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SYNCHROTRON RADIATION IN LORENTZ-VIOLATING EFFECTIVE ELECTRODYNAMICS

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RADIATION IN LORENTZ VIOLATING ELECTRODYNAMICS

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(WORK IN PROGRESS)

MOTIVATION

• Recent interest in quantum gravity as a source of tiny modifications to dynamics in flat space. In particular, modified dispersion relations would arise [Amelino-Camelia et. al., Nature **393**,(1998)763]

$$\omega^2(k) = k^2 \pm \xi \frac{k^3}{M} \; : \; photons$$

$$E^2(p) = p^2 + m^2 + \eta_{R,L} \frac{p^3}{M} \; : \; fermions$$

[Gambini, Pullin (1999); Alfaro, Morales, Urrutia (2000); Thiemann, Salhmann, Winkler (2001); Ellis et. al. (2000); Myers, Pospelov (2003),.....], [redacted]

• Very stringent bounds upon the parameters ξ, η, Θ

- Atomic Physics: $|\Theta_2 + \Theta_4/2| < 10^{-9}$, [Sudarsky, Urrutia, Vucetich (2002)]
- Polarization measurements from astrophysical sources: $\xi < 10^{-4}$, [Gleiser, Kozameh (2001)]; $\xi < 10^{-16}/d_{0.5}$, [Jacobson, Liberati, Mattingly, Stecker (2003)]
- Synchrotron radiation (SR) from CRAB nebulae: either one of $\eta_{R,L} > -7 \times 10^{-8}$, [Jacobson, Liberati, Mattingly (2003)]. Based on reasonable extrapolation of standard SR to LIV case.

• Measure of linear polarization $\Pi = 80 \pm 20\%$ in GRB021206 $\Longrightarrow$ SR models for emission [Coburn, Boghs (2003)]. Also SR models for BL Lac objects: Markarian 421,501 with electrons [Konopelko et. al. (2003)] or with protons [Aharonian (2000) ; Mucke, Protheroe (2000)]

• Possibility that SR, for other astrophysical objects , would impose constraints upon the photon LIV parameter ξ.

MYERS AND POSPELOV EFFECTIVE THEORIES

(Phys. Rev. Lett. **90**(200~~4~~3)211601)

- Actions

$$S_{scalar} = \int d^4x \left[\partial_\mu \varphi^* \partial^\mu \varphi - \mu^2 \varphi^* \varphi + i \frac{\eta}{M} \varphi^* \left(V^\nu \partial_\nu\right)^3 \varphi\right],$$

$$\begin{aligned} S_{photon} \quad &= \quad \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - 4\pi\, J^\mu A_\mu \right. \\ & \left. + \frac{\xi}{M} \left(V^\alpha F_{\alpha\delta}\right) \left(V^\nu \partial_\nu\right) \left(V_\beta \tilde{F}^{\beta\delta}\right)\right]. \end{aligned}$$

- Work in coordinate system where $V^\mu = (1, \vec{0})$.
- Maxwell's equations:

$$\begin{aligned} \nabla \cdot \mathbf{E} \quad &= 4\pi\, \rho, \\ -\frac{\partial \mathbf{E}}{\partial t} + \nabla \times \mathbf{B} + \frac{\xi}{M} \frac{\partial}{\partial t} \left(-\nabla \times \mathbf{E} + \frac{\partial\, \mathbf{B}}{\partial t}\right) \quad &= 4\pi \mathbf{J} \end{aligned}$$

- Particle in constant magnetic field ($\mathbf{v} \perp \mathbf{B}$)

$$\ddot{\mathbf{r}} = \frac{q}{E} \left(1 - \frac{3}{2} \frac{\eta}{M} E + \frac{9}{4} E^2 \left(\frac{\eta}{M}\right)^2\right) \left(\mathbf{v} \times \mathbf{B}\right).$$

$$\omega_0 = \frac{|q| B}{E} \left(1 - \frac{3}{2} \frac{\eta}{M} E + \frac{9}{4} E^2 \left(\frac{\eta}{M}\right)^2\right), \quad R = \frac{\beta}{\omega_0}$$

$$1 - \beta^2 = \frac{\mu^2}{E^2} \left[1 + 2 \frac{\kappa E^3}{\mu^2} - \frac{15}{4} \frac{\kappa^2 E^4}{\mu^2} + O(\kappa^3)\right].$$

MP ELECTRODYNAMICS

- Energy-momentum tensor

$$T^0_{\;0} = \frac{1}{4\pi}\left(\frac{1}{2}(\mathbf{E}^2+\mathbf{B}^2) - \frac{\xi}{M}\mathbf{E}\cdot\frac{\partial\mathbf{B}}{\partial t}\right),$$
$$\mathbf{S} = \frac{1}{4\pi}\left(\mathbf{E}\times\mathbf{B} - \frac{\xi}{M}\mathbf{E}\times\frac{\partial\mathbf{E}}{\partial t}\right).$$

.

- Work with usual potentials in the standard radiation gauge
- Equation for $\vec{A}$

$$\left(-\omega^2 + k^2 - 2i\frac{\xi}{M}\omega^2\,\mathbf{k}\times\right)\mathbf{A}(\mathbf{k},\omega) = 4\pi\,\mathbf{J}_T(\mathbf{k},\omega).$$

- Can be diagonalized in the circular polarizacion basis (birrefringence)

$$\left(-\omega^2 + k^2 \pm 2\frac{\xi}{M}\omega^2 k\right)\mathbf{A}^{\pm} = 4\pi\mathbf{J}^{\pm}_T.$$

- Each mode propagates with velocity ($c=1$)

$$v_\lambda = \frac{1}{n(\lambda z)}, \quad \lambda = \pm, \quad z = \frac{\xi}{M}\omega, \quad n(\lambda z) = \sqrt{1+z^2} + \lambda z$$

- We call $\tilde{\xi} = \xi/M$ in the sequel

GREEN FUNCTIONS AND FIELDS

- The Green function is defined

$$\left[\left(-\omega^2 + k^2\right)\delta_{ik} - 2i\tilde{\xi}\omega^2\,\epsilon_{ijk}\,k_j\right] G_{kl}(\mathbf{k},\omega) = \delta_{il}$$

where

$$G_{kl}(\mathbf{k},\omega) = \frac{1}{U}\left((k^2-\omega^2)\delta_{kl} - \frac{4\tilde{\xi}^2\omega^4}{(k^2-\omega^2)}k_k k_l - 2i\tilde{\xi}\omega^2\epsilon_{klm}k_m\right)$$

$$U = \left(\omega^2\left(1-2k\tilde{\xi}\right) - k^2\right)\left(\omega^2\left(1+2k\tilde{\xi}\right) - k^2\right)$$

- The causal Green function is obtained by $\omega \to \omega + i\epsilon$
- The potential in the radiation approximation is

$$\mathbf{A}(\omega,\hat{\mathbf{n}}) = \frac{1}{r}\frac{1}{\sqrt{1+z^2}}\sum_{\lambda=\pm} n(\lambda z)e^{in(\lambda z)\omega r}\mathbf{J}^{\lambda}(\omega,\mathbf{k}_{\lambda})$$

$$\mathbf{k}_{\lambda} = \omega n(\lambda z)\hat{\mathbf{n}}.$$

- The corresponding electric and magnetic fields are

$$\mathbf{E}(\omega,\hat{\mathbf{n}}) = \frac{1}{r}\frac{i\omega}{\sqrt{1+z^2}}\sum_{\lambda=\pm} n(\lambda z)e^{in(\lambda z)\omega r}\mathbf{J}^{\lambda}(\omega,\mathbf{k}_{\lambda})$$

$$\mathbf{B}(\omega,\hat{\mathbf{n}}) = \sqrt{1+z^2}\,(\hat{\mathbf{n}}\times\mathbf{E}(\omega,\hat{\mathbf{n}})) - iz\,\mathbf{E}(\omega,\hat{\mathbf{n}})$$

- Simplification of the Poynting vector

$$\mathbf{S} = \frac{1}{4\pi}\left(\mathbf{E}\times\mathbf{B} - \frac{\xi}{M}\mathbf{E}\times\frac{\partial\mathbf{E}}{\partial t}\right)$$

with the relation

$$\hat{\mathbf{n}}\times\mathbf{E}(\omega,\mathbf{r}) = \frac{1}{\sqrt{1+z^2}}\left[\mathbf{B}(\omega,\mathbf{r}) + iz\mathbf{E}(\omega,\mathbf{r})\right], \qquad z = \frac{\xi}{M}\omega$$

In Fourier space $\partial/\partial t = -i\omega$, and

$$\begin{aligned}
\mathbf{S} &= \frac{1}{4\pi}\left(\mathbf{E}(-\omega)\times\mathbf{B}(\omega) - \frac{\xi}{M}\mathbf{E}(-\omega)\times(-i\omega)\mathbf{E}(\omega)\right)\\
\mathbf{S} &= \frac{1}{4\pi}\mathbf{E}(-\omega)\times\left[\mathbf{B}(\omega) + iz\mathbf{E}(\omega)\right]\\
\mathbf{S} &= \frac{1}{4\pi}\sqrt{1+z^2}\,\mathbf{E}(-\omega)\times(\hat{\mathbf{n}}\times\mathbf{E}(\omega))\\
\mathbf{S} &= \frac{1}{4\pi}\sqrt{1+z^2}\,(\mathbf{E}(-\omega)\cdot\mathbf{E}(\omega))\,\hat{\mathbf{n}}, \quad \mathbf{E}(-\omega) = (\mathbf{E}(\omega))^*
\end{aligned}$$

- Recalling that

$$\sqrt{1+z^2} = \frac{n(z)+n(-z)}{2}$$

we can rewrite

$$\mathbf{S} = \frac{1}{4\pi}\left(\frac{n(z)+n(-z)}{2}\right)(\mathbf{E}(-\omega)\cdot\mathbf{E}(\omega))\,\hat{\mathbf{n}},$$

- In a standard medium we have

$$\mathbf{S} = \frac{1}{4\pi}n\;(\mathbf{E}(-\omega)\cdot\mathbf{E}(\omega))\;\hat{\mathbf{n}}$$

SYNCHROTRON RADIATION

- General power spectrum

$$\frac{d^2P(T)}{d\omega d\Omega} = \frac{1}{4\pi^2}\frac{\omega^2}{\sqrt{1+z^2}}\int_{-\infty}^{\infty} d\tau\; e^{-i\omega\tau}\sum_{\lambda=\pm}$$

$$\times\left[n^2(\lambda z)J_i^*\left(T+\tau/2,\mathbf{k}_\lambda\right)\; P_{ik}^\lambda\; J_k\left(T-\tau/2,\mathbf{k}_\lambda\right)\right],$$

$$P_{ik}^\lambda = \frac{1}{2}\left(\delta_{ik} - \hat{k}_i\hat{k}_k + \lambda i\epsilon_{ijk}\hat{k}_j\right).$$

- Circular orbit

$$J_k(t,\mathbf{k}) = q\mathbf{v}(t)e^{-i\mathbf{k}\cdot\mathbf{r}(t)}, \quad \mathbf{v}(t) = (-\beta\sin\omega_0 t,\; \beta\cos\omega_0 t,\; 0)$$

- Averaged angular distribution of the m^{th} harmonic

$$\left\langle\frac{d^2P(T)}{d\omega d\Omega}\right\rangle_T = \sum_{\lambda=\pm}\sum_{m=0}^{\infty}\delta(\omega - m\omega_0)\frac{dP_{m,\lambda}}{d\Omega},$$

$$\frac{dP_{m,\lambda}}{d\Omega} = \frac{\omega^2 q^2}{4\pi}\frac{1}{\sqrt{1+z_m^2}}\left[\lambda\beta n(\lambda z_m)J_m'(W_{\lambda m}) + \cot\theta\, J_m(W_{\lambda m})\right]^2$$

$$W_{\lambda m} = mn(\lambda z_m)\beta\sin\theta, \qquad z_m = \tilde{\xi}m\omega_0$$

- Integrated power in the m^{th} harmonic

$$P_{m,\lambda} = \frac{q^2\, m\omega_0^2}{2\sqrt{1+z_m^2}}\beta n(\lambda z_m)\left[2J_{2m}'(2m\,\beta n(\lambda z_m))\right.$$

$$\left. - \left[\frac{1}{[\beta n(\lambda z_m)]^2} - 1\right]\int_0^{2m\,\beta n(\lambda z_m)} dx\; J_{2m}(x)\right].$$

HIGH m EXPANSIONS

- We are in the regime $1 - [\beta n]^2 > 0$
- Integrated power in the m^{th} harmonic

$$P_{\lambda m} = \frac{q^2 m \omega_0}{\sqrt{3}\pi R} \frac{1}{1 + n^2(\lambda z_m)} \left\{ \int_{m/\tilde{m}_c}^{\infty} dx \left(\frac{3}{2\tilde{m}_c} \right)^{2/3} K_{5/3}(x) - 2 \left(\frac{3}{2\tilde{m}_c} \right)^{4/3} K_{2/3}\left(\frac{m}{\tilde{m}_c} \right) \right\}.$$

- The cut-off frequency

$$\tilde{m}_c = \frac{3}{2} \left(1 - [\beta n(\lambda z_m)]^2 \right)^{-3/2}$$

because for $m > \tilde{m}_c$

$$P_{\lambda m} \approx e^{-m/\tilde{m}_c}$$

- Integrated total power in the m^{th} harmonic to second order in $\tilde{\xi}$

$$P_m = \frac{q^2 m \omega_0}{\sqrt{3}\pi R \gamma^2} \left\{ \frac{m_c}{m} \kappa \left(\frac{m}{m_c} \right) - \frac{2}{\gamma^2} K_{2/3}\left(\frac{m}{m_c} \right) + 2\tilde{\xi}^2 (m \omega_0 \beta)^2 \left[\left(\frac{m}{\gamma} \right)^2 - \frac{1}{2} \right] K_{2/3}\left(\frac{m}{m_c} \right) \right\},$$

$$m_c = \frac{3}{2}\gamma^3.$$

T. ERBER

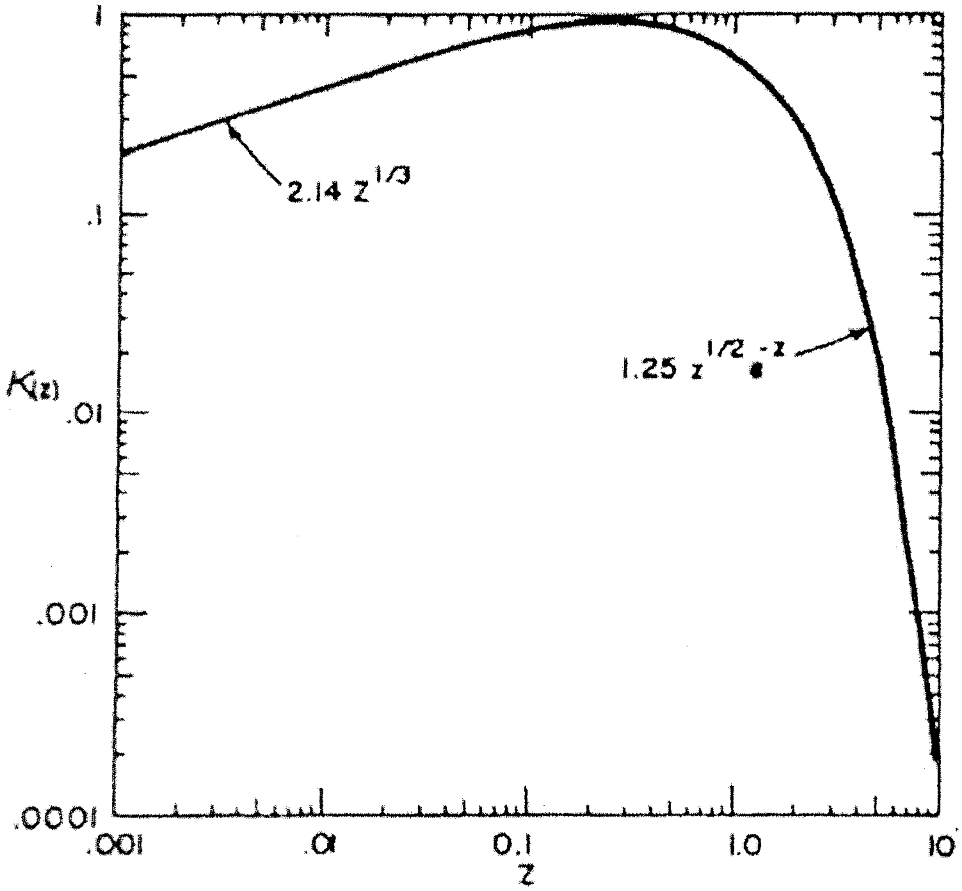

FIG. 3. Graph of the bremsstrahlung function;

$$\kappa(z) = z \int_z^{\infty} dx K_{5/3}(x).$$

$z \to \frac{m}{m_c}$

AVERAGED DEGREE OF CIRCULAR POLARIZATION

- Let us assume that the relativistic electrons have an energy distribution of the type

$$N(E)dE = CE^{-p}dE, \quad 2 < p < 3$$

- Let us define the circular degree of polarization as

$$\Pi_{\odot} = \frac{\langle P_{+}(\omega) - P_{-}(\omega)\rangle}{\langle P_{+}(\omega) + P_{-}(\omega)\rangle}$$

where $P_{\pm}(\omega)$ is the total power distribution per unit frequency and polarization $\lambda = \pm 1$, so that

$$P_{\lambda}(\omega) = \frac{P_{m\lambda}}{\omega_0}$$

- The result is $(p \neq 1)$

$$\Pi_{\odot} = 2\tilde{\xi}\omega \left(\frac{m}{\gamma}\right)\left(\frac{p+1}{p-1}\right)\frac{\Pi(p)}{1 - \frac{3}{2}\left(\frac{\gamma\omega_0}{\omega}\right)(p+1)\,\Pi(p)},$$

$$\Pi(p) = \frac{\Gamma\left(\frac{1}{4}p + \frac{13}{12}\right)\Gamma\left(\frac{1}{4}p + \frac{5}{12}\right)}{\Gamma\left(\frac{1}{4}p + \frac{19}{12}\right)\Gamma\left(\frac{1}{4}p - \frac{1}{12}\right)}$$

- This is the analogous expression for the average of the degree of linear polarization

$$\Pi_{LIN} = \frac{p+1}{p+7/3}$$

LOOKING AT THE DOMINANT AMPLIFYING FACTOR

- We start from

$$P_\lambda(\omega) = \frac{q^2\omega}{2\beta(E)}\frac{2}{1+n^2}\frac{1}{\sqrt{3}\pi}\left[\left(\frac{3}{2}\frac{1}{\tilde{m}_c}\right)^{2/3}\frac{\tilde{m}_c}{m}\kappa(m/\tilde{m}_c) - 2\left(\frac{3}{2\tilde{m}_c}\right)^{4/3}K_{2/3}\left(\frac{m}{\tilde{m}_c}\right)\right]$$

- Most of the radiation comes from $m \approx \tilde{m}_c >>> 1$ where $K_{2/3}(1) = 0.49$, $\kappa(1) \simeq 0.65$. The dominant term is

$$P_\lambda(\omega) = D\left[\left(\frac{1}{\tilde{m}_c}\right)^{2/3}\frac{\tilde{m}_c}{m}\kappa(m/\tilde{m}_c)\right]$$

$$\tilde{m}_c = \frac{3}{2}(1-n(z)^2\beta^2)^{-3/2}, \quad z = \lambda\tilde{\xi}\omega \quad m = \frac{\omega}{\omega_0}, \quad \omega_0 = \frac{qB}{E}.$$

- We are interested in

$$P_\lambda(\omega) = [P_\lambda(\omega)]_{z=0} + \lambda\tilde{\xi}\omega\left(\frac{dP_\lambda(\omega)}{dz}\right)_{z=0} + \ldots\ldots.$$

to calculate

$$\Pi_{\odot} = \tilde{\xi}\omega\frac{\left\langle\left(\frac{dP_\lambda(\omega)}{dz}\right)_{z=0,\kappa=0}\right\rangle}{\left\langle[P_\lambda(\omega)]_{z=0.\kappa=0}\right\rangle} + O(\xi^2,\ \xi\kappa, \ldots\ldots)$$

- It is convenient to change the derivative to

$$\frac{d}{dz} = \frac{2n^2}{1+n^2}\frac{\beta}{n}\frac{d}{d\beta} =$$

because now we can directly take $n = 1$

to obtain

$$\left(\frac{dP_\lambda(\omega)}{dz}\right)_{n=1,\kappa=0} = D\,\frac{d}{d\beta}\left[\left(\frac{1}{m_c}\right)^{2/3}\frac{m_c}{m}\kappa(m/m_c)\right],$$

$$m_c = \frac{3}{2}(1-\beta^2)^{-3/2} = \frac{3}{2}\gamma^3, \quad E = \mu\gamma.$$

- Now we go to the variable x

$$x = \frac{m}{m_c} = \left(\frac{2}{3}\frac{\omega\mu}{qB}\right)\frac{\mu^2}{E^2}, \quad \frac{E}{\mu} = \gamma = A\,x^{-1/2}, \quad A^2 = \frac{2}{3}\frac{\mu\omega}{qB} = \frac{2}{3}\frac{m}{\gamma},$$

$$m_c = \frac{3}{2}A^3\,x^{-3/2}, \qquad \frac{d}{d\beta} = \beta\gamma^3\frac{d}{d\gamma} = -2A^2\frac{d}{d\,x}.$$

- Then

$$\left(\frac{dP_\lambda(\omega)}{dz}\right)_{n=1,\kappa=0} = -\left(\frac{3}{2}A^3\right)^{-2/3} D\;2A^2\,\frac{d\kappa(x)}{dx}$$

$$[P_\lambda(\omega)]_{n=1,\kappa=0} = \left(\frac{3}{2}A^3\right)^{-2/3} D\,\kappa(x)$$

- Finally we get

$$\Pi_{\odot} = -\frac{4}{3}\tilde{\xi}\omega\left(\frac{m}{\gamma}\right)F(p),$$

$$F(p) = \frac{\int_0^\infty x^{(p-3)/2}\,\frac{d\kappa(x)}{dx}\,dx}{\int_0^\infty x^{(p-3)/2}\,\kappa(x)\,dx}.$$

where the energy average is translated into

$$\langle G(x)\rangle = \frac{C}{2}(\mu A)^{-p+1}\int_0^\infty x^{(p-3)/2}\,G(x)\,dx.$$

THE FAR-FIELD APPROXIMATION

- The phase in the Green function is

$$n(\lambda z)\omega\left|\mathbf{r}-\mathbf{r}'\right| \simeq \omega r\left(1-\frac{\mathbf{n}\cdot\mathbf{r}'}{r}+\lambda\tilde{\xi}\omega-\lambda\tilde{\xi}\omega\frac{\mathbf{n}\cdot\mathbf{r}'}{r}+\frac{1}{2}\frac{r'^2}{r^2}\right)$$

- If

$$|\tilde{\xi}\omega|\,\frac{r'}{r} > \left(\frac{r'}{r}\right)^2$$

we can neglect only the term quadratic in $r'$and both ξ-dependent terms remains in the phase:

$$n(\lambda z)\omega\left|\mathbf{r}-\mathbf{r}'\right| \simeq n(\lambda z)\omega(r-\hat{\mathbf{n}}\cdot\mathbf{r}')$$

- Other possibility is that

$$\left(\frac{r'}{r}\right)^2 < |\xi\omega| < \frac{r'}{r}$$

which leads to

$$n(\lambda z)\omega\left|\mathbf{r}-\mathbf{r}'\right| \simeq n(\lambda z)\omega r-\hat{\mathbf{n}}\cdot\mathbf{r}'$$

- Finally, if

$$|\tilde{\xi}\omega| < \left(\frac{r'}{r}\right)^2$$

all the dependence on ξ is negligible in the phase, which reduces to

$$n(\lambda z)\omega\left|\mathbf{r}-\mathbf{r}'\right| \simeq \omega r-\hat{\mathbf{n}}\cdot\mathbf{r}'$$

DATA OF SOME RELEVANT OBJECTS

OBJECT	$r(l.y)$	γ	$B(Gauss)$	$\omega_{obs}(GeV)$	$\omega_0(GeV)$	m	m/γ
$CRAB$	10^4	10^9	10^{-3}	10^{-1}	10^{-30}	10^{29}	10^{20}
$(MARKARIAN)_e$	10^8	10^{11}	10^2	10^4	10^{-26}	10^{30}	10^{19}

OBJECT	$\frac{r'}{r}$	$\tilde{\xi}\omega$	$\tilde{\xi}\omega\frac{r'}{r}$	$\left(\frac{r'}{r}\right)^2$
$CRAB$	10^{-6}	$\xi 10^{-20}$	$\xi 10^{-26}$	10^{-12}
$(MARKARIAN)_e$	10^{-14}	$\xi 10^{-15}$	$\xi 10^{-29}$	10^{-28}

SUMMARY AND OUTLOOK

- EXACT AND COMPLETE DESCRIPTION OF SR IN MP MODEL

(SCHWINGER ET. AL. 1976)

$$\delta\theta \sim m^{-1/3} \sim m_c^{-1/3} \sim \left(1-[\beta(t)n(t)]^2\right)^{1/2}$$

- IN FULL FAR-FIELD APPROXIMATION WE FIND AMPLIFYING FACTORS

$$(\tilde{\xi}\omega)\left(\frac{m}{\gamma}\right) \sim (\tilde{\xi}\omega)\left(\frac{m_c}{\gamma}\right) \sim (\tilde{\xi}\omega)\gamma^2$$

(SIMILAR RESULTS IN NON-COM. SR: CASTORINA, IORIO, ZAPPALA: PRD 69 (2004) 065008)

- **CRAB NEBULAE**

$n(z) = 1$ IN PHASE OF RADIATION FIELD

CORRECTIONS APPEAR ONLY VIA $\beta(E)$; $\tilde{\eta}$

$$\delta\theta \sim \gamma^{-1}(E) \; ; \; \omega_c = \frac{eB}{E}\gamma^3(E)$$

JACOBSON ET. AL. RECOVERED

- **REPEAT ANNALYSIS FOR OTHER ASTROPHYSICAL SOURCES:**
  - (*) AVERAGE CIRCULAR POL.
  - (*) CORRECTIONS TO AVERAGE LINEAR POL.
  - (*) STOKES PARAMETERS

- **GAMBINI – PULLIN** E.D. IS NON-LOCAL IN A_μ. COINCIDES WITH MP TO FIRST ORDER IN $\tilde{\xi}$

- **SR** IN ELLIS ET. AL. E.D. UNDER INVESTIGATION.