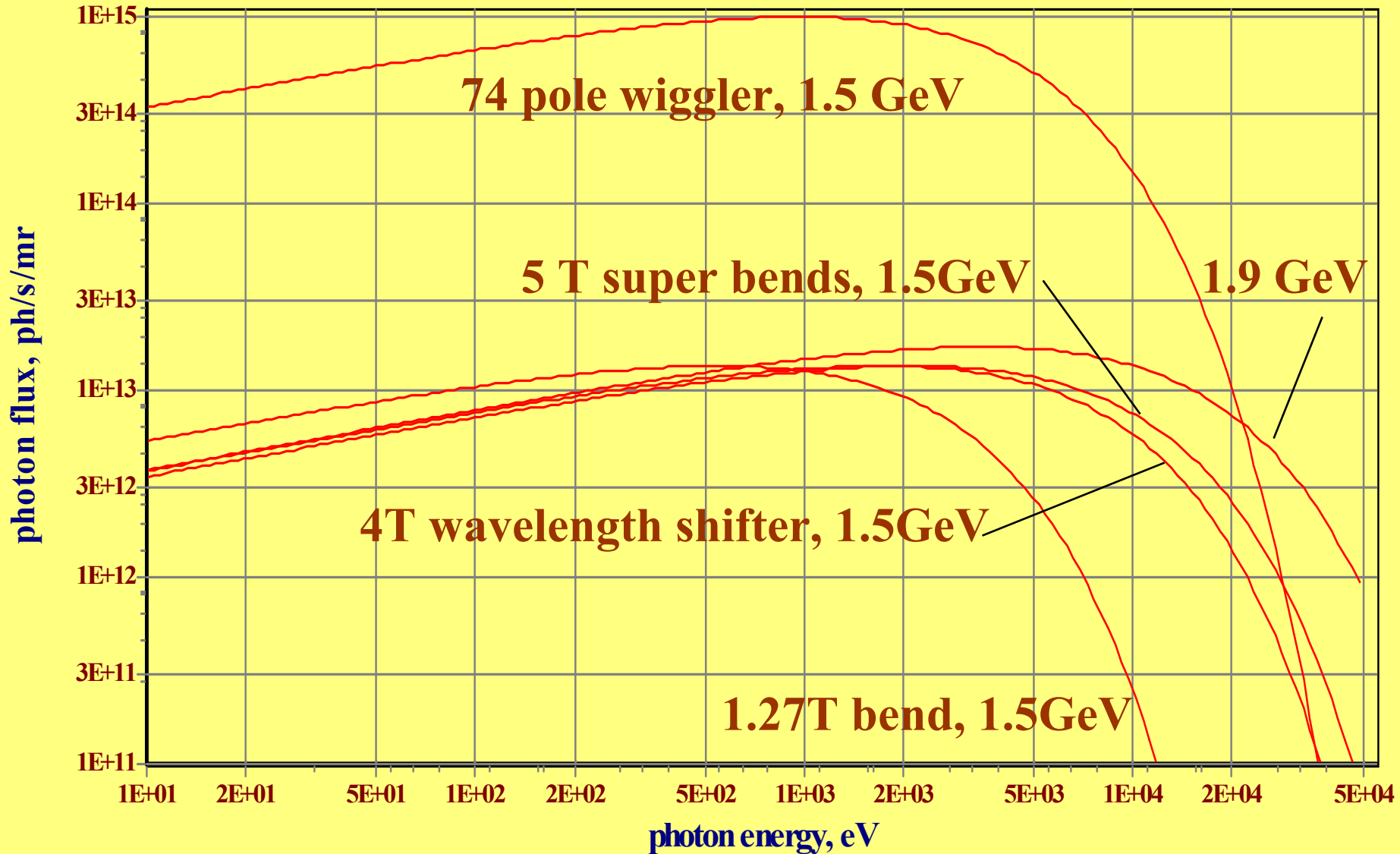
**K: undulator/wiggler
strength parameter**

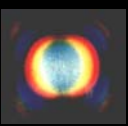
$$K = \frac{eB_0 \lambda_p}{2\pi m c^2 \beta} = 0.934 B_0(\text{T}) \lambda_p(\text{cm})$$



Insertion Device Radiation

Photon fluxes (ph/s/mr) for ALS (400 mA)

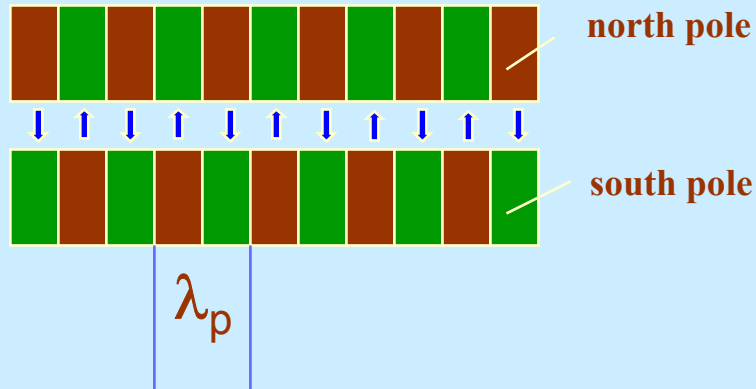




Insertion Device Radiation

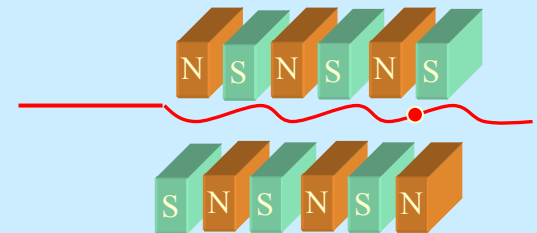
assume a magnetic field

$$B_y(z) = B_0 \cos k_p z$$

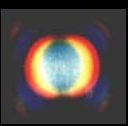


$$k_p = \frac{2\pi}{\lambda}$$

electron performs sinusoidal oscillations



sinusoidal perturbation of field lines



Insertion Device Radiation

sinusoidal perturbation of field lines

N_p undulator periods \iff N_p field oscillations

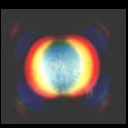
$$E(t) = \begin{cases} E_0 \sin \omega_0 t & \text{for } -\frac{1}{2}N_p T_0 < \omega_0 t < \frac{1}{2}N_p T_0 \\ 0 & \text{elsewhere} \end{cases}$$

spectrum

$$E(\omega) = \int E(t) e^{-i\omega t} dt = \int_{-\infty}^{\infty} E_0 \sin \omega_0 t e^{-i\omega t} dt = E_0 \int_{-\infty}^{\infty} e^{-i(\omega_0 - \omega)t} dt$$

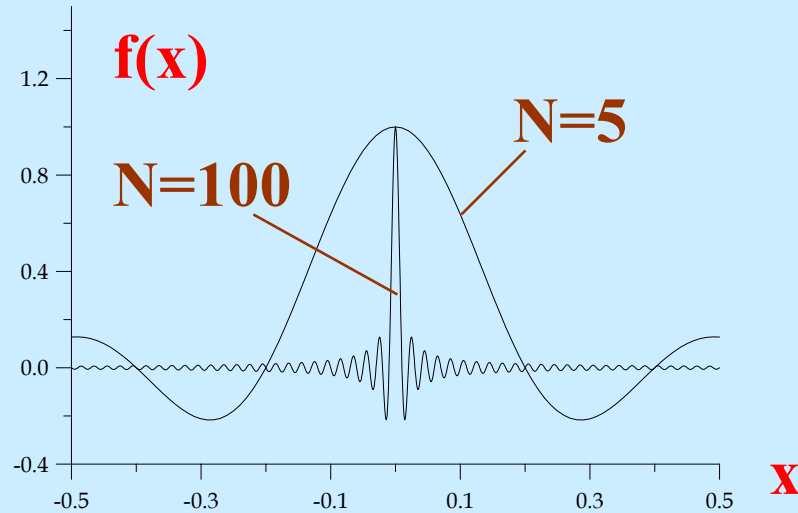
$$E_0 \int_{-\frac{1}{2}N_p T_0}^{\frac{1}{2}N_p T_0} e^{-i(\omega_0 - \omega)t} dt = E_0 \frac{e^{i(\omega_0 - \omega)\frac{1}{2}N_p T_0} - e^{-i(\omega_0 - \omega)\frac{1}{2}N_p T_0}}{i(\omega_0 - \omega)} = E_0 N_p T_0 \frac{\sin(\omega_0 - \omega)\frac{1}{2}N_p T_0}{(\omega_0 - \omega)\frac{1}{2}N_p T_0}$$

$$E(\omega) = E_0 N_p T_0 \frac{\sin(\omega_0 - \omega)\frac{1}{2}N_p T_0}{(\omega_0 - \omega)\frac{1}{2}N_p T_0}$$



Insertion Device Radiation

Sinc-function: $f(x) = \frac{\sin \pi N x}{\pi N x}$



$f(0)=1$ and

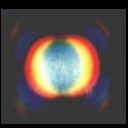
$f(y)=0$ for $y=1/N$

or for

$$(\omega_0 - \omega)^{1/2} N_p T_0 = \pi$$

line width: $\frac{\delta\omega}{\omega_0} = \pm \frac{\omega_0 - \omega}{\omega_0} = \frac{2\pi}{T_0} \frac{1}{N_p} \frac{1}{\omega_0} = \frac{1}{N_p}$

$$\frac{\delta\omega}{\omega_0} = \pm \frac{1}{N_p}$$



Insertion Device Radiation

What is ω_0 ?

undulator period: λ_p

in electron rest system: $\lambda_p^* = \frac{\lambda_p}{\gamma}$

in lab system (Doppler effect): $\omega = \omega_p^* \gamma (1 + \mathbf{n}_z^* \beta)$

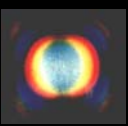
or $\lambda = \frac{\lambda_p}{\gamma^2 (1 + \mathbf{n}_z^* \beta)}$

with $\mathbf{n}_z = \frac{\beta + \mathbf{n}_z^*}{1 + \mathbf{n}_z^* \beta} \iff \lambda = \frac{\lambda_p}{\gamma^2} \frac{\mathbf{n}_z}{\beta + \mathbf{n}_z^*} = \frac{\lambda_p}{\gamma^2} \frac{\cos \theta}{1 + \cos \theta^*}$

$\sin \theta = \frac{\sin \theta^*}{\gamma (1 + \beta \cos \theta^*)} \iff \theta \approx \frac{\sin \theta^*}{\gamma (1 + \beta \cos \theta^*)}$

or

$\gamma^2 \theta^2 = \frac{\sin^2 \theta^*}{(1 + \beta \cos \theta^*)^2} = \frac{1 - \cos \theta^*}{1 + \cos \theta^*} \iff \cos \theta^* = \frac{1 - \gamma^2 \theta^2}{1 + \gamma^2 \theta^2}$



Insertion Device Radiation

$$\lambda = \frac{\lambda_p}{\gamma^2} \frac{\cos \theta}{1 + \cos \theta^*} = \frac{\lambda_p}{\gamma^2} \frac{1}{1 + \frac{1 - \gamma^2 \theta^2}{1 + \gamma^2 \theta^2}} = \frac{\lambda_p}{2\gamma^2} (1 + \gamma^2 \theta^2)$$

$$\theta^2 = (\theta_{\text{und}} + \theta_{\text{obs}})^2 = \theta_{\text{und}}^2 + 2\theta_{\text{und}}\theta_{\text{obs}} + \theta_{\text{obs}}^2$$

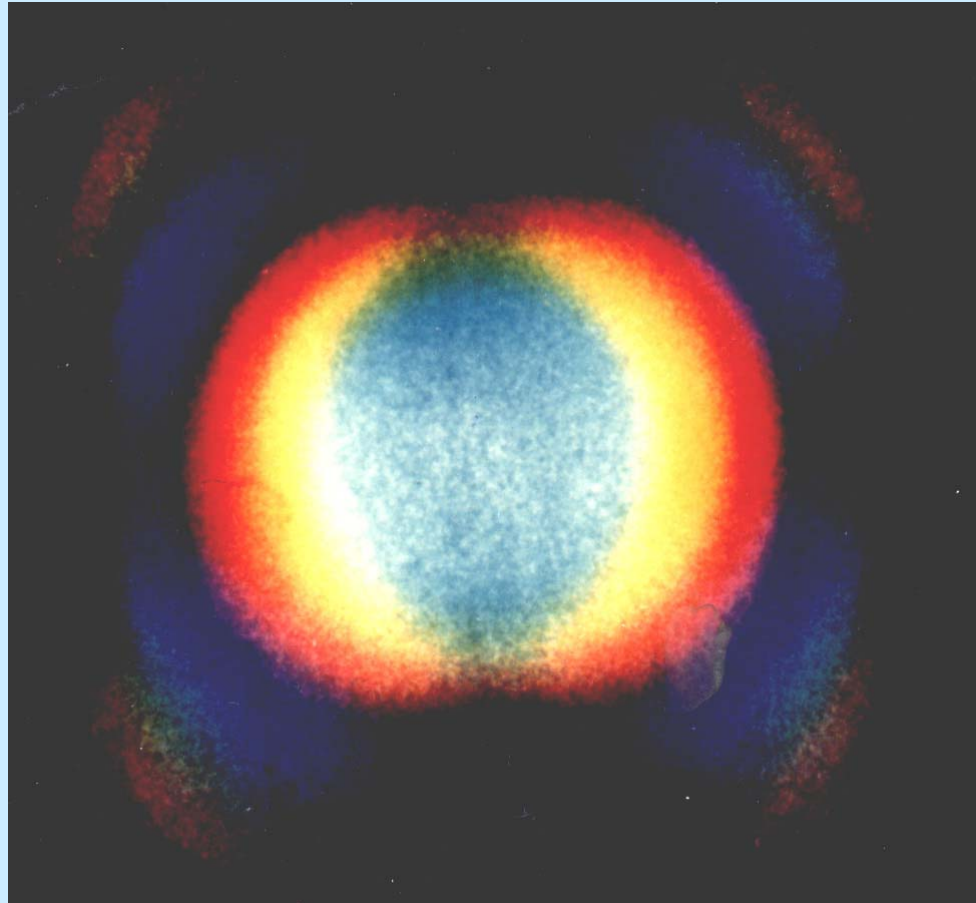
$$\theta_{\text{und}} = \frac{K}{\gamma} \cos k_p z \quad \Leftrightarrow \quad \langle \theta_{\text{und}} \rangle = 0 \quad \text{and} \quad \langle \theta_{\text{und}}^2 \rangle = \frac{1}{2} \frac{K^2}{\gamma^2}$$

fundamental undulator wavelength:

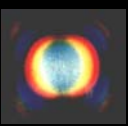
$$\lambda = \frac{\lambda_p}{2\gamma^2} \left(1 + \frac{1}{2} K^2 + \gamma^2 \theta_{\text{obs}}^2 \right)$$



Insertion Device Radiation



$$\lambda = \frac{\lambda_p}{2\gamma^2} \left(1 + \frac{1}{2}K^2 + \gamma^2 \theta_{\text{obs}}^2 \right)$$



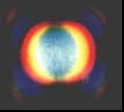
Insertion Device Radiation

$$t_i \quad \frac{t_p}{2i} \leftarrow \frac{1}{2} K^2 \quad \rightarrow$$

$$t_i \quad 1305.6 \frac{t_p}{iE^2} \quad \frac{1}{2} K^2$$

$$P_i \quad 9.4963 \quad \frac{iE^2}{t_p \frac{1}{2} K^2}$$

$$\frac{1}{2i} \sqrt{\frac{1}{2} K^2}{2iN_p}$$



Insertion Device Radiation

$$B_y(z) = B_0 \cos k_p z \quad \text{this is what we want}$$

Maxwell tells us what we can get!

$$B_y(y, z) = B_0 b(y) \cos k_p z$$

$$\nabla \times \mathbf{B} = \mathbf{0} \quad \Leftrightarrow \quad \frac{\partial B_z}{\partial y} = \frac{\partial B_y}{\partial z} = -B_0 b(y) k_p \sin k_p z$$

$$\text{and} \quad B_y = -B_0 b(y) (1 - \cos k_p z)$$

$$\nabla \cdot \mathbf{B} = 0 \quad \Leftrightarrow \quad \frac{\partial B_z}{\partial z} = -B_0 \frac{\partial b(y)}{\partial y} \cos k_p z$$

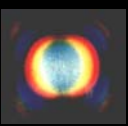
$$\mathbf{B} \neq \mathbf{B}(\mathbf{x})$$

$$\text{form} \quad \frac{\partial^2 B_z}{\partial y \partial z} \quad \longrightarrow \quad \frac{\partial^2 b(y)}{\partial^2 y} = k_p^2 b(y) \quad \Leftrightarrow \quad b(y) = a_1 \cosh k_p y + a_2 \sinh k_p y$$

$$B_x = 0$$

$$B_y = B_0 \cosh k_p y \cos k_p z$$

$$B_z = -B_0 \sinh k_p y \sin k_p z$$



Insertion Device Radiation

beam dynamics

$$\frac{d^2 \mathbf{r}}{ds^2} = \frac{\mathbf{n}}{\rho} = \frac{e}{mc^2 \gamma} \left[\frac{\mathbf{v}}{v} \times \mathbf{B} \right]$$



$$\begin{aligned} \frac{d^2 x}{dt^2} &= -\frac{eB_0}{m c \gamma} \frac{dz}{dt} \cos k_p z \\ \frac{d^2 z}{dt^2} &= +\frac{eB_0}{m c \gamma} \frac{dx}{dt} \cos k_p z \end{aligned}$$

$$\frac{dx}{dt} = -c\beta \frac{K}{\gamma} \sin k_p z$$

$$\frac{dz}{dt} = +c\beta \left(1 - \frac{K^2}{2\gamma^2} \sin^2 k_p z \right)$$



drift velocity

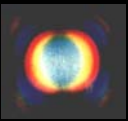
$$\beta_{\text{drift}} = \beta \left(1 - \frac{K^2}{4\gamma^2} \right)$$

$$x(t) = a \cos(k_p c \beta_{\text{drift}} t)$$

$$z(t) = c \beta_{\text{drift}} t + \frac{1}{8} k_p a^2 \sin(2k_p c \beta_{\text{drift}} t)$$

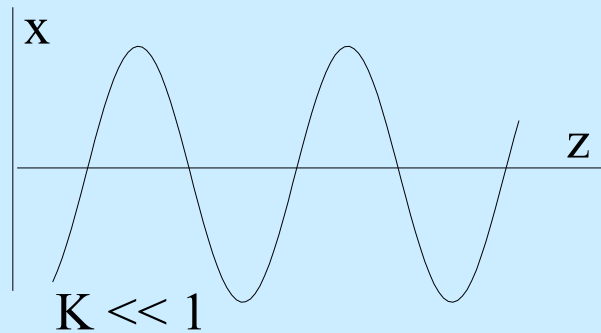


$$a \frac{K}{k_p}$$

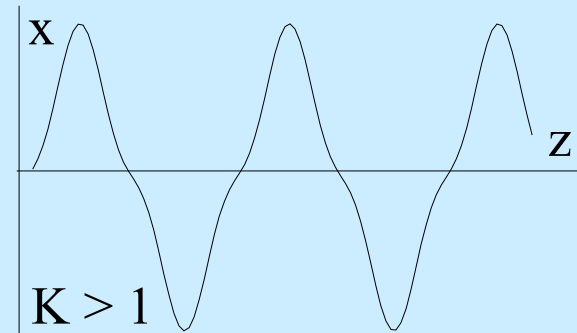


Insertion Device Radiation

Stronger undulator field

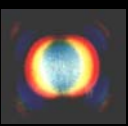


**transverse motion
completely
non-relativistic**



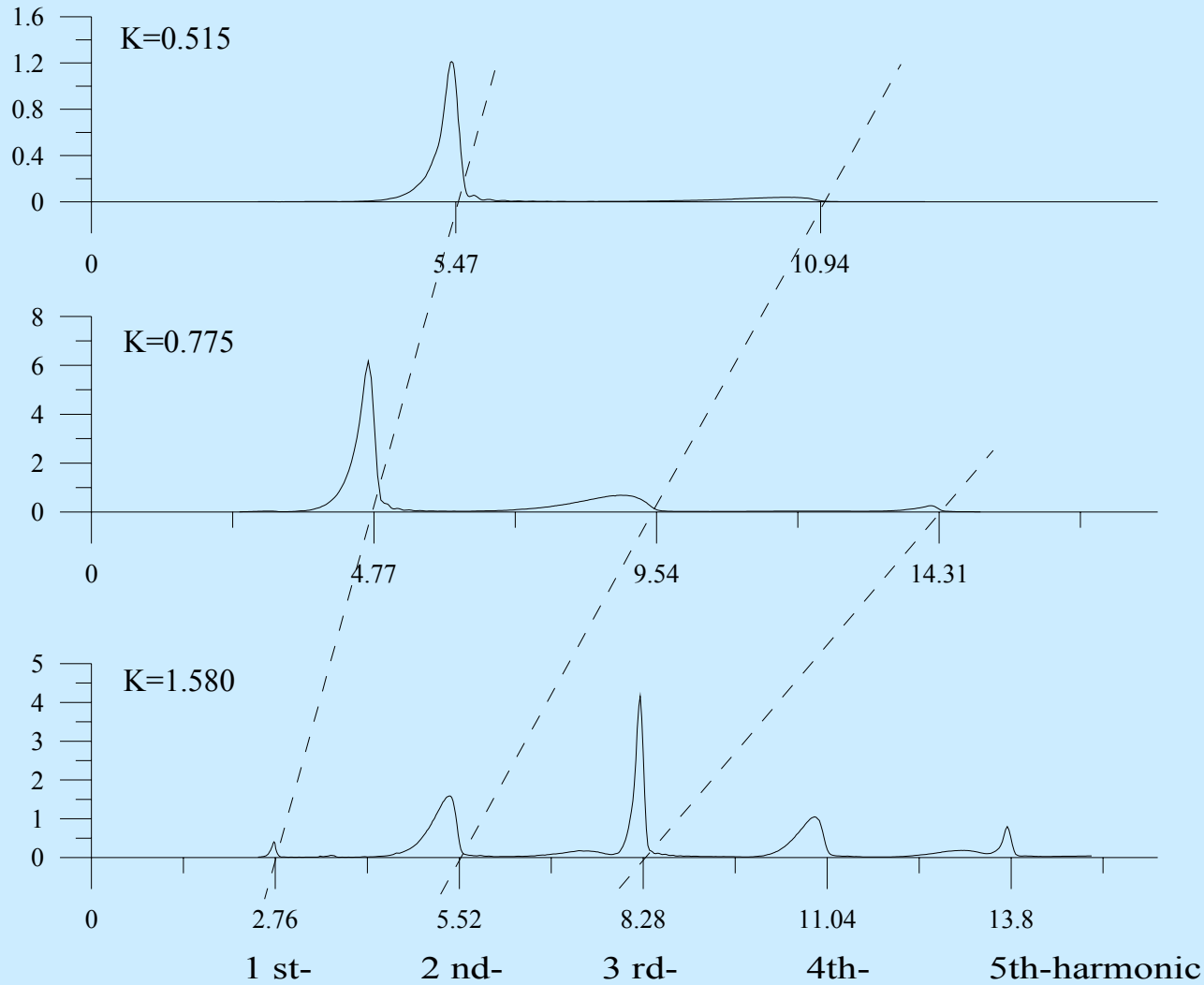
**relativistic effect on
transverse motion**

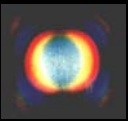
**source of higher harmonics
(only odd harmonics !)**



Insertion Device Radiation

PEP-Undulator: 77mm, 27 periods, 7.1 GeV



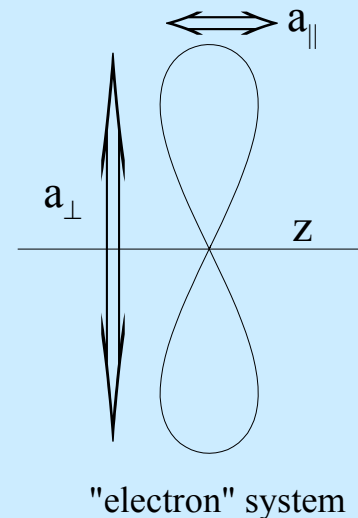
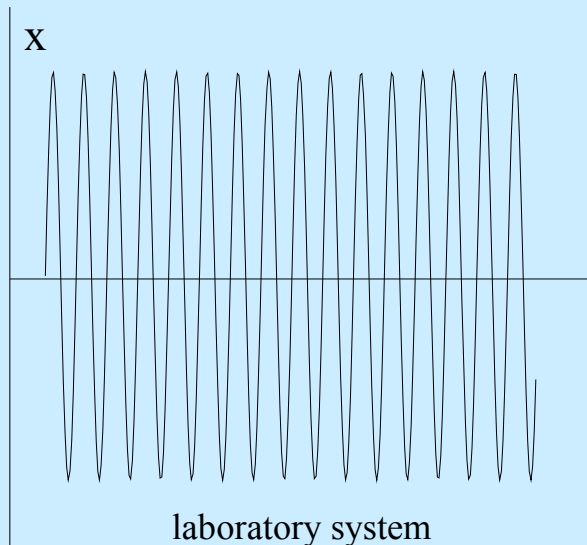


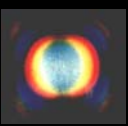
Insertion Device Radiation

now we increase strength parameter **K**

$$x(t) = a \cos(k_p c \beta t)$$

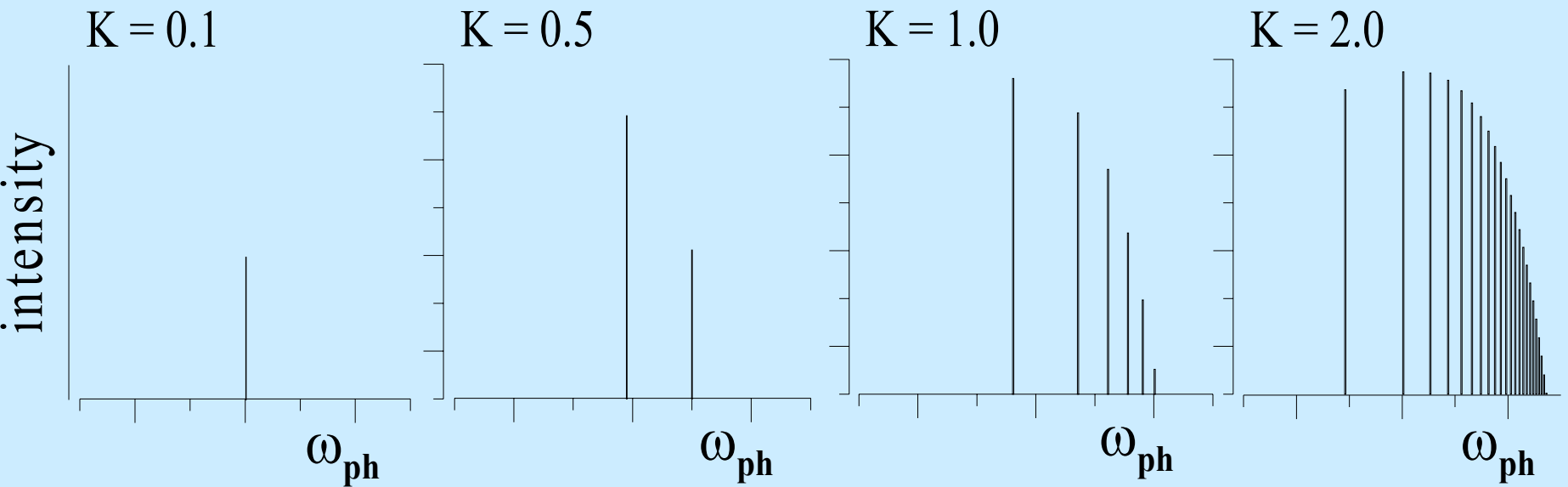
$$z(t) = c \beta t + \frac{1}{8} k_p a^2 \sin(2k_p c \beta t)$$

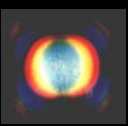




Insertion Device Radiation

transition from undulator to wiggler radiation





Insertion Device Radiation

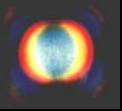
energy loss per undulator pass

$$E_{\text{rad}} = \frac{1}{3} r_c mc^2 K^2 k_p^2 L_u$$

$$E_{\text{rad}} = 0.07257 \frac{E^2 K^2}{k_p^2} L_u$$

tot. radiation power

$$P = 0.07257 \frac{E^2 K^2 NI}{k_p^2}$$



Insertion Device Radiation

$$\frac{dN_{ph}}{d\Omega} = N_p^2 \frac{\eta_0}{\eta_q} \frac{I}{e} \frac{1}{i^2} \text{Sinc}^2 \left(\frac{\sin \theta}{2} K \right)^2$$

$$\text{Sinc}^2 \left(\frac{\sin \theta}{2} K \right)^2$$

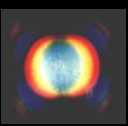
$$F_{\theta} = \frac{2 \cos \theta}{1 - \frac{1}{2} K^2} \quad F_{\varphi} = \frac{2 \sin \theta}{1 - \frac{1}{2} K^2}$$

$$J_m = J_m(i, 2m)$$

$$J_m = J_m(i, 2m)$$

$$u = \frac{\eta_0}{\eta_q} \frac{K^2}{4 \left(1 - \frac{1}{2} K^2\right)}$$

$$v = \frac{\eta_0}{\eta_q} \frac{2 K^2 \cos \theta}{1 - \frac{1}{2} K^2}$$



Insertion Device Radiation

pin hole radiation

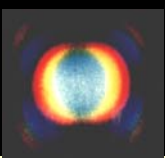
$$\frac{dN_{ph}}{d\Omega} \Big|_i = N_p^2 \frac{\gamma_0}{\gamma} \frac{I}{e} \frac{i^2 K^2 [JJ]}{\left(1 + \frac{1}{2} K^2\right)^2}$$

$$= 1.7466 \times 10^{23} E^2 [\text{GeV}^2] \omega_p^2 \frac{\gamma_0}{\gamma} f_i(K)$$

$$f_i(K) = \frac{i^2 K^2 [JJ]^2}{\left(1 + \frac{1}{2} K^2\right)^2}$$

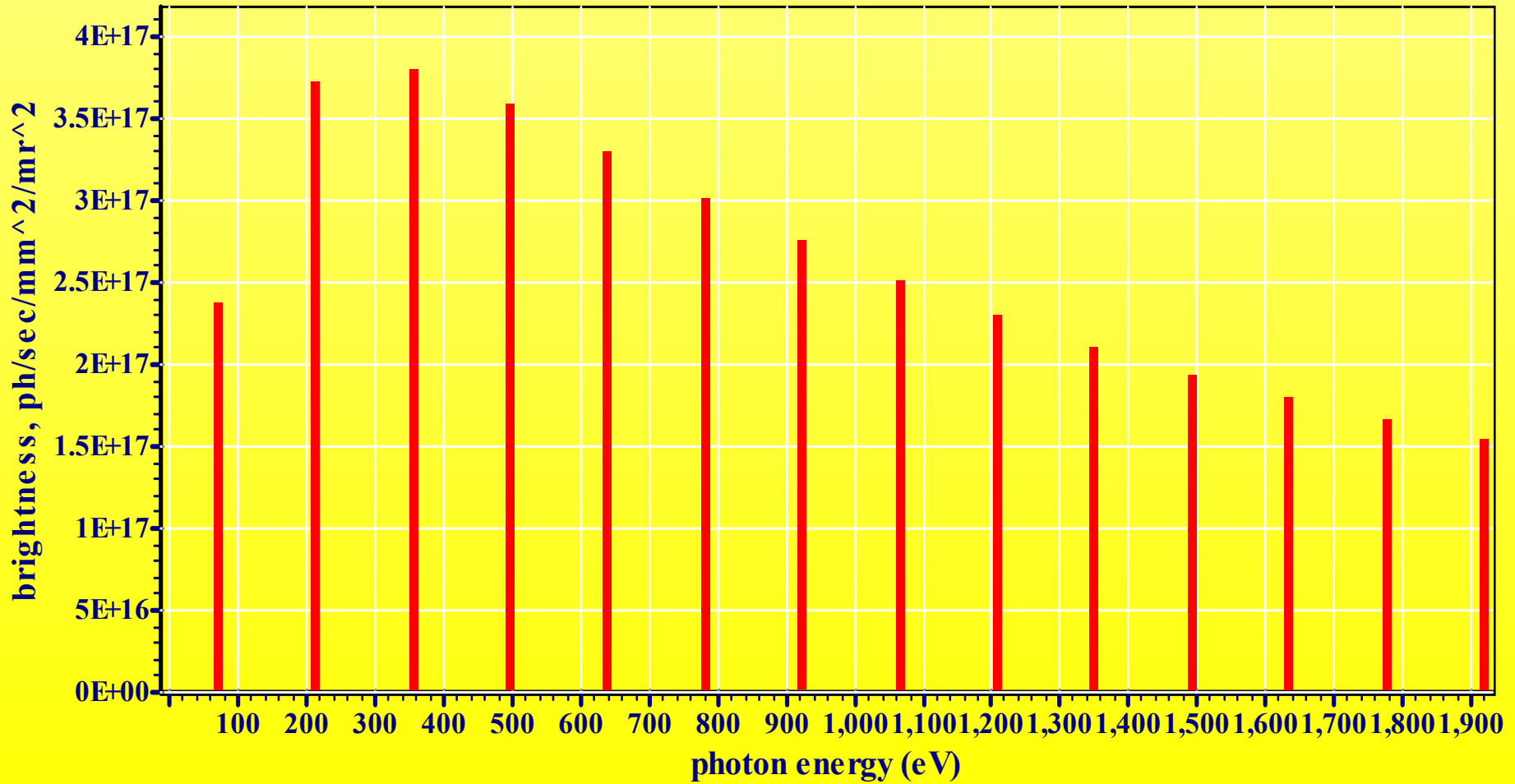
$$[JJ] = \left[J \frac{i\omega}{2} \left(\frac{\omega}{\omega_p} \right) J \frac{i\omega}{2} \left(\frac{\omega}{\omega_p} \right) \right]$$

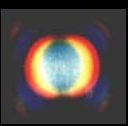
$$x = \frac{iK^2}{4 + 2K^2}$$



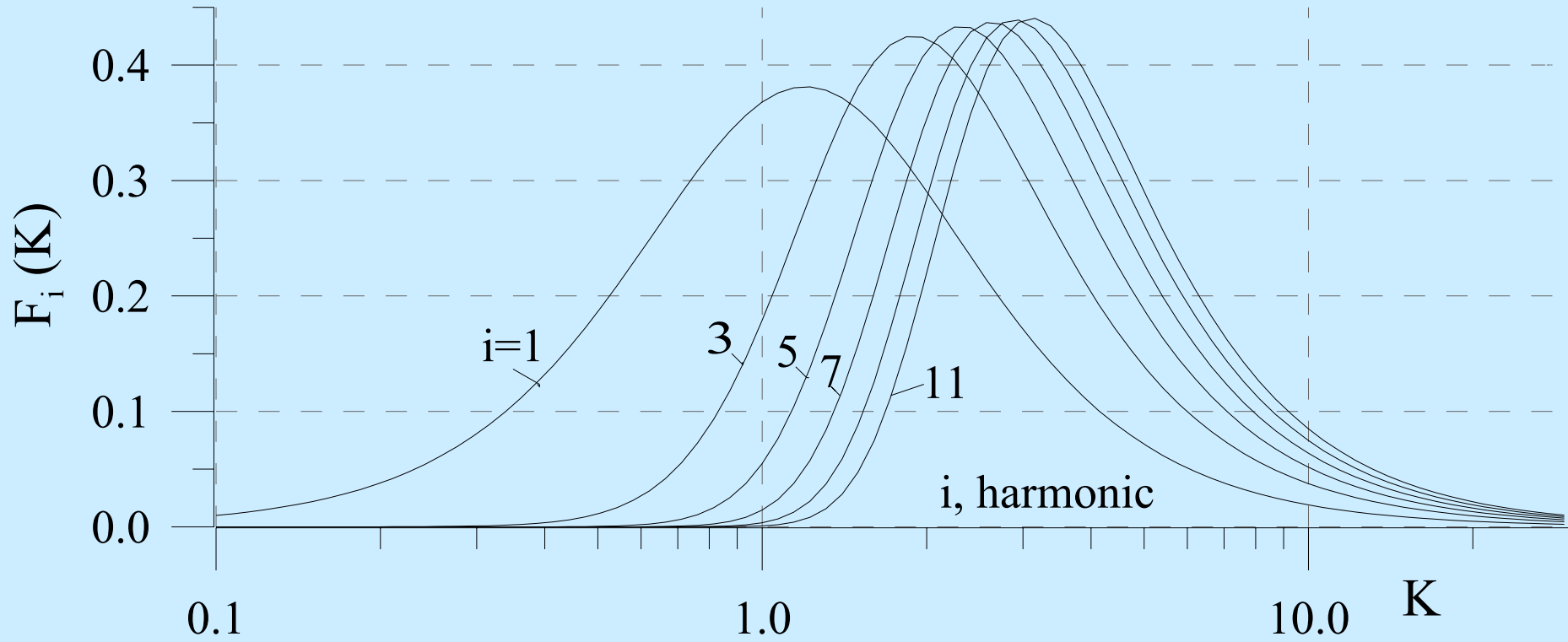
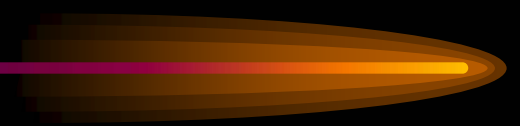
Synchrotron Radiation

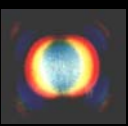
Undulator line spectrum





Insertion Device Radiation

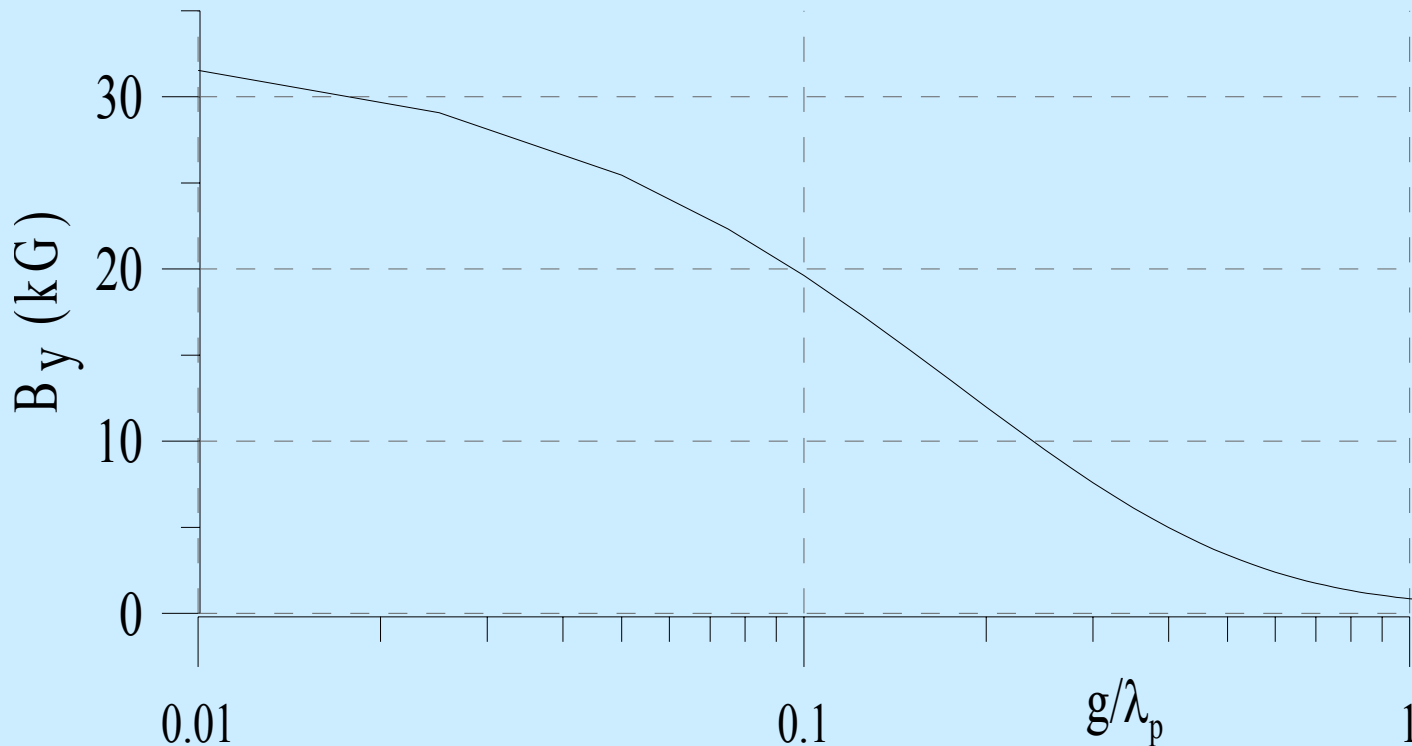


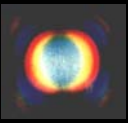


vary field strength by varying gap. For hybrid undulator:

$$B(\text{T}) = 3.3 \exp \left[-\frac{g}{\lambda_p} \left(5.74 - 1.8 \frac{g}{\lambda_p} \right) \right]$$

K.Halbach





$$\lambda = \frac{\lambda_p}{2\gamma^2} \left(1 + \frac{1}{2}K^2 + \gamma^2 \theta_{\text{obs}}^2 \right)$$

make λ_p very short \longrightarrow to get x-rays !?

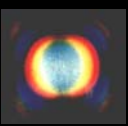
does not work well !

$$K = 0.934 B(\text{T}) \lambda_p (\text{cm})$$

short λ_p leads generally to small value of **K** !

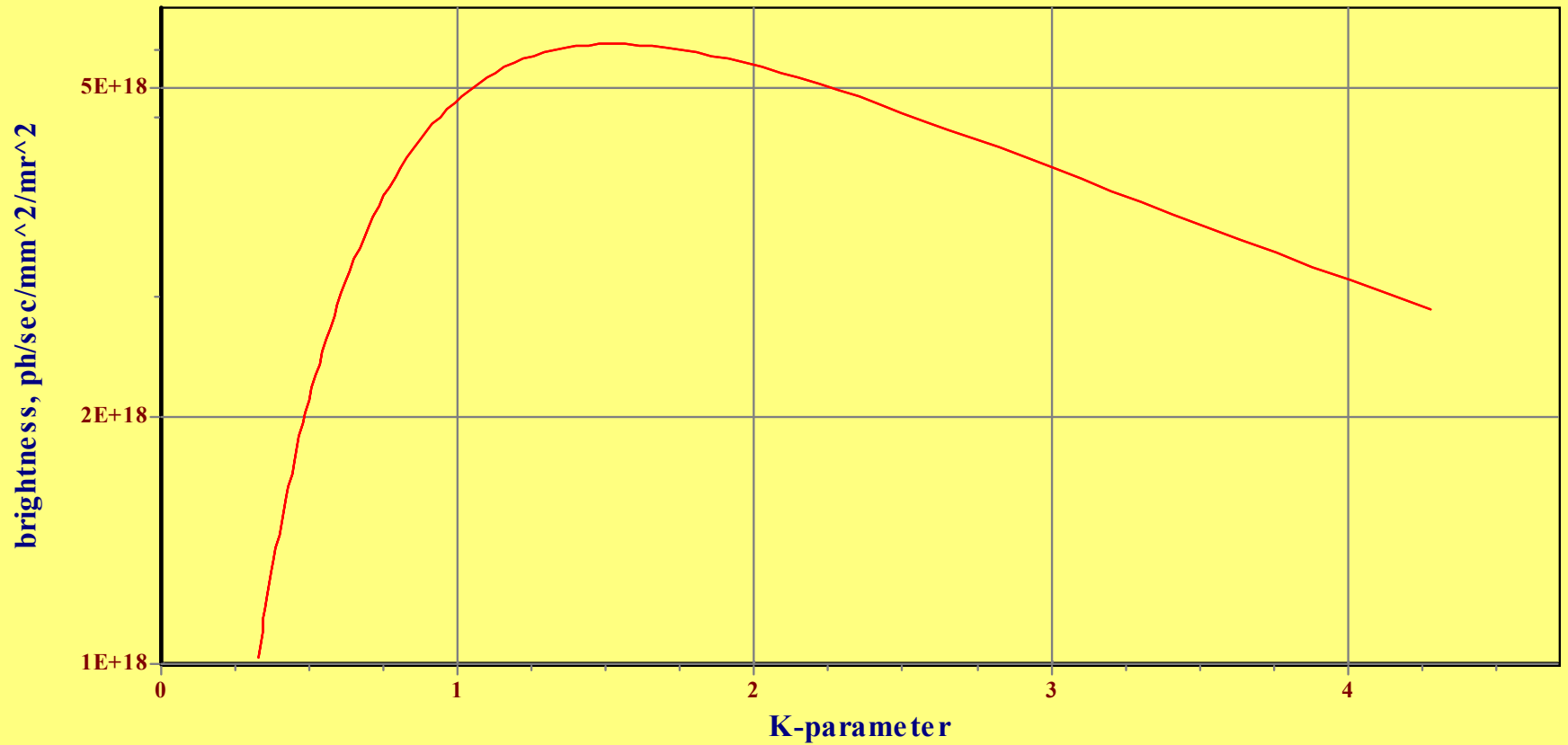
intensity is low

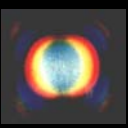
tuning range is narrow



Insertion Device Radiation

graph

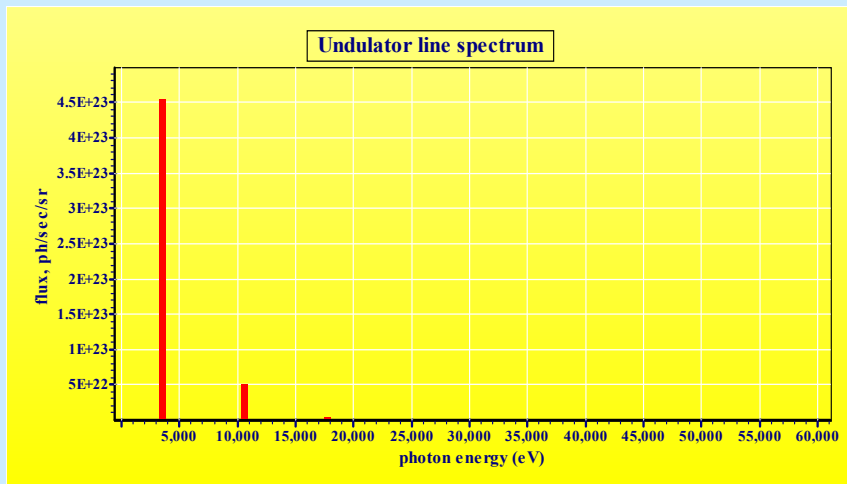




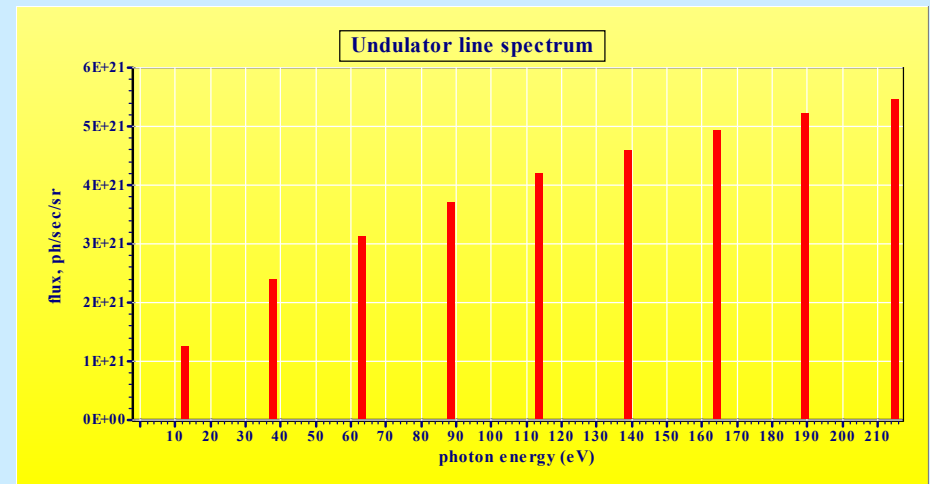
Insertion Device Radiation

tuning range:
$$\lambda = \frac{\lambda_p}{2\gamma^2} \left(1 + \frac{1}{2}K^2 + \gamma^2\theta_{\text{obs}}^2 \right)$$

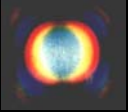
compare two undulators in SPEAR: $\lambda_p = 20$ mm and $=80$ mm



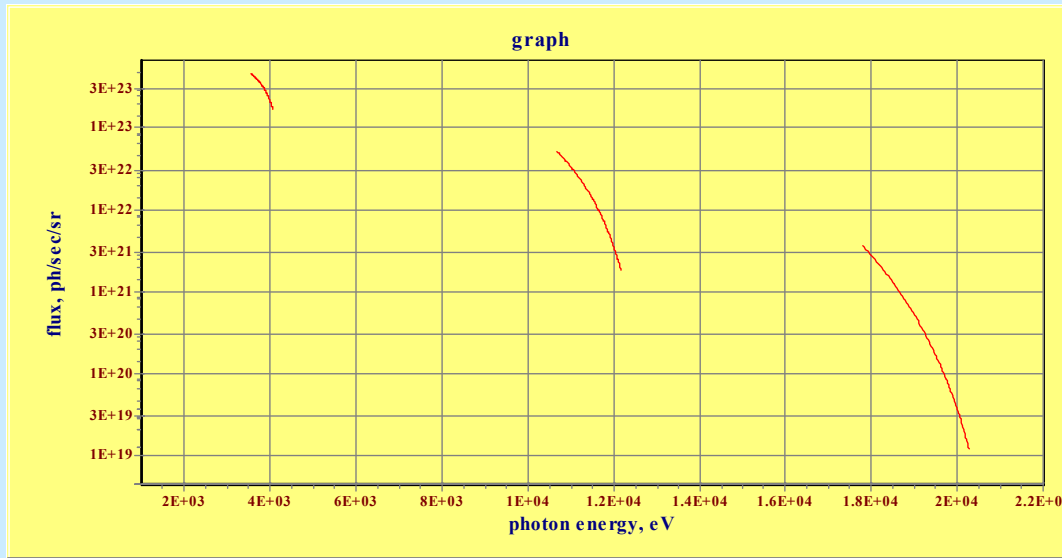
$\lambda_p = 20$ mm, 100 periods
 $0.33 < K < 0.63$



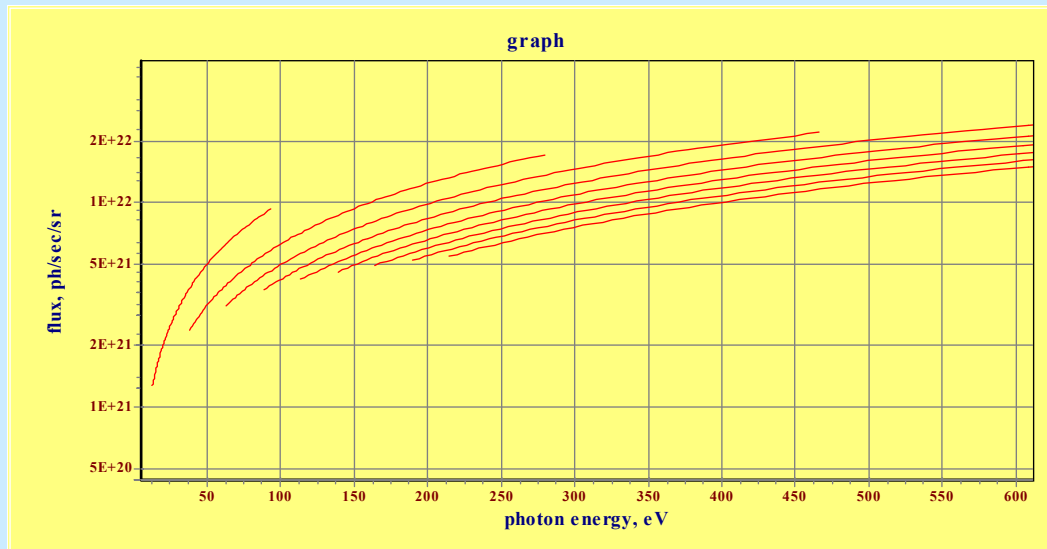
$\lambda_p = 80$ mm, 25 periods
 $4.6 < K < 12.9$



Insertion Device Radiation



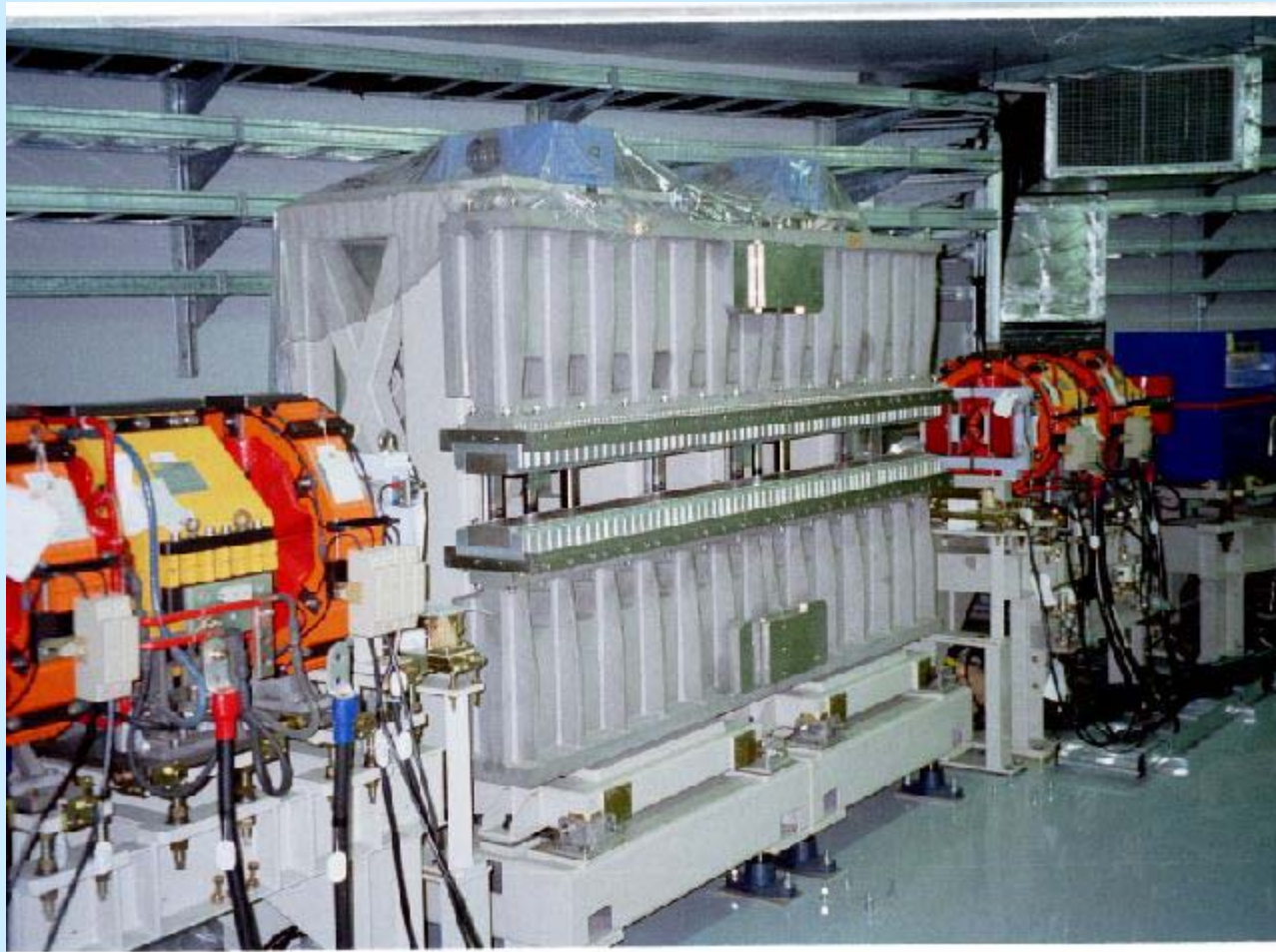
$\lambda_p = 20$ mm, 100 periods
 $0.33 < K < 0.63$



$\lambda_p = 80$ mm, 25 periods
 $4.6 < K < 12.9$



Insertion Device Radiation



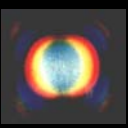
SUBARU : 2.3 m undulator, $\lambda_p = 7.6$ cm, 30 periods



Insertion Device Radiation



SUBARU : 10.8 m undulator, $\lambda_p = 5.4$ cm, 200 periods

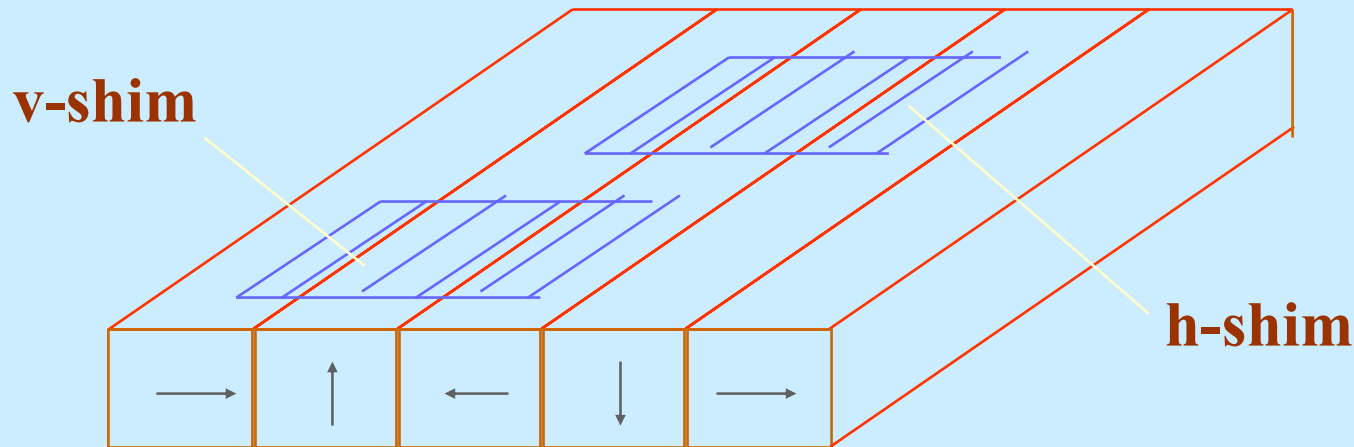


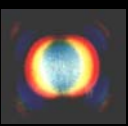
Insertion Device Radiation

use harmonics to reach high photon energies
at low electron energy

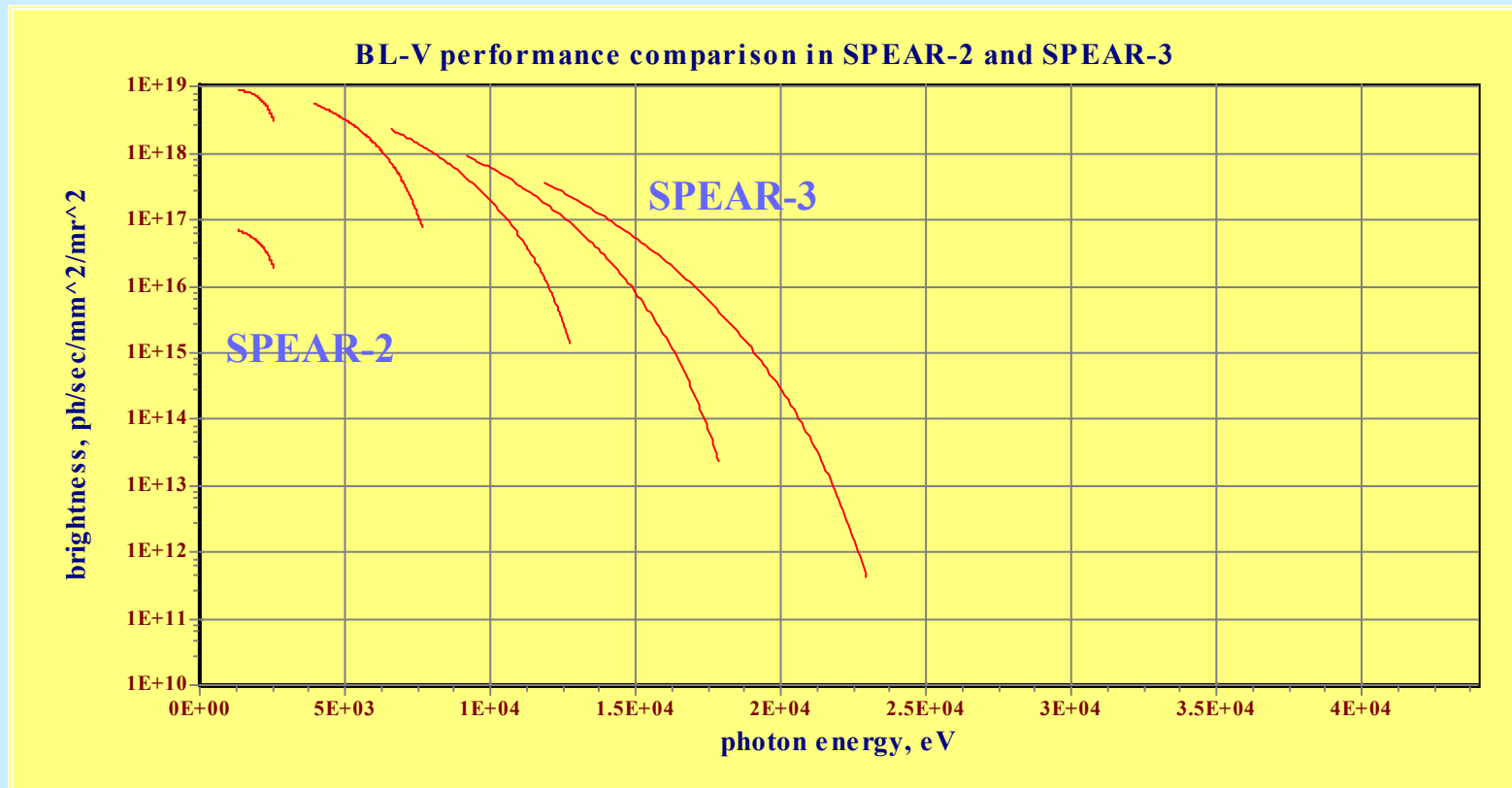
requires high undulator precision:

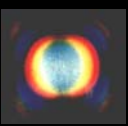
cannot build undulators that precise, but
we can **fix** them by shimming





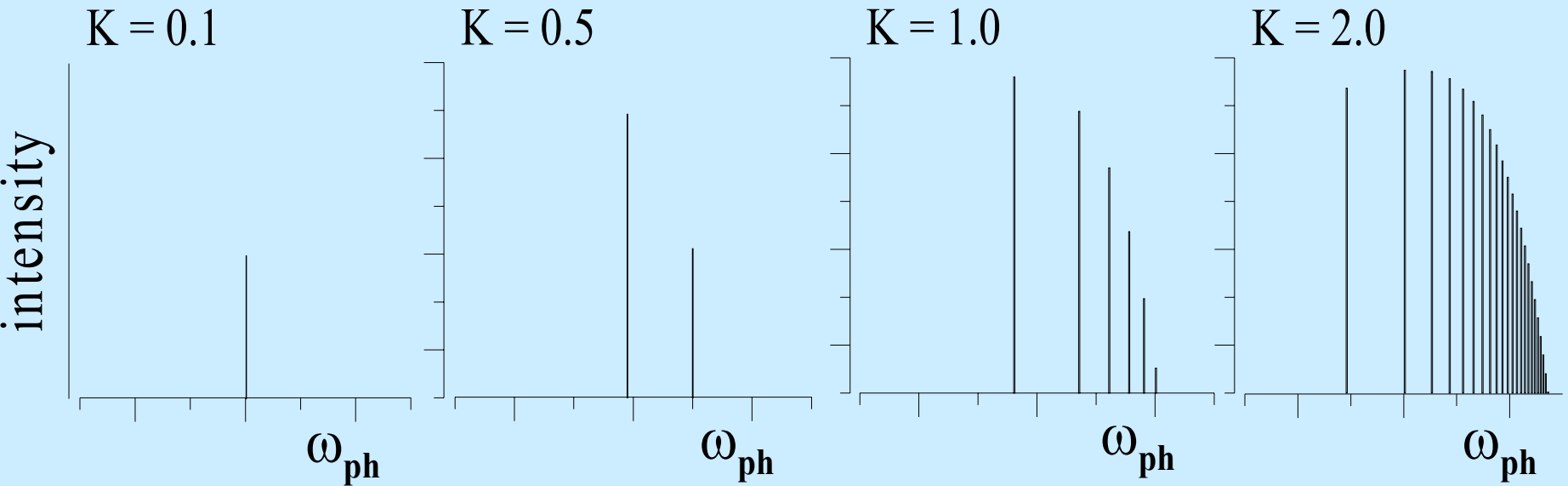
Insertion Device Radiation





Insertion Device Radiation

transition from undulator to wiggler radiation



crit. Photon energy from wiggler magnet at angle ψ with axis

$$P_c \left(\frac{1}{K} \right) \sqrt{1 - \left(\frac{\psi}{K} \right)^2}$$