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ICTP 40th Anniversary

SCHOOL ON SYNCHROTRON RADIATION AND APPLICATIONS
In memory of J.C. Fuggle & L. Fonda

19 April - 21 May 2004

Miramare - Trieste, Italy

1561/8

Introduction to Beamline Design

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Introduction to beamline design

This instruction material is based on the

Lecture notes by
W.B.Peatman, BESSY, Berlin - Germany
for the School on the Use of Synchrotron Radiation in Science and Technology
30 October - 1 December 1995, ICTP, Trieste – Italy

and on the book

W.B.Peatman, "Gratings, mirrors and slits: beamline design for soft X-ray synchrotron radiation sources", Gordon and Breach Science Publishers, Amsterdam, 1997

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review of the sources of synchrotron radiation

the brilliance

geometrical characteristics of the radiation from the different sources:
- bending magnet
- wiggler
- undulator

gratings

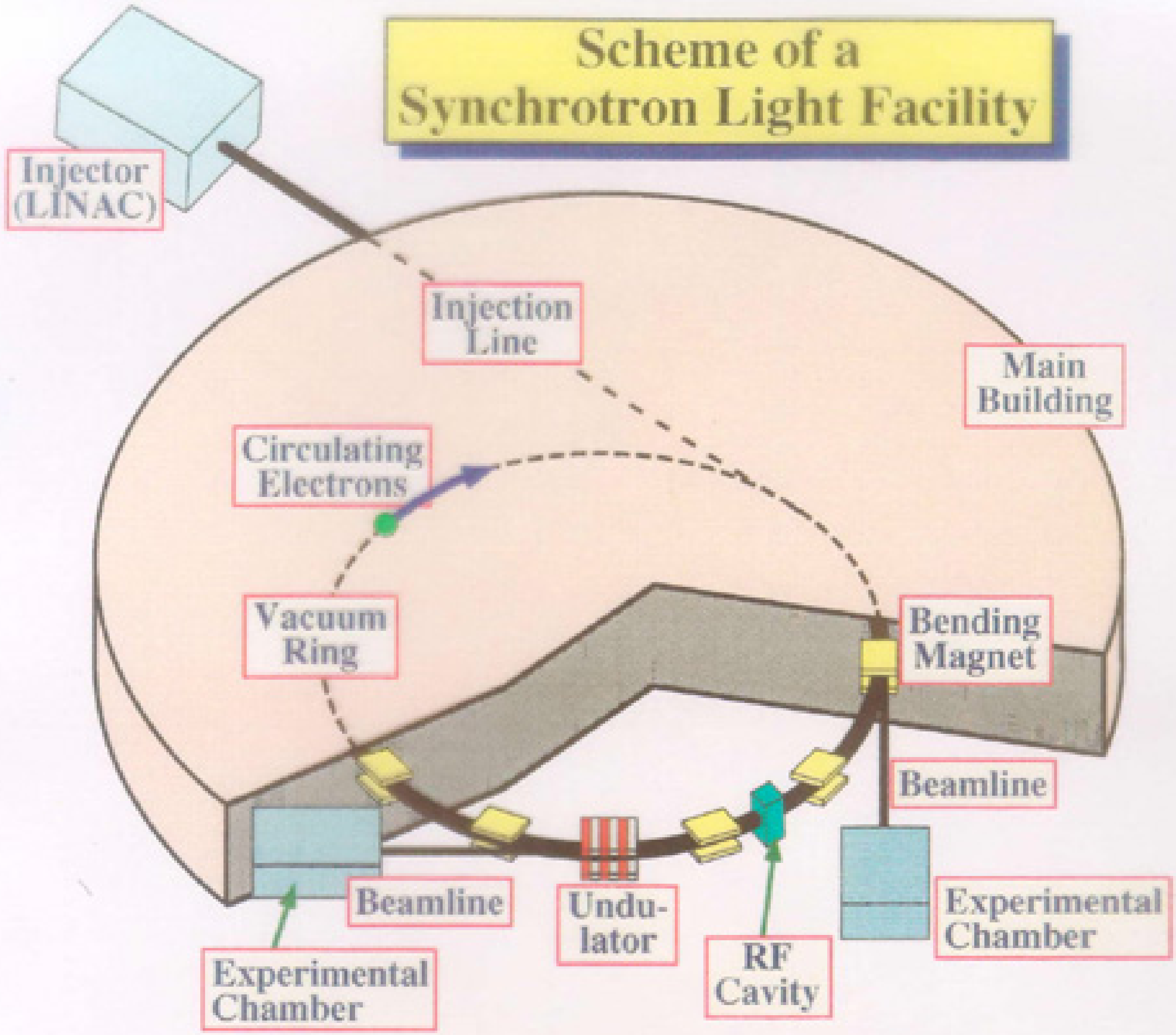
mirrors

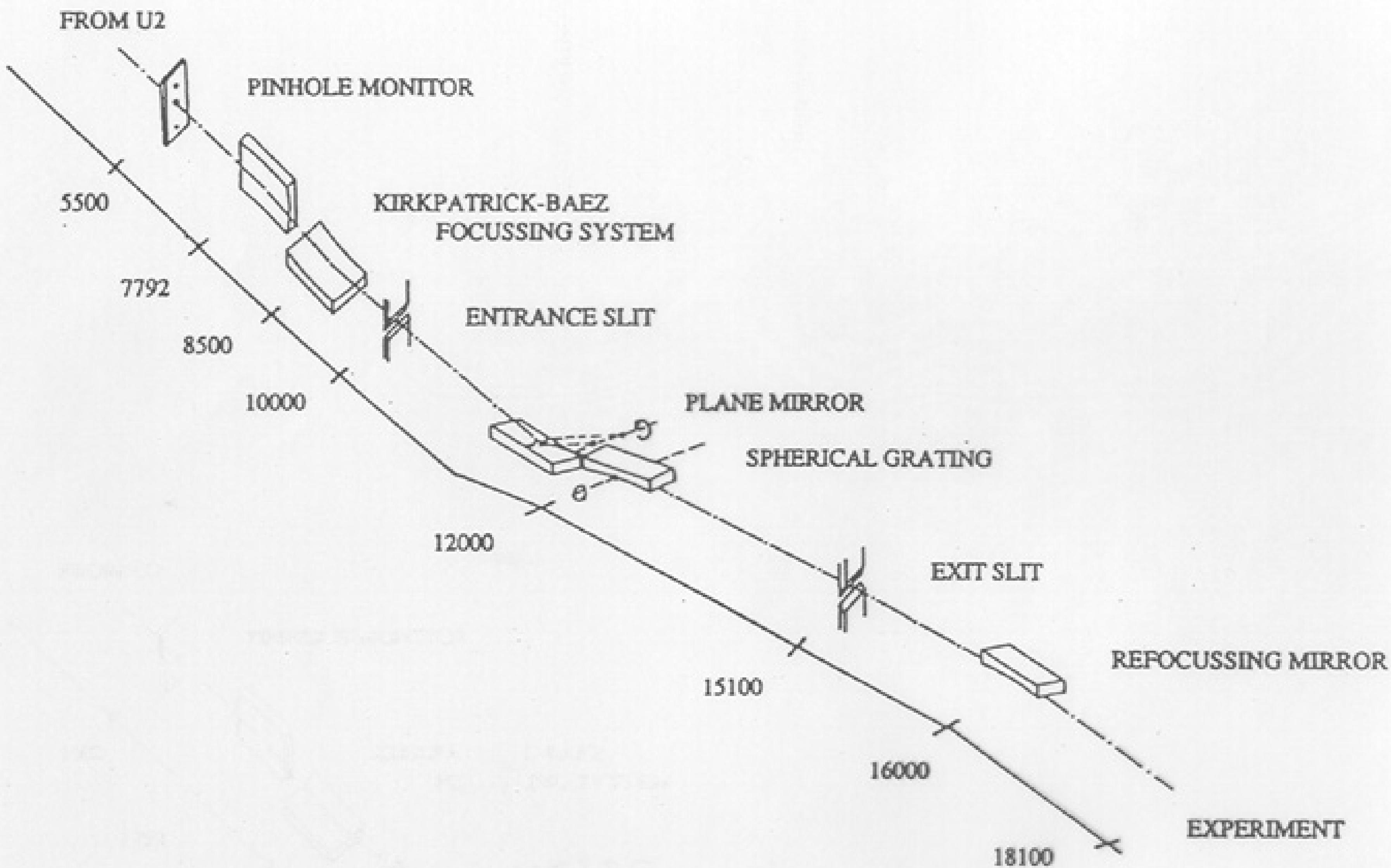
figure accuracy/ heat load on the optical elements

an example of focusing system

guidelines in designing a beamline

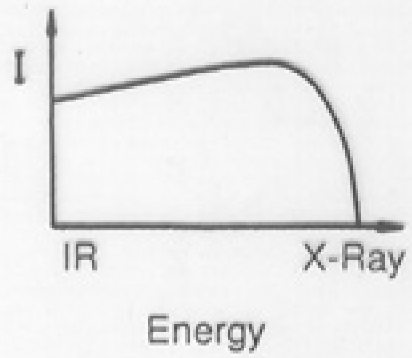
three examples of monochromator/beamlines


Scheme of a
Synchrotron Light Facility
Injector
(LINAC)
Injection
Line
Main
Building
Circulating
Electrons
Vacuum
Ring
Bending
Magnet
Beamline
Beamline
Undu-
lator
RF
Cavity
Experimental
Chamber
Experimental
Chamber

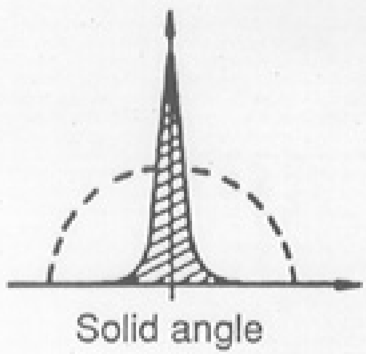


FROM U2
PINHOLE MONITOR
KIRKPATRICK-BAEZ
FOCUSSING SYSTEM
ENTRANCE SLIT
PLANE MIRROR
SPHERICAL GRATING
EXIT SLIT
REFOCUSSING MIRROR
EXPERIMENT
5500
7792
8500
10000
12000
15100
16000
18100

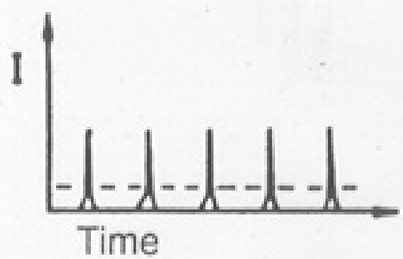
Important Characteristics of Synchrotron Radiation

1) 

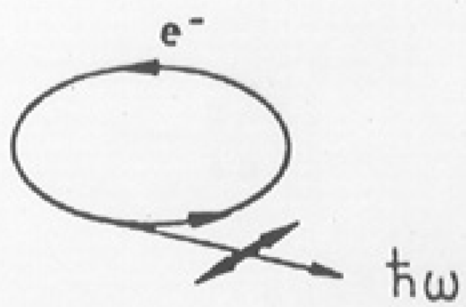
Continuous spectrum

2) 

Emission in small solid angle

3) 

Pulsed time structure

4) 

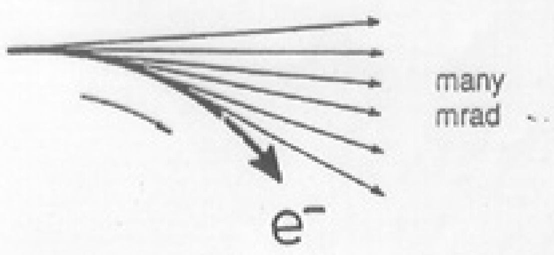
High degree of polarisation

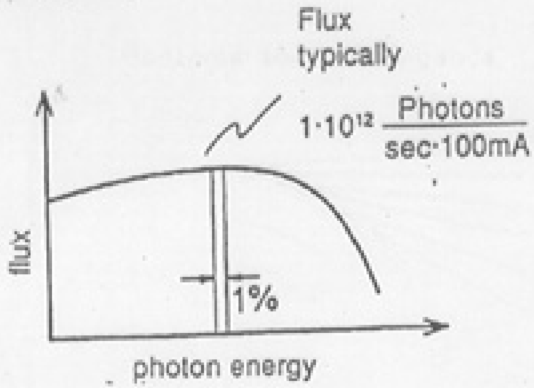
5) Properties can be calculated/predicted

Synchrotron Radiation Sources

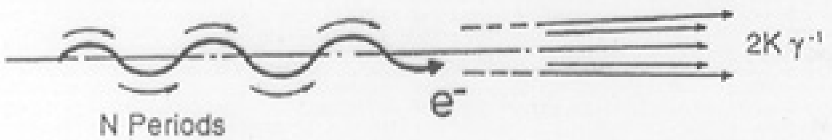
Dipole Magnet

Horizontal source divergence





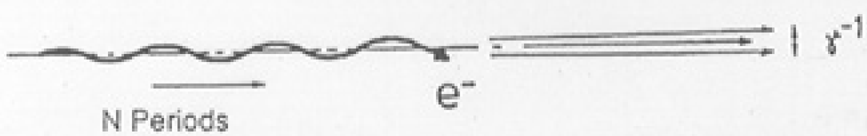
Wiggler



SR-collimated

Intensity N times that of a dipole source with the same number of mrad

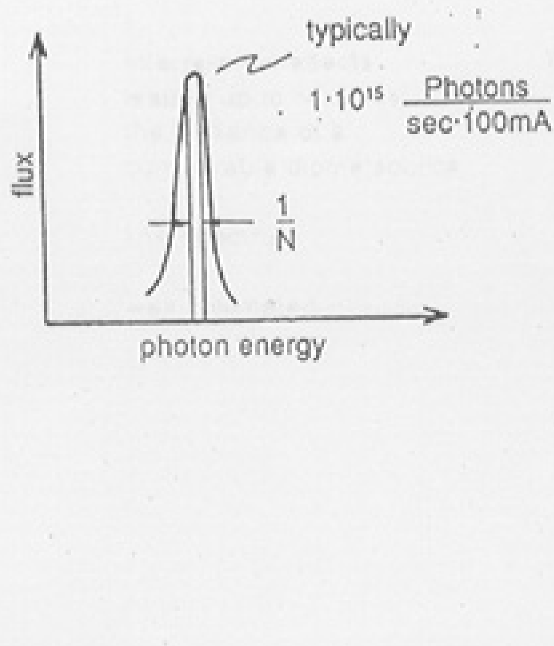
Undulator



Interference effects lead to up to N^2 times the brilliance of a comparable dipole source

line spectrum

well collimated



Flux = photons/sec

$$\text{Brilliance (or Brightness)} = \frac{\text{Flux}}{I}\;\frac{1}{\sigma_x \sigma_y \sigma'_x \sigma'_y \,\text{BW}}$$

I = electron current in the storage ring

$\sigma_x\sigma_y$ = the area from which the SR is emitted

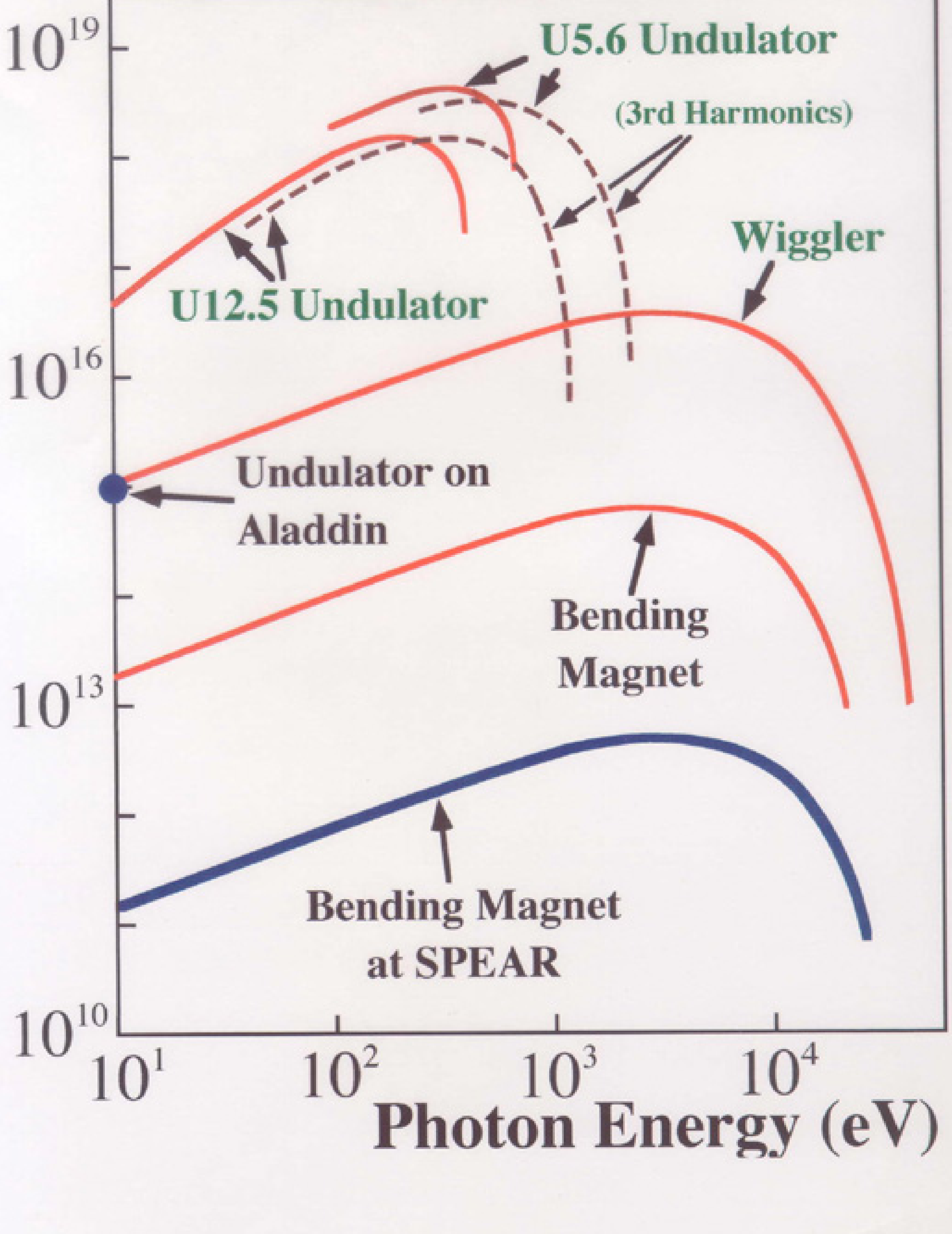
$\sigma'_x\sigma'_y$ = the solid angle into which the SR is emitted

BW = photon energy bandwidth

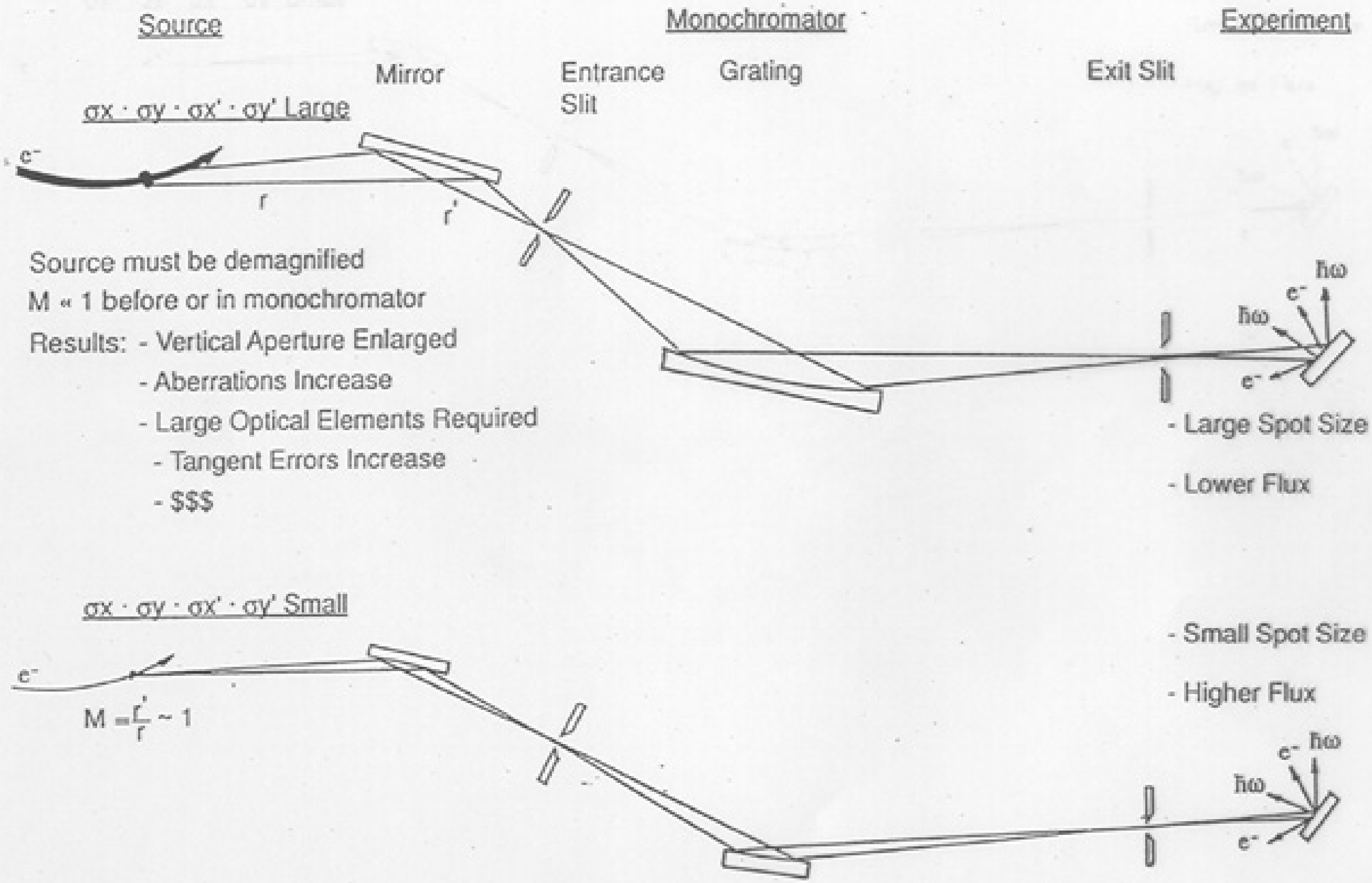
units = photons/sec mm^2 $mrad^2$ 0.1%BW
for given ring current I (typically 100 mA)

ELETTRA's Brightness Curves,

in Photons per second, per mrad2, per mm2, and for 0.1% bandwidth



The Practical Meaning of Brilliance



The electron beam

emittance = $\varepsilon = \sigma_e \sigma'_e$ = constant

$\varepsilon_y = C\,\varepsilon_x$

C = coupling factor (constant), typically 1-10 %

case of ELETTRA

horizontal emittance ε_x = 7.0 nm·rad

coupling factor = 1%

beam dimensions in the straight sections (horizontal/vertical) in µm	241/15
beam divergence in the straight sections (horizontal/vertical) in µrad	29/6

The radiation emission process

effective size σ_r
opening angle σ'_r

Total

$$\sigma_t = \sqrt{s_e^2 + s_r^2}$$

$$\sigma'_t = \sqrt{s'^2_e + s'^2_r}$$

Dipole magnet

effective size

$\sigma_t = \sigma_e$

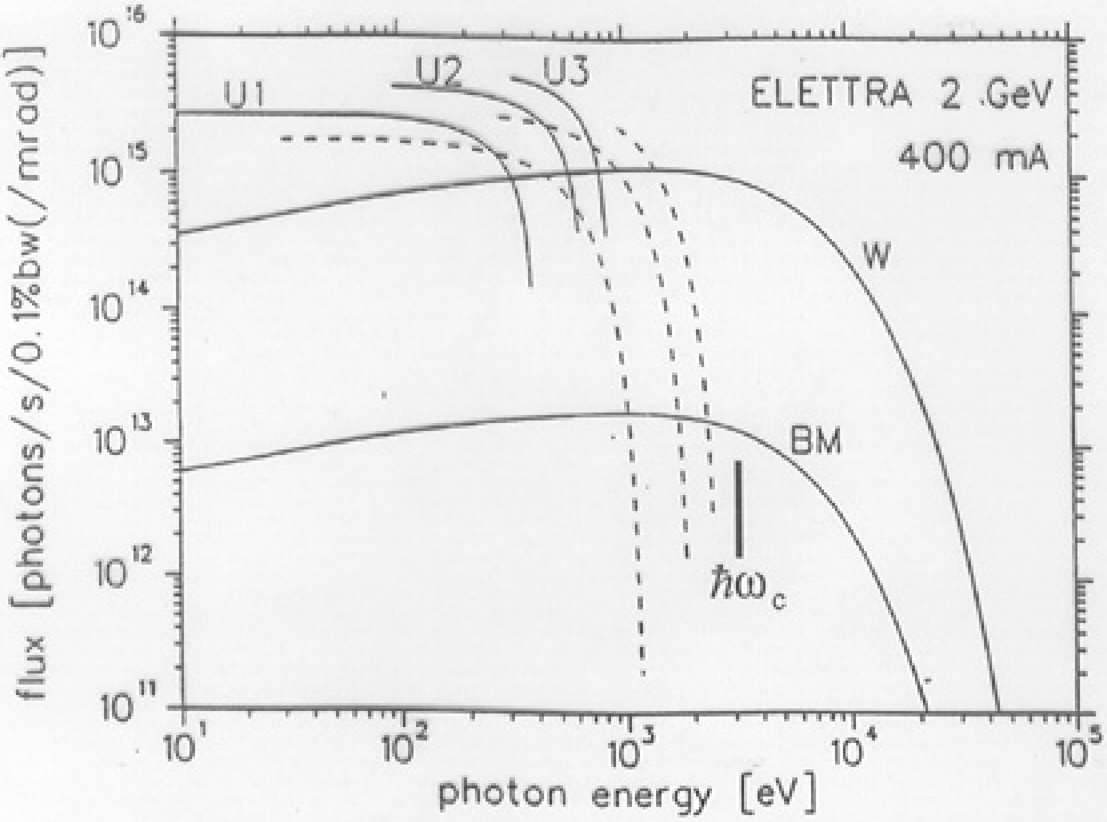
vertical opening angle

$$\sigma'_{rv}\ (\text{rad}) = 0.57 \left(\frac{\lambda}{\lambda_c}\right)^{0.43} \frac{1}{\gamma} \qquad \text{for } 0.2 < \frac{\lambda}{\lambda_c} < 100$$

λ_c = critical wavelength
$\hbar\omega_c$ = critical energy
$\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$ = relativistic factor

$\gamma = 1957\ E$
$\hbar\omega_c = 2218\ \dfrac{E^3}{\rho} = 667\ B\ E^2$

E = energy of the electrons in GeV
ρ = radius of curvature of the electrons in the dipole magnets in meters
B = magnetic field amplitude in Tesla
$\hbar\omega_c$ = critical energy in eV



Spectral flux of radiation sources in the storage ring at 2 GeV.

case of ELETTRA

E = 2 GeV
ρ = 5.5 m
$\hbar\omega_c \approx 3.2$ KeV

$\gamma = 3914$

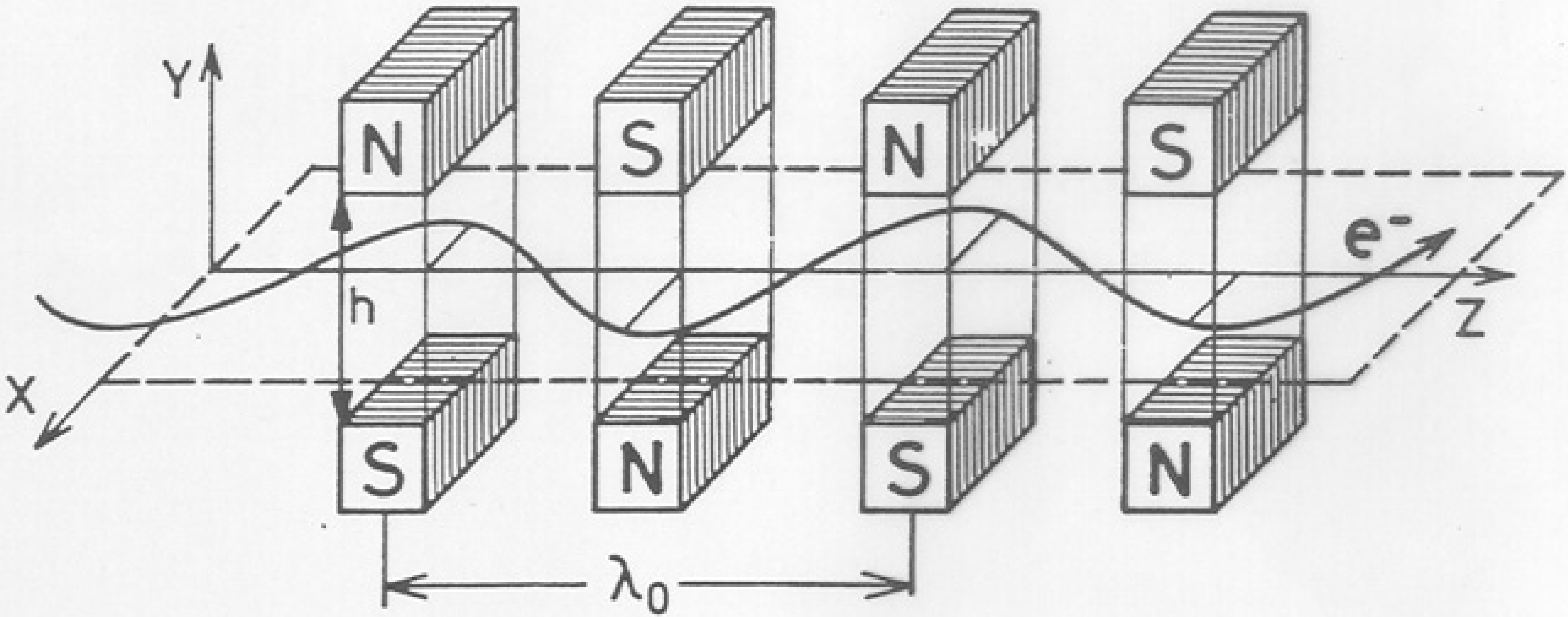
$$\sigma'_{rv}\,(\text{rad}) = 0.57\left(\frac{\lambda}{\lambda_c}\right)^{0.43}\cdot 255\ \mu\text{rad}$$

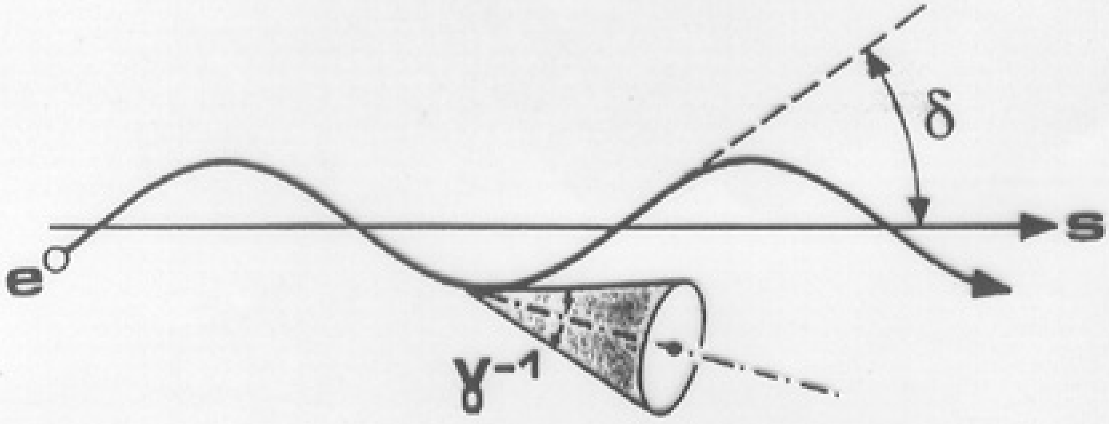
horizontal opening angle

defined by the geometry of the front end

at ELETTRA: 6 or 7 mrad

Layout of a Wiggler, Undulator





magnetic field strength parameter K

$$K = \frac{e\lambda_o B_o}{2\pi mc} = 0.934\ \lambda_o B_o$$

λ_o = length of the undulator period in cm
B_o = magnetic field amplitude in Tesla

$$K = \delta\gamma$$

at $K = 1$ is $\delta = \gamma^{-1}$

Wiggler

$K >> 1$

vertical opening angle

$\sigma'_{rv} = \sigma'_{rv}$ (dipole)

horizontal opening angle

$\sigma'_{rh} = \delta/2 = K/2\gamma$

vertical source size

$$\sigma_{rv} = \left(\frac{\sigma'^2_{ev}}{3} + \frac{\Delta\theta^2}{9} \right)^{1/2} \frac{L}{2}$$

horizontal source size

$$\sigma_{rh} = \left[\left(\frac{K}{\gamma} \frac{\lambda_o}{2\pi} \right)^2 + \left(\frac{\sigma'^2_{eh}}{3} + \frac{\Delta\theta^2}{9} \right) \left(\frac{L}{2} \right)^2 \right]^{1/2}$$

where $\Delta\theta$ is the half opening angle of the optical system

Wiggler W14.0 of ELETTRA at 2 GeV

L = 4.5 m
λ_o = 140 mm
B = 1.5 T, K = 19.6
$2\Delta\theta$ = 1.5 mrad H × 0.28 mrad V

$$x_o = \frac{K}{\gamma}\frac{\lambda_o}{2\pi} = 112\ \mu m$$

separation between the two "eyes"of the wiggler

$\hbar\omega_c$ = 4.0 KeV

$$\sigma'_{rv}\ (rad) = 0.57\left(\frac{\lambda}{\lambda_c}\right)^{0.43}\frac{1}{\gamma} = 108\ \mu rad\ (at\ \hbar\omega = 8\ KeV)$$

σ'_{rh} = 2500 μrad

σ_{rv} = 105 μm

σ_{rh} = 575 μm

Undulator

0 < K = 2-3

$$\lambda = \frac{\lambda_o}{2\gamma^2 k}\left(1+\frac{K^2}{2}+\gamma^2\theta^2\right)$$

$$\frac{\Delta\lambda}{\lambda}=\frac{1}{kN} \text{ or } \frac{\Delta\lambda}{\lambda}=\frac{1}{2kN}$$

k = number of the harmonic
N = number of periods of undulator
$N\lambda_o$ = L = total length of undulator

σ'_r = width of the central radiation cone

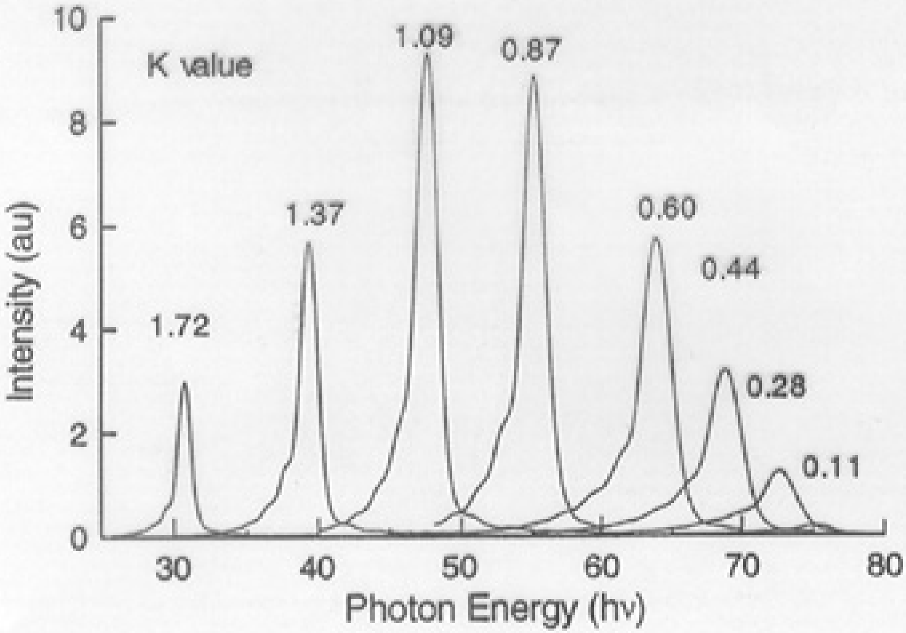
$$\frac{\Delta\lambda}{\lambda}=\frac{1}{kN} \qquad \sigma'_r = \frac{\sqrt{1+\frac{K^2}{2}}}{\gamma}\frac{1}{\sqrt{Nk}}$$

$$\frac{\Delta\lambda}{\lambda}=\frac{1}{2kN} \qquad \sigma'_r = \sqrt{\frac{\lambda}{L}}$$

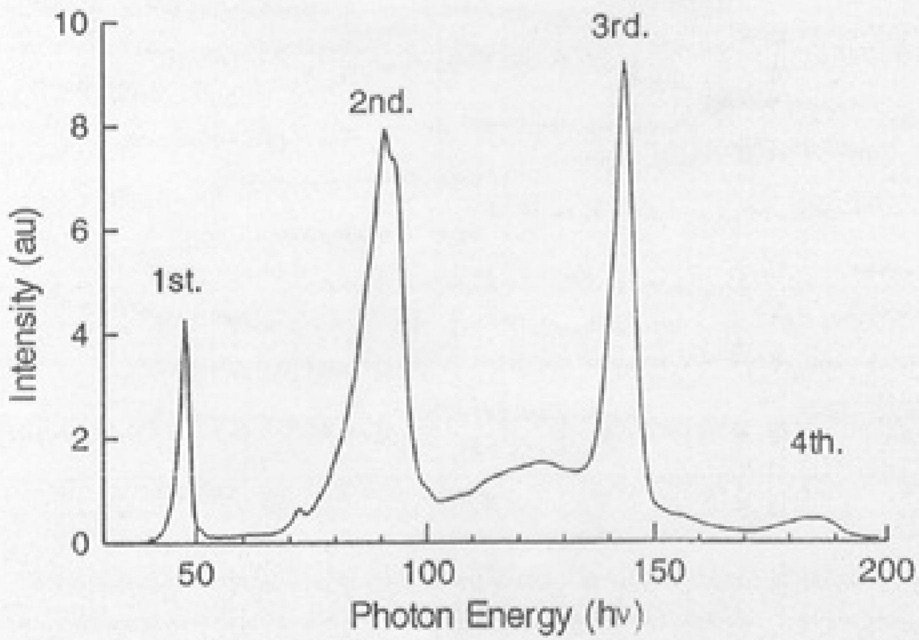
Typical Undulator Spectra

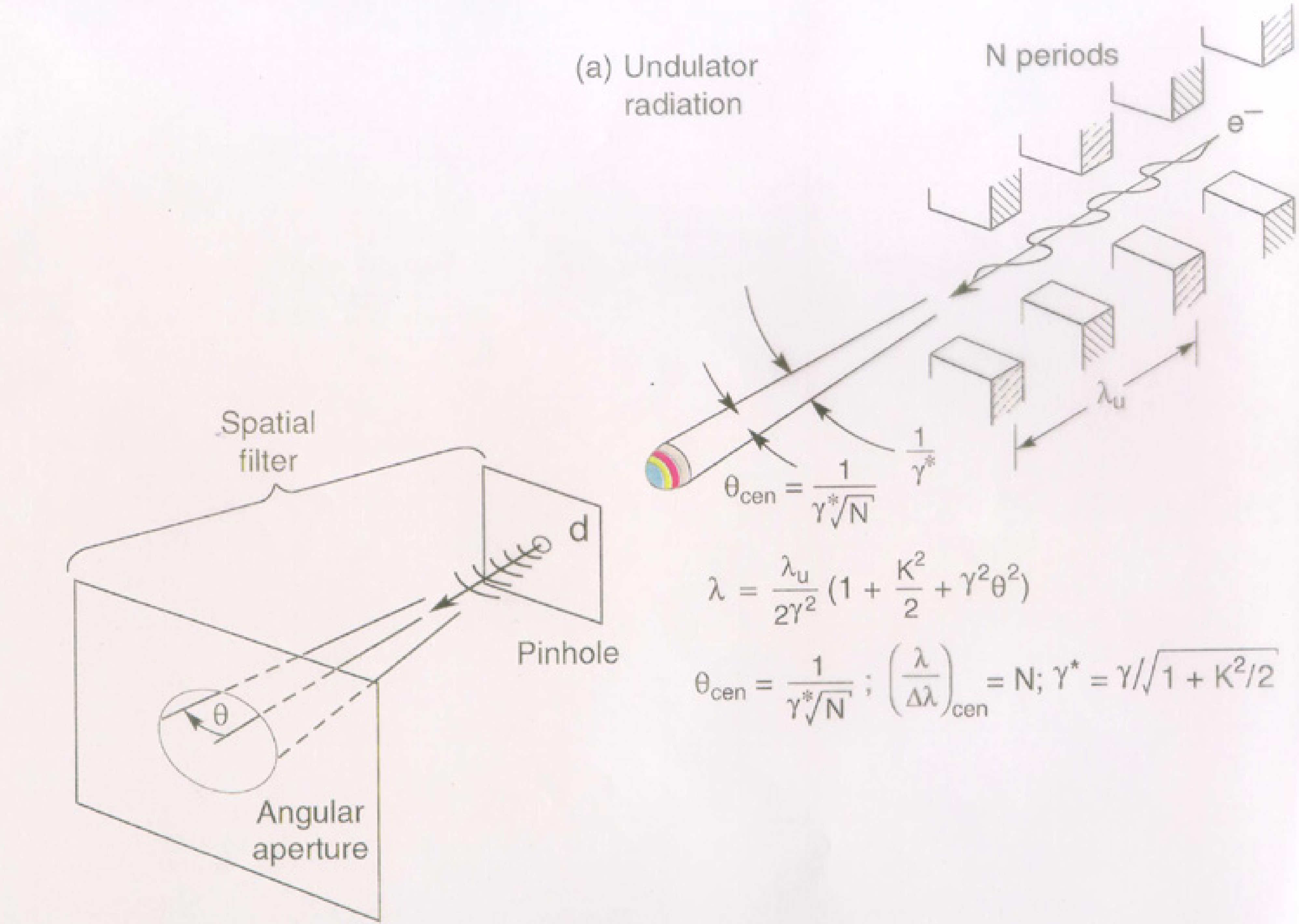
Typical undulator spectra: λ_o = 70 mm, N = 35 and E_{el} = 800 MeV. The envelope of the spectra is determined mainly by the transmission of the monochromator used to record them [2.7].

a. Spectra of the first harmonic for different values of K. The acceptance of the monochromator was 0.13 mrad.



b. The first three harmonics for K = 1.09. The acceptance of the monochromator was 0.13 mrad.





(a) Undulator radiation
N periods
e^-
λ_u
$\frac{1}{\gamma^*}$
$\theta_{cen} = \frac{1}{\gamma^* \sqrt{N}}$
Spatial filter
d
Pinhole
θ
Angular aperture
$\lambda = \frac{\lambda_u}{2\gamma^2}(1 + \frac{K^2}{2} + \gamma^2\theta^2)$
$\theta_{cen} = \frac{1}{\gamma^* \sqrt{N}}$; $\left(\frac{\lambda}{\Delta\lambda}\right)_{cen} = N$; $\gamma^* = \gamma / \sqrt{1 + K^2/2}$

$$\sigma'_r = \sqrt{\frac{\lambda}{L}}$$

diffraction limit case: $\sigma_r \cdot \sigma'_r = \lambda/4\pi$

$$\sigma_r = \frac{1}{4\pi} \sqrt{\lambda L}$$

case of ELETTRA

L = 4.5 m

es. $\hbar\omega$ = 100 eV, that is λ = 12.4 nm

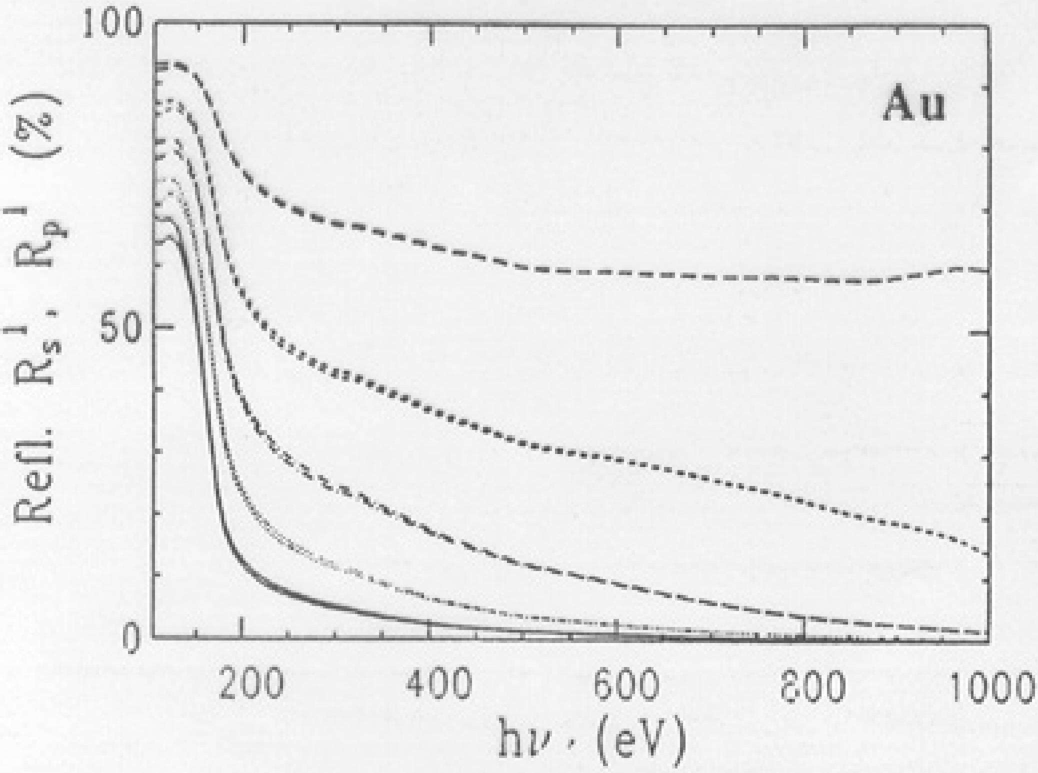
σ'_r = 52 μrad

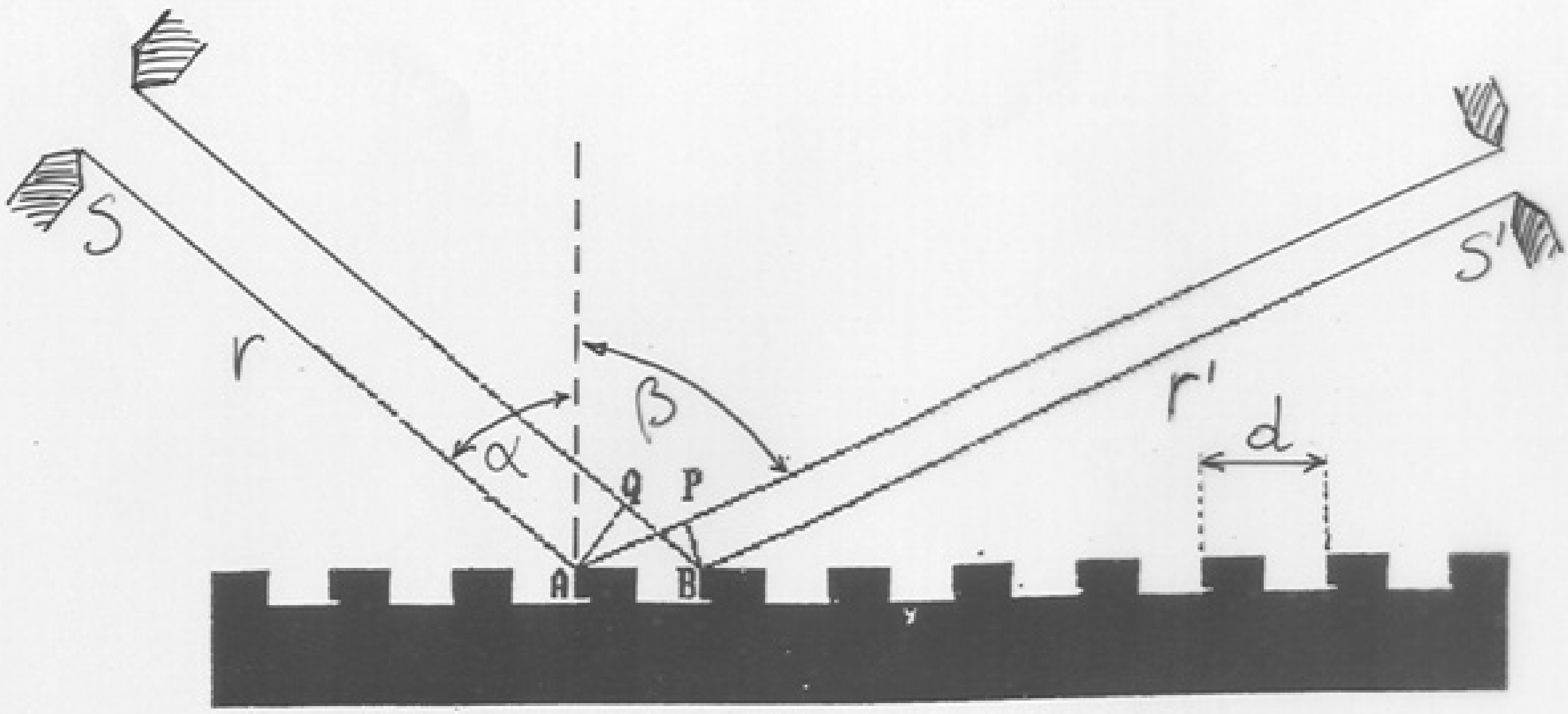
σ_r = 19 μm

Constraints in designing the optical system

- in vacuum operation
- only reflecting optics available
  grazing incidence, limited reflectivity
- no lenses, no prisms
  no multielement optical groups
  for an aberration corrected system

Reflectivity of Au at angles of incidence 80°, 82°, 84°, 86°, 88°





$$d\,(\sin\alpha + \sin\beta) = k\,\lambda$$

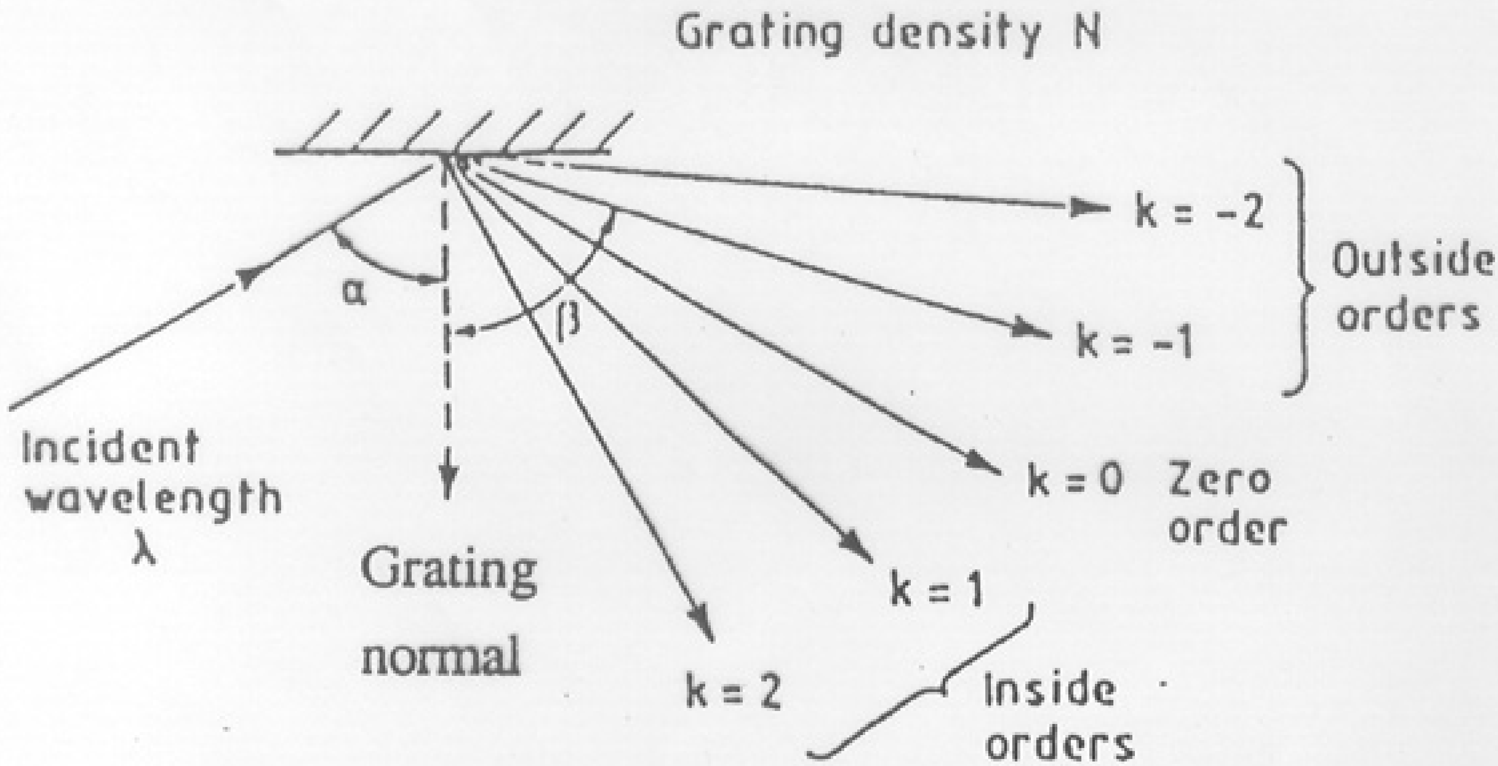
$$k\,N\,\lambda = \sin\alpha + \sin\beta$$

k = order of diffraction: 0, ±1, ±2, etc.
$N = 1/d$ = line density

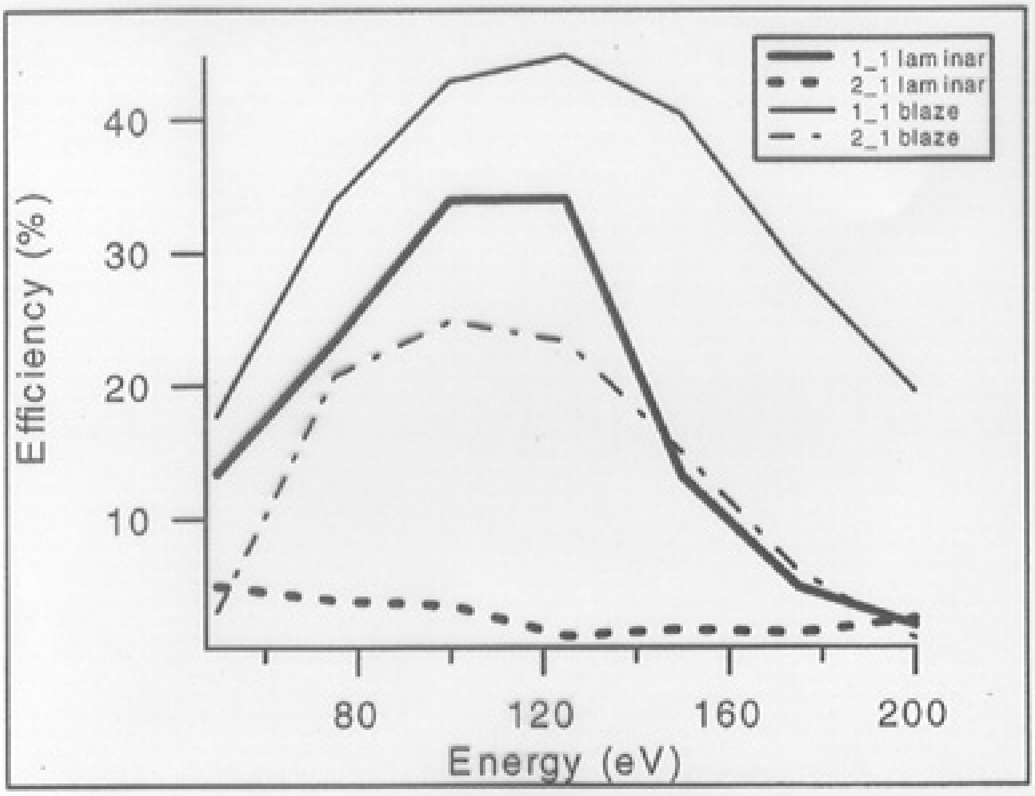
Slit Limited Resolution

Entrance: $\Delta\lambda_{ent} = \frac{1}{Nk}\frac{s}{r}\cos\alpha$

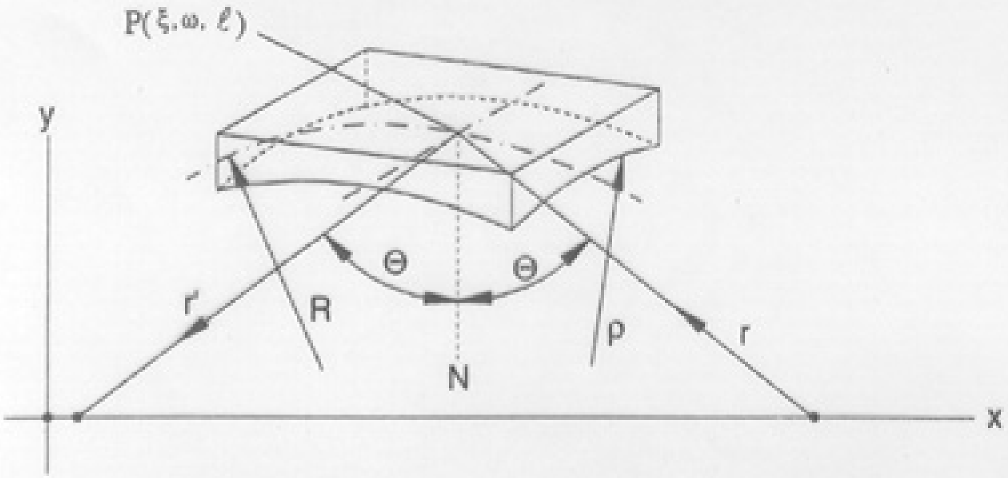
Exit: $\Delta\lambda_{exit} = \frac{1}{Nk}\frac{s'}{r'}\cos\beta$


Grating density N
k = -2
k = -1
Outside orders
α
β
k = 0
Zero order
Incident wavelength λ
Grating normal
k = 1
k = 2
Inside orders

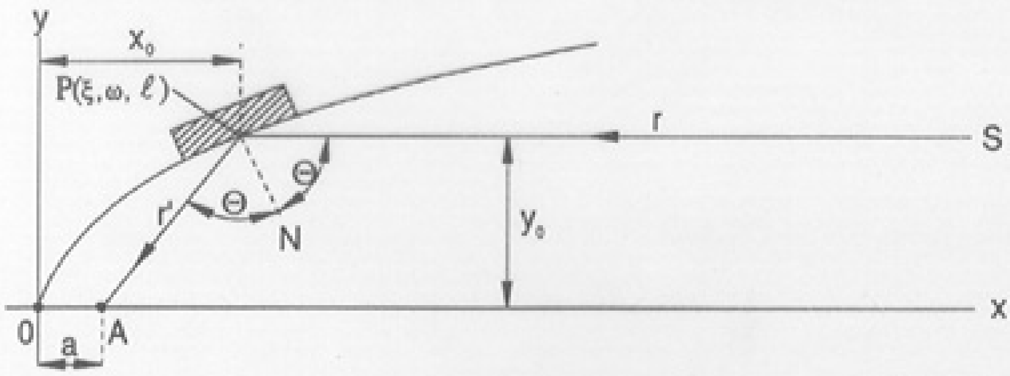
Example: the low energy grating of the BACH beamline at ELETTRA



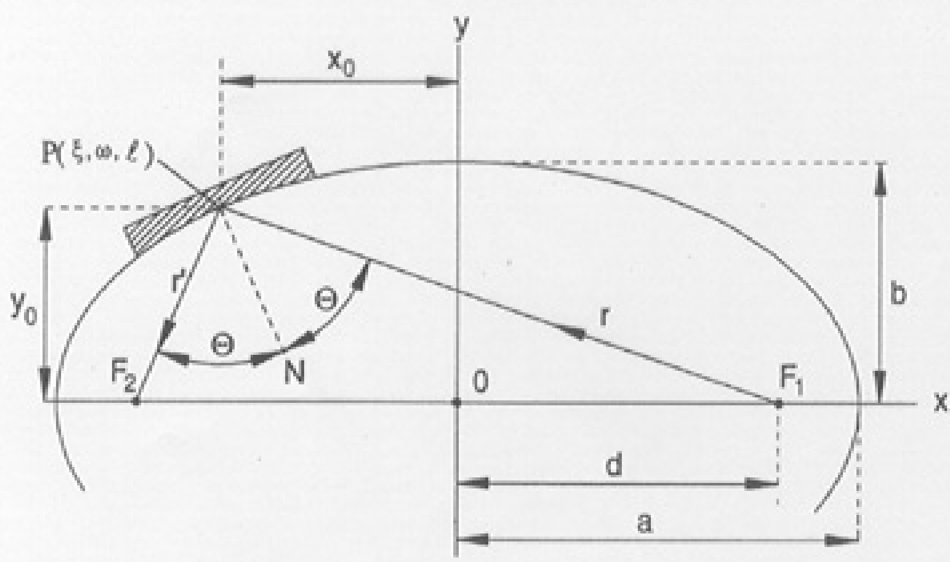
a. Toroid (ρ < R) or sphere (ρ = R)

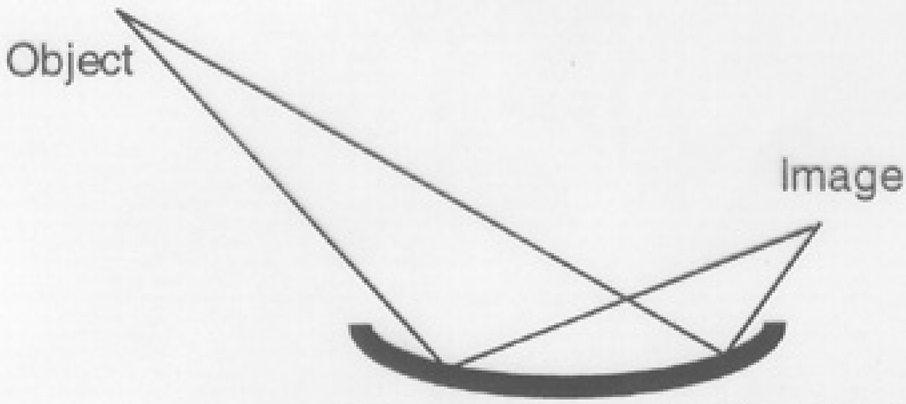


b. Parabola

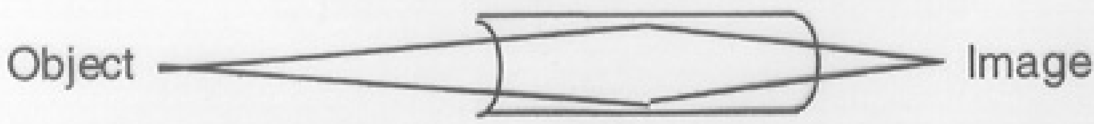


c. Ellipse





meridional or tangential focusing



sagittal focusing

toroid ($\rho < R$) or sphere ($\rho = R$)

meridional focusing $$\left(\frac{1}{r}+\frac{1}{r'}\right)\frac{\cos\theta}{2}=\frac{1}{R} \qquad f=\frac{R\cos\theta}{2}$$

sagittal focusing $$\left(\frac{1}{r}+\frac{1}{r'}\right)\frac{1}{2\cos\theta}=\frac{1}{\rho} \qquad f=\frac{\rho}{2\cos\theta}$$

for $R = \rho$ and $\theta = 87°$ is
$f_s \approx 365\ f_m$

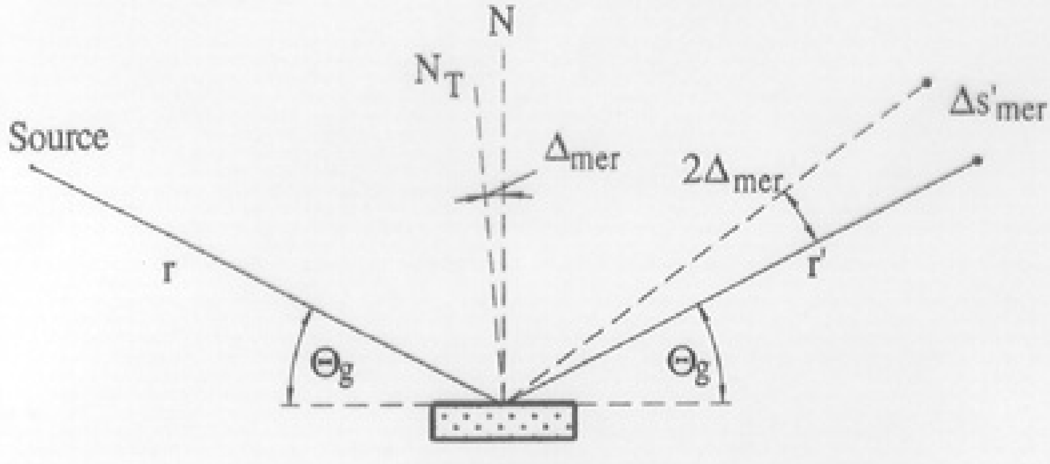
Typical achievable figure accuracy

Surface shape	**Accuracy (arcsec. RMS)**
Flat	<0.1
Spherical	<0.1
Cylindrical	<0.5
Toroidal	<0.5
Elliptical Toroid	<0.5
Paraboloid	<1.0
Ellipsoid	<1.0

remember: 1 sec $\approx$ 5 μrad

Meridional and Sagittal Tangent Errors

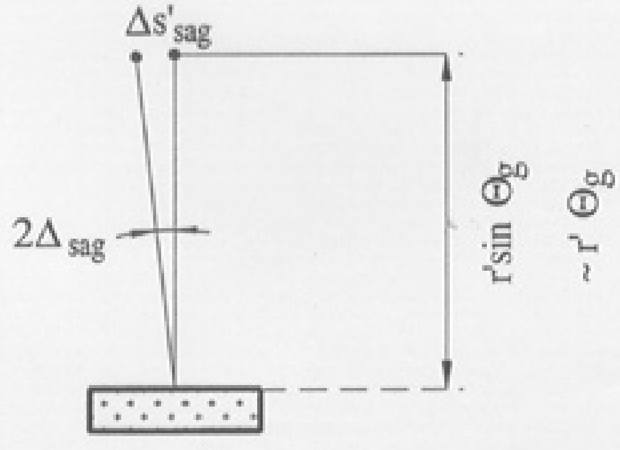
a. Meridional tangent error: Δ_{mer}



Side View

Thus, $\Delta s'_{mer} = 2r'\Delta_{mer}$

b. Sagittal tangent error: Δ_{sag} see figure above



End-on View

$$\Delta s'_{sag} = 2r'\sin\theta_g \cdot \Delta_{sag}$$

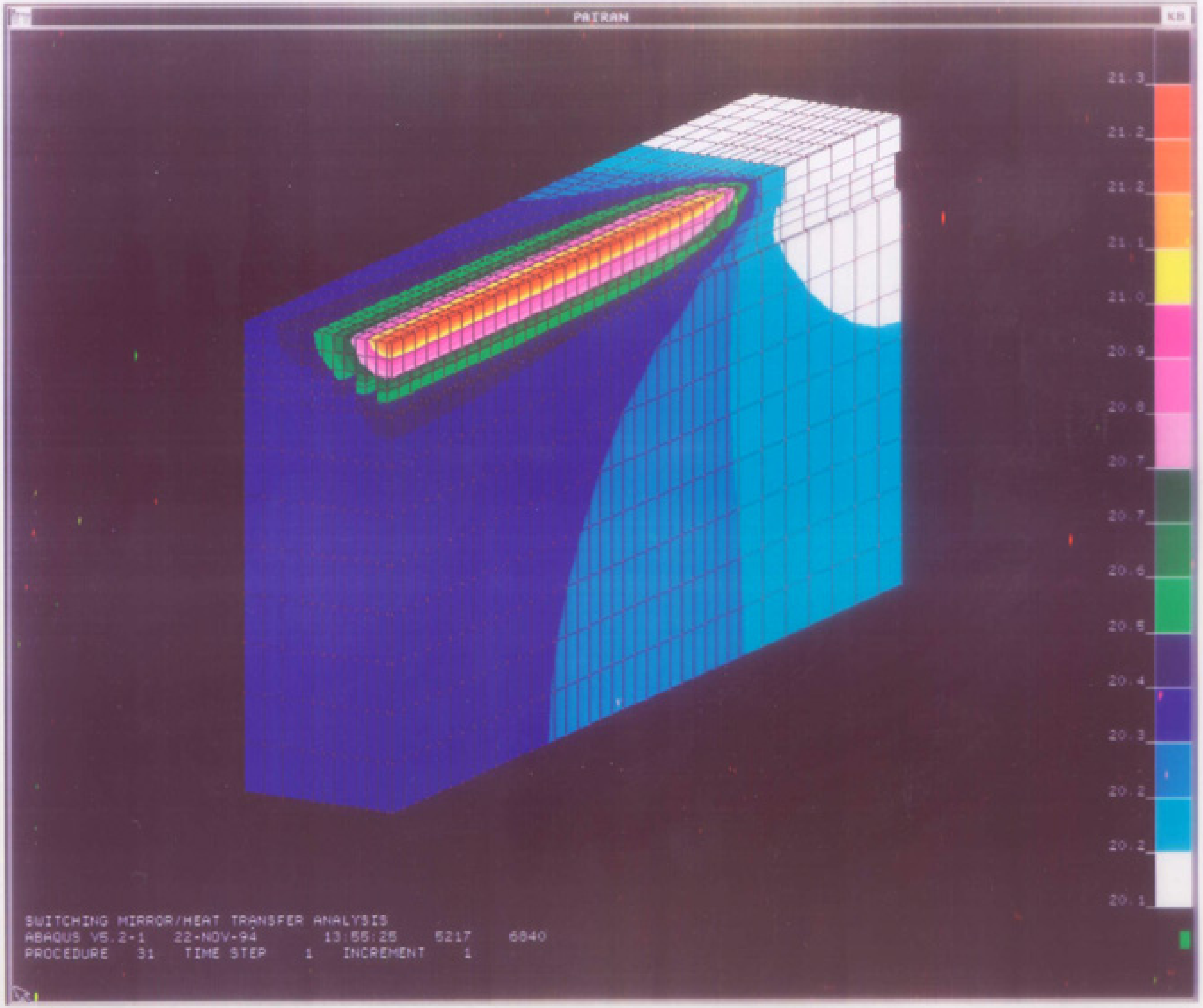
$$\approx 2r'\theta_g \cdot \Delta_{sag}\ .$$

Heat load on the optical elements: cooling is needed

example:
first mirror of the SuperESCA beamline at ELETTRA
source: U5.6 undulator

max absorbed power: 75 W

max induced slope error: 2 arcsec


PATRAN
21.3
21.2
21.2
21.1
21.0
20.9
20.8
20.7
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20.4
20.3
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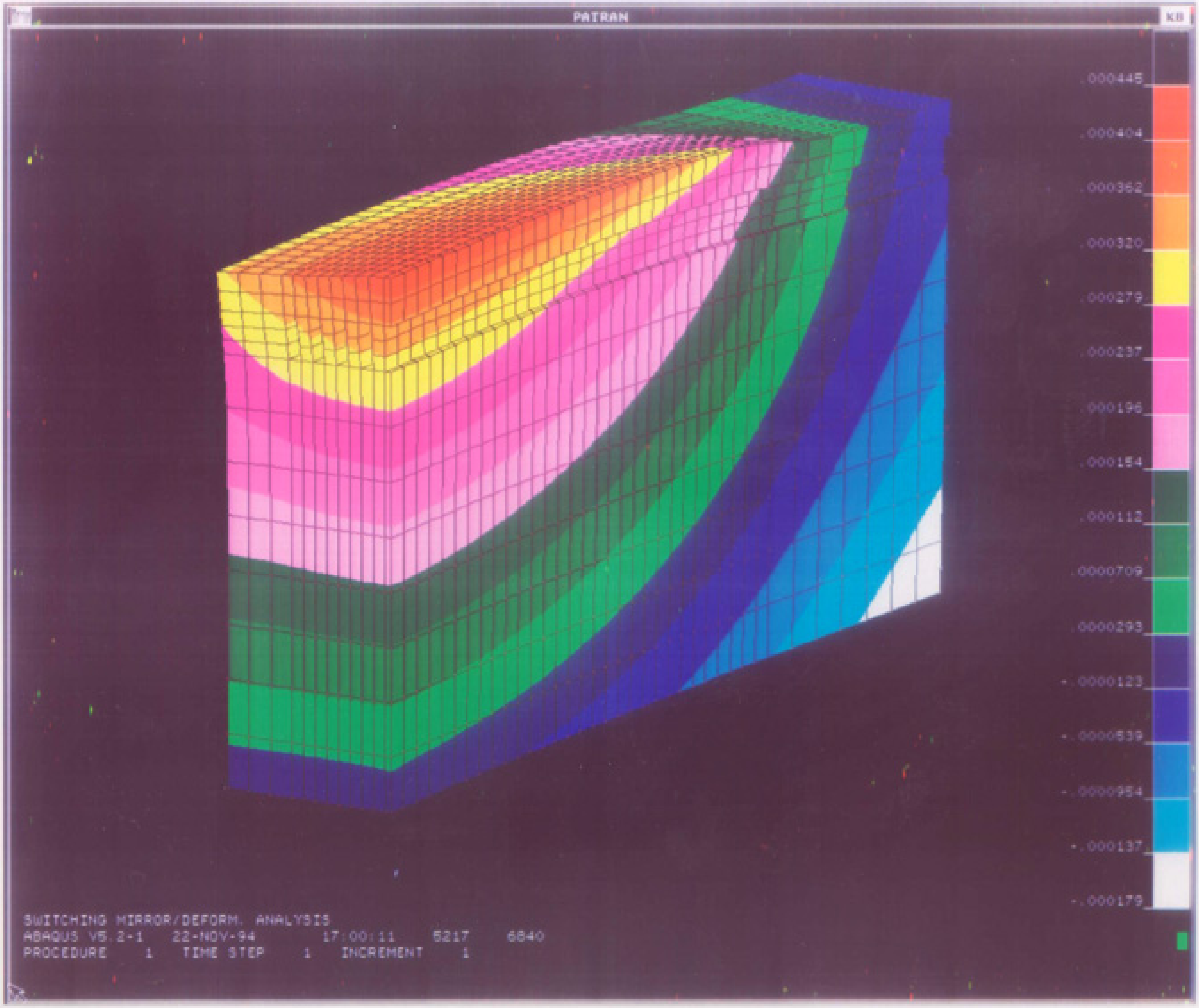
20.2
20.1
SWITCHING MIRROR/HEAT TRANSFER ANALYSIS
ABAQUS V5.2-1 22-NOV-94 13:55:25 5217 6840
PROCEDURE 31 TIME STEP 1 INCREMENT 1


PATRAN
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.000362
.000320
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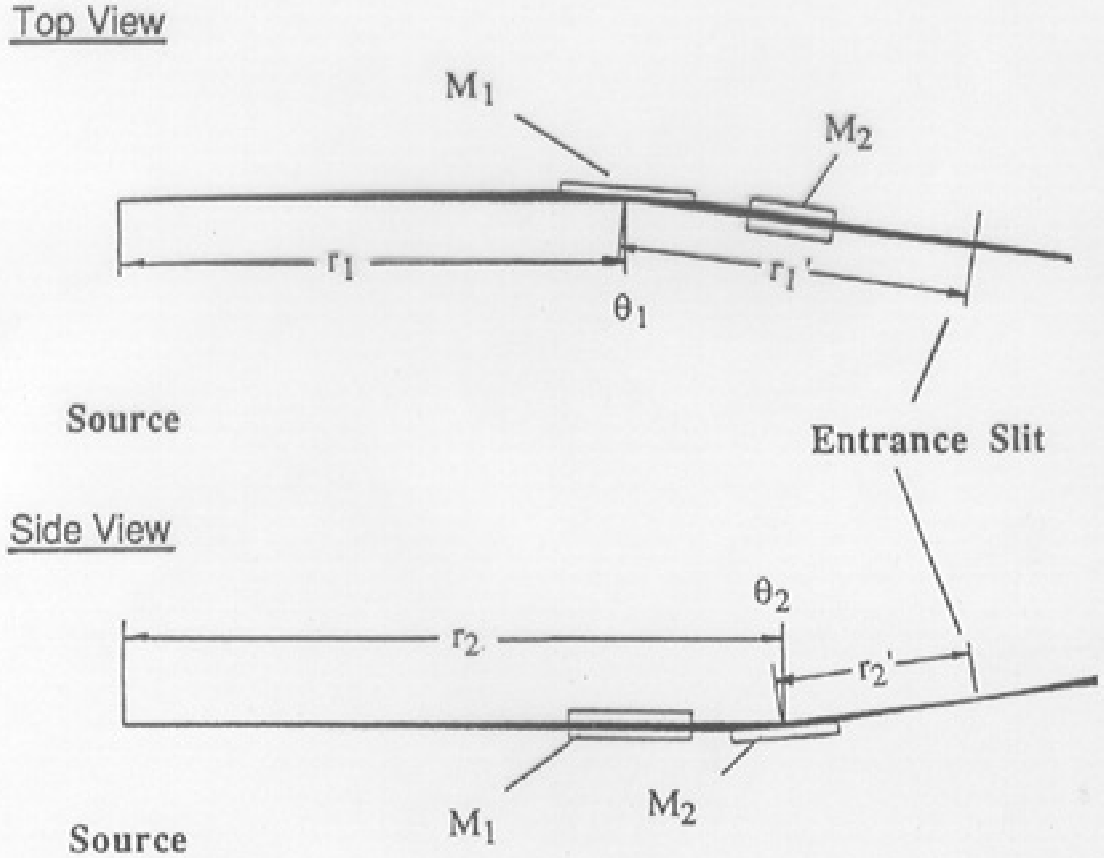
.000154
.000112
.0000709
.0000293
-.0000123
-.0000539
-.0000954
-.000137
-.000179
SWITCHING MIRROR/DEFORM. ANALYSIS
ABAQUS V5.2-1 22-NOV-94 17:00:11 5217 6840
PROCEDURE 1 TIME STEP 1 INCREMENT 1

The Kirkpatrick-Baez Optical System



- separation of horizontal and vertical focusing
- the horizontal focusing mirror comes first: it absorbs most of the heat load

Example of extreme demagnification

$$\text{magnification } M = \frac{r'}{r}$$

$$\text{demagnification} = \frac{1}{\text{magnification}}$$

Source parameters

$\gamma = 3327$ (E = 1.7 GeV)
$\varepsilon_x = 6$ nm·rad
C = 0.10
$\varepsilon_y = 0.6$ nm·rad
$\sigma_{eh} = 219$ µm
$\sigma_{ev} = 42$ µm
$\sigma'_{eh} = 27$ µrad
$\sigma'_{ev} = 14$ µrad

undulator length L = 4100 mm
$\sigma'_{rh} = 160$ µrad
$\sigma'_{rv} = 40$ µrad or 80 µrad
r = 17000 mm

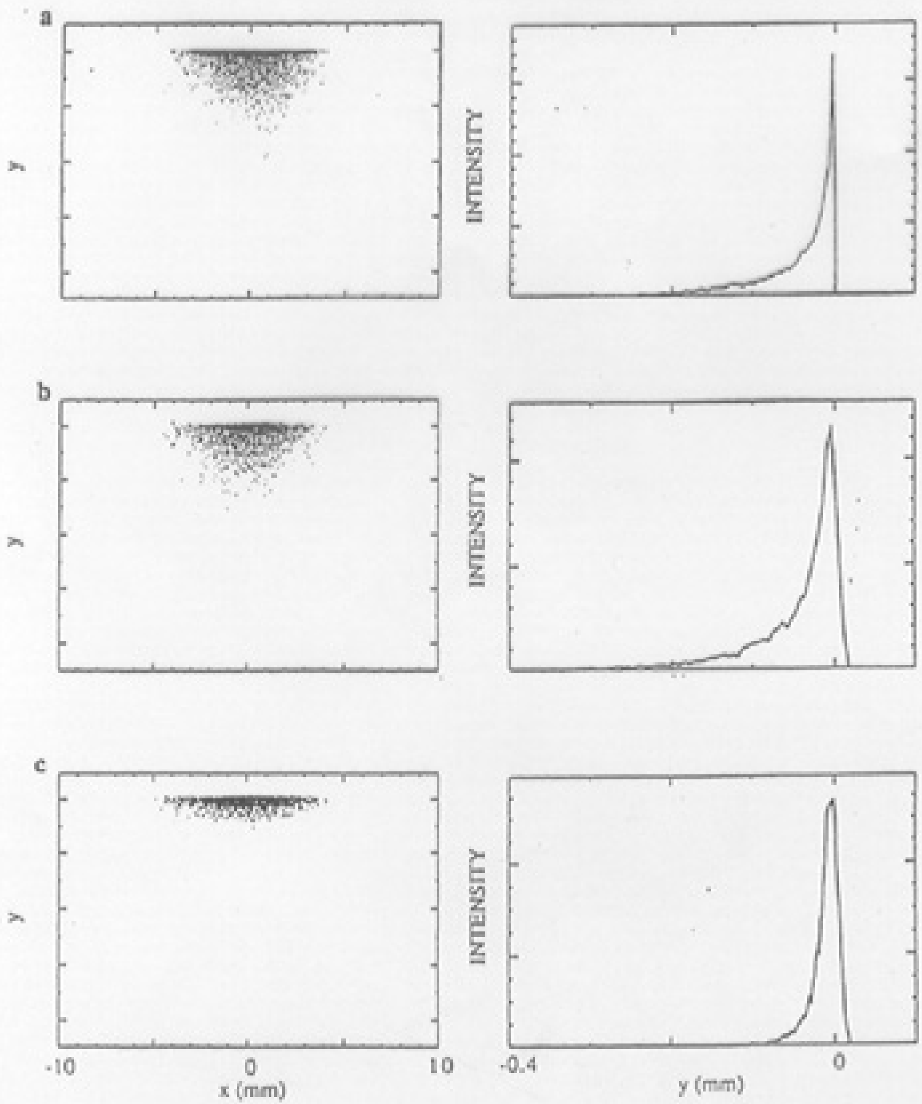
effective source size = $\pm\, 2\sigma_{ev}$ (includes 95% of total flux)
= 168 µm

try nominal demagnification = 24
entrance slit size = 168/24 = 7 µm

Focussing Characteristics of a Spherical Mirror

To the left the spot diagram. To the right, the integrated vertical profile.
The angle of incidence, θ , is 87.5° .

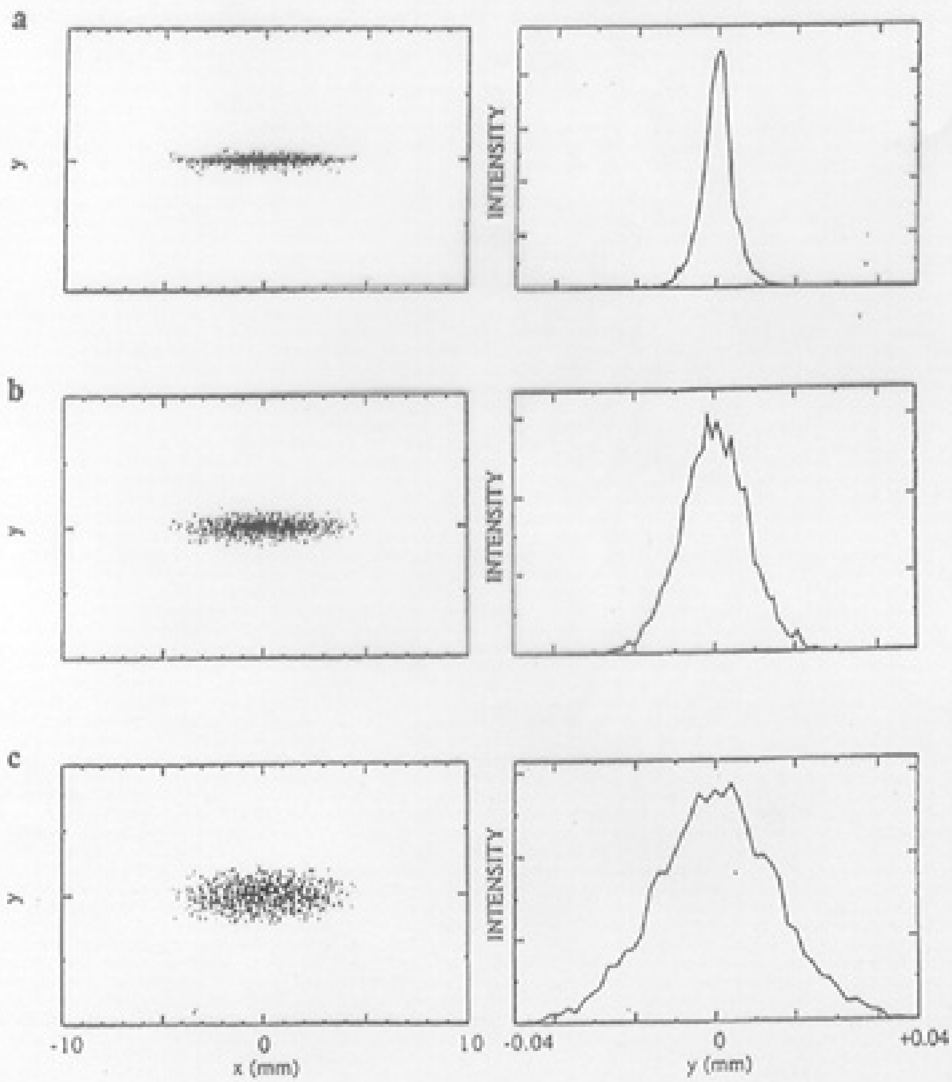
a) Demagnification = 24, σ_{te} = 0 , σ'_{rv} = 80 μrad
b) Demagnification = 24, σ_{te} = 1 sec , σ'_{rv} = 80 μrad
c) Demagnification = 24, σ_{te} = 1 sec , σ'_{rv} = 40 μrad



Focussing Characteristics of a Plane Elliptical Mirror

To the left the spot diagram. To the right, the integrated vertical profile.
The angle of incidence, θ, is 87.5°.

a) Demagnification = 24, $\sigma_{te} = 0$, $\sigma'_{rv} = 80\ \mu rad$
b) Demagnification = 24, $\sigma_{te} = 1$ sec , $\sigma'_{rv} = 80\ \mu rad$
c) Demagnification = 24, $\sigma_{te} = 2$ sec , $\sigma'_{rv} = 80\ \mu rad$



spherical mirror

effective demagnification:

a. 168/37 = 4.5
b. 168/53 = 3.2
c. 168/35 = 4.8

plane elliptical mirror

effective demagnification:

a. 168/12 = 14
b. 168/48 = 3.5
c. 168/80 = 2.1

For nominal demagnification = 10

effective demagnification

sphere	σ_{te} = 0 sec	168/28 = 6.0
	σ_{te} = 1 sec	168/89 = 1.9
plane ellipse	σ_{te} = 0 sec	168/24 = 7.0
	σ_{te} = 1 sec	168/72 = 2.3

in practice nominal demagnification = 5-8

Beamline design

Requests
- Energy range
- Energy resolution
- Number of gratings to be used
  (available energy range without grating change)
- Flux at the sample
(- Spot size at the sample)

Constraints
- The effective source size σ_{tv} is of the order of 20-50 µm which makes $4\sigma_{tv}$ = 80-200 µm.
- For high energy resolution the source must be demagnified somewhere before the exit slit of the monochromator. For very high resolution down to 10 µm.
- Depending on which kind of monochromator is used, a demagnifying mirror system might be necessary in front of the monochromator. Its maximum demagnification is of the order of 8-10.
- The angle of grazing incidence on the optical elements should be small enough to ensure reflectivity.
- Minimization of optical aberrations poses a limit on the grating and other optical elements length, which defines the maximum effective acceptance angles σ'_{tv} and σ'_{th} from the source.
- The finite length of the grating defines α_{max} and hence the low energy limit of the grating.
- The energy dependence of the resolving power of the grating poses a limit on the highest energy which can be reached with a single grating (see below).

Three examples of monochromators/beamlines

toroidal grating monochromator (TGM)

spherical grating monochromator (SGM)

Petersen plane grating monochromator (PGM)

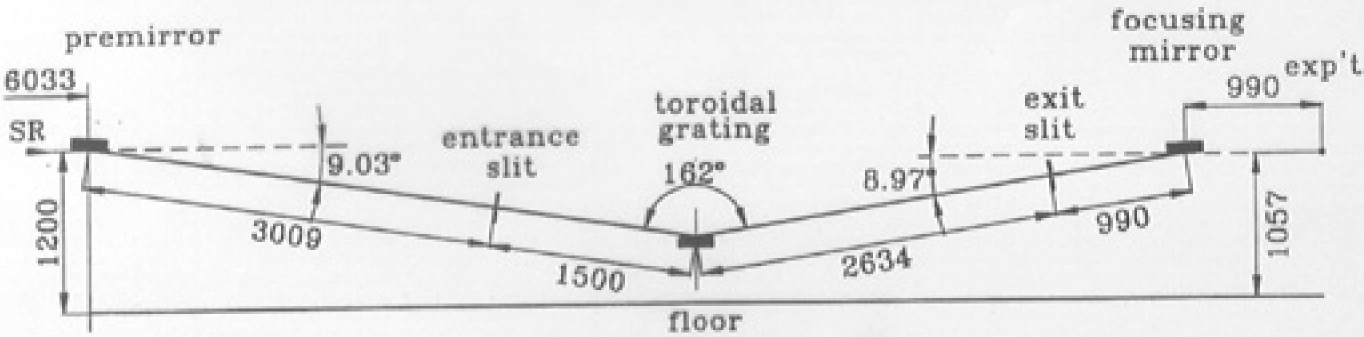
The TGM

the most commonly found type of monochromator at the synchrotron radiation laboratories around the world

simplicity: except for a rotation of the grating everything is fixed in space

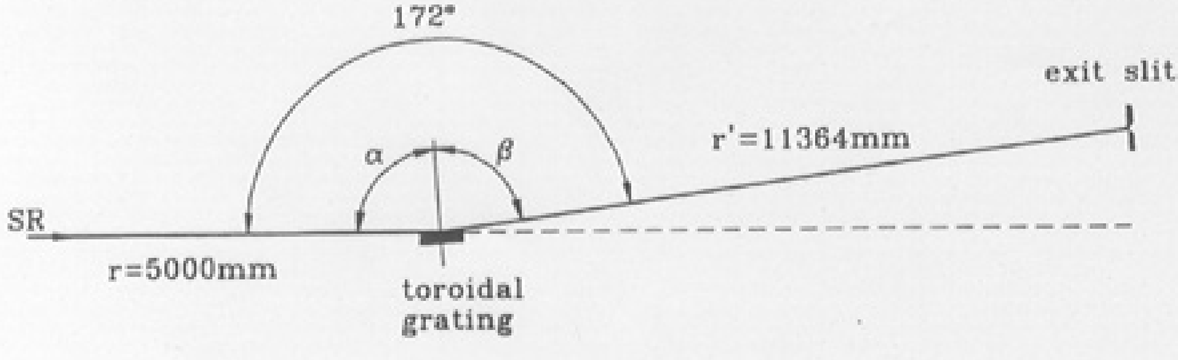
A TGM for Photon Energies from ca. 15 to 160 eV

Shown is a typical toroidal grating monochromator with a 1:2 demagnifying preoptical system and refocussing mirror behind the exit slit. Three gratings are used: 200 ℓ/mm, 600 ℓ/mm and 1800 ℓ/mm. The acceptance is 10 mrad (H) and 2.5 mrad (V) [6.21].



A TGM for Photon Energies from ca. 180 to 1100 eV

A toroidal grating monochromator consisting of a grating and an exit slit. The electron beam itself serves as the source. Two gratings are used: 1100 ℓ/mm and 1500 ℓ/mm. The radii are R = 116000 mm and ρ = 483 mm. The acceptance is 6 mrad (H) and 1 mrad (V) [6.22].



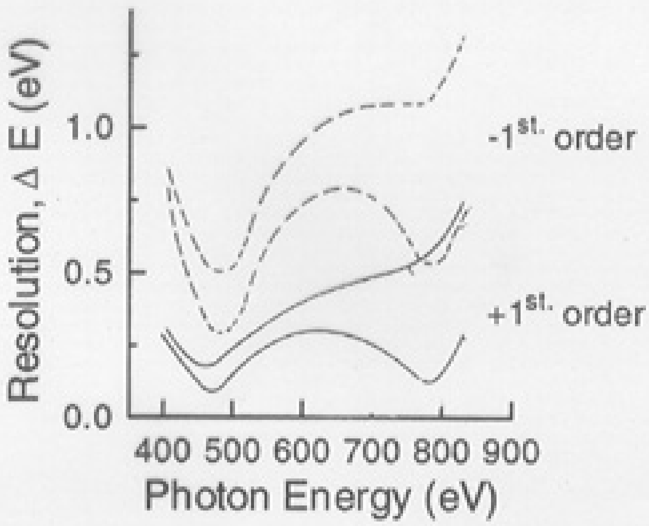
design criterium:
minimize defocussing (F_{200}) and astigmatic coma (F_{120})

$$\Delta\lambda \approx \frac{1}{Nk}\left(\omega F_{200} + \frac{1}{2}\ell^2 F_{120}\right)$$

resolving power $E/\Delta E$ = 200-500

The Performance of a TGM for 180-1100 eV Photons

a. The optimization curves for the toroidal grating monochromator shown in figure 6.2.2 with an 1800 ℓ/mm grating and an acceptance of 2.0 mrad (H) and 1.4 mrad (V) are shown for source sizes of 0.2 mm (upper curves) and 0 mm (point source) lower curves [6.22].



The SGM

the Rowland conditions

set $$\left(\frac{\cos^2\alpha}{r} - \frac{\cos\alpha}{R}\right) = 0$$
$$\left(\frac{\cos^2\beta}{r'} - \frac{\cos\beta}{R}\right) = 0$$

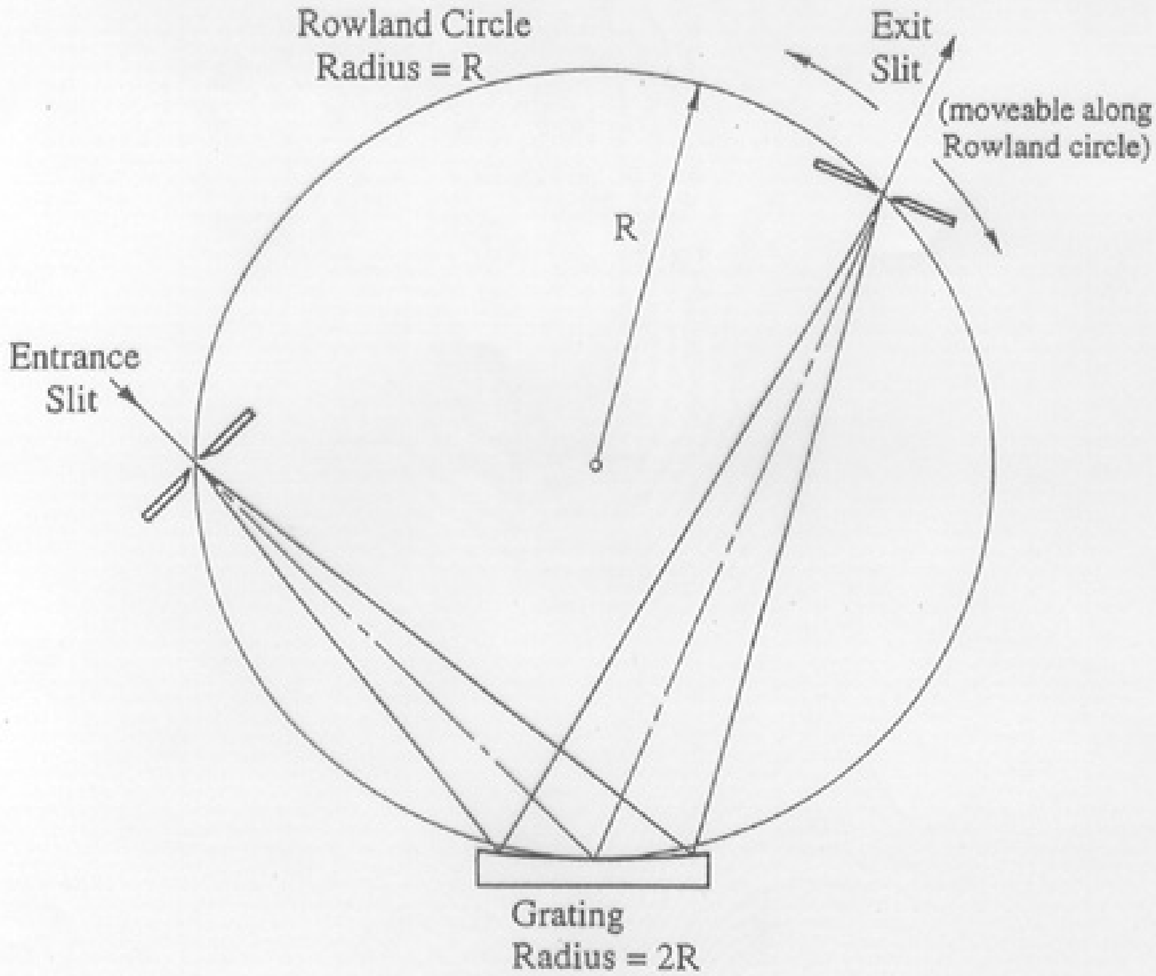
$r = R\cos\alpha$ and $r' = R\cos\beta$

$F_{200} = 0$ meridional focus

$F_{300} = 0$ primary coma

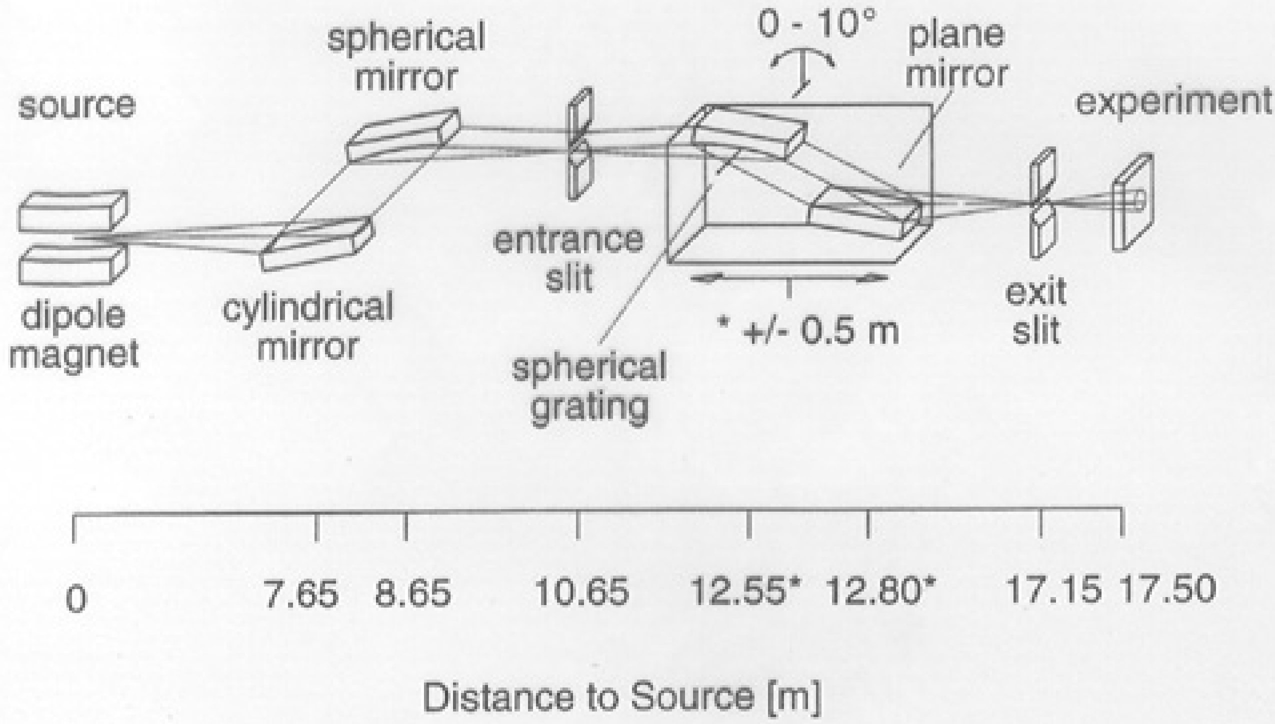
$$F_{400} = -\frac{1}{R^3}(\cos\alpha + \cos\beta) + \frac{1}{R^2}\left(\frac{1}{r} + \frac{1}{r'}\right)$$

The Rowland Circle Monochromator



The Constant Length Rowland Circle Monochromator

In the following design the arm lengths, r and r' are made variable but without moving the slits in order to fulfill the Rowland conditions. This is accomplished by moving the grating back and forth between the slits. Hence the name constant length Rowland circle monochromator [6.32-6.34]. Note that only one additional optical element is required, a plane mirror.



The resolution of a Rowland circle monochromator

$Nk\lambda = \sin\alpha + \sin\beta$

$$\frac{\partial\lambda}{\partial\beta} = \frac{1}{kN}\cos\beta$$

$\lambda = \frac{C}{E}$ C=constant

$$d\lambda = \frac{C}{E^2}\,dE$$

$$d\beta = \frac{s'}{r'}$$

Rowland condition

$$r' = R\cos\beta$$

$$\Delta E = \frac{E^2\cos\beta}{CkN}\cdot\frac{s'}{r'} = \frac{s'E^2}{CkNR}$$

for a SGM several gratings are needed for high resolution in a wide energy range

The magnification of a monochromator

magnification $M(\lambda) = \frac{s'}{s}$

$$\frac{\partial\lambda}{\partial\alpha} = \frac{1}{kN}\cos\alpha$$

$Nk\lambda = \sin\alpha + \sin\beta$

$$\frac{\partial\lambda}{\partial\beta} = \frac{1}{kN}\cos\beta$$

$$\Delta\alpha = \frac{s}{r} \quad \text{and} \quad \Delta\beta = \frac{s'}{r'}$$

$\Delta\lambda$ at the two slits equal

$$\frac{\cos\alpha}{Nk}\Delta\alpha = \frac{\cos\beta}{Nk}\Delta\beta$$

$$\frac{s \cdot \cos\alpha}{r} = \frac{s' \cdot \cos\beta}{r'}$$

$$M(\lambda) = \frac{s'}{s} = \frac{r' \cdot \cos\alpha}{r \cdot \cos\beta}$$

Case of the Rowland circle monochromator

$r = R \cos \alpha$ and $r' = R \cos \beta$

$$M = \frac{R \cdot \cos\beta \cdot \cos\alpha}{R \cdot \cos\alpha \cdot \cos\beta} = 1$$

with a SGM no (or little) demagnification of the source is possible: it is necessary to have a demagnifying mirror system in front of the monochromator

The exactly focussed SGM

$F_{100} = Nk\lambda - (\sin\alpha + \sin\beta) = 0$
(grating equation)

$$F_{200} = \left(\frac{\cos^2\alpha}{r} - \frac{\cos\alpha}{R} \right) + \left(\frac{\cos^2\beta}{r'} - \frac{\cos\beta}{R} \right) = 0$$

(meridional focus)

for all energies

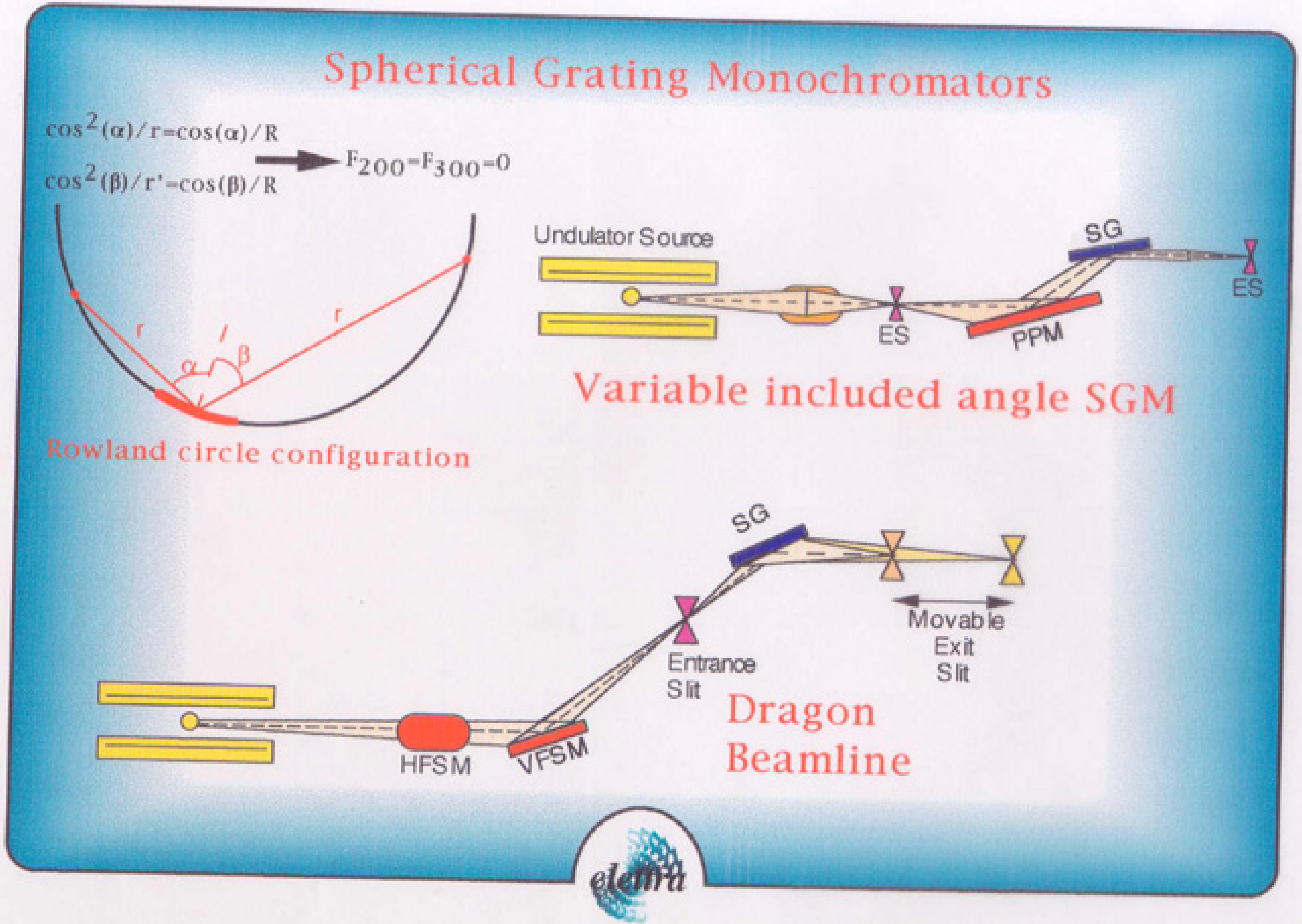
and $F_{300} = 0$ (primary coma)

for one energy inside the grating's range

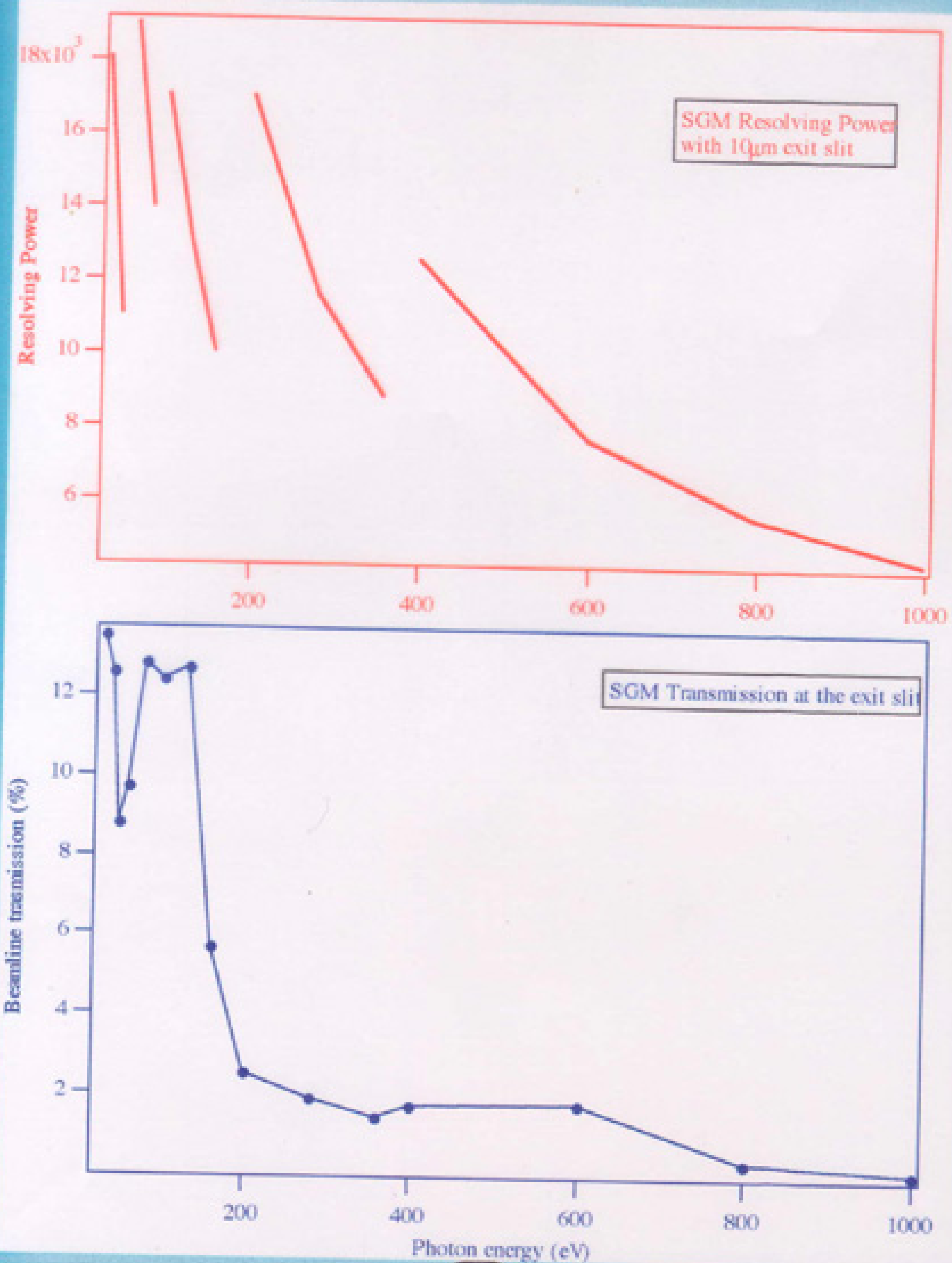
one additional degree of freedom is needed
besides the rotation of the grating

movable exit slit
(Dragon Monochromator)

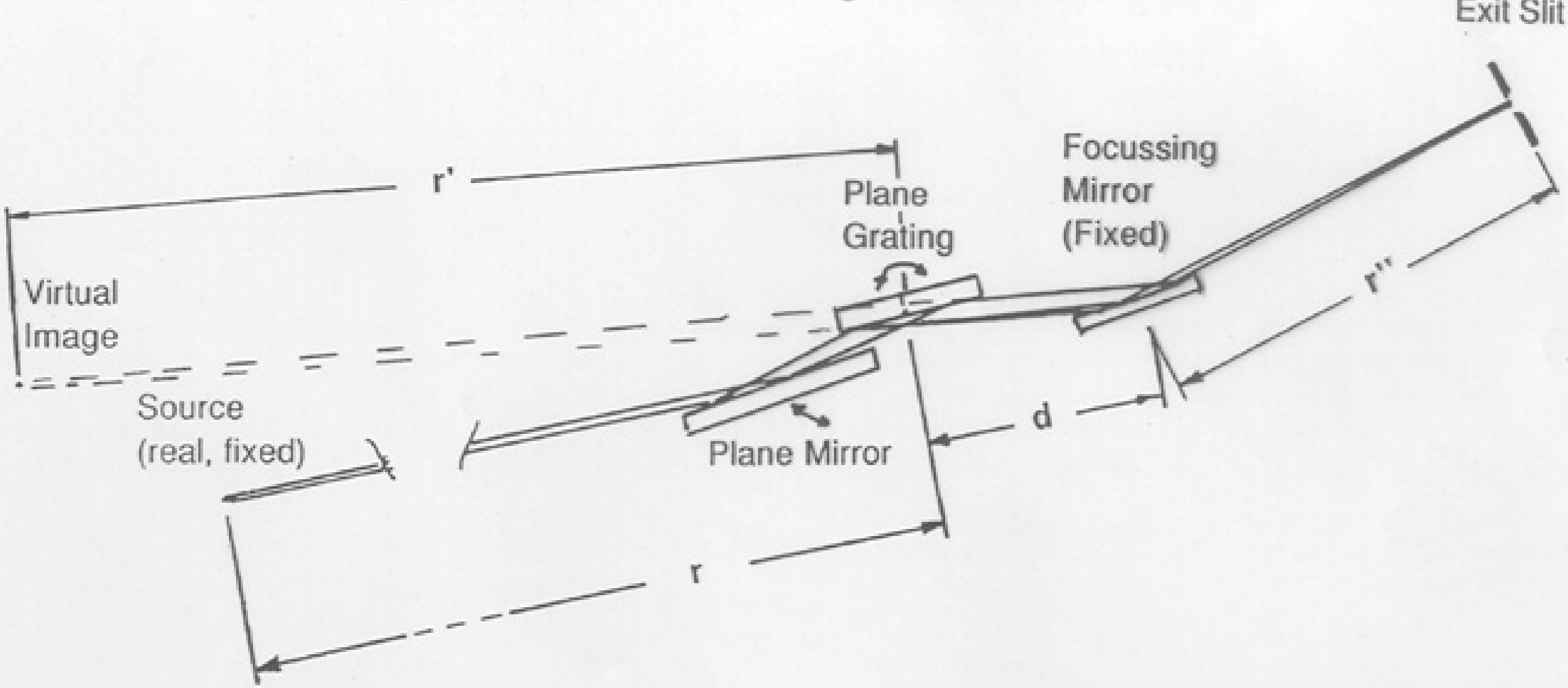
movable plane mirror inside the monochromator
(VASGM: Variable Angle SGM)


Spherical Grating Monochromators
cos2(α)/r=cos(α)/R
cos2(β)/r'=cos(β)/R
F200=F300=0
r
r'
α
β
Rowland circle configuration
Undulator Source
SG
ES
ES
PPM
Variable included angle SGM
SG
Movable
Exit
Slit
Entrance
Slit
HFSM
VFSM
Dragon
Beamline
elettra

SGM resolving power and transmission



Optical Design of the Petersen Plane Grating Monochromator



for a plane grating $R = \infty$

$$F_{200} = 0 \quad \Rightarrow \quad \frac{r'}{r} = -\frac{\cos^2\beta}{\cos^2\alpha} = -c^2_{ff}$$

where c_{ff} is a constant
optimum value $c_{ff} = 2.25$

magnification

$$M_{grating} = M_g = \frac{r' \cdot \cos\alpha}{r \cdot \cos\beta}$$

$$M_{mirror} = M_m = \frac{r''}{r' + d}$$

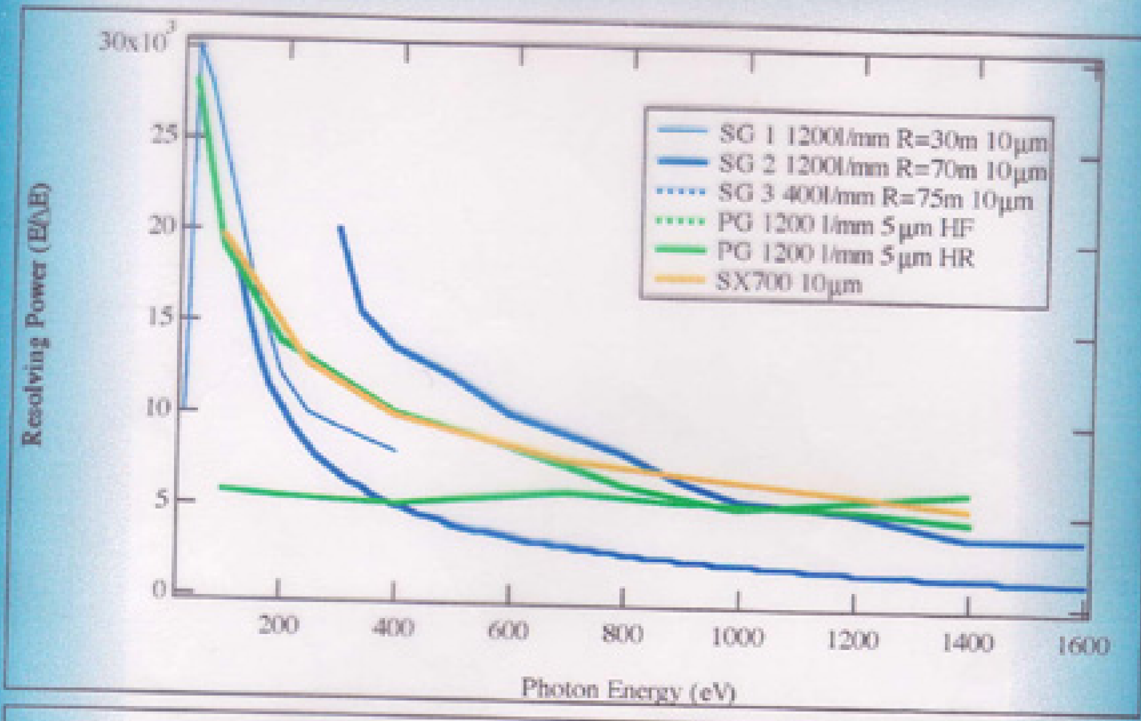
$$M = M_g \cdot M_m \sim \frac{r''}{r \cdot c_{ff}}$$

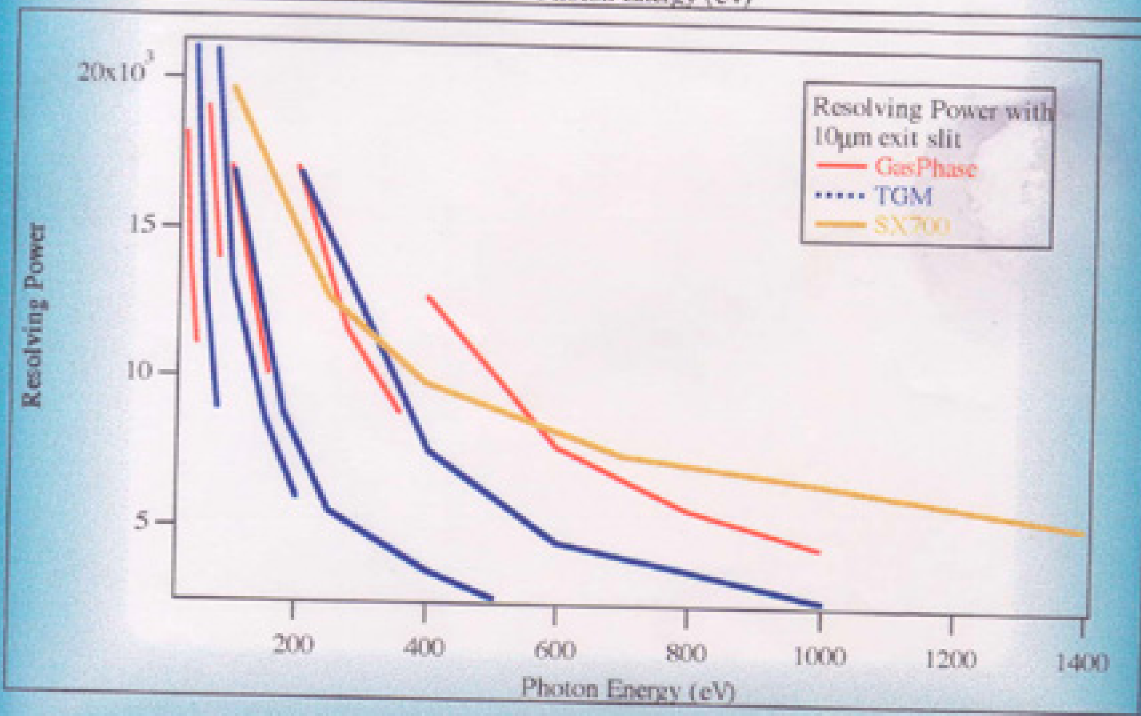
typical values $c_{ff} = 2.25$
$r = 13000$ mm
$r'' = 5000$ mm

$M \sim 1/6$

energy resolution $\Delta E \propto E^{3/2}$

Resolving Power Comparison





elettra