School on Commutative Algebra and interaction with Algebraic Geometry and Combinatorics

THE HILBERT-KUNZ FUNCTION IN GRADED DIMENSION TWO

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Let R denote a two-dimensional normal standard-graded domain over an algebraically closed field of positive characteristic. We prove that the Hilbert-Kunz multiplicity of a primary homogeneous ideal is a rational number. If the field is the algebraic closure of a finite field we show that the Hilbert-Kunz function of such an ideal has no linear term and that its constant term is eventually periodic. We work with locally free sheaves on the corresponding projective curve and use the strong Harder-Narasimhan filtration due to Langer. Our work also suggests a definition for the Hilbert-Kunz multiplicity in characteristic zero.

ON THE ASYMPTOTIC BEHAVIOUR OF COHOMOLOGY

Markus P. Brodmann
Universität Zürich, Switzerland

We present a few results on the asymptotic behaviour of the n-th component of local cohomology of a graded module over a homogeneous noetherian ring for $n \ll 0$. We notably consider the behaviour of the set of associated primes and of various numerical invariants of these components (such as multiplicities, postulation numbers, numbers of generators...). We discuss examples which illustrate the unexpected complexity of this behaviour, even in surprisingly simple situations. Finally, we suggest a list of open problems related to our subject.

WEAKLY STABLE IDEALS AND BOUNDS FOR THE CASTELNUOVO MUMFORD REGULARITY

Giulio Caviglia
University of Kansas, Lawrence, USA

We study monomial ideals, called weakly stable, whose associated primes are lex-segment ideals. We use properties of such ideals to deduce an upper bound for

the Castelnuovo-Mumford regularity of homogeneous ideals in a polynomial ring in terms of the number of variables and the degree of the generators. We give in this way a different proof of a result of the author and E. Sbarra which shows that the known upper bound in characteristic zero holds true also in positive characteristic.

REGULARITY OF IDEALS AND THEIR POWERS

Marc Chardin
Université Pierre et Marie Curie, Paris, France

We will lecture about estimates on the Castelnuovo-Mumford regularity of powers and symmetric powers of ideals, and of Frobenius powers and tensor product of modules that generalizes and unifies several results on the subject.

GENERIC INITIAL IDEALS AND DISTRACTIONS

Aldo Conca
Università di Genova, Italia

The generic initial ideals of a given ideal are rather recent invariants. Not much is known about these objects, and it turns out to be very difficult to compute them. The main purpose of this paper is to study the behaviour of generic initial ideals with respect to the operation of taking distractions. Our main result states that the revlex generic initial ideal of any distraction of a strongly stable ideal I is the ideal I itself. In proving this fact we develop some new results related to distractions, stable and strongly stable ideals. We show, for example, that the gin of a "principal" stable ideal does not depend on the term order and on the characteristic of the base field. This is a joint work with A.Bigatti and L.Robbiano.

ON SEQUENTIALLY COHEN-MACAULAY MODULES

Nguyen Tu Cuong
Institute of Mathematics, Vietnam

Let R be a Noetherian local ring and M a finitely generated R-module. A filtration of submodules

$$0 = M_0 \subset M_1 \subset \ldots \subset M_t = M \qquad (*)$$

is called a dimension filtration of M if M_i is the largest submodule of M_{i+1} such that $\dim M_i < \dim(M_{i+1})$ for all $i = t-1, \ldots, 0$. Then the module M is called a *sequentially Cohen-Macaulay* module if M_{i+1}/M_i is a Cohen-Macaulay module for all i. The aim of this note is to prove a parametrical characterization of sequentially Cohen-Macaulay modules as follows.

Theorem. *Let M be a finitely generated R-module of dimension d with the dimension filtration (*) and $d_i = \dim M_i$. Then the following conditions are equivalent:*

i) M is a sequentially Cohen-Macaulay module.

ii) There exists a system of parameters $x = (x_1, \ldots, x_d)$ of M such that

$$\ell(M/(x_1^2, \ldots, x_d^2)M) = \sum_{i=1}^{t} 2^{d_i} e(x_1, \ldots, x_{d_i}; M_i).$$

iii) There exists a system of parameters $x = (x_1, \ldots, x_d)$ of M such that

$$\ell(M/(x_1^{n_1}, \ldots, x_d^{n_d})M) = \sum_{i=1}^{t} n_1 \ldots n_{d_i} e(x_1, \ldots, x_{d_i}; M_i)$$

for all $n_1, \ldots n_d > 0$.

This is a joint work with Doan Trung Cuong

TOROIDALIZATION OF MORPHISMS OF PROJECTIVE 3-FOLDS

Steven D. Cutkosky
University of Missouri, USA

We discuss our proof birational morphisms of 3-folds can be made toroidal by blowing up nonsingular centers in the domain and target.

INTEGRAL CLOSEDNESS OF MI FOR FINITELY SUPPORTED COMPLETE IDEALS

Clare D'Cruz
Chennai Mathematical Institute, India

Let I be a finitely supported complete ideal in a regular local ring (R, M). Assume that $k = R/M$ is algebraically closed field. We obtain necessary and suffifient conditions for MI to be integrally closed. This is obtained via the higher dimensional analogue of the formula of Hoskin and Deligne for the length of a finitely supported ideal in a regular local ring.

TORIC VARIETIES AND HYPERGEOMETRIC FUNCTIONS

Alicia Dickenstein
Universidad de Buenos Aires, Argentina

I will give an overview of multivariate hypergeometric functions associated with toric varieties, introduced by Gel'fand, Kapranov and Zelevinsky. I will highlight

the connections with commutative algebra and will present some recent results in collaboration with Cattani, Matusevich, Sadykov and Sturmfels.

ASSOCIATED PRIMES OF LOCAL COHOMOLOGY MODULES

Kamran Divaani-Aazar
Institute for Studies in Theoretical Physics and Mathematics, Tehran

In this talk, I will discuss my joint work with Amir Mafi on finiteness of associated primes of local cohomology modules [**2**]. Throughout, let R be a commutative Noetherian ring with identity. For an ideal $\mathfrak{a}$ of R and an R-module M, the i-th local cohomology module of M with respect to $\mathfrak{a}$ is defined as:

$$H^i_{\mathfrak{a}}(M) = \varinjlim_n \operatorname{Ext}^i_R(R/\mathfrak{a}^n, M).$$

In [**3**], Hartshorne defines an R-module M to be $\mathfrak{a}$-cofinite if $\operatorname{Supp}_R M \subseteq \operatorname{V}(\mathfrak{a})$ and $\operatorname{Ext}^i_R(R/\mathfrak{a}, M)$ is finitely generated for all $i \geq 0$. Then asks when the local cohomology modules of a finitely generated module are $\mathfrak{a}$-cofinite. In this regards, the best known result is that for a finitely generated R-module M if either $\mathfrak{a}$ is principal or R is local and $\dim R/\mathfrak{a} = 1$, then $H^i_{\mathfrak{a}}(M)$ are $\mathfrak{a}$-cofinite. Since for an $\mathfrak{a}$-cofinite module N, we have $\operatorname{Ass}_R N = \operatorname{Ass}_R(\operatorname{Hom}_R(R/\mathfrak{a}, N))$, it turns out that $\operatorname{Ass}_R N$ is finite. Huneke [**4**] made the following conjecture: If M is a finitely generated R-module, then the set of associated primes of $H^i_{\mathfrak{a}}(M)$ is finite for all ideals $\mathfrak{a}$ of R and all $i \geq 0$. Singh [**7**] gives a counter-example to this conjecture. However, it is known that this conjecture is true in many situations. For example, Brodmann and Lashgari [**1**, Theorem 2.2] showed that, if for a finitely generated R-module M and an integer t, the local cohomology modules $H^0_{\mathfrak{a}}(M), H^1_{\mathfrak{a}}(M), \ldots, H^{t-1}_{\mathfrak{a}}(M)$ are all finitely generated, then $\operatorname{Ass}_R(H^t_{\mathfrak{a}}(M))$ is finite. Also, for another proof of this result see [**5**]. For a survey of recent developments on finiteness properties of local cohomology, see Lyubeznik's interesting article [**6**].

In this talk, I first introduce the class of weakly Laskerian modules. This class includes all Noetherian modules and also all Artinian modules. Moreover, this class is large enough to contain all Matlis reflexive modules as well as all linear compact modules. Then, we establish the following. Let M be a weakly Laskerian module and $t \in \mathbb{N}$ a given integer. There is a finite subset X of $\operatorname{Spec} R$ such that

$$\operatorname{Ass}_R(H^t_{\mathfrak{a}}(M)) \subseteq \Big(\bigcup_{0 \leq i < t, 0 \leq j \leq t^2+1} \operatorname{Ass}_R(\operatorname{Ext}^j_R(R/\mathfrak{a}, H^i_{\mathfrak{a}}(M))) \Big) \cup X.$$

As an immediate consequence, we deduce that the first non $\mathfrak{a}$-cofinite local cohomology module of M with respect to $\mathfrak{a}$ has only finitely many associated prime ideals. This strengthens the main result of [**1**].

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HOMOLOGY MULTIPLIERS AND THE RELATION-TYPE OF PARAMETER IDEALS

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This is joint work with Ian Aberbach and Huy Tài Hà. The relation type question, raised by C. Huneke, asks whether for a complete equidimensional local ring R there exists a uniform number N such that the relation type of every ideal generated by a system of parameters is at most N. Wang gave a positive answer when the non-Cohen-Macaulay locus X of R has dimension zero. There is an example, due to I. Aberbach, which gives a negative answer when $\dim X \geq 2$. We investigate the remaining situation, i.e., when $\dim X = 1$. We introduce the notion of homology multipliers and show that the question has a positive answer when $R/A(R)$ is a domain, where $A(R)$ is the ideal generated by all homology multipliers in R.

THE INDEX OF REDUCIBILITY OF PARAMETER IDEALS IN FLC RINGS

Shiro Goto

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Let A be a Noetherian local ring with the maximal ideal $\mathfrak{m}$. For each parameter ideal Q in A, we denote by $\ell_A([Q : \mathfrak{m}]/Q)$ the length of $[Q : \mathfrak{m}]/Q$, and call it

the index of reducibility of Q. Let $cr(A) = \sup_Q \ell_A([Q : \mathfrak{m}]/Q)$, where Q runs over parameter ideals in A. When A is a Cohen-Macaulay local ring, the index $\ell_A([Q : \mathfrak{m}]/Q)$ of reducibility of Q is independent of the choice of Q, and one always has the equality $\mathrm{r}(A) = \ell_A([Q : \mathfrak{m}]/Q)$. This is no longer true, unless A is Cohen-Macaulay; one may have $\mathrm{r}(A) = \infty$ ([GSu]). The purpose of my talk is to report some partial answers to the following problem, especially in the case where A has FLC, that is the case where A is a generalized Cohen-Macaulay local ring.

Problem.

(1) *Estimate the supremum* $r(A) = \sup_Q \ell_A([Q : \mathfrak{m}]/Q)$.

(2) *When does the equality* $r(A) = \ell_A([Q : \mathfrak{m}]/Q)$ *hold true for a given parameter ideal* Q *in* A*?*

(3) *Describe the ring-theoretic properties of the Rees algebra* $R(I)$*, the associated graded ring* $G(I)$*, and the fiber cone* $F(I)$ *of the ideal* $I = Q : \mathfrak{m}$ *in the case where* $r(A) = \ell_A([Q : \mathfrak{m}]/Q)$.

The answers will naturally generalize some results of [GSa1, GSa2, GSa3].

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FINITE GENERATION OF SUBALGEBRAS OF A FINITELY GENERATED ALGEBRA

Mitsuyasu Hashimoto

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Let R be a Notherian commutative ring, $f : X \to Y$ a surjective morphism of R-schemes such that X is of finite type over R. We discuss when Y is of finite type over R.

We prove that if R is excellent, Y is reduced, and f is universally open, then Y is of finite type. As an application, we prove that if $X = \operatorname{Spec} S$ and $Y = \operatorname{Spec} A$ are both affine, and if A is a pure subalgebra of S, then Y is of finite type. We

also prove the following theorem, which is very similar to a theorem proved by J. Fogarty. Let G be a flat R-group scheme of finite type. Let X be a G-scheme of finite type over R. If (Y, f) is a universally submersive strict orbit space for the action of G on X, then Y is of finite type over R.

FINITENESS OF HILBERT FUNCTIONS AND BOUNDS FOR CASTELNUOVO-MUMFORD REGULARITY OF INITIAL IDEALS

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Bounds for the Castelnuovo-Mumford regularity of an arbitrary standard graded algebra over a field and its Hilbert coefficients are given in terms of the arithmetic degree (if the algebra is reduced) or in terms of the definning degrees. These bounds depend neither on term orders nor on the coordinates. The one for Castelnuovo-Mumford regularity in terms of the definning degrees is similar to that of [CS]. From this we can deduce two consequences. The first one is related to a result by Kleimann (see [K], [RTV]):

Theorem. *Given two positive integers a and d. Assume that K is an algebraically closed field. Then there exists only a finite number of Hilbert functions associated to reduced K-algebras S such that the arithmetic degree* $\operatorname{adeg} S \leq a$ *and* $\dim S = d$.

The second one strengthens a recent work done in [HH].

Theorem. *Let K be an infinite field and $I \subset R = K[x_1, ..., x_n]$. Let Δ denote the maximal degree Δ of minimal generators of I, $d = \dim R/I$ and $c = n - d$. With respect to any term order and any coordinates we have*

$$\operatorname{reg}(\operatorname{in} I) \leq (2\Delta^c)^{d2^{d-1}}.$$

Moreover, if R/I is a reduced algebra, then we also have

$$\operatorname{reg}(\operatorname{in} I) \leq (\operatorname{adeg}(I))^{(n-1)2^{d-1}}.$$

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NORMALIZATION OF MODULES

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The objective of this work is to study the normalization of the Rees algebra of a finitely generated module over a Noetherian normal domain. As in the case of ideals, we aim at deriving numerical properties of the integral closure of the Rees algebra of a module such as bounds for degrees of the generators. Also, we study the coefficients of the Buchsbaum-Rim polynomials of a module and those of the integral closure of the module. This is joint work with Bernd Ulrich and Wolmer V. Vasconcelos.

GRADED RINGS ASSOCIATED WITH CONCTRACTED IDEALS

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Our main objective is to study depth, Hilbert function and defining equations of the various graded rings (Rees algebra, associated graded ring and fiber cone) of homogeneous contracted ideals in the polynomial ring $R = k[x, y]$ over an algebraically closed field k of characteristic 0. Homogeneous contracted ideals are of two types: ideals with square-free characteristic form and ideals whose characteristic form is a power of a linear form. We show that if I is a contracted ideals with square-free characteristic form, then the Rees algebra, associated graded ring and the fiber cone are Cohen-Macaulay with expected defining equations. Contracted

ideals whose characteristic form is a power of a linear form are called lex-segment ideals. It can be seen that, in $R = k[x, y]$, $in(I)$ is a lex-segment ideal for any $\mathfrak{m}$-primary ideal, where $\mathfrak{m} = (x, y)$. We detect several classes of lex-segment ideals with associated graded ring having maximal and almost maximal depths. In particular, we show that lex-segment ideals occuring as the initial ideal of an $\mathfrak{m}$-primary ideal generated by generic forms have positive depth associated graded rings. We show that $H_I(n) \leq H_{in(I)}(n)$ for all $n \geq 0$, provided depth $\mathrm{gr}_{in(I)}(R) > 0$, where $H_I(n) = \lambda(I^n/I^{n+1})$ denotes the Hilbert function of I. We also prove that certain sub-class of lex-segment ideals corresponding to the generic ideals have normal Rees algebra and we explicitly obtain defining equations of the Rees algebra in such cases.

SEPARATING INVARIANTS

Gregor Kemper
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One of the main objectives of invariant theory is to find invariants that separate orbits under group actions. In this talk the concept of separating invariants, as opposed to generating invariants, will be discussed. It turns out that separating invariants have some surprisingly nice properties. In particular, the behavior of separating invariants under polarization will be discussed. This leads to a new, characteristic-free version of Weyl's theorem.

DIFFERENTIAL SIMPLICITY IN POLYNOMIAL RINGS AND ALGEBRAIC INDEPENDENCE OF POWER SERIES

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Instituto de Matemática Pura e Aplicada, Brasil

Let K be a field of characteristic zero, $f(X, Y), g(X, Y)$ elements of $K[X, Y]$ such that $g(X, Y)$ does not belong to (X, Y) and $d := g(X, Y).d/dX + f(X, Y).d/dy$. A connection is established between the d-simplicity of the localization of $K[X, Y]$ at (X, Y) and the transcendency of the solution in $tK[[t]]$ of the algebraic differential equation $g(t, y(t)).(d(y(t))/dt).y(t) = f(t, y(t))$. This connection is used to obtain some interesting results in the theory of formal power series and to construct new examples of differentially simple rings.

ENDOMORPHISM RINGS OF FINITE GLOBAL DIMENSION

Graham Leuschke
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For a commutative local ring R, consider (possibly noncommutative) R-algebras Λ of the form $\Lambda = \mathrm{End}_R(M)$, where M is a reflexive R-module with nonzero free direct summand. Such algebras Λ of finite global dimension can be viewed as possible

subsitutes for, or analogues of, a resolution of singularities of Spec R. For example, Van den Bergh has shown that a three-dimensional Gorenstein normal $\mathbb{C}$-algebra with terminal singularities has a crepant resolution of singularities if and only if it has such an algebra Λ with finite global dimension and which is maximal Cohen–Macaulay over R (a "noncommutative crepant resolution of singularities"). We produce algebras $\Lambda = \mathrm{End}_R(M)$ having finite global dimension in two contexts: when R is a complete one-dimensional local ring, or when R is a Cohen–Macaulay ring of finite Cohen–Macaulay type. If, in the latter case, R is Gorenstein, then the construction gives a noncommutative crepant resolution of singularities in the sense of Van den Bergh.

A SURPRISING FACT ABOUT D-MODULES IN CHARACTERISTICS $p > 0$

Gennady Lyubeznik
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Let k be a field of characteristic $p > 0$, let R be the ring of polynomials in a finite number of variables over k, let D be the ring of k-linear differential operators of R and let f be a non-zero polynomial in R. We prove that $R[1/f]$ obtained from R by inverting f is generated as a D-module by $1/f$. This is a striking fact considering that counterexamples to an analogous statement in characteristic 0 have long been known. This is joint work with Josep Alvarez Montaner of the University of Barcelona, Spain.

SINGULARITIES OF MODULI SPACES OF VECTOR BUNDLES IN CHARACTERISTIC 0 AND P

Vikram Mehta
Tata Institute of Fundamental Research, India
(joint work with Venkata Balaji)

We show that these moduli spaces in char 0 specialize correctly to the moduli spaces in char p , using a result of Hashimoto.We then conclude that the moduli varieties in char 0 have canonical singularities, using a result of Hara- Watanabe.

RELATIVELY COMPRESSED ALGEBRAS

Juan Migliore
University of Notre Dame, USA

I will discuss some recent joint work with Rosa Miró-Roig and Uwe Nagel. Most of the work deals with level algebras, so we begin with this context. A level Artinian algebra $A = R/I$ of socle degree s and socle type c is *compressed* if its Hilbert function is the maximum possible, given s and c. It is known precisely what this

maximum Hilbert function is. Except in special cases, though (i.e. roughly "half" of the Gorenstein cases, $c = 1$), the minimal free resolution of A is not known.

Generalizing this, suppose that $\mathfrak{a}$ is a sufficiently general complete intersection and that we are given s and c. Then a quotient A of $R/\mathfrak{a}$, that is level of type c and socle degree s, is said to be a *relatively compressed level algebra (with respect to* $\mathfrak{a}$*)* if the Hilbert function of A is maximal among all level quotients of $R/\mathfrak{a}$ with the same socle degree and socle type.

We are interested in the Hilbert function, and especially the minimal free resolution, of such algebras. For the Hilbert function, there is a natural upper bound analogous to the compressed case, which one proves with inverse systems. Unlike the compressed case, however, this bound is not always achieved. We discuss the correct "expected" Hilbert function. Furthermore, we extend this discussion beyond the level case to arbitrary socle type, and we show that the prediction for the Hilbert function in the general case is equivalent to Fröberg's conjecture for the Hilbert function of an ideal of general forms of fixed degrees.

Turning to minimal free resolutions, our main results are for Gorenstein algebras, but we also discuss the case of larger socle type. In the case of relatively compressed Gorenstein algebras of even socle degree, we explicitly compute the entire minimal free resolution and show that the graded Betti numbers are uniquely determined by the Hilbert function. This is analogous to the known results for compressed Gorenstein algebras. In the case of relatively compressed Gorenstein algebras of odd socle degree we give a slightly less precise result, but show by example that unexpected behavior can occur. We also give the precise minimal free resolution for odd socle degree in certain cases, relating it to a conjecture of Mustata.

In the case of higher socle type, we find that in some situations there are forced to be redundant ("ghost") terms, so the graded Betti numbers are not those that one would naively expect from the Hilbert function. This is not surprising when these ghost terms come because of Koszul relations (and duality), but we show that they can come for other reasons as well.

REES RING, FIBER RING OF CODIMENSION TWO LATTICES IDEALS

Marcel Morales
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In this lecture we blow up a variety described by a lattice ideal and we study the fiber over the origin, this is called vertical fiber by B. Teissier. More precisely, lattice ideals are a generalization of toric ideals. A lattice ideal is a binomial ideal in a polynomial ring $I \subset k[x_1, ..., x_n]$ such that any variable $x_1, ..., x_n$ is a non zero divisor modulo I. In this lecture we introduce the syzygy graph of a codimension two ideal, then we describe a presentation of the fiber ring combinatorial from the syzygy graph. In particular we prove that the analytic spread $l(I)$ is ≤ 3, the fiber ring and the Rees ring of I are Cohen-Macaulay. An explicit reduction of I is also given and I can be defined up to radical by less than 3 elements. This result

extends previous result in the simplicial case by Gimenez, Morales and Simis, and Barile-Morales.

AN EAKIN-SATHAYE THEOREM FOR COMPLETE AND JOINT REDUCTIONS

Liam O'Carroll
University of Edinburgh, Scotland

We present an Eakin-Sathaye Theorem for complete reductions (and so also for joint reductions) in the sense of Rees. Applications are outlined.

A CRITERION FOR REGULAR SEQUENCES

D. P. Patil
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Let R be a commutative noetherian ring and $f_1, \ldots, f_r \in R$. In this article we give a criterion for $f_1, \ldots, f_r$ to be a (locally) regular sequence for a finitely generated module over R which strengthens and generalises a result in [1]. As an immediate consequence we deduce that if $V(g_1, \ldots, g_r) \subseteq V(f_1, \ldots, f_r)$ and if $f_1, \ldots, f_r$ is a (locally) regular sequence in R, then $g_1, \ldots, g_r$ is also a (locally) regular sequence in R. More precisely, we prove the following :

Theorem. Let R be a commutative noetherian ring, $f_1, \ldots, f_r \in R$ and let M be a finitely generated R-module. Then the following statements are equivalent :

(i) $f_1, \ldots, f_r$ is a regular sequence on M.

(ii) $\operatorname{depth}_{R_\mathfrak{p}}(M_\mathfrak{p}) \geq r$ for every $\mathfrak{p} \in \operatorname{Supp}(M/(f_1, \ldots, f_r)M)$.

(iii) $\operatorname{depth}_{R_\mathfrak{p}}(M_\mathfrak{p}) \geq r$ for every $\mathfrak{p} \in \operatorname{Ass}(M/(f_1, \ldots, f_r)M)$.

Corollary 1. (Eisenbud, Herrmann, Vogel [1; Corollary 1]) *Let R be a commutative noetherian ring, $f_1, \ldots, f_r \in R$ and let M be a finitely generated R-module. Then $f_1, \ldots, f_r$ is a regular sequence on M if and only if $f_1, \ldots, f_r$ is a regular sequence on $M_\mathfrak{p}$ for every $\mathfrak{p} \in \operatorname{Ass}(M/(f_1, \ldots, f_r)M)$.*

Corollary 2. *Let R be a commutative noetherian ring and let $f_1, \ldots, f_r$, $g_1, \ldots, g_r \in R$. Let M be a finitely generated R-module with*

$$\operatorname{Supp}(M/(g_1, \ldots, g_r)M) \subseteq \operatorname{Supp}(M/(f_1, \ldots, f_r)M).$$

Suppose that $f_1, \ldots, f_r$ is a regular sequence on M. Then $g_1, \ldots, g_r$ is also a regular sequence on M. In particular, if $V(g_1, \ldots, g_r) \subseteq V(f_1, \ldots, f_r)$ and if $f_1, \ldots, f_r$ is a regular sequence in R, then $g_1, \ldots, g_r$ is also a regular sequence in R.

Corollary 3. *If M is a finitely generated Cohen-Macaulay module over a noetherian local ring R, then every system of parameters of M is a regular sequence on*

M . In particular, in a Cohen-Macaulay local ring every system of parameters is a regular sequence.

This is joint work with U. Storch and J. Stückrad.

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UNIFORM BOUNDS FOR THE RELATION TYPE

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Let A be a noetherian ring and $N \subseteq M$ be two finitely generated A-modules. The Artin-Rees lemma states that if I is an ideal of A, then there exists an integer $s \geq 1$, depending on I, M and N, such that for all integers $n \geq s$, $I^n M \cap N = I^{n-s}(I^s M \cap N)$. It is said that the pair (N, M) has the strong uniform Artin-Rees property with respect to a set of ideals $\mathcal{W}$ of A and with strong uniform Artin-Rees number $s = s(N, M; \mathcal{W})$ if, for all integers $n \geq s$ and for all ideals I of $\mathcal{W}$, $I^n M \cap N = I^{n-s}(I^s M \cap N)$. Using the fact that the strong uniform Artin-Rees number $s(N, M; \mathcal{W})$ is bounded above by the relation type of all ideals of $\mathcal{W}$, it can be deduced the strong uniform Artin-Rees property for some specific sets of ideals $\mathcal{W}$. This is joint work with J. M. Giral.

NORMALIZATION OF IDEALS

Claudia Polini

University of Notre Dame, USA

We will discuss constructions and devices to compute the integral closure of ideals. This is joint work with Ulrich, Vasconcelos and Villarreal.

NESTED SETS AND HOMOLOGY OF HYPERPLANE ARRANGEMENTS

Claudio Procesi

Università di Roma La Sapienza, Italia

The theory of wonderful compactifications of hyperplane arrangements allows to construct a homology basis dual to the no broken circuit basis, which is useful for explicit computations. We will present these results.

HILBERT COEFFICIENTS OF THE CANONICAL MODULE

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Let$(A, \mathfrak{m})$ be a Cohen-Macaulay module with a cannonical module ω_A. It is well known that both ω_A and A have the same multiplicity i.e $e_0(\omega_A) = e_0(A)$. A natural question is to find the relations between other Hilbert coeffecients of A and ω_A. Set $\mu(\omega_A) = \tau$, the Cohen-Macaulay type of A. We show

$$\tau^{-1}e_1(\omega_A) \leq e_1(A) \leq \tau e_1(\omega_A). \tag{1}$$

Furthermore we prove that

$$e_1(\omega_A) = \tau e_1(A) \text{ iff } A \text{ is Gorenstein.} \tag{2}$$
$$e_1(A) = \tau e_1(\omega_A) \text{ iff } A \text{ is Gorenstein or } A \text{ has minimal multiplicity.} \tag{3}$$

If $\dim A = 1$ and $G(A)$ is Cohen-Macaulay we also prove

$$e_i(A) \leq \tau e_i(\omega_A) \text{ for each } i \geq 1. \tag{4}$$

THE SUSLIN MATRICES

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In his doctoral thesis A. Suslin showed that a unimodular vector of the form $(a_0, a_1, a_2^2, \cdots, a_r^r)$ can be completed to an invertible matrix. Given a pair of vectors $v, w \in M_{1\ r+1}(R)$ he defines a matrix $S_r(v, w) \in M_{2^r}(R)$, with determinant equal to the inner product $v.w^T$. Consequently, if the dot product is 1 then $S_r(v, w) \in Sl_{2^r}(R)$. The actual completion is shown to be in the class $S_r(v, w)E_{2^r}(R)$, where $v = (a_0, a_1, a_2, \cdots, a_r)$, and for any w with $wv^t = 1$.

The Suslin matrices have proved useful in several contexts. We mention some examples.

• A. Suslin showed that

$$SK_1(\frac{k[x_1, \cdots, x_n]}{(\sum_{i=1}^n x_i y_i - 1)}) \simeq \mathbb{Z},$$

with generator $[S_{n-1}((x_1, \cdots, x_n), (y_1, \cdots, y_n))]$.

• Let $\sum_{i=1}^{n} x_i y_i = 1$. Let P, P^* be the projective modules corresponding to the unimodular rows $(x_1, \cdots, x_n)$, $(y_1, \cdots, y_n)$ respectively. Then $P^* \simeq Hom_R(P, R)$, the dual of P. If n is even then $P \simeq P^*$. However, if $n > 1$ is odd then M.V. Nori, and R.G.Swan independently showed (using topological arguments) that P, P^* need not be isomorphic. This can also be shown using the Suslin matrices, when $n > 3$.

• Inspired by the proof of the Serre's conjecture on the freeness of projective modules over a polynomial extension $k[x_1, \cdots, x_n]$ of a field k, and N. Mohan Kumar's theorems on set theoretic complete intersections, M. Boratanski showed that any ideal I in a polynomial ring R over a field can be generated upto radical by $m = \mu(\frac{I}{I^2})$ elements. Here the μ denotes the minimal number of generators of the R/I-module I/I^2. Thus, one has $\sqrt{I} = \sqrt{(f_1, \cdots, f_m)}$, for some $f_1, \cdots, f_m \in R$.
• The orbit spaces $Um_n(R)/E_n(R)$ have a nice Witt group structures, if R is a noetherian ring of Krull dimension d, in which, say, -1 is a square, and if $n \geq \max\{3, \frac{d+3}{2}\}$.

We briefly recapitulate these applications, and some Fundamental properties of these matrices, which allows us to relate the group generated by the Suslin matrices with Spinor groups.

BOUNDS FOR MULTIPLICITIES

Tim Römer
University of Osnabrück, Germany

Let $S = K[x_1, ..., x_n]$ be a polynomial ring and $R = S/I$ be a graded K-algebra where I is a graded ideal in S. Herzog, Huneke and Srinivasan have conjectured that the multiplicity of R is bounded above by a function of the maximal shifts in the minimal graded free resolution of R over S. We prove the conjecture in the case that $codim(R) = 2$ which generalizes results of Herzog, Srinivasan and Gold.

BLOWUP ALGEBRAS AND FILTRATIONS

Maria Evelina Rossi
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We present an unifying approach for studying numerical and homological characters of several graded algebras associated to an ideal of a local ring. As we already did in a joint paper with J.Herzog and G.Valla for the Symmetric Algebra and following an idea of Cortadellas concerning the Fiber Cone, we take advantage of the theory of the associated graded module to general filtrations in order to prove several results on certain blowup algebras which are not associated to any filtration.

We need to extend classical results in the theory of local rings to the case of filtrations on a module. Even if of intrinsic interest, the extension to modules has not been so studied. However some of the work done by D. Northcott, J.Fillmore, C. Rhodes, D. Kirby, H. Meheran and more recently T.Cortadellas and S. Zarzuela, J.Verma and A. Jayanthan, T. Puthenpurakal was carried over in the general setting.

UNIFORM BEHAVIOUR OF THE FROBENIUS CLOSURES OF IDEALS GENERATED BY REGULAR SEQUENCES

Rodney Y. Sharp
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This talk will describe joint work with Mordechai Katzman. Let R be a commutative Noetherian ring of prime characteristic p, and let I be a proper ideal of R. For $n \in \mathbb{N}_0$ (the set of non-negative integers), the *n-th Frobenius power* $I^{[p^n]}$ of I is the ideal of R generated by all p^n-th powers of elements of I. The *Frobenius closure* I^F *of* I, defined as

$$I^F := \left\{ r \in R : \text{ there exists } n \in \mathbb{N}_0 \text{ such that } r^{p^n} \in I^{[p^n]} \right\},$$

is an ideal of R contained in the tight closure of I. Since I^F is finitely generated, there exists $m_0 \in \mathbb{N}_0$ such that $(I^F)^{[p^{m_0}]} = I^{[p^{m_0}]}$, and we define $Q(I)$ to be the smallest power of p with this property. An interesting question is whether the set $\{Q(J) : J \text{ is a proper ideal of } R\}$ of powers of p is bounded. A simpler question is whether, for a given proper ideal I of R, the set $\{Q(I^{[p^n]}) : n \in \mathbb{N}_0\}$ is bounded. One of the results that will be described in this talk shows that the latter question has an affirmative answer when I is generated by a regular sequence. The method of proof employs a result of R. Hartshorne and R. Speiser, subsequently extended by G. Lyubeznik, that can be applied to a left module over the skew polynomial ring $R[x, f]$ that is Artinian (that is, 'cofinite' in the language of Hartshorne and Speiser) as an R-module.

LEXICOGRAPHIC GENERIC INITIAL IDEALS OF COMPLETE INTERSECTION SPACE CURVES

Jessica Sidman
Mt. Holyoke College, USA

The lexicographic term ordering is appealing both because we are accustomed to using the dictionary ordering in our daily lives and because it lends itself to a pretty geometric interpretation. Via a lexicographic Groebner basis for an ideal I, we can determine projections of $V(I)$ to rings with fewer variables.

In practice, computations with the lexicographic term ordering are often extremely unwieldy even for codimension 2 complete intersection ideals. I will discuss the Castelnuovo-Mumford regularity of the lexicographic generic initial ideals of generic complete intersection curves in $\mathbb{P}^3$. I will use the partial elimination ideals defined by M. Green to show the connection between regularity and geometry. This is joint work with Aldo Conca.

LEFSCHETZ THEOREMS FOR THE DIVISOR CLASS GROUP

Vasudevan Srinivas
Tata Institute of Fundamental Research, India

This is a report on some joint work with G. V. Ravindra, on Lefschetz theorems for the divisor class group of a normal projective variety, and an application to 1-motives.

ASSOCIATED PRIMES OF LOCAL COHOMOLOGY MODULES AND OF PROBENIUS POWERS

Irena Swanson
New Mexico State University

This is joint work with Anurag Singh. We construct normal hypersurfaces whose local cohomology modules have infinitely many associated primes. These include unique factorization domains of characteristic zero with rational singularities, as well as F-regular unique factorization domains of positive characteristic. In the same rings we construct ideals for which the set of associated primes of Frobenius powers is infinite. As a consequence, we answer a question on the associated primes of certain families of ideals which arose from the localization problem in tight closure theory.

ON THE BETTI NUMBERS OF SHIFTED COMPLEXES OF STABLE SIMPLICIAL COMPLEXES

Zhongming Tang
Suzhou University, P. R. China

Let Δ be a stable simplicial complex on n vertexes. Over an arbitrary base field K, the symmetric algebraic shifted complex Δ^s of Δ is defined. It is proved that the Betti numbers of the Stanley-Reisner ideals in the polynomial ring $K[x_1, x_2, \ldots, x_n]$ of the symmetric algebraic shifted, exterior algebraic shifted and combinatorial shifted complexes of Δ are equal.

THE GEOMETRY OF THE CONE AND THE ARITHMETICAL RANK OF AFFINE TORIC VARIETIES

Apostolos Thoma
University of Ioannina, Greece

We discuss how the geometry of the cone of an affine toric variety affects the arithmetical rank of the toric ideal. This is joint work with Marcel Morales.

SHARP UPPER BOUND FOR THE REGULARITY INDEX OF ZERO-SCHEMES

Phan Van Thiên
Normal University of Hue, Vietnam

Let $P_1, \ldots, P_s$ be distinct points in the projective space $\mathbb{P}^n := \mathbb{P}^n(k)$, where k is an arbitrary field. Denote by $\wp_1, \ldots, \wp_s$ the prime ideals in the polynomial ring $R := k[X_0, \ldots, X_n]$ corresponding to the points $P_1, \ldots, P_s$. Let $m_1, \ldots, m_s$ be positive integers and

$$A := R/(\wp_1^{m_1} \cap \cdots \cap \wp_s^{m_s}).$$

There has been much interest to find sharp upper bounds for the regularity $reg(A)$ in terms $m_1, \ldots, m_s$. In this talk I will discuss the following conjecture of Trung:

Conjecture. For $j = 1, \ldots, n$ put

$$T_j = \max\left\{\left[\frac{m_{i_1} + \cdots + m_{i_q} + j - 2}{j}\right] \mid P_{i_1}, \ldots, P_{i_q} \text{ lie on a } j\text{-plane}\right\}.$$

Then $reg(A) \leq \max\{T_j \mid j = 1, \ldots, n\}$.

This conjecture has an affirmative answer in the following cases:

1. The points are in general position (Catalisano, Trung and Valla [CTV]).
2. $n = 2$ (Fatabbi [Fa], see also [Th1]),
3. $n = 3$ (Thien [Th2], see also [FL]),
4. $n = 4$ with $m_1 = \cdots = m_s = 2$ (Thien [Th3]).

These cases can be proved by using an algebraic method presented in [T].

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MINIMALITY OF HILBERT-KUNZ MULTIPLICITY

Keiichi Watanabe
Nihon University, Japan

This is a joint work with Ken-ichi Yoshida. Let (A, m) be a Noetherian local ring of characteristic $p > 0$ of dimension d. We always assume A is unmixed. We denote by $e_{HK}(A)$ the Hilbert-Kunz multiplicity of A. Then always $e_{HK}(A) \geq 1$ and if $e_{HK}(A) = 1$, then A is regular. If A is not regular, what is the next value of $e_{HK}(A)$? We answer this question in case $d \leq 4$. Namely, the minimal value is taken if and only if A is an ordinary quadric.

Another minimality question is the following: Let $J \subsetneqq I$ be two m-primary ideals. Then what is the minimal value of $e_{HK}(J) - e_{HK}(I)$? We call this minimal value the minimal relative Hilbert-Kunz multiplicity of A and denote by $m_{HK}(A)$. We discuss some properties of $m_{HK}(A)$. For example, if $A = B^G$ is the invariant subring of a finite group G acting linearly on a polynomial ring B without pseudo-reflections, then $m_{HK}(A) = 1/|G|$.

CHARACTERIZATION OF GORENSTEIN RINGS VIA GORENSTEIN INJECTIVE DIMENSION

Siamak Yassemi
IPM and University of Tehran, Iran

Let $(R, \mathfrak{m}, k)$ be a Noetherian local ring. It is well-known that R is regular if and only if the injective dimension of k is finite. In this talk it is shown that R is Gorenstein if and only if the Gorenstein injective dimension of k is finite.

MODULES OF GENERALIZED FRACTIONS AND LOCAL COHOMOLOGY MODULES

Hossein Zakeri
Teacher Training University, Tehran, Iran

In this talk, I establish a quasi isomorphism of complexes,which is homogeneous in graded situation, from a given Cech complex to a certain complex of modules of generalized fractions.Also,we present some results on the vanishing and non-vanishing of top local cohomology modules.

ARITHMETIC PROPERTIES OF THE REES ALGBERA OF A MODULE

Santiago Zarzuela
University of Barcelona, Spain

Let R be a local ring and E a finitely generated R-module with finite rank $e > 0$. The Rees algebra of E is then defined as the symmetric algebra of E modulo its R-torsion: $R(E) = Sym(E)/T_R(Sym(E))$. Following B. Ulrich, A. Simis and W. V.

Vasconcelos [1], for a given reduction U of E and a system of generators of U, one may construct a Nagata extension S of R such that there exists a free S-submodule F of $E' = E \otimes_R S$ with rank $e-1$. When the quotient E'/F is isomorphic to an ideal I of S it is then said that I is a generic Bourbaki ideal of E. Under suitable conditions, the Rees algebra of E and the Rees algebra of a generic Bourbaki ideal I of E are well related and it is possible to study the properties of $R(E)$ by means of $R(I)$. In this talk we want to go further in this approach, mainly to study the depth of the Rees algebra of a module. This is joint work with Ana Luísa Branco Correia.

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