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SUMMER SCHOOL AND CONFERENCE ON DYNAMICAL SYSTEMS

The global dynamics of generic diffeomorphisms

(Lecture 2)

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4 C^1-Generic diffeomorphisms and periodic orbits

The study of the global dynamics of generic diffeomorphisms has been possible because it is well reflected by the periodic orbits.

4.1 Perturbation of the trajectories

All the perturbations lemmas come from the following elementary lemma:

Lemma 4.1. *Let M be a compact riemannian manifold. There is $C > 0$ such that, for any $\varepsilon > 0$, for any points x, y verifying $2d(x, y) < \varepsilon$, there is a diffeomorphism h verifying:*

1. $\|h\|_1 \leq \varepsilon$
2. *h coincides with id out of the ball $B(x, C \cdot d(x, y))$*
3. $h(x) = y$

That is, for the C^1-topology, the security zone one need for doing an ε-perturbation of a trajectory has a size is proportional to ε (and inversally proportional to the C^1-size of the perturbation). If we consider the C^2-metrics on the set of diffeomorphisms, one need to replace the condition $Cd(x, y) < \varepsilon$ by $C\sqrt{d(x, y)} < \varepsilon$: the security zone has a size proportional to $\sqrt{\varepsilon}$, which is huge in comparition to ε. This simple remark is the reason that all trajectories perturbations lemmas hold for the C^1 topology, and are unknown for C^r-topology, $r > 1$.

Theorem 4.1. *(The (local)connecting lemma) (see [Ar$_1$, Ha, WX]) Let f be a diffeomorphism on a compact manifold. Let $\mathcal{U}$ be a C^1-neighborhood of f. Then there is $N > 0$, $\delta_0 > 0$ and $0 < \lambda < 1$ with the following property:*

Let z be a point and $0 < \delta < \delta_0$ such that the ball $B(z, \delta)$ is disjoint from $f^i(B(z, \delta))$ for every $i \in \{1, \ldots, N\}$. Let $V = \bigcup_{i=0}^{N} f^i(B(z, \delta))$. Let x, y be two points out of V, and assume that x admits a forward iterate $f^n(x) \in B(z, \lambda\delta)$, $n > 0$ and that y admits a backward iterate $f^{-m}(y) \in B(z, \lambda\delta))$.

Then, there is $k \in \{1, \ldots, m+n\}$ and $g \in \mathcal{U}$ coinciding with f out of V such that $g^k(x) = y$.

I will admit this lemma for the momment, postponing if possible the proof to the last lecture.

This lemma implies most of the known perturbations lemma

Theorem 4.2. *(The closing lemma, Pugh[Pu, PR]) Let f be diffeomorphism of a compact manifold and x a non-wandering point of f. Then there is g, C^1 close to f, such that $x \in Per(g)$*

proof : Le $x \in \Omega(f)$. If x is periodic there is nothing to do. So assume that x is not periodic. Fix a neighorhood of f, and then consider the constant λ δ_0, and N. Choose $\delta \in]0, \delta_0$ such that the δ-ball around x is disjoint from its $2N$ first iterates. Now there is a point $y \in f^{N+1}(B(x, \delta))$ such having positive and negative iterates in $B(x, \lambda\delta)$. Applying the local connecting lemma one gets that y becomes periodic for a diffeomorphism in the choosen neighborhood. Furthermore its orbits is passig very clos to x (in $B(x, \delta)$) so that a small conjugacy allows to get that this orbit passes through x. □

Theorem 4.3. *(Hayashi's connecting lemma)[Ha] Let f be a diffeomorphism on a compact manifold. Assume that p, q are hyperbolic periodic points and $x \in W^u(p)$ and $y \in W^s(q)$ verifies that there is a sequence x_i converging to x and positive numbers n_i such that $f^{n_i}(x_i)$ converges to y_i. Then there is arbitrarily small perturbations of f such that $W^u(p)$ cuts $W^s(q)$ along the orbit of x and that there is $k > 0$ such that $f^k(x) = y$.*

proof : Fix a neighborhood $\mathcal{U}$ of f. By shrinking U if necessary, one can assume that if two perturbations of f have disjoint support and are each in U then the composed perturbation is too in U.

Then consider δ_0, λ, N given by Theorem 4.1. Consider $\delta < \delta_0$ such that the N first iterates of $B(x, \delta)$ and $B(y, \delta)$ are pairwize disjoint. Consider i such that $x_i \in B(x, \lambda\delta)$ and $f^{n_i}(x_i) \in B(y, \lambda\delta)$.

Consider m_0 such that $a = f^{-m_0}(x) \in W^u_{loc}(p)$ where the local unstable manifold of p is suficiently small for being disjoint from the N iterates of $B(x, \delta) \cup B(y, \delta)$. Now Theorem 4.1 allows to build a perturbation g (in $\mathcal{U}$) with support in $\bigcup_0^{N-1}(f^j(B(x, \delta)$ and such that $g^k(a) = f^{n_i}(x_i) \in B(y, \lambda\delta)$. Now let b be a positive iterate of y in a small local stable manifold of q, dijoint from the N iterates of the balls, and using once more Theorem 4.1 one gets now that b becmes a positive iterate of a . □

Exercise 17. *Find where I was not rigorous in the previous proof!!!*

In fact,in the last lecture, we will see a more precise (an much more technical) version of Theorem 4.1, which is in fact the way Hayashi, Arnaud, Wen&Xia proved Theorem 4.1. Using this stronger version and global arguments we prove the following global pertubation lemma:

Theorem 4.4. *(The globalconnecting lemma, B-, Crovisier, [BC]). Let f be a diffeomorphism such that every periodic point is hyperbolic. Let x, y be two points such that $x \dashv y$ (that is, one can go from x to y by pseudo-orbits of arbitrarily small jumps). Then for any C^1-neighborhood $\mathcal{U}$ of f there is $g \in \mathcal{U}$ and $n > 0$ such that $g^n(x) = y$.*

Question 2. *Is Theorem 4.4 true for all diffeomorphism (without the hypothesis "all the periodic orbits are hyperbolic")?*

In fact we allready generalize this theorem defining the notion of avoidable periodic orbits (including the hyperbolic, and the elliptic orbits with no resonnance condition)

We are very far to understand what kind of perturbation are possible and what are not. For example Lan Wen ask us the following question:

Question 3. *Let f be a diffeomorphism of a compact manifold and assume that x, y are two hyperbolic fixed points such that $x \in \overline{W^s(y)} \cap \overline{W^u(y)}$. Is it possible to create a cycle involving x and y by a C^1-small perturbation of f?*

Another quetion, by Flavio Abdenur:

Question 4. *If x is recurrent for f^k for some $k > 0$. Is it possible to create a periodic orbit of period k through x, by a C^1-small perturbation?*

For a very long time I try to answer to the following question:

Question 5. *Given any x, is it possible to create a hyperbolic periodic orbit p such that $x \in W^s(p)$?*

4.2 The chain recurrence set of C^1-generic diffeomorphisms

Theorem 4.5. *For C^1-generic diffeomorphisms, $\overline{Per_{Hyp}(f)} = \overline{Rec(f)} = \overline{Lim(f)} = \Omega(f) = \mathcal{R}(f)$*

proof : The closure of the set of hyperbolic periodic points varies lower semi-continuously (because each of the hyperbolic periodic points varies locally continuously). Then Theorem 3.4 ensures the existence of $\mathcal{R} \subset \text{Diff}^1(M)$ residual such that the diffeomorphisms $f \in \mathcal{R}$ are continuity points of $g \mapsto \overline{\text{Per}_{Hyp}(g)}$. Then for $f \in \mathcal{R}$, $\overline{\text{Per}_{Hyp}(f)} = \mathcal{R}(f)$. Assume (arguing by absurd) that $x \in \mathcal{R}(f) \setminus \overline{\text{Per}_{Hyp}(f)}$. Then Theorem 4.4 ensures that diffeomorphisms g arbirarily C^1-close to f admits x has a periodic point. An elementary perturbation (using for instance Franks'lemma Lemma 5.1) allows to get x as an hyperbolic periodic point, contradicting the continuity. □

4.3 The chain recurrence classes

Let introduce the following relation: $x \prec y$ if, for any neighborhood U, V of x, y , respectively, there is $n > 0$ such that $f^n(U) \cap V \neq \emptyset$. Notice that this relation is more restrictive than $\dashv$: $x \prec y \Longrightarrow x \dashv y$. Furthermore, $x \prec x$ if and only if $x \in \Omega(f)$.

Theorem 4.6. *[BC] For any C^1-generic diffeomorphisms the relations $\prec$ and $\dashv$ coincide.*

proof : Consider a countable base $\mathcal{O}$ of the topology of M. For any $U, V \in \mathcal{O}$ let $A(U, V)$ be the set of diffeomorphisms such that some positive iterate $f^n(U)$ meets V. Notice that $A(U, V)$ is an open set. Let $B(U, V)$ be the complement of the closure of $A(U, V)$ and $C(U, V) = A(U, V) \cup B(U, V)$. By construction, $C(U, V)$ is open and dense in $\text{Diff}^1(M)$. Consider $\mathcal{R} = \bigcap_{U,V \in \mathcal{O}} C(U, V)$. It is a residual subset of $\text{Diff}^1(M)$.

A characterisation of f being in $\mathcal{R}$ is: if the positive iterates of some open set $U \in \mathcal{O}$ are disjoint from $V \in \mathcal{O}$ then this property is robust.

Assume that $x \not\prec y$. Then there are neighborhoods $U, V \in \mathcal{O}$ of x, y such that positive iterates of U are disjoint from V. Then this property is robust. But if $x \dashv y$ then Theorem 4.4 asserts that $g^n(x) = y$ with $n > 0$ for arbitrarily small perturbation g of f. So we proved

$$x \not\prec y \Longrightarrow x \not\dashv y.$$

□

An invariant compact set Λ is called *weakly transitive* is for any two point $x, y \in \Lambda$ one has $x \prec y$. As the closure of the union of an increasing family of weakly transitive set is a weakly transitive set, Zorn lemma ensures the existence of *maximal weakly transitive sets.*

Corollary 4.2. *For f generic, the chain recurrence classes are the maximal weakly transitive sets.*

Question 6. *For f generic, are the chain recurrence classes transitive?*

Theorem 4.4 is very far to be a result of "shadowing by perturbation": the segment of orbit of g joining x and y is not close to a segment of the initial pseudo-orbit. In that direction there is a result by Sylvain Crovisier, based on Theorem 4.1.

Theorem 4.7. *(Crovisier)[Cr]Let f be a diffeomorphism whose periodic orbits are hyperbolic and $\mathcal{U}$ be a C^1-neighborhood of f. Le K be a weakly transitive set. Then for any $\varepsilon > 0$ there is $g \in \mathcal{U}$ coinciding with f out of an arbitrary neighborhood og K and having a periodic orbit γ such that the Hausdorff distance between γ and K is less than ε.*

Corollary 4.3. *For f generic, any weakly transitive set (and in particular any chain recurrence class) is the Hausdorff limit of a seqence of periodic orbits.*

This corollary is very usefull if you want to induce on a chain recurrence class some property (like a dominated splitting) of the nearby periodic orbits.

4.4 Homoclinic classes

Let f be a diffeomorphism of a compact manifold M, and p be a hyperbolic periodic point of f. We denote by $W^s(p)$ and $W^u(p)$ the stable and unstable manifolds of the orbit of p.

Two hyperbolic periodic points p and q are *homoclinically related* if $W^s(p) \cap W^u(q)$ and $W^u(p) \cap W^s(q)$ contain tranversal intersection points. This define an equivalence relation on the set of hyperbolic periodic orbits. The *homoclinic class* $H(f,p)$ of p is the closure of its equivalence class.

Let recall the classical properties of the homoclinic classes:

Proposition 4.4. *The homoclinic class of p verifies the following properties:*

- $$H(f,p) = \overline{W^s(p) \pitchfork W^u(p)}.$$
- *$H(f,p)$ is an invariant compact set which is transitive.*
- *If $E \subset H(p,f)$ is a finite set such that any $q \in E$ is either a periodic point homoclinically related to p or a transverse homoclinic intersection of p. Then E is contained in a transitive hyperbolic basic set.*

The two first item are classical consequences of the Lambda-lemma. A good reference for the third one may be the book by Palis and Takens [PT].

4.5 Heteroclinic intersection for C^1-generic diffeomorphisms

Theorem 4.8. *For f generic, assume that p, q are periodic points such that $dimW^u(p) + dimW^s(q) \geq dimM$. If $p \prec q$ (or equivalently $p \dashv q$) then $W^u(p)$ cuts transversally $W^s(p)$.*

proof : Consider the set H_k of pair (p,q) of hyperbolic periodic orbits of period less that k such that $W^u(p)$ cuts transversally $W^s(p)$. This set varies lower semi-continuously and then there is a residual se $\mathcal{R}_k$ of continuity points for this set. Assume $f \in \mathcal{R}_k$ and assume that $(p,q) \notin H_k$ but $p \prec q$.

Then Theorem 4.3 ensures that, for an arbitrarily C^1-small perturbation g of f, the pair p, q belongs to $H_k(g)$ contradicting $f \in \mathcal{R}_k$. The announced residual set is $\bigcap_k \mathcal{R}_k$.

□

4.6 Homoclinic class and chain recurrence classes

Theorem 4.9. *[Ar_1, CMP, BC] For f generic, the homoclinic classes are chain recurrence classes. In other words, a chain recurrence class containing a periodic point p is the homoclinic class of p.*

Lemma 4.5. *Let f be a generic diffeomorphism. Let p be an hyperbolic periodic point and $z \in \overline{W^s(p)} \cap \overline{W^u(p)}$. Then there are C^1-small perturbations of f for which z is an transverse homoclinic intersection associated to the orbit of x*

proof : If the point z is not periodic, this is a direct application of the connecting lemma: one fixe the neighborhood V of a segment of orbit $z, \dots f^N(V)$. There is $x \in W^u_{loc}(p)$ and $y \in W^s_{loc}(p)$ out of V, having positive and negative iterates, respectively, in the neighborhood of size λ times smaller around z. Then $g^k(x) = y$ for a small perturbation of f in V. The point x and y remains in the local unstable and stable manifold of p (because they iare disjoint from V) so that we create a homoclinic orbit passing through V and therefore arbitrarily close to z. A new perturbation produce a transverse homoclinic orbit through z.

Assume now that z is periodic. Notice that $z \prec p$ and $p \prec z$ so that Theorem4.8 implies that one of the invariant manifold of p intersect transversally one of those of z. Let assume for instance that $W^s(p)$ cuts transversally $W^u(z)$. As $p \prec z$, Theorem 4.3 allows to create an intersection (may be non transverse) between $W^u(p)$ and $W^s(z)$ by an arbitrary C^1-small perturbation of f. This perturbation being very small i keep the ther bifurcation creating a cycle betweem p and z.

Now an arbitrarily small perturbation creates a homoclinic orbit of p close to z, and this intersection may be turned to be transverse. □

Corollary 4.6. *([Ar_1, CMP]) For f generic for any periodic point p one has :*

$$H(p, f) = \overline{W^s(p)} \cap \overline{W^u(p)}.$$

proof : Let $\mathcal{G}$ be the residual set on which the lemma above holds.

It is enough to prove the corollary for periodic orbits of period less that k. There is a dense open set $\mathcal{V}_k$ of diffeomorphisms having finitely many periodic orbis of period less than k all of them hyperbolic and the number being locally constant. On each connecting components of this open and dense set the periodic orbits of period less than k vary continuously with he diffeomorphisms. Let $\mathcal{H}_k(f)$ be the collection of homoclinic classes associated to periodic orbits of period less than k. These compacts sets vary lower semicontinuously with f (because transverse homoclinic intersections persists by C^1-perturbations) so that there is a residual set $\mathcal{R}_k$ on which all homoclinic class varies continuously.

Let $f \in \mathcal{R}_\cap \mathcal{G}$ and assume that p with period less that k verifies $H(p) \neq \overline{W^s(p)} \cap \overline{W^u(p)}$ Let $z \in \overline{W^s(p)} \cap \overline{W^u(p)} \setminus H(p)$. Then the lemma above allows to create a a new homoclinic intersection at z contradicting the continuity of $H(p)$.

□

Lemma 4.7. *[CMP] For f generic, for any periodic point p, the closure $\overline{W}^u(p)$ is Lyapunov stable.*

proof : By an analogous argument as in the corollary above, for f generic the closure of the unstable manifold of any periodic point varies continuously with f. Assume that $\overline{W^u(p)}$ is not Lyapunov stable. Then there is $y \in W^u(p) \setminus \text{orb}(p)$ and $z \notin \overline{W^u(p)}$ such that $y \dashv z$. As f is generic, then $\prec = \dashv$ so that $y \prec z$. So Theorem 4.3 allows o make arbitrarily small perturbatios of f such that z will belon to $W^u(p)$ contradicting the continuity of $\overline{W^u(p)}$ □

Lemma 4.8. *Let K be a Lyapunov stable set. Assume that $x \in K$ and that $x \prec y$. Then $y \in K$.*

proof : K admits a base of positively invariant neighborhoods. Consider $y \notin K$ and let W_0, V be two disjoint open sets $K \subset W_0$ and $y \in V$. Let $W \subset W_0$ be a positively invariant neighborhood of K. Now for any $x \in K$, W is a neighborhood whose positive iterates ar dijoint from W. So $x \not\prec y$. □

End of proof of Theorem 4.9 For f generic any homoclinic class $H(p)$ is the intersection of $\overline{W^u(p)}$ which is Lyapunov stable and of $\overline{W^s(p)}$ which is Lyapunov stable for f^{-1}. So any weakly transitive set meeting $H(p)$ is include in $\overline{W^u(p)}$ by Lemma 4.8 and in $\overline{W^s(p)}$ so in $H(p)$ (as f is generic).

As f is generic the chain recurrence class of p is weakly transitive, and so coincide with $H(p)$.

Exercise 18. *For any diffeomorphism f and any x let $W^u_{\dashv}(x, f) = \{y \in M \mid x \dashv y\}$. Prove that, for any C^1-generic diffeomorphism f of a compact manifold M, for any periodic point $x \in Per(f)$ one has $W^u_{\dashv}(x, f) = \overline{W^u(x, f)}$.*

4.7 Isolated classes

Definition 4.9. *A chain recurrence class C is* isolated *if there is a neighborhood U of C such that $\mathcal{R}(f) \cap U = C$.*

It is robustly isolated *is there is a neighborhood $\mathcal{U}$ of f such that for any $g \in \mathcal{U}$ the intersection $\mathcal{R}(g) \cap U$ consists in a unique chain recurrence class.*

Remark 4.10. *If C is isolated and U is a neighborhood as in the definition, then for every compact neighborhood $V \subset U$ of C, C is the maximal invariant set*

$$C = \bigcap_{n \in \mathbb{Z}} f^n(V).$$

In the same way, C is robustly isolated if and only if there is a neighborhood V of C and a neighborhood $\mathcal{U}$ of f such that, for any $g \in \mathcal{U}$ the maximal invariant set $C = \bigcap_{n \in \mathbb{Z}} g^n(V)$ is chain recurrent. In other word, C is robustly chain recurrent.

Notice that, for f generic, the isolated chain recurrence classes are homoclinic classes, because the periodic orbits are dense in $\mathcal{R}(f)$.

The aim of this section is to prove:

Theorem 4.10. *[Ab, BC] For f generic, any isolated chain recurrence class is robustly isolated.*

(this result is essencially due to F. Abdenur in [Ab] and adapted to this context by [BC].)

Lemma 4.11. *For f generic, given any two periodic orbits p,q the homoclinic classes $H(p)$ and $H(q)$ are equal or disjoint. Furthermore, if they are disjoint they are robustly disjoint: there is a C^1-neighborhood $\mathcal{U}$ of f such that for any $g \in \mathcal{U}$ the homoclinic classes of the continuations of p and q are disjoint.*

proof : For f generic, the homoclinic class are chain recurrence classes so that coincide or are disjoint. Furthermore, if there are dioint, there is a pair attractor repellor such that one of the class, say $H(p)$ is contained in the attractor and the other in the repellor. Let U be an isolating neighborhood of the attractor. Then $f(\bar{U}) \subset U$ and there is a C^1-neighborhood $\mathcal{U}$ of f such that $g \in \mathcal{U}$ verifies $g(\bar{U}) \subset U$. Then the homoclinic class $H(p, g$ is contained in the corresponding attractor for g and $H(q, g)$ in the repellor so that they are disjoint. □

We are now ready to prove Theorem 4.10

proof : Lemma 4.11 implies that there is a residual subset $\mathcal{R}$ of $\text{Diff}^1(M)$ on which one has:

for any open set U of M the number $N(f, U) \in \mathbb{N} \cup \{\infty\})$ of homoclinic classes intersecting U is lower semi continuous. Then there is a residual set $\mathcal{R}_U \subset \mathcal{R}$ on which it is continuous. Notice that $\mathcal{R}_U$ is residual in $\text{Diff}^1(M)$.

Consider a countable base $\{U_n\}$ of the topology of M and $\mathcal{G} = \bigcap_{n\in\mathbb{N}} \mathcal{R}_{U_n}$ is residual. For $f \in \mathcal{G}$ let $H(p)$ be an isolated class. Then there is a covering of $H(p)$ by finitely many open set $U_i, i \in I$ such that $U_i \cap \mathcal{R}(f) \subset H(p)$. In particular $N(f, U_i) = 1$.

So one get an open neighborhood $\mathcal{V}$ of f on which $N(f, U_i) = 1$ and $U_i \cap \mathcal{R}(g) \subset H(p, g)$ for any $g \in \mathcal{R} \cap \mathcal{V}$. Assume that g (not generic!) in $\mathcal{V}$ verifies that there is a chain recurent class C which does not contain $H(p_g, g)$, such that $C \cap U_j \neq \emptyset$ for some $j \in I$. Then there is a pair attractor repelor such that C (for instance) is contained in the attractor and $H(p_g, g)$ is contained in the repellor. Consider $x \in C$. By Theorem 4.4 an arbitrarily small perturbation of g may turn x to be an hyperbolic periodic point. When the pertubation is small enough, the chain recurrent class of x remains in the attractor associated to an isolating neighborhood, and $H(p)$ remain in the repellor. Now this remain the case for any smll perturbation in particular for some generic diffemorphisms.

This contradicts the continuity of $N(f, U_j)$.

□

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