Insertion Device Radiation

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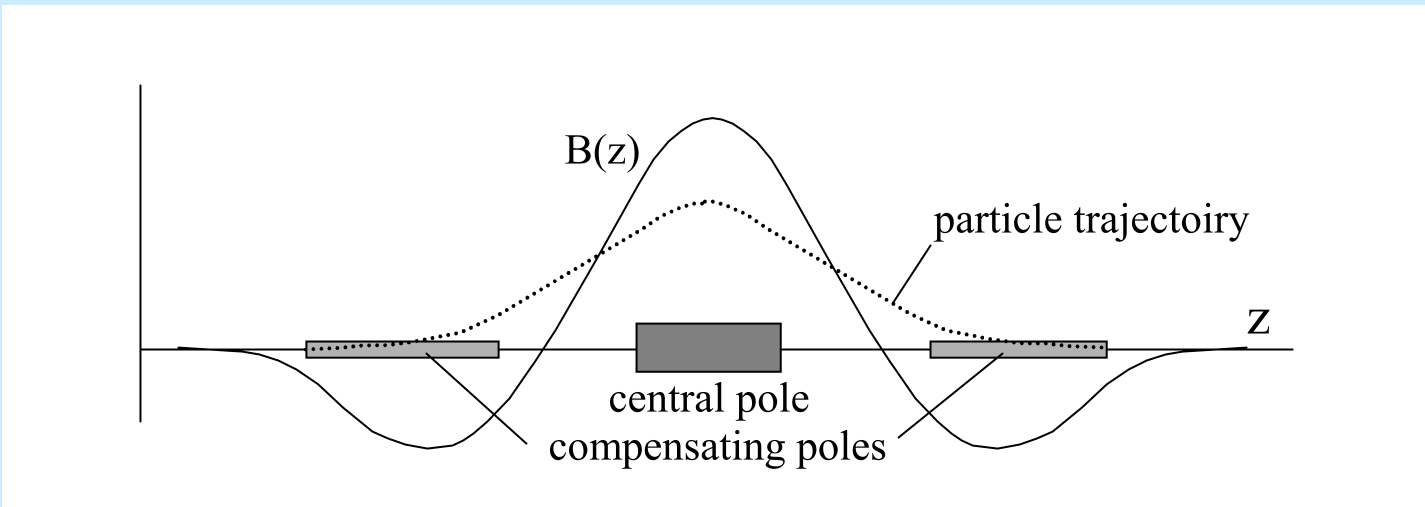
<u>Insertion devices do not change the shape of the storage ring!</u>

$$\int_{-\infty}^{\infty} B_y(x=0,z)\,\mathrm{d}z = 0$$

- **Wavelength shifter**
- **Wiggler magnet**
- **Undulators**
- **Super bends**

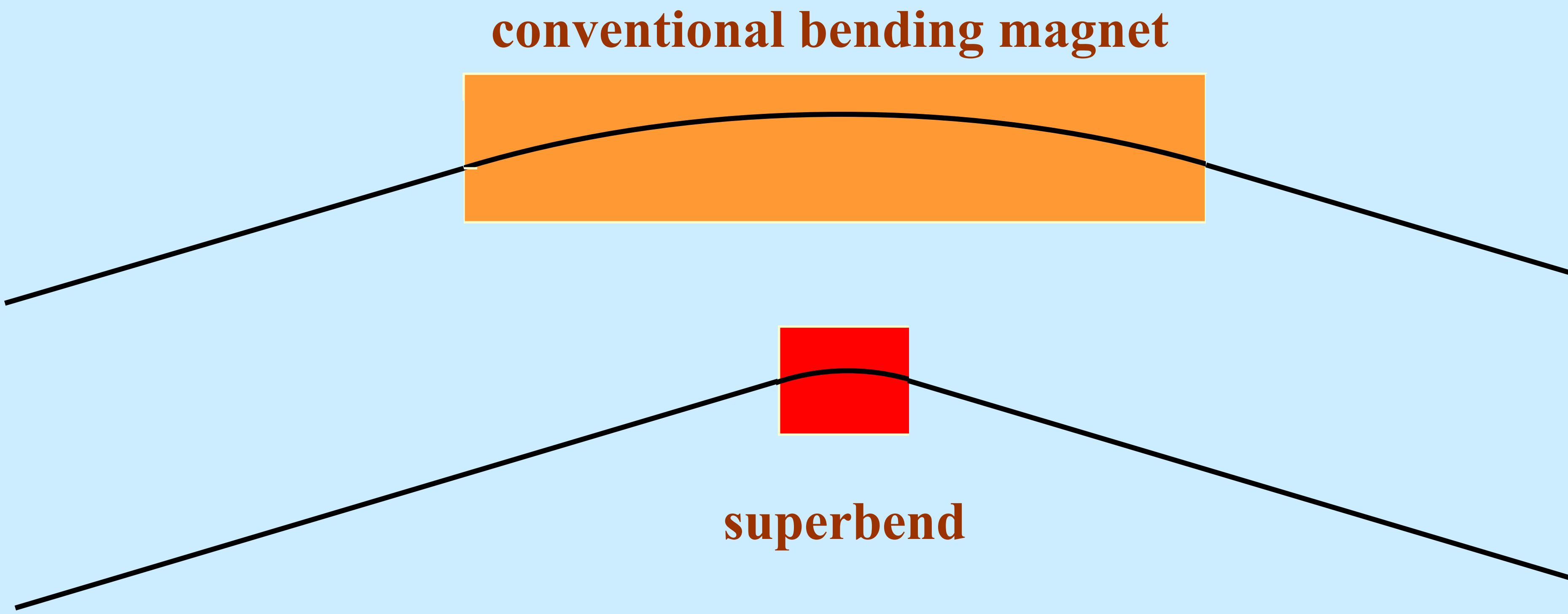
Purpose: **harden radiation**
increase intensity
high brightness monochromatic radiation
elliptically polarized radiation

wave length shifter



$$\int_{-\infty}^{+\infty} B_y(y=0,z)\,dz = 0$$

Super bends:
replace conventional bending magnets with super conducting bending magnets causing the same deflection angle.



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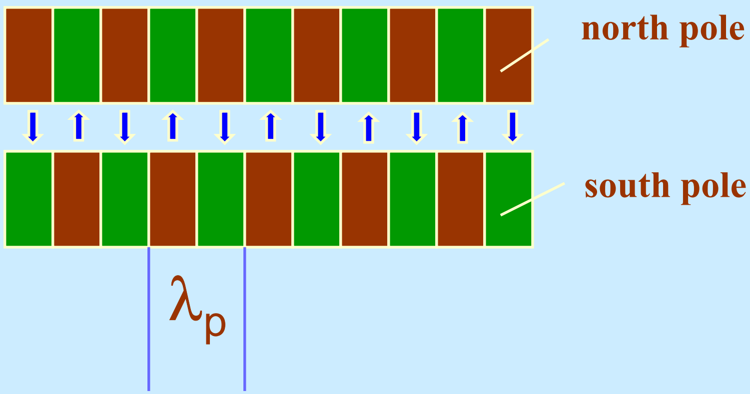
Periodically deflecting magnets:



Wiggler magnets, strong field

Undulators, weak field

Wiggler magnets produce ordinary, broad band synchrotron radiation;

Intensity increased by factor N_p (# of poles)

deflection angle per half-pole

$$B_y(z) = B_0 \cos k_p z$$

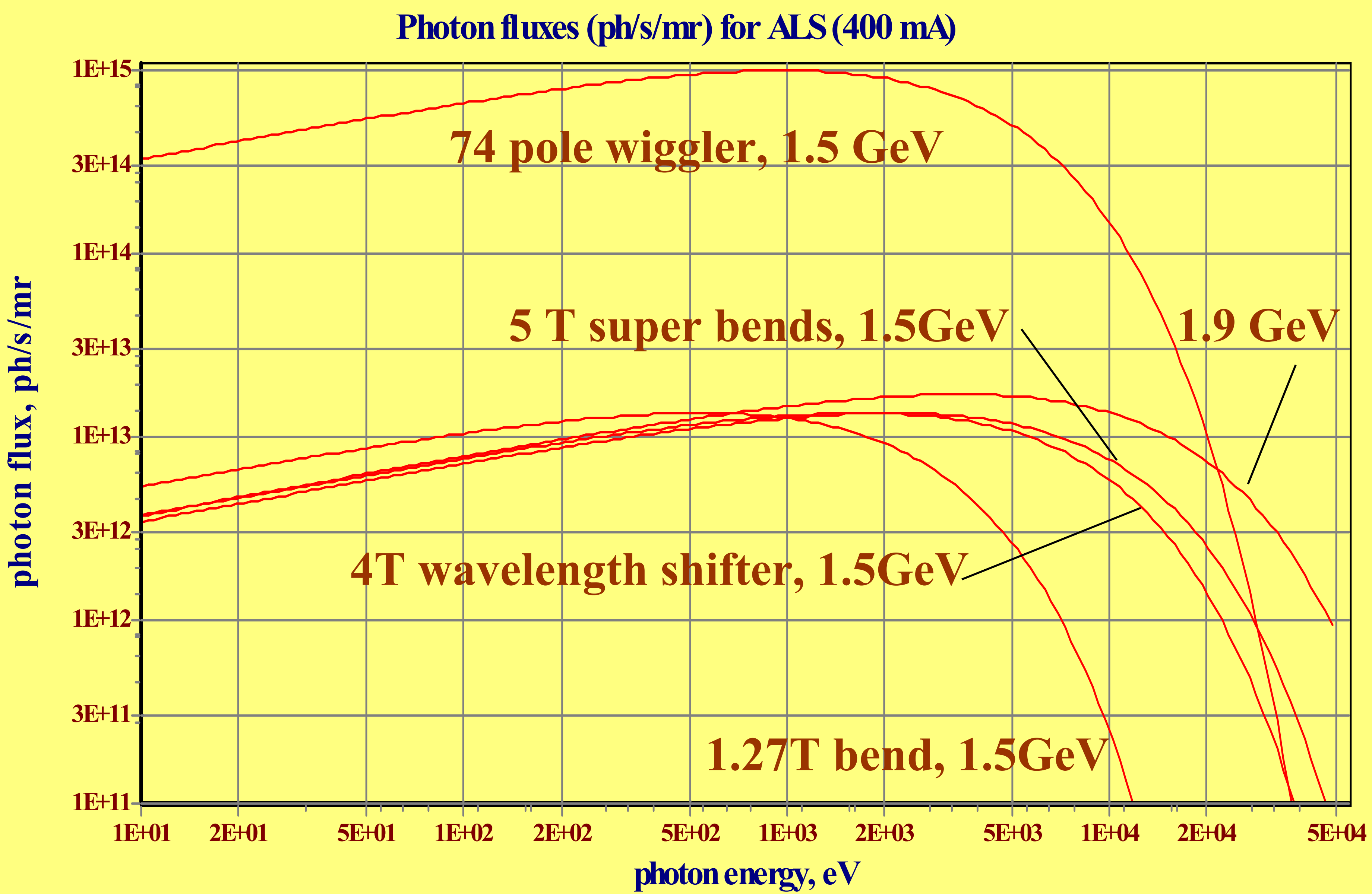
$$d\theta = \frac{dz}{\rho} = \frac{eB}{cp} dz \quad \Rightarrow \quad \theta = \int \frac{dz}{\rho} = \frac{eB}{cp} \int_0^{\lambda_p/4} \cos k_p z \, dz = \frac{eB_0 \lambda_p}{2\pi cp}$$

$$\theta = \frac{eB_0 \lambda_p}{2\pi cp} = \frac{K}{\gamma}$$

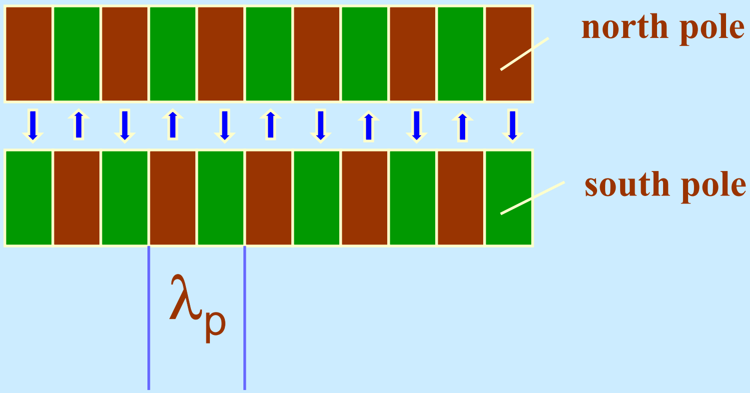
K: undulator/wiggler strength parameter

$$K = \frac{eB_0 \lambda_p}{2\pi mc^2 \beta} = 0.934\, B_0(\mathrm{T})\, \lambda_p(\mathrm{cm})$$

Insertion Device Radiation

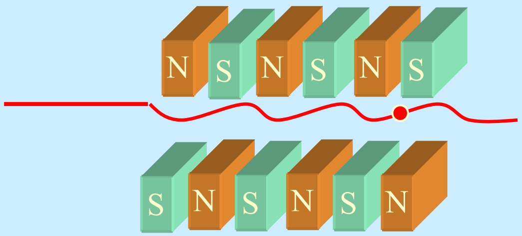


assume a magnetic field $B_y(z) = B_0 \cos k_p z$



$$k_p = \frac{2\pi}{\lambda}$$

electron performs sinusoidal oscillations



sinusoidal perturbation of field lines

sinusoidal perturbation of field lines

N_p undulator periods ⟷ N_p field oscillations

$$E(t) = \begin{cases} E_0 \sin\omega_0 t & \text{for } -\tfrac{1}{2}N_p T_0 < \omega_0 t < \tfrac{1}{2}N_p T_0 \\ 0 & \text{elsewhere} \end{cases}$$

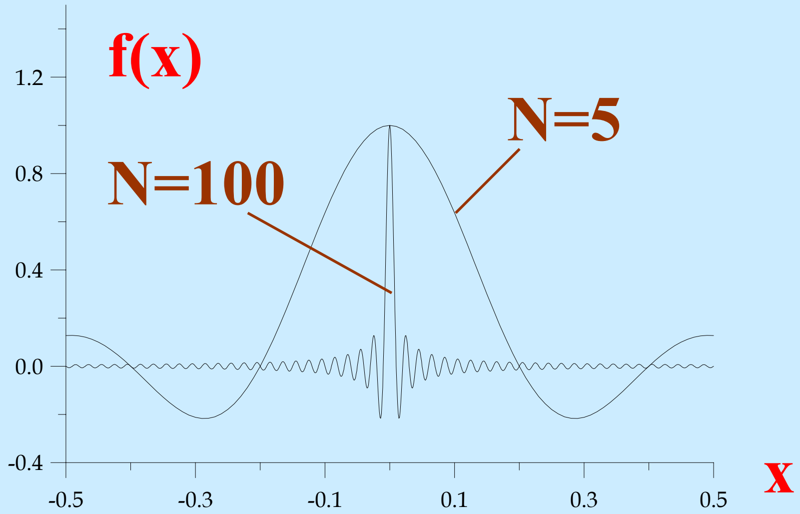
spectrum

$$E(\omega) = \int E(t)\, e^{-i\omega t} dt = \int_{-\infty}^{\infty} E_0 \sin\omega_0 t\, e^{-i\omega t} dt = E_0 \int_{-\infty}^{\infty} e^{-i(\omega_0-\omega)t} dt$$

$$E_0 \int_{-\frac{1}{2}N_p T_0}^{\frac{1}{2}N_p T_0} e^{-i(\omega_0-\omega)t} dt = E_0 \frac{e^{i(\omega_0-\omega)\frac{1}{2}N_p T_0} - e^{-i(\omega_0-\omega)\frac{1}{2}N_p T_0}}{i(\omega_0-\omega)} = E_0 N_p T_0 \frac{\sin(\omega_0-\omega)\frac{1}{2}N_p T_0}{(\omega_0-\omega)\frac{1}{2}N_p T_0}$$

$$E(\omega) = E_0 N_p T_0 \frac{\sin(\omega_0-\omega)\frac{1}{2}N_p T_0}{(\omega_0-\omega)\frac{1}{2}N_p T_0}$$

Sinc-function: $$f(x) = \frac{\sin \pi N x}{\pi N x}$$



f(0)=1 and

f(y)=0 for y=1/N

or for

$$(\omega_0 - \omega)\tfrac{1}{2} N_p T_0 = \pi$$

line width: $$\frac{\delta\omega}{\omega_0} = \pm\frac{\omega_0 - \omega}{\omega_0} = \frac{2\pi}{T_0}\frac{1}{N_p}\frac{1}{\omega_0} = \frac{1}{N_p}$$

$$\frac{\delta\omega}{\omega_0} = \pm\frac{1}{N_p}$$

What is ω_0 ?

undulator period: λ_p

in electron rest system: $\lambda_p^* = \frac{\lambda_p}{\gamma}$

in lab system (Doppler effect): $\omega = \omega_p^* \gamma (1 + \mathbf{n}_z^* \beta)$

or $\lambda = \frac{\lambda_p}{\gamma^2 (1 + \mathbf{n}_z^* \beta)}$

with $\mathbf{n}_z = \frac{\beta + \mathbf{n}_z^*}{1 + \mathbf{n}_z^* \beta}$ ⇐ $\lambda = \frac{\lambda_p}{\gamma^2} \frac{\mathbf{n}_z}{\beta + \mathbf{n}_z^*} = \frac{\lambda_p}{\gamma^2} \frac{\cos\theta}{1 + \cos\theta^*}$

$\sin\theta = \frac{\sin\theta^*}{\gamma(1 + \beta\cos\theta^*)}$ ⇒ $\theta \approx \frac{\sin\theta^*}{\gamma(1 + \beta\cos\theta^*)}$

or

$\gamma^2\theta^2 = \frac{\sin^2\theta^*}{(1 + \beta\cos\theta^*)^2} = \frac{1 - \cos\theta^*}{1 + \cos\theta^*}$ ⇒ $\cos\theta^* = \frac{1 - \gamma^2\theta^2}{1 + \gamma^2\theta^2}$

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$$\lambda = \frac{\lambda_p}{\gamma^2}\frac{\cos\theta}{1+\cos\theta^*} = \frac{\lambda_p}{\gamma^2}\frac{1}{1+\frac{1-\gamma^2\theta^2}{1+\gamma^2\theta^2}} = \frac{\lambda_p}{2\gamma^2}\left(1+\gamma^2\theta^2\right)$$

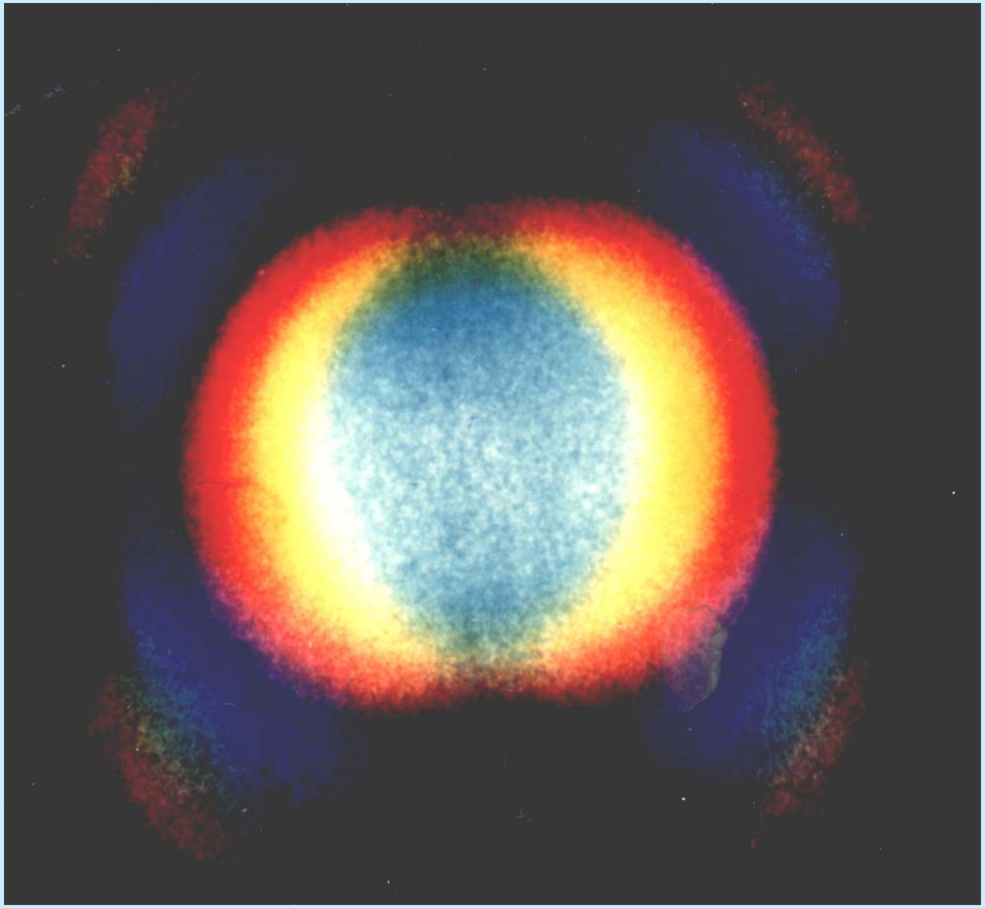
$$\theta^2 = (\theta_{und}+\theta_{obs})^2 = \theta_{und}^2 + 2\theta_{und}\theta_{obs} + \theta_{obs}^2$$

$$\theta_{und} = \frac{K}{\gamma}\cos k_p z \qquad \langle\theta_{und}\rangle = 0 \quad \text{and} \quad \langle\theta_{und}^2\rangle = \frac{1}{2}\frac{K^2}{\gamma^2}$$

fundamental undulator wavelength:

$$\lambda = \frac{\lambda_p}{2\gamma^2}\left(1+\frac{1}{2}K^2+\gamma^2\theta_{obs}^2\right)$$

Insertion Device Radiation



$$\lambda = \frac{\lambda_p}{2\gamma^2}\left(1 + \frac{1}{2}K^2 + \gamma^2\theta_{obs}^2\right)$$

$$\lambda_i = \frac{\lambda_p}{2i\gamma^2}\left(1 + \frac{1}{2}K^2 + \gamma^2\theta^2\right)$$

$$\lambda_i(\text{Å}) = 1305.6\,\frac{\lambda_p}{iE^2}\left(1 + \frac{1}{2}K^2\right)$$

$$\varepsilon_i(\text{eV}) = 9.4963 \times \frac{iE^2}{\lambda_p\left(1 + \frac{1}{2}K^2\right)}$$

$$\theta = \frac{1}{\gamma}\sqrt{\frac{1 + \frac{1}{2}K^2}{2iN_p}}$$

$$B_y(z) = B_0 \cos k_{\mathrm{p}} z$$

this is what we want

Maxwell tells us what we can get!

$$B_y(y,z) = B_0\, b(y) \cos k_{\mathrm{p}} z$$

$\nabla \times \mathbf{B} = \mathbf{0}$

$$\frac{\partial B_z}{\partial y} = \frac{\partial B_y}{\partial z} = -B_0\, b(y)\, k_{\mathrm{p}} \sin k_{\mathrm{p}} z$$

and $B_y = -B_0\, b(y)\,(1 - \cos k_{\mathrm{p}} z)$

∇ ⌛$\mathbf{B} = 0$

$\mathbf{B} \neq \mathbf{B}(\mathbf{x})$

$$\frac{\partial B_z}{\partial z} = -B_0 \frac{\partial b(y)}{\partial y} \cos k_{\mathrm{p}} z$$

form $\dfrac{\partial^2 B_z}{\partial y \partial z}$ ➡ $\dfrac{\partial^2 b(y)}{\partial^2 y} = k_{\mathrm{p}}^2\, b(y)$ $\quad b(y) = a_1 \cosh k_{\mathrm{p}} y + a_2 \sinh k_{\mathrm{p}} y$

$$B_x = 0$$
$$B_y = B_0 \cosh k_{\mathrm{p}} y \cos k_{\mathrm{p}} z$$
$$B_z = -B_0 \sinh k_{\mathrm{p}} y \sin k_{\mathrm{p}} z$$

beam dynamics

$$\frac{d^2\mathbf{r}}{ds^2} = \frac{\mathbf{n}}{\rho} = \frac{e}{mc^2\gamma}\left[\frac{\mathbf{v}}{v}\times\mathbf{B}\right]$$

$$\frac{d^2x}{dt^2} = -\frac{eB_0}{mc\gamma}\frac{dz}{dt}\cos k_p z$$

$$\frac{d^2z}{dt^2} = +\frac{eB_0}{mc\gamma}\frac{dx}{dt}\cos k_p z$$

$$\frac{dx}{dt} = -c\beta\frac{K}{\gamma}\sin k_p z$$

$$\frac{dz}{dt} = +c\beta\left(1 - \frac{K^2}{2\gamma^2}\sin^2 k_p z\right)$$

drift velocity

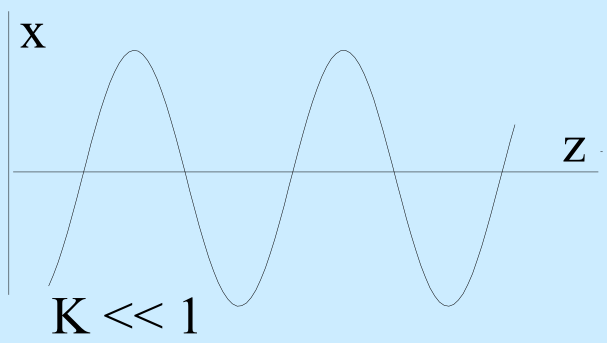
$$\bar{\beta} = \beta\left(1 - \frac{K^2}{4\gamma^2}\right)$$

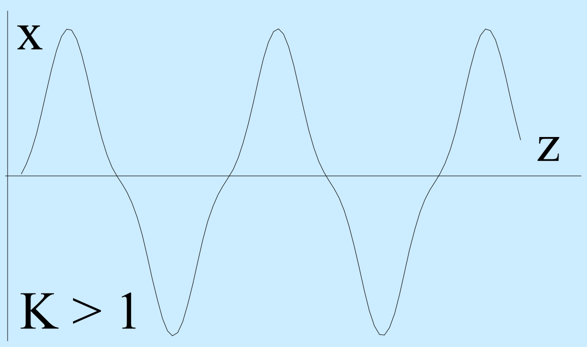
$$x(t) = a\cos(k_p c\bar{\beta}t)$$

$$z(t) = c\bar{\beta}t + \frac{1}{8}k_p a^2\sin(2k_p c\bar{\beta}t)$$

$$a \approx \frac{K}{\gamma k_p}$$

Stronger undulator field



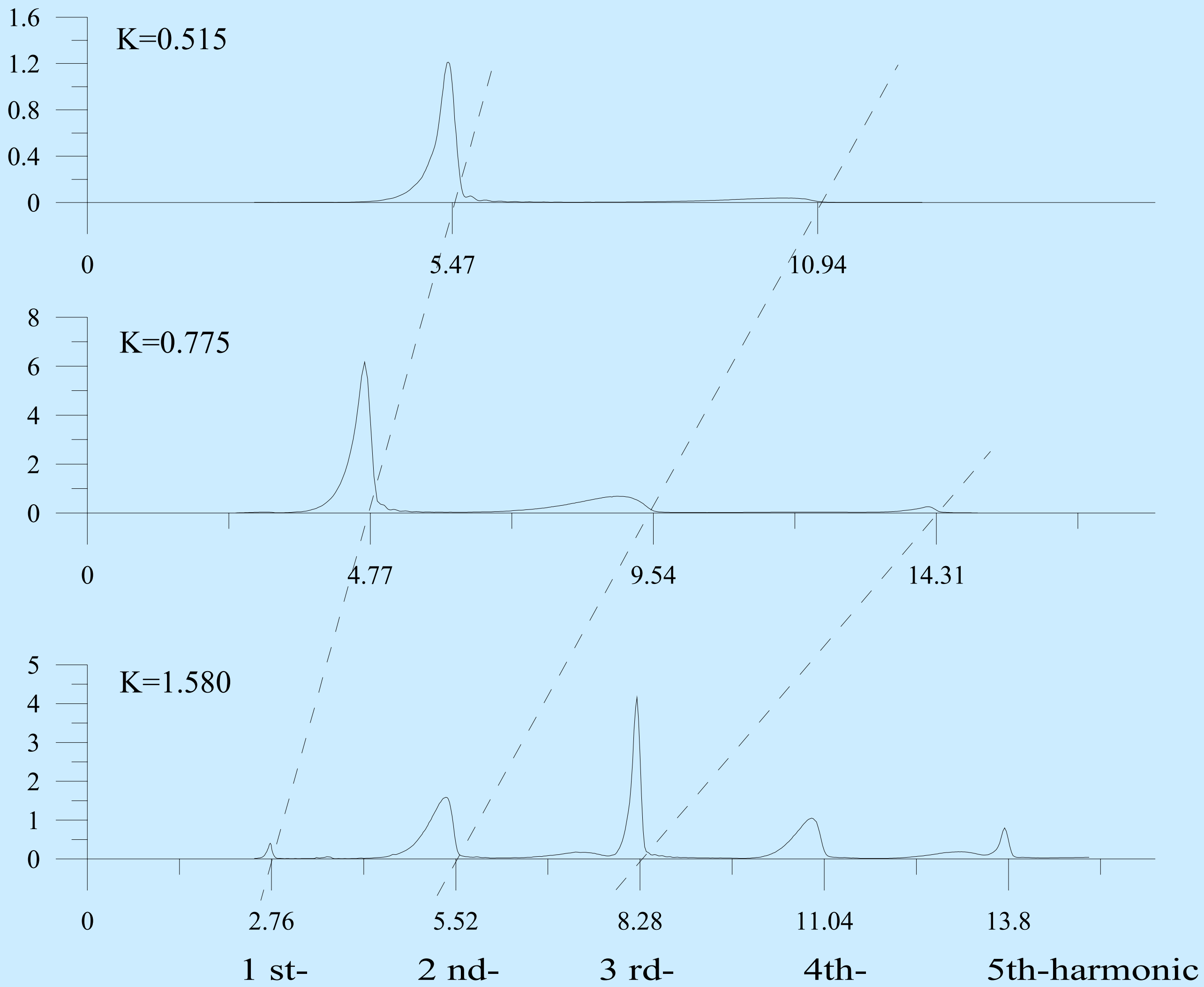


transverse motion completely non-relativistic

relativistic effect on transverse motion

source of higher harmonics (only odd harmonics !)

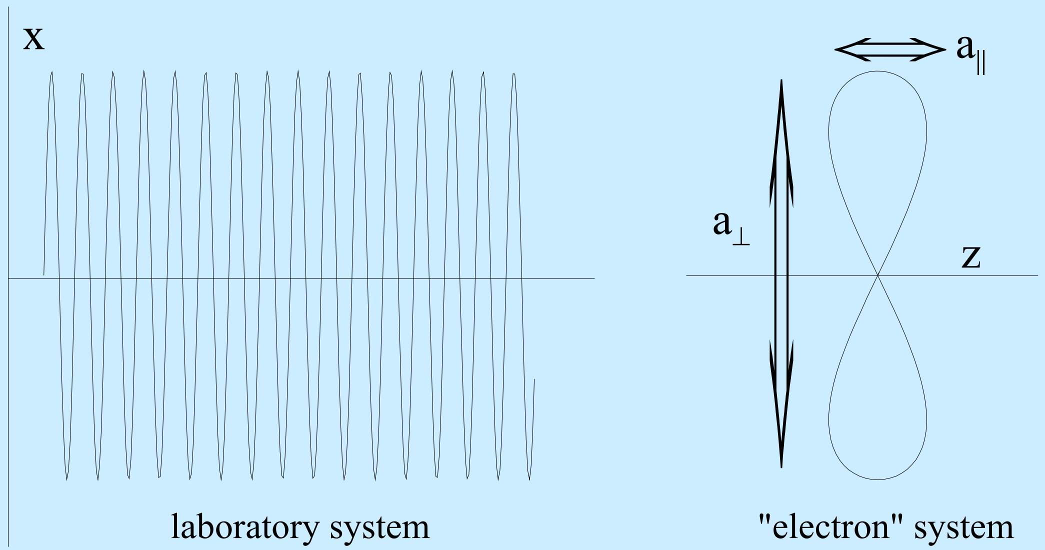
PEP-Undulator: 77mm, 27 periods, 7.1 GeV



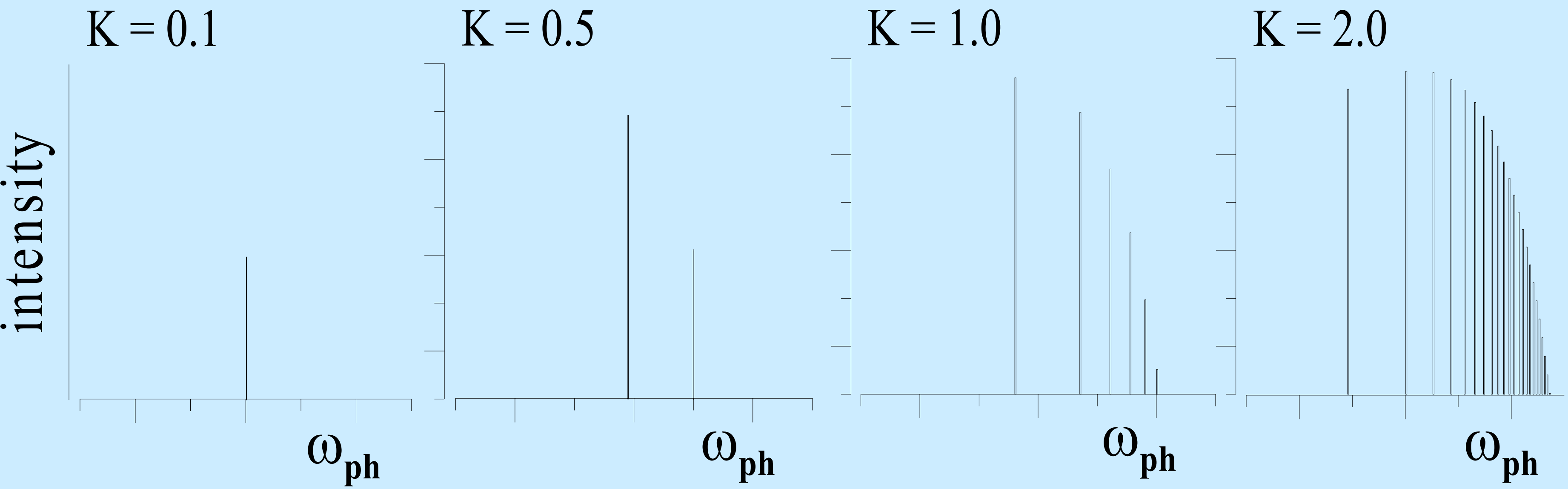
now we increase strength parameter K

$$x(t) = a\cos(k_p c\beta t)$$

$$z(t) = c\beta t + \frac{1}{8}k_p a^2 \sin(2k_p c\beta t)$$



transition from undulator to wiggler radiation



energy loss per undulator pass

$$\Delta E_{\mathrm{rad}} = \frac{1}{3} r_{\mathrm{c}} mc^2 \gamma^2 K^2 k_{\mathrm{p}}^2 L_{\mathrm{u}}$$

$$\Delta E_{\mathrm{rad}}(\mathrm{eV}) = 0.07257 \frac{\mathrm{E}^2 \mathrm{K}^2}{\lambda_{\mathrm{p}}^2} L_{\mathrm{u}}$$

tot. radiation power

$$P(\mathrm{W}) = 0.07257 \frac{\mathrm{E}^2 \mathrm{K}^2 \mathrm{N} \mathrm{I}}{\lambda_{\mathrm{p}}}$$

Insertion Device Radiation

$$\frac{dN_{ph}(\omega)}{d\Omega} = \alpha N_p^2 \frac{\Delta\omega}{\omega}\frac{I}{e}\sum_{i=1}^{\infty} i^2 \,\mathrm{Sinc}\,\left(F_\sigma^2 + F_\pi^2\right)$$

$$\mathrm{Sinc} = \left(\frac{\sin(\pi N_p \Delta\omega/\omega_1)}{\pi N_p \Delta\omega/\omega_1}\right)^2$$

$$F_\sigma = \frac{2\gamma\theta\Sigma_1 \cos\varphi - K\Sigma_2}{\left(1+\frac{1}{2}K^2+\gamma^2\theta^2\right)^2} \qquad F_\pi = \frac{2\gamma\theta\Sigma_1 \sin\varphi}{\left(1+\frac{1}{2}K^2+\gamma^2\theta^2\right)^2}$$

$$\Sigma_{1,i} = \sum_{m=-\infty}^{\infty} J_{-m}(u)\, J_{i-2m}(v)$$

$$\Sigma_{2,i} = \sum_{m=-\infty}^{\infty} J_{-m}(u)\left(J_{i-2m-1}(v) + J_{i-2m+1}(v)\right)$$

$$u = \frac{\omega}{\omega_1}\frac{iK^2}{4\left(1+\frac{1}{2}K^2+\gamma^2\theta^2\right)} \qquad v = \frac{\omega}{\omega_1}\frac{2iK\gamma\theta\cos\varphi}{1+\frac{1}{2}K^2+\gamma^2\theta^2}$$

pin hole radiation

$$\left.\frac{\mathrm{d}N_{ph}(\omega)}{\mathrm{d}\Omega}\right|_i = \alpha N_p^2 \gamma^2 \frac{\Delta\omega}{\omega} \frac{I}{e} \frac{i^2 K^2 [JJ]^2}{\left(1+\frac{1}{2}K^2\right)^2}$$

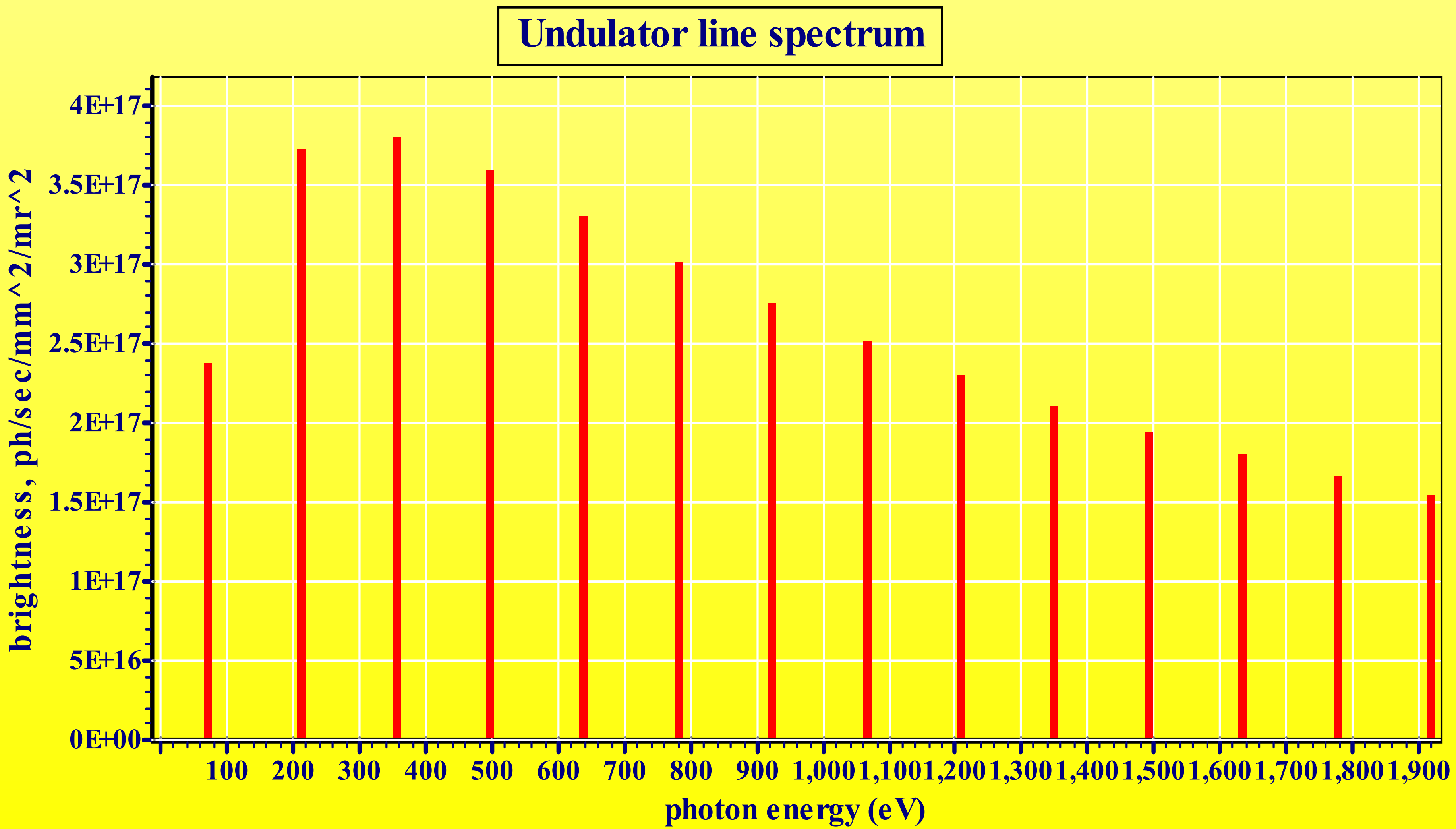
$$= 1.7466 \times 10^{23} E^2 (\mathrm{GeV}^2)\, I(\mathrm{A})\, N_p^2 \frac{\Delta\omega}{\omega} f_i(K),$$

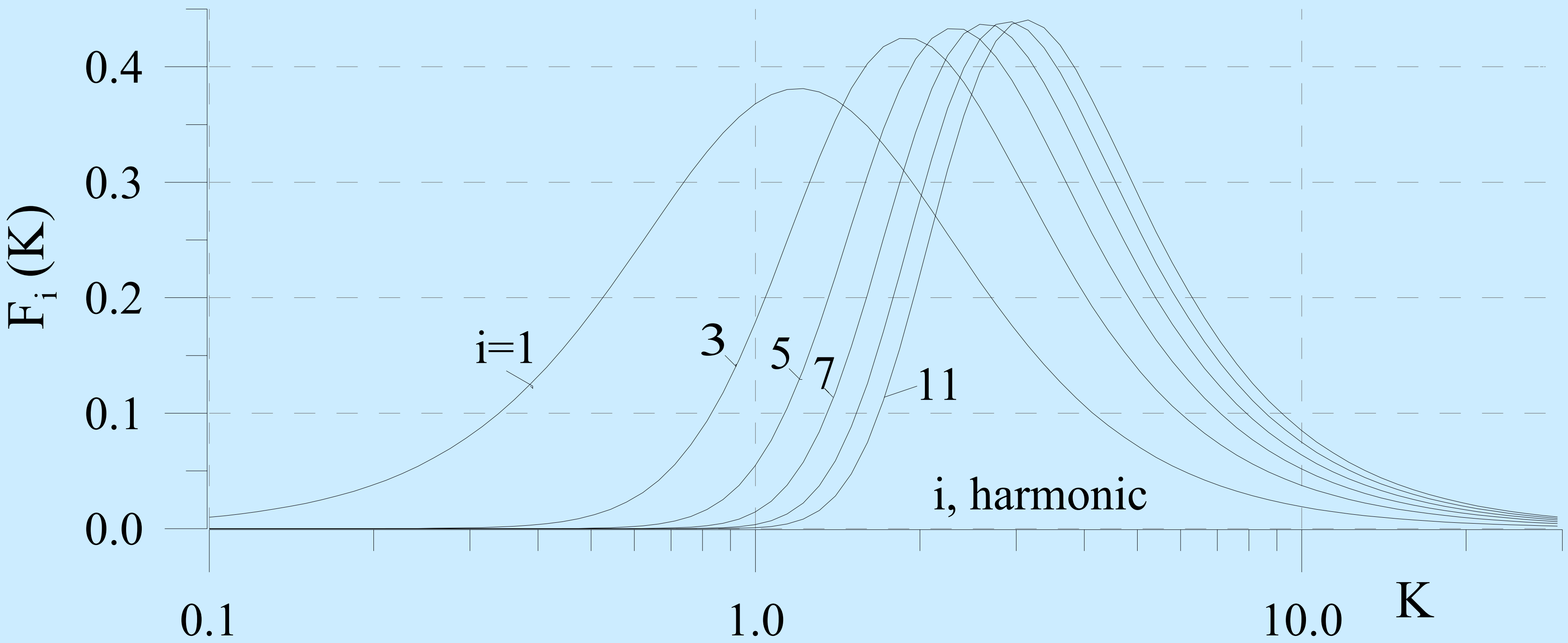
$$f_i(K) = \frac{i^2 K^2 [JJ]^2}{\left(1+\frac{1}{2}K^2\right)^2}$$

$$[JJ] = \left[J_{\frac{i-1}{2}}(ix) - J_{\frac{i+1}{2}}(ix)\right]$$

$$x = \frac{iK^2}{4+2K^2}$$

Synchrotron Radiation



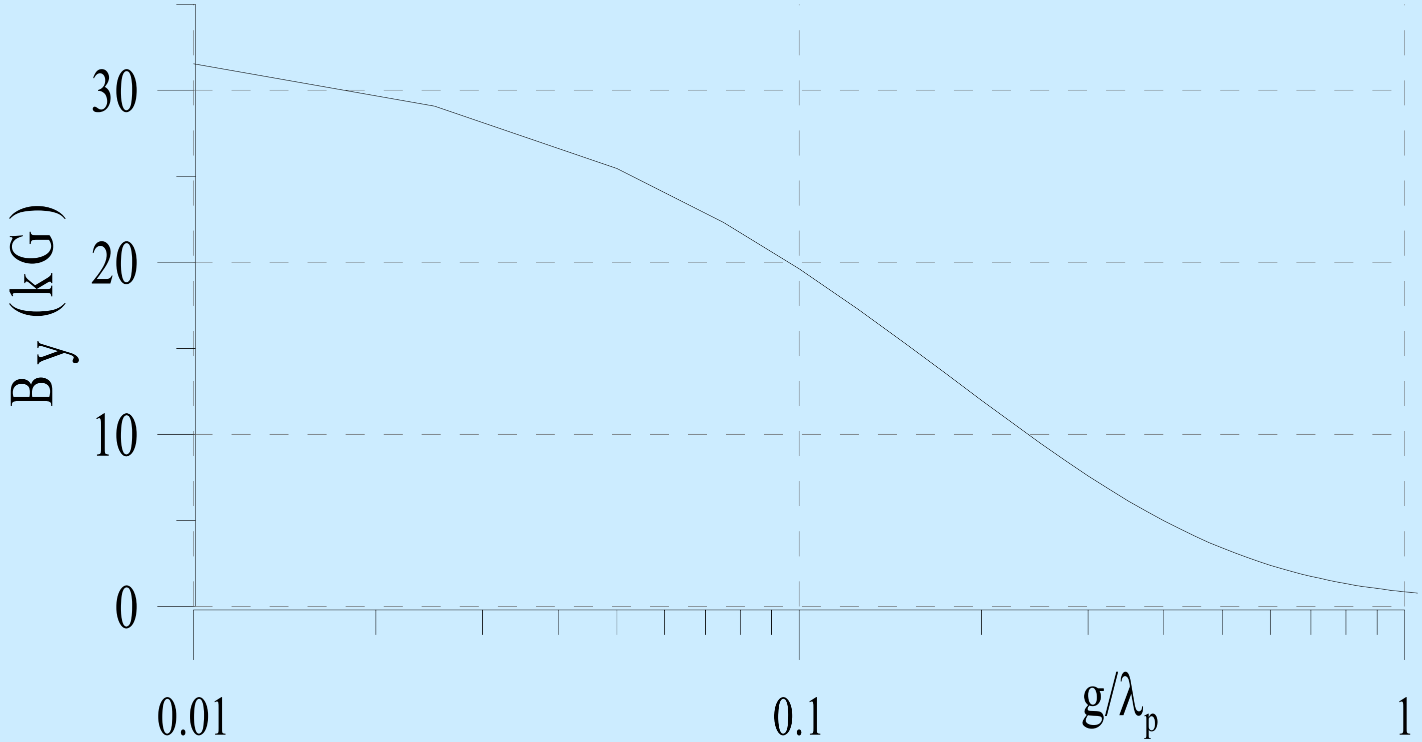


0.4
0.3
0.2
0.1
0.0
F_i (K)
i=1
3
5
7
11
i, harmonic
0.1
1.0
10.0
K

vary field strength by varying gap. For hybrid undulator:

$$B(\mathrm{T}) = 3.3\exp\left[-\frac{g}{\lambda_p}\left(5.74 - 1.8\frac{g}{\lambda_p}\right)\right]$$

K.Halbach



$$\lambda = \frac{\lambda_p}{2\gamma^2}\left(1 + \frac{1}{2}K^2 + \gamma^2\theta^2_{obs}\right)$$

make λ_p very short → **to get x-rays !?**

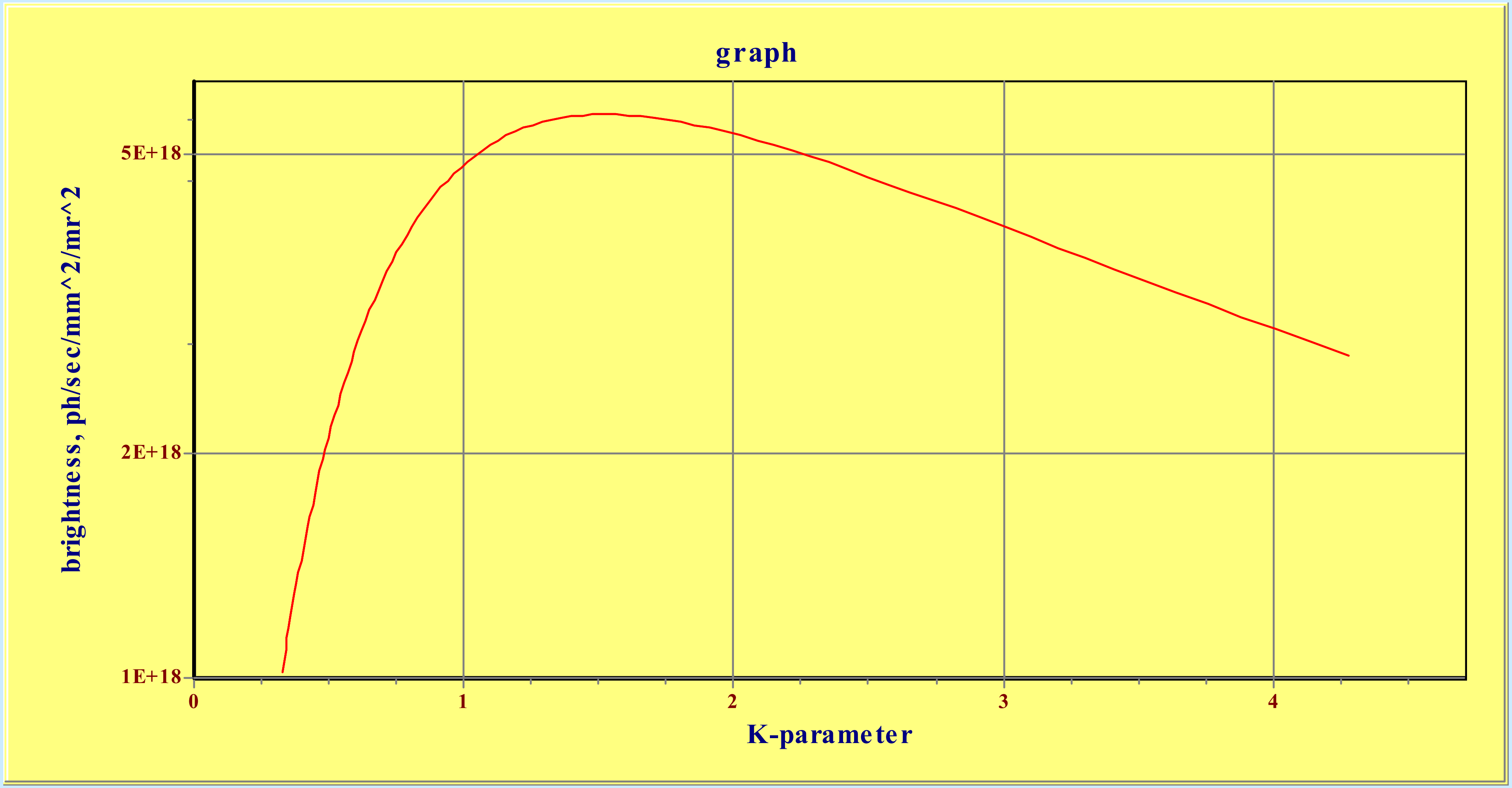
does not work well !

$$K = 0.934\,B(\mathrm{T})\,\lambda_p(\mathrm{cm})$$

short λ_p leads generally to small value of K !

intensity is low
tuning range is narrow

Insertion Device Radiation

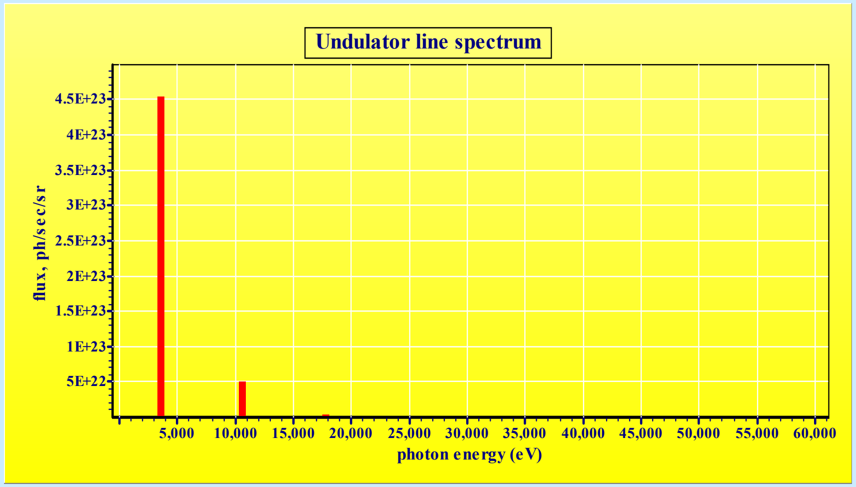


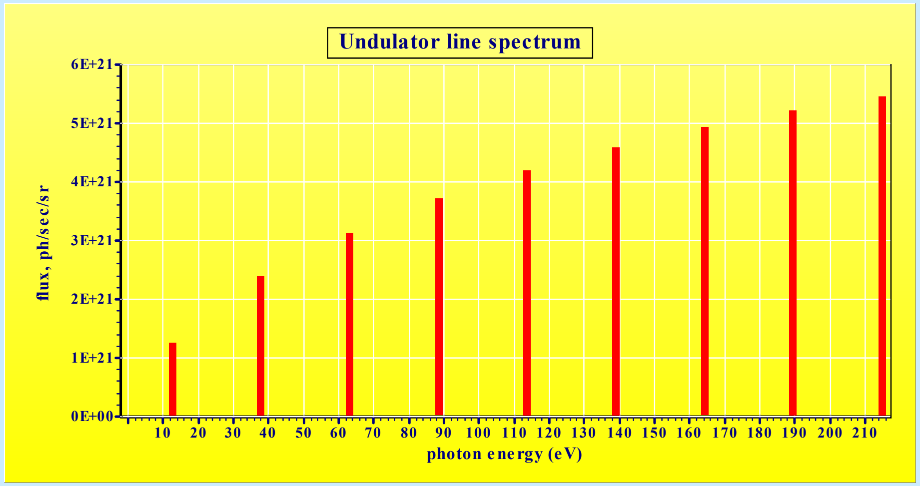
Insertion Device Radiation

tuning range:

$$\lambda = \frac{\lambda_p}{2\gamma^2}\left(1 + \frac{1}{2}K^2 + \gamma^2\theta_{obs}^2\right)$$

compare two undulators in SPEAR: λ_p= 20 mm and =80 mm

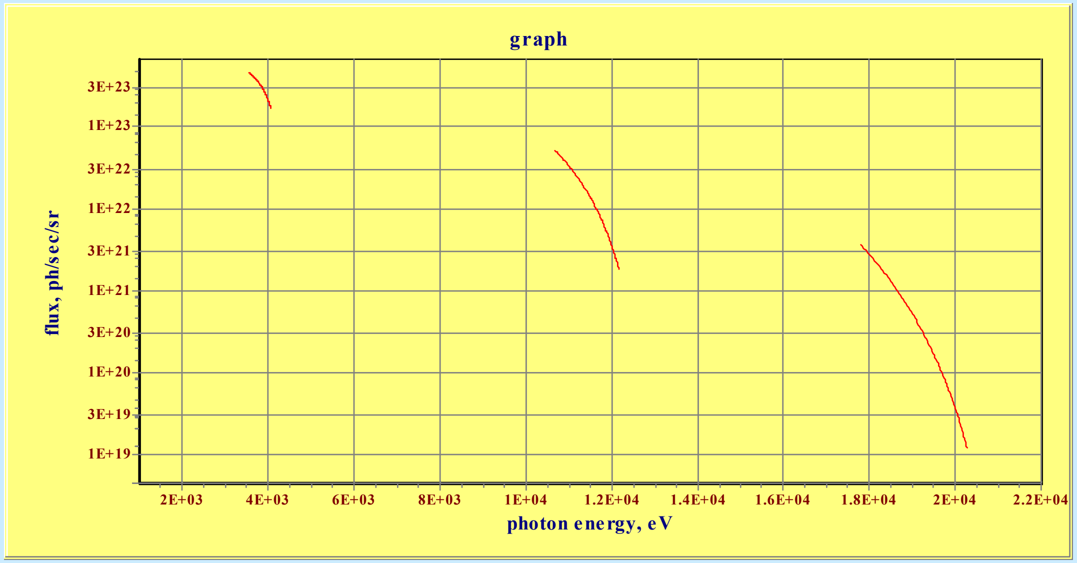




λ_p= 20 mm, 100 periods
0.33 < K < 0.63

λ_p= 80 mm, 25 periods
4.6 < K < 12.9

Insertion Device Radiation



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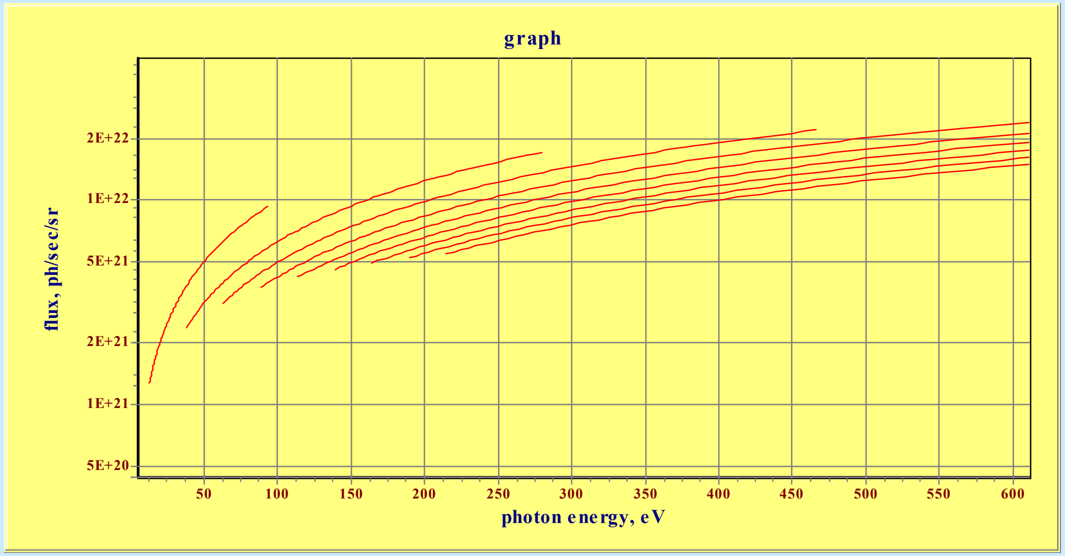
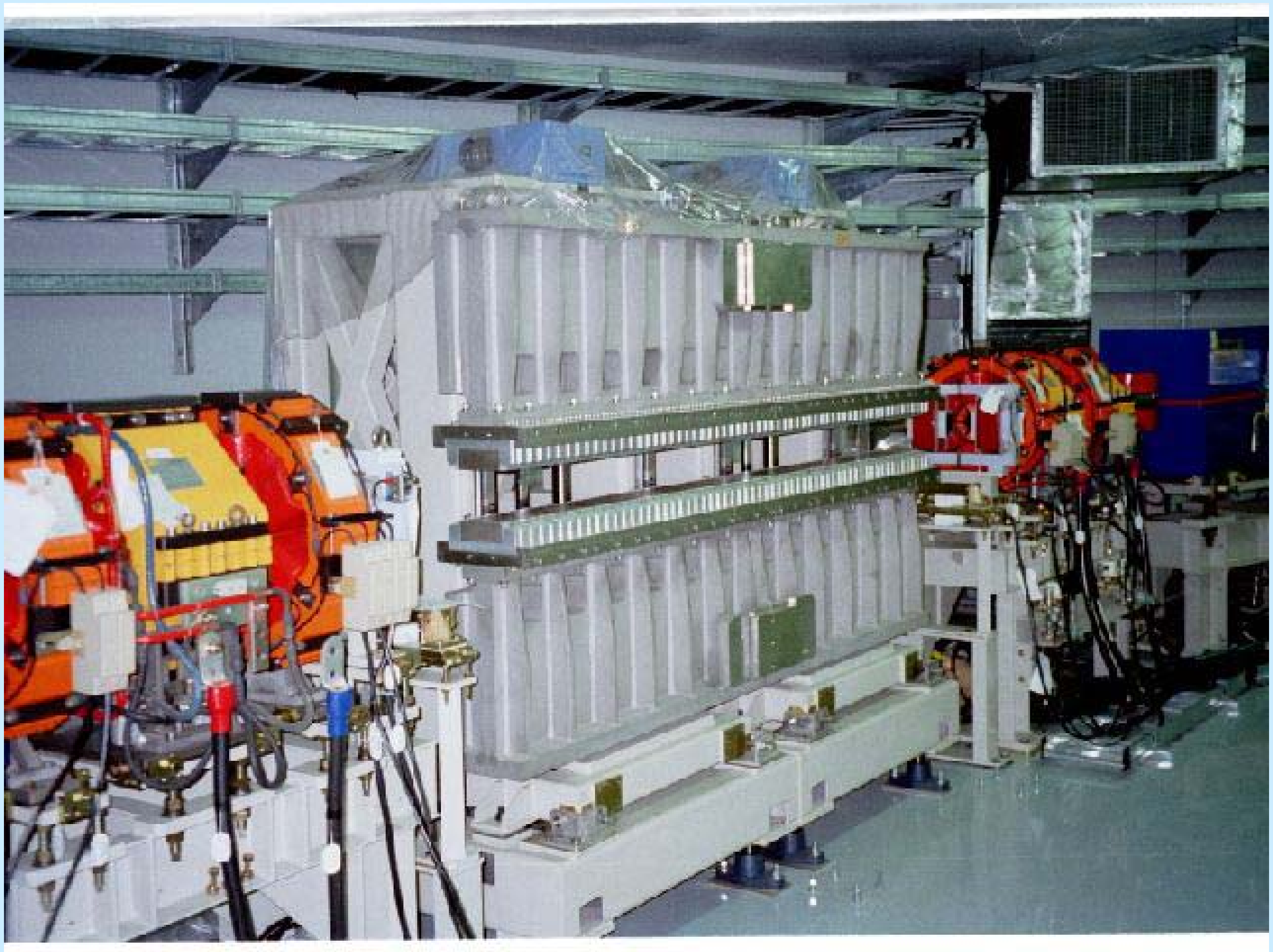


λ_p= 80 mm, 25 periods
4.6 < K < 12.9

Insertion Device Radiation



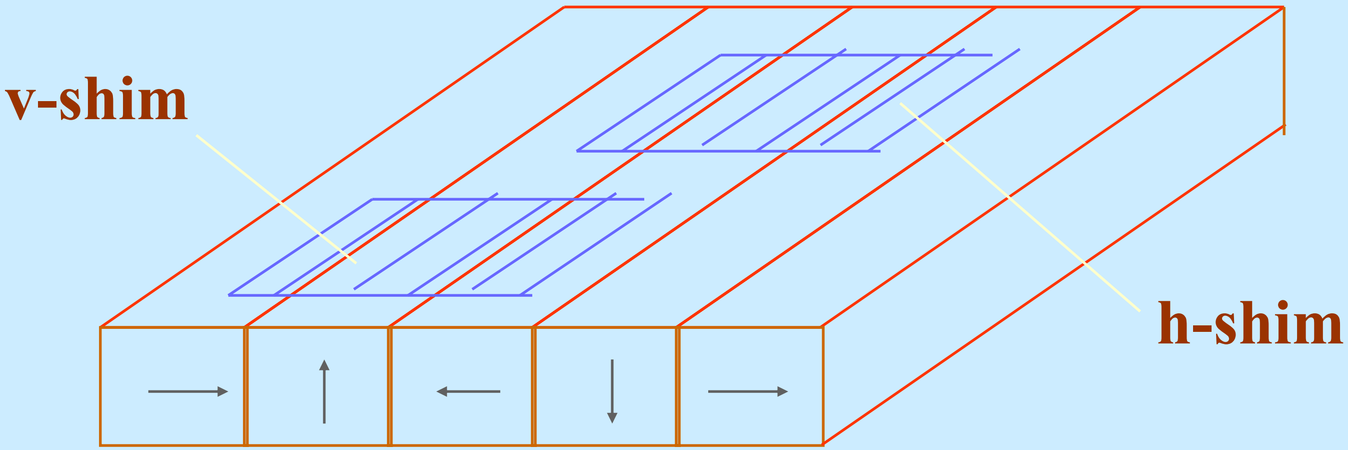
SUBARU : 2.3 m undulator, λ_p = 7.6 cm, 30 periods

SUBARU : 10.8 m undulator, λ_p = 5.4 cm, 200 periods

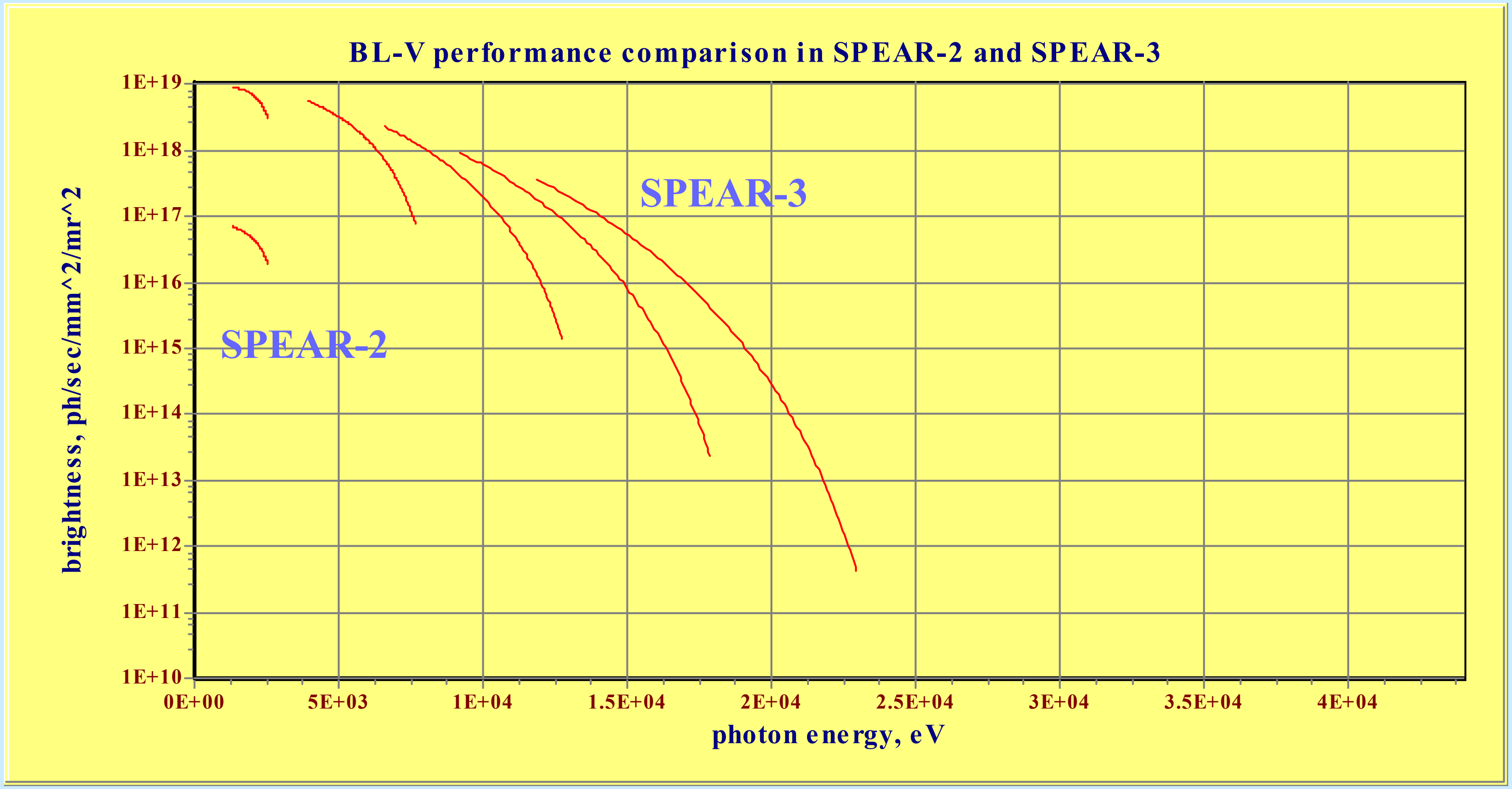
use harmonics to reach high photon energies
at low electron energy

requires high undulator precision:

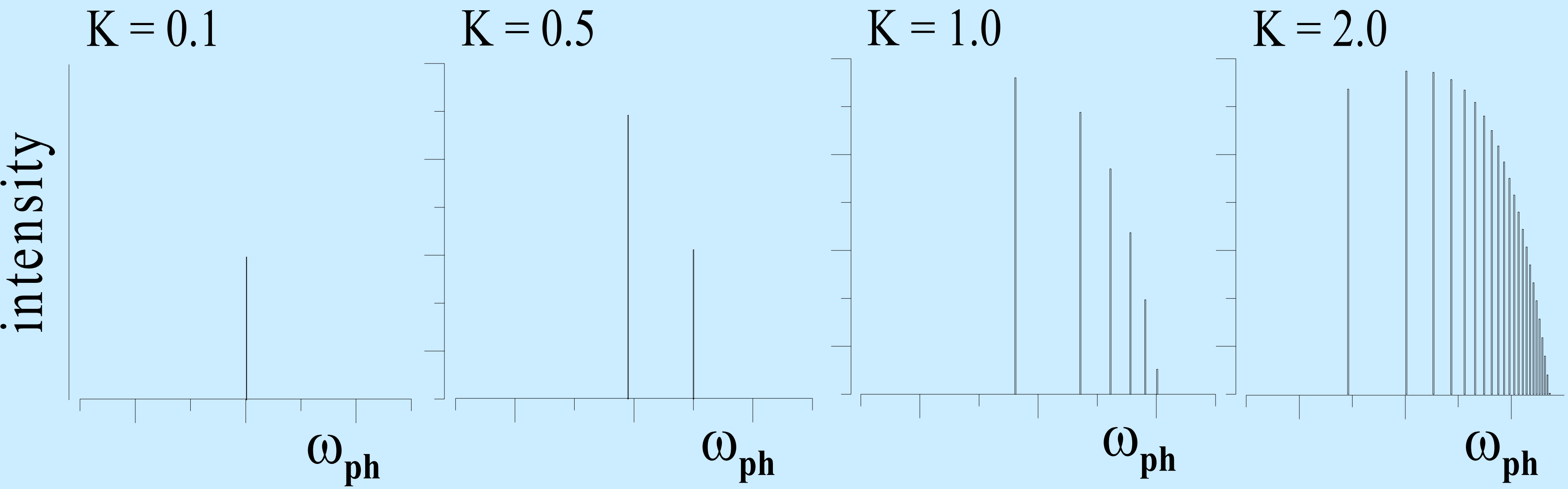
cannot build undulators that precise, but
we can fix them by shimming



Insertion Device Radiation



transition from undulator to wiggler radiation



crit. Photon energy from wiggler magnet at angle ψ with axis

$$\hbar\omega_c(\psi) = \hbar\omega_c(0)\sqrt{1-\left(\frac{\gamma\psi}{K}\right)^2}$$