

SCHOOL ON SYNCHROTRON RADIATION AND APPLICATIONS
In memory of J.C. Fuggle & L. Fonda

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Miramare - Trieste, Italy

1561/8

Introduction to Beamline Design

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Introduction to beamline design

This instruction material is based on the

Lecture notes by

W.B.Peatman, BESSY, Berlin - Germany

for the School on the Use of Synchrotron Radiation in
Science and Technology

30 October - 1 December 1995, ICTP, Trieste – Italy

and on the book

W.B.Peatman, "Gratings, mirrors and slits: beamline design
for soft X-ray synchrotron radiation sources", Gordon and
Breach Science Publishers, Amsterdam, 1997

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review of the sources of synchrotron radiation

the brilliance

geometrical characteristics of the radiation from the different sources:

- bending magnet
- wiggler
- undulator

gratings

mirrors

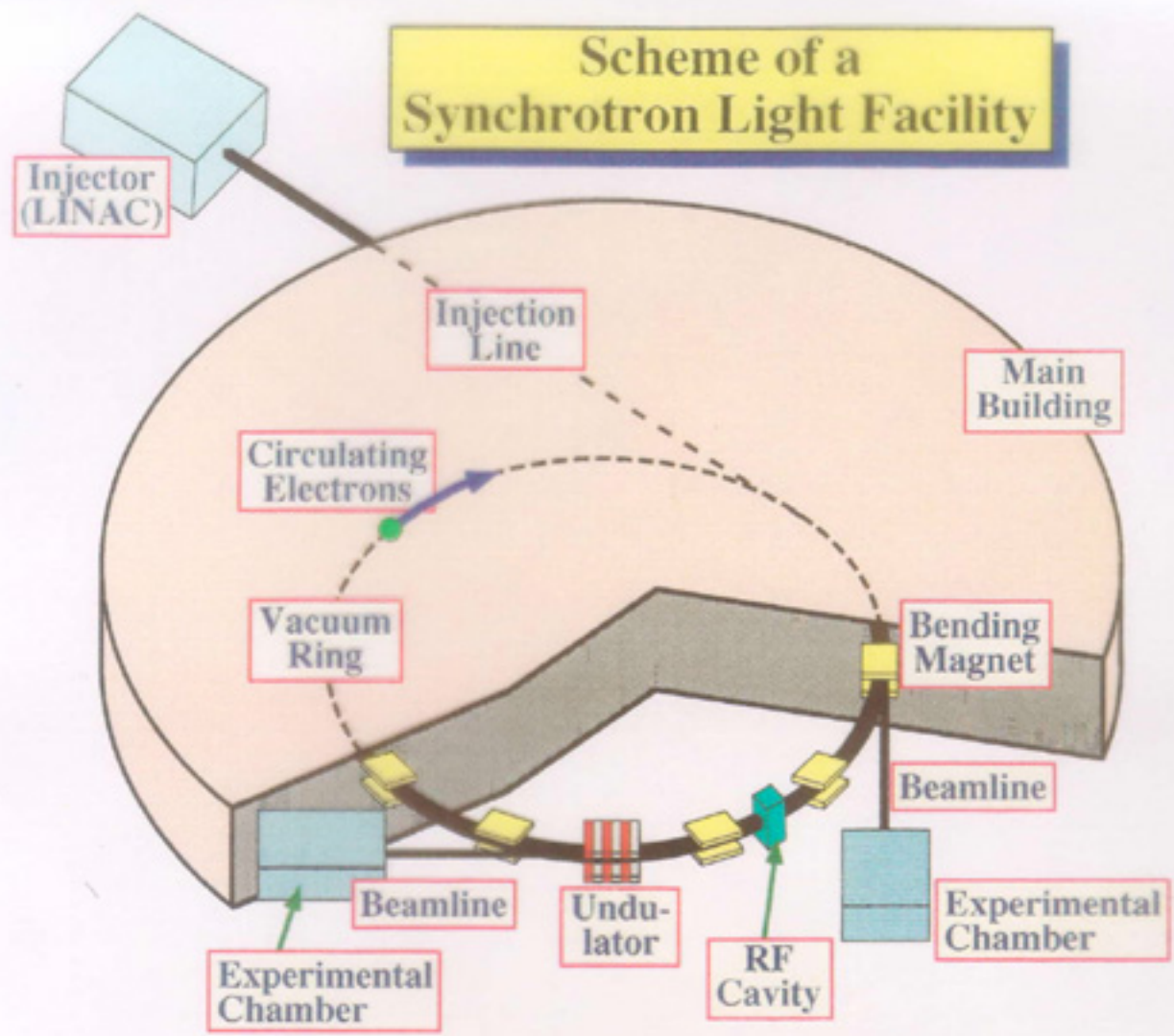
figure accuracy/ heat load on the optical elements

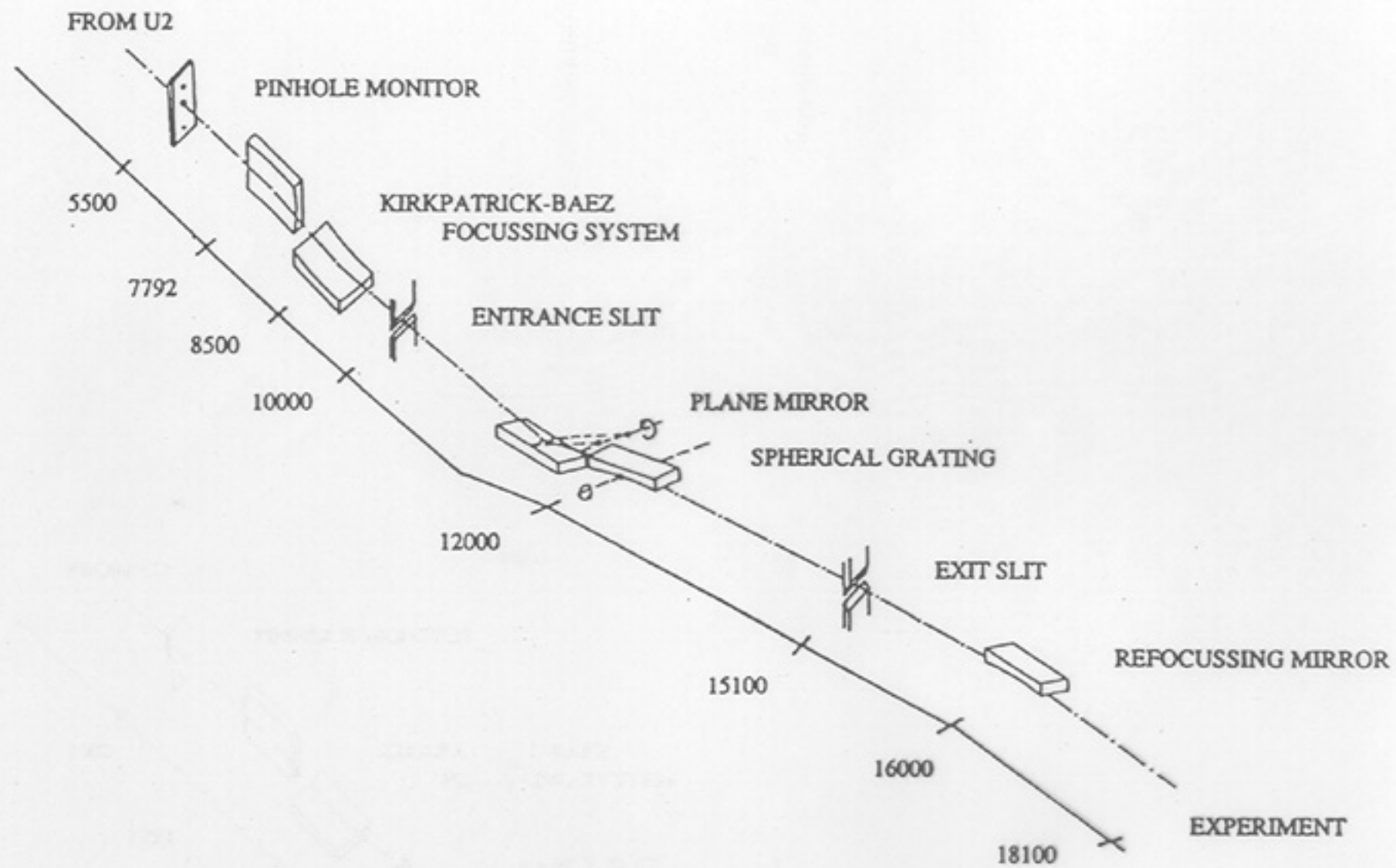
an example of focusing system

guidelines in designing a beamline

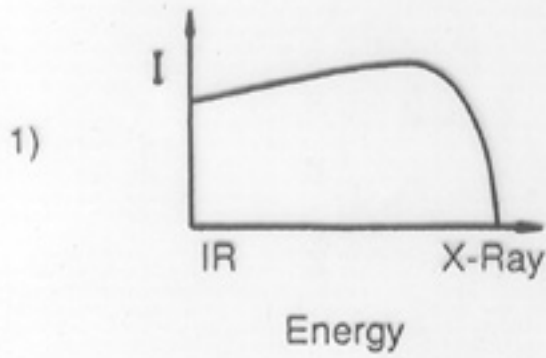
three examples of monochromator/beamlines

Scheme of a Synchrotron Light Facility

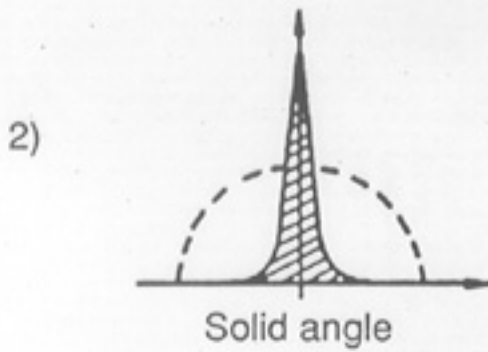




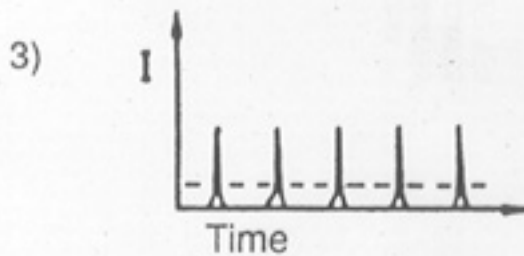
Important Characteristics of Synchrotron Radiation



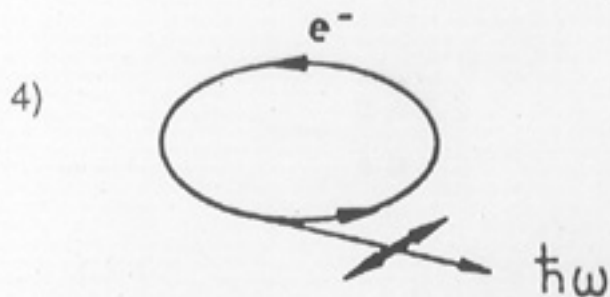
Continuous spectrum



Emission in small solid angle



Pulsed time structure



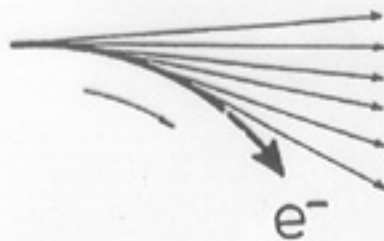
High degree of polarisation

5) Properties can be calculated/predicted

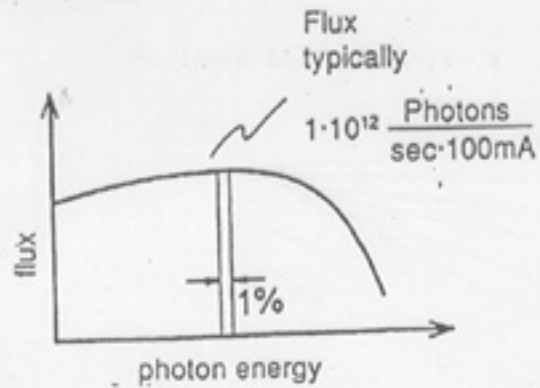
Synchrotron Radiation Sources

Dipole Magnet

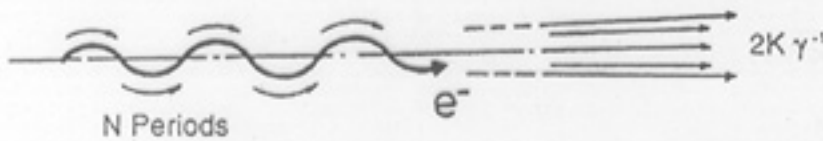
Horizontal source divergence



many mrad



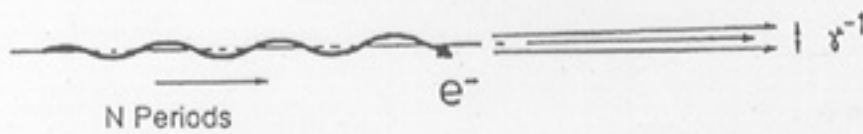
Wiggler



SR-collimated

Intensity N times that of a dipole source with the same number of mrad

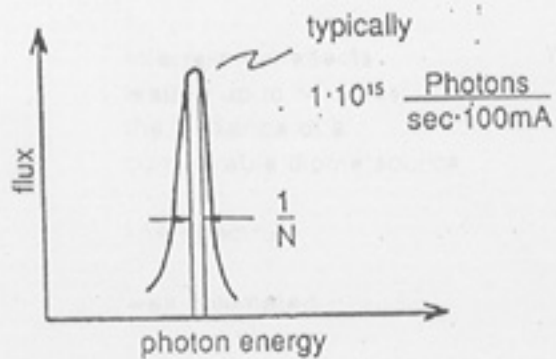
Undulator



Interference effects lead to up to N^2 times the brilliance of a comparable dipole source

line spectrum

well collimated



Flux = photons/sec

$$\text{Brilliance (or Brightness)} = \frac{\text{Flux}}{I} \frac{1}{s_x s_y s'_x s'_y \text{ BW}}$$

I = electron current in the storage ring

$\sigma_x \sigma_y$ = the area from which the SR is emitted

$\sigma'_x \sigma'_y$ = the solid angle into which the SR is emitted

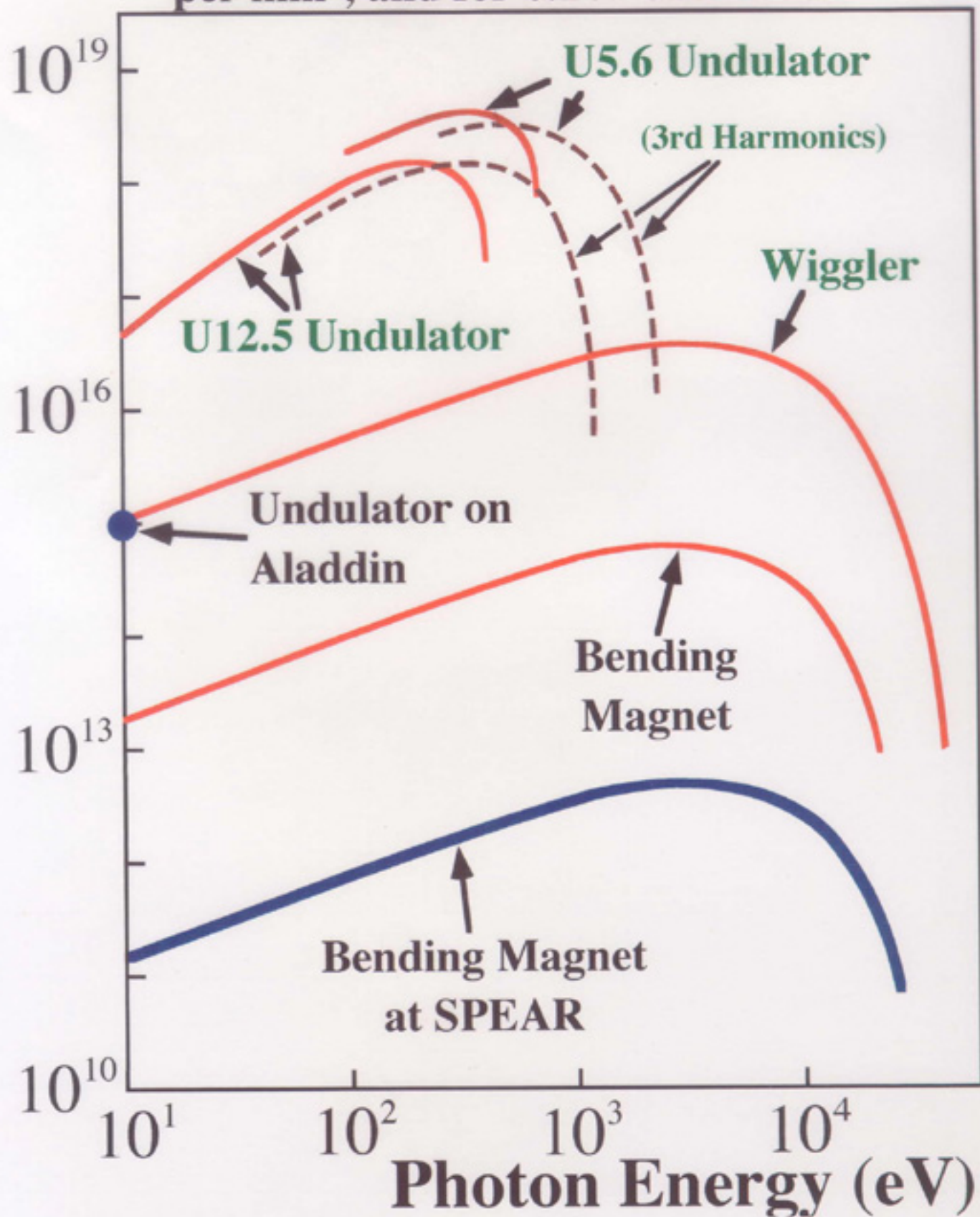
BW = photon energy bandwidth

units = photons/sec mm² mrad² 0.1% BW

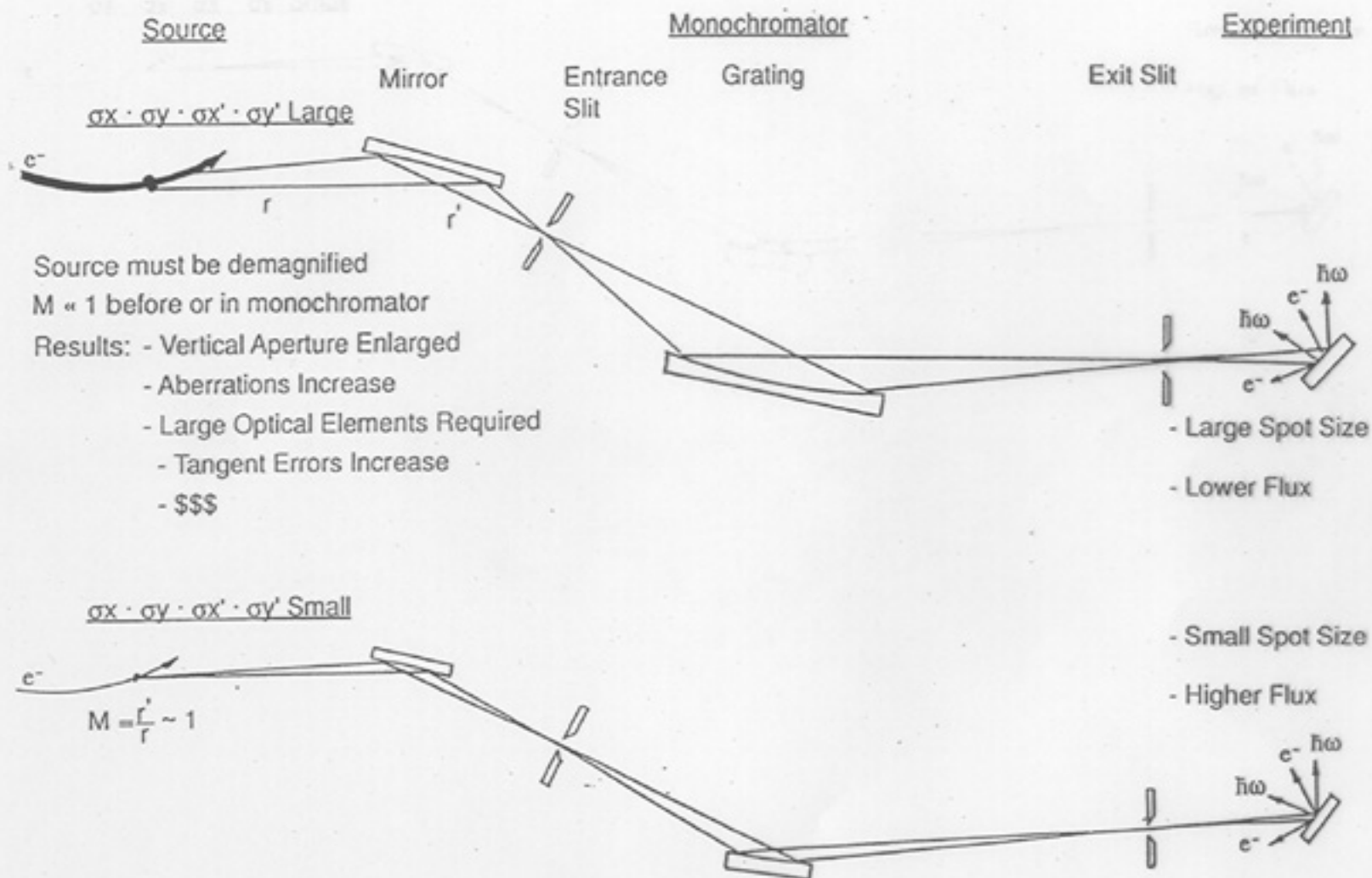
for given ring current I (typically 100 mA)

ELETRA's Brightness Curves,

in Photons per second, per mrad²,
per mm², and for 0.1% bandwidth



The Practical Meaning of Brilliance



The electron beam

emittance = $\varepsilon = \sigma_e \sigma'_e = \text{constant}$

$$\varepsilon_y = C \varepsilon_x$$

C = coupling factor (constant), typically 1-10 %

case of ELETTRA

horizontal emittance $\varepsilon_x = 7.0 \text{ nm}\cdot\text{rad}$

coupling factor = 1%

beam dimensions in the straight sections
(horizontal/vertical) in μm

241/15

beam divergence in the straight sections
(horizontal/vertical) in μrad

29/6

The radiation emission process

effective size σ_r

opening angle σ'_r

Total

$$\sigma_t = \sqrt{\mathbf{s}_e^2 + \mathbf{s}_r^2}$$

$$\sigma'_t = \sqrt{\mathbf{s}'_e^2 + \mathbf{s}'_r^2}$$

Dipole magnet

effective size

$$\sigma_t = \sigma_e$$

vertical opening angle

$$\sigma'_{rv} \text{ (rad)} = 0.57 \left(\frac{\lambda}{\lambda_c} \right)^{0.43} \frac{1}{\gamma} \quad \text{for } 0.2 < \frac{\lambda}{\lambda_c} < 100$$

λ_c = critical wavelength

$\hbar\omega_c$ = critical energy

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} = \text{relativistic factor}$$

$$\gamma = 1957 E$$

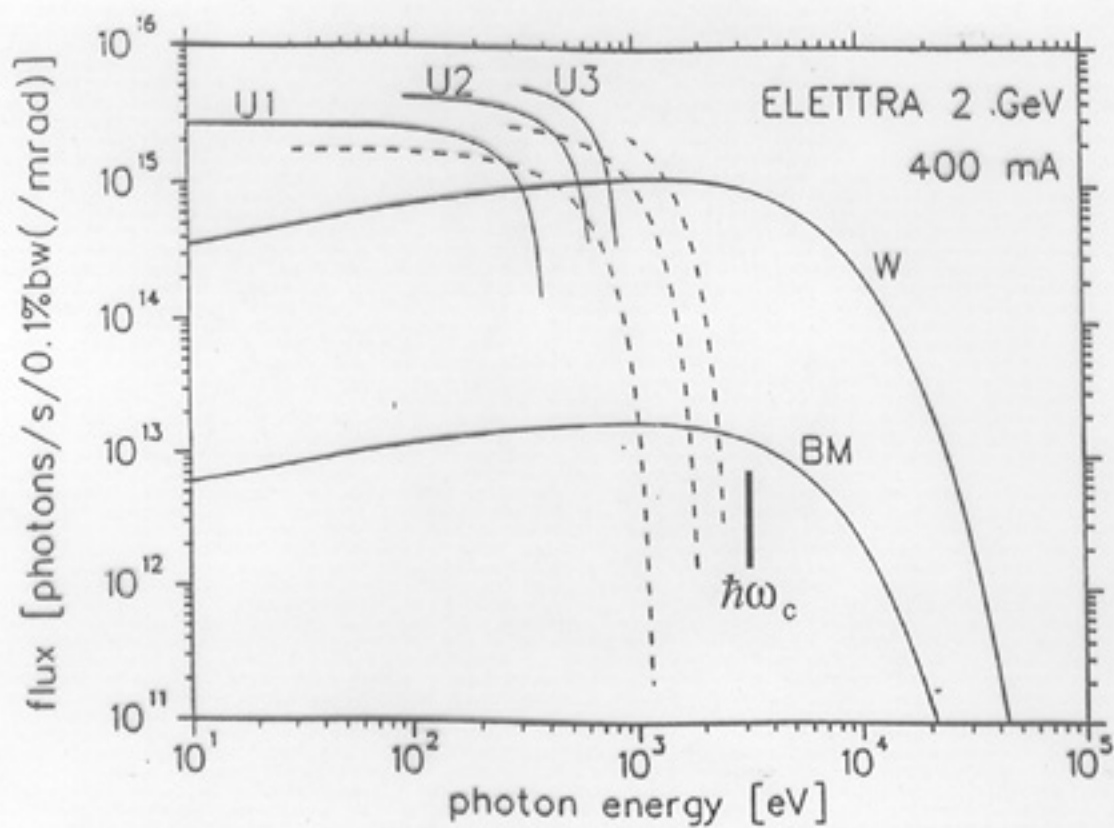
$$\hbar\omega_c = 2218 \frac{E^3}{\rho} = 667 B E^2$$

E = energy of the electrons in GeV

ρ = radius of curvature of the electrons in the dipole magnets in meters

B = magnetic field amplitude in Tesla

$\hbar\omega_c$ = critical energy in eV



Spectral flux of radiation sources in the storage ring at 2 GeV.

case of ELETTRA

$$\begin{aligned}
 E &= 2 \text{ GeV} \\
 \rho &= 5.5 \text{ m} \\
 \hbar\omega_c &\approx 3.2 \text{ KeV}
 \end{aligned}$$

$$\gamma = 3914$$

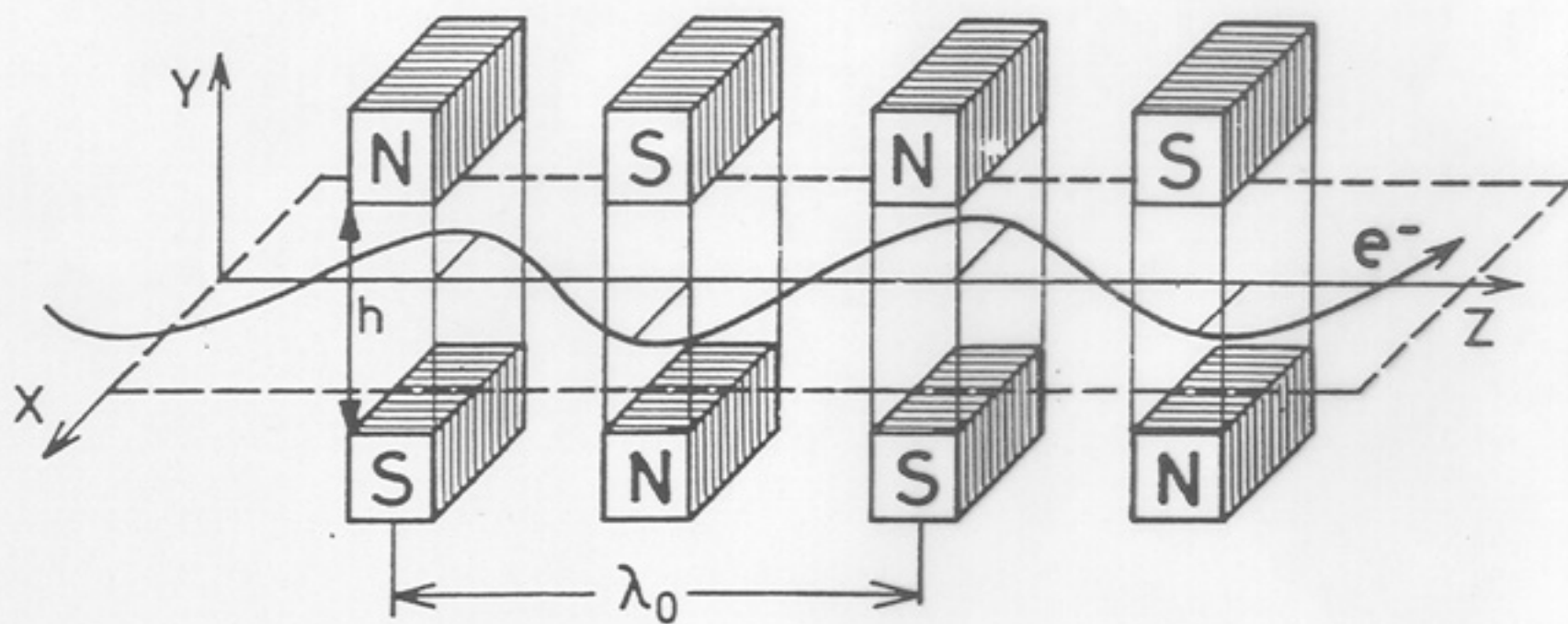
$$\sigma'_{rv} \text{ (rad)} = 0.57 \left(\frac{\lambda}{\lambda_c} \right)^{0.43} \cdot 255 \text{ } \mu\text{rad}$$

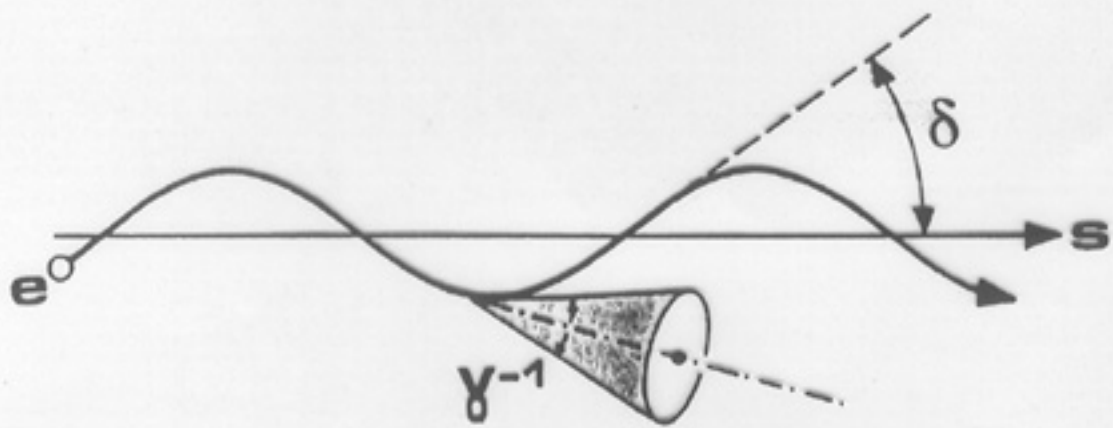
horizontal opening angle

defined by the geometry of the front end

at ELETTRA: 6 or 7 mrad

Layout of a Wiggler, Undulator





magnetic field strength parameter K

$$K = \frac{e\lambda_0 B_0}{2\pi mc} = 0.934 \lambda_0 B_0$$

λ_0 = length of the undulator period in cm

B_0 = magnetic field amplitude in Tesla

$$K = \delta\gamma$$

at $K = 1$ is $\delta = \gamma^{-1}$

Wiggler

$$K \gg 1$$

vertical opening angle

$$\sigma'_{rv} = \sigma'_{rv} \text{ (dipole)}$$

horizontal opening angle

$$\sigma'_{rh} = \delta/2 = K/2\gamma$$

vertical source size

$$\sigma_{rv} = \left(\frac{\sigma'^2_{ev}}{3} + \frac{\Delta\theta^2}{9} \right)^{1/2} \frac{L}{2}$$

horizontal source size

$$\sigma_{rh} = \left[\left(\frac{K \lambda_o}{\gamma 2\pi} \right)^2 + \left(\frac{\sigma'^2_{eh}}{3} + \frac{\Delta\theta^2}{9} \right) \left(\frac{L}{2} \right)^2 \right]^{1/2}$$

where $\Delta\theta$ is the half opening angle of the optical system

Wiggler W14.0 of ELETTRA at 2 GeV

$$L = 4.5 \text{ m}$$

$$\lambda_o = 140 \text{ mm}$$

$$B = 1.5 \text{ T}, K = 19.6$$

$$2\Delta\theta = 1.5 \text{ mrad H} \times 0.28 \text{ mrad V}$$

$$x_o = \frac{K \lambda_o}{\gamma 2\pi} = 112 \text{ } \mu\text{m}$$

separation between the two "eyes" of the wiggler

$$\hbar\omega_c = 4.0 \text{ KeV}$$

$$\sigma'_{rv} \text{ (rad)} = 0.57 \left(\frac{\lambda}{\lambda_c} \right)^{0.43} \frac{1}{\gamma} = 108 \text{ } \mu\text{rad (at } \hbar\omega = 8 \text{ KeV)}$$

$$\sigma'_{rh} = 2500 \text{ } \mu\text{rad}$$

$$\sigma_{rv} = 105 \text{ } \mu\text{m}$$

$$\sigma_{rh} = 575 \text{ } \mu\text{m}$$

Undulator

$$0 < K = 2-3$$

$$\lambda = \frac{\lambda_o}{2\gamma^2 k} \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right)$$

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{kN} \quad \text{or} \quad \frac{\Delta\lambda}{\lambda} = \frac{1}{2kN}$$

k = number of the harmonic

N = number of periods of undulator

$N\lambda_o = L$ = total length of undulator

σ'_r = width of the central radiation cone

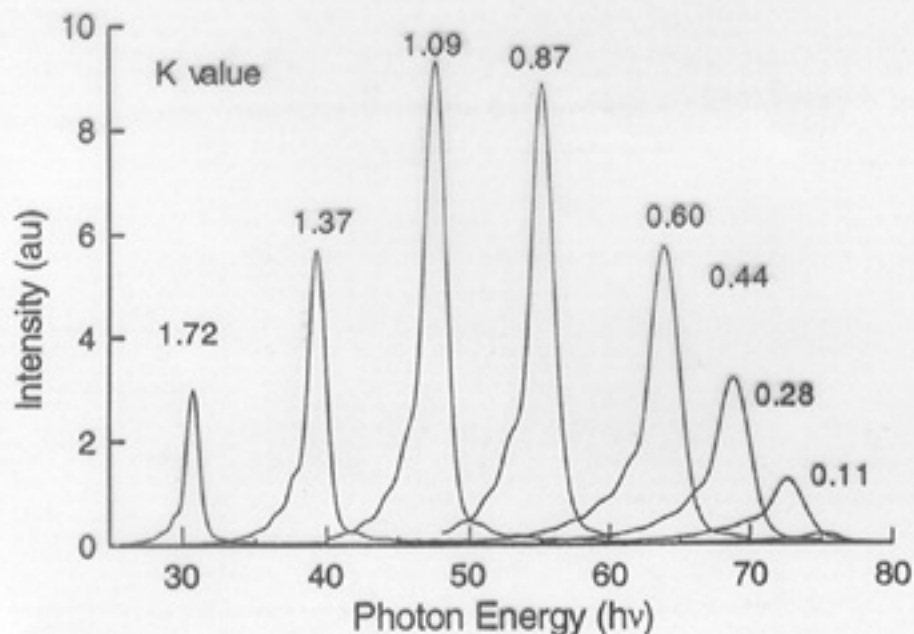
$$\frac{\Delta\lambda}{\lambda} = \frac{1}{kN} \quad \sigma'_r = \frac{\sqrt{1 + \frac{K^2}{2}}}{\gamma} \frac{1}{\sqrt{Nk}}$$

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{2kN} \quad \sigma'_r = \sqrt{\frac{\lambda}{L}}$$

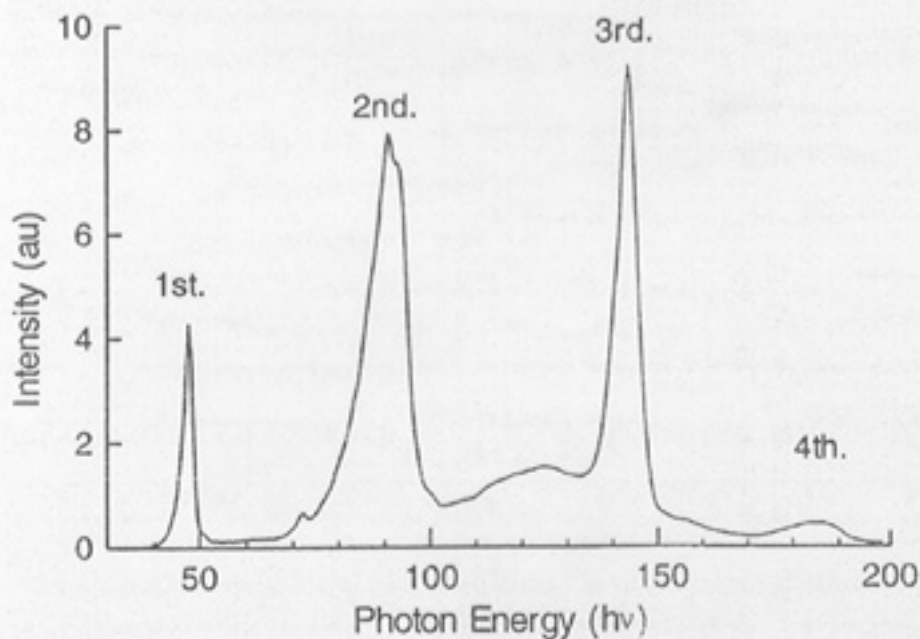
Typical Undulator Spectra

Typical undulator spectra: $\lambda_0 = 70$ mm, $N = 35$ and $E_{e1} = 800$ MeV. The envelope of the spectra is determined mainly by the transmission of the monochromator used to record them [2.7].

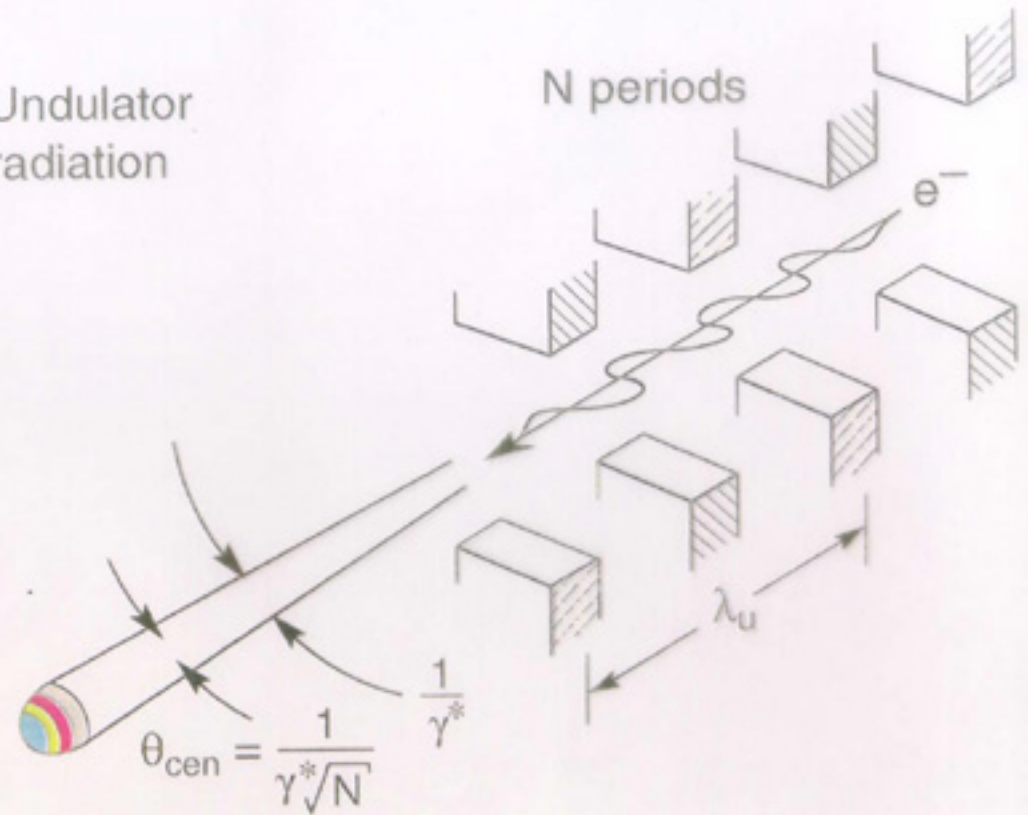
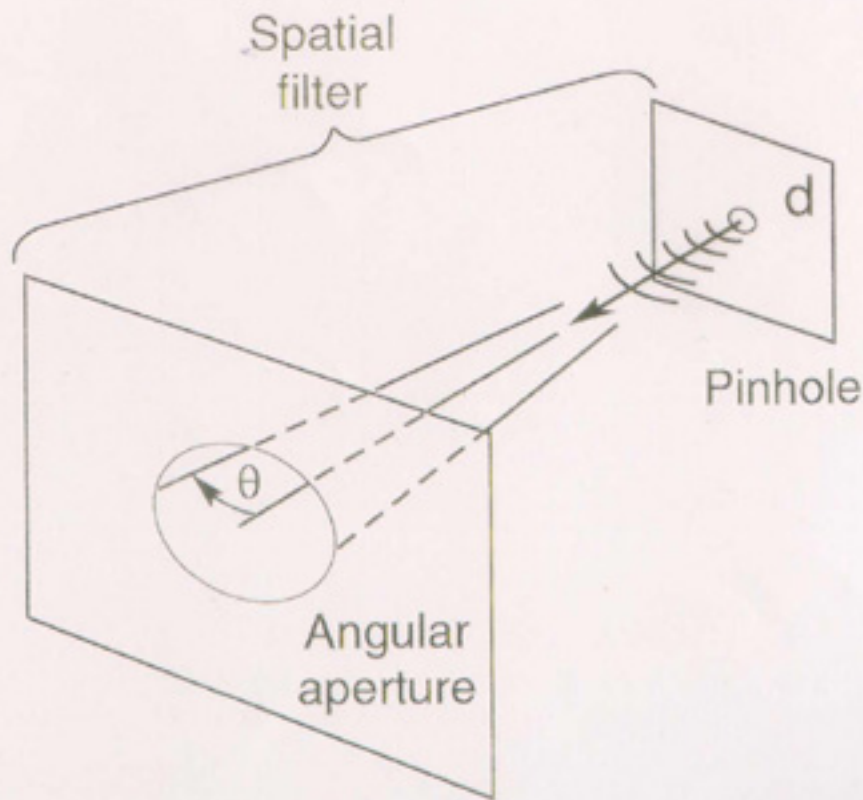
a. Spectra of the first harmonic for different values of K . The acceptance of the monochromator was 0.13 mrad.



b. The first three harmonics for $K = 1.09$. The acceptance of the monochromator was 0.13 mrad.



(a) Undulator radiation



$$\lambda = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right)$$

$$\theta_{cen} = \frac{1}{\gamma^* \sqrt{N}} ; \left(\frac{\lambda}{\Delta\lambda} \right)_{cen} = N ; \gamma^* = \gamma / \sqrt{1 + K^2/2}$$

$$\sigma'_r = \sqrt{\frac{\lambda}{L}}$$

diffraction limit case: $\sigma_r \cdot \sigma'_r = \lambda/4\pi$

$$\sigma_r = \frac{1}{4\pi} \sqrt{\lambda L}$$

case of ELETTRA

$$L = 4.5 \text{ m}$$

es. $\hbar\omega = 100 \text{ eV}$, that is $\lambda = 12.4 \text{ nm}$

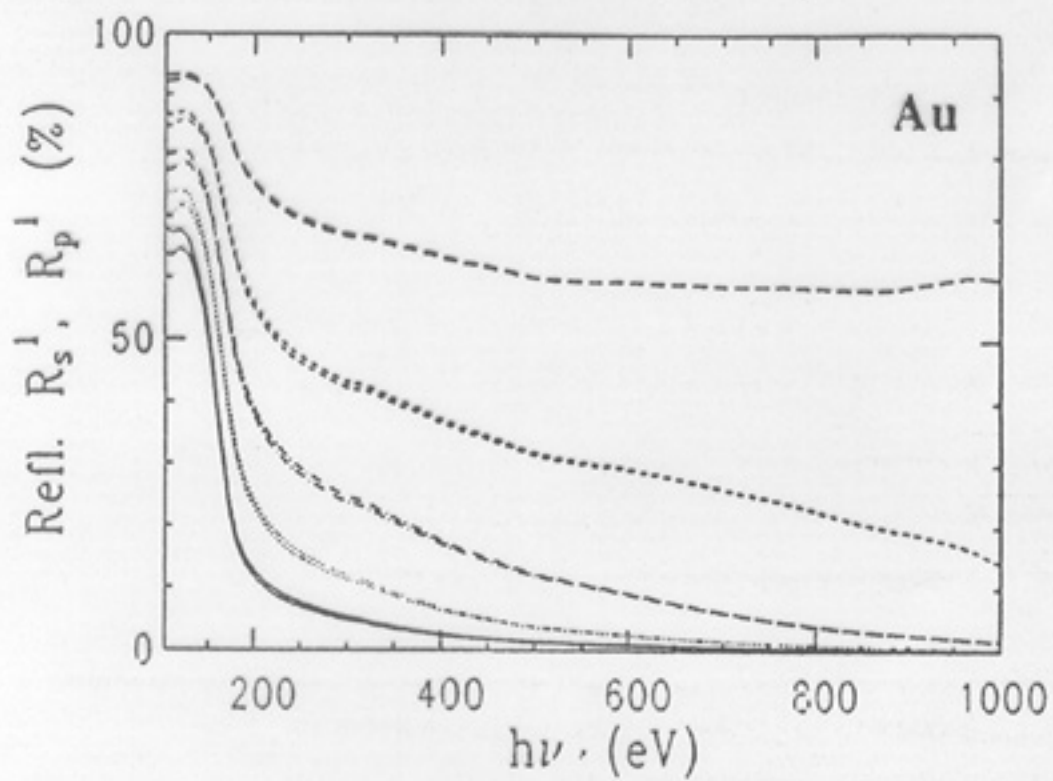
$$\sigma'_r = 52 \text{ } \mu\text{rad}$$

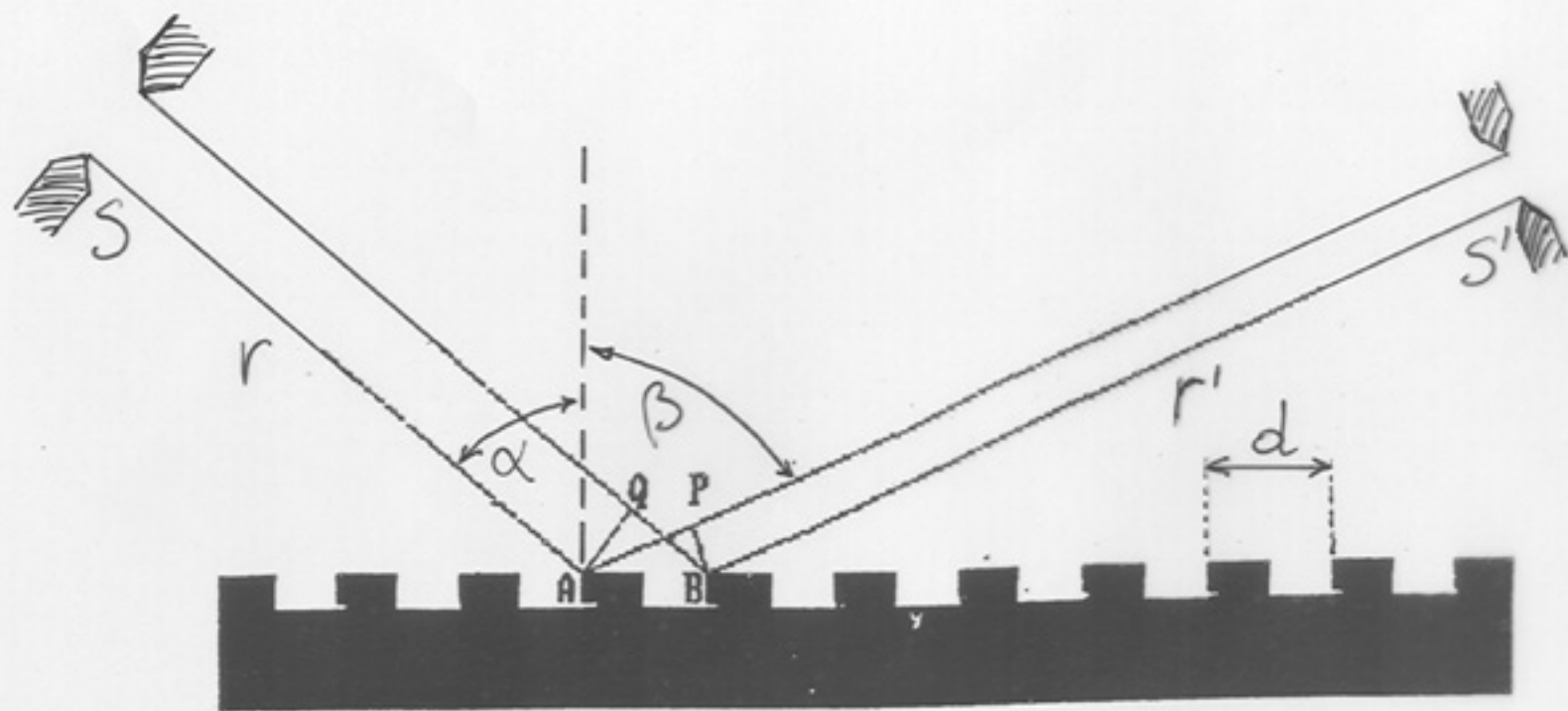
$$\sigma_r = 19 \text{ } \mu\text{m}$$

Constraints in designing the optical system

- in vacuum operation
- only reflecting optics available
grazing incidence, limited reflectivity
- no lenses, no prisms
no multielement optical groups
for an aberration corrected system

Reflectivity of Au at angles of incidence
 $80^\circ, 82^\circ, 84^\circ, 86^\circ, 88^\circ$





$$d (\sin \alpha + \sin \beta) = k \lambda$$

$$k N \lambda = \sin \alpha + \sin \beta$$

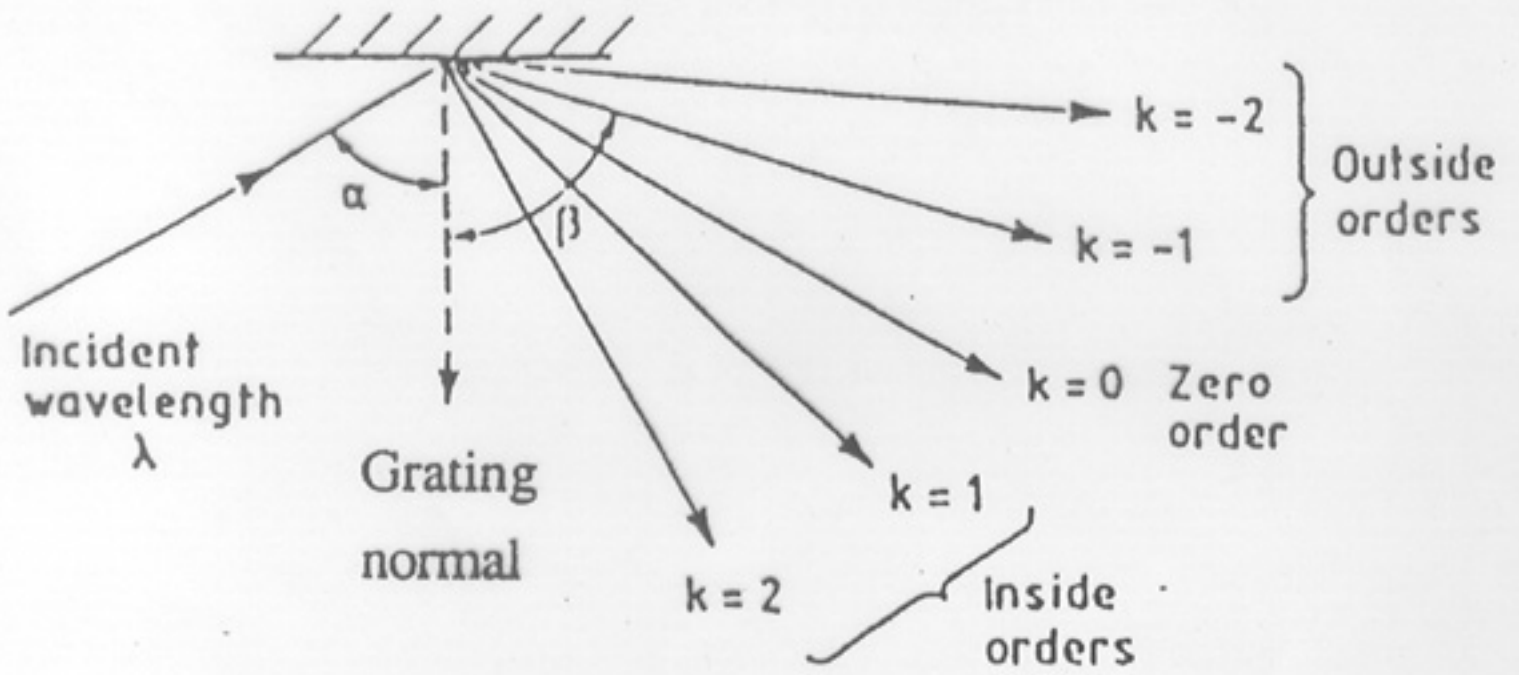
k = order of diffraction: $0, \pm 1, \pm 2$, etc.
 $N = 1/d =$ line density

Slit Limited Resolution

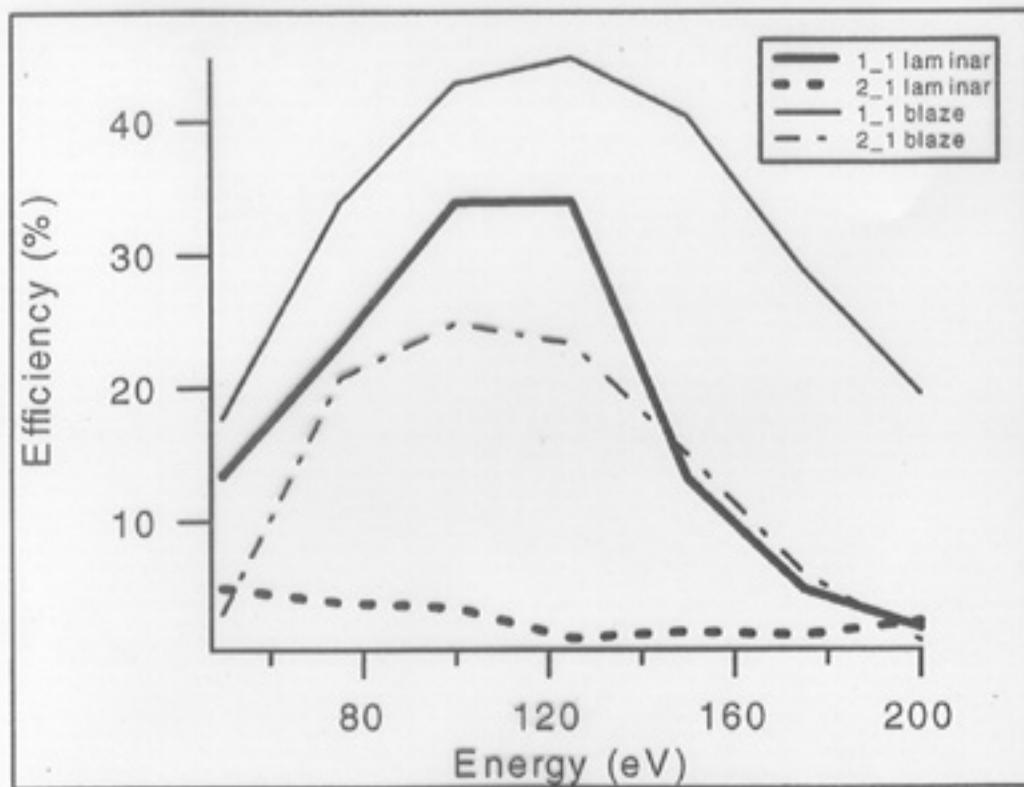
$$\text{Entrance: } \Delta\lambda_{\text{ent}} = \frac{1}{Nk} \frac{s}{r} \cos\alpha$$

$$\text{Exit: } \Delta\lambda_{\text{exit}} = \frac{1}{Nk} \frac{s'}{r'} \cos\beta$$

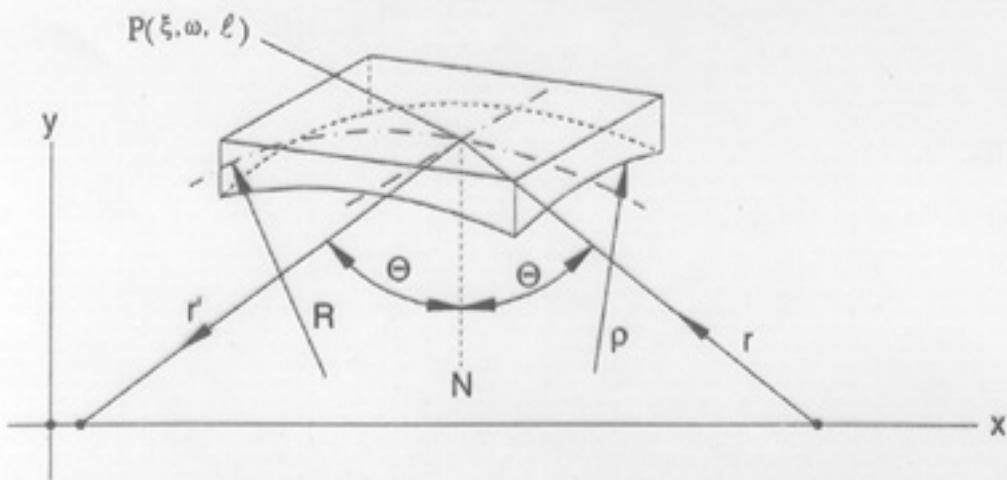
Grating density N



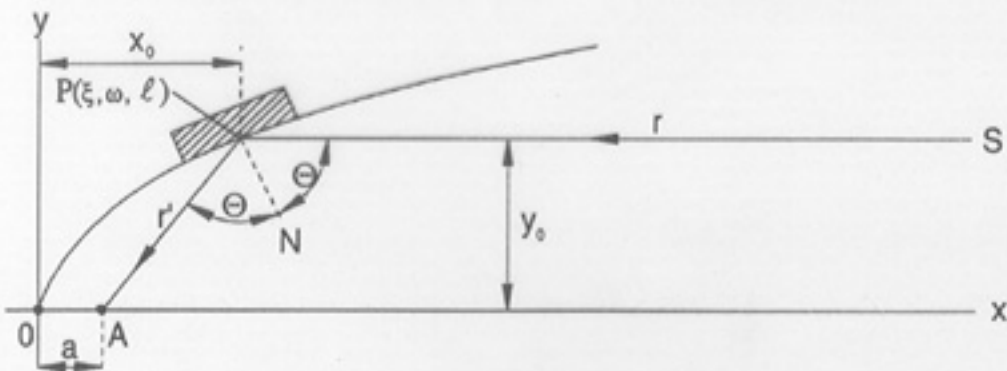
Example: the low energy grating of the
BACH beamline at ELETTRA



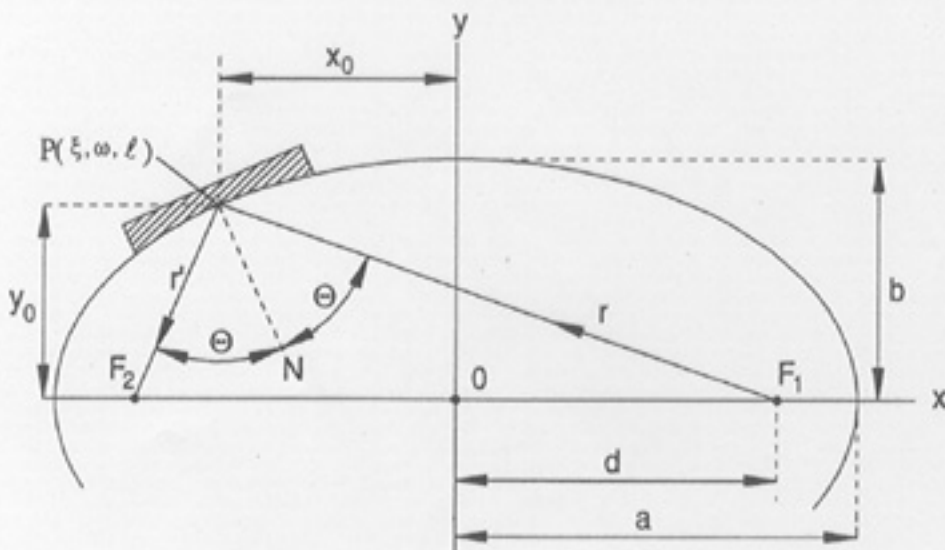
a. Toroid ($\rho < R$) or sphere ($\rho = R$)

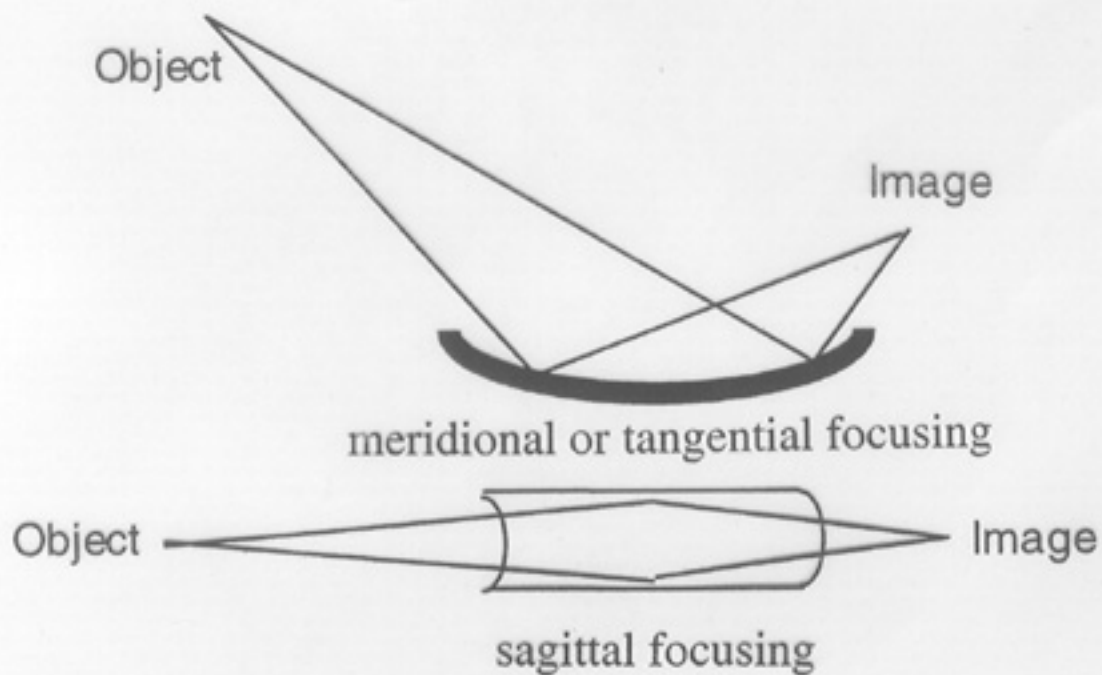


b. Parabola



c. Ellipse





toroid ($\rho < R$) or sphere ($\rho = R$)

meridional focusing $\left(\frac{1}{r} + \frac{1}{r'}\right) \frac{\cos\theta}{2} = \frac{1}{R} \quad f = \frac{R \cos\theta}{2}$

sagittal focusing $\left(\frac{1}{r} + \frac{1}{r'}\right) \frac{1}{2 \cos\theta} = \frac{1}{\rho} \quad f = \frac{\rho}{2 \cos\theta}$

for $R = \rho$ and $\theta = 87^\circ$ is
 $f_s \approx 365 f_m$

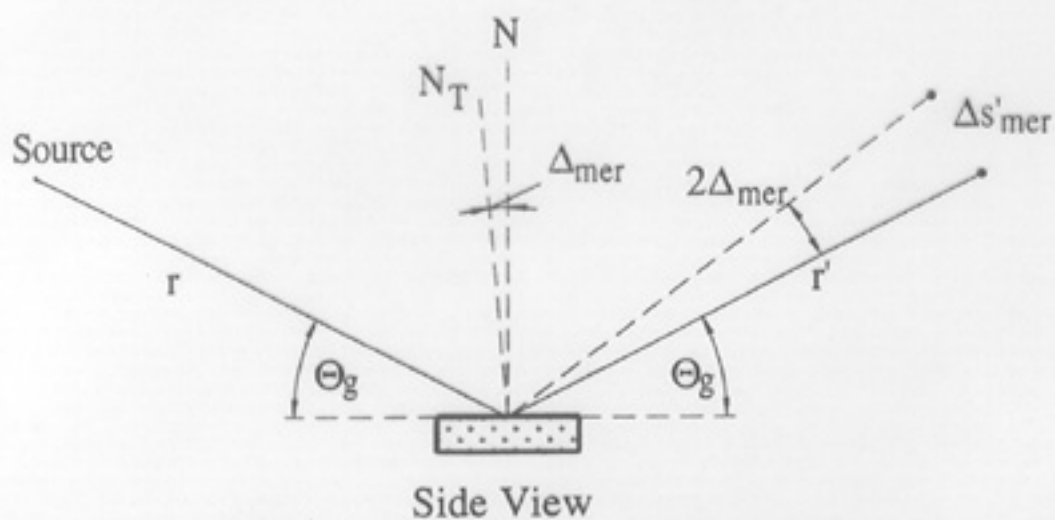
Typical achievable figure accuracy

Surface shape	Accuracy (arcsec. RMS)
Flat	< 0.1
Spherical	< 0.1
Cylindrical	< 0.5
Toroidal	< 0.5
Elliptical Toroid	< 0.5
Paraboloid	< 1.0
Ellipsoid	< 1.0

remember: 1 sec \approx 5 μ rad

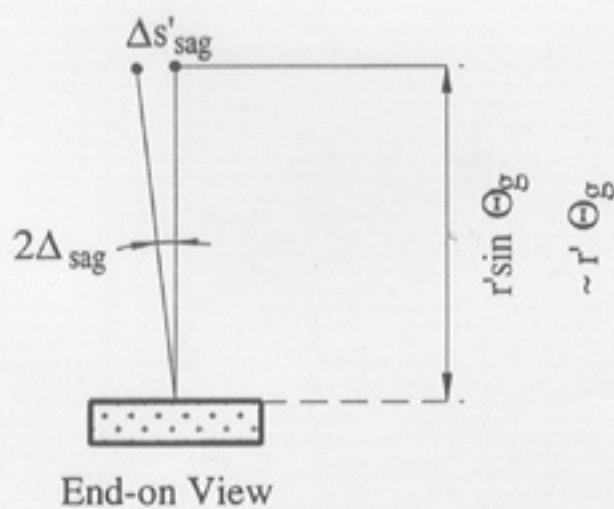
Meridional and Sagittal Tangent Errors

a. Meridional tangent error: Δ_{mer}



$$\text{Thus, } \Delta s'_{mer} = 2r'\Delta_{mer}$$

b. Sagittal tangent error: Δ_{sag} see figure above



$$\Delta s'_{sag} = 2r'\sin\theta_g \cdot \Delta_{sag}$$

$$\approx 2r'\theta_g \cdot \Delta_{sag}$$

Heat load on the optical elements: cooling is needed

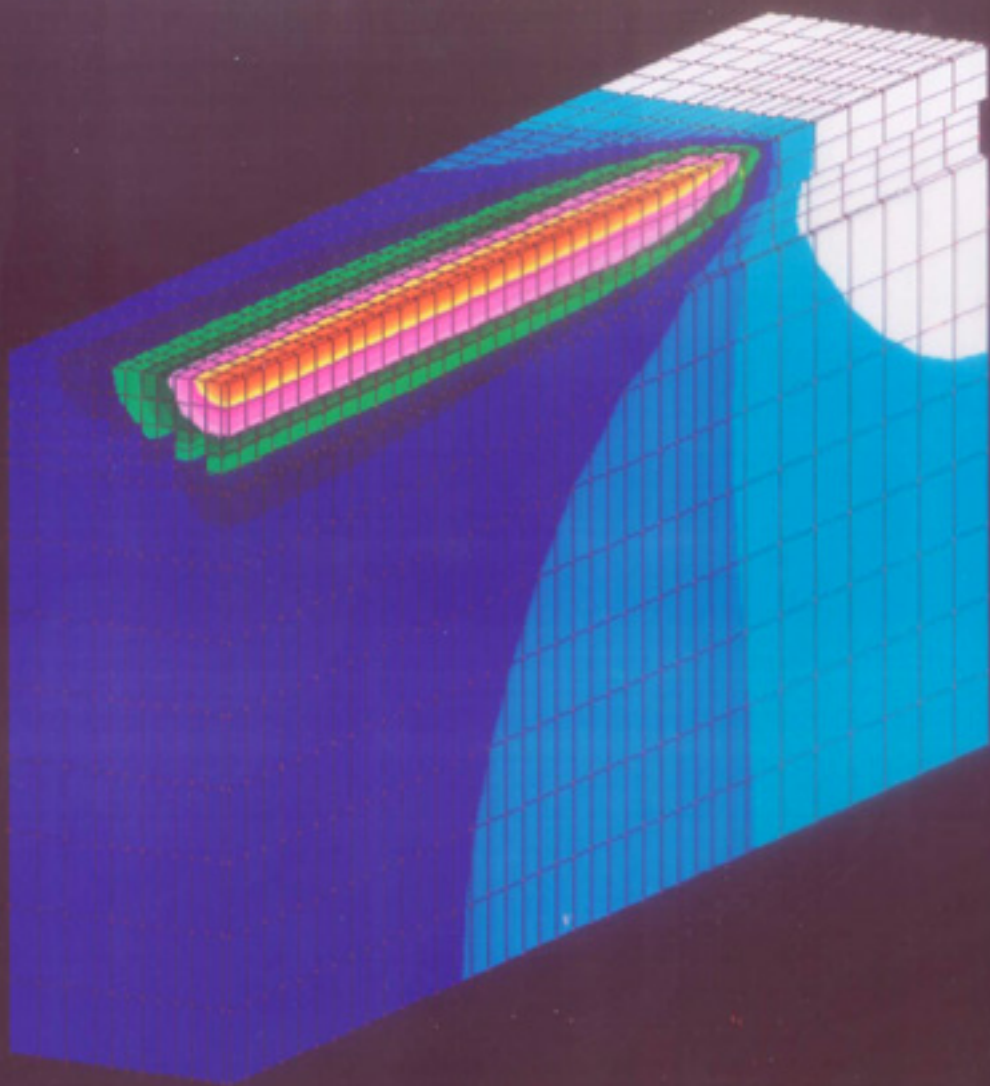
example:

first mirror of the SuperESCA beamline at ELETTRA

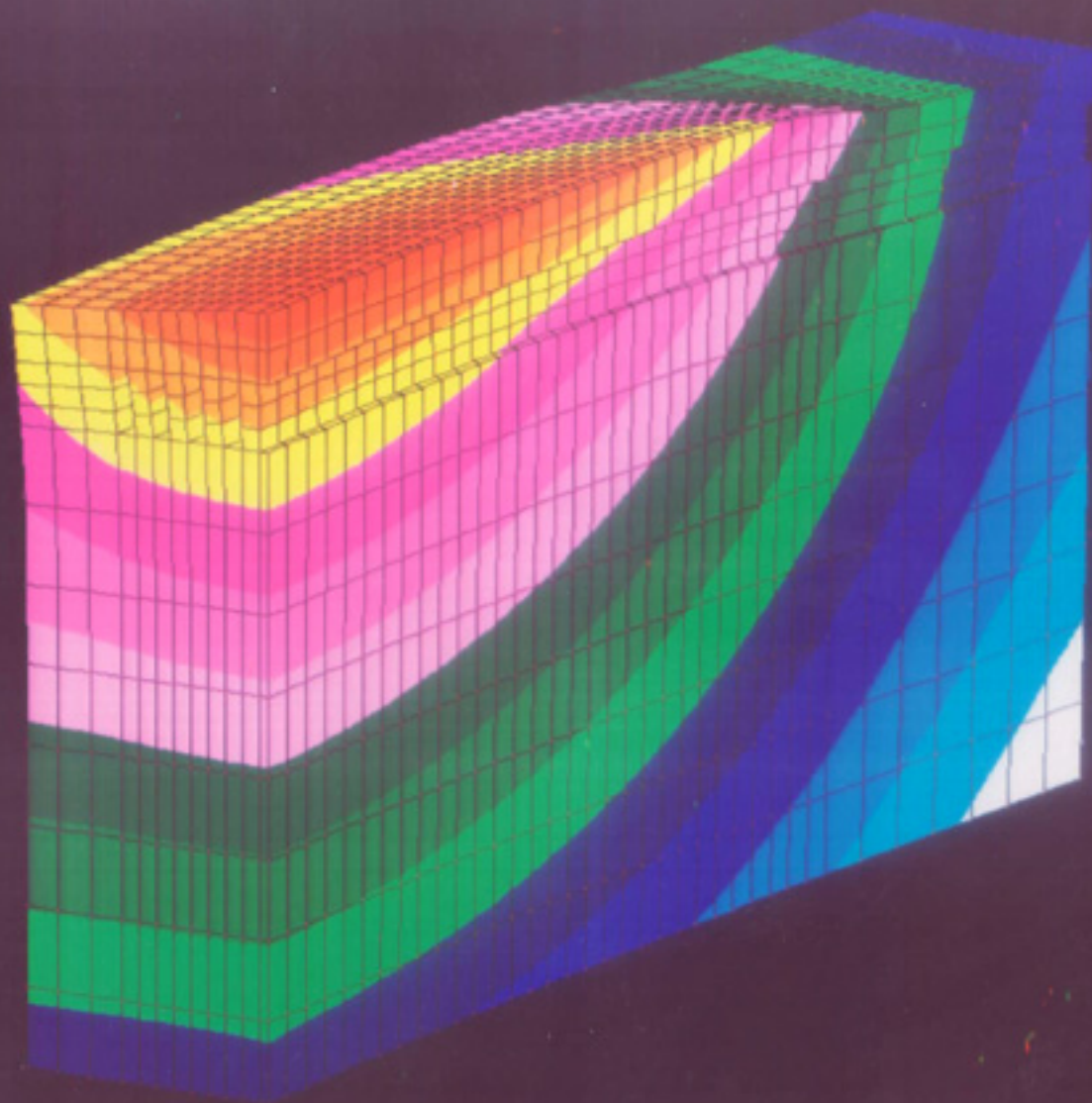
source: U5.6 undulator

max absorbed power: 75 W

max induced slope error: 2 arcsec



SWITCHING MIRROR/HEAT TRANSFER ANALYSIS
 ABAQUS V5.2-1 22-NOV-94 13:55:25 5217 6840
 PROCEDURE 31 TIME STEP 1 INCREMENT 1



.000445

.000404

.000362

.000320

.000279

.000237

.000196

.000154

.000112

.0000709

.0000293

+ .0000123

+ .0000539

+ .0000954

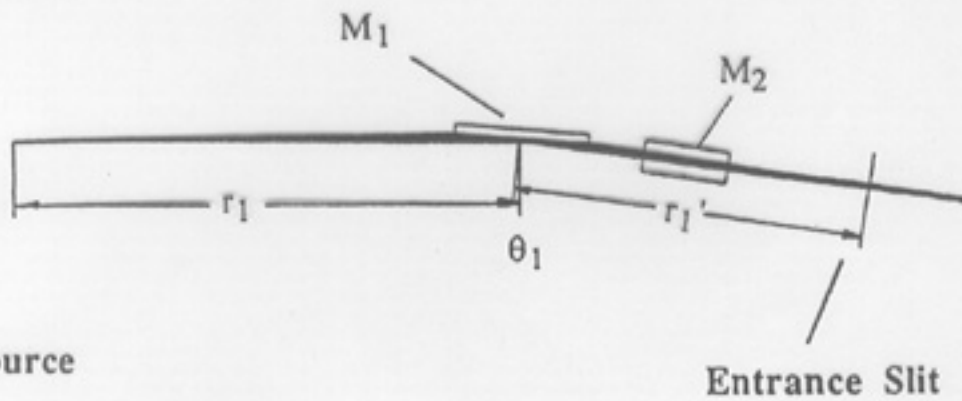
+ .000137

+ .000179

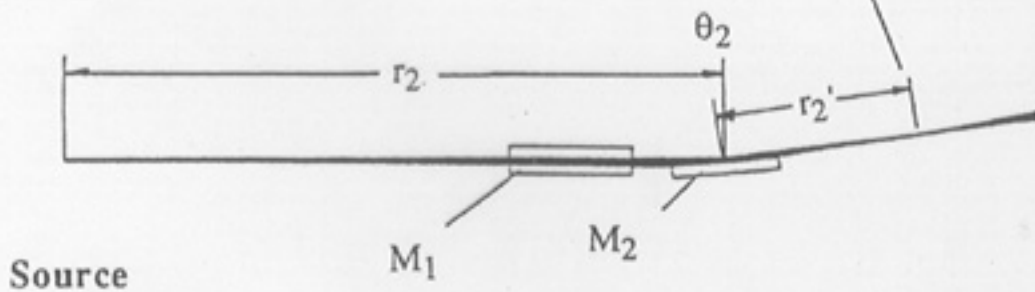
SWITCHING MIRROR/DEFORM. ANALYSIS
ABAQUS V5.2-1 22-NOV-94 17:00:11 5217 6840
PROCEDURE 1 TIME STEP 1 INCREMENT 1

The Kirkpatrick-Baez Optical System

Top View



Side View



- separation of horizontal and vertical focusing
- the horizontal focusing mirror comes first:
it absorbs most of the heat load

Example of extreme demagnification

$$\text{magnification } M = \frac{r'}{r}$$
$$\text{demagnification} = \frac{1}{\text{magnification}}$$

Source parameters

$$\gamma = 3327 \text{ (E = 1.7 GeV)}$$

$$\varepsilon_x = 6 \text{ nm}\cdot\text{rad}$$

$$C = 0.10$$

$$\varepsilon_y = 0.6 \text{ nm}\cdot\text{rad}$$

$$\sigma_{eh} = 219 \text{ }\mu\text{m}$$

$$\sigma_{ev} = 42 \text{ }\mu\text{m}$$

$$\sigma'_{eh} = 27 \text{ }\mu\text{rad}$$

$$\sigma'_{ev} = 14 \text{ }\mu\text{rad}$$

$$\text{undulator length } L = 4100 \text{ mm}$$

$$\sigma'_{rh} = 160 \text{ }\mu\text{rad}$$

$$\sigma'_{rv} = 40 \text{ }\mu\text{rad or } 80 \text{ }\mu\text{rad}$$

$$r = 17000 \text{ mm}$$

$$\begin{aligned} \text{effective source size} &= \pm 2\sigma_{ev} \text{ (includes 95\% of total flux)} \\ &= 168 \text{ }\mu\text{m} \end{aligned}$$

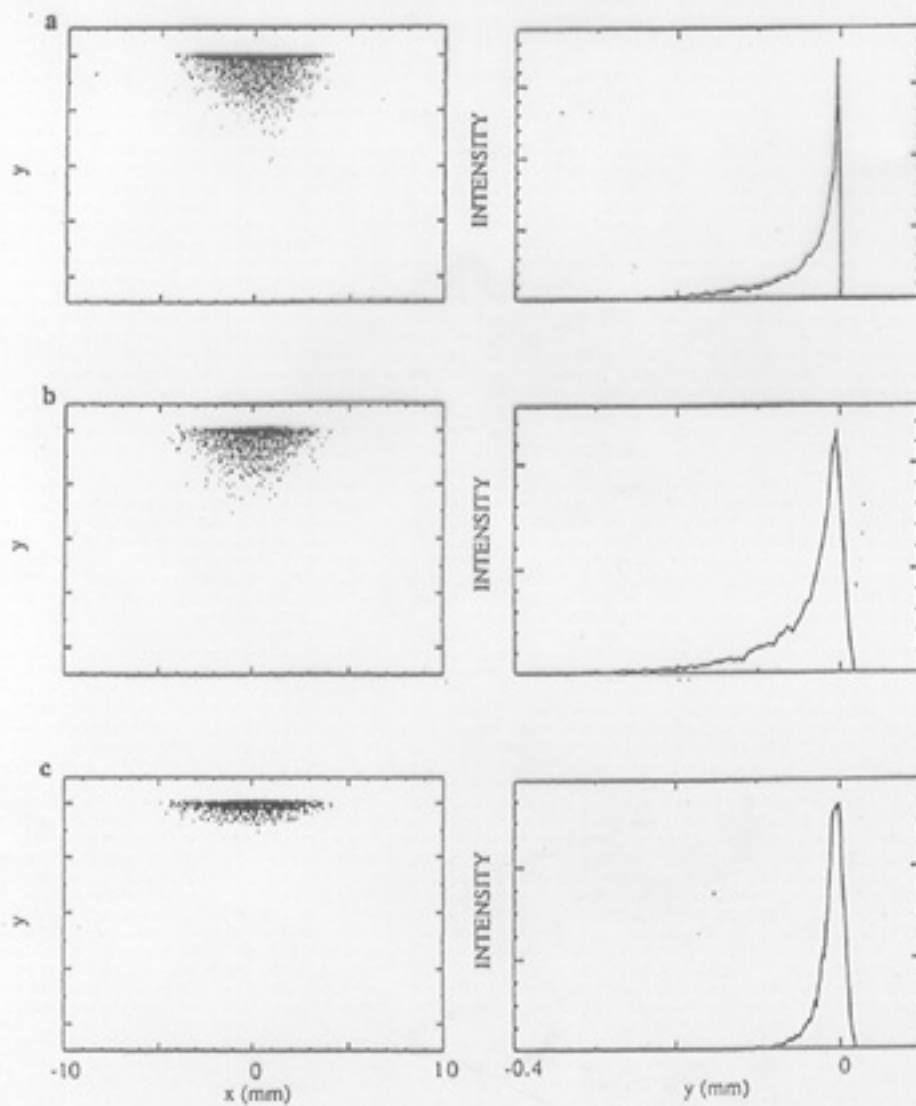
$$\text{try nominal demagnification} = 24$$

$$\text{entrance slit size} = 168/24 = 7 \text{ }\mu\text{m}$$

Focussing Characteristics of a Spherical Mirror

To the left the spot diagram. To the right, the integrated vertical profile.
 The angle of incidence, θ , is 87.5° .

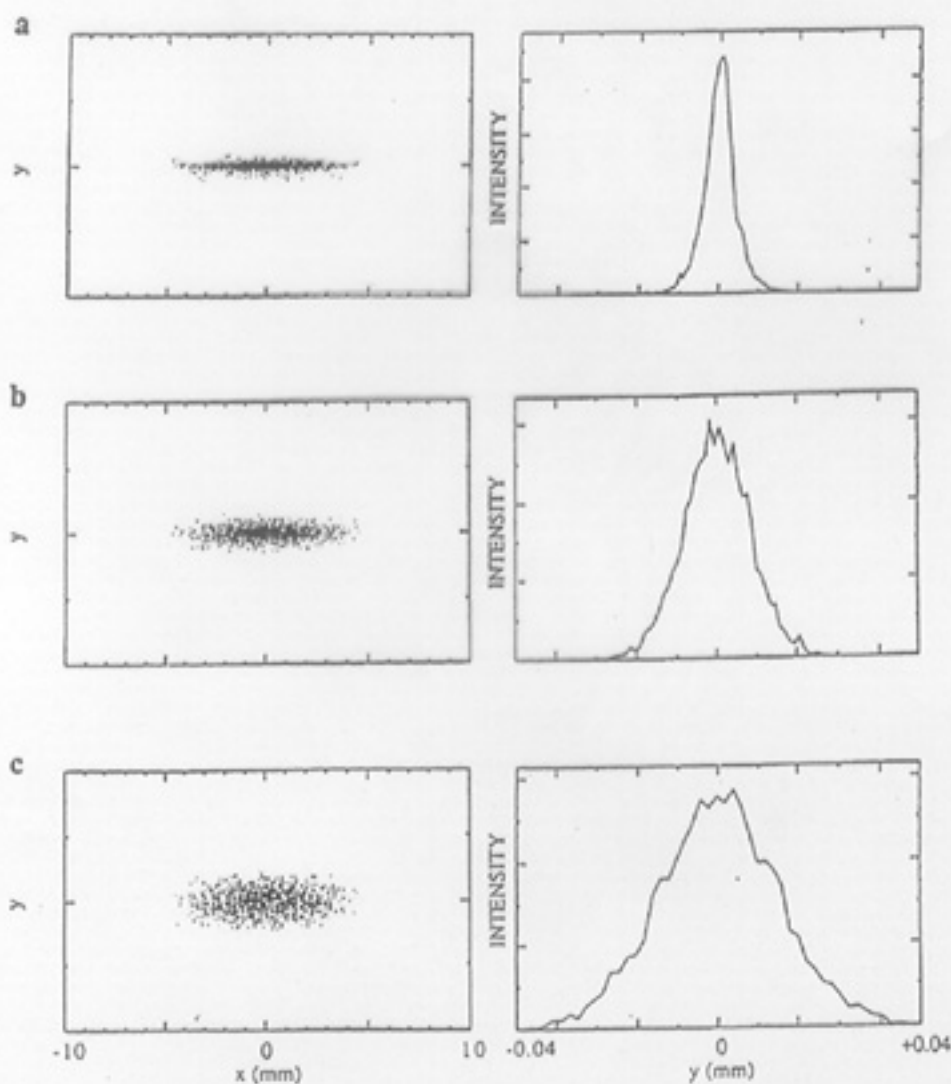
- | | | |
|--------------------------|---------------------------------|-----------------------------------|
| a) Demagnification = 24, | $\sigma_{te} = 0$, | $\sigma'_{rv} = 80 \mu\text{rad}$ |
| b) Demagnification = 24, | $\sigma_{te} = 1 \text{ sec}$, | $\sigma'_{rv} = 80 \mu\text{rad}$ |
| c) Demagnification = 24, | $\sigma_{te} = 1 \text{ sec}$, | $\sigma'_{rv} = 40 \mu\text{rad}$ |



Focussing Characteristics of a Plane Elliptical Mirror

To the left the spot diagram. To the right, the integrated vertical profile.
The angle of incidence, θ , is 87.5° .

- a) Demagnification = 24, $\sigma_{te} = 0$, $\sigma'_{rv} = 80 \mu\text{rad}$
- b) Demagnification = 24, $\sigma_{te} = 1 \text{ sec}$, $\sigma'_{rv} = 80 \mu\text{rad}$
- c) Demagnification = 24, $\sigma_{te} = 2 \text{ sec}$, $\sigma'_{rv} = 80 \mu\text{rad}$



spherical mirror

effective demagnification:

a. $168/37 = 4.5$

b. $168/53 = 3.2$

c. $168/35 = 4.8$

plane elliptical mirror

effective demagnification:

a. $168/12 = 14$

b. $168/48 = 3.5$

c. $168/80 = 2.1$

For nominal demagnification = 10

effective demagnification

sphere	$\sigma_{te} = 0 \text{ sec}$	$168/28 = 6.0$
	$\sigma_{te} = 1 \text{ sec}$	$168/89 = 1.9$

plane ellipse	$\sigma_{te} = 0 \text{ sec}$	$168/24 = 7.0$
	$\sigma_{te} = 1 \text{ sec}$	$168/72 = 2.3$

in practice nominal demagnification = 5-8

Beamline design

Requests

- Energy range
- Energy resolution
- Number of gratings to be used
(available energy range without grating change)
- Flux at the sample
- (- Spot size at the sample)

Constraints

- The effective source size σ_{tv} is of the order of 20-50 μm which makes $4\sigma_{tv} = 80-200 \mu\text{m}$.
- For high energy resolution the source must be demagnified somewhere before the exit slit of the monochromator. For very high resolution down to 10 μm .
- Depending on which kind of monochromator is used, a demagnifying mirror system might be necessary in front of the monochromator. Its maximum demagnification is of the order of 8-10.
- The angle of grazing incidence on the optical elements should be small enough to ensure reflectivity.
- Minimization of optical aberrations poses a limit on the grating and other optical elements length, which defines the maximum effective acceptance angles σ'_{tv} and σ'_{th} from the source.
- The finite length of the grating defines α_{max} and hence the low energy limit of the grating.
- The energy dependence of the resolving power of the grating poses a limit on the highest energy which can be reached with a single grating (see below).

Three examples of monochromators/beamlines

toroidal grating monochromator (TGM)

spherical grating monochromator (SGM)

Petersen plane grating monochromator (PGM)

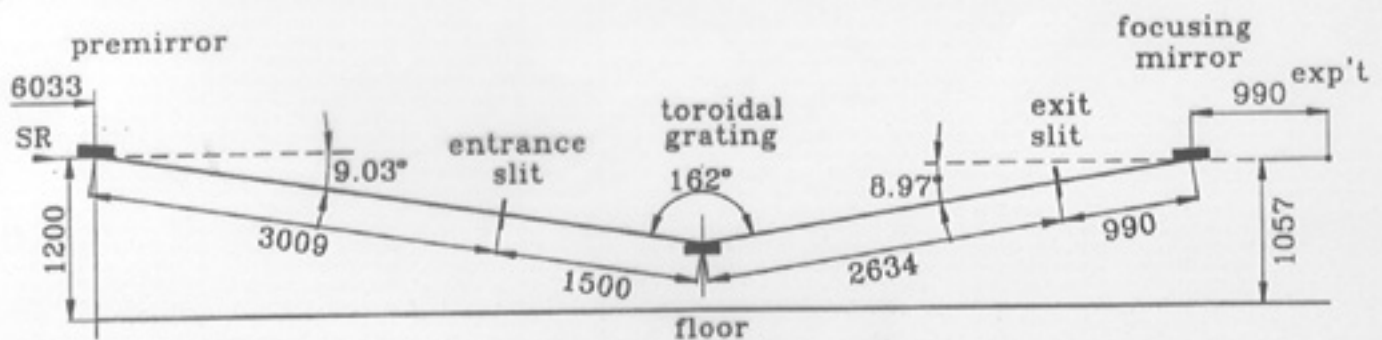
The TGM

the most commonly found type of monochromator at the synchrotron radiation laboratories around the world

simplicity: except for a rotation of the grating everything is fixed in space

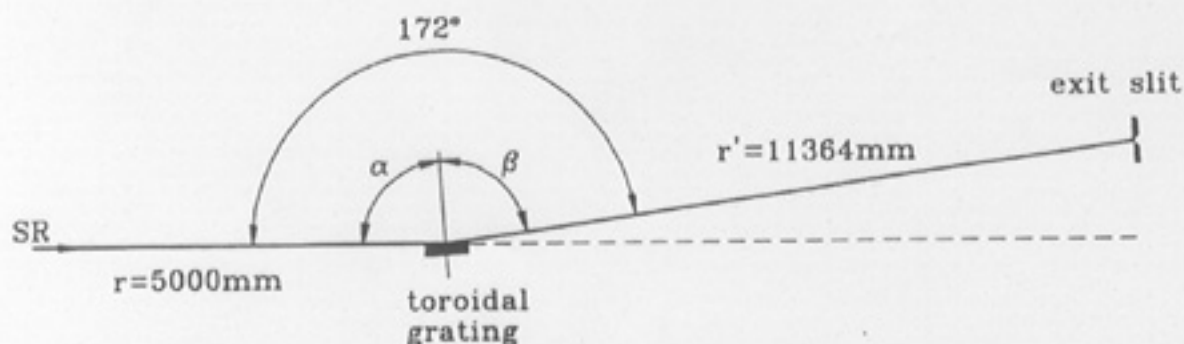
A TGM for Photon Energies from ca. 15 to 160 eV

Shown is a typical toroidal grating monochromator with a 1:2 demagnifying preoptical system and refocussing mirror behind the exit slit. Three gratings are used: 200 ℓ/mm , 600 ℓ/mm and 1800 ℓ/mm . The acceptance is 10 mrad (H) and 2.5 mrad (V) [6.21].



A TGM for Photon Energies from ca. 180 to 1100 eV

A toroidal grating monochromator consisting of a grating and an exit slit. The electron beam itself serves as the source. Two gratings are used: 1100 ℓ/mm and 1500 ℓ/mm . The radii are $R = 116000$ mm and $\rho = 483$ mm. The acceptance is 6 mrad (H) and 1 mrad (V) [6.22].



design criterium:

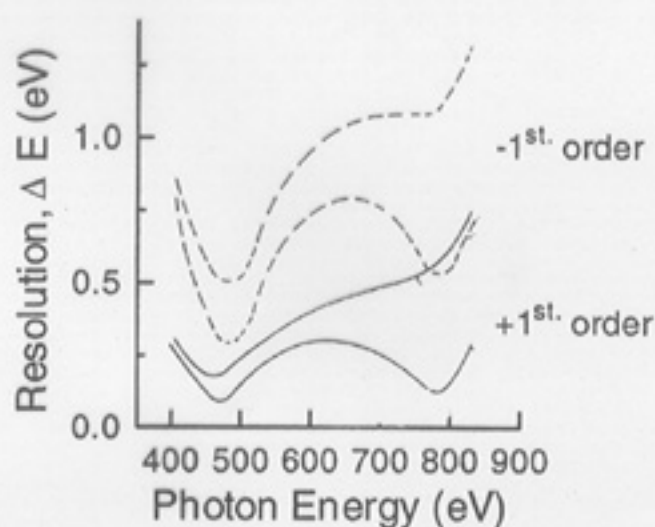
minimize defocussing (F_{200}) and astigmatic coma (F_{120})

$$\Delta\lambda \approx \frac{1}{Nk} \left(\omega F_{200} + \frac{1}{2} \ell^2 F_{120} \right)$$

resolving power $E/\Delta E = 200-500$

The Performance of a TGM for 180-1100 eV Photons

a. The optimization curves for the toroidal grating monochromator shown in figure 6.2.2 with an 1800 ℓ/mm grating and an acceptance of 2.0 mrad (H) and 1.4 mrad (V) are shown for source sizes of 0.2 mm (upper curves) and 0 mm (point source) lower curves [6.22].



The SGM

the Rowland conditions

$$\text{set } \left(\frac{\cos^2 \alpha}{r} - \frac{\cos \alpha}{R} \right) = 0$$
$$\left(\frac{\cos^2 \beta}{r'} - \frac{\cos \beta}{R} \right) = 0$$

$$r = R \cos \alpha \quad \text{and} \quad r' = R \cos \beta$$

$$F_{200} = 0$$

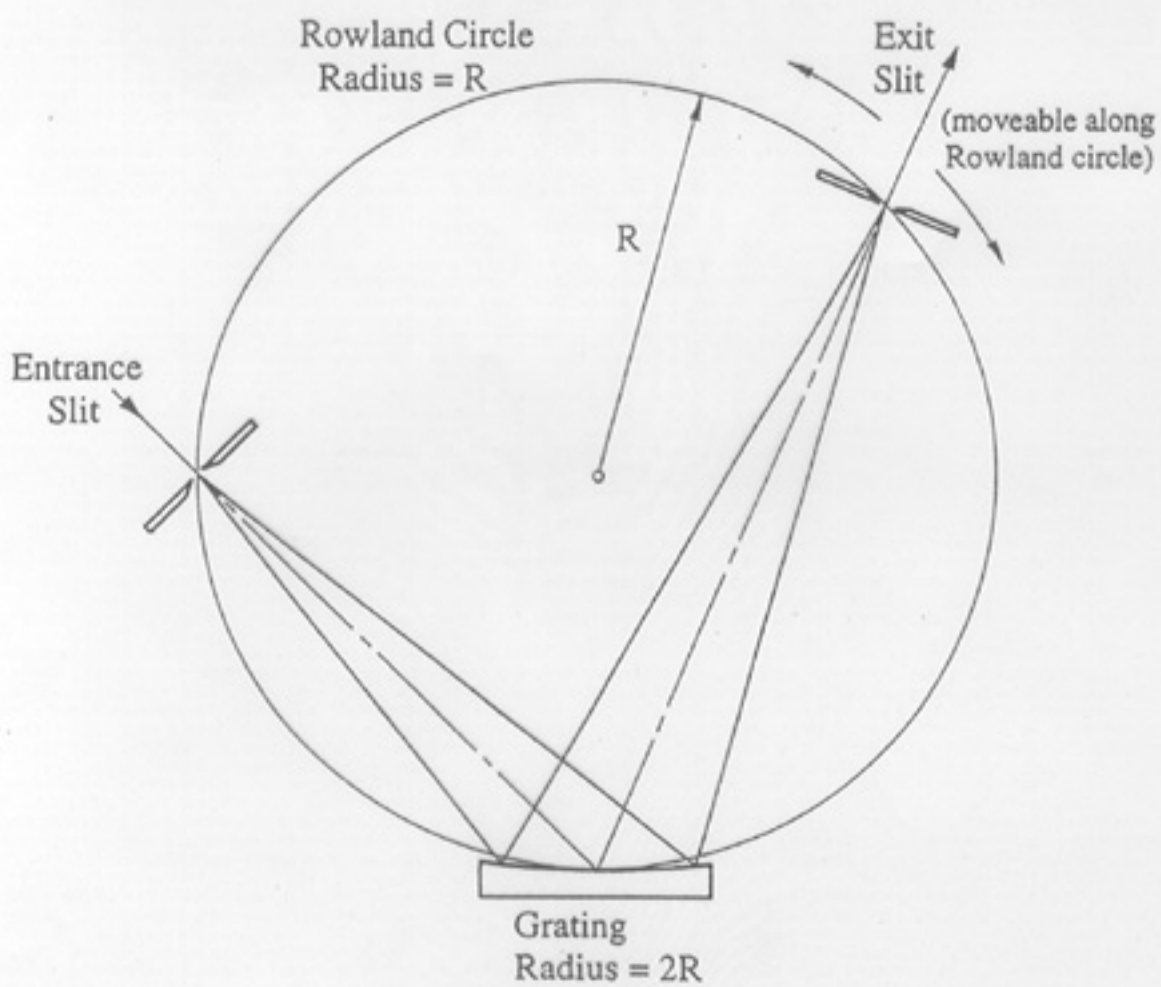
meridional focus

$$F_{300} = 0$$

primary coma

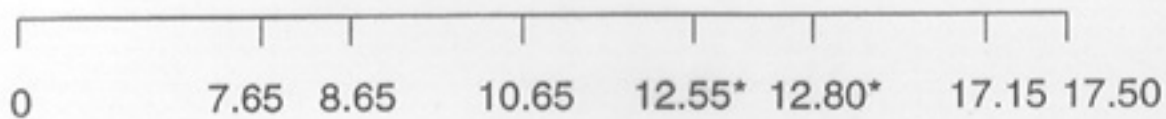
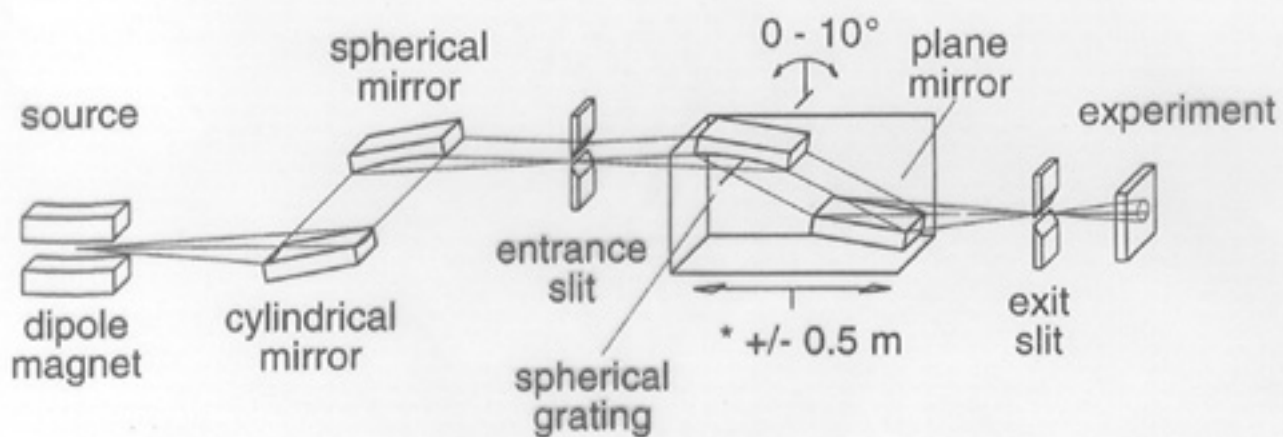
$$F_{400} = -\frac{1}{R^3} (\cos \alpha + \cos \beta) + \frac{1}{R^2} \left(\frac{1}{r} + \frac{1}{r'} \right)$$

The Rowland Circle Monochromator



The Constant Length Rowland Circle Monochromator

In the following design the arm lengths, r and r' are made variable but without moving the slits in order to fulfill the Rowland conditions. This is accomplished by moving the grating back and forth between the slits. Hence the name constant length Rowland circle monochromator [6.32-6.34]. Note that only one additional optical element is required, a plane mirror.



Distance to Source [m]

The resolution of a Rowland circle monochromator

$$Nk\lambda = \sin \alpha + \sin \beta \qquad \frac{\partial \lambda}{\partial \beta} = \frac{1}{kN} \cos \beta$$

$$\lambda = \frac{C}{E} \quad C=\text{constant} \qquad d\lambda = \frac{C}{E^2} dE$$

$$d\beta = \frac{s'}{r'}$$

$$\text{Rowland condition} \qquad r' = R \cos \beta$$

$$\Delta E = \frac{E^2 \cos \beta}{CkN} \cdot \frac{s'}{r'} = \frac{s' E^2}{CkNR}$$

for a SGM several gratings are needed for high resolution in a wide energy range

The magnification of a monochromator

$$\text{magnification } M(\lambda) = \frac{s'}{s}$$

$$Nk\lambda = \sin \alpha + \sin \beta$$

$$\frac{\partial \lambda}{\partial \alpha} = \frac{1}{kN} \cos \alpha$$

$$\frac{\partial \lambda}{\partial \beta} = \frac{1}{kN} \cos \beta$$

$$\Delta \alpha = \frac{s}{r} \quad \text{and} \quad \Delta \beta = \frac{s'}{r'}$$

$$\Delta \lambda \text{ at the two slits equal} \quad \frac{\cos \alpha}{Nk} \Delta \alpha = \frac{\cos \beta}{Nk} \Delta \beta$$

$$\frac{s \cdot \cos \alpha}{r} = \frac{s' \cdot \cos \beta}{r'}$$

$$M(\lambda) = \frac{s'}{s} = \frac{r' \cdot \cos \alpha}{r \cdot \cos \beta}$$

Case of the Rowland circle monochromator

$$r = R \cos \alpha \quad \text{and} \quad r' = R \cos \beta$$

$$M = \frac{R \cdot \cos \beta \cdot \cos \alpha}{R \cdot \cos \alpha \cdot \cos \beta} = 1$$

with a SGM no (or little) demagnification of the source is possible: it is necessary to have a demagnifying mirror system in front of the monochromator

The exactly focussed SGM

$$F_{100} = Nk\lambda - (\sin \alpha + \sin \beta) = 0$$

(grating equation)

$$F_{200} = \left(\frac{\cos^2 \alpha}{r} - \frac{\cos \alpha}{R} \right) + \left(\frac{\cos^2 \beta}{r'} - \frac{\cos \beta}{R} \right) = 0$$

(meridional focus)

for all energies

and $F_{300} = 0$ (primary coma)

for one energy inside the grating's range

one additional degree of freedom is needed
besides the rotation of the grating

movable exit slit
(Dragon Monochromator)

movable plane mirror inside the monochromator
(VASGM: Variable Angle SGM)

Spherical Grating Monochromators

$$\cos^2(\alpha)/r = \cos(\alpha)/R$$

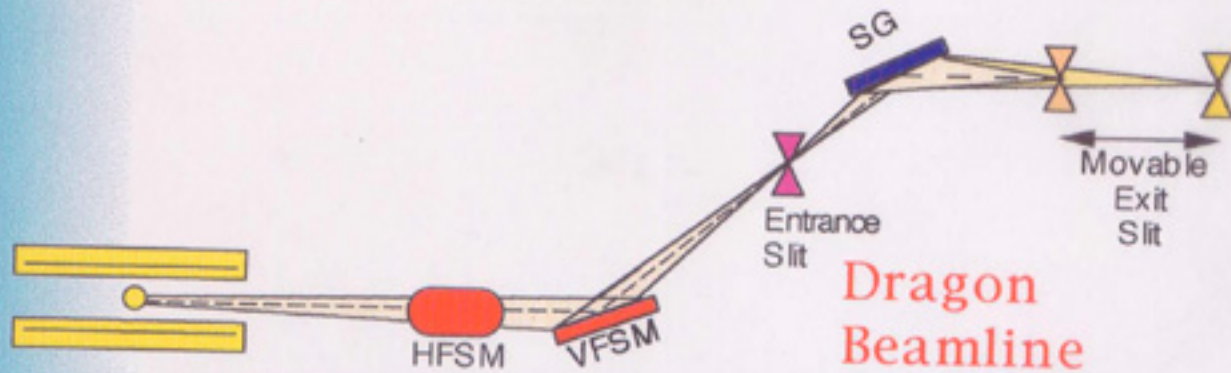
$$\cos^2(\beta)/r' = \cos(\beta)/R \rightarrow F_{200} = F_{300} = 0$$



Rowland circle configuration



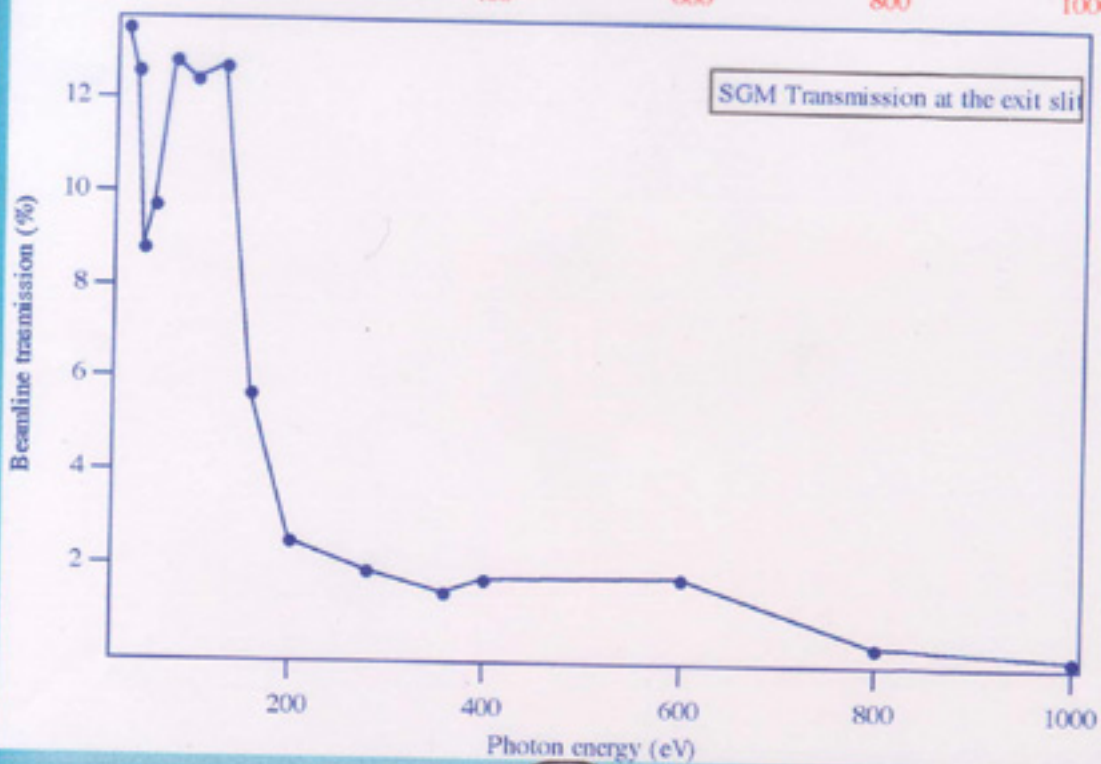
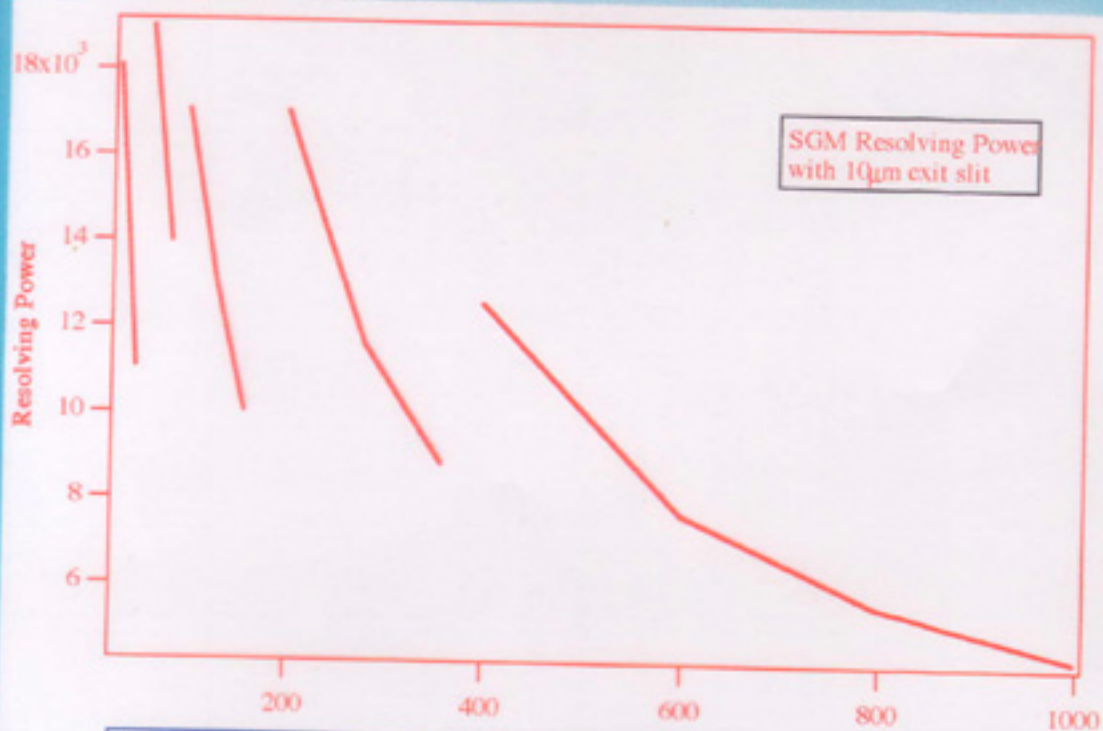
Variable included angle SGM



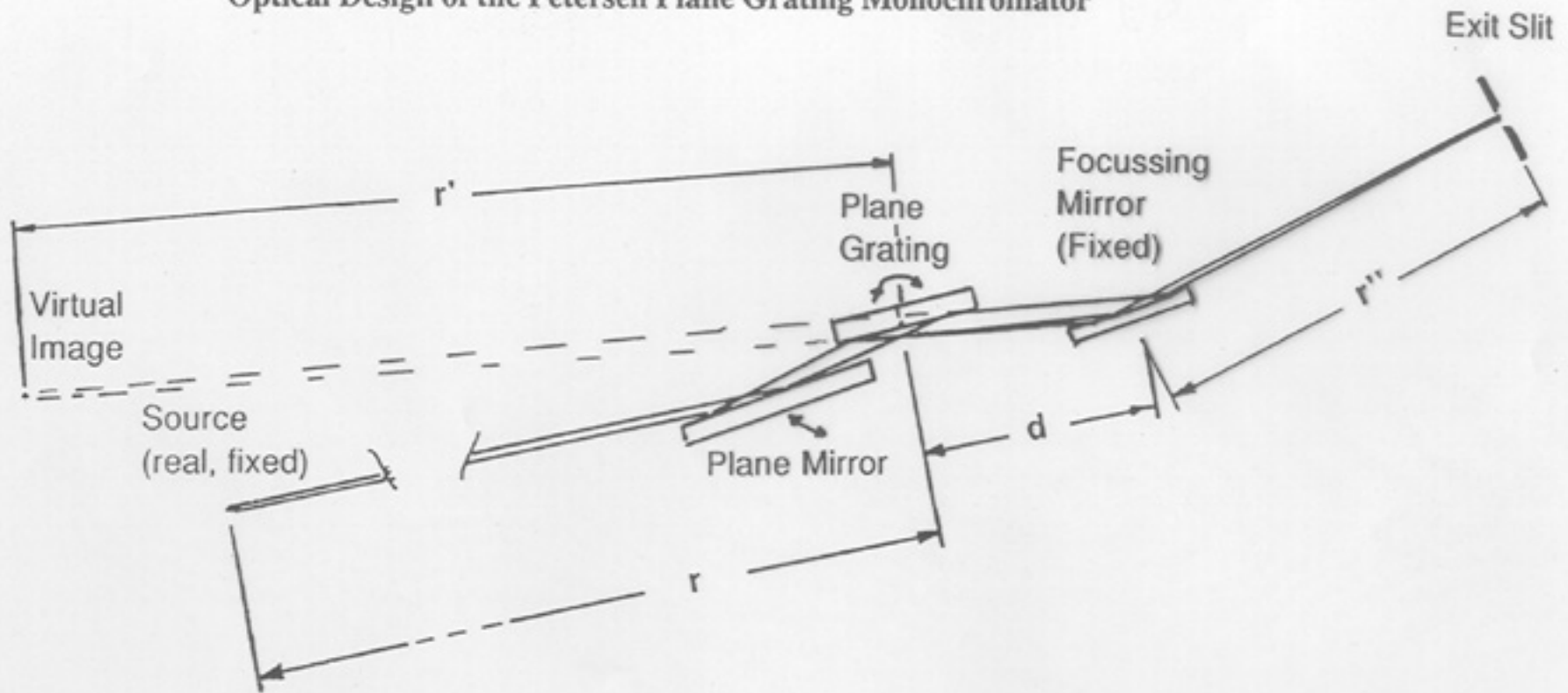
Dragon Beamline



SGM resolving power and transmission



Optical Design of the Petersen Plane Grating Monochromator



for a plane grating $R = \infty$

$$F_{200} = 0 \Rightarrow \frac{r'}{r} = -\frac{\cos^2 \beta}{\cos^2 \alpha} = -c_{\text{ff}}$$

where c_{ff} is a constant
optimum value $c_{\text{ff}} = 2.25$

magnification

$$M_{\text{grating}} = M_{\text{g}} = \frac{r' \cdot \cos \alpha}{r \cdot \cos \beta}$$

$$M_{\text{mirror}} = M_{\text{m}} = \frac{r''}{r' + d}$$

$$M = M_{\text{g}} \cdot M_{\text{m}} \sim \frac{r''}{r \cdot c_{\text{ff}}}$$

typical values $c_{\text{ff}} = 2.25$
 $r = 13000 \text{ mm}$
 $r'' = 5000 \text{ mm}$

$$M \sim 1/6$$

energy resolution $\Delta E \propto E^{3/2}$

Resolving Power Comparison

