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"VII School on Non-Accelerator Astroparticle Physics"

26 July - 6 August 2004

Standard Model and Beyond - II

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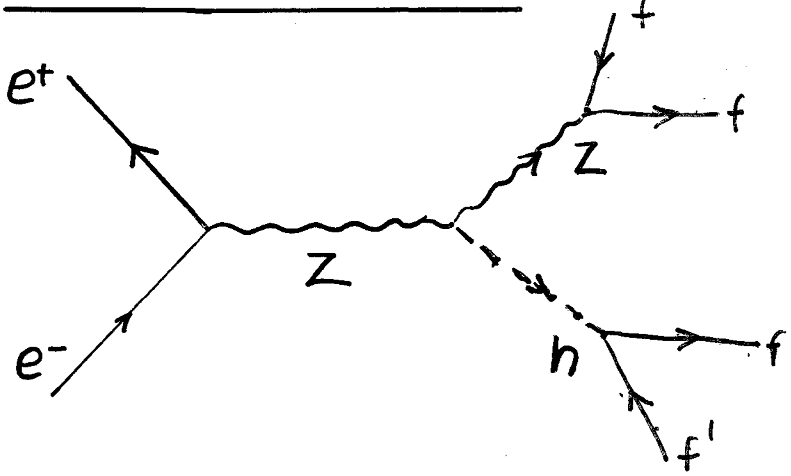
Higgs Boson Search

LEP, Linear Collider:



Björken process
"Higgsstrahlung

$$e^+e^- \longrightarrow Z\,h$$

- $h \hookrightarrow \tau^+\tau^-$, $q\bar{q}$ (mainly $b\bar{b}$)
- $Z \hookrightarrow \nu\bar{\nu}$, $\ell^+\ell^-$, $q\bar{q}$

Expected mass reach $m_h \lesssim \sqrt{s} - 100\,\text{GeV}$

TEVATRON:

$$p\bar{p} \longrightarrow W^\pm h \longrightarrow \ell^\pm q\bar{q}\, \not{p}_T$$

- $h \hookrightarrow q\bar{q}$ (mainly $b\bar{b}$)
- $W^\pm \hookrightarrow \ell^\pm\nu$

most promising channel

Mass reach depends on luminosity:

for 10 fb^{-1} : $m_h \lesssim 120\,\text{GeV}$

(Main Injector, detector upgrades)

Higgs search at LEP, Linear Collider

$e^+e^- \to hZ \to h\nu\bar{\nu},\ h\ell^+\ell^-,\ hq\bar{q},\ q\bar{q}\tau^+\tau^-,\ \tau^+\tau^-q\bar{q}$

BR =	20.0%	6.7%	64.6%	3.4%	5.3%

$BR(h \to b\bar{b}) \approx 85\%$ b-tagging is important

(i) $\not{E}_T$ channel:

2 acoplanar b-jets, large $\not{p}_T$, missing mass $\approx m_Z$

(ii) Lepton channel: $\ell = e, \mu$

2 isolated leptons with $m_{e^+e^-} \approx m_Z$, 2 b-jets

(iii) 4-jet channel:

4 jets, 2 or 4 of them b-jets, one jet pair with mass $\approx m_Z$

(iv) τ channel:

4 jets, 2 of them with low multiplicity, $\not{E}_T$, 2 b-jets

$m_{\tau\tau}$ or $m_{qq} \approx m_Z$

Background: $e^+e^- \to q\bar{q}$, ZZ, WW, 4-fermion processes, processes with $\not{p}_T$

No signal found: $m_h > 114$ GeV LEP

Linear Collider: $e^+e^- \to Z\,H$, $e^+e^- \to \bar{\nu}_e \nu_e H$

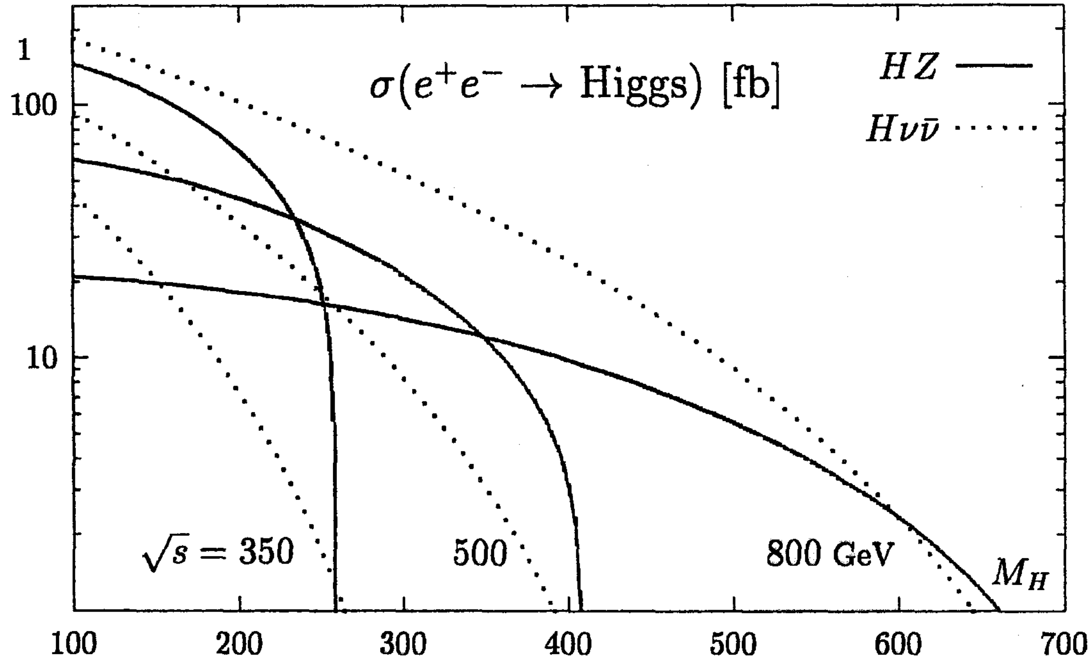
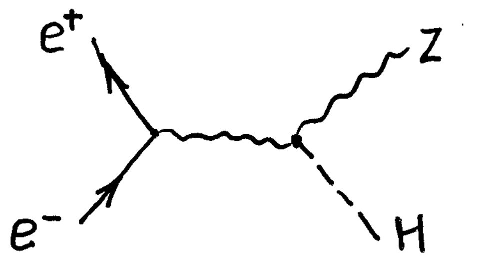
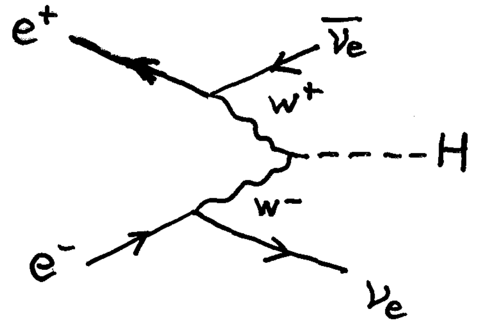


Figure 2.1.3: *The Higgs-strahlung and WW fusion production cross–sections vs.* M_H *for* $\sqrt{s}$ *= 350 GeV, 500 GeV and 800 GeV.*





$\sigma(e^+e^- \to e^+e^- Z)$
can contribute
up to $\approx 10\%$
at $\sqrt{s} \gtrsim 800$ GeV

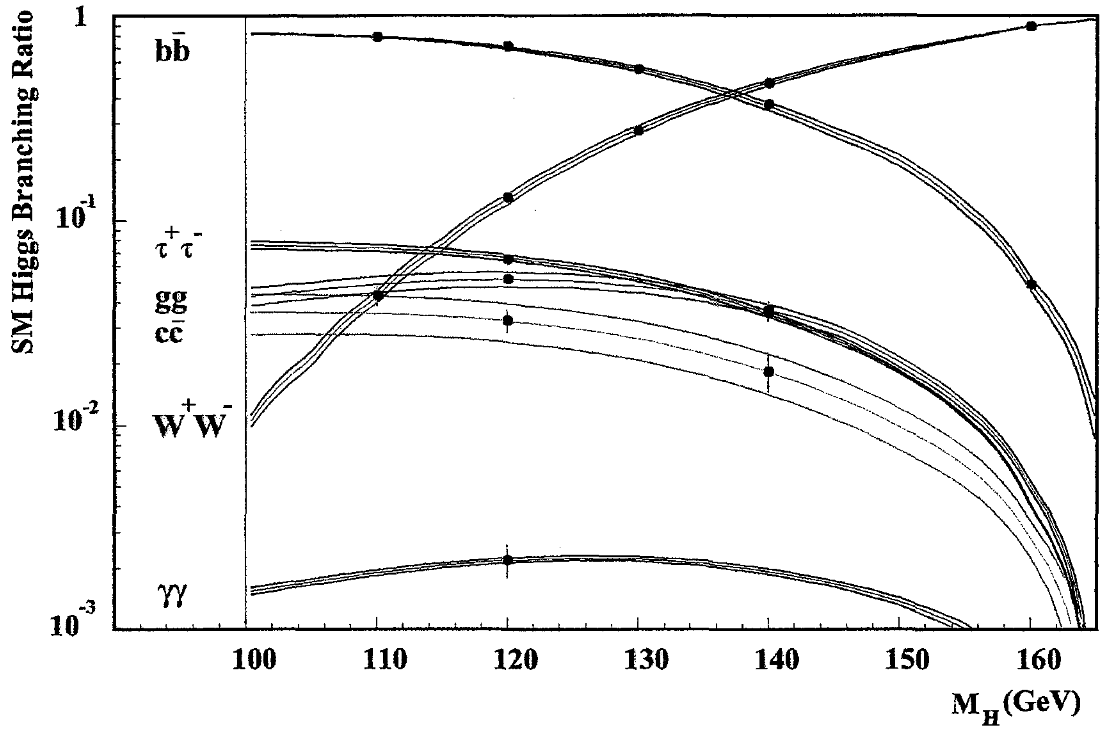


Figure 2.2.4: *The predicted SM Higgs boson branching ratios. Points with error bars show the expected experimental accuracy, while the lines show the estimated uncertainties on the SM predictions.*

SM Higgs Search at Tevatron

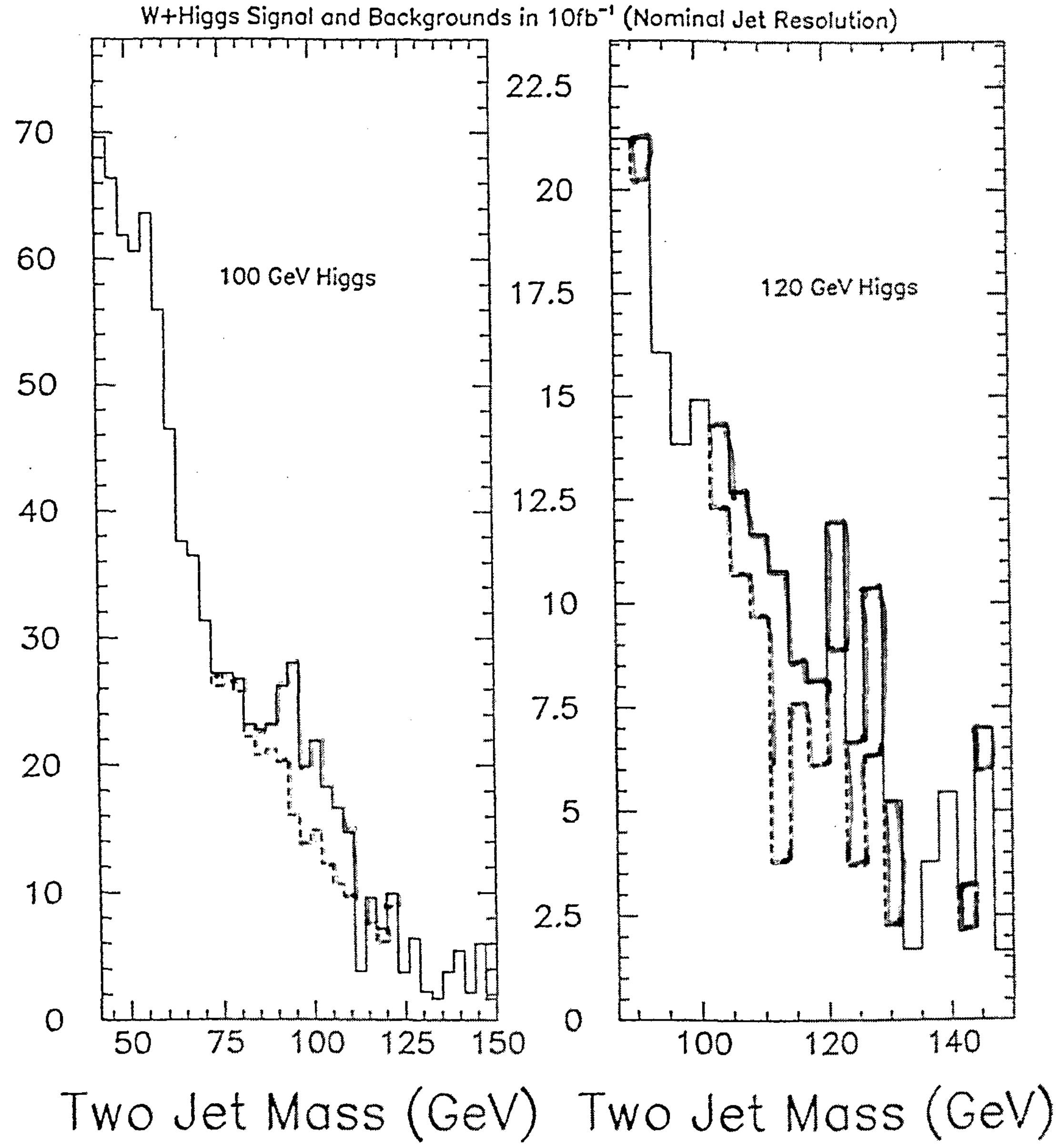
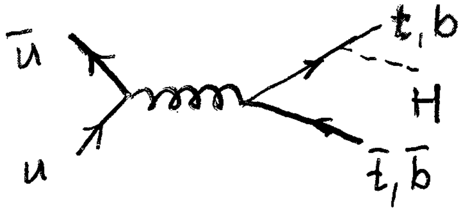

W+Higgs Signal and Backgrounds in 10fb^{-1} (Nominal Jet Resolution)

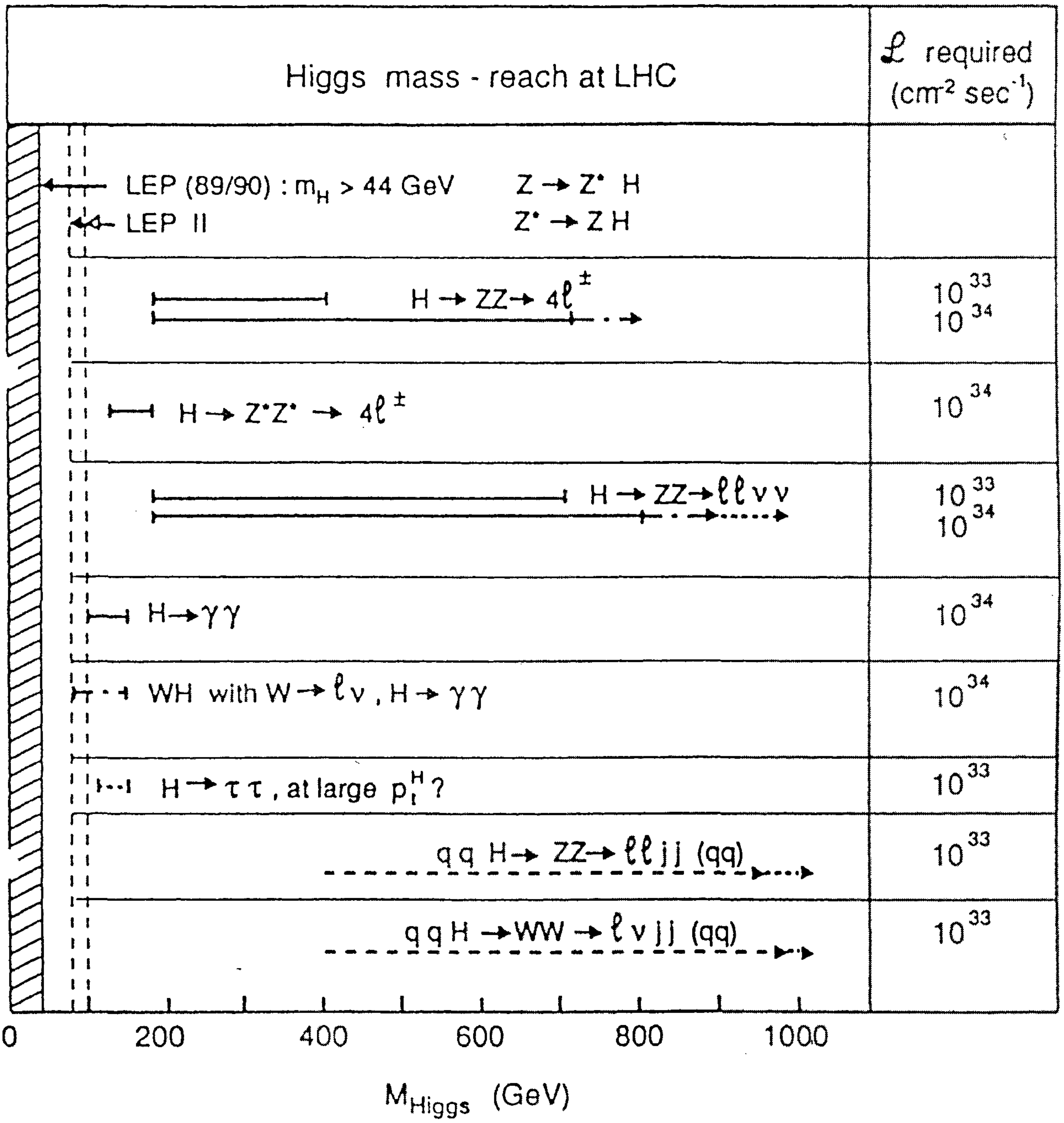
Figure 5.13: Same as the last figure but only the highest two masses.

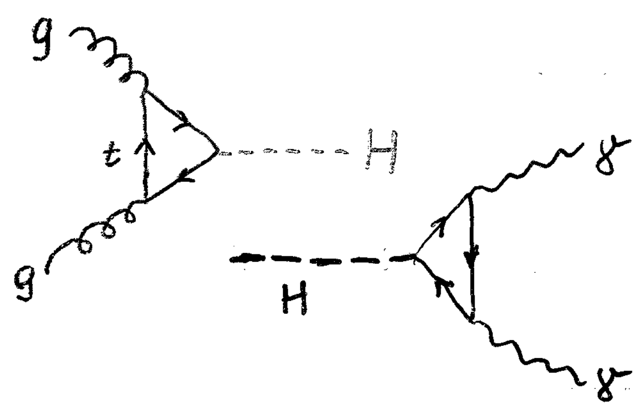
$$p\bar{p} \to W^+ + H + X$$
$$\hookrightarrow e^+\nu_e \quad \hookrightarrow b\bar{b}, q\bar{q} \ldots$$

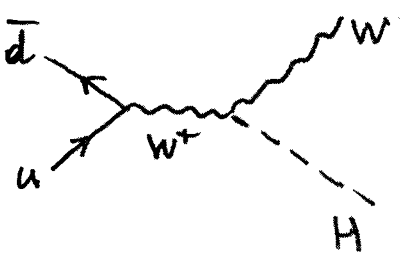


$$p\bar{p} \to t\bar{t}H,\ b\bar{b}H$$

SM Higgs at LHC







Grand Unification GUT

SM gauge group $SU(3)\times SU(2)\times U(1)$ has rank 4

$SU(5)$ is "smallest" Lie group with rank 4 and

$$SU(5) \supset SU(3)\times SU(2)\times U(1)$$

(Pati, Salam)

Other possible choices: $SU(2)_L\times SU(2)_R\times SU(4)_c$, $SO(10)$, $E(6)$, ...

SU(5) : 24 Generators $\frac{1}{2}\Lambda^a$, $a = 1 \dots 24$

$a = 1 \dots 8$: $\Lambda^a = \left(\begin{array}{c|cc} \lambda^a & 0 & 0 \\ \hline 0 & & \\ 0 & & 0 \end{array}\right)$ λ^a : Gell-Mann matrices

$a = 22, 23$: $\Lambda^a = \left(\begin{array}{c|c} 0 & 0\ 0 \\ \hline 0 & \\ 0 & \tau^{1,2} \end{array}\right)$ $\tau^{1,2}$: Pauli matrices

Diagonal: $\Lambda^{15} = \frac{1}{\sqrt{6}}\begin{pmatrix} 1 & & & & \\ & 1 & & 0 & \\ & & 1 & & \\ & 0 & & -3 & \\ & & & & 0 \end{pmatrix}$, $\Lambda^{24} = \frac{1}{\sqrt{10}}\begin{pmatrix} 1 & & & & \\ & 1 & & 0 & \\ & & 1 & & \\ & 0 & & 1 & \\ & & & & -4 \end{pmatrix}$

$\Lambda^9 \dots \Lambda^{14}$: $\left(\begin{array}{c|c} 0 & *\ 0 \\ \hline * & \\ 0 & 0 \end{array}\right)$

$$\mathrm{Tr}\,(\Lambda^a\Lambda^b) = 2\delta^{ab}$$

$\Lambda^{16} \dots \Lambda^{21}$: $\left(\begin{array}{c|c} & 0\ * \\ \hline 0 & \\ * & \end{array}\right)$

$Q = I^3 + Y$:

$$Q = -\sqrt{\tfrac{2}{3}}\,\Lambda^{15}, \quad I^3 = \tfrac{1}{8}\left(\sqrt{10}\,\Lambda^{24} - \sqrt{6}\,\Lambda^{15}\right), \quad Y = -\tfrac{1}{8}\left(\sqrt{10}\,\Lambda^{24} + \sqrt{6}\,\Lambda^{15}\right)$$

Q is generator of group

Gauge bosons $\mathcal{A}^a_\mu$, $a=1\dots24$

$a=1\dots8$: $\mathcal{A}^a_\mu = G^a_\mu$ gluons

$$\left.\begin{array}{l} B_\mu = -\frac{1}{2}\left(\sqrt{\frac{5}{2}}\mathcal{A}^{15}_\mu + \sqrt{\frac{3}{2}}\mathcal{A}^{24}_\mu\right) \\ W^3_\mu = \frac{1}{4}\ \sqrt{10}\mathcal{A}^{25}_\mu - \sqrt{6}\mathcal{A}^{15}_\mu) \\ W^\pm = \frac{1}{\sqrt{2}}\left(\mathcal{A}^{22}_\mu \mp i\mathcal{A}^{23}_\mu\right) \end{array}\right\} \text{week bosons}$$

new bosons, lepto-quarks, diquarks

$$\overbrace{\begin{array}{ll} X^1_\mu = \frac{1}{\sqrt{2}}\left(\mathcal{A}^9_\mu + i\mathcal{A}^{10}_\mu\right) & Y^1_\mu = \frac{1}{\sqrt{2}}\left(\mathcal{A}^{16}_\mu + i\mathcal{A}^{17}_\mu\right) \\ X^2_\mu = \frac{1}{\sqrt{2}}\left(\mathcal{A}^{11}_\mu + i\mathcal{A}^{12}_\mu\right) & Y^2_\mu = \frac{1}{\sqrt{2}}\left(\mathcal{A}^{18}_\mu + i\mathcal{A}^{19}_\mu\right) \\ X^3_\mu = \frac{1}{\sqrt{2}}\left(\mathcal{A}^{13}_\mu + i\mathcal{A}^{14}_\mu\right) & Y^3_\mu = \frac{1}{\sqrt{2}}\left(\mathcal{A}^{20}_\mu + i\mathcal{A}^{21}_\mu\right) \end{array}}$$

$$\mathcal{A}_\mu = \mathcal{A}^a_\mu T^a = \mathcal{A}^a_\mu \frac{\Lambda^a}{2} =$$

$$\frac{1}{\sqrt{2}}\left(\begin{array}{ccc|cc} \frac{1}{\sqrt{2}}G^3_\mu + \frac{1}{\sqrt{6}}G^8_\mu - \frac{2}{\sqrt{30}}B_\mu & \frac{1}{\sqrt{2}}G^{1-i2}_\mu & \frac{1}{\sqrt{2}}G^{4-i5}_\mu & \bar{X}^1_\mu & \bar{Y}^1_\mu \\ \frac{1}{\sqrt{2}}G^{1+i2}_\mu & -\frac{G^3_\mu}{\sqrt{2}} + \frac{G^8_\mu}{\sqrt{6}} - \frac{2B_\mu}{\sqrt{30}} & \frac{1}{\sqrt{2}}G^{6-i7}_\mu & \bar{X}^2_\mu & \bar{Y}^2_\mu \\ \frac{1}{\sqrt{2}}G^{4+i5}_\mu & \frac{1}{\sqrt{2}}G^{6+i7}_\mu & -\sqrt{\frac{2}{3}}G^8_\mu - \frac{2B_\mu}{\sqrt{30}} & \bar{X}^3_\mu & \bar{Y}^3_\mu \\ \hline X^1_\mu & X^2_\mu & X^3_\mu & \frac{W^3_\mu}{\sqrt{2}} + \frac{3B_\mu}{\sqrt{30}} & W^+_\mu \\ Y^1_\mu & Y^2_\mu & Y^3_\mu & W^-_\mu & -\frac{W^3_\mu}{\sqrt{2}} + \frac{3B_\mu}{\sqrt{30}} \end{array}\right)$$

Fermions: 15 helicity states per family:

$$\underset{\substack{\uparrow\\ \text{colour}}}{2\times 3} + \underset{\substack{\uparrow}}{2\times 3} + 2 + \underset{\substack{\uparrow\\ \nu \text{ massles}}}{1} = 15$$

Put into $\overline{5}$ and 10:

$$\overline{5}: \quad (\psi_q)_L = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ \nu_e \end{pmatrix}_L \quad \text{colour index} \qquad q = 1 \ldots 5$$

$$10: \quad \chi_L^{pq} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_3^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^c \\ d_1 & d_2 & d_3 & e^c & 0 \end{pmatrix}_L$$

$p,q=$

NB: $(e_L)^c = e_R$

Generators in these representation:

$$T^a = -\tfrac{1}{2}\Lambda^{a+} \qquad \text{for } \overline{5}$$

$$\tilde{T}^a = \underset{\substack{\uparrow\\ \text{acts on}\\ \text{index } p}}{\frac{\Lambda^a}{2}} \otimes \mathbb{1} + \mathbb{1} \otimes \underset{\substack{\uparrow\\ \text{acts on}\\ \text{index } q}}{\frac{\Lambda^a}{2}} \qquad \text{for } 10$$

Q is $SU(5)$ generator $\Rightarrow Tr(Q) = 0$

$$\Rightarrow \overline{5}: \quad -3Q_d + Q_e = 0 \quad \Rightarrow \quad Q_d = \tfrac{1}{3} Q_e$$

$$\Rightarrow 10: \quad 3Q_d - Q_e = 0 \quad \Rightarrow \quad Q_d = \tfrac{1}{3} Q_e$$

Gauge interaction of fermions:

$$\mathcal{L}_f = i(\overline{\Psi}_q)_L \gamma^\mu D_\mu(\Psi_q)_L + i\overline{X}^{pq}_L \gamma^\mu D_\mu X^{pq}_L$$

$$D_\mu(\Psi_q)_L = \partial_\mu(\Psi_q)_L - i g_G \frac{(\Lambda^{a*})_{qp}}{2}(\Psi_p)_L \mathcal{A}^a_\mu$$

$$D_\mu X^{pq}_L = \partial_\mu X^{pq}_L + 2 i g_G \frac{\Lambda^a_{pr}}{2} X^{rq}_L \mathcal{A}^a_\mu$$

g_G : SU(5) gauge coupling constant

Assume SU(5) symmetry is exact at scale $Q = M_G$ and higher energies.

SU(3) × SU(2) × U(1) is embedded in SU(5) at scale M_G

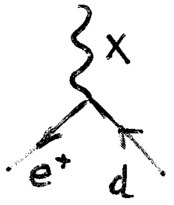
G_μ and $W^\pm_\mu$ interaction terms

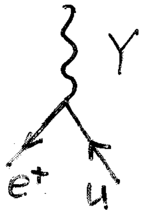
$$\Rightarrow g_s = g_G\ ,\quad g = g_G \quad \text{at } M_G$$

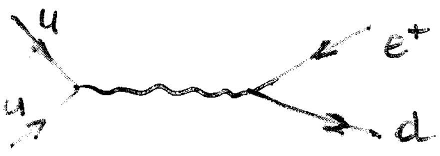
B_μ interaction terms $\Rightarrow g' = \sqrt{\frac{3}{5}}\, g_G$ at M_G

$$\Rightarrow \sin^2\theta_W = \frac{3}{8} \quad \text{at } M_G$$

X^i_μ and Y^i_μ couple to leptons and quarks;





Proton decay: 

$p \to \pi^0 e^+$

Spontaneous breaking $SU(5) \xrightarrow{?} U(1)_{em}$

Result of breaking mechanism must be

$m_X, m_Y \approx M_G \approx 10^{14} - 10^{16}\,\mathrm{GeV}, \quad m_W, m_Z \approx 100\,\mathrm{GeV}$

Higgs mechanism: Higgs multiplets with very different VEV's are necessary (factor $10^{12} - 10^{14}$)

SSB in 2 steps:

$$SU(5) \xrightarrow[\underline{24}\,:\ \phi(x)]{\qquad\qquad} SU(3)\times SU(2)\times U(1) \xrightarrow[\underline{5}\,:\ H(x)]{} SU_c(3)\times U(1)_{em}$$

$$\phi(x) = \sum_{a=1}^{24} \phi^a(x)\, T^a \qquad\qquad T^a = \frac{\Lambda^a}{2} \quad SU(5) \text{ generators}$$

$$H(x) = \begin{pmatrix} H_1(x) \\ H_2(x) \\ H_3(x) \\ H_4(x) \\ H_5(x) \end{pmatrix}$$

$$\mathcal{L}_{Higgs} = \mathcal{L}_\phi + \mathcal{L}_H + \mathcal{L}_{\phi H}$$

$$\mathcal{L}_\phi = \mathrm{Tr}\,(D_\mu \phi)^2 \underbrace{- \mu_1^2\, \mathrm{Tr}\,\phi^2 - \lambda_1 (\mathrm{Tr}\,\phi^2)^2 - \lambda_2\, \mathrm{Tr}\,\phi^4}_{-V_\phi}$$

$$D_\mu \phi = \partial_\mu \phi + i g_G [A_\mu, \phi]$$

$$\mathcal{L}_H = (D_\mu H)^\dagger (D^\mu H) \underbrace{- \tfrac{1}{2}\mu_2^2 H^\dagger H - \lambda_3 (H^\dagger H)^2}_{-V_H}$$

$$\mathcal{L}_{\phi H} = - V_{\phi H} = - \varkappa_1 H^\dagger H \, \mathrm{Tr}\, \phi^2 - \varkappa_2 H^\dagger \phi^2 H$$

Minimize $V_{Higgs} = V_\phi + V_H + V_{\phi H}$.

1^{st} step of SSB $SU(5) \longrightarrow SU(3) \times SU(2) \times U(1)$ is achieved by

$$\langle 0|\phi|0\rangle = \frac{v_\phi}{\sqrt{15}} \begin{pmatrix} 1 & & & & \\ & 1 & & 0 & \\ & & 1 & & \\ & 0 & & -\frac{3}{2}-\frac{\varepsilon}{2} & \\ & & & & -\frac{3}{2}+\frac{\varepsilon}{2} \end{pmatrix}$$

$\mathrm{Tr}(D_\mu \phi)^2$ gives mass terms for X and Y bosons:

$$\mathcal{L}_{XY} = \frac{5}{12} g_G^2 v_\phi^2 \sum_{i=1}^{3} \left(\bar{X}^i_\mu X^{i\mu} + \bar{Y}^i_\mu Y^{i\mu} \right)$$

$$m_X^2 = m_Y^2 = \frac{5}{12} g_G^2 v_\phi^2$$

$$m_X = m_Y \approx M_G \Rightarrow \underline{v_\phi \approx 10^{15}\,\mathrm{GeV}}$$

2^{nd} step of SSB $SU(3) \times SU(2) \times U(1) \longrightarrow SU(3) \times U(1)_{em}$ is achieved by

$$\langle 0|H|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v_H \end{pmatrix}$$

$$\Rightarrow m_W^2 = \frac{g^2 v_H^2}{4}, \quad m_Z^2 = \frac{g^2 v_H^2}{4\cos^2\theta_W}, \quad v_H \rightarrow v$$

How large is M_G ?

$g_s(Q)$, $g(Q)$, $g'(Q)$ depend on scale Q following the <u>renormalization group equations:</u>

From QCD and QFD:

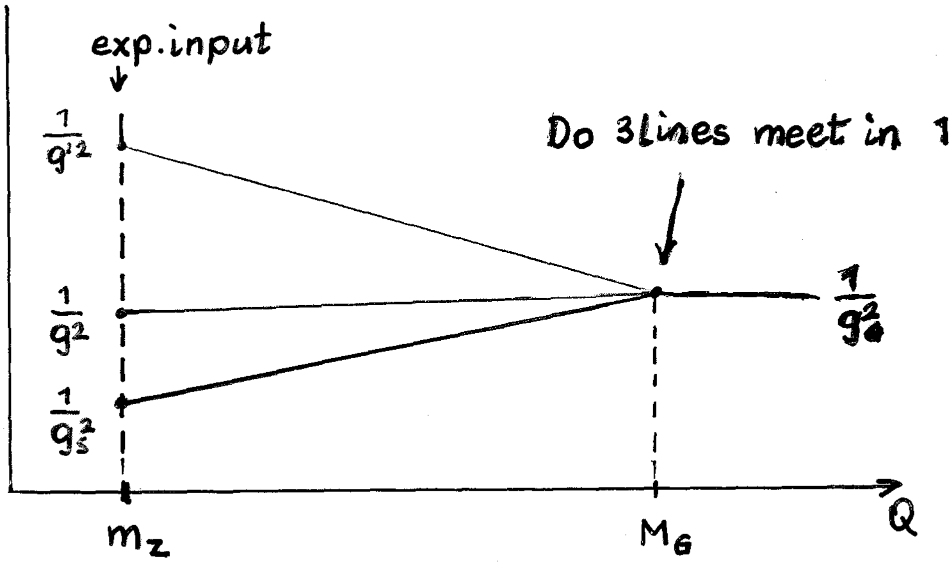
$$\frac{1}{g'^2(Q)} - \frac{1}{g'^2(M_G)} = 2b_1 \ln\frac{Q}{M_G}$$

$$\frac{1}{g^2(Q)} - \frac{1}{g^2(M_G)} = 2b_2 \ln\frac{Q}{M_G}$$

$$\frac{1}{g_s^2(Q)} - \frac{1}{g_s^2(M_G)} = 2b_3 \ln\frac{Q}{M_G}$$

$b_1 = \frac{1}{16\pi^2}\left(-\frac{20}{9}n_f\right)$, $b_2 = \frac{1}{16\pi^2}\left(\frac{22}{3} - \frac{4n_f}{3}\right)$, $b_3 = \frac{1}{16\pi^2}\left(11 - \frac{4n_f}{3}\right)$

n_f — number of families



Start with input for $g_s(Q)$ and $g(Q)$ at $Q = m_Z$. Calculate M_G from $g_s(M_G) = g(M_G)$. Scale $g'(Q)$ from $Q = M_G$ to $Q = m_Z$

$\alpha_s(m_Z) = \frac{g_s^2(m_Z)}{4\pi} = 0.12$, $\alpha(m_Z) = \frac{1}{128.89}$

Unification of gauge couplings

Amaldi et al. 1991, Standard Mode

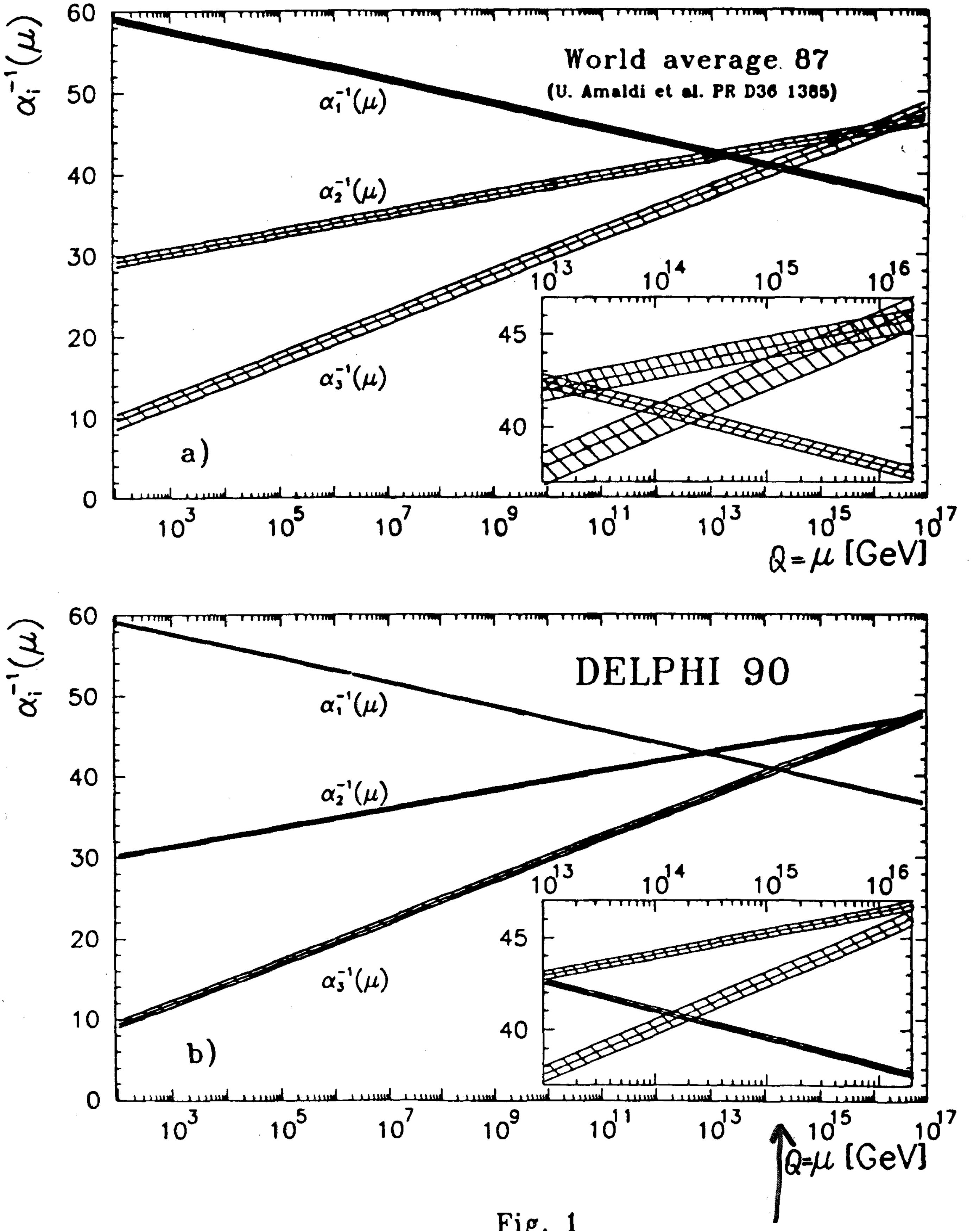
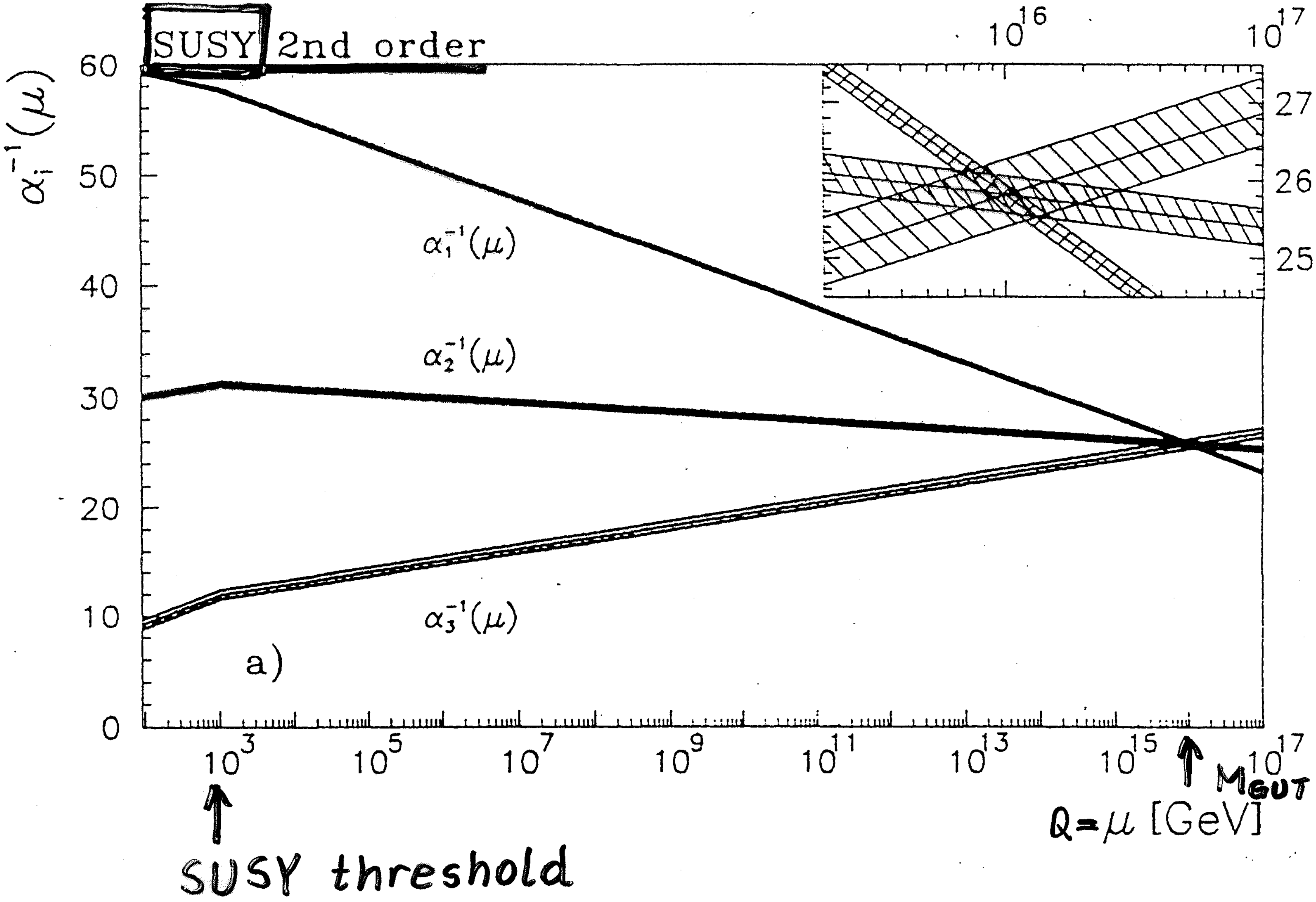


Fig. 1

"Unification" not exact, M_{GUT} too low,

Unification of gauge couplings

Amaldi et al. 1991



Results:

$M_G \approx 5 \times 10^{+14}\,\mathrm{GeV}$, $\alpha_G(M_G) = \frac{g_G^2(M_G)}{4\pi} \cong 2.4 \times 10^{-2}$

$\sin^2\theta_W(m_Z) \simeq 0.20 \neq \frac{3}{8}$

This value of M_G leads to proton decay with proton lifetime $(m_X \approx m_G)$

$$\tau_p \approx \frac{1}{\alpha_G^2} \frac{m_X^4}{m_p^5} \approx 3 \times 10^{30}\,\text{years}$$

in contradiction with experimental lower bound $\tau_p > 6 \times 10^{33}$ years.

Also $\sin^2\theta_W(m_Z) \approx 0.2$ is in disagreement with present experimental value.

[In SUSY GUT [SU(5), SO(10)] good agreement can be achieved].

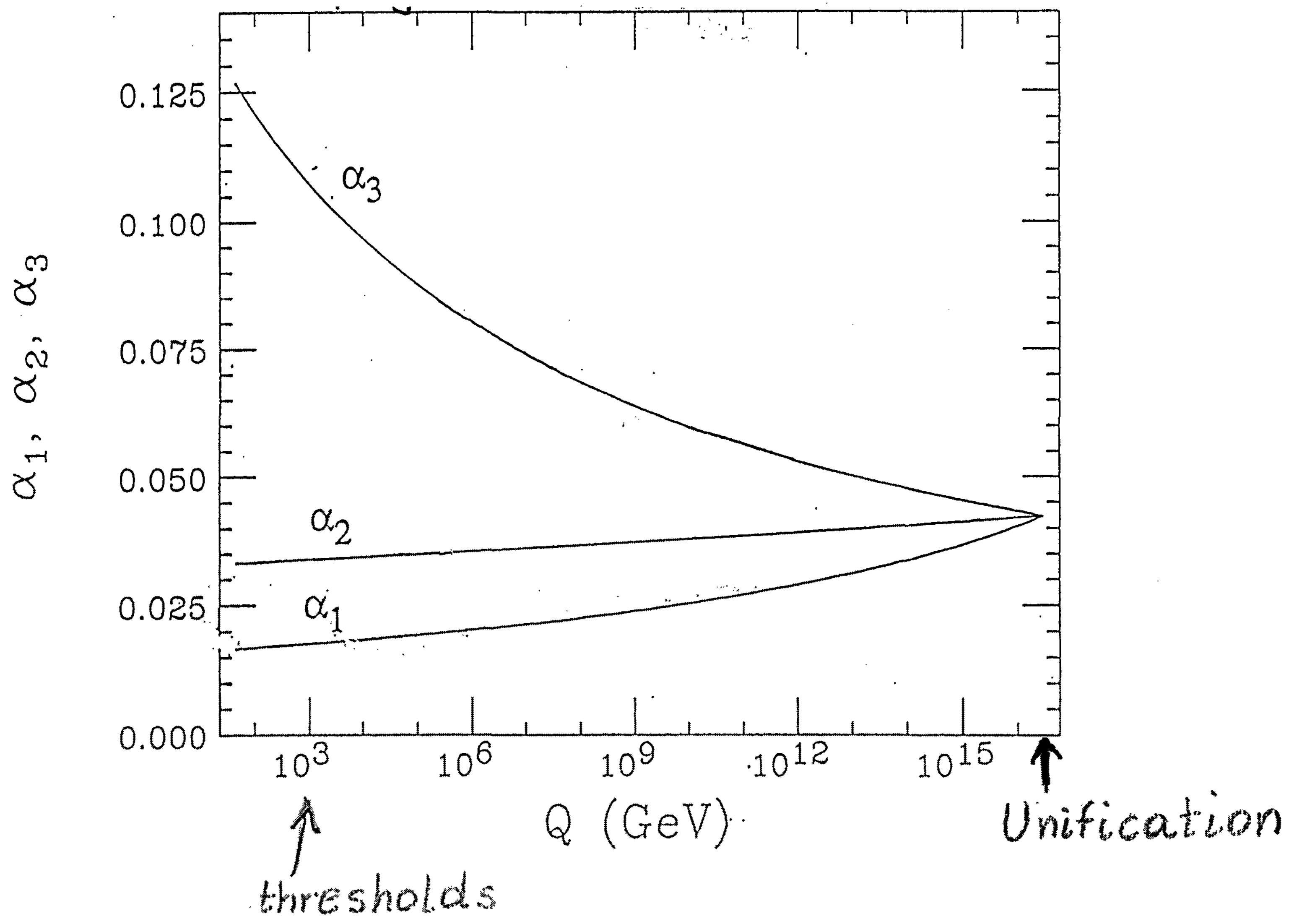
NB: $\Delta(B-L) = 0$

$\sin^2\theta_W = 0.23150 \pm 0.00016$ exp

$\sin^2\theta_W = 0.2100 \pm 0.0026$ without SUSY

$\sin^2\theta_W = 0.2355 \pm 0.0017$ in MSSM

Bagger et al.: Unification of couplings $\alpha_{1,2,3}$



Unification at $Q \approx 10^{16}$ GeV if $100\,\text{GeV} \lesssim M(\text{SUSY particles}) \lesssim 10\,\text{TeV}$

"large" α_3 (M_Z)

Gauge sector of Standard Model extremely well tested by experiments at TEVATRON and LEP, SLD

Experimental accuracy $\approx 0.1\% - 1\%$,

e.g. $\frac{\Delta m_Z}{m_Z} \approx 10^{-5}$, $\frac{\Delta \Gamma_Z}{\Gamma_Z} \approx 10^{-3}$

Theory: most 1-loop corrections calculated,
also some 2-loop corrections

Central Problem:

What is the mechanism of electroweak symmetry breaking?

In SM: Higgs mechanism

Higgs mass constrained by precision data:
$m_H \lesssim 250$ GeV
Direct search: $m_H \gtrsim 114$ GeV

SM Higgs will be found at LHC (Tevatron?)

Precise nature of electroweak symmetry breaking is expected to be clarified at an
e^+e^- linear collider

Open questions in Standard Model

Origin of electroweak symmetry breaking
    Scalar Higgs field

Origin of masses ($M_W = 80\,GeV$)

Unification of gauge couplings ($M_{GUT} \approx 10^{16} GeV$)
GUT ⇒ how to stabilize mass of Higgs?
fine-tuning problem
how to relate highly diffent scales?
$M_{GUT} \sim M_W$
hierarchy problem

SUSY solves fine-tuning problem and
hierarchy problem in GUT

If scalar Higgs field is elementary, then SUSY may be only consistent framework in a GUT (in 4-dim space-time)

Radiative symmetry breaking:
Electroweak $U(2) \times U(1)$ spontaneously broken simultaneously with SUGRA

Note: $M_{Higgs} \lesssim 1 TeV$, Unitarity

Possible ways to solve hierarchy problem:

Higgs field is

(i) elementary

SUSY

Large compactified
extra dimensions

Little Higgs model

(ii) not elementary

Compositeness

Strong electroweak
symmetry breaking

Technicolour

Higgsless
extra dim. models