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SMR 1564 - 20

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**SPRING COLLEGE ON SCIENCE AT THE NANOSCALE**  
**(24 May - 11 June 2004)**

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**SINGLE MOLECULE BIOPHYSICS - Part I**

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*These are preliminary lecture notes, intended only for distribution to participants.*

# Protein Mechanics

(or, successful nano-mechanics for the past 3.5 billion years)

“Everything in Biology is mechanical”

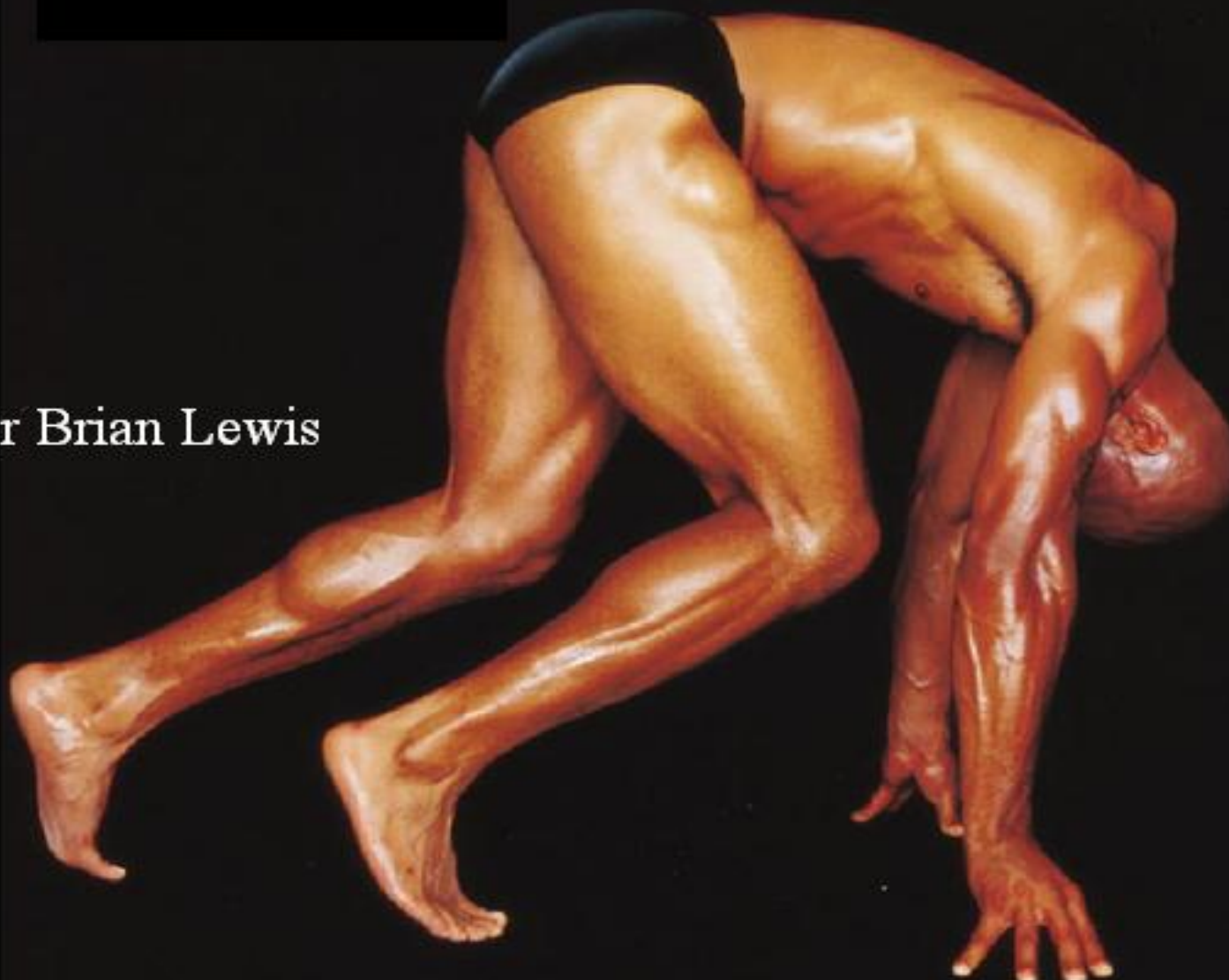
Engineering protein designs using molecular biological tools

Examination of mechanical properties of the resulting molecules using single molecule force probes and electron microscopy

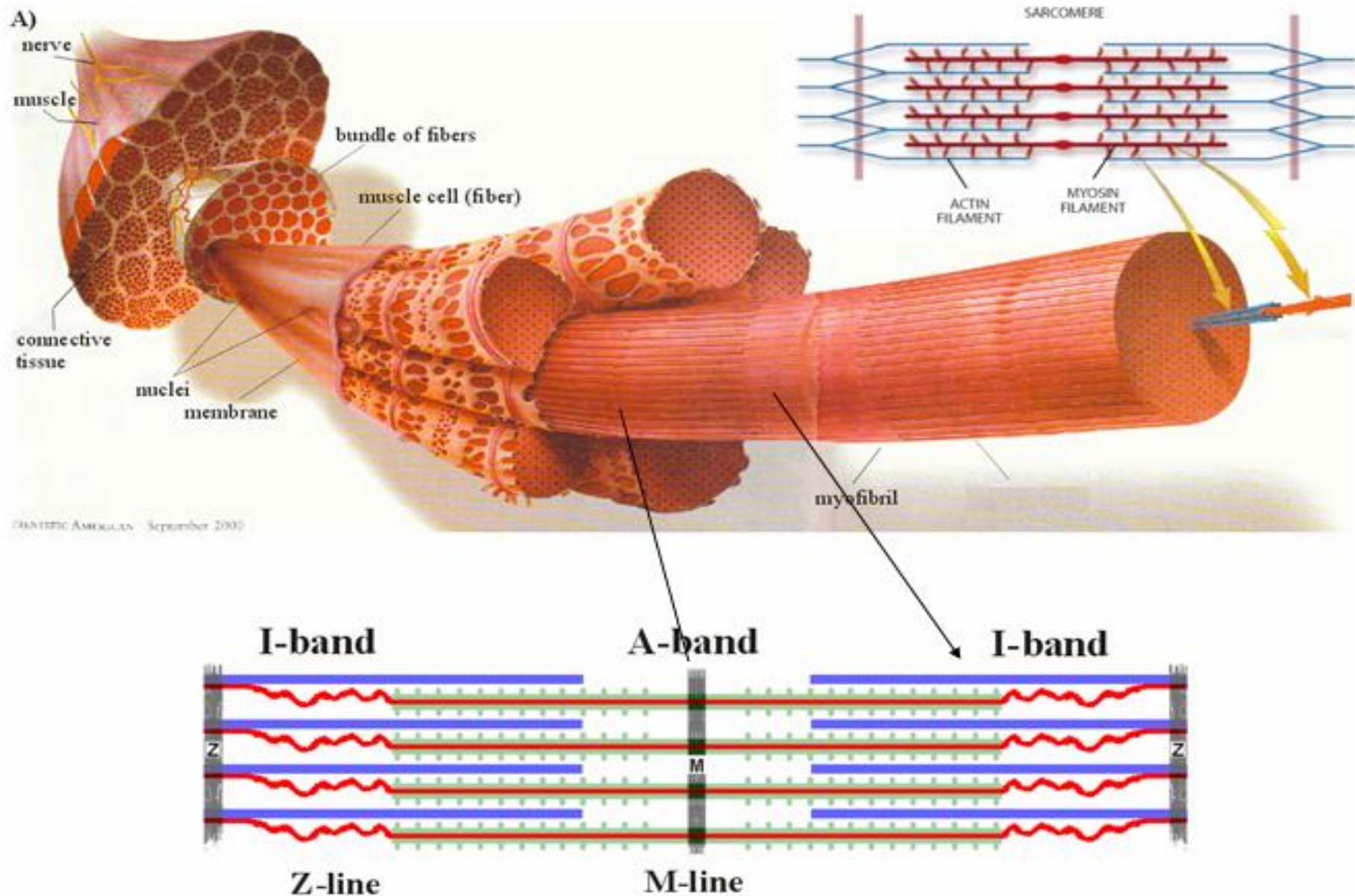
Use this approach to probe the mechanics of biological tissues  
At the single molecule level.

The elasticity of muscle is due to the **Brownian motion** driven collapse of a protein named **titin**

Sprinter Brian Lewis

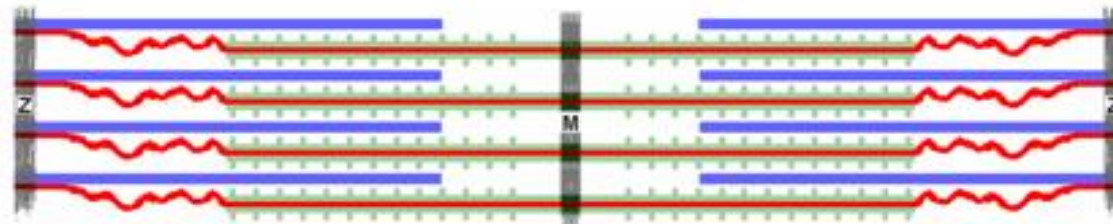


# Muscle can contract and also can extend elastically

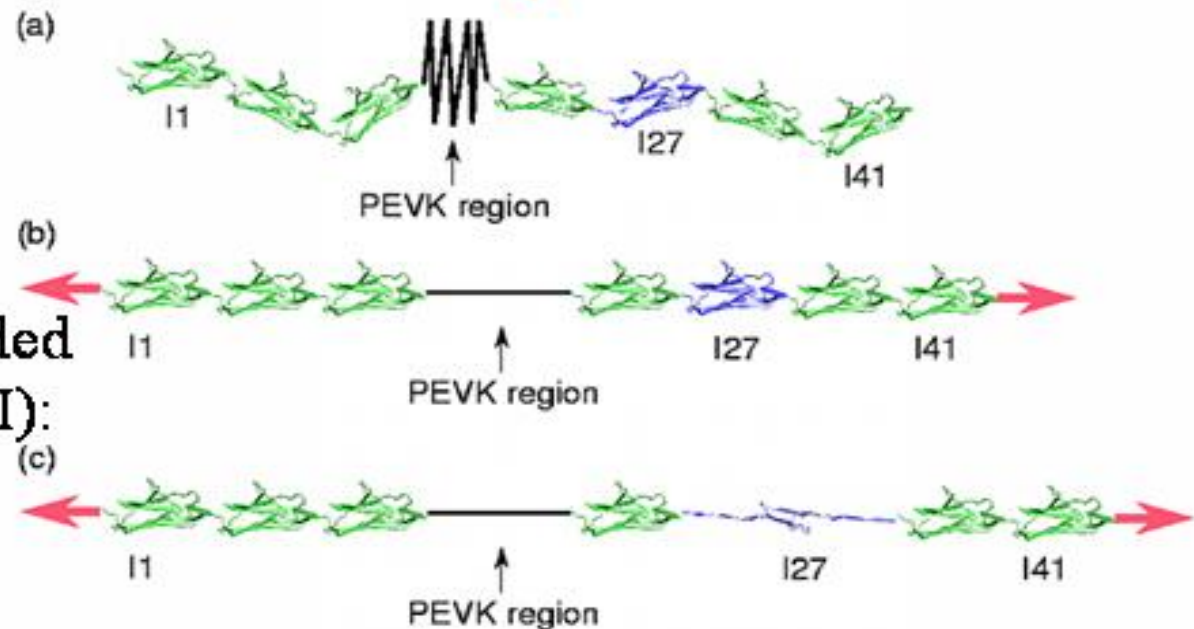




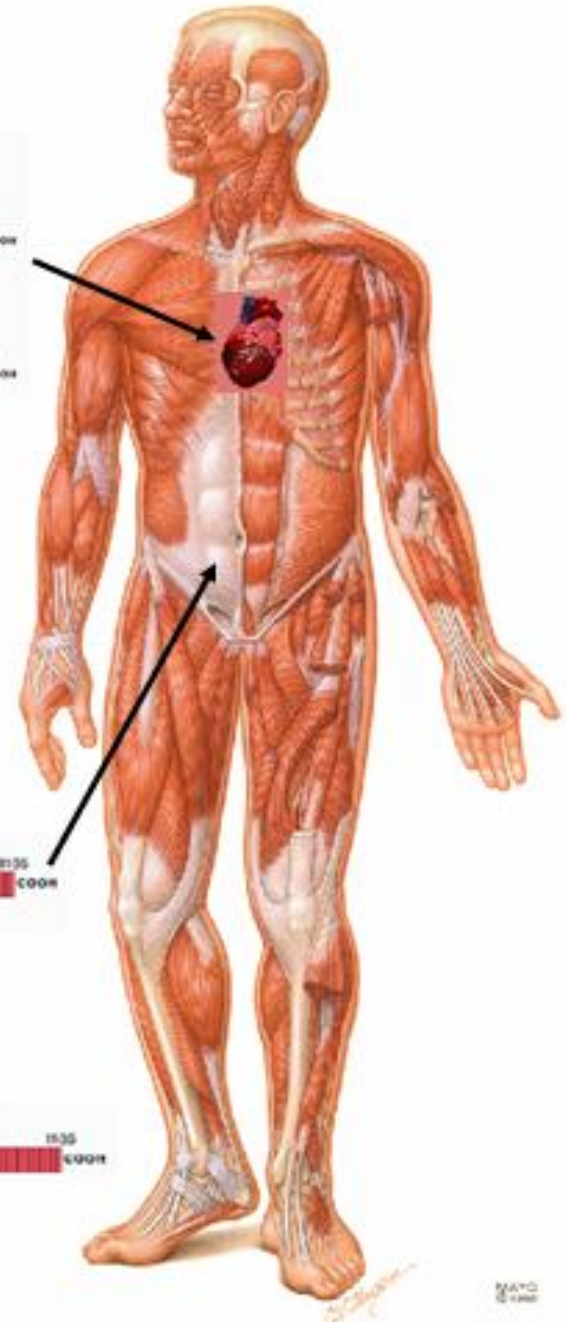
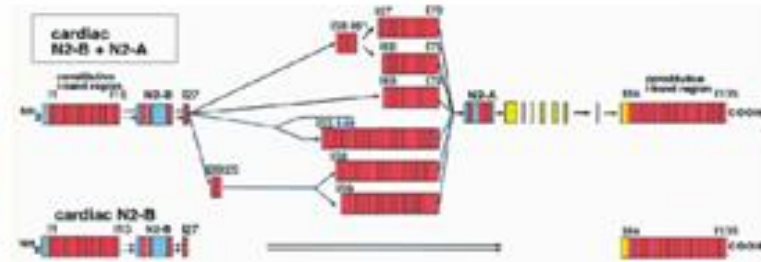
The elasticity of muscle results from the properties of a giant protein named Titin (from Titan!)



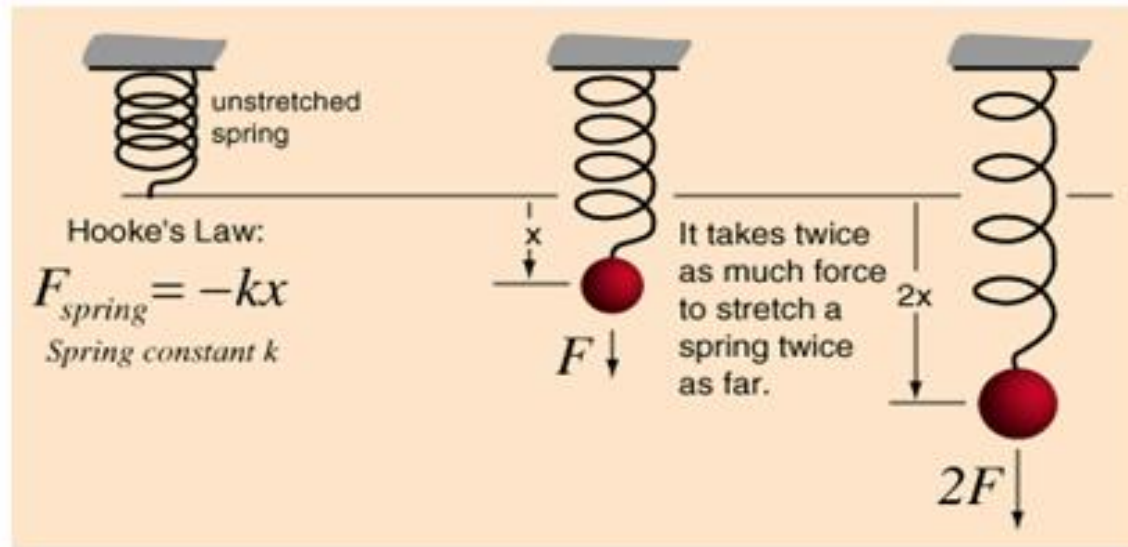
Titin has a coiled region (PEVK) and a region folded into individual modules (I):



## *Machina Carnis*

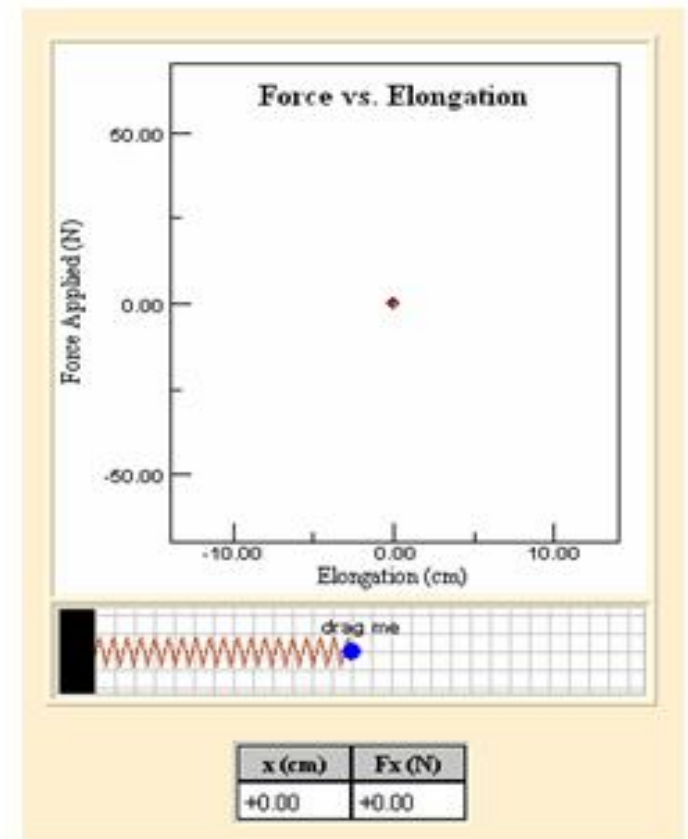


## Is the elasticity of titin like that of a spring?



The elasticity of a metal spring results from the stretching of the bonds between the metal atoms.

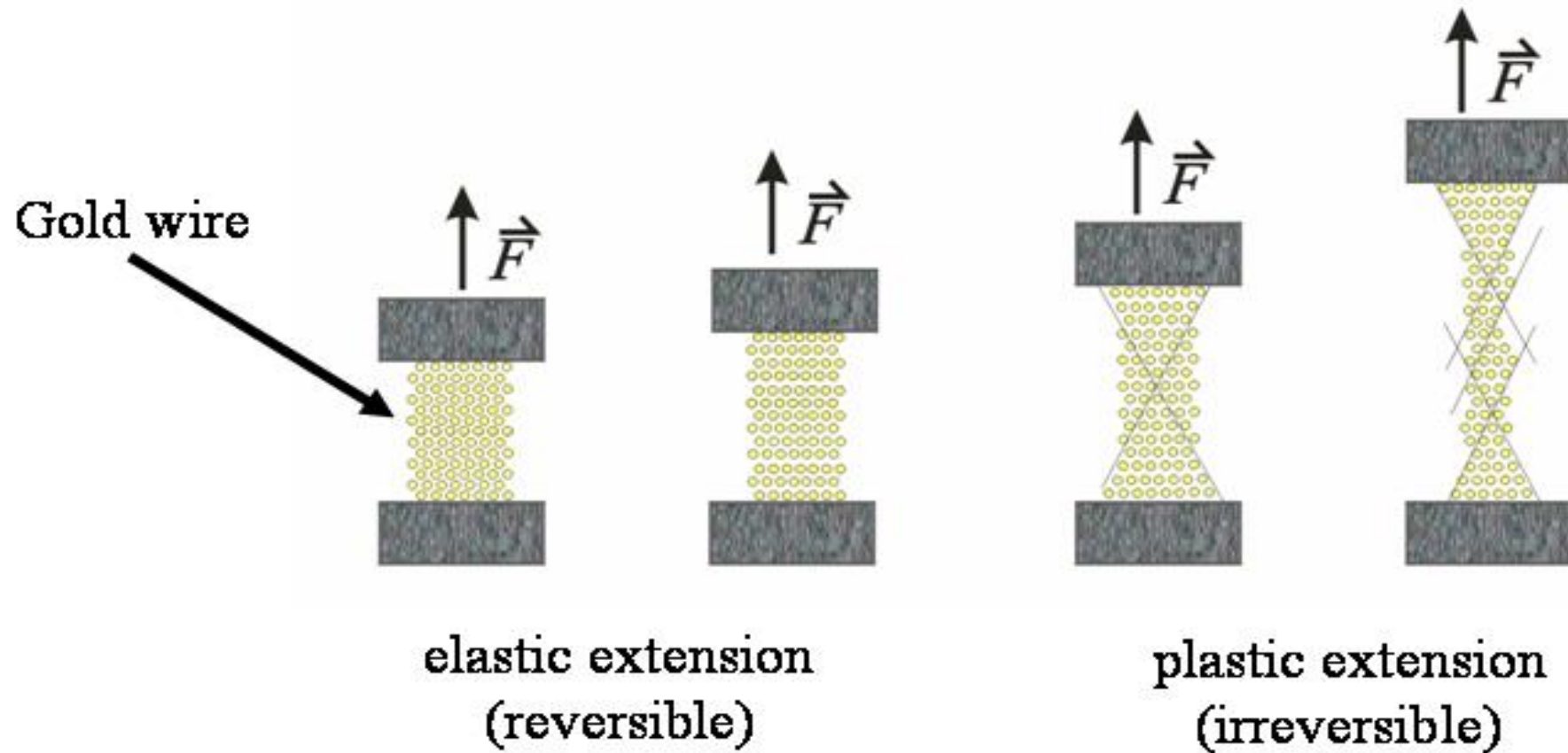
Is this how titin works?



Will titin also break if we pull it too far?

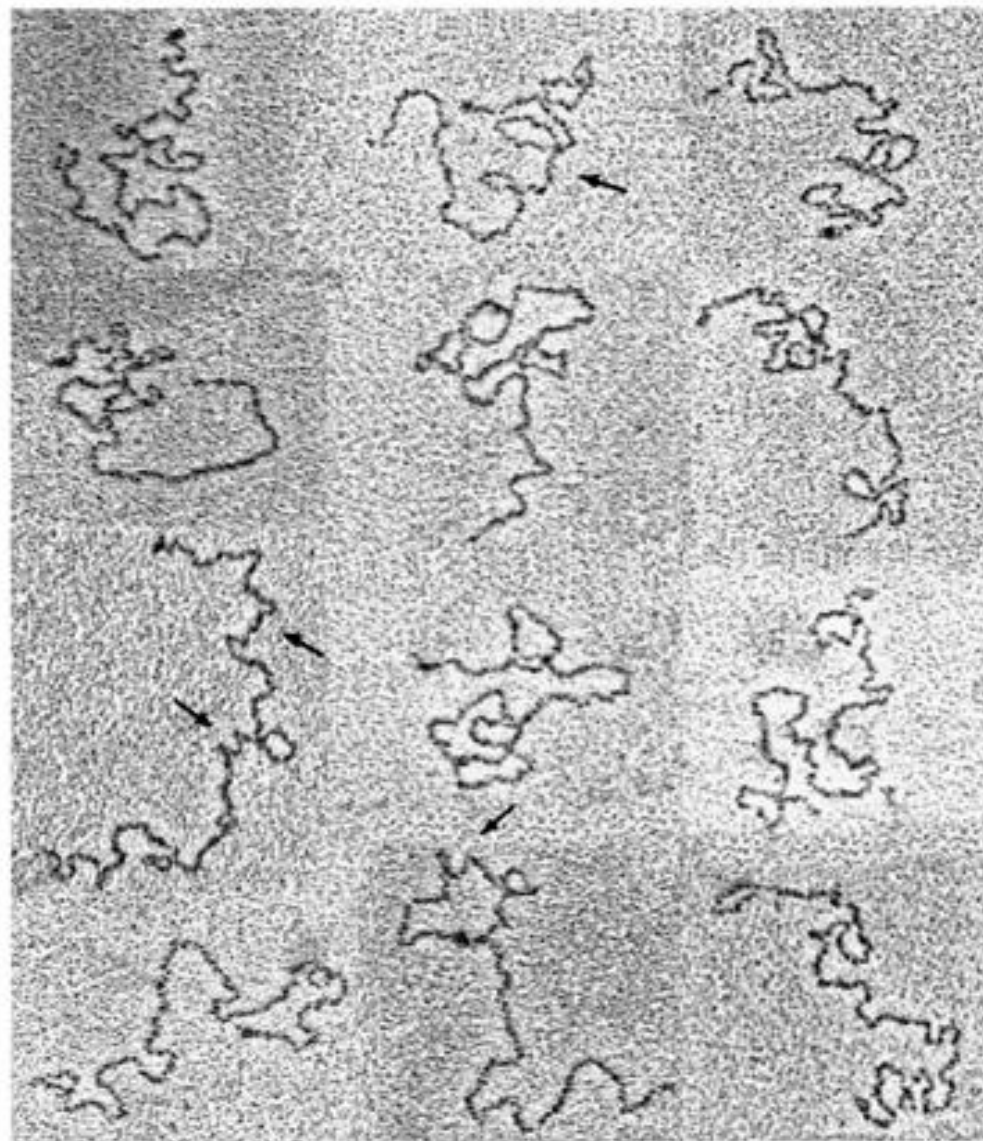


A metal wire in a spring extends by bond stretching and breaks by irreversibly disrupting its atomic arrangements

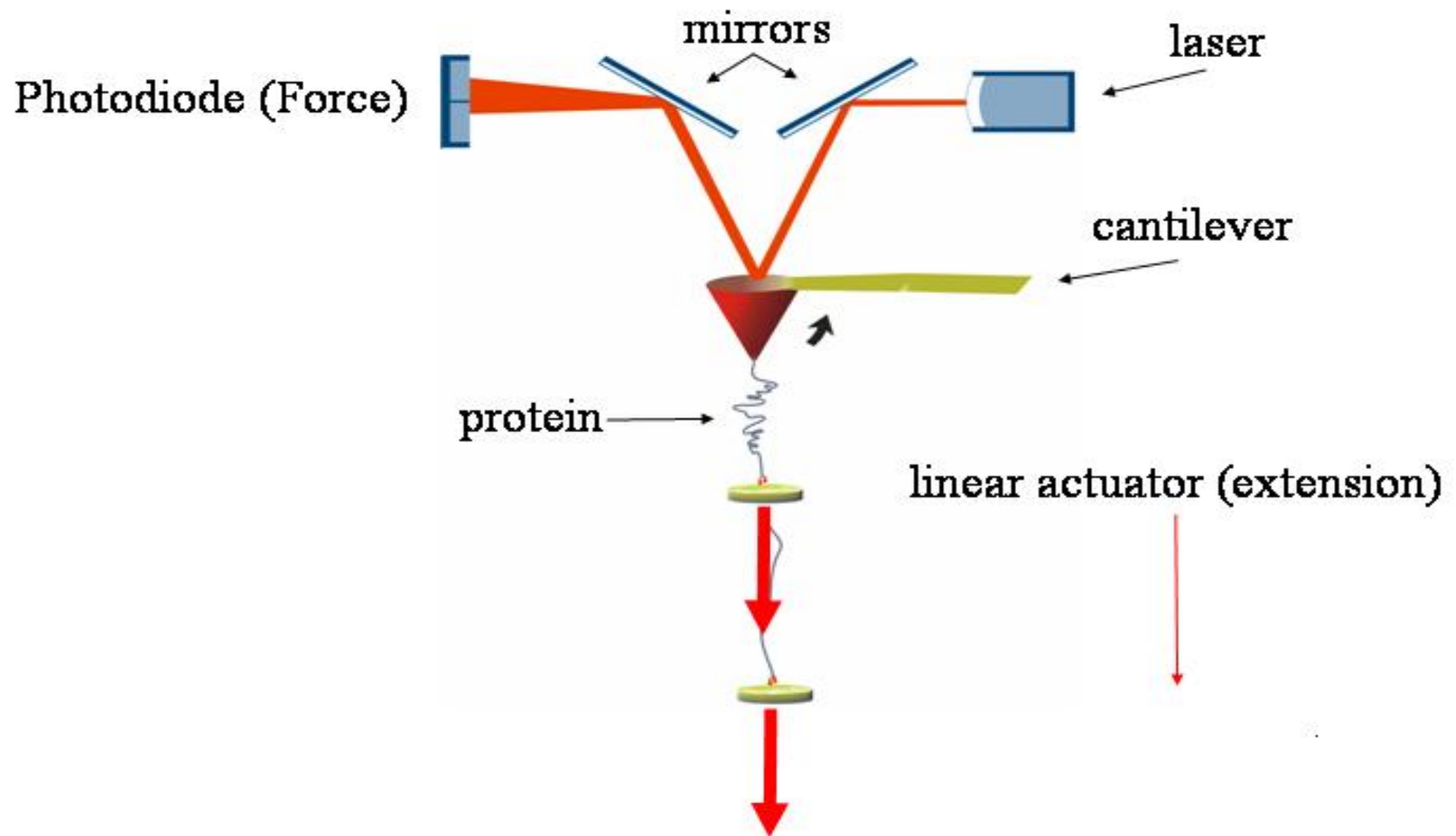


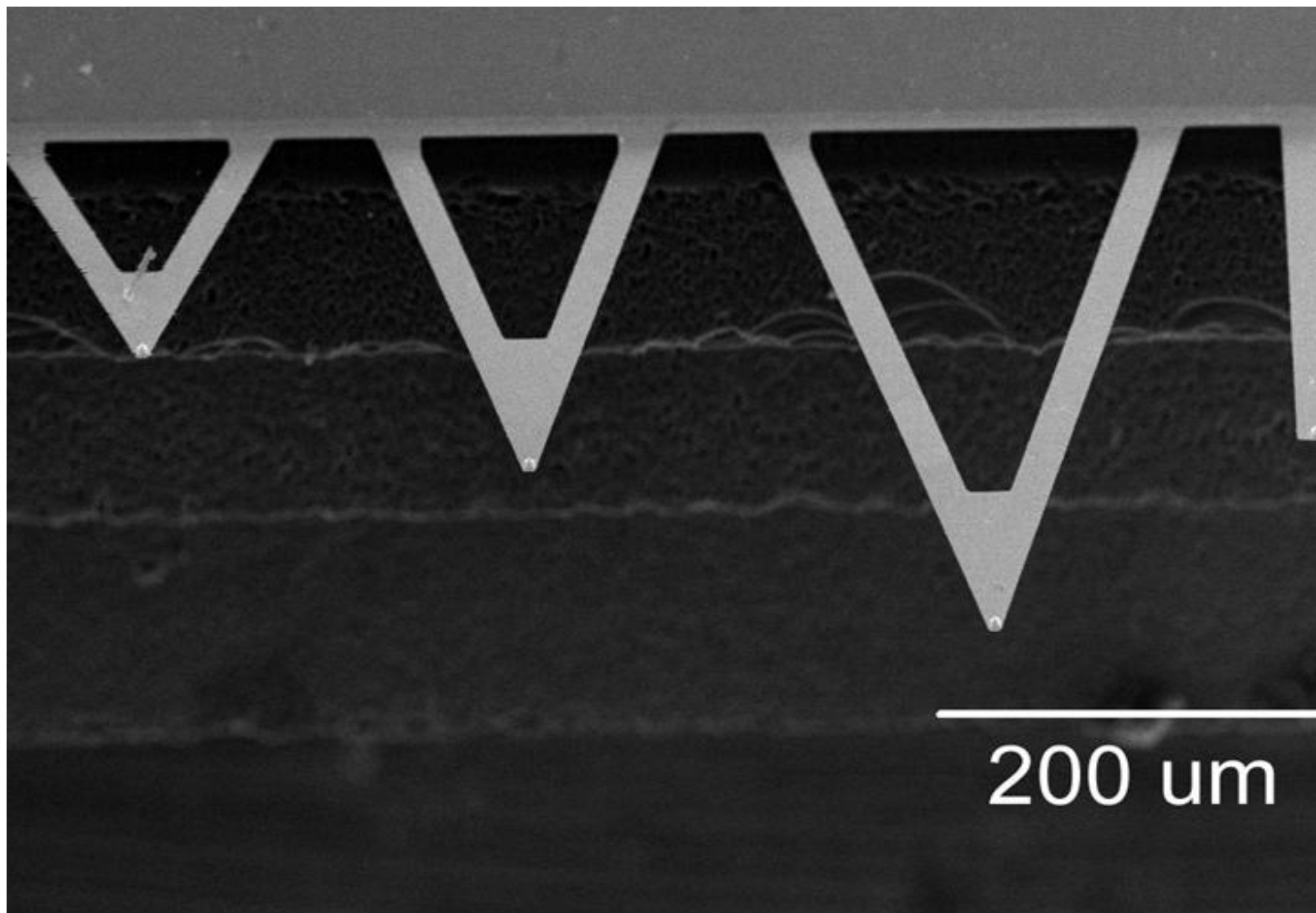


## Electron micrographs of isolated titin molecules



We can stretch a single protein and measure how does the restoring force changes with the extension.



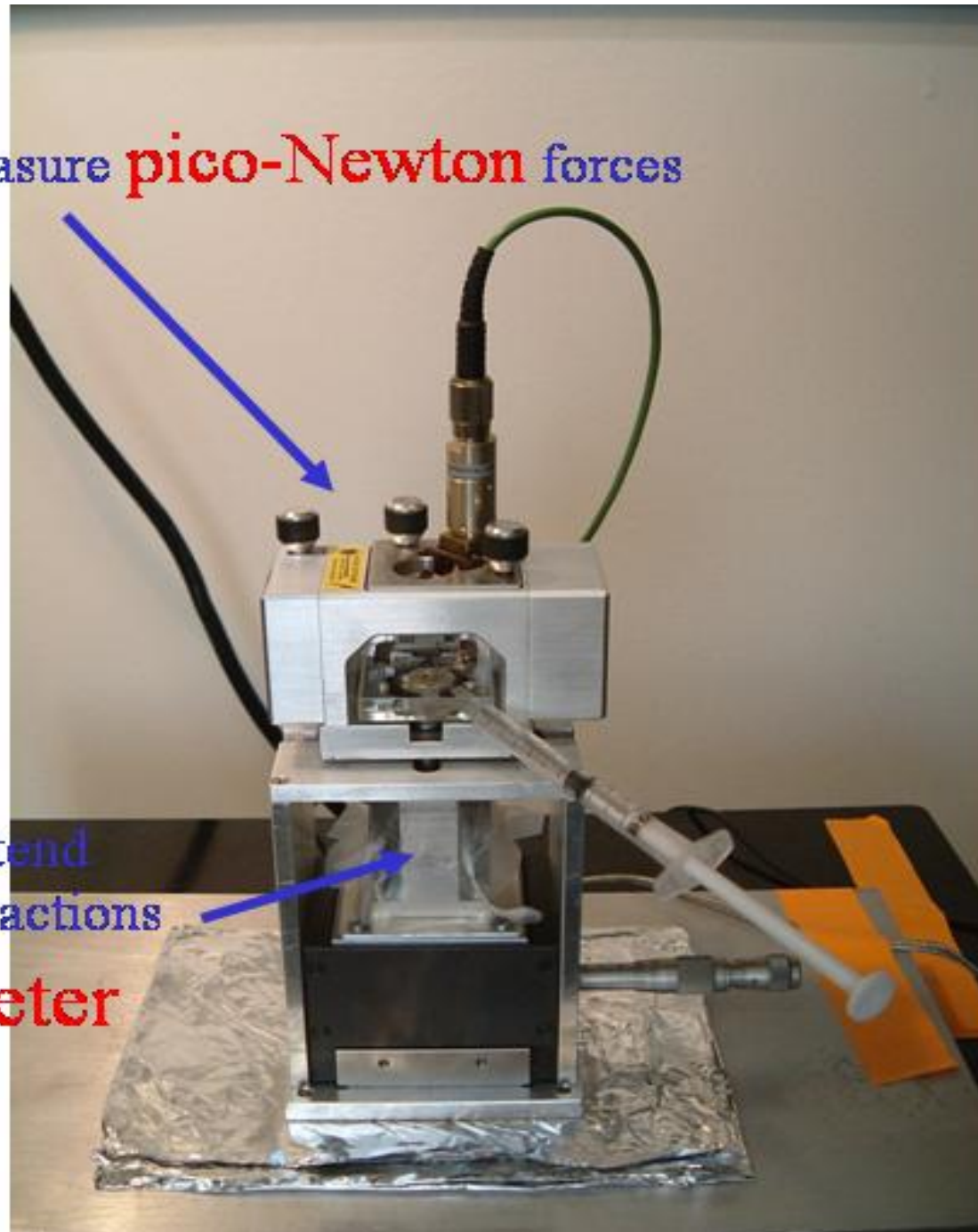




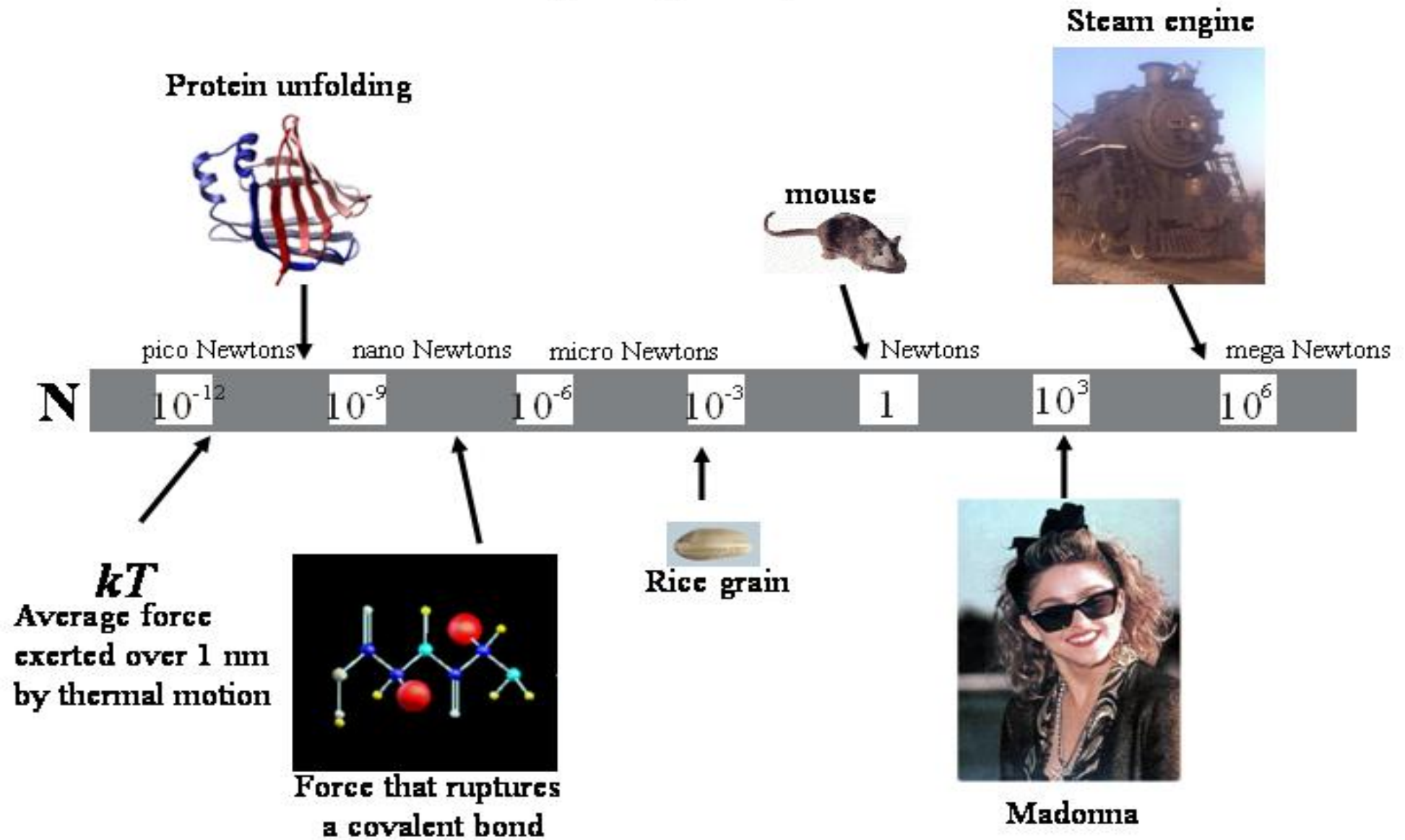


detector can measure **pico-Newton** forces

Actuator can extend  
a molecule by fractions  
of a **nano-meter**

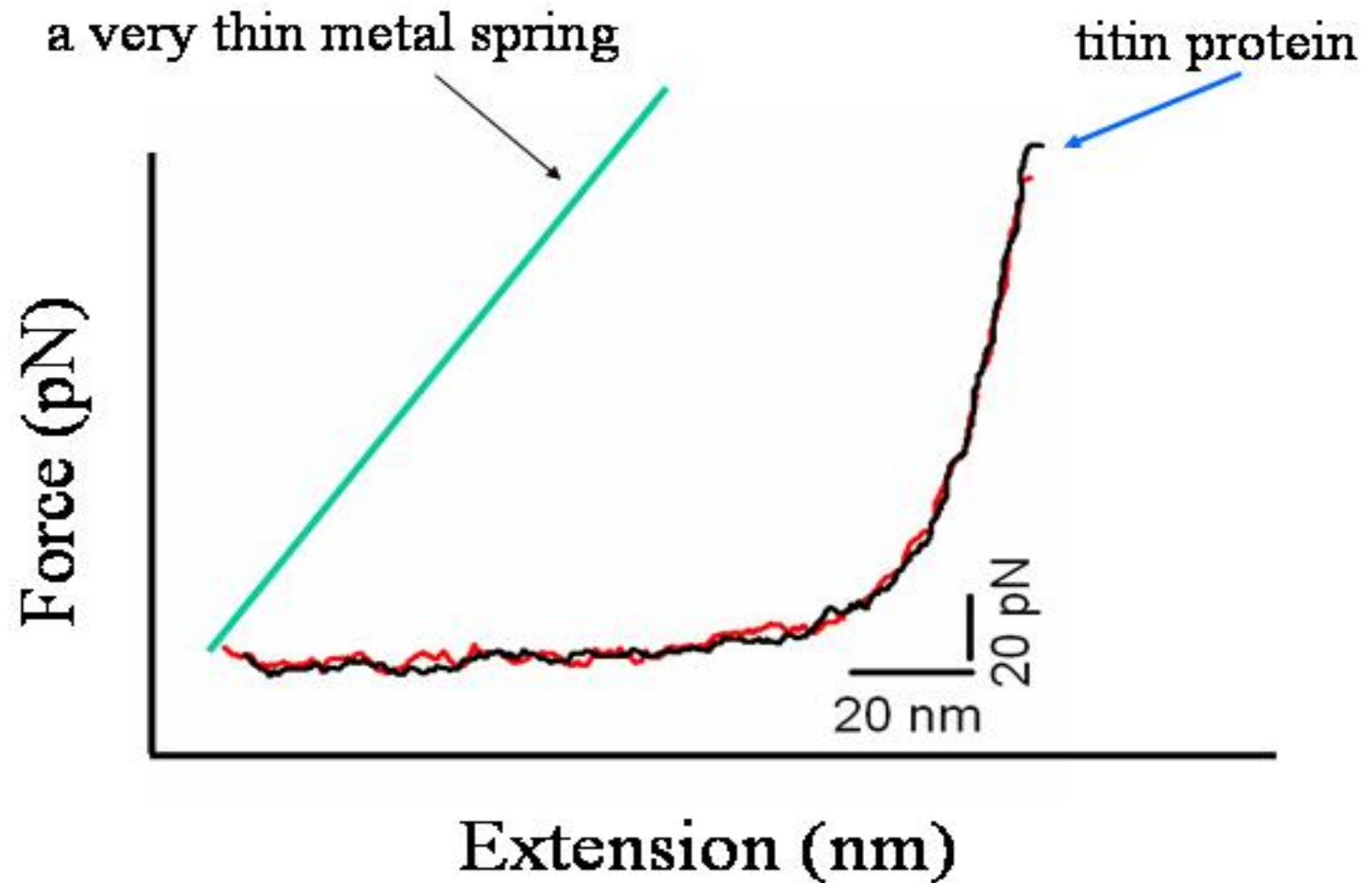


# How much is a pico ( $10^{-12}$ ) Newton of Force?



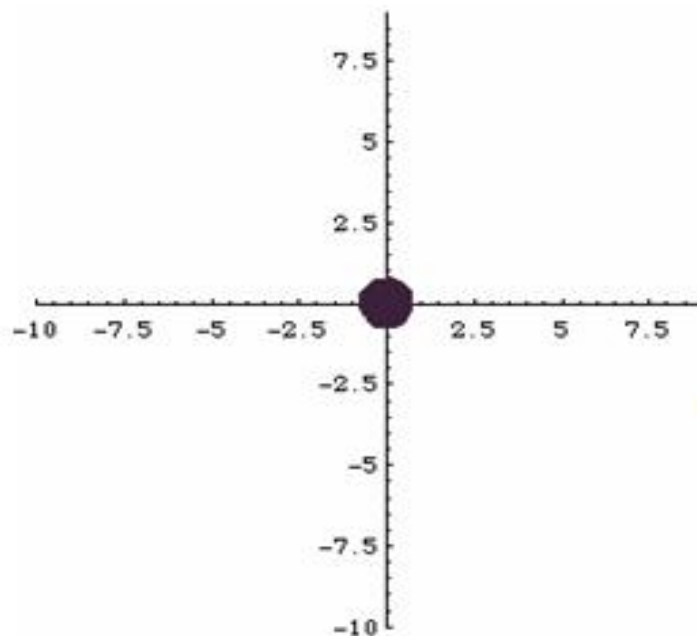


If we stretch a single titin protein we obtain force-extension curves that is very different from Hooke's law.



To understand how titin works,  
we must first understand Brownian motion.

Robert Brown (1773-1858 ), a  
botanist, reported in 1828 his  
observation that pollen grains in  
water underwent incessant motion.



A single pollen grain observed  
with a microscope is being moved  
about by mysterious forces.

**Albert Einstein, *Annalen der Physik (Paris)*, 17:549, 1905**

**“In this paper it will be shown that, according to the molecular kinetic theory of heat, bodies of a microscopically visible size suspended in liquids must, as a result of thermal molecular motions, perform motions of such magnitudes that they can be easily observed with a microscope. “**



Smoluchowski was a Polish physicist that published a paper in 1906 where he also arrived at a very similar conclusion.



*M. Smoluchowski*

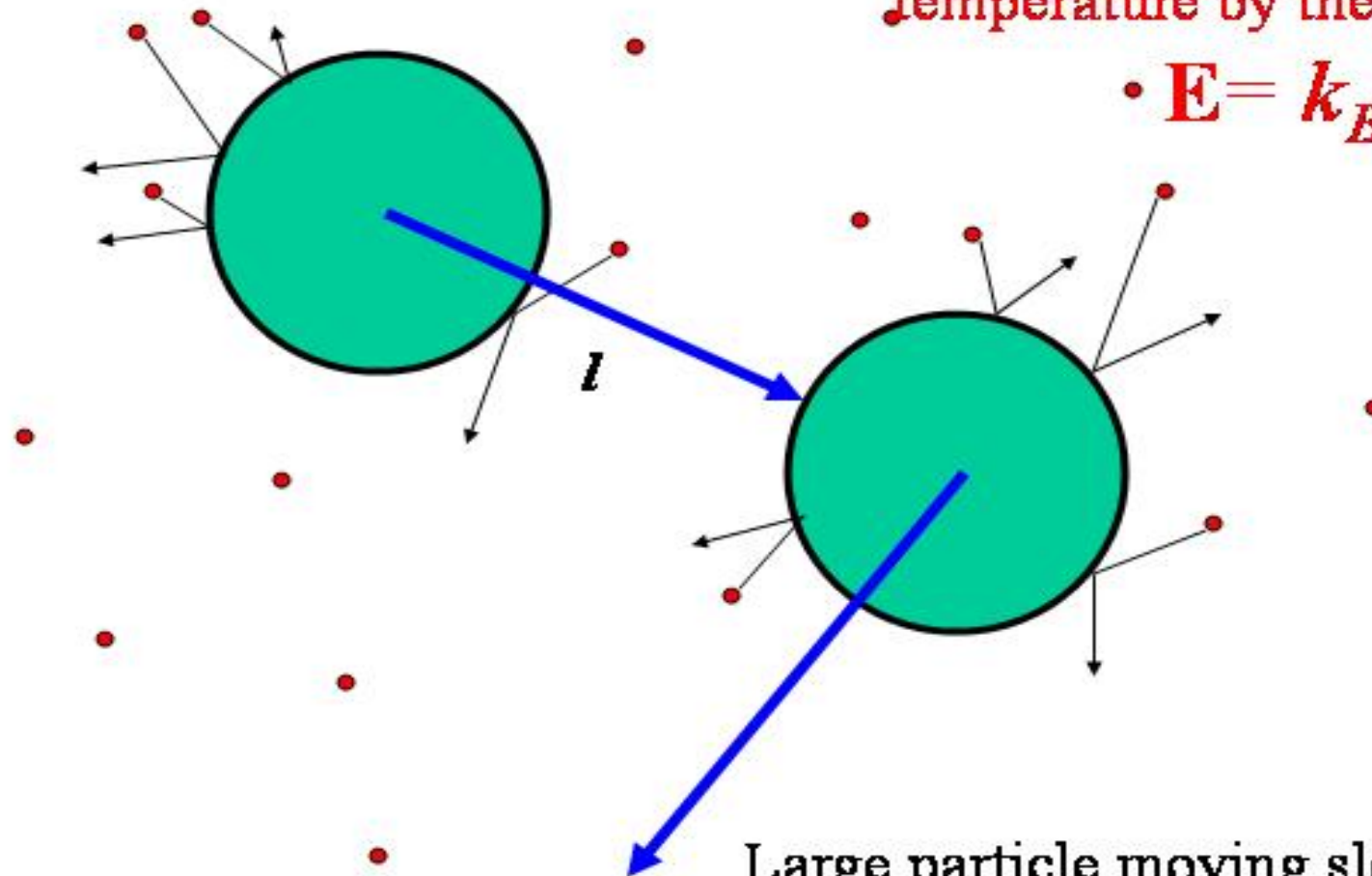
**Marian Smoluchowski, *Annalen der Physik (Paris)*, 21:772, 1906**



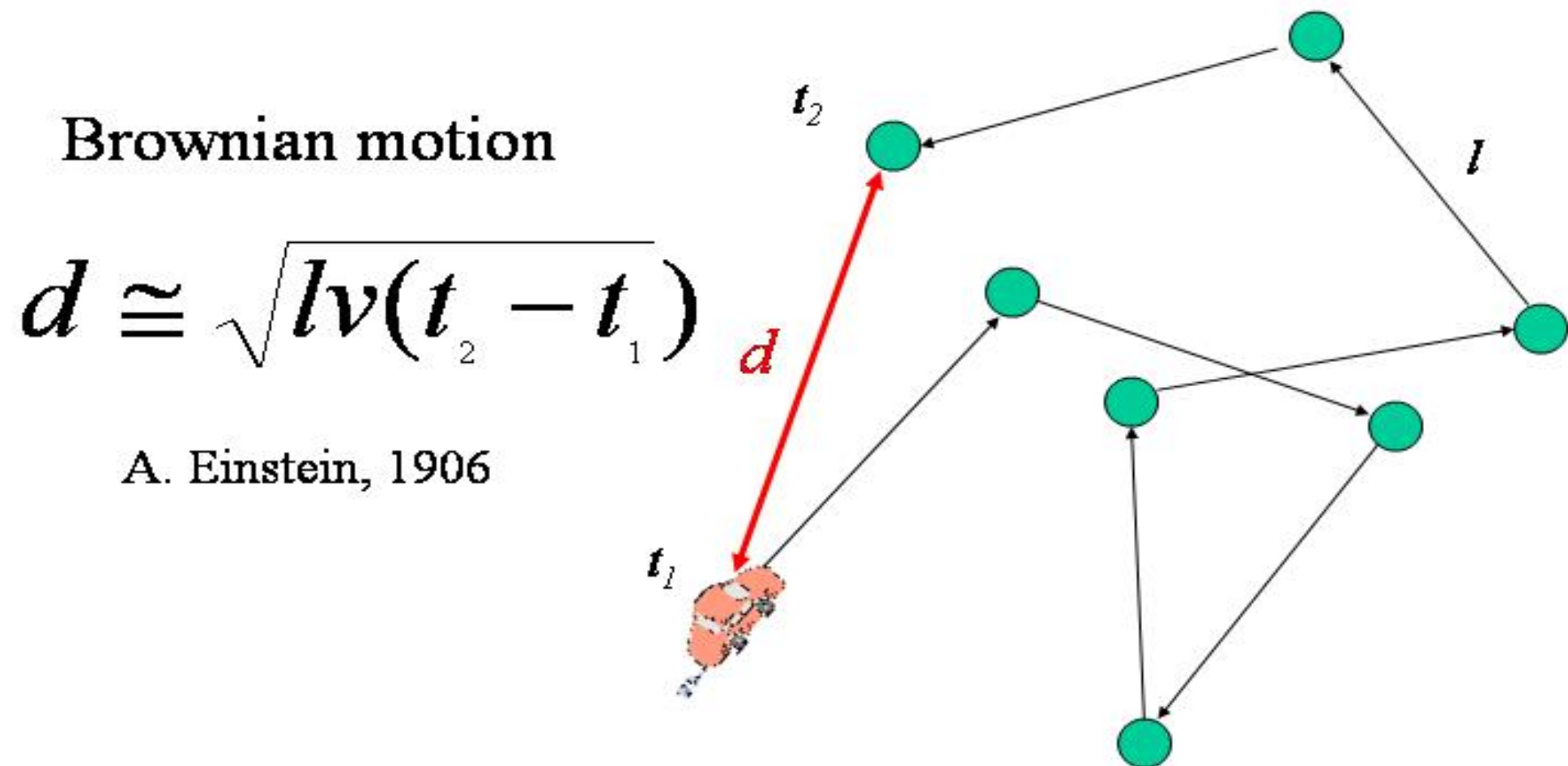
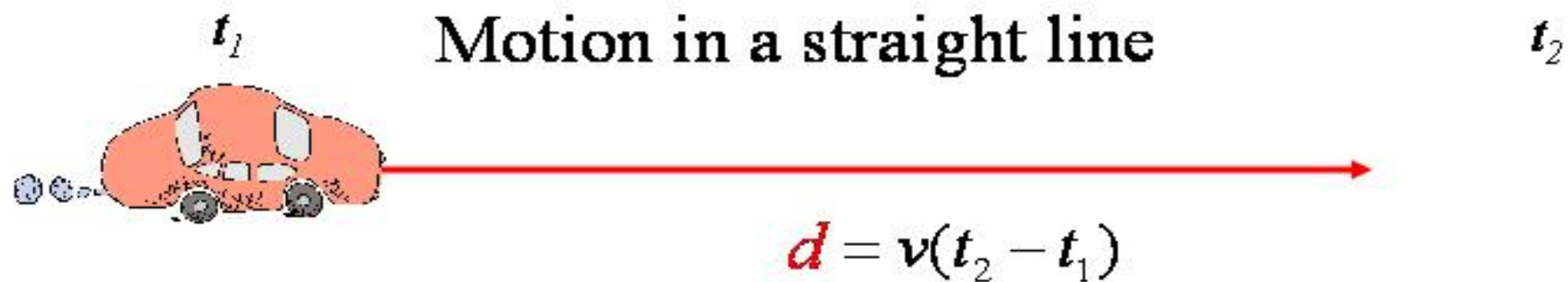
Small particles moving fast  
(red water molecules)

The average kinetic energy of  
each molecules is related to  
temperature by the relationship:

$$E = k_B T$$



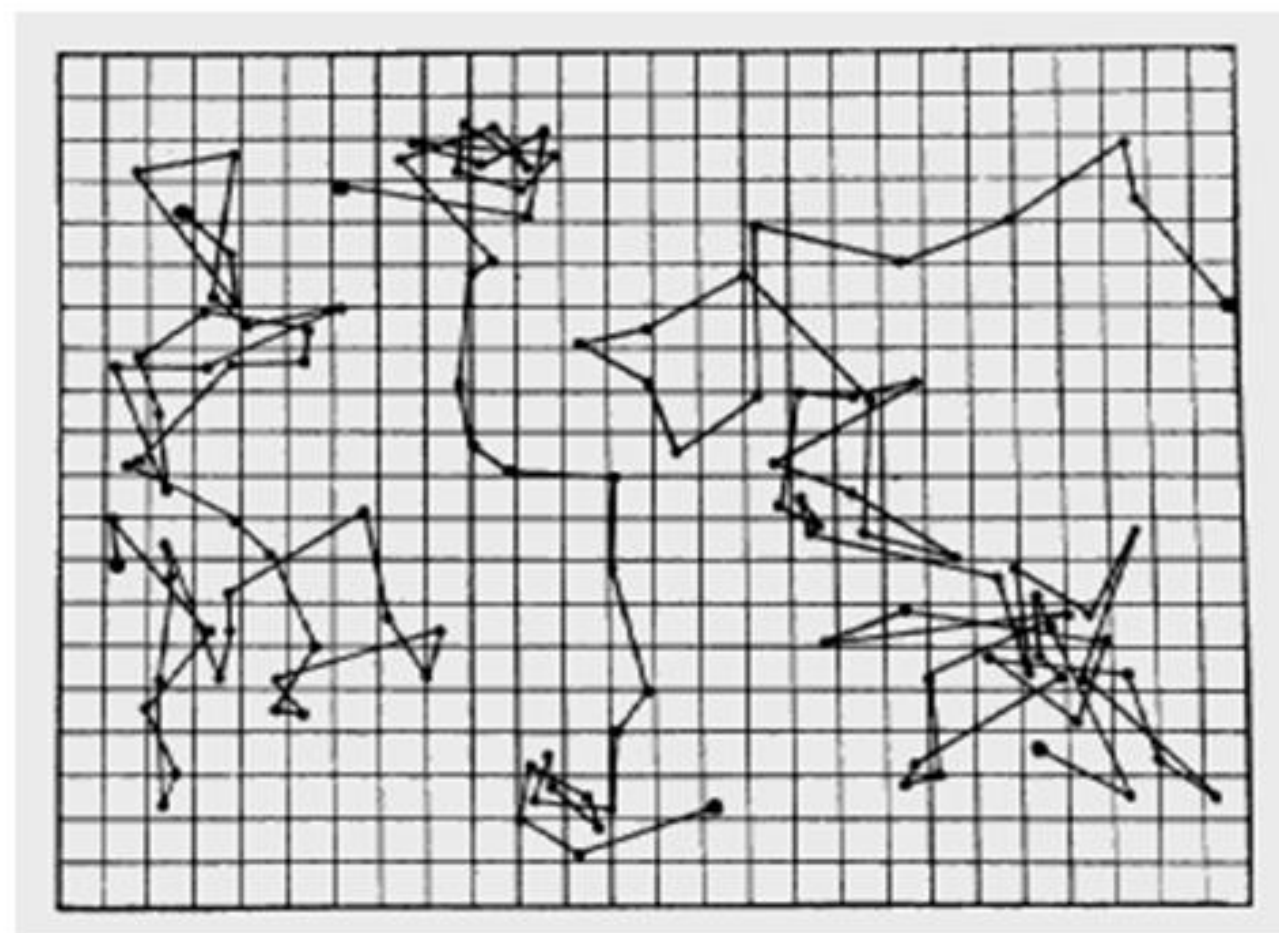
Large particle moving slowly at an  
average velocity  $v$ , on a random path  
of steps of length  $l$  (green pollen grain)



In 1908, the French scientist Jean Babbtiste Perrin (who was awarded the Nobel prize in 1926), experimentally confirmed the predictions made by Einstein and Smoluchowski.



Three tracings of the motion of particles of radius  $0.53\mu\text{m}$ , as seen under the microscope by Perrin. Positions every 30 seconds are joined by lines (the mesh size is  $3.2\mu\text{m}$ ).





A computer simulation of the Einstein-Smoluchowski  
view of Brownian motion

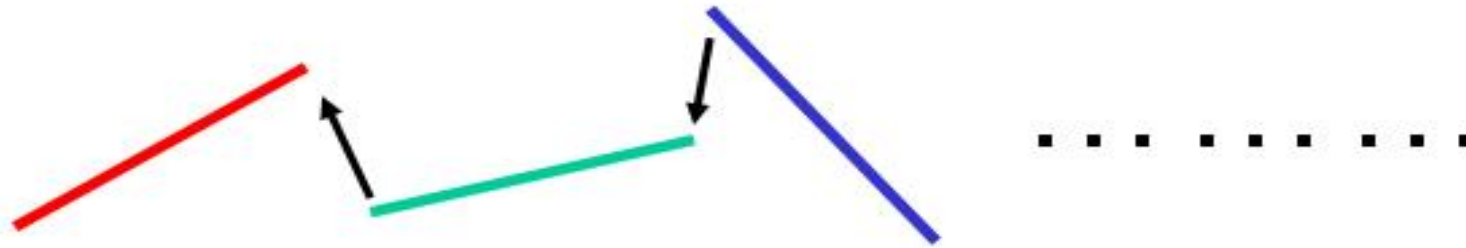


$$d \sim \sqrt{\Delta t}$$

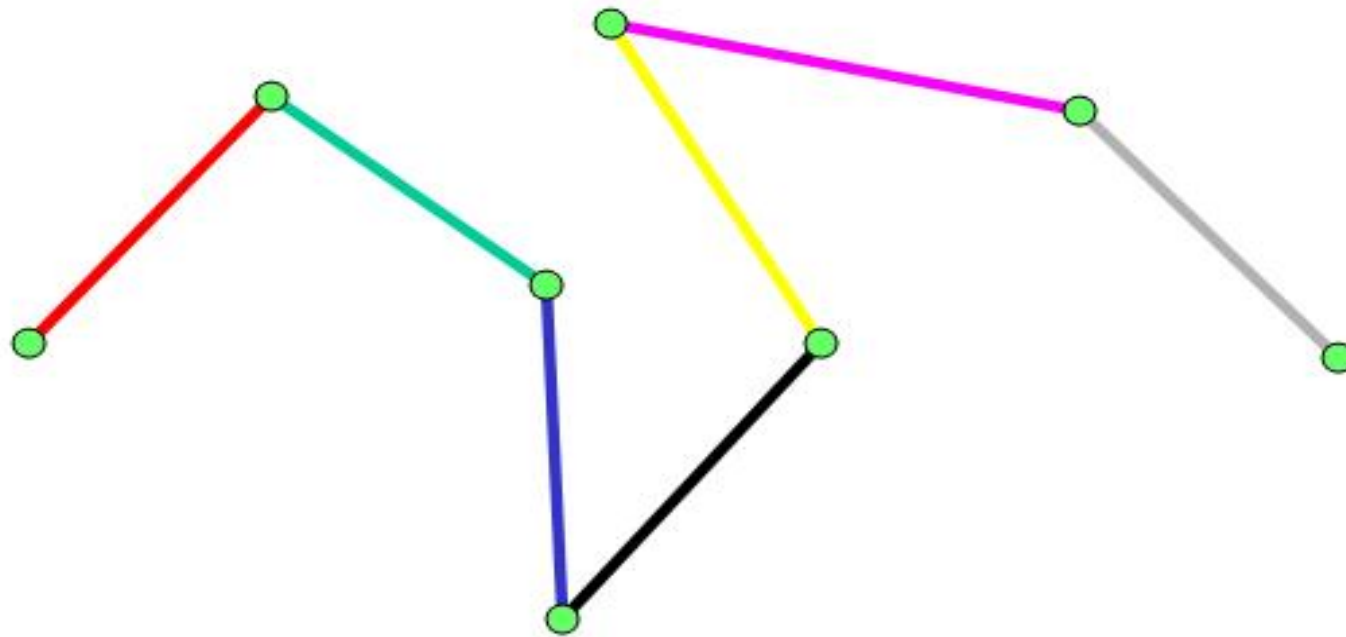
Titin is a polymer.

What does Brownian motion do  
to a polymer?

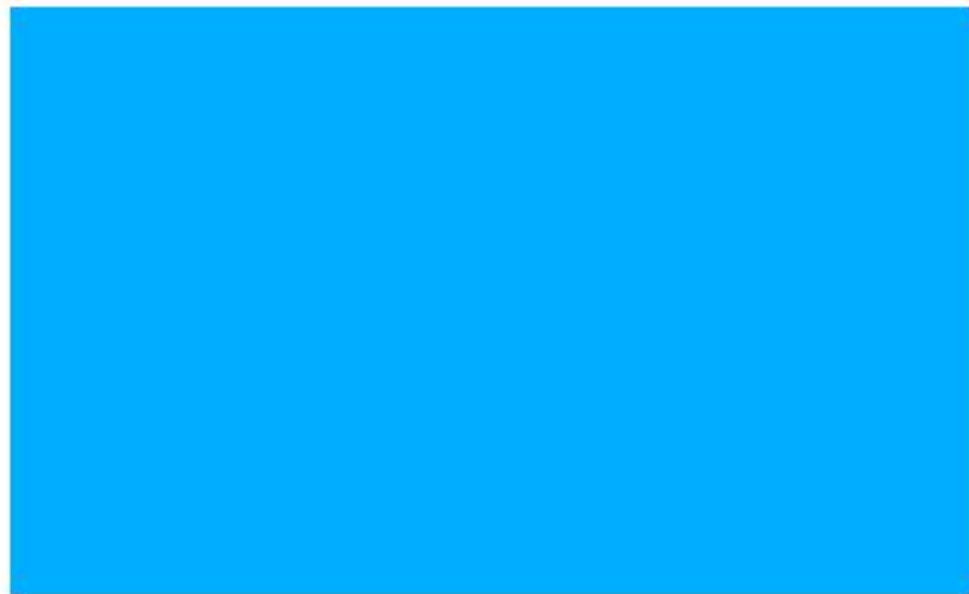
The linking of many individual molecules called monomers



Results in the formation of a **POLYMER**



Building a very simple polymer: polyethylene

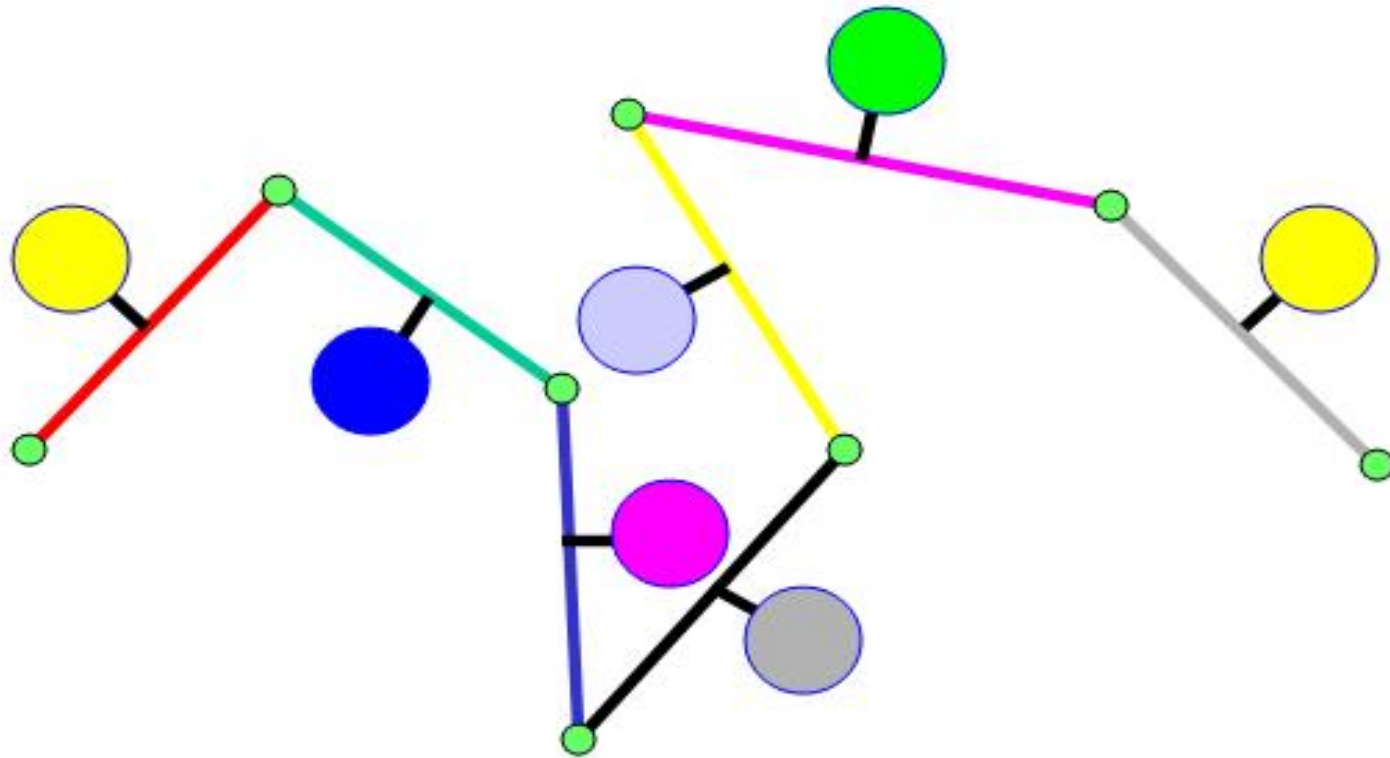




Proteins are polymers of individual amino-acids. There are 20 different amino-acids. Combinations of these 20 amino acids can give rise to extremely complex polymers called polypeptides.

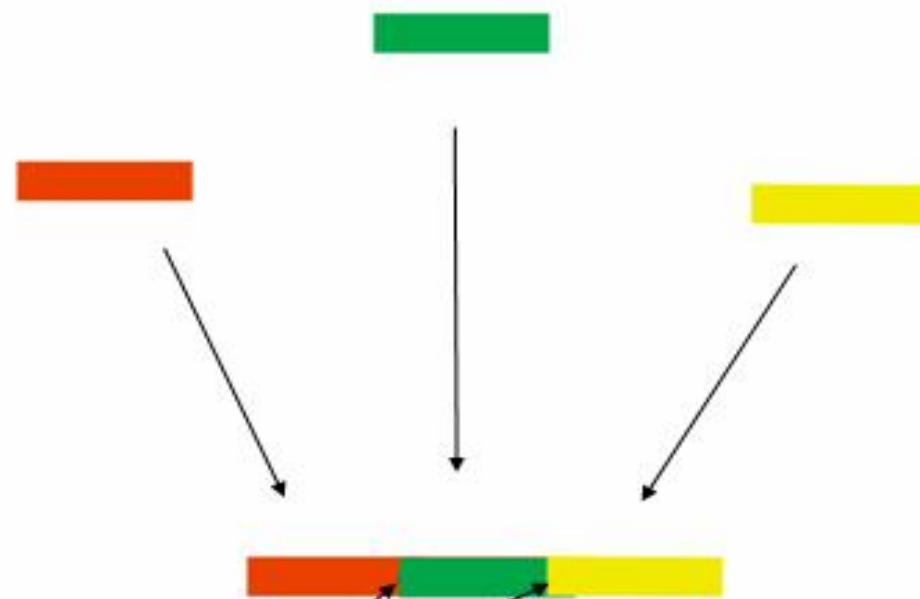


Polymers then, are freely jointed segments with complex chemical groups branching out on the sides.

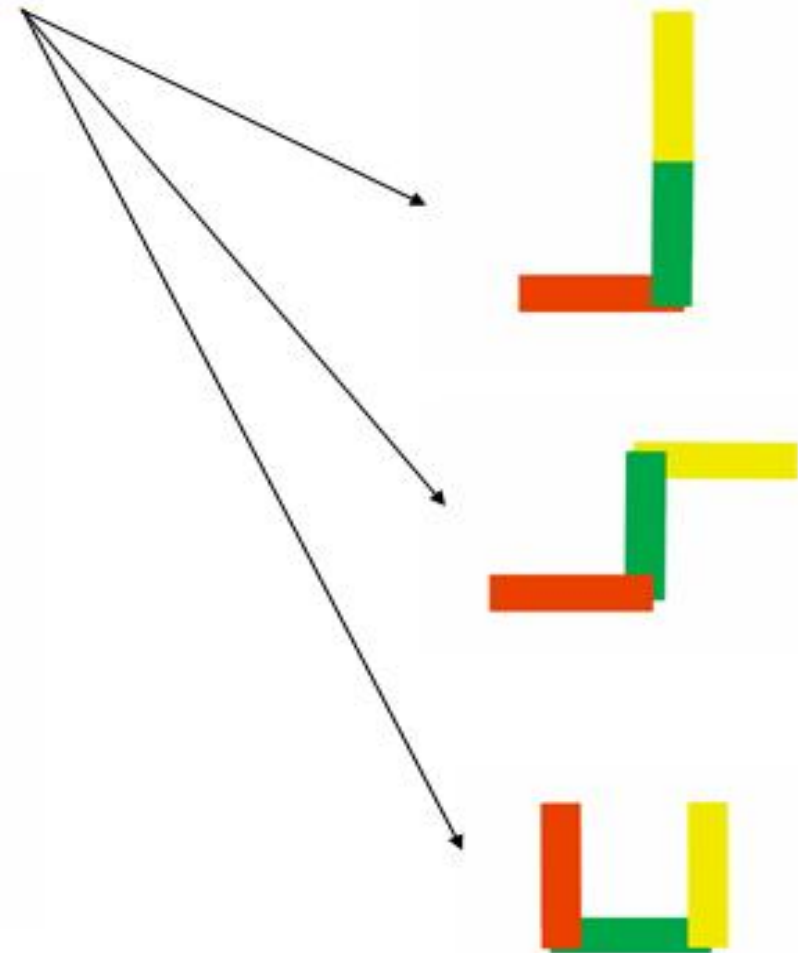


# Polymers can have many different conformations

Making a simple **toy** polymer



joints

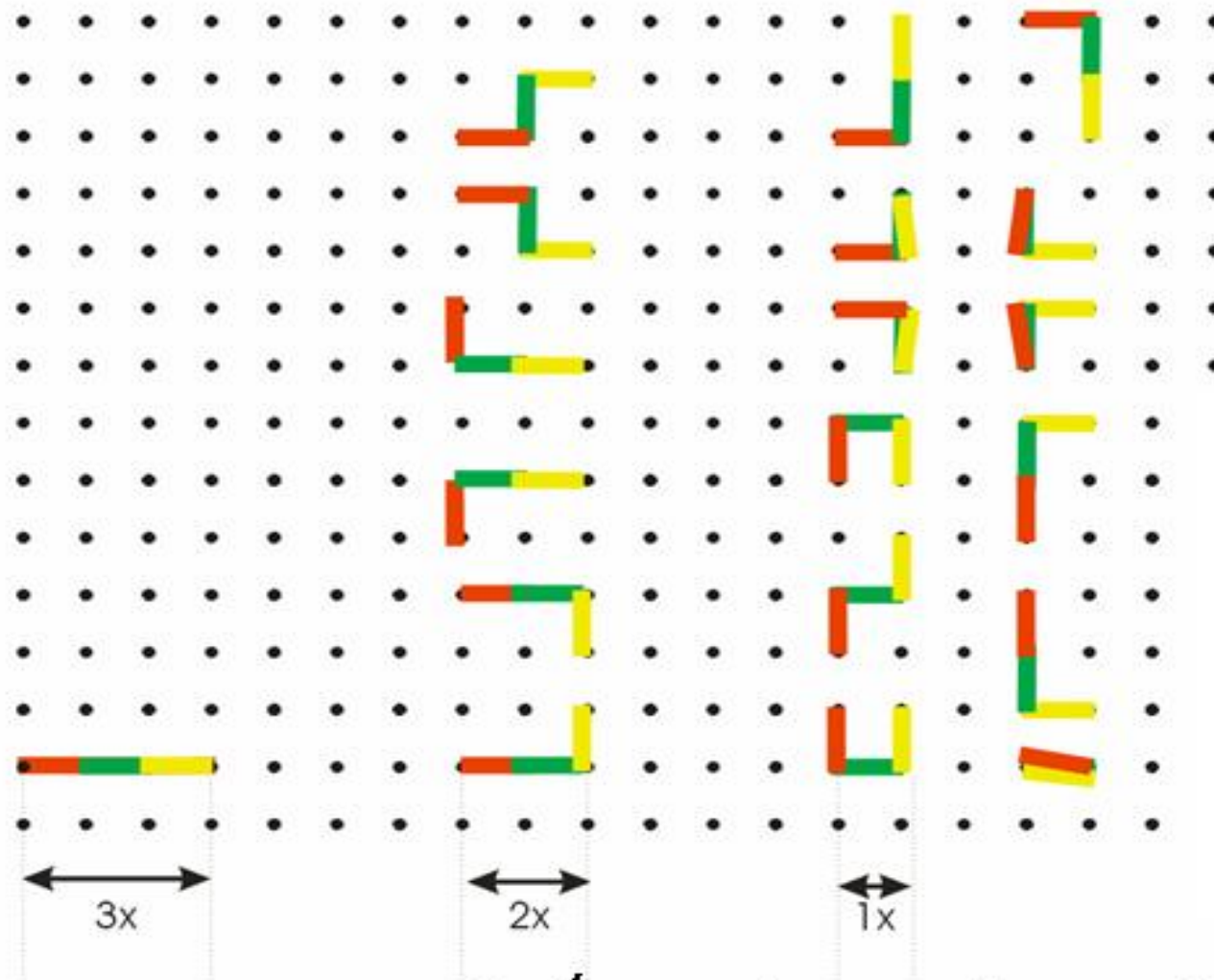


Polymers move towards situations that permit the largest number of conformations



Ludwig Boltzmann  
(1844-1906)

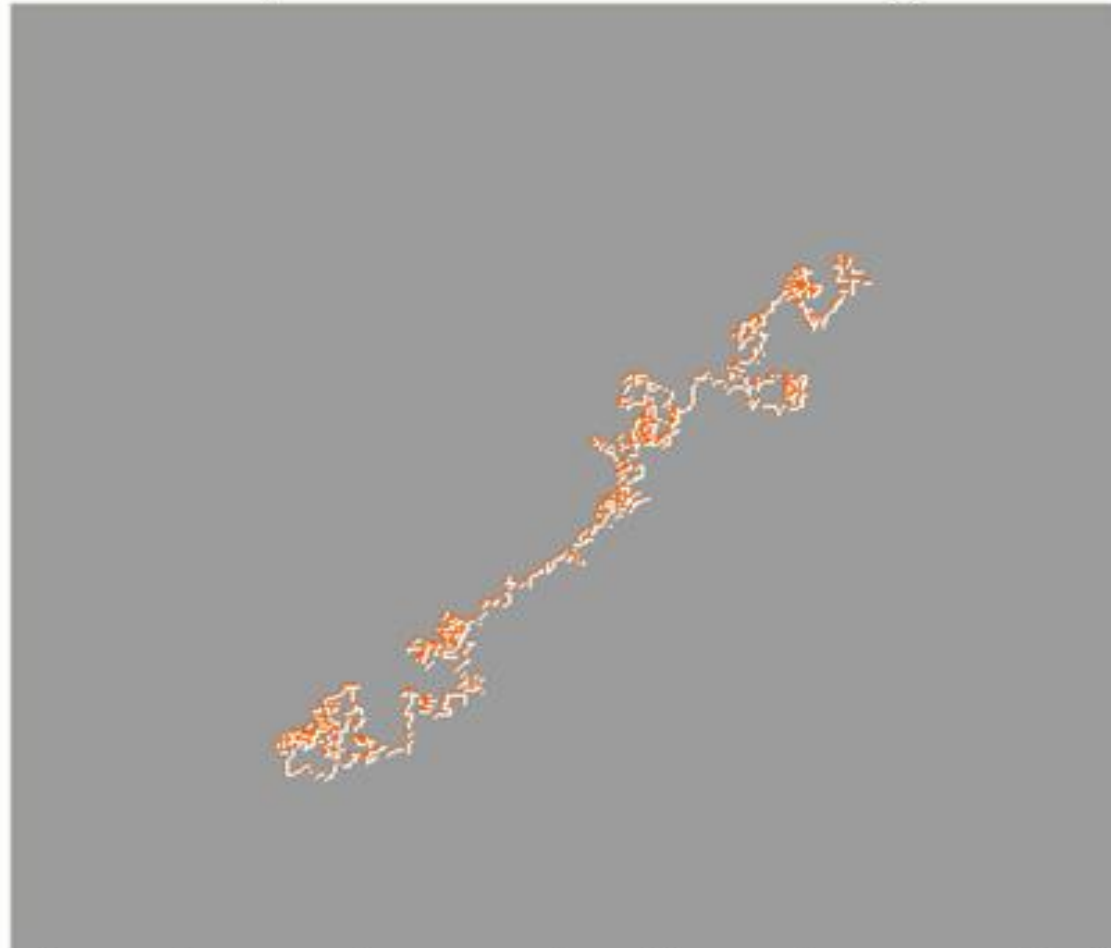
Austrian physicist who established the relationship between entropy and the statistical analysis of molecular motions



$$Entropy \cong \ln(\text{number of conformations})$$



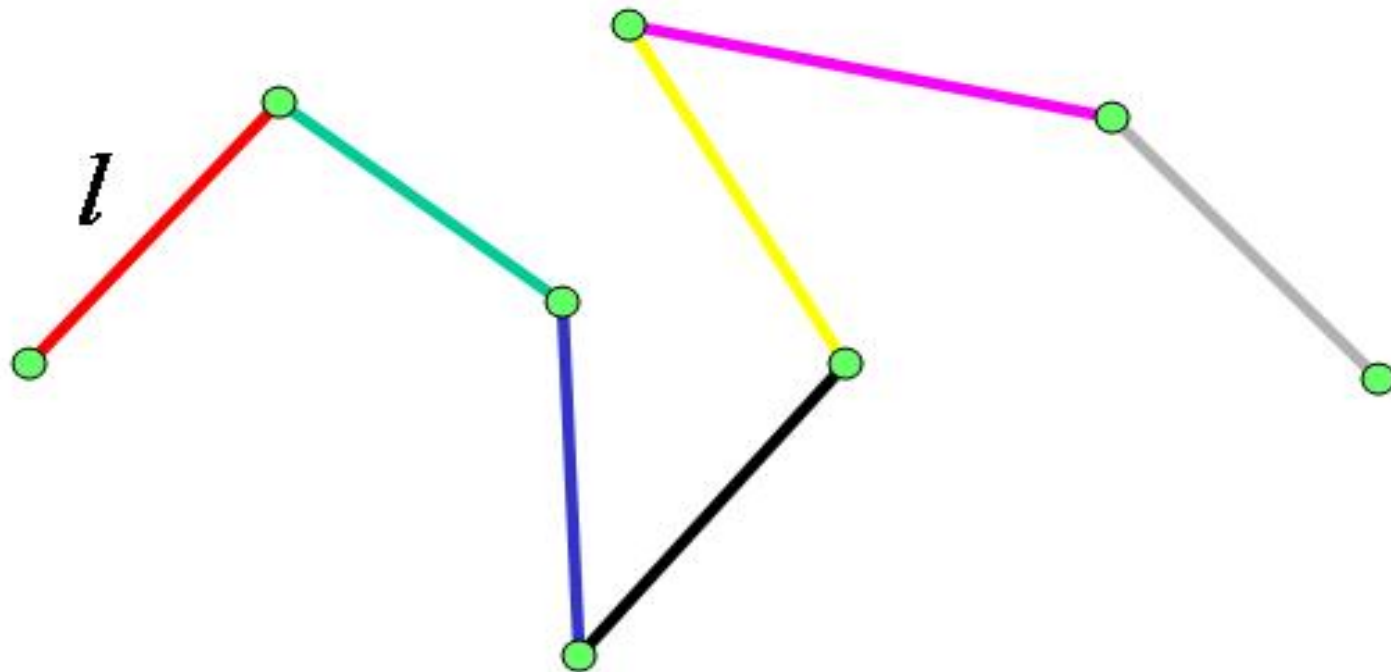
Increased entropy makes polymers look crooked  
and pushes them to collapse



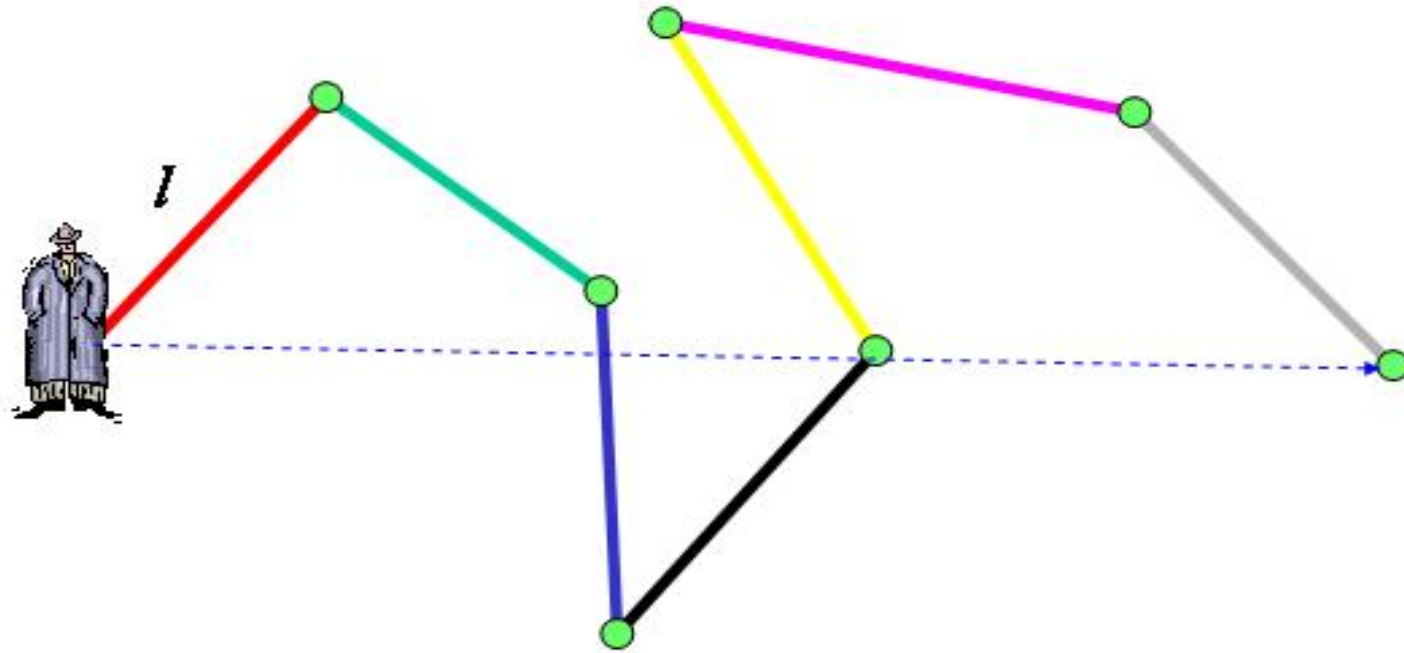
Simulation of a polymer increasing its entropy as it shrinks from  
a more extended conformation

For a polymer made of  $N$  segments of equal length  $l$ , the contour length is defined as:

$$L_c = N \cdot l$$

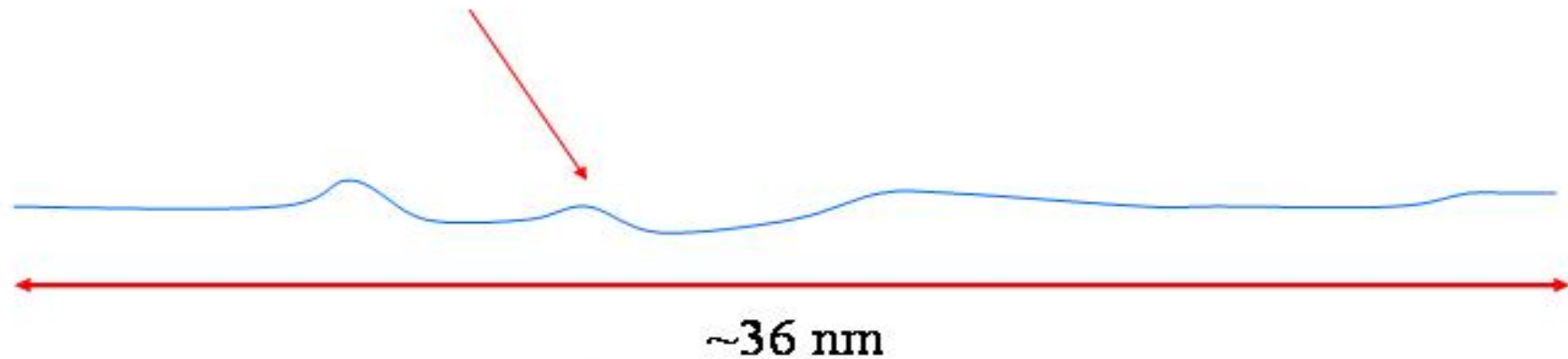


A polymer looks like the path of a particle  
in Brownian motion



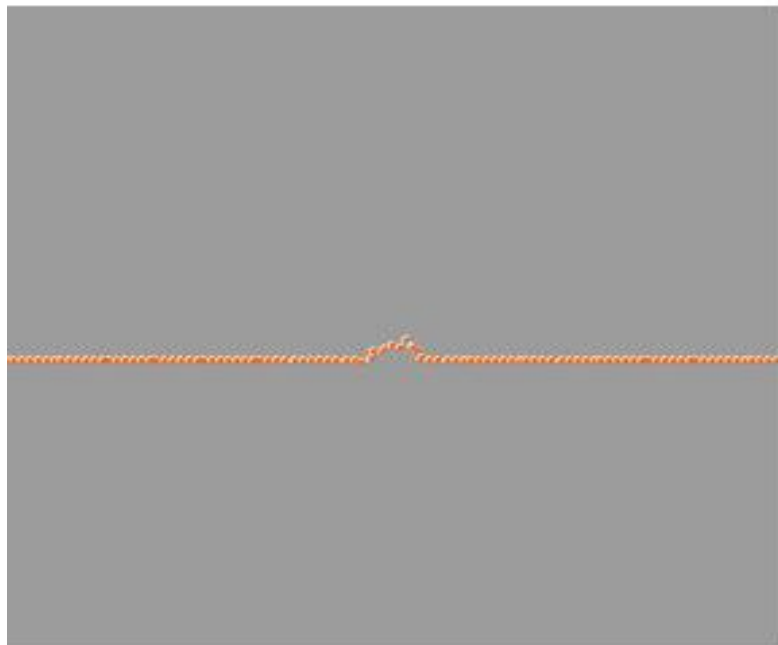
$$d = \sqrt{lv(t_2 - t_1)} = \sqrt{lL_c}$$

The contour length,  $L_c$ , of a single protein molecule composed of 100 amino acids is 36 nm



Answer:

$$d = \sqrt{L_c l} \cong 3.6 \text{ nm}$$



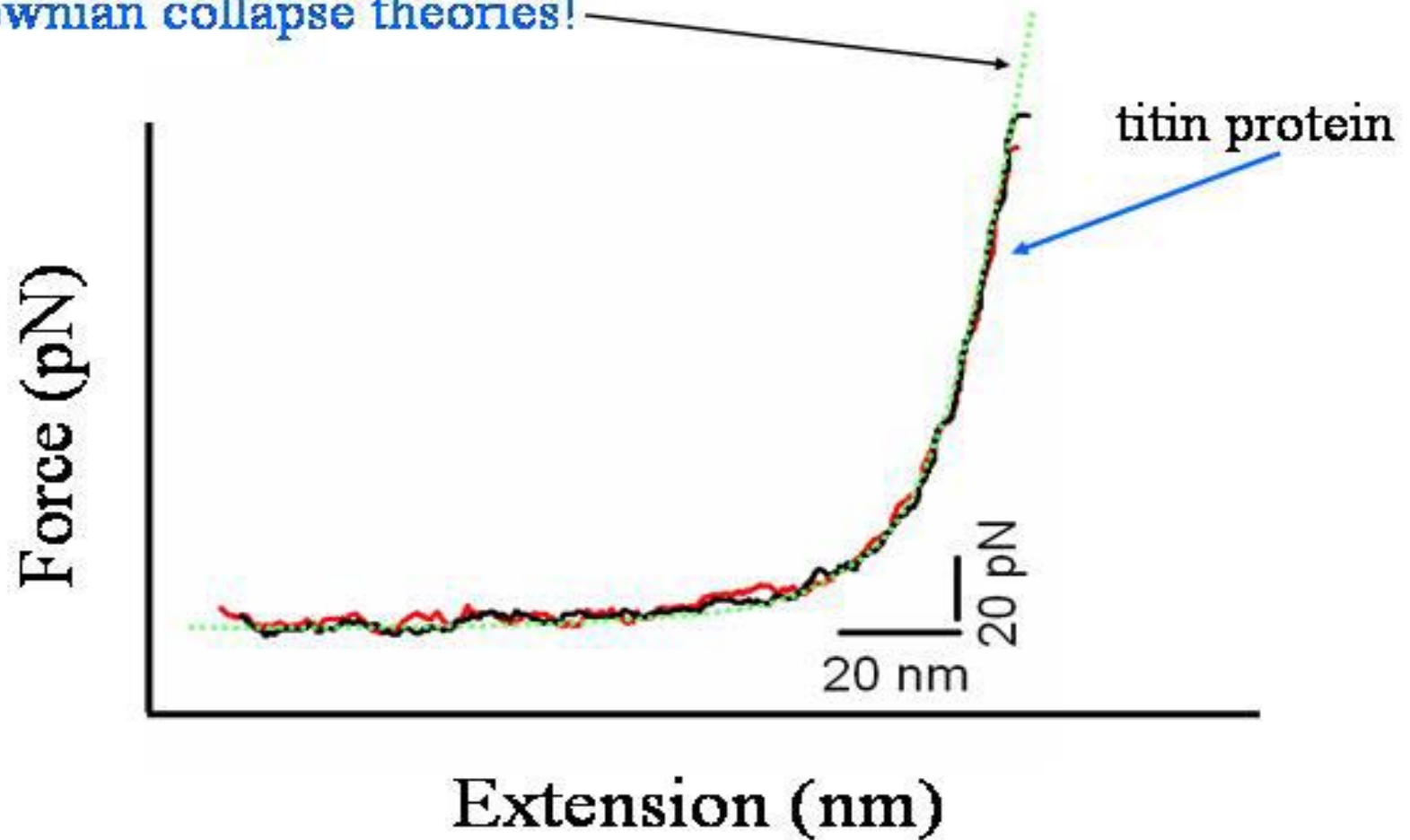


## Derivation of the elasticity of a freely jointed polymer chain

We calculate how much a polymer extends at a given pulling force.

We assume that the polymer chain is in equilibrium, with randomly oriented segments.

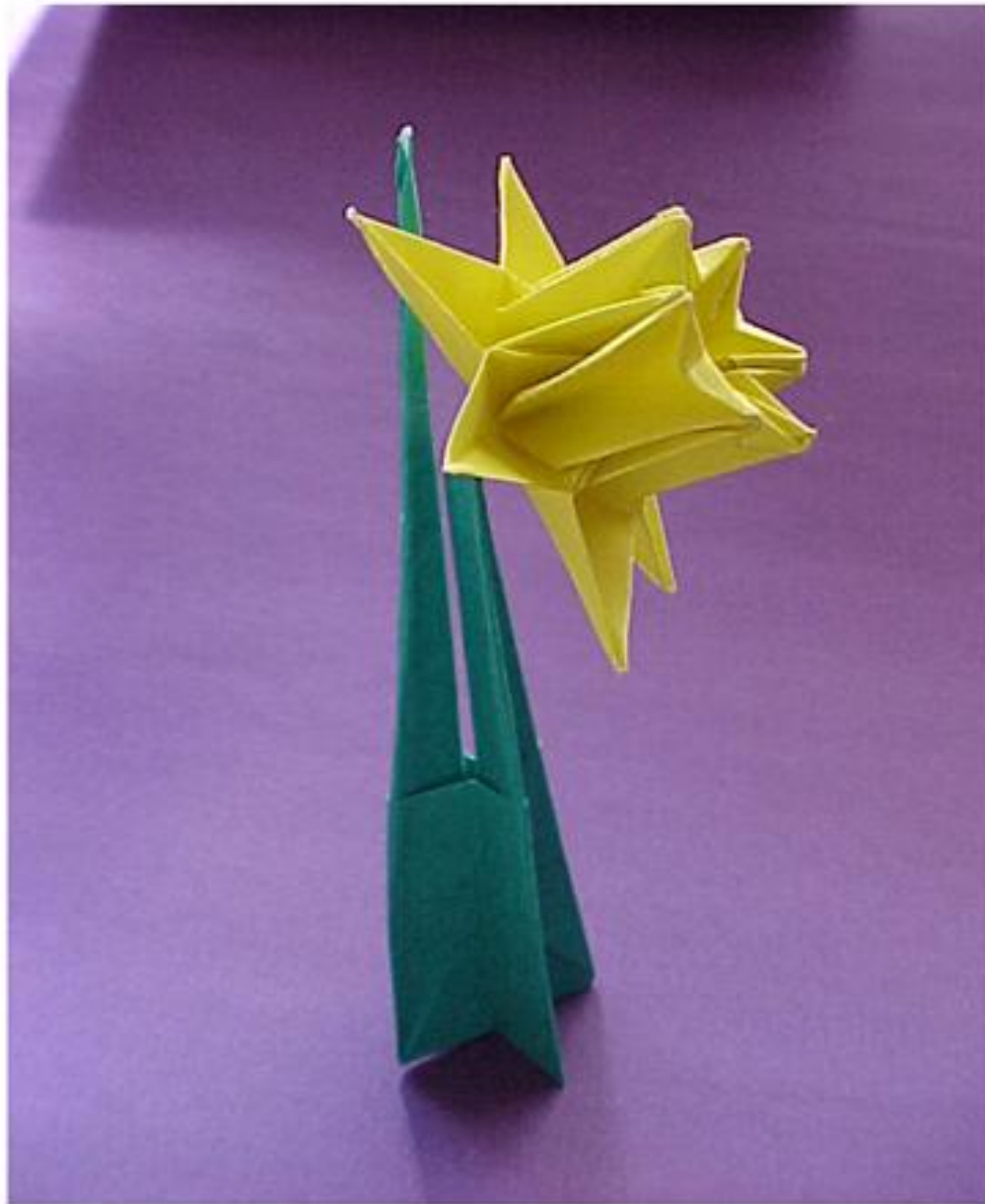
If we stretch a single titin protein we obtain force-extension curves that are perfectly explained by the Brownian collapse theories!



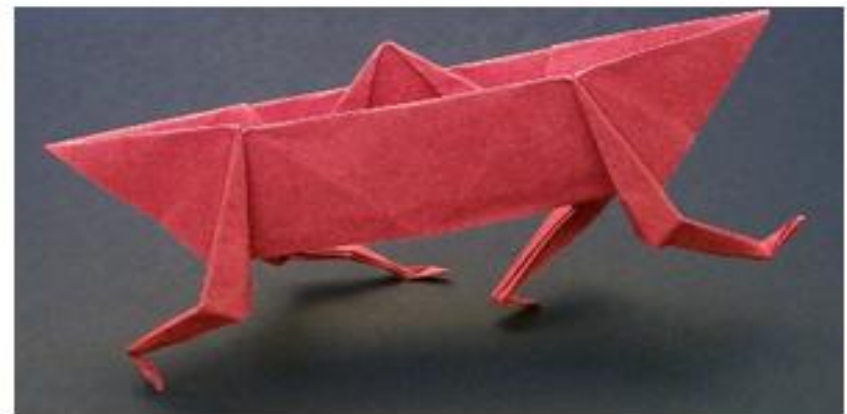
Ok, so we have discovered that muscle elasticity results from the Brownian motion of titin.

What happens to titin if we stretch it too much?  
Will it brake like the metal spring?

No! Certain parts of titin will just reversibly unfold.

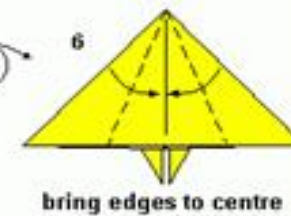
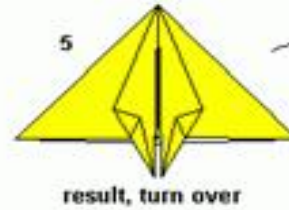
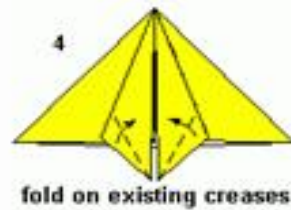
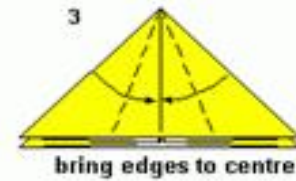
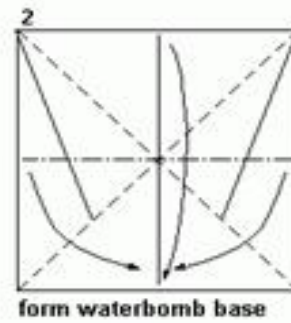
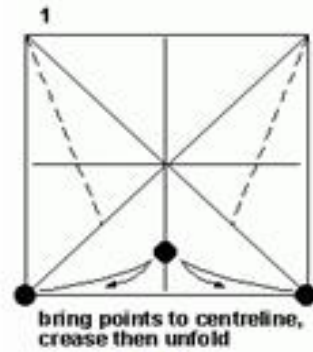


**Origami:** The art of paper folding

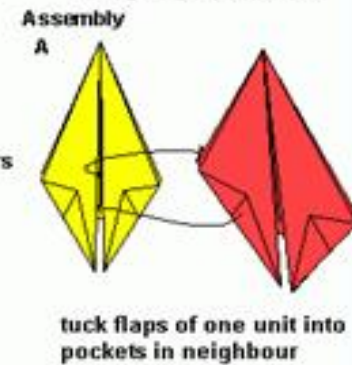




# TUVWXYZ



make 6 units in each of 7 different colours

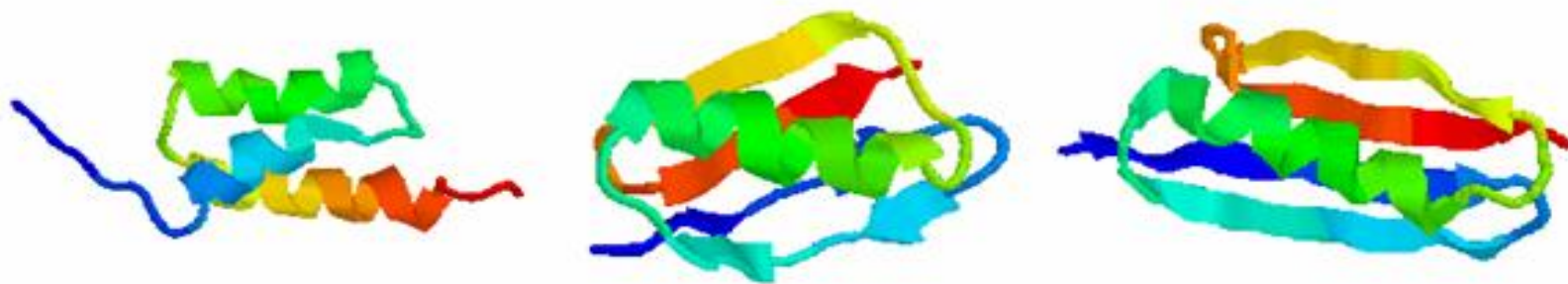


build a ring of 6 differently coloured units, then extend each unit into a 6 point star shape. There is another ring of 6 units (coloured white in drawing) which is added during construction.



model © M.Mukhopadhyay  
diagrams © D.Petty

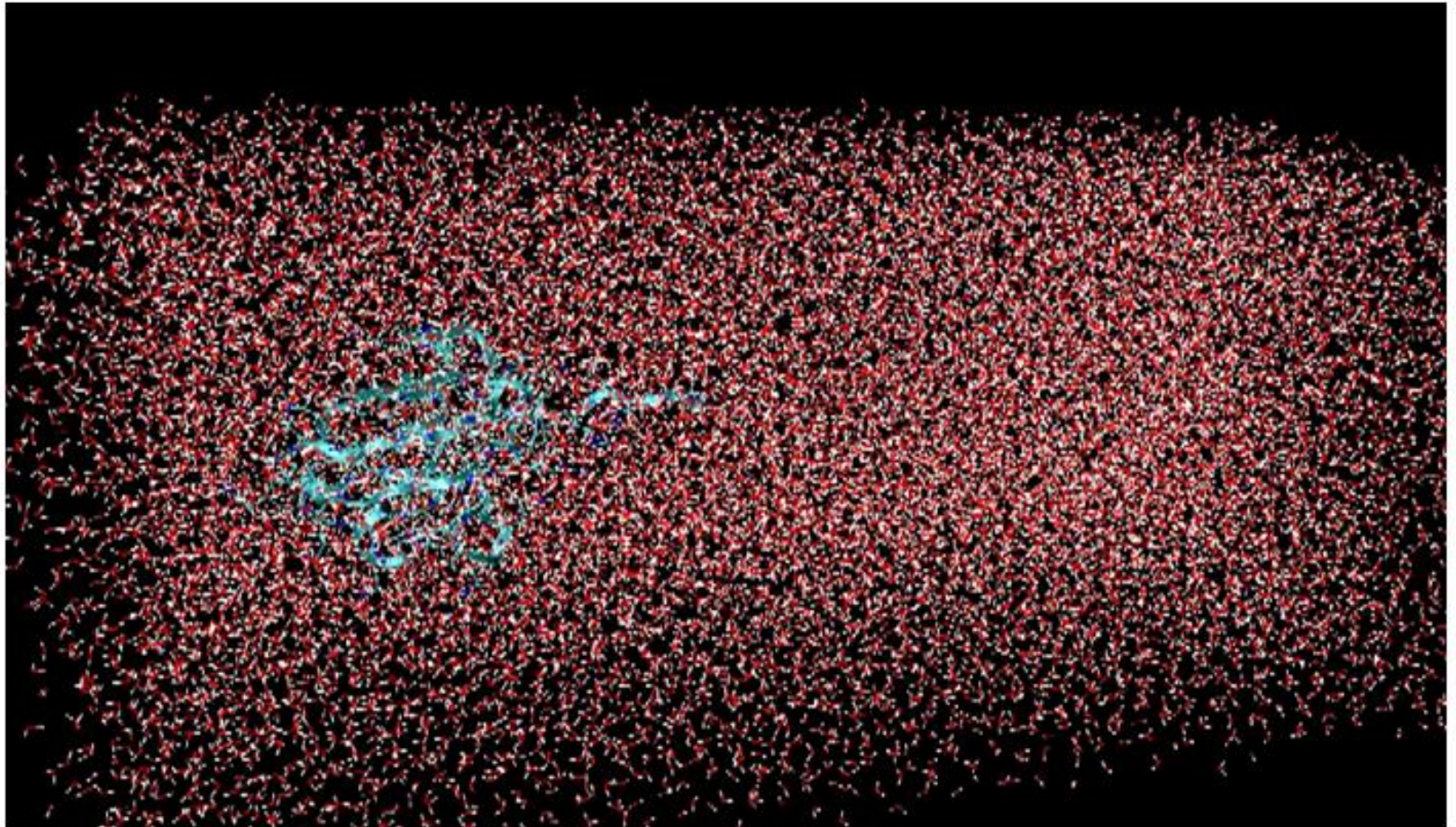
Likewise there are folding instructions for proteins



We know they exist, but we do not know what they are.

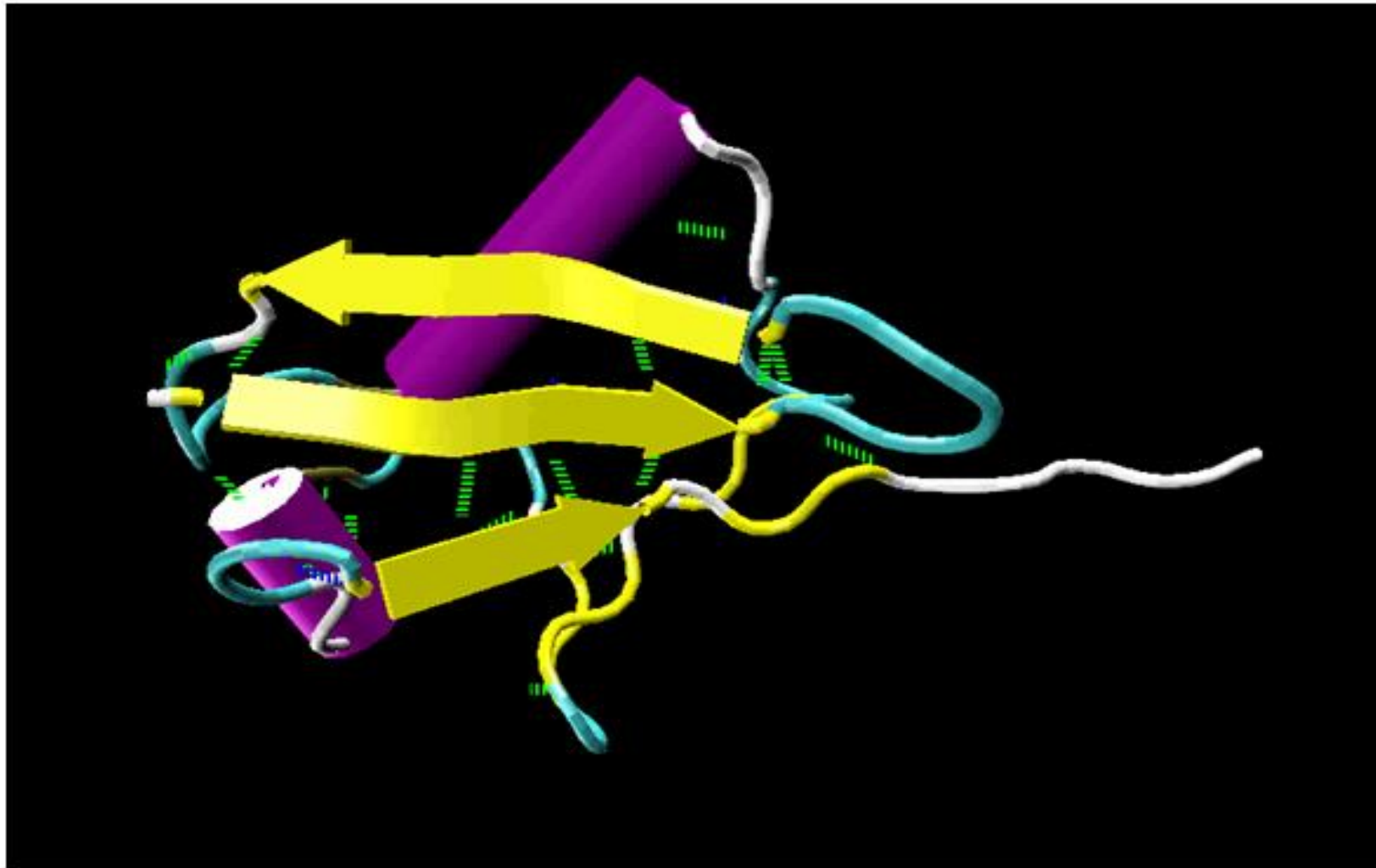


Folded proteins have a **bonded structure** that helps them resist the constant bombardment caused by Brownian motion.



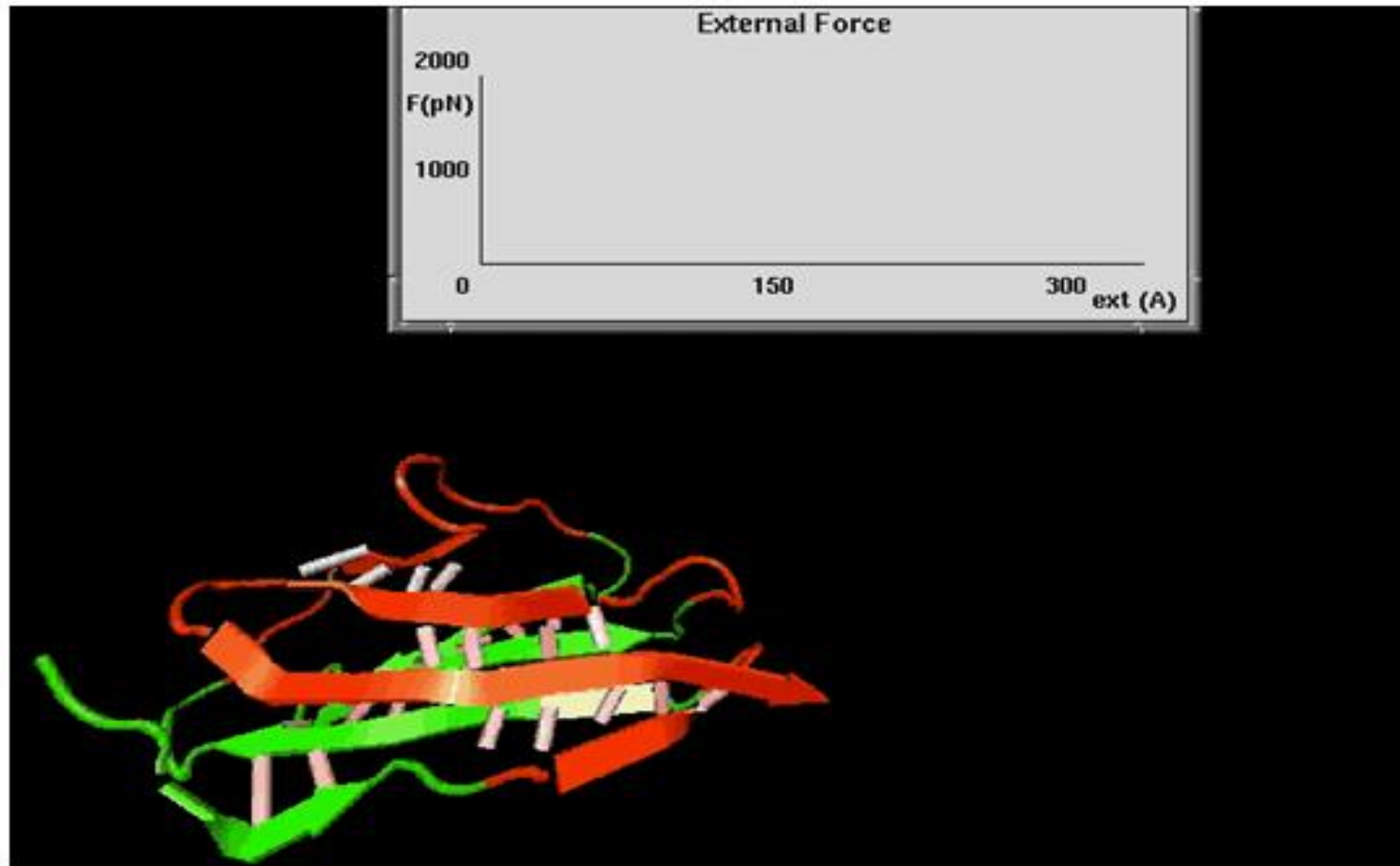


The Bonded structure is dynamic, as it gets bombarded, the structure bends and bonds break and reform

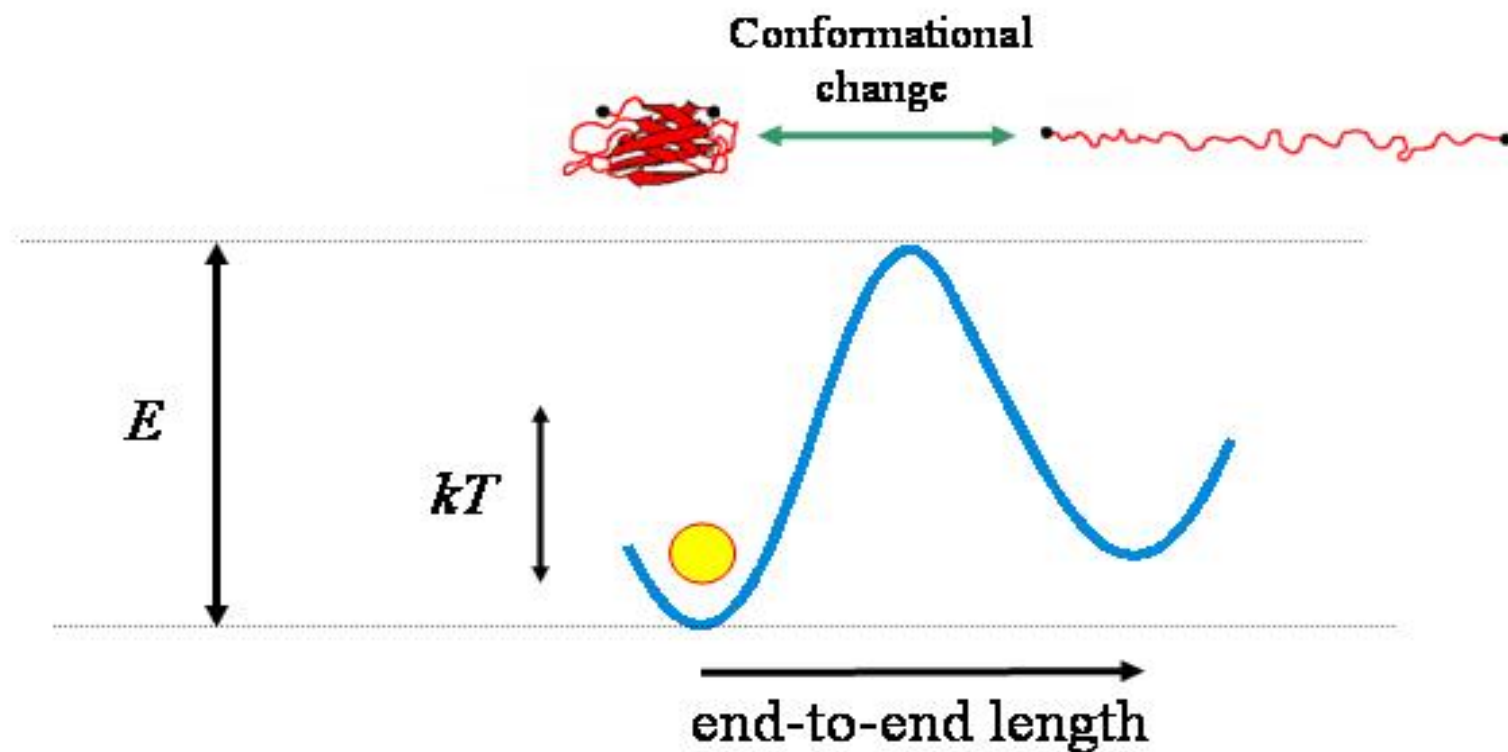




The bonds create an energy barrier



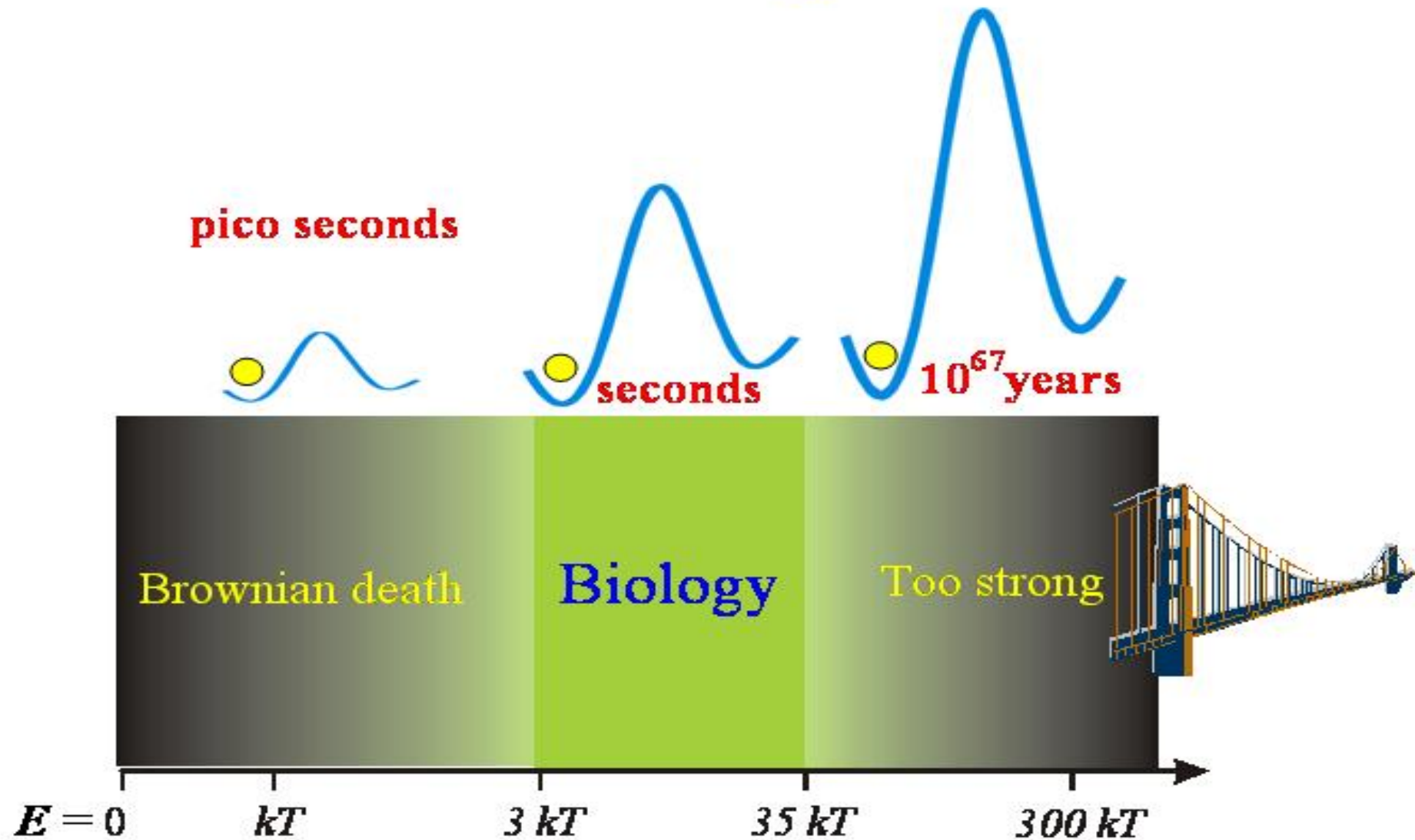
How often does Brownian motion overcome the energy barrier and causes the protein to unfold spontaneously ?



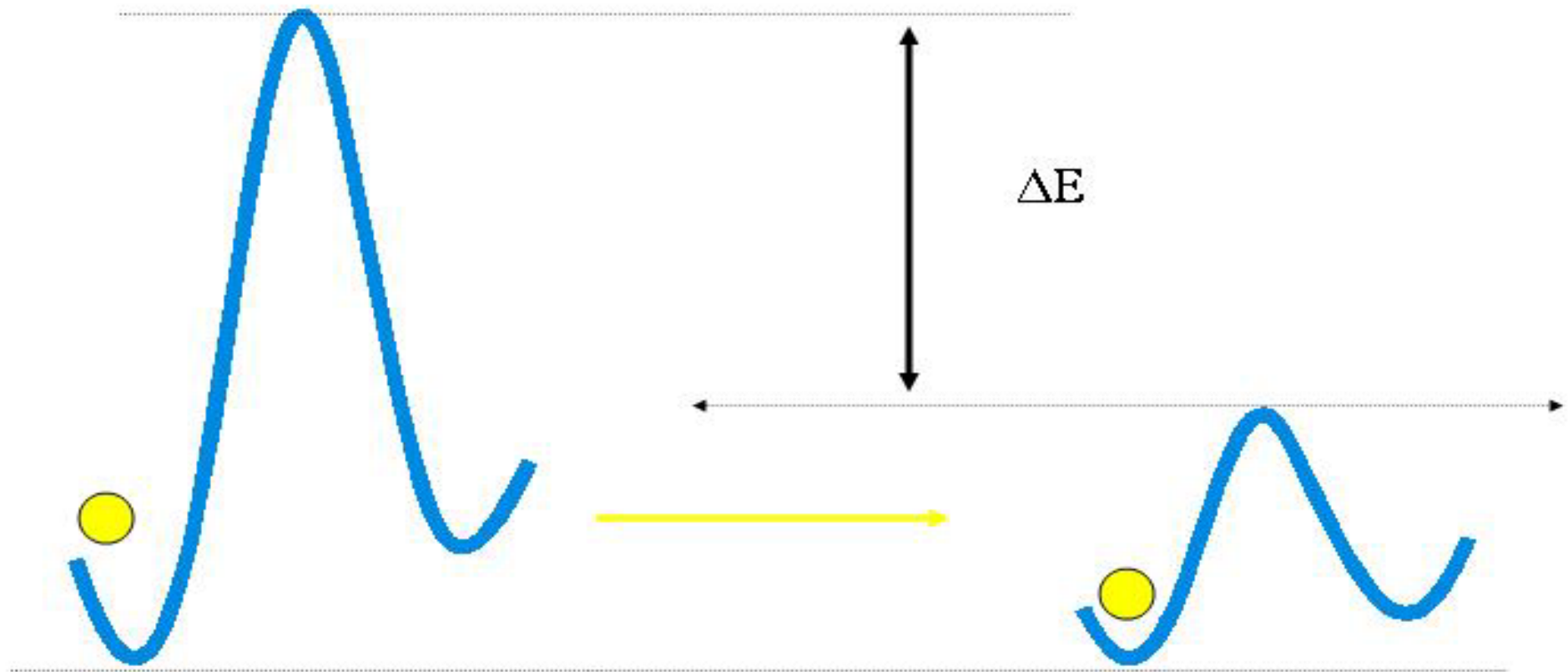
$$t \cong t_0 \cdot e^{\frac{E}{kT}}$$

$$t_0 \sim 10^{-12} \text{ seconds}$$

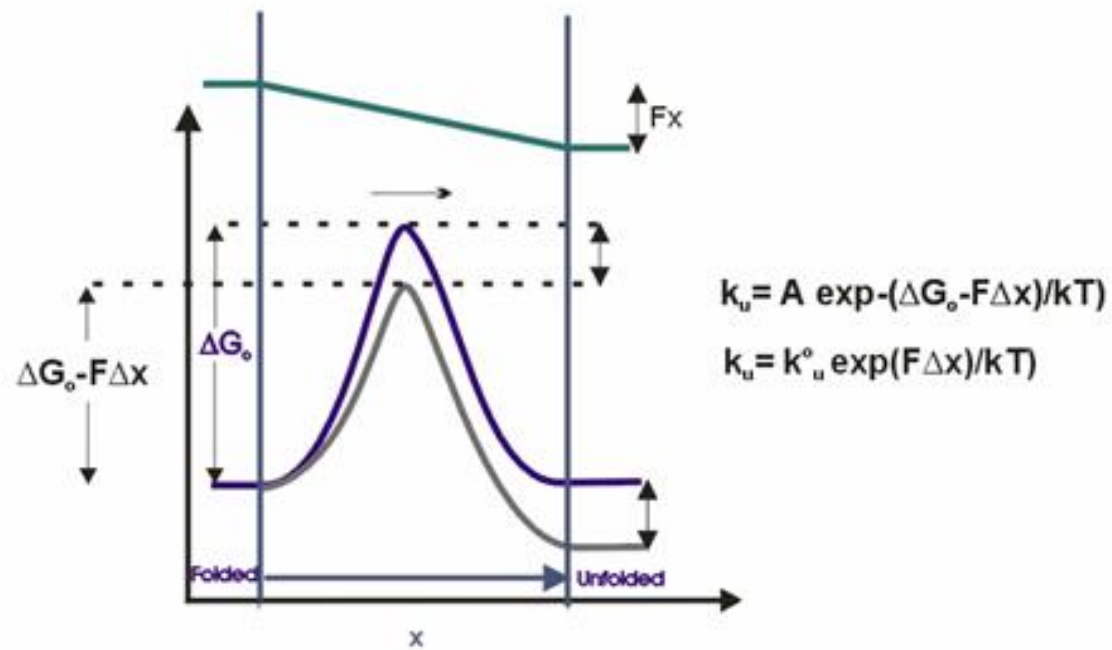
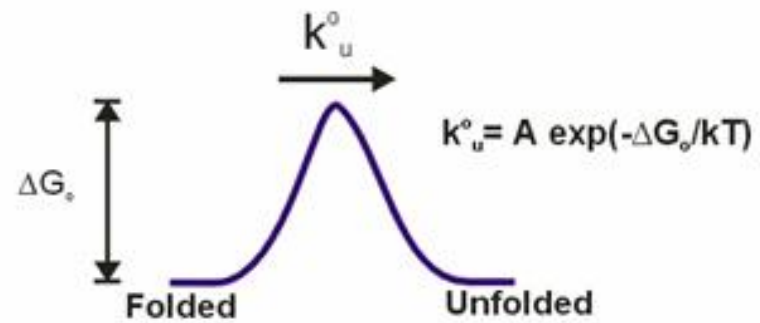
How long do we have to wait for Brownian motion to overcome an energy barrier ?



Control in Biology is accomplished by  
reducing energy barriers. Then, unlikely events  
will occur over short time periods



## Effect of an external force on a rate constant





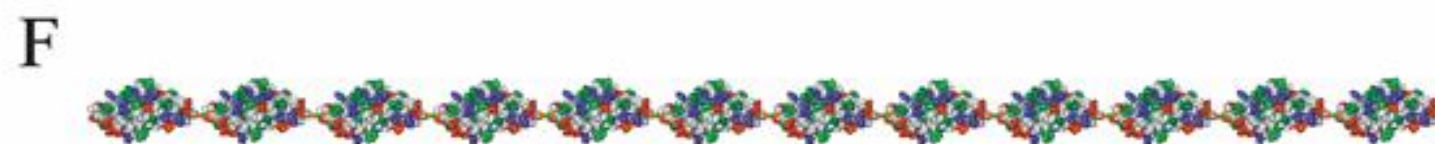
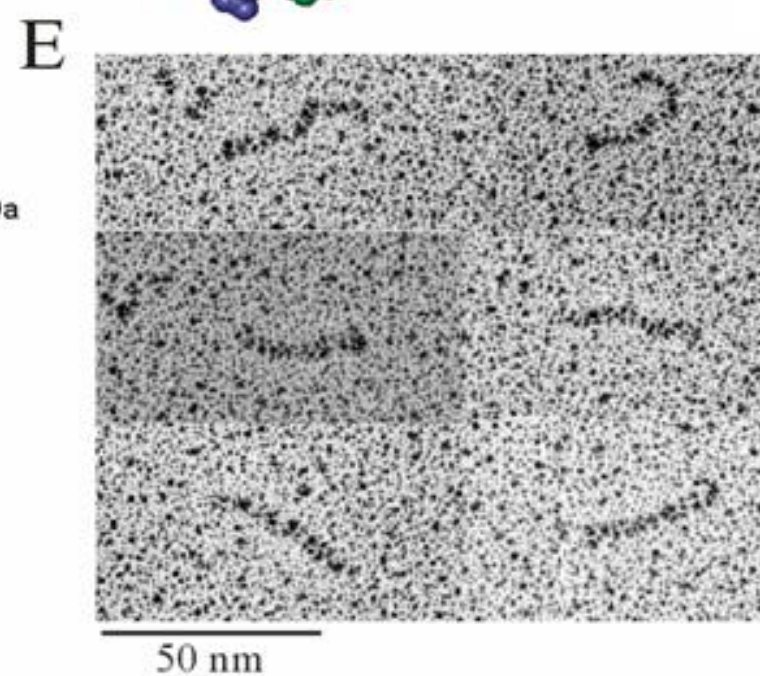
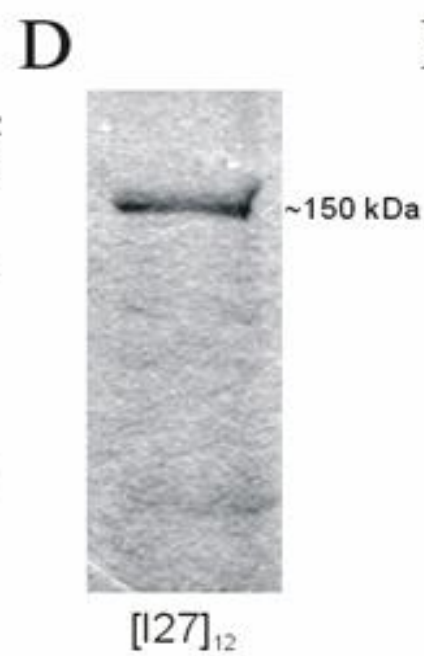
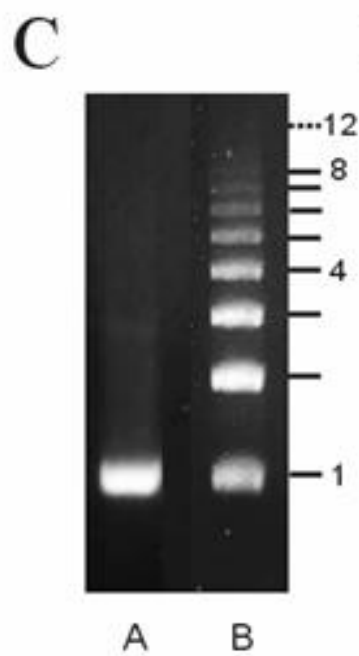
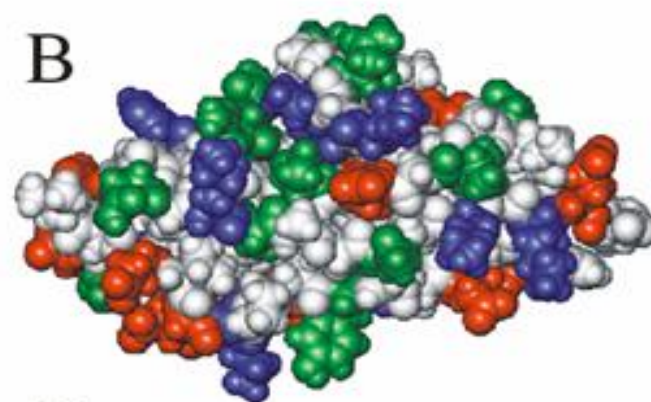
If we apply a mechanical force to the protein, we can accelerate unfolding by reducing the energy barrier.

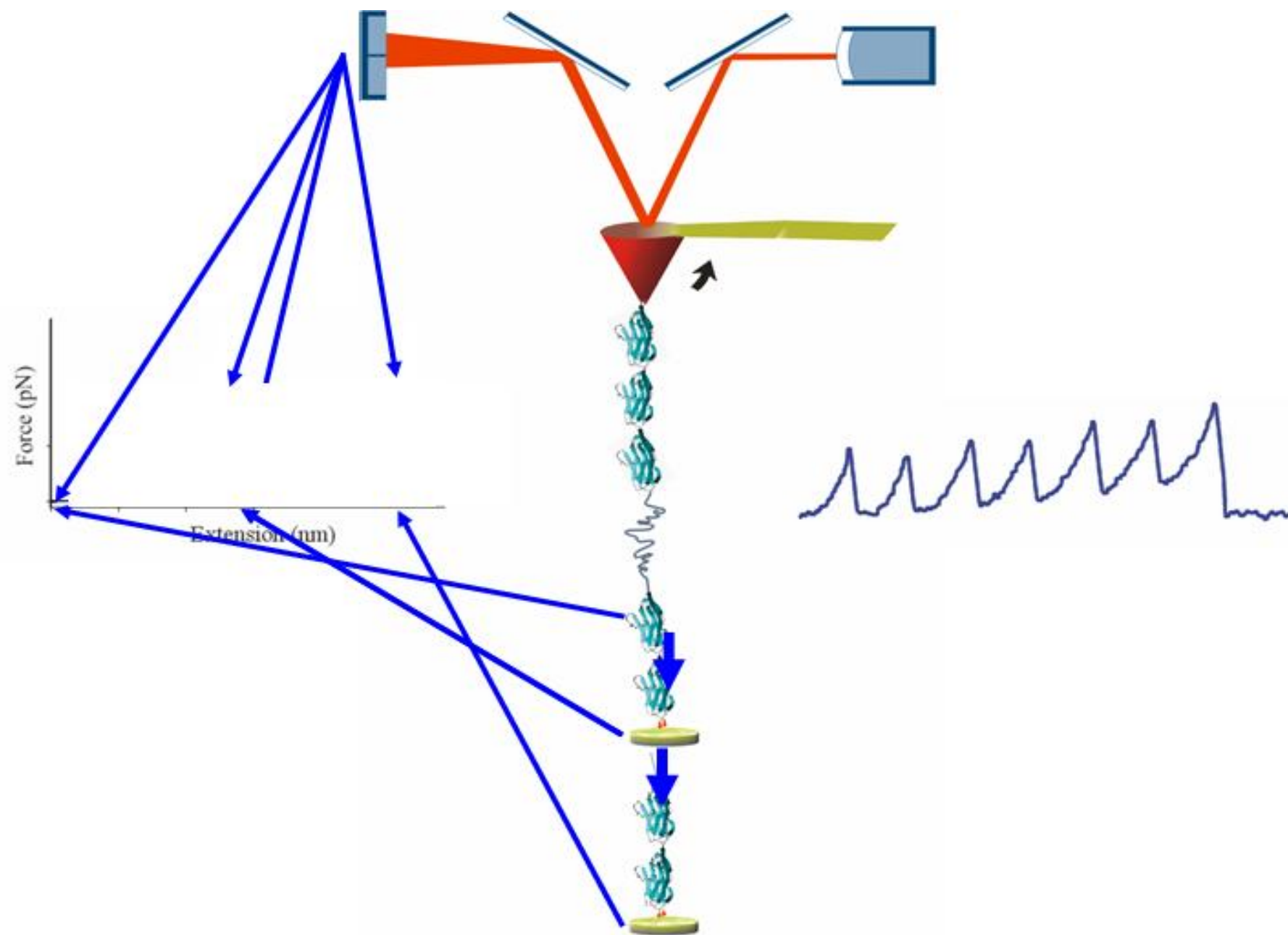


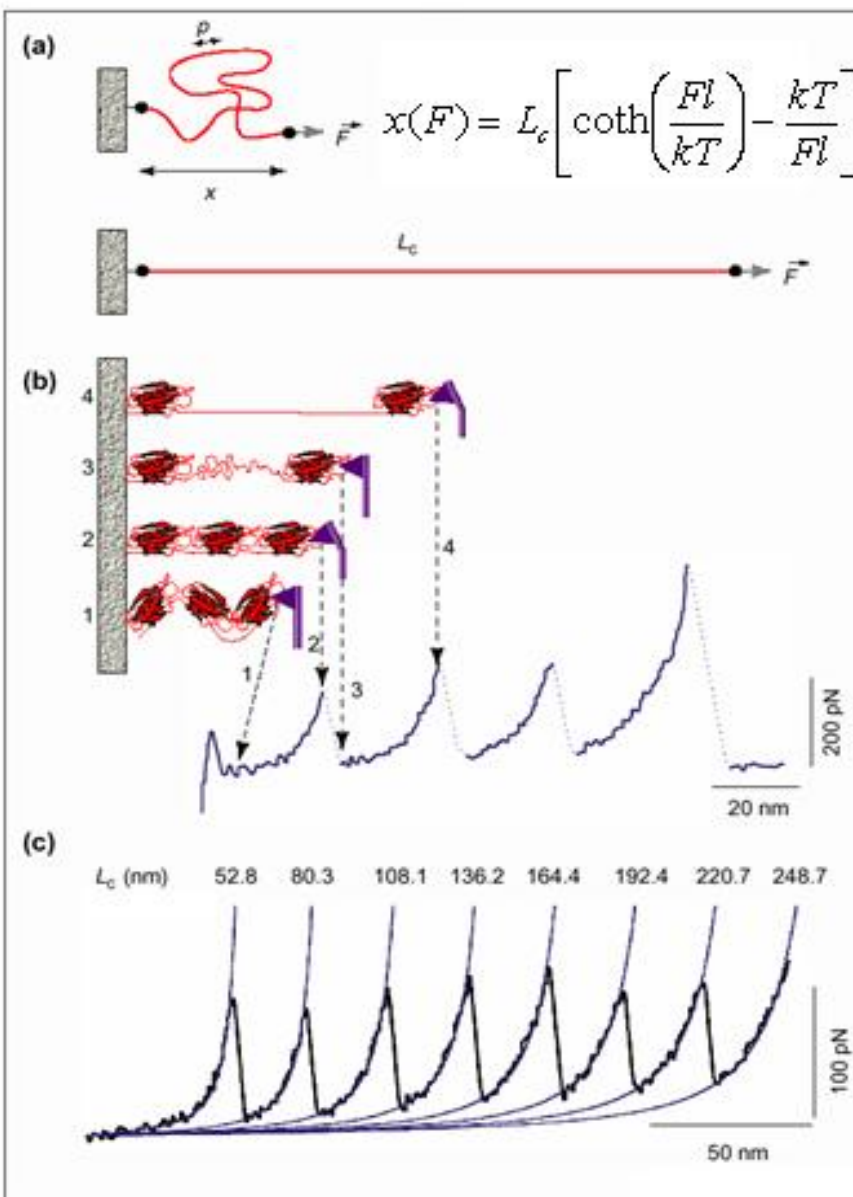
If  $F\Delta x = 0$  then  $t=40$  minutes!

If  $F\Delta x = 3kT$  then  $t=2$  minutes!

If  $F\Delta x = 35kT$  then  $t < \text{picosecond} \sim 0$





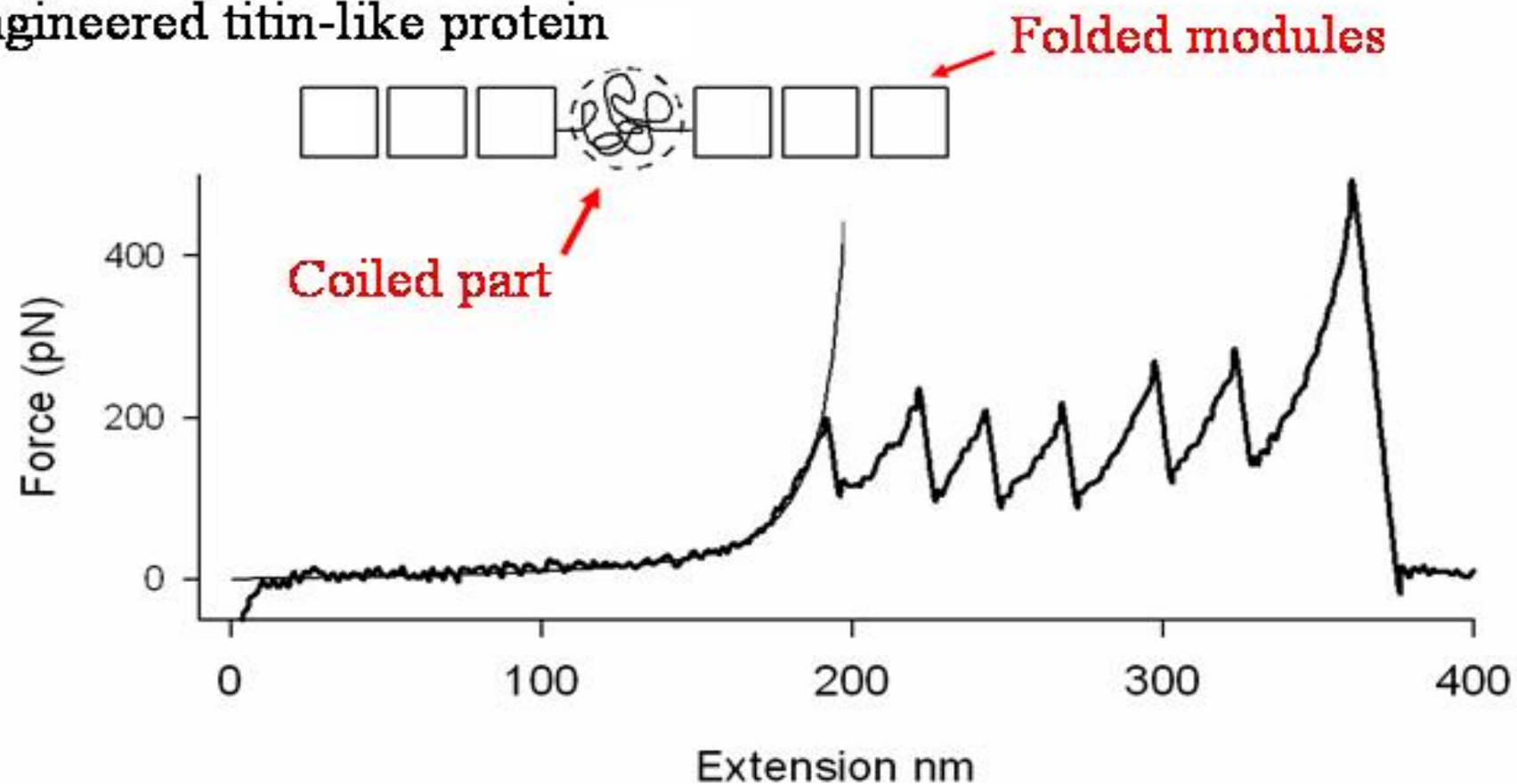


# Igor Demonstration of actual data and its analysis with models of polymer elasticity



**We can understand how titin works by engineering a protein that imitates its properties.**

Engineered titin-like protein



# Conclusions

Brownian motion explains muscle elasticity in humans



Helps understand the effect of mutations on the elasticity



We may be able to test and design better titin molecules



Source: Orina