

the **abdus salam** international centre for theoretical physics

ICTP 40th Anniversary

SMR 1564 - 5

SPRING COLLEGE ON SCIENCE AT THE NANOSCALE (24 May - 11 June 2004)

**TDDFT THEORY:** 

APPLICATIONS TO NANO AND BIO-STRUCTURES

Optical Properties of Nanostructures: Extended systems: problems and new developments

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These are preliminary lecture notes, intended only for distribution to participants.



# Optical properties of nanostrutures Angel Rubio

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I. Motivation. Basic concepts. Foundations TDDFT.

II. Illustration of the physics for nano- and bio structures

III. Extended systems: problems and new developments



ICTP Spring College on Science at the Nanoscale, Trieste May 24<sup>th</sup> -June 11<sup>th</sup> 2004

## **Optical properties of nanostrutures**

## III. Extended systems: problems and new developments

Introduction:

how to handle the electron dynamics in extended systems under the influence of an external electromagnetic field?

TDDFT:

 Problems with standard exchange-correlation functionals
 A new fxc derived from Many-body perturbation theory proper description of excitonic effecs!!!
 Applications to poliacetilene as one-dimensional system

G. Onida, L. Reining and AR, Rev. Mod. Phys. 74, 601 (2002)

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#### Time-dependent approach for extended systems: a gauge formalism

G.F. Bertsch, J.I. Iwata, AR, K. Yabana, PRB62, 7998 (2000)

The Lagrangian of a periodic system in a volume V under a uniform field is

$$L = \sum_{i} \langle \psi_{i} | i \hbar \frac{\partial}{\partial t} - \frac{1}{2m} (\vec{p} + \frac{e}{c}\vec{A})^{2} - V_{ion} | \psi_{i} \rangle - E_{Hartree} - E_{xc} + \frac{V}{8\pi c^{2}} (\frac{d\vec{A}}{dt})^{2}$$

The equation of motion are:

$$i\hbar\frac{\partial}{\partial t}\psi_{i} = \left[\frac{1}{2m}\left(\vec{p} + \frac{e}{c}\vec{A}\right)^{2} + V_{ion} + V_{H} + V_{xc}\right]\psi_{i}$$
$$\frac{d^{2}\vec{A}}{dt^{2}} = -4\pi e^{2}\frac{n}{m}\vec{A} - 4\pi c\frac{e}{V}\sum_{i} \langle\psi_{i}|\frac{\vec{p}}{m}|\psi_{i}\rangle$$

For the electric field is:  $\vec{E}(t) = \frac{-1}{c} \frac{d\vec{A}}{dt}$   $\frac{d\vec{E}}{dt} = -4\pi\vec{j} \quad \text{and} \quad \vec{j} = \frac{-e}{V} \sum_{i} \langle \psi_{i} | \frac{\vec{p}}{m} | \psi_{i} \rangle - \frac{e^{2}}{c} n\vec{A}$ 

$$\bigvee V(x) + \Delta V_1$$

#### Lithium

Diamond



#### Non-local fxc for extended systems:

## Motivation

#### BSE vs TDLDA comparison on EEL



The LDA Kernel is not able to reproduce Optical Properties in Solids



The LDA Kernel already offers a good representation of the Electron Energy Loss (EEL) spectrum in Solids

See for a review: G. Onida, L. Reining and AR, Rev. Mod. Phys. 74, 601 (2002)

#### Why a non-local (static?) fxc for extended systems:

$$f_{xc} = -\alpha(\omega)/q^2$$

- In the EEL spectra  $f_{xc}$  is added to the full coulomb that already contains a long range contribution

$$EEL\alpha\epsilon^{-1}(\omega) = 1 + vX$$
$$X(\omega) = X_0(\omega) + X_0(\omega)(v + f_{xc}(\omega))X(\omega)$$

- In the absorption spectra  $f_{xc}$  is added to the full coulomb that does not contains the long range contribution (q=0)

$$\epsilon_{M}(\omega) = 1 - v \overline{X}$$
  $\epsilon_{M}^{RPA} \equiv 1/[1 - v X_{0}]_{G=G'=0}$ 

$$\bar{X}(\omega) = \chi_0^{GW}(\omega) + \chi_0^{GW}(\omega)(\bar{v} + f_{xc}(\omega))\bar{X}(\omega)$$

The lack of a long range term in  $f_{xc}^{LDA}$  is relatively weightless in the EEL but is crucial in the absorption spectra!!!

L. Reining, V. Olevano, AR, G. Onida, PRL88, 0664041 (2002); S. Botti et al, PRB (2004).

#### Density Functional versus Many-body perturbation theory



Band-gap problem!!!!!

## Density Functional Theory and Many-Body Perturbation Theory

$$\begin{bmatrix} -\frac{\nabla^2}{2} + V_{ext} (\mathbf{r}) + V_{Hartree} ([n], \mathbf{r}) + V_{xc} ([n], \mathbf{r}) \end{bmatrix} \phi_i (\mathbf{r}) = \epsilon_i \phi_i (\mathbf{r})$$
R. O. Jones and O. Gunnarsson, Rev. Mod. Phys. **61**, 689 (1989)
$$\underbrace{V_{xc} ([n], \mathbf{r})}_{V_{xc}} = \frac{\delta E_{xc} [n]}{\delta n (\mathbf{r})} \qquad E_{xc}^{LDA} [n] = \int d\mathbf{r} n (\mathbf{r}) \epsilon_{xc}^{hom} ([n]; \mathbf{r})$$
Density Functional Theory
Exchange-correlation Potential: Real, Local in space, Frequency independent
Self-Energy: Complex, Non-local in space, Frequency dependent

Self-Energy: Complex, Non-local in space, Frequency dependent

$$\left[\mathcal{H}_{KS}-V_{xc}\left(\mathbf{r}\right)\right]\left(\mathbf{r}\right)\phi_{i}\left(\mathbf{r};E_{\lambda}\right)+\int\,d\mathbf{r}'\Sigma\left(\mathbf{r},\mathbf{r}';E_{\lambda}\right)\phi_{i}\left(\mathbf{r}';E_{\lambda}\right)=E_{\lambda}\left(\omega\right)\phi_{\lambda}\left(\mathbf{r},E_{\lambda}\right)$$

Many-Body Perturbation Theory

*G. Onida, L. Reining and AR, Rev. Mod. Phys.* **74**, 601 (2002) F. Aryasetiawan, Rep. Prog. Phys. **61**, 237-312 (1998)



## G<sub>0</sub>W<sub>0</sub> Band Structures of Insulators



From "Quasiparticle calculations in solids", W.G. Aulbur, L. Jönsson and J.W. Wilkins, Solid State Physics 54 1 (2000), also available in preprint form at http://www.physics.ohio-state.edu/~wilkins/vita/publications.html#reviews



G. Onida, L. Reining and AR, Rev. Mod. Phys. 74, 601 (2002)



$$TDDFT \dots$$

$$t \left( \tilde{P}_{\mathbf{G}_{1}\mathbf{G}_{2}}(t) \right) = \left( \tilde{P}_{\mathbf{G}_{1},\mathbf{G}_{2}}^{0}(t) + \left( \tilde{P}_{\mathbf{G}_{1},\mathbf{G}_{3}}^{0}(t-t_{1}) \right) \left( f_{\mathbf{G}_{s},\mathbf{G}_{4}}^{c}(t_{1}-t_{2}) \right) \right) \left( \tilde{P}_{\mathbf{G}_{4}\mathbf{G}_{2}}(t_{2}) \right)$$

$$\iint d\mathbf{r}_{2} d\mathbf{r}_{3}\chi_{s} \left( \mathbf{r}_{1}, \mathbf{r}_{2}; \omega \right) \left[ \frac{1}{|\mathbf{r}_{1} - \mathbf{r}_{2}|} + f^{xc} \left( \mathbf{r}_{2}, \mathbf{r}_{3}; \omega \right) \right] \xi \left( \mathbf{r}_{3}; \omega \right) = \lambda \left( \omega \right) \xi \left( \mathbf{r}_{1}; \omega \right) \quad \lambda \left( E_{\lambda} \right) = 1$$

$$t \underbrace{\tilde{L}_{\mathbf{K}_{1}\mathbf{K}_{2}}(t)}_{\tilde{P}_{\mathbf{G}_{1},\mathbf{G}_{2}}(t)} = \underbrace{\tilde{L}_{\mathbf{K}_{1}}^{0}(t)\,\delta_{\mathbf{K}_{1}\mathbf{K}_{2}}}_{\tilde{L}_{\mathbf{K}_{3}}(t)} + \sum_{\mathbf{K}_{3}}i\int dt_{1} \underbrace{\tilde{L}_{\mathbf{K}_{3}\mathbf{K}_{2}}(t_{1})}_{t_{1}} \mathbf{K}_{1} \equiv \{c_{1}, v_{1}, \mathbf{k}_{1}\}$$

$$\tilde{P}_{\mathbf{G}_{1},\mathbf{G}_{2}}(\omega) \propto \sum_{\mathbf{K}_{1},\mathbf{K}_{2}}\Phi_{\mathbf{K}_{1}}^{*}(\mathbf{G}_{1})\,L_{\mathbf{K}_{1},\mathbf{K}_{2}}(\omega)\,\Phi_{\mathbf{K}_{2}}(\mathbf{G}_{2}) \qquad \Phi_{\mathbf{K}_{1}}(\mathbf{G}_{1}) = \langle c_{1}\mathbf{k}_{1}|e^{\mathbf{G}_{1}\cdot\mathbf{r}}|v_{1}\mathbf{k}_{1}\rangle$$

$$\tilde{P}_{\mathbf{G}_{1},\mathbf{G}_{2}}(\omega) \propto \sum_{\lambda}\frac{\Phi_{\lambda}^{*}(\mathbf{G}_{1})\,\Phi_{\lambda}(\mathbf{G}_{2})}{\omega - E_{\lambda}} \qquad \mathbf{H}|\lambda\rangle = E_{\lambda}|\lambda\rangle \qquad H_{\mathbf{K}_{1},\mathbf{K}_{2}} = (\epsilon_{c_{1}\mathbf{k}_{1}} - \epsilon_{v_{1}\mathbf{k}_{1}})\,\delta_{\mathbf{K}_{1},\mathbf{K}_{2}} + iW_{\mathbf{K}_{1},\mathbf{K}_{2}}$$

... and Many-Body Perturbation Theory

# Many-Body approach to the Exchange-Correlation Kernel of TDDFT

A diagrammatic approach

Hypothesis

It exists a "many-body xc-kernel" such that the TDDFT and Many-Body polarization functions are identical

Consequently TDDFT equation can be used as an equation for the xc-kernel and as a formal solution can be found in terms of an iterative equation for the nth order contribution

A. Marini, R. Del Sole and AR, PRL (2003)

#### Many-Body approach to the Exchange-Correlation Kernel of TDDFT

$$\begin{split} \overrightarrow{\textbf{P}} \mathbf{T} \\ \widetilde{\mathbf{P}}\left(\mathbf{q},\omega\right) &= \mathbf{P}^{(0)}\left(\mathbf{q},\omega\right) + \mathbf{P}^{(0)}\left(\mathbf{q},\omega\right) \mathbf{f}_{xc}\left(\mathbf{q},\omega\right) \widetilde{\mathbf{P}}\left(\mathbf{q},\omega\right) \\ \widetilde{\mathbf{P}}_{\mathbf{G}_{1},\mathbf{G}_{2}}\left(\mathbf{q},\omega\right) &= const. \sum_{\mathbf{K}_{1},\mathbf{K}_{2}} \Phi_{\mathbf{K}_{1}}^{*}\left(\mathbf{q},\mathbf{G}_{1}\right) \widetilde{S}_{\mathbf{K}_{1},\mathbf{K}_{2}}\left(\mathbf{q},\omega\right) \Phi_{\mathbf{K}_{2}}\left(\mathbf{q},\mathbf{G}_{2}\right) \\ \hline \\ & \overbrace{\mathbf{M}BPT} \\ \widetilde{\mathbf{S}}\left(\mathbf{q},\omega\right) &= \mathbf{S}^{(0)}\left(\mathbf{q},\omega\right) + \mathbf{S}^{(0)}\left(\mathbf{q},\omega\right) \mathbf{W}\left(\mathbf{q}\right) \widetilde{\mathbf{S}}\left(\mathbf{q},\omega\right) \\ \Phi_{\mathbf{K}}\left(\mathbf{q},\mathbf{G}\right) &= \langle c\mathbf{k} | e^{i(\mathbf{q}+\mathbf{G})\cdot\mathbf{r}} | v\mathbf{k} - \mathbf{q} \rangle \quad \mathbf{K} := (c v \mathbf{k}) \\ \hline \\ & Bethe-Salpeter Equation \\ \hline \\ & \mathbf{f}_{xc}^{(n)}\left(\mathbf{q},\omega\right) &= \frac{1}{\mathbf{P}^{(0)}\left(\mathbf{q},\omega\right)} \left[ \delta \widetilde{\mathbf{P}}^{(n)}\left(\mathbf{q},\omega\right) \left(\mathbf{P}^{(0)}\left(\mathbf{q},\omega\right)\right)^{-1} - \sum_{m=1,n-1} (-1)^{m} \delta \widetilde{\mathbf{P}}^{(m)}\left(\mathbf{q},\omega\right) \mathbf{f}_{xc}^{(n-m)}\left(\mathbf{q},\omega\right) \right] \\ & \mathbf{f}_{xc}\left(\mathbf{q},\omega\right) &= \sum_{n} \mathbf{f}_{xc}^{(n)}\left(\mathbf{q},\omega\right) \quad \mathbf{f}_{xc}^{(0)}\left(\mathbf{q},\omega\right) = 0 \\ \hline \\ & A. Marini, R. Del Sole and AR, PRL (2003) \\ \hline \end{aligned}$$

## Bound excitons in TDDFT



 $\overline{22}$ 

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A. Marini, R. Del Sole and AR, PRL (2003)

## How many terms ?



## A many-body causal TDDFT kernel



## Low dimensional sytems (1D): polyacetylane



FIG. 1. ALDA and VK static axial polarizability of polyacetylene compared with restricted Hartree-Fock [18] and MP2 [22] results.

M. van Faassen et al. PRL 88 186401 (2002)

#### **Electric field dependence of the XC**

#### **Potenital in Molecular Chains**



FIG. 2. Changes, due to an electric field of 0.001 a.u. in response and hole potentials for  $H_2$ - $H_2$ , constructed from multireference CI singles doubles density with a large (cc-pV6Z without *d* and *f* functions) basis set, compared to the applied field (potential  $v_E$ ).

S.J.A. Van Gisbergen PRL 83 694 (1999)

#### In LDA and GGA xc potential lack of a term counteracting the applied electric field

$$f_{xc}^{BSE}(r,r',\omega)$$

Isolated infinite Polyacetylene chain





$$\left[ R_{\mathbf{K}}^{(\mathbf{q})} \right]_{\mathbf{G}_{1},\mathbf{G}_{2}} = \sum_{\mathbf{K}', E_{\mathbf{K}'}^{(\mathbf{q})} \neq E_{\mathbf{K}}^{(\mathbf{q})}} \frac{\Phi_{\mathbf{K}}^{*}\left(\mathbf{q}, \mathbf{G}_{1}\right) W_{\mathbf{K},\mathbf{K}'}\left(\mathbf{q}\right) \Phi_{\mathbf{K}'}\left(\mathbf{q}, \mathbf{G}_{2}\right)}{E_{\mathbf{K}}^{(\mathbf{q})} - E_{\mathbf{K}'}^{(\mathbf{q})}} \qquad \left[ Q_{\mathbf{K}}^{(\mathbf{q})} \right]_{\mathbf{G}_{1},\mathbf{G}_{2}} = \sum_{\mathbf{K}', E_{\mathbf{K}'}^{(\mathbf{q})} = E_{\mathbf{K}}^{(\mathbf{q})}} \Phi_{\mathbf{K}}^{*}\left(\mathbf{q}, \mathbf{G}_{1}\right) W_{\mathbf{K},\mathbf{K}'}\left(\mathbf{q}\right) \Phi_{\mathbf{K}'}\left(\mathbf{q}, \mathbf{G}_{2}\right)$$

•When the only optical spectra is calculated TDDFT is as time consuming as BSE...

•...but when the full dielectric matrix is needed TDDFT is more favorable than BSE

# What about the description of decaying quasiparticle processes within TDDFT?

G. Onida, L. Reining and AR, Rev. Mod. Phys. 74, 601 (2002)

## Lifetime of quasiparticles





interactions between quasiparticles limit how long the corresponding quantum states retain their identity, i.e., the **lifetime** of the excitation. In combination with the velocity, this lifetime determines the **mean free path**, a measure of influence of the excitation

#### **Importance of lifetime**

- screening in an electron gassurface photochemistry
- electron-phonon coupling- electron transfer across interfaces
- localization electron dynamics and energy transfer

# $\mathbf{PHYSICS \ OF \ LIFETIME}$ $\tau^{1} = 2 \sum \int d\mathbf{r} \int d\mathbf{r}' \phi_{i}^{*}(\mathbf{r}) \phi_{f}^{*}(\mathbf{r}') \operatorname{Im}W(\mathbf{r},\mathbf{r}';\omega) \phi_{i}(\mathbf{r}') \phi_{f}(\mathbf{r})$

#### Density of States (DOS) versus Screening





from F.Reinert et al., PRB 63 (2001) 115415.

# The 2-point vertex function Suppose to have a good approximation fot the (TD)DFT *potential*... $\hat{H} = \int d\mathbf{x} \hat{\psi}^{\dagger}(\mathbf{x}) \left[h_0\left(\mathbf{x}\right) + v_{xc}\left(\mathbf{x}\right)\right] \hat{\psi}(\mathbf{x}) + H_{interaction} - \int d\mathbf{x} \hat{\psi}^{\dagger}(\mathbf{x}) v_{xc}\left(\mathbf{x}\right) \hat{\psi}(\mathbf{x})$ $\chi(1,2) = \tilde{\chi}(1,2) + \int d34\tilde{\chi}(1,2) \left[v(3,4) + f_{xc}(3,4)\right] \chi(4,2)$ $f_{rc}(1,2) \equiv \delta v_{rc}(1) / \delta \rho(2)$ $\tilde{\Gamma}(1,2;3) = \delta(1,2)\,\delta(2,3) + \int d4567\,\Xi(1,5;2,4)\,G(4,6)\,G(7,5)\,\tilde{\Gamma}(6,7;3)$ $\Xi(1,4;2,3) \approx W(1,2)\,\delta(1,3)\,\delta(2,4) - f_{xc}(1,3)\delta(1,2)\,\delta(3,4)$ $\Sigma_{G_0W_0} \to i \int d3 W^{TDDFT} (1^+, 3) \tilde{\Gamma}_{loc} (3, 2) G (1, 2) \qquad \tilde{\Gamma}_{loc} (1, 2) = \int d3 \chi_0^{-1} (1, 3) \tilde{\chi} (3, 2)$

No difference with GoWo using ALDA or similar approaches PRL 62, 2718 (1989); PRB 49, 8024 (1994); PRB 56, 12832 (1997).

**BUT IS**  $\tilde{\Gamma}(1,2;3) = \delta(1,2)\delta(2,3) + \tilde{\Gamma}_{3}(1,2;3) - \tilde{\Gamma}_{loc}(1,3)\delta(1,2) \sim \delta(1,2)\delta(2,3)$ ?

PRL 91, 056402 (2003); PRL 91, 256402 (2003). PRL 88, 066404 (2002) etc etc

$$\begin{aligned} & \textbf{The 3-point vertex function} \\ \mathbf{f}_{xc}^{(1)}(\mathbf{q},\omega) = \mathbf{f}_{xc}^{(0)}(\mathbf{q},\omega) = \mathbf{f}_{xc}^{(0)}(\mathbf{q},\omega) = \mathbf{f}_{xc}^{(0)}(\mathbf{q},\omega) \\ & \bar{\chi}(1,2) = \chi_0(1,2) + \int d34\chi_0(1,3) \mathbf{f}_{xc}^{(0)}(1,3) \mathbf{f}_{xc}^{(0)}(1,2) + \int d34G(1,3)G(4,1)\widetilde{\Gamma}_{TDDFT}^{(0)}(3,4;2) \\ & \textbf{If} \\ \hline \widetilde{\Gamma}_{TDDFT}^{(1)}(1,2;3) \equiv \delta(1,2)\delta(2,3) + iW_0(1,2) \int d4G_0(1,4)G_0(4,2)\widetilde{\Gamma}_{loc}(4,3) \\ & \tilde{\Gamma}_{TDDFT}^{(0)}(1,2;3) \equiv \delta(1,2)\delta(2,3) + i \underbrace{\int}_{\sigma_{xc}}^{\sigma_{xc}\sigma_{xc}} \widetilde{\chi}(4,3) \\ & \textbf{Closed expression for the ''on mass-shell'' electronic lifetime \\ & \tau_{c\mathbf{k}}^{-1} = -2\Omega^{-1} \sum_{\mathbf{G}_{1},\mathbf{G}_{2}} \sum_{\mathbf{q},c'} \rho_{cc'}(\mathbf{kqG}_{1}) \rho_{cc'}^{*}(\mathbf{kqG}_{2}) Im \left[W_{\mathbf{G}_{1}\mathbf{G}_{2}}^{TDDFT}(\mathbf{q},\epsilon_{c\mathbf{k}}-\epsilon_{c'\mathbf{k}-q})\right], \\ & \Delta \tau_{c\mathbf{k},0}^{-1} = -2\Omega^{-1} \sum_{\mathbf{G}_{1},\mathbf{G}_{2}} \sum_{\mathbf{q},c'} Re\left[\left(\Gamma_{cc'}^{ec'}(\mathbf{kqG}_{1}) + \Gamma_{cc'}^{vc}(\mathbf{kqG}_{1})\right)\rho_{cc'}^{*}(\mathbf{kqG}_{2})\right] Im \left[W_{\mathbf{G}_{1}\mathbf{G}_{2}}^{TDDFT}(\mathbf{q},\epsilon_{c\mathbf{k}}-\epsilon_{c'\mathbf{k}-q})\right] \end{aligned}$$

# Excitonic effects (via TDDFT) on the lifetimes of LiF



# Acknowledgements

#### A. Castro, A. Marini, X. López, L Wirtz, D. Varsano

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It is one of the first duties of a professor, in any subject, to exaggerate a little both the importance of his subject and his own importance in it.

**G.H. Hardy (A Mathematician's Apology)** 



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