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Theory of Gravitational Waves

Transparencies - II

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THEORY OF GRAVITATIONAL WAVES – II

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How is gravitational wave emission linked to changes in the source?

Changes in a spherically symmetric source which maintain spherical symmetry cause no change in the external field

- Birkhoff's theorem (true in Newtonian theory and GR)

$\Rightarrow$ No gravitational waves come from this

Need time-dependent, non-spherical motion to get gravitational waves

Note similarities with electromagnetism:

em: $A_0 = 0$ $\quad A_{i,i} = 0$ $\quad \Box A_i = 0$

GW: $h^{TT}_{0\mu} = 0$ $\quad h^{TT}_{jk,k} = 0$ $\quad \Box h^{TT}_{jk} = 0$

but there are also important differences

Recall

$$\Box \bar{h}_{\mu\nu} = - \frac{16\pi G}{c^4} T_{\mu\nu}$$

Integrating this, one gets after some manipulation

$$h^{TT}_{jk}(t, \underline{x}) = \frac{2}{r} \frac{G}{c^4} \ddot{I}^{TT}_{jk}\left(t - \frac{r}{c}\right)$$

where I_{jk} is the mass quadrupole moment given by:

$$I_{jk} = \sum_A m_A \left[x_j^A x_k^A - \frac{1}{3} \delta_{jk} (x^A)^2 \right]$$

(definition of Misner, Thorne & Wheeler - 1973)

The energy flux is given by

$$T_{0r} = \frac{1}{32\pi} \frac{c^4}{G} \left\langle h^{TT}_{jk,0} \, h^{TT}_{jk,r} \right\rangle$$

where $\langle \; \rangle$ means average over several cycles

Putting in the expression for h_{jk}^{TT} and integrating over the solid angle, get the luminosity:

$$L_{GW} \equiv -\frac{dE}{dt} = \frac{1}{5}\frac{G}{c^5}\left\langle \dddot{I}_{jk}\,\dddot{I}_{jk}\right\rangle$$

This is known as the <u>quadrupole formula</u>

- derived for weak field but has wider validity

For <u>non-axisymmetric</u> motion, gravitational waves can also carry away <u>angular momentum</u>

$$\frac{dJ_i}{dt} = -\frac{2}{5}\frac{G}{c^5}\,\varepsilon_{ijk}\left\langle \ddot{I}_{jm}\,\dddot{I}_{km}\right\rangle$$

where ε_{ijk} is the permutation tensor

Order of magnitude estimates

$$\dddot{I}_{jk} \sim \frac{MR^2}{T^3} \sim \frac{Mv^3}{R}$$

M, R, T, v are characterist[ic] mass, size, timescale and velocity of the source

From the earlier formula:

$$L_{GW} \sim \frac{G}{c^5}\left(\frac{M}{R}\right)^2 v^6$$

$$\sim L_0 \left(\frac{R_s}{R}\right)^2 \left(\frac{v}{c}\right)^6$$

with $L_0 \equiv \frac{c^5}{G}$

$= 3 \cdot 6 \times 10^{59}$ erg/s

R_s is the Schwarzschild radius of the source $\left(= \frac{2GM}{c^2}\right)$

$\Rightarrow$ the most powerful sources will be <u>compact</u> $R \sim R_s$ and <u>fast moving</u> $v \sim c$

For detectors, the important quantity is h

From $$h_{jk}^{TT} = \frac{2}{r}\frac{G}{c^4}\ddot{I}_{jk}^{TT}$$

we have that

$$h \sim \left(\frac{R_s}{R}\right)\left(\frac{v^2}{c^2}\right)\frac{R}{r}$$

($h \propto \frac{1}{r}$ is a general feature)

There is a connection between compactness of the source and speed of motion:

— often have

Kinetic energy $\sim$ Gravitational PE

$$\frac{1}{2}v^2 \sim \frac{GM}{R}$$

$$\frac{v^2}{c^2} \sim \left(\frac{R_s}{R}\right)$$

Then $$h \sim \left(\frac{R_s}{R}\right)^2 \frac{R}{r}$$

Relative sizes

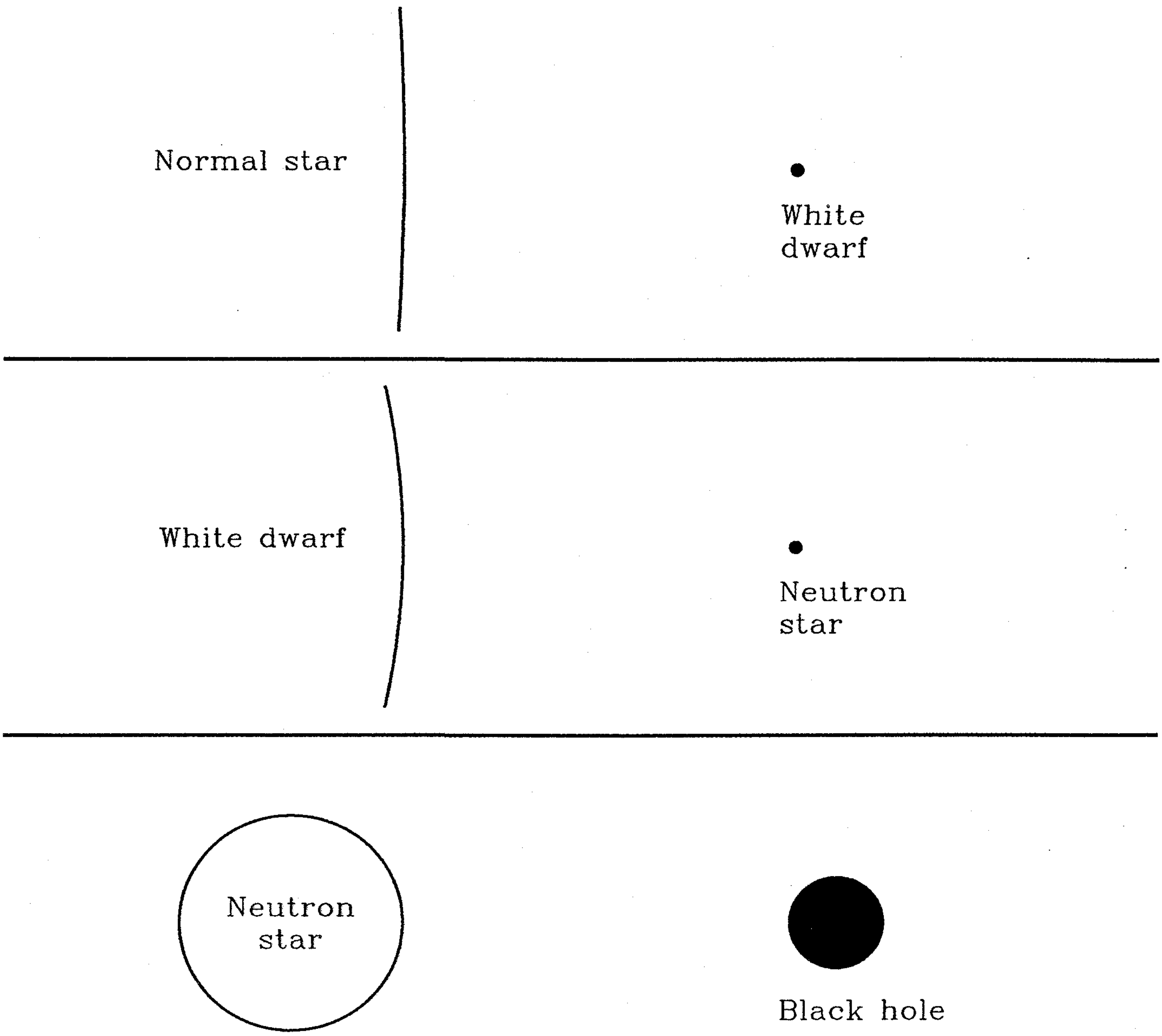
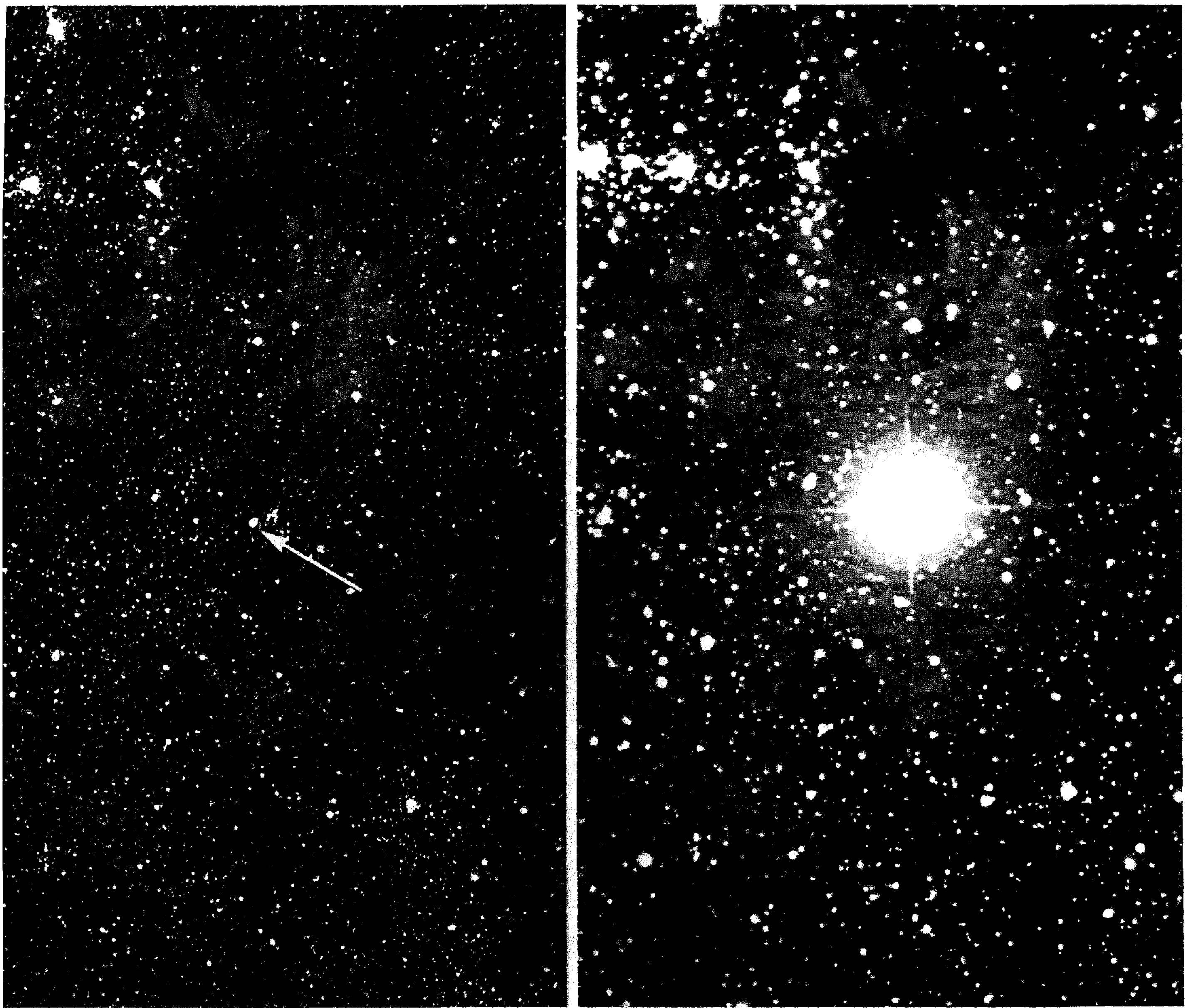
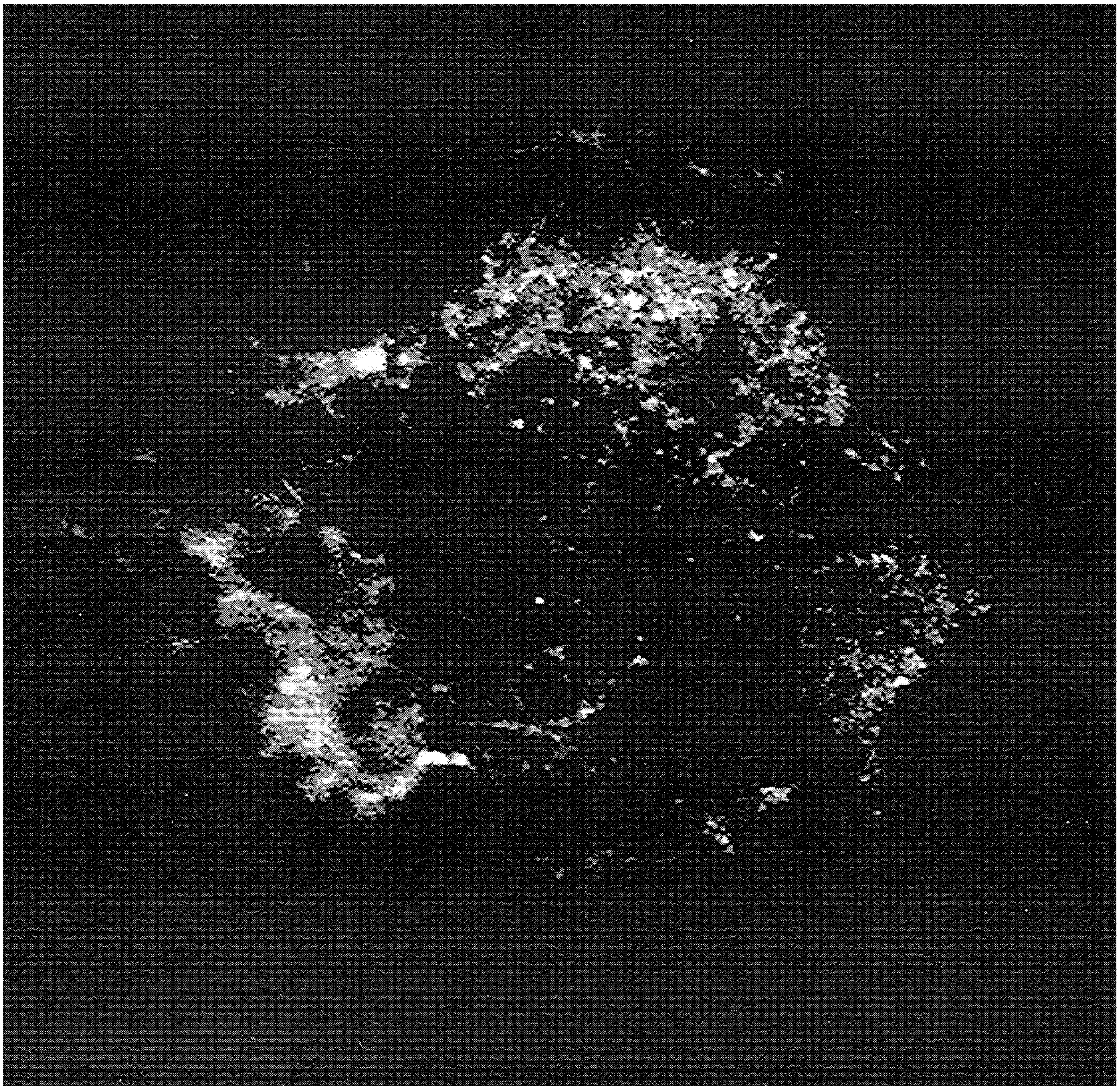


Figure 1: Relative sizes of normal stars, white dwarfs, neutron stars and black holes having similar masses (we have taken $1.4\,M_{\odot}$). Note that while white dwarfs are much more compact than normal stars, they are not nearly as compact as neutron stars or black holes which, however, come rather close together.



Before and After Supernova 1987a. In February 1987, a supernova (the catastrophic death of a very massive star) exploded in the Large Magellanic Cloud (a small companion of our own Milky Way Galaxy). This event was the nearest observed supernova since the invention of the telescope, and hence has caused great and continuing excitement amongst astronomers. Here we see photographs of the region before (left) and after the explosion. The image of the supergiant star which exploded to create the supernova (arrowed) is clearly elongated. This does not necessarily indicate any particular pecularity or a close companion, rather it is the effect of stars being by chance aligned along similar lines of sight. Several other examples can be seen in this picture and other, different, blended images are seen in the photograph of the same field taken two weeks after the supernova appeared. The difference in image quality (seeing) between these pictures is an effect of the Earth's atmosphere which was steadier when the plates used to make the pre-supernova picture were taken.

hs: David Malin, Anglo-Australian Telescope. © Anglo-Australian Telescope Board (1987)



http://chandra.harvard.edu

Irrotational case M/Mmax = 0·72

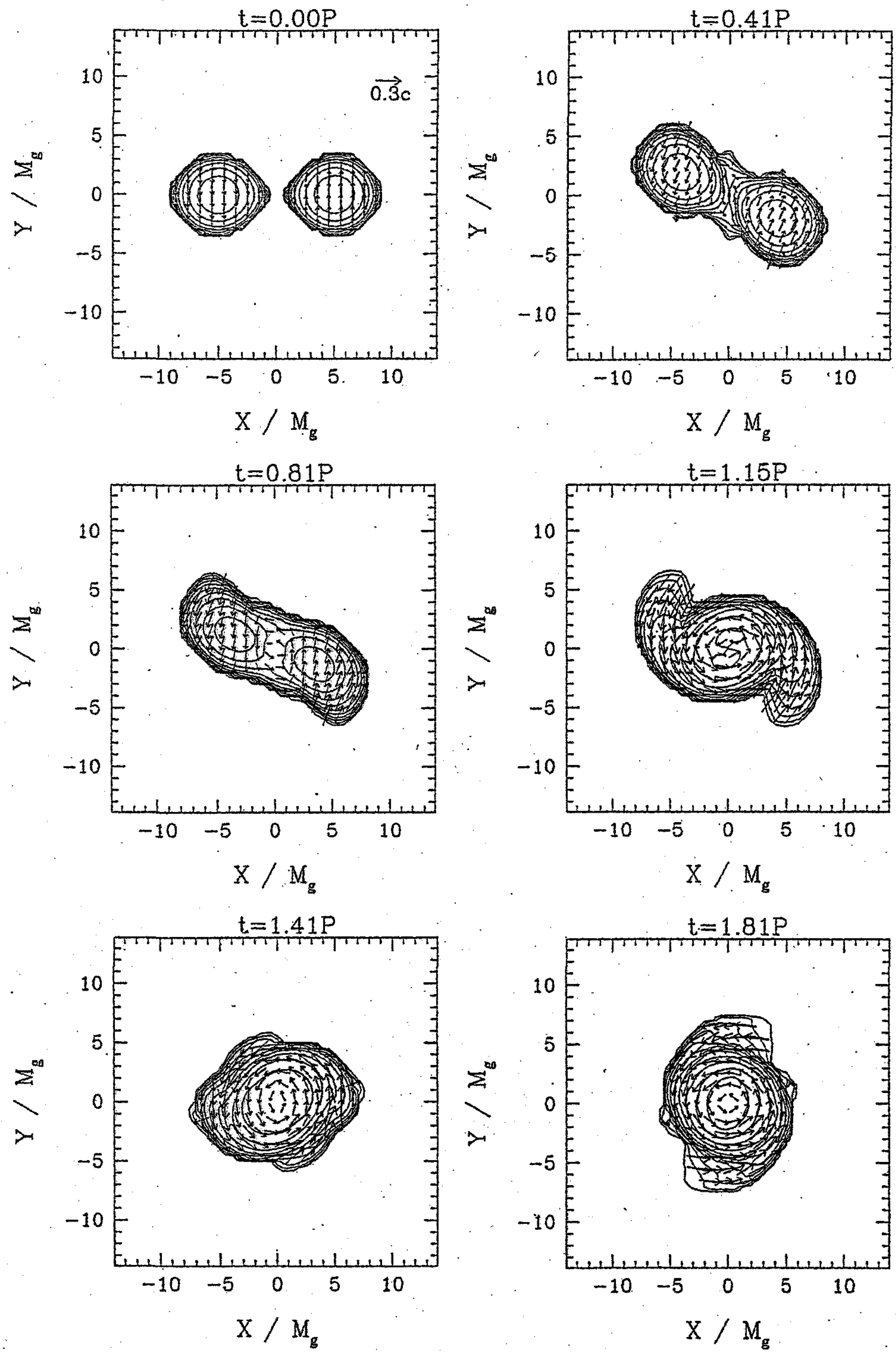


FIG. 9. The same as Fig. 2, but for model (I1). The contour lines are drawn for $\rho_* / \rho_{* \max} = 10^{-0.3j}$, where $\rho_{* \max} = 0.00401$, for $j = 0,1,2,\ldots,10$.

If Mmax = 1·86 M☉ PSR 1534+12 would be this case

ummary of astronomical sources of gravitational waves

urst sources ($h \gtrsim 10^{-22}$ for detection by advanced LIGO)

- collapse to form stellar mass black holes and neutron stars - associated with supernovae
- coalescence of neutron star and black hole binaries
- star falling into a (large) black hole

Periodic sources ($h \gtrsim 10^{-28}$)

- rotating neutron stars
  - young ones in supernovae
  - spin-up of millisecond pulsars
  - misaligned strong magnetic field
- binary stars

Stochastic sources ($h \gtrsim 10^{-25}$)

- binary stars
- supernovae
- early universe, cosmic strings, phase transitions
- Population III stars

Best bets for early detection by laser interferometers

- coalescing neutron star binaries
- rotating neutron stars
- coalescing black hole binaries

For all of these it is important to produce templates of the expected wave signals to aid extraction of signals from noise

- experiment and theory need to walk closely together!

Anticipated output from gravitational wave observations for basic physics:

- confirmation of existence of black holes
- better understanding of gravity
- neutron stars → physics of high density matter
- early universe → ultra-high energy physics

Gravitational waves and the binary pulsar PSR 1913 + 16

- object discovered by Hulse and Taylor (1974)
- observed pulsar with period 59 ms
- Doppler shifts in frequency
  $\rightarrow$ orbital motion with period ~ 8 hours round unseen companion
- orbital velocity ~ 300 km/s
  $\rightarrow \frac{v}{c} \sim 10^{-3}$

With > 20 years of observations, now know all system parameters to high accuracy

The orbital period is decreasing at a rate $\dot{P} = -2{\cdot}425 \times 10^{-12}$ ($\sim 0{\cdot}1$ ms/y)

- consistent with gravitationa[l] waves taking away orbital energy at the rate predicted by GR!