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CONFERENCE ON FUNDAMENTAL SYMMETRIES AND FUNDAMENTAL CONSTANTS

15 - 18 September 2004

ASPECTS OF CPT VIOLATION AND ITS PHENOMENOLOGY

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Neutrinos from Another Perspective:
Decoherence, CPT Violation, Dark Energy and LSND

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ICTP, fundamental symmetries, September 2004

ESSENTIAL QUESTIONS

- **Are there theories which allow CPT breaking?**
- **How (un)likely is it that somebody someday finds CPT violation, and why?**
- **What formalism? How can we be sure of observing CPT Violation ? our current phenomenology is based on CPT invariance**
- How should we compare various "figures of merit" of CPT tests (direct mass measurement, K^0-$\overline{K}^0$ mass difference a la CPLEAR, Decoherence Effects, EPR states, neutrino mixing, electron g-2 and cyclotron frequency comparison, neutrino spin-flavour conversion (for Lorentz violating models) *etc.*)

OUTLINE

- WHAT IS CPT SYMMETRY.
- WHY CPT VIOLATION ?

  Theoretical models and ideas: **Quantum Gravity Models violating Lorentz symmetry and/or quantum coherence**:

  (i) space-time foam (local f.t., non-critical strings),

  (ii) (non supersymmetric) string-inspired standard model extension with Lorentz Violation.

  (iii) Loop Quantum Gravity

- HOW CAN WE DETECT CPT VIOLATION? :
  SENSITIVE PARTICLE PHYSICS PROBES:
  (i) Neutral Mesons: KAONS, B-MESONS, entangled states in ϕ and B factories.
  (ii) anti-matter factories: antihydrogen (precision spectroscopic tests on free and trapped molecules),
  (iii) Low energy atomic physics experiments.
  * *(iv) Neutrino Physics*
  (v) Astrophysical Tests (especially Lorentz-Invariance violation tests, via modified dispersion relations of matter probes *etc.*)

  *: *This talk.*

CPT THEOREM

C(harge) -P(arity=reflection) -T(ime reversal) INVARIANCE is a property of any quantum field theory in Flat space times which respects:
(i) Locality, (ii) Unitarity and (iii) Lorentz Symmetry.

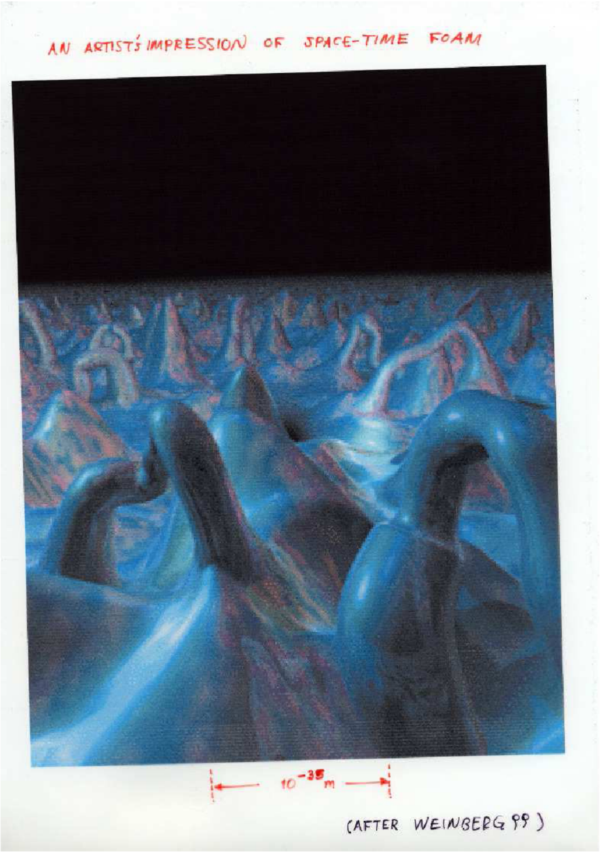
$$\Theta\mathcal{L}(x)\Theta^\dagger = \mathcal{L}(-x) \ ,$$
$$\Theta = CPT \ , \ \mathcal{L} = \mathcal{L}^\dagger \ \text{(Lagrangian)}$$

Theories with HIGHLY CURVED SPACE TIMES , with space time boundaries of black-hole horizon type, may violate (ii) & (iii) and hence CPT.

E.g.: SPACE-TIME FOAMY SITUATIONS IN SOME QUANTUM GRAVITY MODELS.

SPACE-TIME FOAM

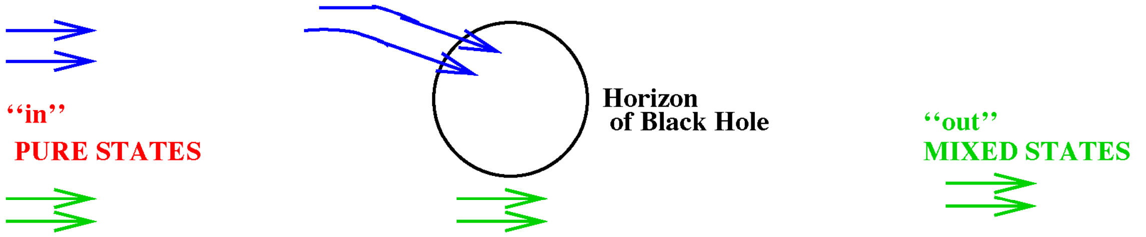
Space-time MAY BE DISCRETE at scales $10^{-35}\ m$ (Planck) → LORENTZ VIOLATION (LV)? (and hence CPTV); also there may be ENVIRONMENT of GRAVITATIONAL d.o.f. INACCESSIBLE to low-energy experiments (non-propagating d.o.f., no scattering) → CPT VIOLATION (and may be LV)



FOAM AND UNITARITY VIOLATION

SPACE-TIME FOAM: Quantum Gravity SINGULAR Fluctuations (microscopic (Planck size) black holes etc) may imply “environment” → evolution of initially pure states to mixed ones:

SPACE–TIME FOAMY SITUATIONS
NON UNITARY (CPT VIOLATING) EVOLUTION
OF PURE STATES TO MIXED ONES



$| \dots >$

ρ_{out} = density matrix

$= \mathrm{Tr}_{unobs} | \psi >< \psi |$

modified temporal evolution of ρ:

$$\frac{d}{dt}\rho = i\,[\,\rho\,,\,H\,] + \Delta H(\rho)\,\rho$$

quantum mechanical terms

quantum mechanics violating term

$\rho_{out} = \mathrm{Tr}_{\mathrm{unobs}} |out >< out| = \$\ \rho_{in}$

$\$ \ne S S^\dagger$, $S = e^{iHt}$ =scattering matrix \$ non invertible, unitarity lost in effective theory.

CPT VIOLATION (CPTV) AND \$ $\neq SS^\dagger$

A THEOREM BY R. WALD: **If \$ $\neq S\ S^\dagger$, then CPT is violated, at least in its strong form.**

PROOF:

Suppose CPT is conserved, then there exists unitary, invertible opearator Θ : $\Theta\overline{\rho}_{in} = \rho_{out}$

$$\rho_{out} = \$\ \rho_{in} \rightarrow \Theta\overline{\rho}_{in} = \$\ \Theta^{-1}\overline{\rho}_{out} \rightarrow \overline{\rho}_{in} = \Theta^{-1}\$\ \Theta^{-1}\overline{\rho}_{out}.$$

But $\overline{\rho}_{out} = \$\overline{\rho}_{in}$, hence :

$$\overline{\rho}_{in} = \Theta^{-1}\$\Theta^{-1}\ \$\ \overline{\rho}_{in}$$

BUT THIS IMPLIES THAT \$ HAS AN INVERSE- $\Theta^{-1}\$\Theta^{-1}$, IMPOSSIBLE (information loss), hence CPT MUST BE VIOLATED (at least in its strong form).

NB: My preferred way of CPTV by Quantum Gravity

COSMOLOGICAL CPTV?

(NM, hep-ph/0309221)

Recent Astrophysical Evidence for Dark Energy (acceleration of the Universe (SnIA), CMB anisotropies (WMAP...))

Best fit models of the Universe consistent with non-zero **cosmological constant** $\Lambda \neq 0$ (de Sitter)

Λ-universe will eternally accelerate, as it will enter in an inflationary phase again: $a(t) \sim e^{\sqrt{\Lambda/3t}}$, $t \to \infty$, there is **cosmological Horizon**.

Horizon implies incompatibility with S-matrix: no proper definition of asymptotic state vectors, environment of d.o.f. crossing the horizon (c.f. dual picture of black hole, now observer is inside the horizon).

Theorem by Wald on $-matrix and CPTV: CPT is violated due to $\Lambda > 0$:

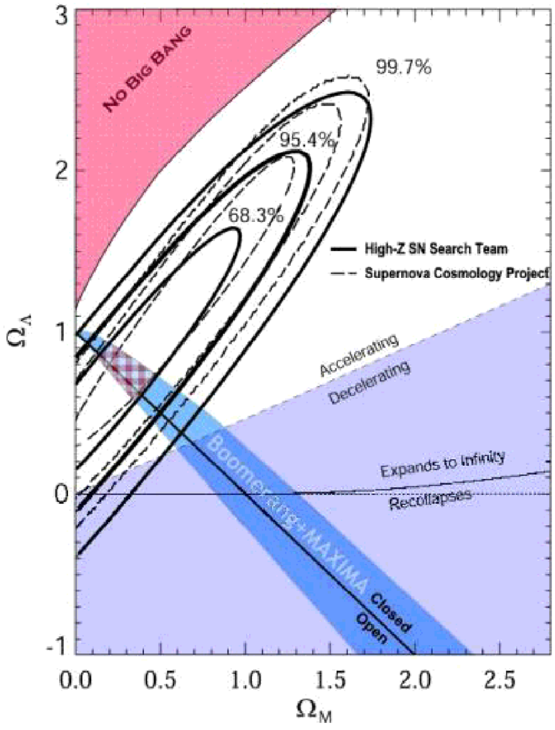
$$\partial_t \rho = [\rho, H] + \mathcal{O}(\Lambda/M_P^3)\rho$$

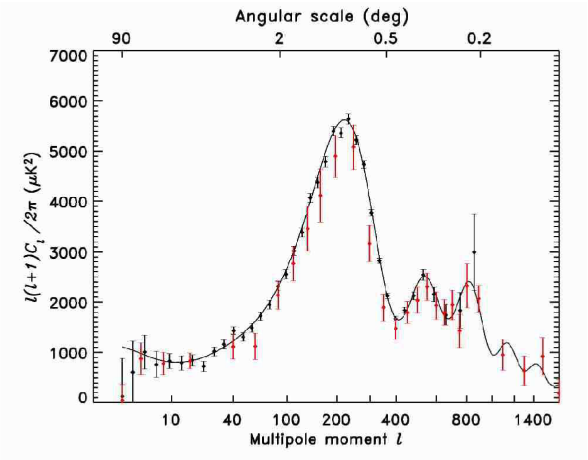
Tiny cosmological CPTV effects, but detected through Universe acceleration!

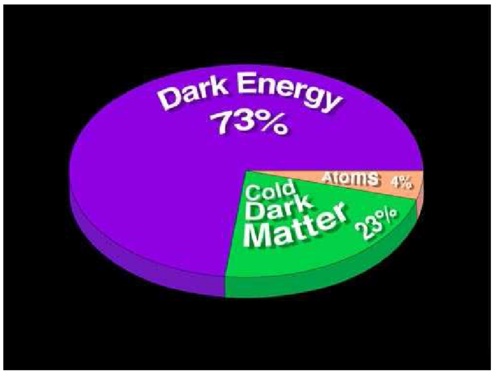
Issue: Quantize de Sitter space as an open system? or use Relaxation models for Dark Energy, where S-matrix is OK?.

Evidence for Dark Energy

WMAP improved results on CMB: $\Omega_{\rm total} = 1.02 \pm 0.02$, high precision measurement of secondary (two more) acoustic peaks (c.f. new determination of Ω_b). Agreement with SnIa Data. Best Fit : $\Omega_\Lambda = 0.73$, $\Omega_{\rm Matter} = 0.23$



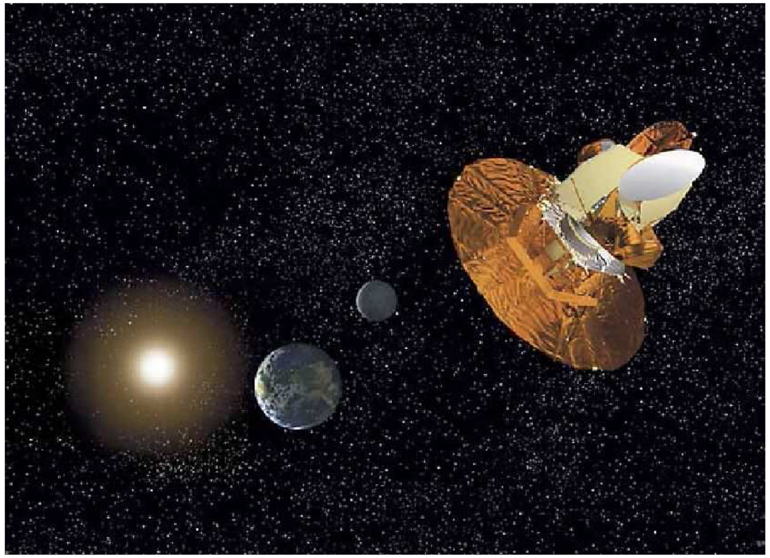


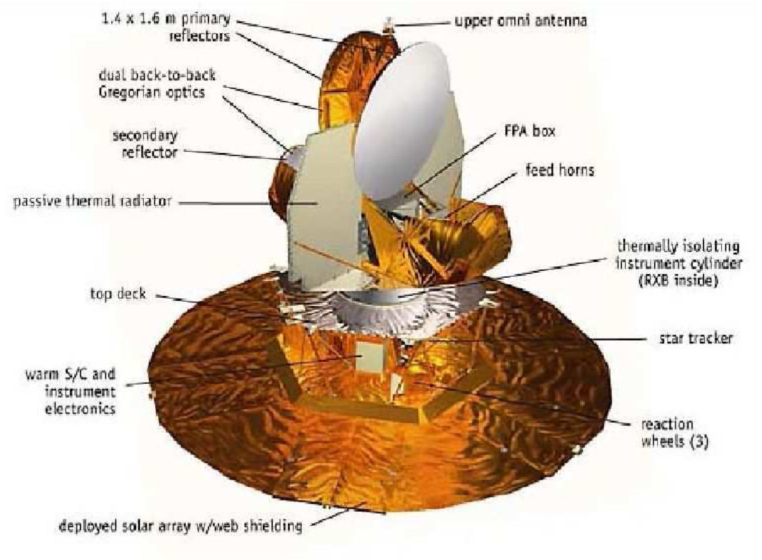


WHAT IS WMAP

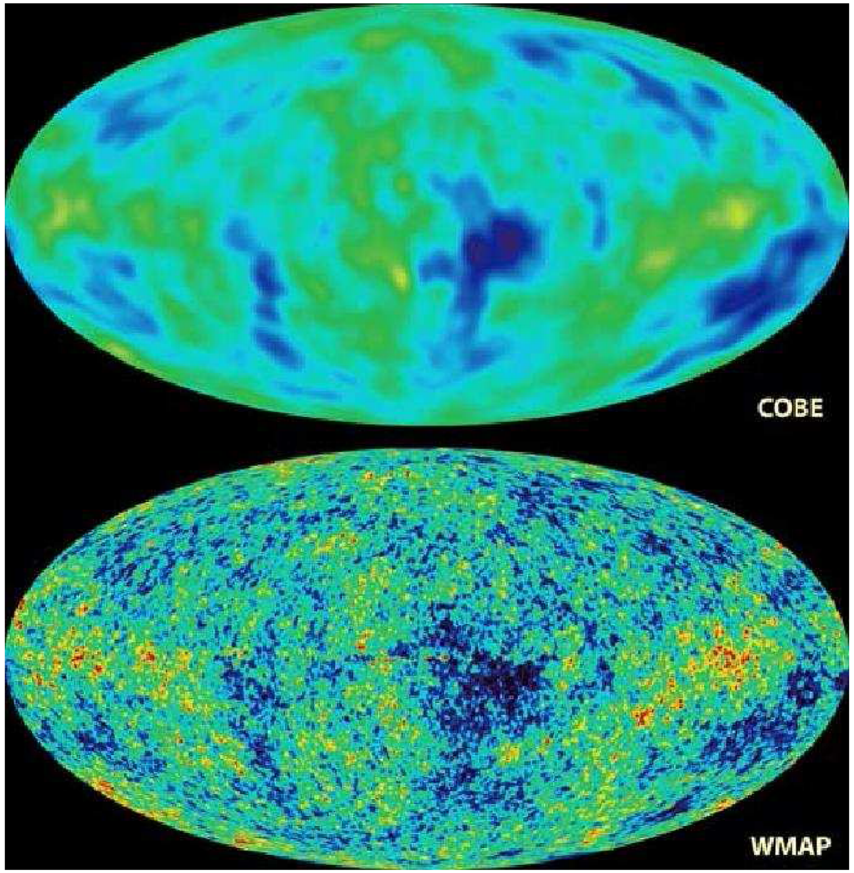
Wilkinson Microwave Anisotropy Probe (http://map.gsfc.nasa.gov/)

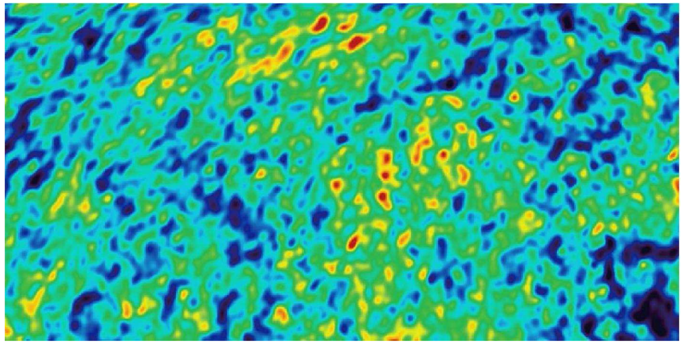
Satellite dedicated to probing Cosmic Microwave Background Anisotropies - named in honour of its first director (D. Wilkinson (1935-2002)).





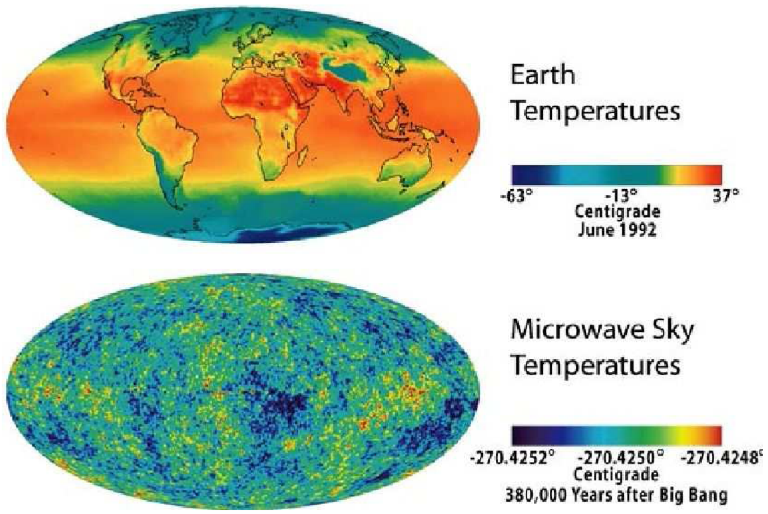
WMAP ACHIEVEMENTS IN ONE YEAR

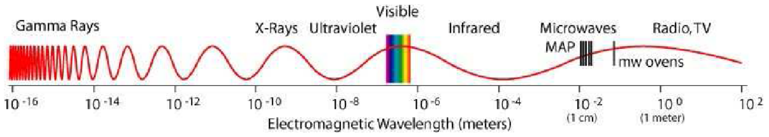




WMAPping the Universe and comparison with COBE (left), Importance of Detail (right).

WHAT THIS PICTURE MEANS





The cosmic microwave background is the afterglow radiation left over from the hot Big Bang. Its temperature is extremely uniform all over the sky. However, tiny temperature variations or fluctuations (at the part per million level) can offer great insight into the origin, evolution, and content of the universe. The light that is reaching us has been stretched out as the universe has stretched, so light that was once beyond gamma rays is now reaching us in the form of microwaves (longer wavelength).

ORDER OF MAGNITUDE of CPTV

Tiny cosmological (global) CPTV effects may be much smaller than QG (local) space-time effects (foam etc).

Naively, Quantum Gravity (QG) has a dimensionful constant: $G_N \sim 1/M_P^2$, $M_P = 10^{19}$ GeV. Hence, CPT violating and decoherening effects may be expected to be suppressed by E^3/M_P^2 , where E is a typical energy scale of the low-energy probe. This would be practically undetectable in neutral mesons, but neutrinons might be sensitive ! (e.g. modified dispersion relations (m.d.r.) for ultrahigh energy ν from GRB's (Ellis, NM, Nanopoulos, Volkov)) Also in some astrophysical cases, e.g. Crab Nebula or Vela pulsar synchrotron radiation constraints electron m.d.r. of this order (Jacobson, Liberati, Mattingly, Ellis, NM, Sakharov)

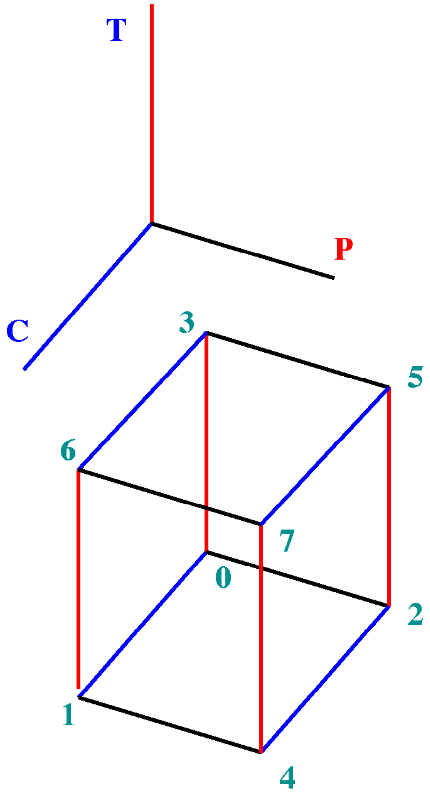
HOWEVER: RESUMMATION & OTHER EFFECTS in theoretical models may result in much larger effects of order: $\frac{E^2}{M_P}$.
(This happens, e.g., loop gravity, some stringy models of QG involving open string excitations ...)

SUCH LARGE EFFECTS ARE definitely ACCESSIBLE/FALSIFIABLE BY CURRENT AND IMMEDIATE FUTURE EXPERIMENTS.

PHENOMENOLOGY of CPTV:

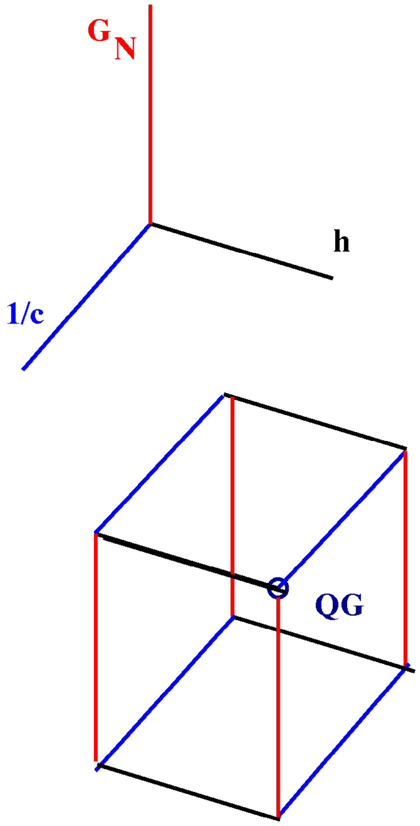
MNEMONIC CUBES

OKUN'S CUBE



CPT: 0, 4, 5, 6 even ; 1, 2, 3, 7 odd

PENROSE'S



QG MAY VIOLATE CPT ?

LORENTZ-VIOLATION AND CPT:

STANDARD MODEL EXTENSION (SME)

V.A. Kostelecký, R. Bluhm, D. Colladay, R. Potting, N. Russell

In this case Lorentz symmetry is violated and hence CPT, but no quantum decoherence or unitarity loss.
String theory (non supersymmetric) → Tachyonic instabilities, coupling with tensorial fields (gauge etc), $\rightarrow < A_\mu > \neq 0 \; , < T_{\mu_1 \dots \mu_n} > \neq 0 \; ,$
Spontaneous breaking of Lorentz symmetry by (exotic) string vacua
MODIFIED DIRAC EQUATION in SME: for FREE Hydrogen H (anti-hydrogen $\overline{H}$): spinor ψ reps. electron (positron) with charge $q = -|e| (q = |e|)$ around a proton (antiporoton) of charge $-q$:

$$(i\gamma^\mu D^\mu - M - a_\mu \gamma^\mu - b_\mu \gamma_5 \gamma^\mu - \\ -\frac{1}{2} H_{\mu\nu} \sigma^{\mu\nu} + ic_{\mu\nu} \gamma^\mu D^\nu + id_{\mu\nu} \gamma_5 \gamma^\mu D^\nu)\psi = 0$$

where $D_\mu = \partial_\mu - qA_\mu$, $A_\mu = (-q/4\pi r, 0)$ Coulomb potential. CPT & Lorentz violation: $a_\mu \; , b_\mu \; .$
Lorentz violation only: $c_{\mu\nu} \; , d_{\mu\nu} \; , H_{\mu\nu}$

PHENOMENOLOGY of CPTV:

HOW CAN WE DETECT LORENTZ (and CPT) VIOLATION? :

Direct SME Tests & Modified Dispersion relations (M.D.R.)

Many LV Models of QG predict modified dispersion relations (d.r.) for matter probes, inclusive of ν (Amelino-Camelia, Ellis, NM, Nanopoulos, Sarkar, Volkov, Gambini, Pullin, ...).

This leads to one class of tests using ν: each mass eigenstate has QG deformed dispersion relations, may be the same for all flavours, may be not:

$E^2 = \vec{k}^2 + m_i^2 + f_i(E, M_{qg}, \vec{k})$, e.g. $f_i = \sum_\alpha C_\alpha \vec{k}^2 (\frac{|\vec{k}|}{M_{qg}})^\alpha$.

Stringent bounds on f_i from oscillation experiments.

GENERIC TESTS of M.D.R. &/OR S.M.E.

(i) astrophysics tests - arrival time fluctuations for photons (model independent analysis of astrophysical GRB data)

(ii) Nuclear/Atomic Physics precision measurements (clock comparison, spectroscopic tests on free and trapped molecules, quadrupole moments *etc*).

(iii) antihydrogen (precision spectroscopic tests on free and trapped molecules: e.g. $1S \to 2S$ forbidden transitions),

*(iii) Neutrino mixing, spin-flavour conversion.

PLANCK SCALE LV-SME BOUNDS

LOW-ENERGY ATOMIC PHYSICS EXPERIMENTS:

LEADING ORDER BOUNDS

EXPER.	SECTOR	PARAMS. (J=X,Y)	BOUND (GeV)
Penning Trap	electron	$\bar{b}_J^e$	5×10^{-25}
Hg-Cs clock comparison	electron	$\bar{b}_J^e$	10^{-27}
	proton	$\bar{b}_J^p$	10^{-27}
	neutron	$\bar{b}_J^n$	10^{-30}
H Maser	electron	$\bar{b}_J^e$	10^{-27}
	proton	$\bar{b}_J^p$	10^{-27}
spin polarized matter	electron	$\bar{b}_J^e \,/\, \bar{b}_Z^e$	$10^{-29} \,/\, 10^{-28}$
He-Xe Maser	neutron	$\bar{b}_J^n$	10^{-31}
Muonium	muon	$\bar{b}_J^\mu$	2×10^{-23}
Muon g-2	muon	$\bar{b}_J^\mu$	5×10^{-25} (estimated)

X,Y.Z celestial equatorial coordinates $\bar{b}_J = b_3 - m d_{30} - H_{12}$

(Bluhm, hep-ph/0111323)

Neutrinos & SME

SME-LV+CPTV (phenomenological) model for ν (Kostelecký & Mewes 20003)

$\mathcal{L}_{SME} \ni \frac{1}{2} i \overline{\psi}_{a,L} \gamma^\mu D_\mu \psi_{a,L} - (a_L)_{\mu ab} \overline{\psi}_{a,L} \gamma^\mu \psi_{b,L} + \frac{1}{2} i (c_L)_{\mu\nu ab} \overline{\psi}_{a,L} \gamma^\mu D^\nu \psi_{b,L}$

a, b flavour indices, No ν mass differences.

Presence of LV induces directional dependence (sidereal effects)!

Effective Hamiltonian:

$(H_{\rm eff})_{ab} = |\vec{p}|\delta_{ab} + \frac{1}{|\vec{p}|}((a_L)^\mu p_\mu - (c_L)^{\mu\nu} p_\mu p_\nu)_{ab}$

NB: ν Oscillations now are controlled by (dimensionless) $a_L L$ & $c_L LE$ (L=oscill. length). Contrast conventional case: $\Delta m^2 L/E$

Imporant SME feature: despite CPTV, oscillation probs $P_{\nu_x \to \nu_y} = P_{\bar{\nu}_x \to \bar{\nu}_y}$ (if no mass differences).

Bind LV+CPTV SME experimentally. E.g.: High energy long baseline expts: no evidence for $\nu_{e,\mu} \to \nu_\tau$ at $E \sim 100$ GeV , $L \sim 10^{-18}$ GeV^{-1} $\to a_L < 10^{-18}$ GeV, $c_L < 10^{-20}$.

For LSND anomaly: Mass-squared difference required: $\Delta m^2 = 10^{-19}$ GeV$^2 = 10^{-1}$ eV^2, $a_L \sim 10^{-18}$ GeV, $c_L \sim 10^{-17}$. Affect other expts. . No good for LSND.

Experimental Sensitivities for ν's

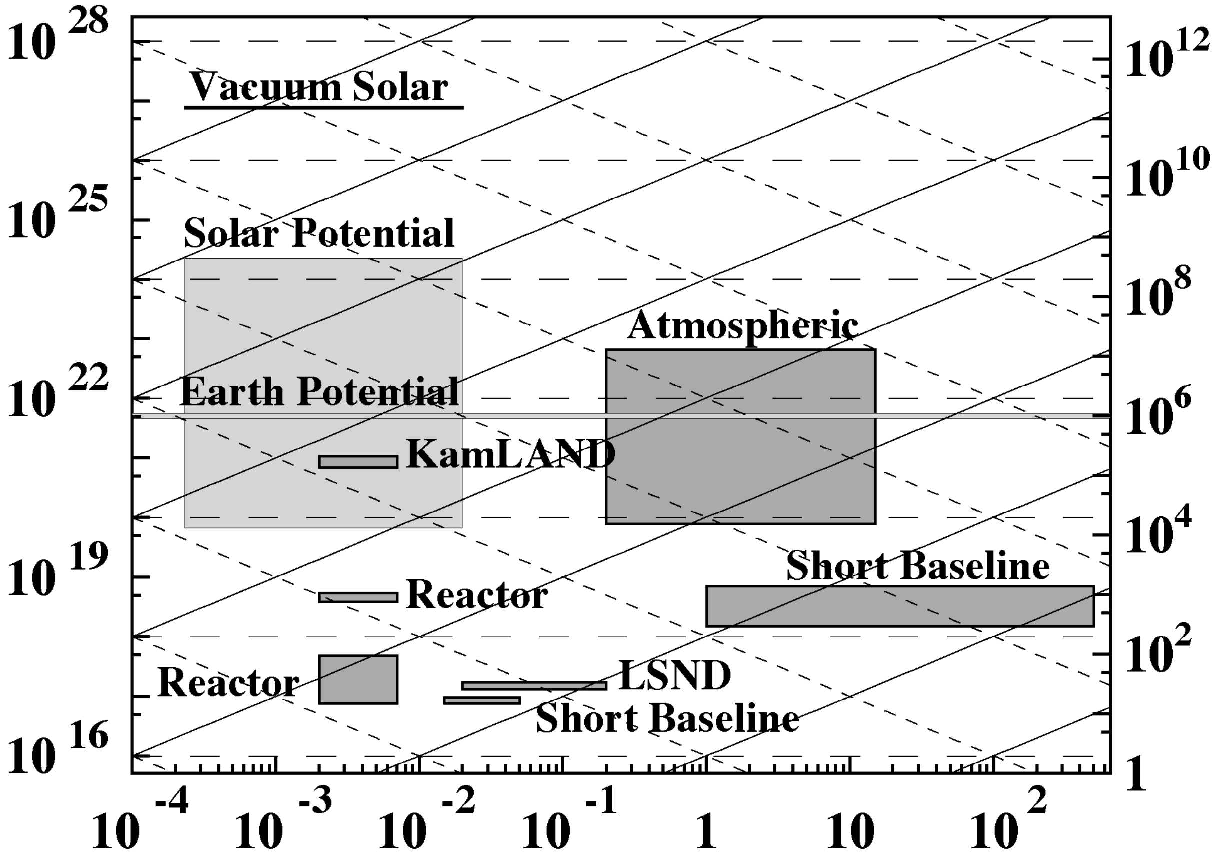


Figure 1: Approximate experimental sensitivities. Lines of constant L/E (solid), L (dashed), and LE (dotted) are shown, which give sensitivities to Δm^2, a_L, and c_L, respectively. (Kostelecky & Mews hep-ph/0308300)

Lorentz Invariance & ν spin-flavor conversion:

Loop Gravity Example: (Alfaro, Morales-Tecotl and Urrutia 2000, 2002) : $E_\pm^2 = A_p^2 p^2 + \eta p^4 \pm 2\lambda p + m^2$

$A_p = 1 + \kappa_1 \frac{\ell_P}{\mathcal{L}}$, $\eta = \kappa_3 \ell_P^2$, $\lambda = \kappa_5 \frac{\ell_P}{2\mathcal{L}^2}$, and $\mathcal{L} \sim E^{-1}$ or const., a characteristic scale.

Weak interaction Effects of ν propagating in a medium (Mikheyev Smirnov Wolfenstein effect): energy shift $\sqrt{2}G_F(2n_e - n_n)$. Plus interaction with external magnetic field B via radiatively induced magnetic moment μ: $\mu\overline{\psi}\sigma^{\mu\nu}F_{\mu\nu}\psi$.

SPIN-FLAVOUR CONVERSION:

$$i\frac{d}{dr}\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{eR} \\ \nu_{\mu R} \end{pmatrix} = \mathcal{H}\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{eR} \\ \nu_{\mu R} \end{pmatrix},$$

$\mathcal{H} =$ effective. Hamiltonian $\ni$ λ-effects.

(NB: For Majorana ν replace: $\nu_{iL} \to \nu_i$, $\nu_{iR} \to \overline{\nu}_i$).

Resonant Condition for Flavour-Spin-flip:

$$\nu_{eL} \to \nu_{\mu R} : \quad 2\lambda + \frac{\Delta m^2}{2p}\cos 2\theta - \sqrt{2}G_F n_e(r_{res}) = 0$$

$$\nu_{\mu L} \to \nu_{eR} : \quad 2\lambda - \frac{\Delta m^2}{2p}\cos 2\theta - \sqrt{2}G_F n_e(r_{res}) = 0$$

Lorentz Invariance & ν spin-flavor conversion:

Use above conditions to obtain bounds for λ, κ_i via:

$$P_{\nu_{eL}\to\nu_{\mu R}} = \frac{1}{2}(1 - \cos2\tilde{\theta}\cos2\theta),$$

$$\tan2\tilde{\theta}(r) = \frac{4\mu B(r)E}{|\Delta m^2|\cos2\theta - 4E\lambda + 2\sqrt{2}G_F E n_e(r)}.$$

Assumptions: Reasonable profiles for solar $n_e \sim n_0 e^{-10.5r/R_\odot}$, $n_0 = 85 N_A \mathrm{cm}^{-3}$. Also: $\mu \sim 10^{-11}\mu_B$.

Upper bound on λ:

$$\lambda \leq \frac{1}{2}\left(10^{-12}e^{-10.5r_{res}/R_\odot}\mathrm{eV} + \frac{|\Delta m^2|}{2E}\right)$$

- (i) $\mathcal{L}$=universal constant:

  Photon dispersion GRB, AGN tests: $\mathcal{L} \sim 10^{-18}$ eV^{-1}

  Best fit SOLAR ν-oscillations induced by MSW, use expt values of Δm^2, $\sin^2 2\theta$, and bind κ_i: $\kappa_5 < 10^{-25}$.

  From ATMOSHERIC oscillations, in particular LSND experiment, $\nu_\mu \to \nu_e$ fits the data with: $|\Delta m^2| \sim eV^2$, $\sin^2 2\theta \sim (0.2 - 3) \times 10^{-3}$, $E_{\max} \sim 10$ MeV, then $\kappa_5 < 10^{-17}$.

- (ii) $\mathcal{L} \sim p^{-1}$ (mobile scale) : From SOLAR oscillations, with $p \sim 1 - 10\mathrm{MeV}$. one gets $\kappa_5 = \mathcal{O}(1 - 100)$, natural range of values.

  From ATMOSHERIC oscillations,for the maximum ν $E \sim 10$ MeV, and $\mathcal{L} \sim E^{-1}$, one gets $\kappa_5 \sim 10^4$ (very weak bound). **CONCLUSION: (ii) is favoured.**

ν-mixing & modified Lorentz Invariance (LI):

A Peculiar way flavour ν states experience LI?

Deformed dispersion relations for ν flavor states (Blasone, Magueijo & Pires-Pacheco (BMP) (2003)): flavor states are *superposition of mass eigenstates* with standard d.r. of *different mass.*

$E_e =< \nu_e|H|\nu_e >= \omega_{k,1}\cos^2\theta + \omega_{k,2}\sin^2\theta$

$E_\mu =< \nu_\mu|H|\nu_\mu >= \omega_{k,2}\cos^2\theta + \omega_{k,1}\sin^2\theta$

$H|\nu_i >= \omega_i|\nu_i >$, $\omega_{k,i} = \sqrt{\vec{k}^2 + m_i^2}$. sum of two square roots in not in general a square root, hence modified d.r. for flavour states → LI?

Idea of BMP: Avoid using preferred frame by non-linear modified Lorentz transformations to ensure observer independence (Amelino-Camelia, Magueijo & Smolin):

$E_i^2 f_i^2(E_i) - \vec{k}^2 g_i^2(E_i) = M_i^2 \quad i = e, \mu$

Determine m(odified). d.r. $f_i(E_i, \theta, m_i), g_i(E_i, \theta, m_i), M_i(m_i, \theta)$ by comparing with $E_i = E_(\omega_i, m_i)$ above.

Identify non-linear Lorentz trnsf that leaves the m.d.r. invariant: $U \circ (E, \vec{k}) = (Ef, \vec{k}g)$.

ν-mixing & LI: Experiment

Test these ideas experimentally? β DECAY

Energy conservation in β-decay: $N_1 \rightarrow N_2 + e^- + \bar{\nu}_e$, (e.g. $N_1 =^3 H$, $N_2 =^3 He$)

Non-linear LI case:

$E_{N_1} = E_{N_2} + E + E_e f_e(E_e)$, ($E$=energy of e) vs

Preferred frame case: $E_{N_1} = E_{N_2} + E + E_e$,

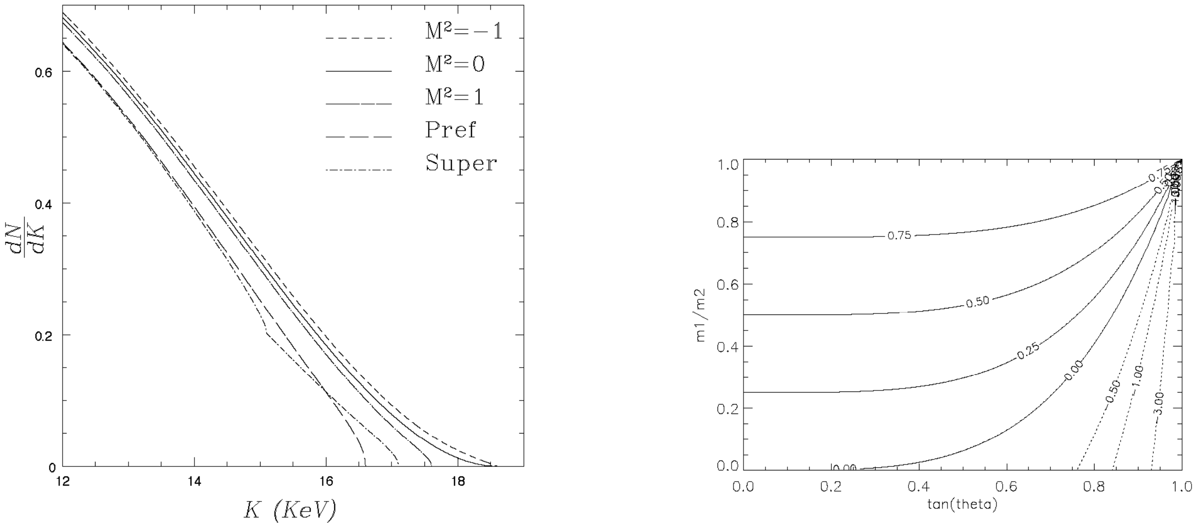


Figure 2: Left: Tail of tritium β-decay spectrum, for massless ν (solid) and for LI flavour states (dashed and dot-long-dashed). Also plotted is the preferred frame case. Right: Likelihood Contours of M^2 (in units of m_2^2) upon which β-decay depends.

PHENOMENOLOGY of CPTV (cont'd):

HOW CAN WE DETECT CPT VIOLATION? :
Departure from Quantum Mechanical evolution (QMV): QG Decoherence (c.f. open systems -gravitational 'environment')
Not necessarily Lorentz Violating (Millburn 2003)

SENSITIVE PARTICLE PHYSICS PROBES of QMV:

(i) neutral kaons and B-mesons (Ellis, Hagelin, Nanopoulos, Srednicki, (1984), + NM, Lopez (1992-95)), and ϕ-, B-factories (novel CPT tests for EPR states) (Bernabeu, NM, Papavassiliou 2003)

(ii) ultracold (slow) neutrons in Earth's gravitational field ?

$\star$ (iii) Neutrino flavour mixing ?

QG Decoherence and neutrino mixing

Quantum Gravity (QG) may induce oscillations between neutrino flavours independently of masses (Liu et al., 1997, Chang et al., 1998, Lisi et al., Benatti & Floreanini 2000).

$$\partial_t \rho = i[\rho, H] + \delta \not{H} \rho$$

where (Ellis, Hagelin, Nanopoulos, Srednicki 1984)

$$\delta \not{H}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -2\alpha & -2\beta & 0 \\ 0 & -2\beta & -2\gamma & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

for energy and lepton number conservation. and

$$\delta \not{H}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2\alpha & -2\beta \\ 0 & 0 & -2\beta & -2\gamma \end{pmatrix}$$

if energy and lepton number violated, but flavour conserved (σ_1 Pauli matrix).

Positivity of ρ requires: $\alpha, \gamma > 0, \qquad \alpha\gamma > \beta^2$.

α, β, γ **violate CPT** (Ellis, NM, Nanopoulos 1992, Lopez + EMN 1995).

QG Decoherence and neutrino mixing

OSCILLATION PROBABILITIES

(i) For flavour conserving case:

Example: $\nu_e \to \nu_x$ $(x = \mu, \tau$ or sterile):

$$P_{\nu_e \to \nu_x} = \frac{1}{2} - \frac{1}{2}e^{-\gamma L}\cos^2 2\theta_v - \frac{1}{2}e^{-\alpha L}\sin^2 2\theta_v \cos(\frac{|m^2_{\nu_1} - m^2_{\nu_2}|}{2E_\nu}L)$$

Here L is oscillation length, θ_v is the mixing angle:
$\theta_v \neq 0 \longleftrightarrow$ massless ν's,

in mass basis: $|\nu_e >= \mathrm{cos}\theta_v|\nu_1 > + \mathrm{sin}\theta_v|\nu_2 >$,
$|\nu_\mu >= -\mathrm{sin}\theta_v|\nu_1 > + \mathrm{cos}\theta_v|\nu_2 >$.

NB: flavour oscillations even in massless case, due to γ compatible with flavour conserving formalism:

$$< \nu_e|\sigma_1|\nu_e >= - < \nu_\mu|\sigma_1|\nu_\mu >= 2\mathrm{sin}\theta_v\mathrm{cos}\theta_v.$$

NB: $P_{\nu_e \to \nu_x}(t \to \infty) = \frac{1}{n}$, for n generations.

QG Decoherence and neutrino mixing

OSCILLATION PROBABILITIES

(ii) For Energy and Lepton number conserving case:

Example: $\nu_e \longrightarrow \nu_x \quad (x = \mu, \tau$ or sterile):

$$P_{\nu_e \to \nu_x} = \frac{1}{2}\sin^2 2\theta_v \left(1 - e^{-(\alpha+\gamma)L}\cos(\frac{|m_{\nu_1}^2 - m_{\nu_2}^2|}{2E_\nu}L)\right)$$

assuming $\alpha, \beta, \gamma \ll \frac{|m_{\nu_1}^2 - m_{\nu_2}^2|}{2E_\nu}$.

NB: $P_{\nu_e \to \nu_x}(t \to \infty) = \frac{1}{n}\sin^2 2\theta_v$, for n generations, **in contrast with case (i) above**.

EXPERIMENTAL SENSITIVITY

Use results (i) & (ii) to bind experimentally $\xi \equiv \{\alpha, \beta, \gamma\}$.

Theoretical Models Predictions vs. Experiment:

Optimistic: (Ellis, NM, Nanopoulos, ...)
$\xi \sim \xi_0(\frac{E}{\mathrm{GeV}})^n, n = 0, 2, -1.$

Pessimistic: (Adler 2000) $\xi \sim \frac{(\Delta m^2)^2}{E^2 M_{qg}}$,
($M_{qg} \sim M_P \sim 10^{19}$ **GeV).**

QG Decoherence and neutrino mixing

In some models of QG Decoherence, complete positivity of $\rho(t)$ for composite systems, such as ϕ or B mesons, may be imposed (Benatti & Floreannini). This results in ideal Markov environment, with:

$\alpha = \beta = 0, \gamma > 0.$

If this model assumed for ν (Lisi et all., PRL 85 (2000), 1166), then phenomenological parametrisation:

$\gamma = \gamma_0 (E/\mathrm{GeV})^n$, $n = 0, 2, -1.$

with E the neutrino energy.

From Atmosperic ν data → Bounds:

$n = 0$, $\gamma_0 < 3.5 \times 10^{-23}$ GeV

$n = 2$, $\gamma_0 < 0.9 \times 10^{-27}$ GeV (c.f. CPLEAR bound for Kaons: $\gamma < 10^{-21}$ GeV (PLB364 (1995) 239))

$n = -1$, $\gamma_0 < 2 \times 10^{-21}$ GeV.

NB: Tests on ν-mixing from Decoherence exhibit much greater sensitivity than neutral mesons.

QG Decoherence and neutrino mixing

FITTING THE DATA (Lisi et al. PRL 85 (2000), 1166)

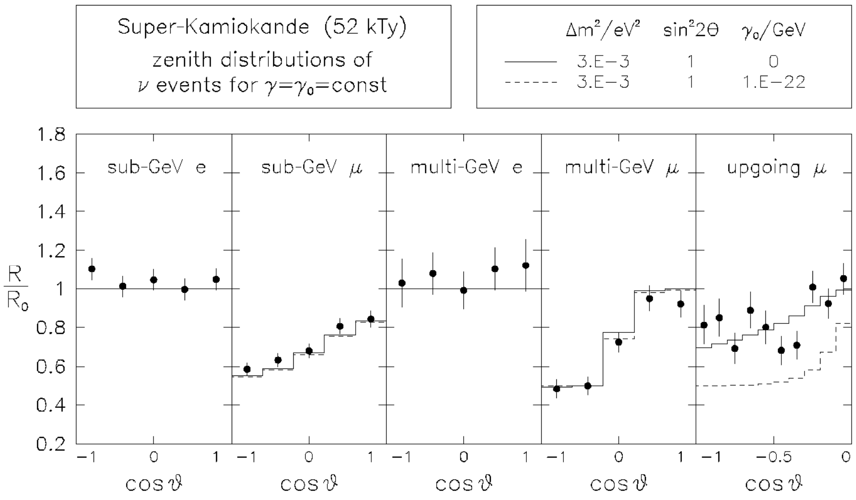


Figure 3: Effects of decoherence $(\gamma = \gamma_0 = \text{const} \neq 0)$ on the distributions of lepton events as a function of the zenith angle ϑ

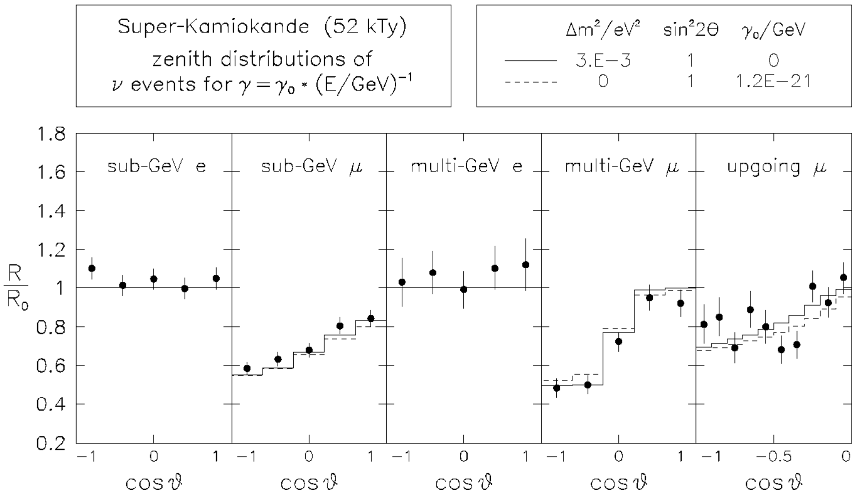


Figure 4: Best-fit scenarios for pure oscillations $(\gamma = 0)$ (solid line) and for pure decoherence with $\gamma \propto 1/E$ (dashed line).

PHENOMENOLOGY of CPTV (cont'd):

HOW CAN WE DETECT CPT VIOLATION? (Cont'd)

***Departure from Locality (and Neutrinos)**

- (v) Neutrino Physics Anomalies (LSND) and CPT Violation: Instead of Sterile neutrino introduce CPT violation (CPTV) in neutrino sector.
- Existing models (Barenboim and Lykken 2001) violate LOCALITY (and probably Lorentz symmetry **OFF-SHELL** (Greenberg) 2002.)
- Current ExperimentaL Situation:
  WMAP + all other data on neutrinos, including KamLAND: Simplest model of CPTV with Dirac neutrino masses is **marginally ruled out**?
- But this is **only one way of CPTV**. Remember departure from Quantum Mechanical Evolution (QMV) (as in SOME QG models) QMV-CPTV may induce neutrino oscillations and CPTV not related to masses (Fogli, Lisi, Marrone 2000).
- stringent limits on CPTV-ν by $\beta\beta$-decay if true (Klapdor-Kleingrothaus, Paes and Sarkar 2000)

NEUTRINO ANOMALIES & CPTV

Interpreting the LSND anomaly as CPT-violation in the neutrino sector

LSND anomaly : observed oscillations in $\bar{\nu}_\mu - \bar{\nu}_e$

(initial 2.6σ hint for $\nu_\mu - \nu_e$ decreased to 0.6σ)

Proposal of Murayama and Yanagida (hep-ph/0010178) (see also: Barenboim + Lykken, Barenboim *et al.* hep-ph/0108199):

CPT-Violating (CPTV) (Dirac) Mass spectrum* : $m_\nu \neq m_{\bar{\nu}}$.

Expts like MiniBooNE (looks for $\nu_\mu \to \nu_e$) can directly test such hypotheses in the immediate future.

If CPTV then signals in atmospheric & solar ν oscillations.

Recent results from KamLAND (hep-ex/0212021) + ATMOSPHERIC DATA

$\rightarrow$ disfavour marginally CPTV-Dirac mass scenario (A. Strumia, hep-ph/0201134 v4 (April 03), Addendum).

CPTV-ν-mass scenario and LSND anomaly

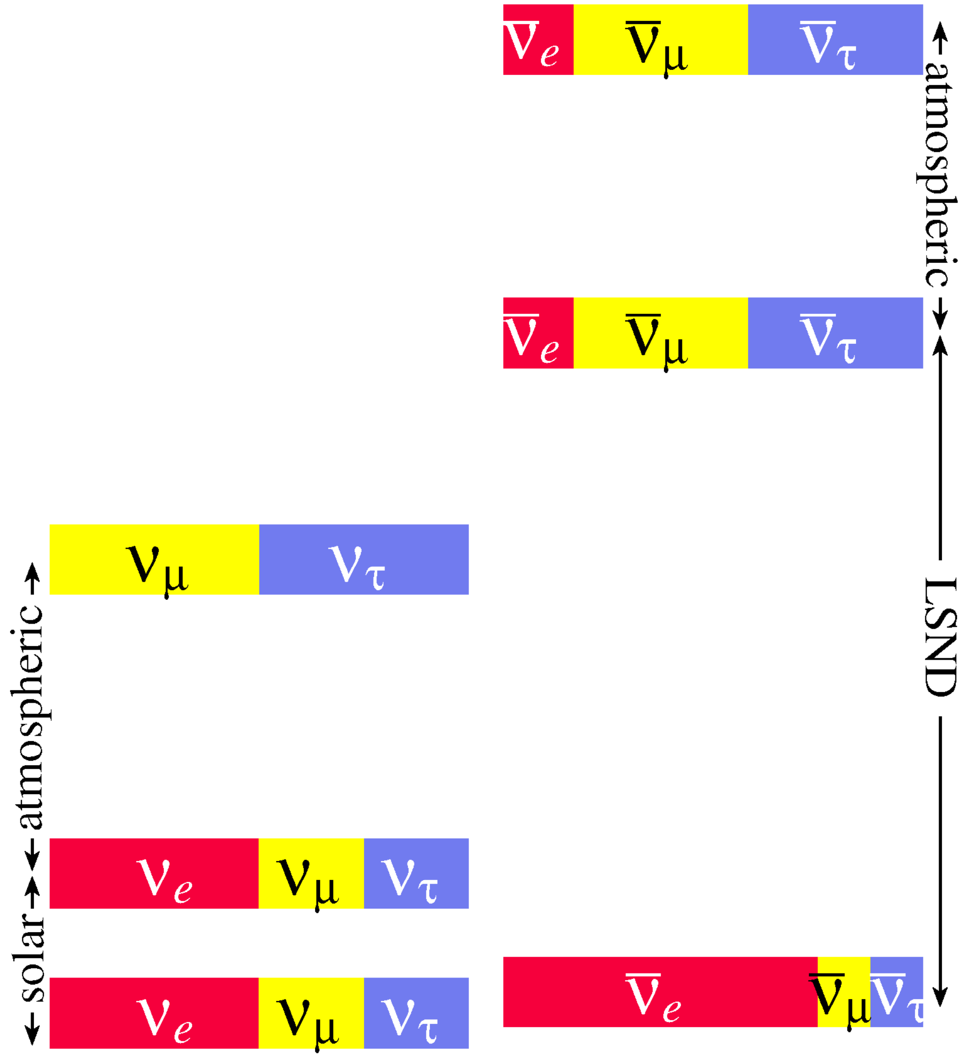


Figure 5: The CPTV spectrum proposed by Murayama-Yanagida. To explain LSND we need $m_\nu^2 - m_{\bar{\nu}}^2 \sim 0.1 \text{ ev}^{-2} = 10^{-19} \text{ GeV}^2$.

$\star$ ν CPTV (Dirac) Mass Spectrum

(Barenboim, Borisov, Lykken, Smirnov hep-ph/0108199)

Relax Locality (but maintain Lorentz Invariance), hence CPTV natural.

Dirac Mass: $m_i = \tan\beta \overline{m}_i$, $i = 1, 2, 3$ (three neutrino flavours) $\tan\beta = 0$ or $\cot\beta = 0$ MAXIMAL CPTV. $\tan\beta = 1$ CPT conserved.

Explain LSND without Sterile ν.

why CPTV only in ν? Brane world scenaria (ν can propagate in the bulk like gravity ?)

No knowledge of mixing matrix

Specific Model (BARENBOIM, LYKKEN hep-ph/0210041): "Homeotic ν" (like Dirac theory both +ve,-ve energies):

$$\psi_+(x) = u_+(p)e^{-ip\cdot x}, \quad p^2 = m^2,\ p_0 > 0$$

$$\psi_-(x) = u_-(p)e^{-ip\cdot x}, \quad p^2 = m^2,\ p_0 < 0$$

$$(p_\mu \gamma^\mu - m\epsilon(p_0))u_\pm(p) = 0,\ \ \epsilon(p_0) = \text{sign function}$$

$$S = \int d^4x \bar{\psi} i\partial_\mu \gamma^\mu \psi + \frac{im}{2\pi}\int d^3x dt dt' \bar{\psi}(t)\frac{1}{t-t'}\psi(t')$$

NB: Lorentz invariance maintained ON SHELL only. $(\epsilon(p_0))$ but Locality relaxed. OFF-SHELL LV (Greenberg 2002)

Fit of SK & K2K Data

(A. Strumia, hep-ph/0201134 v4, addendum.)

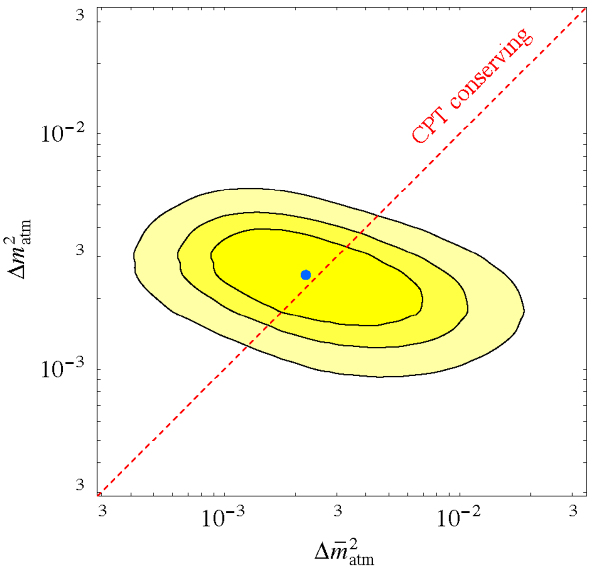


Figure 6: Atmospheric $m_\nu - m_{\overline{\nu}}$ (68, 90, 99 %, 2 d.o.f.)

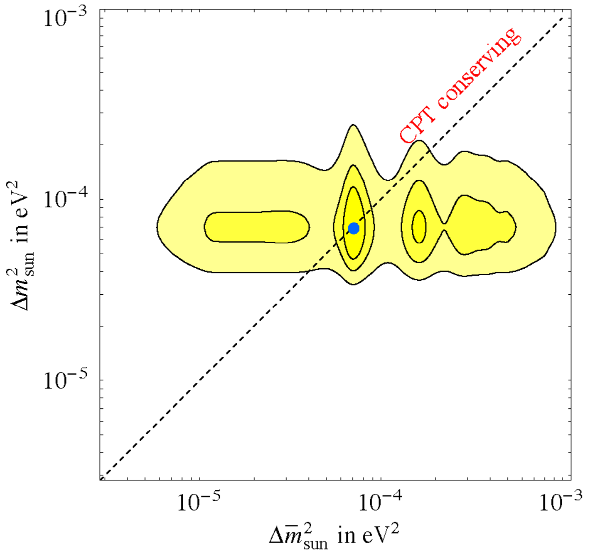


Figure 7: For solar & reactor data (68, 90, 99 %, 2 d.o.f.)

CPTV mass scenario marginally disfavoured?

WMAP Data and Neutrinos

Cosmology (WMAP): $\sum m_\nu < 0.69$ eV (95% CL, 1 d.o.f.) (one can include sterile ν)

$$P_{\nu_e \to \nu_e} = 1 - S\sin^2\theta_{es}, \quad P_{\nu_\mu \to \nu_\mu} = 1 - S\sin^2\theta_{\mu s},$$

$$P_{\nu_e \to \nu_\mu} = S\sin^2\theta_{LSND} \ (\sin^2\theta_{LSND} \simeq \tfrac{1}{4}\sin^2 2\theta_{es}\sin^2 2\theta_{\mu s})$$

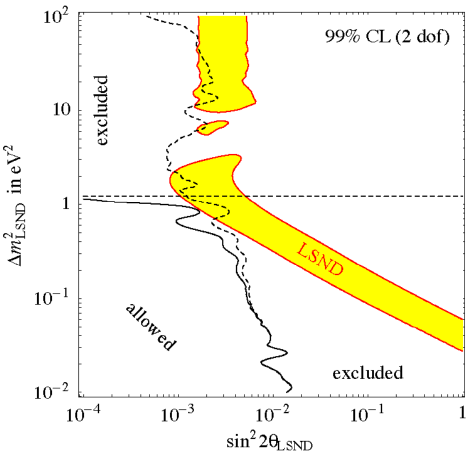


Figure 8: Upper half plane disfavoured by WMAP. Dashed curved line: upper bound from all other ν expts.

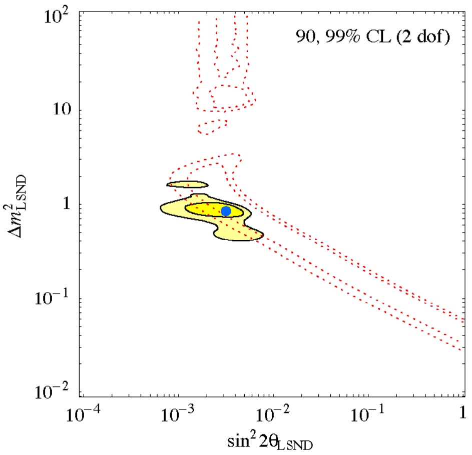


Figure 9: Best fit , all data; 3 + 1 sterile ν solution (CPT cons.) disfavoured by WMAP since $(\Delta m^2_{LSND})^{1/2} \simeq \sum m_\nu$.

(A. Strumia, hep-ph/0201134 v4, addendum.)

Data Summary and Interpretations

model & no. of free parameters		$\Delta\chi^2$	mainly incompatible with	main future test
ideal fit (no known model)		0		?
$\Delta L = 2$ decay $\bar{\mu} \to \bar{e}\bar{\nu}_\mu\bar{\nu}_e$	6	12	KARMEN	TWIST
$3+1: \ \Delta m^2_{\rm sterile} = \Delta m^2_{\rm LSND}$	9	$6+9?$	BUGEY + cosmology?	MINIBOONE
3 ν and CPTV (no $\Delta\bar{m}^2_{\rm sun}$)	10	15	KamLAND	KAMLAND
3 ν and CPTV (no $\Delta\bar{m}^2_{\rm atm}$)	10	25	SK atmospheric	$\bar{\nu}_\mu$ LBL?
normal 3 neutrinos	5	25	LSND	MINIBOONE
$2+2: \ \Delta m^2_{\rm sterile} = \Delta m^2_{\rm sun}$	9	30	SNO	SNO
$2+2: \ \Delta m^2_{\rm sterile} = \Delta m^2_{\rm atm}$	9	50	SK atmospheric	ν_μ LBL

Table 1: Interpretations of solar, atmospheric and LSND data, ordered according to the quality of their global fit. A $\Delta\chi^2 = n^2$ roughly signals an incompatibility at n standard deviations. (A. Strumia, hep-ex/0304039)

What about four $(3+1$ or $2+2)$ ν and CPTV ?

Four ν + CPTV

(Barger, Marfatia & Whisnant 2003)

CPTV $\longrightarrow$ different flavor mixing between ν, $\bar{\nu}$:

$\nu_a = \sum_{i=1}^{4} U^*_{ai}\nu_i, \qquad \bar{\nu}_a = \sum_{i=1}^{4} \bar{U}_{ai}\bar{\nu}_i,$

with $U \ne \bar{U}$ due to CPTV.

- 3 + 1 models: one ν mass well separated from others, sterile ν couples only to isolated state.

  Oscillation probs: $P_{\nu_i \to \nu_i}(|U_{ij}|^2) \ne P_{\bar{\nu}_i \to \bar{\nu}_i}(|\bar{U}_{ij}|^2)$

  Bugey binds $|\bar{U}_{e4}|$ and CDHSW binds $U_{\mu 4}$ but no tight constraints for $|\bar{U}_{\mu 4}|$, U_{e4}. (Contrast with $(3+1)\nu$ CPT conserving models where $U = \bar{U}$.).

  Hence $(3+1)\nu$ + CPTV still viable.

- 2 + 2 models: sterile ν couples to solar and atmospheric ν oscillations. This structure is only permitted in $\bar{\nu}$ sector. Even with CPTV 2+2 models strongly disfavoured by data.

3+1 ν + CPTV

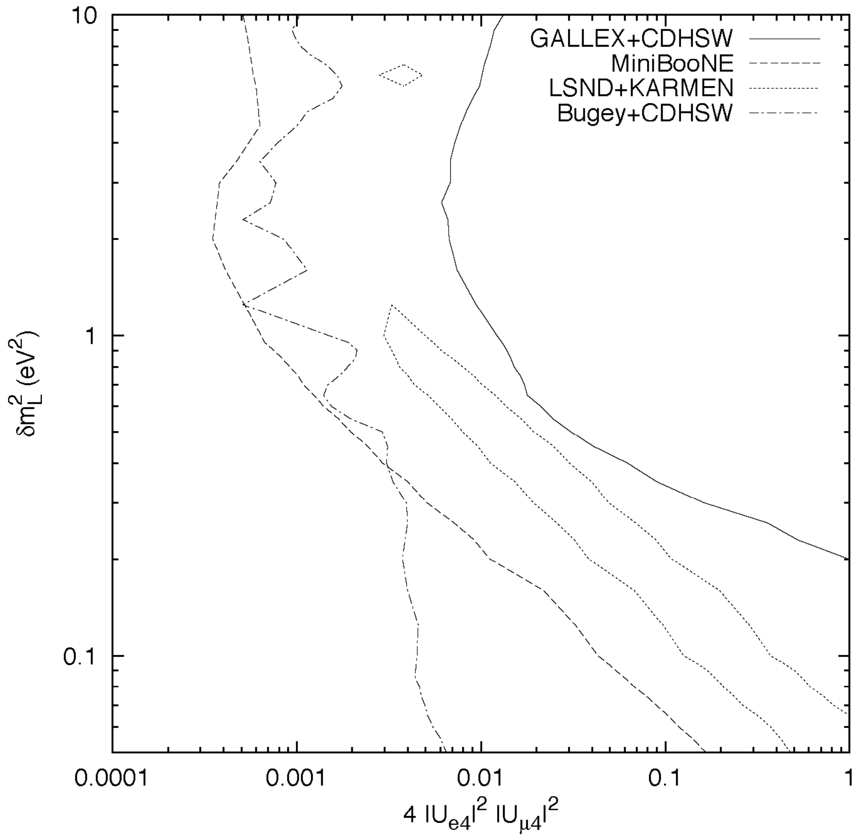


Figure 10: Upper bound (solid) on the $\nu_\mu \longrightarrow \nu_e$ oscillation amplitude $4|U_{e4}|^2|U_{\mu 4}|^2$ from the GALLEX limit on $|U_{e4}|$ and the CDHSW limit on $|U_{\mu 4}|$ (90% C. L. results are used in both cases). The dot-dashed line is the 99% C. L. upper bound from Bugey and CDHSW if CPT is conserved. Also shown are the expected sensitivity (dashed) of the MiniBooNE experiment and, for comparison, the allowed region (within the dotted lines) for $4|\bar{U}_{e4}|^2|\bar{U}_{\mu 4}|^2$ from a combined analysis of LSND and KARMEN data, both at the 90% C. L .

3+1 $\bar{\nu}$ + CPTV

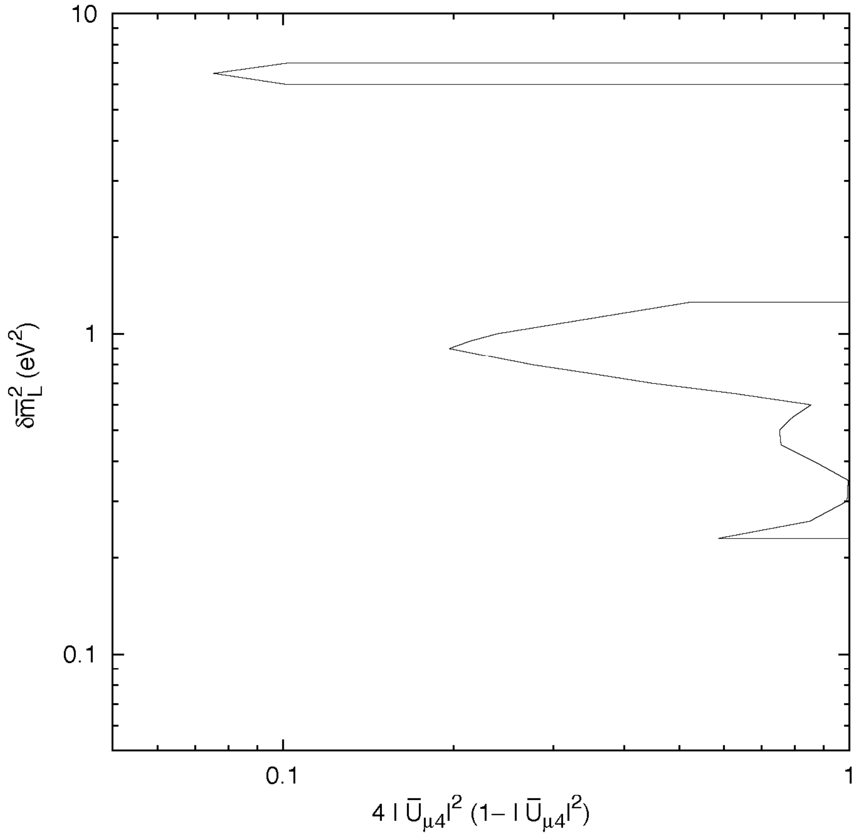


Figure 11: Lower bounds on $4|\bar{U}_{\mu 4}|^2(1-|\bar{U}_{\mu 4}|^2)$ (the amplitude for atmospheric $\bar{\nu}_\mu$ survival at the LSND mass scale) from the Bugey limit on $\bar{\nu}_e$ disappearance and the $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation amplitude indicated by LSND and KARMEN (90% C. L. results are used in both cases).

2+2 ν + CPTV

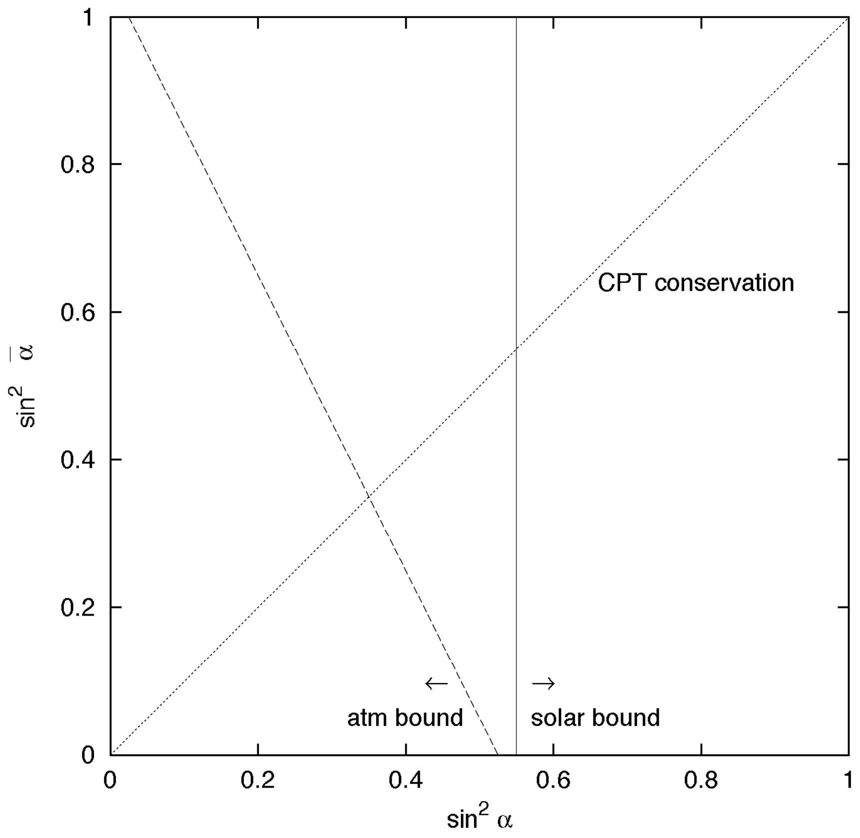


Figure 12: Constraints on sterile neutrino mixing angles α and $\bar{\alpha}$ from solar (solid) and atmospheric (dashed) data. The dotted line is the prediction if CPT is conserved.

Three ν Generations, Decoherence and LSND

G. Barenboim & N.M. (March 2004)

Generic Decohering Evolution:

$$\frac{\partial \rho_\mu}{\partial t} \sum_{ij} h_i \rho_j f_{ij\mu} + \sum_{\nu} \mathcal{L}_{\mu\nu} \rho_\mu \ ,$$

$$\mu, \nu = 0, \ldots N^2 - 1, \quad i, j = 1, \ldots N^2 - 1 \qquad (1)$$

for N-level systems. For us $N = 3$, f_{ijk} structure constants of SU(3).

Entropy increase requirement:

$$\mathcal{L}_{0\mu} = \mathcal{L}_{\mu 0} = 0 \ ,$$

$$\mathcal{L}_{ij} = \frac{1}{2} \sum_{k,\ell,m} b_m^{(n)} b_k^{(n)} f_{imk} f_{\ell kj} \ ,$$

with the notation $b_j \equiv \sum_\mu b_\mu^{(j)} \mathcal{J}_\mu$, b_i Lindblad (entanglement) operators, $\mathcal{J}_\mu$, $\mu = 0, \ldots 8(3)$ be a set of SU(3) generators.

Three ν Generations, Decoherence and LSND

A.M. Gago et al., hep-ph/0208166.

In terms of the generators $\mathcal{J}_\mu$, $\mu = 0, \ldots 8$ of the SU(3) group, the Hamiltonian $H_{\rm eff}$ can be expanded as :

$$\mathcal{H}_{\rm eff} = \frac{1}{2p}\sqrt{2/3}\left(6p^2 + \sum_{i=1}^{3} m_i^2\right)\mathcal{J}_0 + \frac{1}{2p}(\Delta m_{12}^2)\mathcal{J}_3 + \frac{1}{2\sqrt{3}p}\left(\Delta m_{13}^2 + \Delta m_{23}^2\right)\mathcal{J}_8 \ , \tag{2}$$

with $\Delta m_{ij}^2 = m_i^2 - m_j^2$, $i, j = 1, 2, 3.$

Assume diagonal form for decoherence matrix $\mathcal{L}$ in (1):

$$[\mathcal{L}_{\mu\nu}] = {\rm Diag}\left(0, -\gamma_1, -\gamma_2, -\gamma_3, -\gamma_4, -\gamma_5, -\gamma_6, -\gamma_7, -\gamma_8\right)$$

in direct analogy with the two-level case of complete positivity (Lisi et al., Benatti-Floreanini).

No strong physical motivation behind such restricted form; leads to the simplest possible decoherence models; will be used to fit neutrino data. If successful (will do!) just adds more in favor of decoherence models due to restricted number of available parameters for the fit.

Transition Probabilities

Probability conservation: decouples $\rho_0(t) = \sqrt{2/3}$.

Evolution can be written: $\partial_t \rho = \mathcal{M}_{ij}\rho_j$

$$P(\nu_\alpha \to \nu_\beta) = \mathrm{Tr}[\rho^\alpha(t)\rho^\beta] = \frac{1}{3} + \frac{1}{2}\sum_{i,k,j} e^{\lambda_k t}\mathcal{D}_{ik}\mathcal{D}^{-1}_{kj}\rho^\alpha_j(0)\rho^\beta_i \qquad (3)$$

where $\alpha, \beta = e, \mu, \tau$, and Latin indices: $1, \ldots 8$. λ_k are the eigenvalues of the matrix $\mathcal{M}$ in evolution (1). The matrices $\mathcal{D}_{ij}$ are the matrices that diagonalize $\mathcal{M}_{ij}$. λ_k depend on both the decoherence parameters γ_i and the mass differences Δm^2_{ij}, e.g.
$\lambda_1 = \frac{1}{2}[-(\gamma_1 + \gamma_2) - \sqrt{(\gamma_2 - \gamma_1)^2 - 4\Delta^2_{12}}]$, with $\Delta_{ij} \equiv \Delta m^2_{ij}/2p$, $i, j = 1, 2, 3$.

Generic feature λ_k to depend on:

$$\begin{aligned} \Omega_{12} &= \sqrt{(\gamma_2 - \gamma_1)^2 - 4\Delta^2_{12}} \\ \Omega_{13} &= \sqrt{(\gamma_5 - \gamma_4)^2 - 4\Delta^2_{13}} \\ \Omega_{23} &= \sqrt{(\gamma_7 - \gamma_6)^2 - 4\Delta^2_{23}} \end{aligned} \qquad (4)$$

Distinguish two cases according to the relative magnitudes of Δ_{ij} and $\Delta\gamma_{kl} \equiv \gamma_k - \gamma_l$: (i) $2|\Delta_{ij}| \geq |\Delta\gamma_{k\ell}|$, ((3) $\ni$ sine and cosine), and (ii) $2|\Delta_{ij}| < |\Delta\gamma_{k\ell}|$ ((3) $\ni$ sinh and cosh).

Neutrino Mixing

Assuming mixing between the flavors, amounts to expressing neutrino flavor eigenstates $|\nu_\alpha >$, $\alpha = e, \mu, \tau$ in terms of mass eigenstates $|\nu_i >$, $i = 1, 2, 3$ through a (unitary) matrix U:

$$|\nu_\alpha >= \sum_{i=1}^{3} U^*_{\alpha i} |\nu_i > \quad .$$

This implies:

$$\rho^\alpha = |\nu_\alpha >< \nu_\alpha| = \sum_{i,j} U^*_{\alpha i} U_{\alpha j} |\nu_i >< \nu_j| \ .$$

From this: $\rho^\alpha_\mu = 2\mathrm{Tr}(\rho^\alpha \mathcal{J}_\mu)$.

Transition Probabilities & Mixing

Barenboim & N.M. (2004):

$$\bar{\gamma}_i = \bar{\gamma}_{i+1} \text{ for } i = 1,4,6 \qquad \text{and} \qquad \bar{\gamma}_3 = \bar{\gamma}_8$$

$$P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} = \frac{1}{3} + \frac{1}{2}\left\{\rho_1^\alpha \rho_1^\beta \cos\left(\frac{|\Omega_{12}|t}{2}\right) e^{-\bar{\gamma}_1 t} \right.$$
$$+\rho_4^\alpha \rho_4^\beta \cos\left(\frac{|\Omega_{13}|t}{2}\right) e^{-\bar{\gamma}_4 t} + \rho_6^\alpha \rho_6^\beta \cos\left(\frac{|\Omega_{23}|t}{2}\right) e^{-\bar{\gamma}_6 t}$$
$$\left. +e^{-\bar{\gamma}_3 t}\left(\rho_3^\alpha \rho_3^\beta + \rho_8^\alpha \rho_8^\beta\right)\right\}. \qquad (5)$$

where Ω_{ij} same in both sectors (due to choice of γ_i's).

$$\begin{aligned}
\rho_1^\alpha &= 2\,\mathrm{Re}(U_{\alpha 1}^* U_{\alpha 2}) \\
\rho_3^\alpha &= |U_{\alpha 1}|^2 - |U_{\alpha 2}|^2 \\
\rho_4^\alpha &= 2\,\mathrm{Re}(U_{\alpha 1}^* U_{\alpha 3}) \\
\rho_6^\alpha &= 2\,\mathrm{Re}(U_{\alpha 2}^* U_{\alpha 3}) \\
\rho_8^\alpha &= \frac{1}{\sqrt{3}}(|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2 - 2|U_{\alpha 3}|^2)
\end{aligned} \qquad (6)$$

where the mixing matrices are the same as in the neutrino sector. For the neutrino sector, as there are no decoherence effects, the standard expression for the transition probability is valid.

Three ν Generations, Decoherence and LSND

Parameters of model:

CPT violation is driven by, and restricted to, the decoherence parameters, and hence masses and mixing angles are the same in both sectors, and selected to be

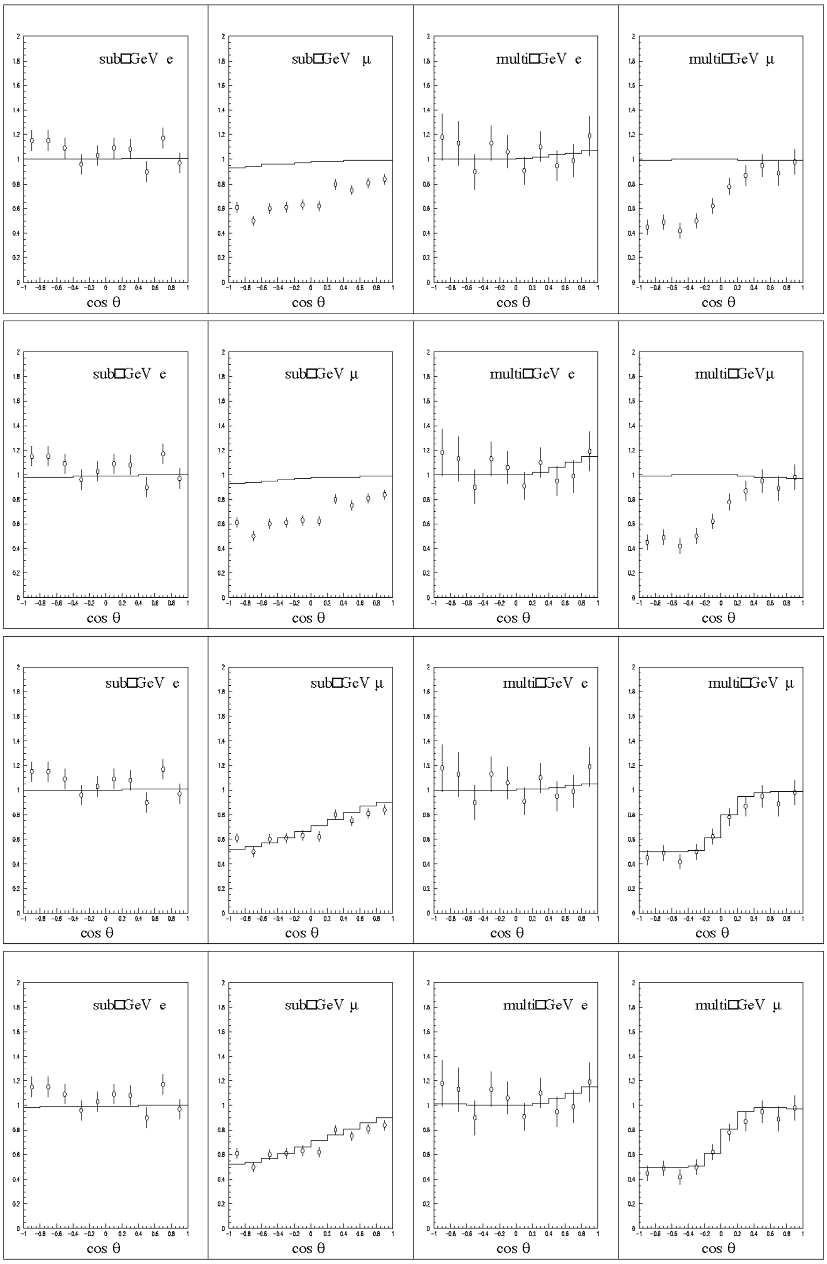
$$\Delta m_{12}^2 = \Delta \overline{m_{12}}^2 = 7 \cdot 10^{-5}\ \mathrm{eV}^2,$$
$$\Delta m_{23}^2 = \Delta \overline{m_{23}}^2 = 2.5 \cdot 10^{-3}\ \mathrm{eV}^2,$$
$$\theta_{23} = \overline{\theta_{23}} = \pi/4,\ \theta_{12} = \overline{\theta_{12}} = .45,$$
$$\theta_{13} = \overline{\theta_{13}} = .05,$$

as indicated by the state of the art analysis.

For the decoherence parameters we have chosen:

$$\overline{\gamma_1} = \overline{\gamma_2} = 2 \cdot 10^{-18} \cdot E \quad \text{and} \quad \overline{\gamma_3} = \overline{\gamma_8} = 1 \cdot 10^{-24}/E\ , \quad (7)$$

where E is the neutrino energy, and barred quantities refer to the antineutrinos. All others vanish.



From top to bottom: (a) pure decoherence in antineutrino sector, (b) pure decoherence in both sectors, (c) mixing plus decoherence in the antineutrino sector only, (d) mixing plus decoherence in both sectors. The dots correspond to SK data. (Barenboim & NM (2004))

χ^2 Values

As bare eye comparisons can be misleading, we have also calculated the χ^2 value for each of the cases, defining the atmospheric χ^2 as

$$\chi^2_{\rm atm} = \sum_{M,S} \sum_{\alpha=e,\mu} \sum_{i=1}^{10} \frac{(R^{\rm exp}_{\alpha,i} - R^{\rm th}_{\alpha,i})^2}{\sigma^2_{\alpha i}} \quad . \qquad (8)$$

Here $\sigma_{\alpha,i}$ are the statistical errors, the ratios $R_{\alpha,i}$ between the observed and predicted signal can be written as

$$R^{\rm exp}_{\alpha,i} = N^{\rm exp}_{\alpha,i} / N^{\rm MC}_{\alpha,i} \qquad (9)$$

(with α indicating the lepton flavor and i counting the different bins, ten in total) and M, S stand for the multi-GeV and sub-GeV data respectively

χ^2 Values

model	χ^2 without LSND	χ^2 including LSND
(a)	980.7	980.8
(b)	979.8	980.0
(c)	52.2	52.3
(d)	54.4	54.6
(e)	53.9	60.7

Table 2: χ^2 obtained for (a) pure decoherence in antineutrino sector, (b) pure decoherence in both sectors, (c) mixing plus decoherence in the antineutrino sector only, (d) mixing plus decoherence in both sectors, (e) standard scenario with and without the LSND result

NB: CPTV Decoherence in antineutrino sector only fits all data, including LSND (then fit is best)!

Theoretical Understanding of Phenomenological model essential. In progress...

Predictions for Upcoming Expts.

Mixing + CPTV Decoherence in $\overline{\nu}$ sector, for three generation models, is the only one scenario (?) that at present fits all data including LSND.

Decoherence in BOTH sectors affects solar ν (seizable unobserved effects, since they are weighted by distance travelled)

Experimental Prospects for Testing...

- KamLAND: look for electron ν survival probability $P_{\overline{\nu}_e \to \overline{\nu}_e} \simeq 0.63$in OUR MODEL:Perfect agreement.
- MiniBOONE: according to OUR MODEL it should confirm LSND only when running in antineutrino sector. Most significant future Expt.
- MINOS: smaller but experimentally accessible signatures may be seen by comparing conjugated channels (muon survival probability)

CONSEQUENCES

SPECULATIONS: IF CPTV DECOHERENVE scenario in $\overline{\nu}$ TURNS OUT TO BE RIGHT...

- Is QG Decoherence in $\overline{\nu}$ of order $\gamma_0 E$, γ_0/E, $\gamma_0 \sim \Delta m^2 \sim 10^{-5}$ eV responsible for a common origin of neutrino, antineutrino mass differences between flavours?
- Dark Energy in the Universe (Cosmological Constant (?)) Notice: the allegedly "observed" value $\Lambda \lesssim 10^{-122} M_p^4$ is CLOSE ENOUGH TO: $(\Delta m^2)^2$ for $\Delta m^2 \sim 10^{-5}$ eV$^2 \sim 10^{-61} M_P$, $M_P \sim 10^{19}$ GeV !!
- Is Λ due to QG decoherence in $\overline{\nu}$?

  Question: If true how can it be that decoherence is large in $\overline{\nu}$ but vanishingly small in ν sector?
- Is also Matter-Antimatter Asymmetry in the Universe due to Decoherence in (anti)neutrino sector?

Some attempt to link all these: Barenboim + NM, hep-ph/0406035

Decoherence in $\overline{\nu}$ and Δm^2

MSW effects: neutrinos in matter $\Delta m^2 \sim G_F n_e k$,

n_e electronic density of medium, G_F Fermi's (weak interaction) constant, k momentum scale of neutrino
Medium discriminates between flavour: only ν_e interact with charged currents.

Idea (Barenboim + NM (2004)): what about QG foam effects? charged black hole antiblack hole pairs can create by Hawking radiation and Hawking absorption (CPT mirror process for antiblack holes) local fluctuations in the density of charged foam particles in the medium. Stochastic fluctuations of these charge densities due to back reaction effects on foam (metric fluctuations)

MSW in stochastic media already studied (Loreti and Balantekin, PRD 50 (1994)).Neutrinos interact with such charge densities of particle emitted by the QG foam, assume stochastic Gaussian medium with density fluctuations about a mean value $n_0(k) \propto k^{-1}$: the higher the momentum the lesser the number of foam particles the ν ($\overline{\nu}$) interacts with.

Gravitational MSW effect: $(\Delta m^2)_{\rm foam} \sim G_N n_0 k$,
$n_0(k)k \sim k$-independent, $G_N \sim 1/M_P^2$.

Stochastic QG fluctuations origin of Δm^2 COMMON in both sectors (average n_0 same in BOTH secotrs)

Foam-Density Fluctuations and CPTV

But there are fluctuations (Gaussian) of $n(r)$:

$\langle n^c_{\rm bh}(r) n^c_{\rm bh}(r')\rangle \sim \Omega^2 n_0^2 \delta(r-r')$, $< n^c_{\rm bh}(r) >= n_0$,

Effective neutrino Hamiltonian will assume the generic form $H_{\rm eff} = H + n^c_{\rm bh}(r) H_I$, where $H_I = G_N J_{fxf}$, is an appropriate constant $f \times f$ matrix, whose entries depend on the details of the foam/neutrino interactions

Evolution of neutrino density matrix in such media:

$$\partial_t \langle\rho\rangle = i[H + n_0 H_I, \langle\rho\rangle] - \Omega^2 n_0^2 [H_I, [H_I, \langle\rho\rangle]] =$$
$$iH^- \langle\rho\rangle - i\langle\rho\rangle H^+ + 2\Omega^2 n_0^2 H_I \langle\rho\rangle H_I,$$

where $H^\pm = H_{\rm eff} \pm i\Omega^2 n_0^2 H_I^2$, and $\langle \ldots \rangle$ indicates average with respect to the stochastic effects.

The Hamiltonian part: space-time foam-induced mass-squared MSW-like splittings for neutrinos (mean field). The double commutator fluctuation decoherence part: is *time irreversible*, unrelated in principle to CP properties, and thus CPT violating. Similar to energy-driven decoherence models (Houghston 1996, Adler 2000)

Foam-Density Fluctuations and CPTV

Due to CPTV:

$$\overline{\Omega} \neq \Omega$$

while maintaining $\langle n^c_{\rm bh} \rangle \equiv n_0$ the same in both sectors.

This is physically meaningful, since it implies that for the same momenta for neutrinos and antineutrinos, and hence the same average number of foam particles they interact with, their back reaction (interaction) with the foam, which causes the foam-particle density fluctuations, is different, as a result of CPT violation.

Decoherence parameters in the antiparticle sector: (in Planck units)

$$\overline{\Omega}^2 G_N^2 n_0^2$$

Reproduce the decoherence FIT to LSND with equality of masses and widths between neutrinos and antineutrinos, but with $\overline{\gamma}_j \sim 10^{-18} \cdot E$ in $\overline{\nu}$ sector, by:

$$\overline{\Omega}^2 \propto 10^{-18} k^3/((\Delta m^2)_{\rm foam})^2 \sim 10^{28} (k/GeV)^3 (GeV)^{-1}.$$

The increase of the fluctuations with the (antineutrino) momentum scale is reasonable, since the higher the momentum, the stronger the back reaction onto the space time foam (but the average number $n_0 \propto k^{-1}$).

Dark Energy and Δm^2

Flavour space and Quantization: some problems

Quantum field theory (QFT) requires infinite volume limit. In contrast to quantum mechanical treatment of fixed volume (Pontecorvo), the neutrino *flavour* states are *orthogonal* to the *energy* eigenstates.

They define two inequivalent vacua related to each other by a *non unitary* transformation $G^{-1}(\theta, t)$ (Blasone, Vitiello 1995):

$$|0(t)\rangle_f = G_\theta^{-1}(t)|0(t)\rangle_m,$$

where θ is the mixing angle, t is the time, and the suffix f(m) denotes flavour(energy) eigenstates.
$G_\theta(t) = \exp\left(\theta \int d^3x[\nu_1^\dagger(x)\nu_2(x) - \nu_2^\dagger(x)\nu_1(x)]\right)$.

Bogolubov transformation connecting the creation and annihilation operator coefficients appearing energy or flavour eigenstates. Of the two Bogolubov coefficients concentrate on the one expressing condensate content of the flavour vacuum, $V_{\vec{k}} = |V_{\vec{k}}|e^{i(\omega_{k,1}+\omega_{k,2})t}$, $\omega_{k,i} = \sqrt{k^2 + m_i^2}$. with ${}_f\langle 0|\alpha^{r\dagger}_{\vec{k},i}\alpha^r_{\vec{k},i}|0\rangle_f ={}_f \langle 0|\beta^{r\dagger}_{\vec{k},i}\beta^r_{\vec{k},i}|0\rangle_f = \sin^2\theta|V_{\vec{k}}|^2$ in the two-generation scenario. For three geerations there are various V_{ij}.

Properties of Flavour Condensate

$|V_{\vec{k}}| = 0$ for $m_1 = m_2$, has a maximum at $k^2 = m_1 m_2$, and for $k \gg \sqrt{m_1 m_2}$

$$|V_{\vec{k}}| \sim \frac{(m_1 - m_2)^2}{4|\vec{k}|^2}, \quad k \equiv |\vec{k}| \gg \sqrt{m_1 m_2}$$

Flavour vacuum $|0\rangle$, is the correct one to be used for vacuum energy contributions, since otherwise the probability is not conserved (Blasone, Henning, Vitiello 1999).

Cosmological Constant and Δm^2

The energy-momentum tensor $T_{\mu\nu}$ of a Dirac fermion field in the Robertson-Walker space-time background can be calculated straightforwardly. The flavour-vacuum average of T_{00} is:

$$
\begin{aligned}
&{}_f\langle 0|T_{00}|0\rangle_f = \langle \rho_{\rm vac}^{\nu-{\rm mix}}\rangle \eta_{00} \equiv \Lambda \eta_{00} \\
&= \sum_{i,r}\int d^3k\omega_{k,i}\left({}_f\langle 0|\alpha_{\vec{k},i}^{r\dagger}\alpha_{\vec{k},i}^{r}|0\rangle_f +_f \langle 0|\beta_{\vec{k},i}^{r\dagger}\beta_{\vec{k},i}^{r}|0\rangle_f\right) \\
&= 8{\rm sin}^2\theta \int_0^K d^3k(\omega_{k,1}+\omega_{k,2})|V_{\vec{k}}|^2.
\end{aligned}
$$

where $\eta_{00} = 1$ in a Robertson-Walker (cosmological) metric background.

Consistent choice of cutoff scale, $K \equiv k_0 = m_1 + m_2$ (Barenboim + NM 2004) compatible with our decoherence-induced mass difference scenario.

Cosmological Constant and Δm^2

For hierarchical neutrino models, i.e. $m_1 \gg m_2 \rightarrow k_0 \gg \sqrt{m_1 m_2}$, modes near the cutoff contribute most to the vacuum energy (divergence),

$$\Lambda \equiv \langle \rho_{\rm vac}^{\nu-{\rm mix}} \rangle \sim 8\pi {\rm sin}^2\theta (m_1 - m_2)^2 (m_1 + m_2)^2 \times \left(\sqrt{2} + 1 + \mathcal{O}(\frac{m_2^2}{m_1^2}) \right) \propto \sin^2\theta (\Delta m^2)^2$$

NB: for $k \sim m_1 + m_2$ (dominant modes for Λ) the induced foam mass splittings $\Delta m^2_{\rm foam} \sim G_N \langle n^c_{\rm bh} \rangle (m_1 + m_2)$ from which $m_1 - m_2 \sim \langle n^c_{\rm bh} \rangle / M_P^2$. If we assume there are $\mathcal{N}_c$ charged foam-induced objects per Planck volume, $V_P \sim M_P^{-3}$ then, $\mathcal{N}_{c,{\rm max}} \sim m_1 - m_2 / M_P$ (small).

Λ and Decoherence

String theory considerations (Mavromatos 2003) suggest that the temporal evolution of the matter density matrix ρ in such a de-Sitter Universe, will be decoherent:

$$\partial_t \rho = i[\rho, H] + :\Lambda g_{\mu\nu}[g^{\mu\nu}, \rho]:,$$

where $\Lambda \sim (\Delta m^2)^2$, and $:\cdots:$ denotes quantum ordering. Antisymmetric ordering yields $[g_{\mu\nu}, [g^{\mu\nu}, \rho]]$, which yields metric variances $(\Delta g_{\mu\nu})^2$, expressing quantum fluctuations of the space time geometry, as a result of the interactions of neutrinos with the foam.

The terms proportional to Λ lead to a decoherent evolution of a pure quantum mechanical state to a mixed one. Wald's Theorem → a strong form of CPT Violation, CPT operator is not well defined, → probably different decoherent parameters between particles and antiparticles (different ways of interaction with the foam)

Hence: $(\overline{\Delta} g_{\mu\nu})^2 \gg (\Delta g_{\mu\nu})^2$. Actually $\Omega^2 \propto (\Delta g_{\mu\nu})^2$ (back reaction effects of foam).

However, in our case, Ω-fluctuation decoherence terms much larger than Λ-induced terms, e.g. in three generation scenaria if $\overline{\Omega}^2_{ij}$ postulated to parametrise $\overline{\gamma}_{1,2}$, then: $\overline{\gamma}_{\Lambda,1} \sim 10^{-18} k^3/M_p^2$ and $\overline{\gamma}_{\Lambda,2} \Delta m^2 k/M_p^2$, i.e. suppressed.

Matter-Antimatter Asymmetry and Decoherence

Sphaleron transitions occurring at and after the electroweak phase transition induce violations of $B + L$, which efficiently wipe out any pre-existing $B + L$ asymmetry. Leptogenesis models evade this problem by generating an early asymmetry in L, which is then converted to a baryon asymmetry by the $B - L$ conserving sphaleron processes.

To avoid sphaleron dilution of $B + L$, and to satisfy the Sakharov conditions for baryogenesis, standard leptogenesis models require strongly out-of-equilibrium processes and new sources of CP violation beyond the Standard Model.

Our model of decoherence on the contrary provides a novel and extremely economical mechanism to generate the observed baryon asymmetry, through a process of equilibrium electroweak leptogenesis (the fact that it violates CPT obviates the need for two of the three Sakharov conditions, namely the requirements of out-of-equilibrium and CP violating processes).

By CPTV we have violations of the index theorem that relates the Chern-Simons winding number of the sphaleron configuration to a change in $B + L$.

Matter-Antimatter Asymmetry and Decoherence

It is difficult to do a precise calculation of this effect, but it is easy to derive an order of magnitude estimate. In Kaon physics the asymmetries between semileptonic decays of K_0 and those of $\overline{K}_0$ turned out to depend linearly on dimensionless decoherence parameters such as $\widehat{\gamma} = \gamma/\Delta\Gamma$; in the parametrization of Ellis, Hagelin, Nanopoulos, Srednicki, where $\Delta\Gamma = \Gamma_L - \Gamma_S$ was a characteristic energy scale associated with energy eigenstates of the kaon system.

In similar spirit, in our case of lepton-antilepton number asymmetries, one expects the corresponding asymmetry to depend, to leading order, linearly on the quantity $\widehat{\gamma} = \gamma/\sqrt{\Delta m^2}$, since the quantity $\sqrt{\Delta m^2}$ is the characteristic energy scale in the neutrino case, playing a role analogous to $\Delta\Gamma$ in the kaon case, but of course no zeroth order terms. Thus, matter-antimatter asymmetry is proportional to the dimensionless decoherence parameter $\widehat{\gamma}$

$$\mathcal{A} = \frac{\langle \nu \rangle - \langle \overline{\nu} \rangle}{\langle \nu \rangle + \langle \overline{\nu} \rangle} \simeq \widehat{\gamma_1} \simeq 10^{-6}$$

where $\widehat{\gamma} \to \widehat{\gamma_1} = 10^{-18} \cdot E/\sqrt{\Delta m^2}$ is the dominant decoherence term. The coefficient 10^{-18} may be thought of as T/M_P with T the temperature, whose value gets frozen at the electroweak symmetry breaking temperature.

Baryon Asymmetry and Decoherence

Thus,

$$B = \frac{n_\nu - \bar{n_\nu}}{s} \sim \frac{\mathcal{A} n_\nu}{g_* n_\gamma}$$

with n_ν $(\bar{n_\nu})$ the number density of (anti) neutrinos, n_γ the number density of photons and g_* the effective number of degrees of freedom (at the temperature where the asymmetry is developed) which depends on the exact matter content of the model but it ranges from 10^2 to 10^3 in our case.

This implies a residual baryon asymmetry of order 10^{-10}, roughly the desired magnitude.

Genuine vs “Fake” CPTV & Decoherence Effects

Important to distinguish: Intrinsic (genuine, due to QG) from Extrinsic (“fake”) CPTV effects due to matter influences.

EXTRINSIC CPTV:

- (i) in neutral mesons: e.g. K^0, $\overline{K}^0$ in regenerator
- (ii) in neutrinos: ν, $\overline{\nu}$ in matter media.

(i) Matter regenerator scatters K^0 differently from $\overline{K}^0$, this implies. e.g. ASYMMETRY:

$$A^r_{CPT} = 2\Delta T e^{-\frac{1}{2}(\Gamma_S - \Gamma_L)t_r} \sin(\Delta m t_r)$$

bf NB: no dependance (to second order) on α, β, γ decoh. parameters, CAN DISENTANGLE from genuine QG (!)

[Notation: $\Delta T = \int dt(T - \overline{T});$, $T = \frac{2\pi\mathcal{N}}{m_K}\mathcal{M}$, $\overline{T} = \frac{2\pi\mathcal{N}}{m_K}\overline{\mathcal{M}}$, $\mathcal{M} \equiv \langle K^0|A|K^0\rangle$, $A =$forward scatt. amplitude, $N =$nuclear regenerator density. $T \neq \overline{T}$]

Genuine vs “Fake” CPTV & neutrinos

(ii) EXTRINSIC CPTV IN ν,$\overline{\nu}$ IN MATTER

NEUTRINO OSCILLATIONS IN MATTER (MSW EFFECT)

In standard treatments:

$$i\frac{d}{dt}|\nu(t)\rangle = \mathcal{H}(t)|\nu(t)\rangle,$$

$$|\nu(t)\rangle = S(t,t_0)|\nu(t_0)\rangle,$$

$S(t,t_0) = e^{-i\int_{t_0}^{t}\mathcal{H}(t')dt'}$ (S-matrix),

$S(t,t_0) = S(t,t_1)S(t_1,t_0), \qquad S(t_0,t_0) = 1, \qquad S(t,t_0) = S^\dagger(t,t_0)$ (unitarity),

$$i\frac{d}{dt}S(t,t_0) = \mathcal{H}(t)S(t,t_0)$$

FLAVOUR BASIS

$$i\frac{d}{dt}S_f(t,t_0) = \mathcal{H}_f(t)S_f(t,t_0)$$

Genuine vs “Fake” CPTV & neutrinos

SOME NOMENCLATURE

Probability differences:

$P_{\alpha\beta} = P(\nu_\alpha \to \nu_\beta), P_{\overline{\alpha}\overline{\beta}} = P(\overline{\nu_\alpha} \to \overline{\nu_\beta})$, Greek indices=flavour.

- (I) CP: $\Delta P^{\rm CP}_{\alpha\beta} = P_{\alpha\beta} - P_{\overline{\alpha}\overline{\beta}}$
- (II) T: $\Delta P^{\rm T}_{\alpha\beta} = P_{\alpha\beta} - P_{\beta\alpha}$
- (III) CPT: $\Delta P^{\rm CPT}_{\alpha\beta} = P_{\alpha\beta} - P_{\overline{\beta}\overline{\alpha}}$

Probability Conservation:

$$\sum_{\alpha=e,\mu,\tau,\ldots} \Delta P^{\rm CPT}_{\alpha\beta} = \sum_{\beta=e,\mu,\tau,\ldots} \Delta P^{\rm CPT}_{\alpha\beta} = 0$$

If assume CPT theorem is valid, i.e. only “fake” CPTV exists, then: fopr CP, T: intrinsic & extrinsic effects, but for CPT: a non-zero $\Delta P^{\rm CPT}$ would quantify only matter effects

L/E dependence of $\Delta P^{\rm CPT}$ due to matter would distinguish it from QG effects, where one might have enhancement with ν energy E (c.f. below).

“Fake” CPTV & neutrinos

HAMILTONIAN FOR MATTER PROPAGATION OF ν

$$\mathcal{H}(t) = H_f(t) + V_f(t) = UH_mU^\dagger + V_f(t)$$

subindex m(f)=mass (flavour) eigenstate basis, $V_f(t) = \sqrt{2}G_F n_e(t)$ MSW effect, matter potential, G_F weak interaction (Fermi) const., $n_e(t)$ density of electrons in matter.

HAMILTONIAN FOR MATTER PROPAGATION OF $\overline{\nu}$

$$\mathcal{H}(t) = H_f(t) - V_f(t) = UH_mU^\dagger - V_f(t)$$

“FAKE” CPTV: ± difference between ν, $\overline{\nu}$ prop. in matter is the origin for “fake” CPTV

$$P(\nu_\alpha \to \nu_\beta) - P(\overline{\nu}_\beta - \overline{\nu}_\alpha) \neq 0$$

$P_{\alpha\beta} = \mathrm{Tr}(\rho_\beta(t)\rho_\alpha) = |\langle \nu_\beta(t)|\nu_\alpha(t_0)\rangle|^2$

Systematic study (various matter profiles ...): Jacobson, Ohlsson, hep-ph/0305064: RESULT: “Fake” CPTV increases with oscillation length L, decreases with ν energy E, and vanishes for $E \to \infty$.

“Fake” CPTV & neutrinos

FORMALLY: if ONLY Extrinsic CPTV effects are present:

$$\Delta P^{\rm CPT}_{\alpha\beta} = -\Delta P^{\rm CPT}_{\overline{\beta}\overline{\alpha}}$$

i.e. probability difference for $\overline{\nu}$ do not give further information. CONTRAST WITH GENUINE CPTV where $\Delta P^{\rm CPT}_{\alpha\beta} \ne \Delta P^{\rm CPT}_{\overline{\beta}\overline{\alpha}}$ due to different decoherence parameters between ν and $\overline{\nu}$ sectors.

One has:

$$\Delta P^{\rm CPT}_{\alpha\beta} = |[S_f(t,t_0)]_{\beta\alpha}|^2 - |[\overline{S_f}(t,t_0)]_{\alpha\beta}|^2 \qquad (10)$$

$S_f(t,t_0) \rightarrow$ evolution for ν: $S_f = e^{-i\int_{t_0}^t \mathcal{H}(t)dt}$

$\overline{S_f}(t,t_0) \rightarrow$ evolution for $\overline{\nu}$: $\overline{S_f} = e^{-i\int_{t_0}^t \overline{\mathcal{H}}(t)dt}$

Systematic Computations: Jacobson-Ohlsson, hep-ph/0305064

“Fake” CPTV & neutrinos

Experiment	CPT probability differences	
	Quantities	Numerical value
BNL NWG	$\Delta P_{\mu e}^{\rm CPT}$	0.010
BNL NWG	$\Delta P_{\mu e}^{\rm CPT}$	0.032
BooNE	$\Delta P_{\mu e}^{\rm CPT}$	$6.6 \cdot 10^{-13}$
MiniBooNE	$\Delta P_{\mu e}^{\rm CPT}$	$4.1 \cdot 10^{-14}$
CHOOZ	$\Delta P_{ee}^{\rm CPT}$	$-3.6 \cdot 10^{-5}$
ICARUS	$\Delta P_{\mu e}^{\rm CPT}$	$4.0 \cdot 10^{-5}$
	$\Delta P_{\mu \tau}^{\rm CPT}$	$-3.8 \cdot 10^{-5}$
JHF-Kamioka	$\Delta P_{\mu e}^{\rm CPT}$	$3.8 \cdot 10^{-3}$
	$\Delta P_{\mu \mu}^{\rm CPT}$	$-1.3 \cdot 10^{-4}$
K2K	$\Delta P_{\mu e}^{\rm CPT}$	$1.0 \cdot 10^{-3}$
	$\Delta P_{\mu \mu}^{\rm CPT}$	$-5.3 \cdot 10^{-5}$

Experiment	CPT probability differences	
	Quantities	Numerical value
KamLAND	$\Delta P_{ee}^{\rm CPT}$	-0.033
LSND	$\Delta P_{\mu e}^{\rm CPT}$	$4.8 \cdot 10^{-15}$
MINOS	$\Delta P_{\mu e}^{\rm CPT}$	$1.9 \cdot 10^{-4}$
	$\Delta P_{\mu \mu}^{\rm CPT}$	$-1.1 \cdot 10^{-5}$
NuMI I	$\Delta P_{\mu e}^{\rm CPT}$	0.026
NuMI II	$\Delta P_{\mu e}^{\rm CPT}$	$2.6 \cdot 10^{-3}$
NuTeV	$\Delta P_{\mu e}^{\rm CPT}$	$1.6 \cdot 10^{-18}$
NuTeV	$\Delta P_{\mu e}^{\rm CPT}$	$8.2 \cdot 10^{-20}$
OPERA	$\Delta P_{\mu \tau}^{\rm CPT}$	$-3.8 \cdot 10^{-5}$
Palo Verde	$\Delta P_{ee}^{\rm CPT}$	$-1.2 \cdot 10^{-5}$
Palo Verde	$\Delta P_{ee}^{\rm CPT}$	$-2.2 \cdot 10^{-5}$

Table 3: Extrinsic CPT pds for some past, present, and fututre long-baseline experiments (Jacobson-Ohlsson, hep-ph/0305064).

NB: Extrinsic CPTV negligible for future ν factories ($\sim 10^{-5}$), sensitive to genuine CPTV? (study for 2 cases: $L \sim 3000\ Km, 7000\ Km, hep-ph/0305064$)

“Fake” Decoherence & Neutrinos

Another “Fake” Effect: Gaussian Averaged ν-oscillations can produced Decoherence (T. Ohlsson, hep-ph/0012272)

Recall scillation formula:

$$P_{\alpha\beta} = P_{\alpha\beta}(L,E) =$$

$$\delta_{\alpha\beta} - 4\sum_{a=1}^{n}\sum_{\beta=1,a<b}^{n} \mathrm{Re}\left(U^*_{\alpha a}U_{\beta a}U_{\alpha b}U^*_{\beta b}\right)\sin^2\left(\frac{\Delta m^2_{ab}L}{4E}\right) -$$

$$2\sum_{a=1}^{n}\sum_{b=1,a<b}^{n} \mathrm{Im}\left(U^*_{\alpha a}U_{\beta a}U_{\alpha b}U^*_{\beta b}\right)\sin\left(\frac{\Delta m^2_{ab}L}{2E}\right)$$

where $\alpha,\beta = e,\mu,\tau,...,\ a,b = 1,2,...n,$ $\Delta m^2_{ab} = m^2_a - m^2_b$

BUT...UNCERTAINTIES for E IN PRODUCTION OF nu-WAVE; Also: NOT WELL-DEFINED PROPAGATION LENGTH L :

$$\Delta E \neq 0, \qquad \Delta L \neq 0$$

Hence, have to AVERAGE Oscillation Probabilitty P over L/E Dependance.

“Fake” Decoherence & Neutrinos

GAUSSIAN AVERAGE: Approximate $\langle L/E \rangle \simeq \langle L \rangle / \langle E \rangle$

$$\langle P \rangle = \int_{-\infty}^{\infty} \mathrm{d}x \; P(x) \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\ell)^2}{2\sigma^2}}$$

$\ell \equiv \langle x \rangle$, $\sigma = \sqrt{\langle (x - \langle x \rangle)^2}$, $x = L/4E$.

AVERAGE $\langle P_{\alpha\beta} \rangle$:

$$\langle P_{\alpha\beta} \rangle = \delta_{\alpha\beta} -$$
$$2 \sum_{a=1}^{n} \sum_{\beta=1, a<b}^{n} \mathrm{Re}\left(U^*_{\alpha a} U_{\beta a} U_{\alpha b} U^*_{\beta b} \right) \left(1 - \cos(2\ell \Delta m^2_{ab}) e^{-2\sigma^2 (\Delta m^2_{ab})^2} \right)$$
$$-2 \sum_{a=1}^{n} \sum_{b=1, a<b}^{n} \mathrm{Im}\left(U^*_{\alpha a} U_{\beta a} U_{\alpha b} U^*_{\beta b} \right) \sin(2\ell \Delta m^2_{ab}) e^{-2\sigma^2 (\Delta m^2_{ab})^2}$$

NB: Damping factors due to σ (!)

“Fake” Decoherence & Neutrinos

EXAMPLE: TWO FLAVOURS

$$\langle P_{\alpha\beta}\rangle = \frac{1}{2}\sin^2 2\theta \left(1 - e^{-2\sigma^2(\Delta m^2)^2}\cos(2\ell\Delta m^2)\right) , \; \ell = \frac{\langle L\rangle}{4\langle E\rangle}$$

Bounds on σ (T. Ohlsson)

- Pessimistic: $\sigma \simeq \Delta x \simeq \Delta\frac{L}{4E} \leq \frac{\langle L\rangle}{4\langle E\rangle}\left(\frac{\Delta L}{\langle L\rangle} + \frac{\Delta E}{\langle E\rangle}\right)$
- Optimistic: $\sigma \leq \frac{\langle L\rangle}{4\langle E\rangle}\left([\frac{\Delta L}{\langle L\rangle}]^2 + [\frac{\Delta E}{\langle E\rangle}]^2\right)^{1/2}$

"Fake" Decoherence & Neutrinos

Equivalence with decoherence:

Lindblad: $\dot{\rho} = i[\rho, H] + \mathcal{D}[\rho]$,

$$\mathcal{D}[\rho] = \sum_{i=1}^{n} [D_i, [D_i, \rho]]$$

(if $D_i^\dagger = D_i$, energy is conserved on average, and the ρ is a completely positive map) (Adler 2000)

Example: TWO FLAVOURS: One Decoherence Coefficient γ:

$$P_{e\mu}(L, E) = \frac{1}{2}\sin^2 2\theta \left(1 - e^{-\gamma L}\cos(\frac{\Delta m^2 L}{2E})\right)$$

$(L = t, c = 1)$.

COMPARE WITH "FAKE" GAUSSIAN AVERAGE:

$$2\sigma^2(\Delta m^2)^2 = \gamma L \quad \rightarrow \quad \gamma = \frac{(\Delta m^2)^2}{8E^2} L r^2$$

with $\sigma = (L/4E)r$, $r = \frac{\Delta L}{L} + \frac{\Delta E}{E}$ (pessimistic), or $r = \sqrt{(\frac{\Delta L}{L})^2 + (\frac{\Delta E}{E})^2}$ (optimistic).

For atmospheric ν: $\sigma_{\rm atm} \sim 1.5 \times 10^3$ eV^2 (for $L \sim 12000\ Km$), $r \sim \mathcal{O}(1)$, hence

$$\gamma_{\rm atm,fake} < 10^{-24}\ rmGeV$$

COMPARE WITH QG: (i) optimistic (Ellis, NM, Nanopoulos) : $\gamma_{QG} \sim E^2/M_{QG}$, (ii) pessimistic: (Adler) $\gamma_{QG} \sim \frac{(\Delta m^2)^2}{E^2 M_{QG}}$.

NB: In QG NO L Dependence, but $1/M_{QG}$ (in 4-dim $M_{QG} \sim M_P \sim 10^{19}$ GeV) CAN DISENTANGLE (!)

“Fake” Decoherence & Neutrinos

NB: GAUSSIAN AVERAGE ALSO DUE TO QUANTUM-GRAVITY UNCERTAINTIES:

If Δ/L is due to “Fuzzyness” od space time due to quantum fluctuations, then (Van Dam, Ng, Ellis, NM, Nanopoulos)

$$\frac{\Delta L}{L}, \quad \frac{\Delta E}{E} \sim \beta \left(\frac{E}{M_{QG}}\right)^{\alpha},$$

α some positive integer, $\alpha \geq 1$, β some coefficient. In this case $r \sim \beta \left(\frac{E}{M_{QG}}\right)^{\alpha}$.

Then, from Gaussian Average we get for Decoherence:

$$\gamma \sim \frac{(\Delta m^2)^2}{8E^2} \beta \left(\frac{E}{M_{QG}}\right)^{\alpha} L$$

NB: modified E-dependence, but still $\propto L$.

INTERESTING TO EXPLORE FURTHER...

HOWEVER, IN GENERAL SUCH EFFECTS CAN BE DISENTANGLED FROM OTHER α, β, γ COEFFICIENTS OR STOCHASTIC-MEDIUM EFFECTS BY THEIR L DEPENDENCE...

“Fake” Decoherence & Neutrinos

NEUTRINO OSCILLATIONS IN (NOISY) MEDIA

NEUTRINO PROPAGATION IN A MEDIUM WITH, SAY, ELECTRON DENSITY n_e (e.g. the Sun environment) (Mikheyev-Smirnov (1986), Wolfenstein (1978))

MSW EFFECT

MASS-SQUARED DIFFERENCE (and other effects, e.g. spin precession) BETWEEN ν FLAVOURS IS DEVELOPED AS A RESULT OF THE PASSAGE OF ν THROUGH MATTER, EVEN IF ν WERE DEGENERATE IN MASS IN VACUO.

Mixing angle: $\sin^2 2\tilde{\theta} = \sin^2 2\theta \left(\frac{\Delta m^2}{\Delta \tilde{m}^2}\right)$

Mass-Squared Difference:
$\Delta \tilde{m}^2 = \sqrt{(D - \Delta m^2 \cos 2\theta)^2 + (\Delta m^2 \sin 2\theta)^2}$

Tilde= Medium quantity, Untilde= vacuum quantity.

$D = \sqrt{2} G_F n_e k$, (G_F =Fermi's const, k =momentum scale)

PHYSICALLY: Charged current interact only with ν_e:
$\frac{G_F}{\sqrt{2}} \overline{\nu_e} \gamma_\lambda (1 + \gamma_5) e \overline{e} \gamma^\lambda (1 + \gamma_5) \nu_e$.

"Fake" Decoherence & Neutrinos

MSW EFFECT CAN BE GENERALISED TO STOCHASTICALLY FLUCTUATING MEDIA (Loreti, Balantekin (1994))

FLUCTUATING (in time) ELECTRON DENSITY in MEDIUM:

$$\langle n_e \rangle = n_{e,0} \equiv n_0$$

$$\langle n_e(t) n_e(t') \rangle = n_0^2 \Omega^2 \delta(t - t') + \text{higher correlations}$$

We set from now on $t = r$ $(c = 1)$.

QUANTUM EFFECTS IN OPEN-SYSTEM QUANTUM MECHANICS ARE DESCRIBED BY TEMPORAL EVOLUTION OF DENSITY MATRIX OF MATTER SYSTEM (ν in our example)

$$\rho = \mathrm{Tr}|\psi\rangle\langle\psi| \equiv \psi \otimes \psi^\dagger$$

Tr =unobserved degrees of freedom.

If ψ obeys Schrödinger eq. $i\frac{d}{dt}\psi = \hat{H}\psi$, $\psi^T(t) = (\psi_1, \psi_2, \ldots \psi_N)$ for N-level system, then:

$$i\frac{d\hat{\rho}}{dt} = [\hat{H}, \hat{\rho}]; \quad \hat{H} = \hat{H}_0 + B(t)\hat{M}'$$

H_0 mean field effects, $\hat{M}'$ independent of time, $B(t)$ fluctuating field, $\langle B(t) \rangle = 0$, $\langle B(t_1)B(t_2) \rangle = \alpha^2 f(|t_1 - t_2|)$.

GAUSSIAN FIELD: $\langle B(t_1)B(t_2) \rangle = 2\tau\alpha^2\delta(t_1 - t_2) \ldots$

"Fake" Decoherence & Neutrinos

EVOLUTION EQUATION FOR DENSITY MATRIX IN STOCHASTIC MEDIA

$$\begin{aligned} \hat{\rho}_I &= \hat{U}_0^\dagger \hat{\rho} \hat{U}_0 \\ \hat{M} &= \hat{U}_0^\dagger \hat{M}' \hat{U}_0 , \qquad \text{with} \quad i\frac{d}{dt}\hat{U}_0 = \hat{H}_0 \hat{U}_0 \\ \hat{H}_I &= B(t)\hat{M} \end{aligned}$$

EVOLUTION: $i\frac{\hat{\rho}_I}{dt} = [\hat{H}_I, \hat{\rho}_I]$

SOLVE BY ITERATION:

$$\hat{\rho}(t) = \hat{\rho}_0 - i\int_{t_0}^{t} dt_1 B(t_1)[\hat{M}(t_1), \hat{\rho}_0] -$$

$$\int_0^t \int_0^{t_1} dt_1 dt_2 B(t_1) B(t_2) [\hat{M}(t_1), [\hat{M}(t_2), \hat{\rho}_0]] + \ldots$$

Average over Random Field: $\langle g(B) \rangle = \int_{-\infty}^{\infty} \mathcal{D}[B(t)] g(B) e^{-\int_0^t \frac{B^2}{2k} dt}$,
$\mathcal{D}[B] = \Pi_i dB(t_i)\sqrt{\frac{\Delta t}{2k\pi}}$, $k = 2\alpha^2 \tau$

$$\frac{d}{dt}\langle \hat{\rho}(t) \rangle = -i[\hat{H}_0, \langle \hat{\rho}(t) \rangle] - \alpha^2 \tau [\hat{M}', [\hat{M}', \langle \hat{\rho}(t) \rangle]]$$

NB: DOUBLE COMMUTATOR IS TIME IRREVERSIBLE, unrelated to CP ⟶ CPT Violating due to matter.

CONCLUSIONS

Neutrino Physics may provide a very useful guide in our quest for a theory of Quantum Gravity, in particular stringent constraints on CPT Violation. The latter may not be an academic issue, but a real feature of QG.

Neutrino oscillation experiments provide stringent bounds on many quantum gravity models, entailing Lorentz Invariance Violation. There are also plenty of low energy nuclear and atomic physics experiments which yield stringent bounds in models with Lorentz (LV) and CPT violation. Frame dependence of LV effects crucial.

But Quantum Gravity may exhibit Lorentz Invariant (and hence frame independent) CPTV Decoherence. Theoretically the presence of an environment may be consistent with Lorentz Invariance (Millburn 2003). The scenario of three-generation antineutrino decoherence + mixing is still compatible with all the existing ν data, including LSND, and also yields interesting estimates for Dark Energy and matter-antimatter asymmetry, compatible with known estimates and limits, observations.

More work (Theory & Expt) to be done before conclusions are reached...