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AND FUNDAMENTAL CONSTANTS**

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**TESTING PLANCK SCALE LORENTZ VIOLATIONS  
WITH HIGH ENERGY ASTROPHYSICS**

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QuickTime™ and a  
TIFF (Uncompressed) decompressor  
are needed to see this picture.

# Testing Planck scale Lorentz violations with high energy astrophysics

**T. Jacobson, SL, D. Mattingly: PRD 66, 081302 (2002); PRD 67, 124011-12 (2003)**

**T. Jacobson, SL, D. Mattingly: Nature 424, 1019 (2003)**

**T. Jacobson, SL, D. Mattingly, F. Stecker: PRL 93 (2004) 021101**

**Collaborators:**

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# QG Phenomenology?

To eventually understand QG, we will need to observe phenomena that depend on QG  
extract reliable predictions from candidate theories & compare with observations

Motivated by tentative theories, partial calculations, potential symmetry violation, hunches, philosophy...

- Primordial gravitons from the vacuum
- Loss of quantum coherence or state collapse
- QG imprint on initial cosmological perturbations
- Scalar moduli or other new field(s)
- Extra dimensions and low-scale QG:  $M_p^2 = R^n M_{p(4+n)}^{n+2}$ 
  - dev. from Newton's law
  - collider black holes
- Violation of global internal symmetries
- Violation of spacetime symmetries

# Lorentz violation as the first evidence of QG?

LI linked to **scale-free spacetime**: unbounded boosts expose ultra-short distances...

**Suggestions for Lorentz violation come from:**

- need to cut off UV divergences of QFT & BH entropy
- tentative calculations in various QG scenarios, e.g.
  - semiclassical spin-network calculations in Loop QG
  - string theory tensor VEVs
  - spacetime foam
  - non-commutative geometry
  - some brane-world backgrounds
  - condensed matter analogues of “emergent gravity”

Very different approaches but common prediction of modified dispersion relations for elementary particles

# QG phenomenology via modified dispersion relations

$$E^2 = p^2 + m^2 + \Delta(p, M, \mu)$$

$\mu$  = some particle mass scale

$$M = 10^{19} \text{ GeV} \approx M_{\text{Planck}}$$

Almost all of the above cited framework do lead to modified dispersion relations that can be cast in this form

If we presume that any Lorentz violation is associated with quantum gravity and suppressed by at least one inverse power of the Planck scale  $M$  and we violate only boost symmetry

$$\Delta(p) = \tilde{\eta}_1 p + \tilde{\eta}_2 p^2 + \tilde{\eta}_3 p^3 + \tilde{\eta}_4 p^4 + \dots + \tilde{\eta}_n p^n$$

$$\tilde{\eta}_1 = \eta_1 \frac{\mu^2}{M}, \quad \tilde{\eta}_2 = \eta_2 \frac{\mu}{M}, \quad \tilde{\eta}_3 = \eta_3 \frac{1}{M}, \quad \tilde{\eta}_4 = \eta_4 \frac{1}{M^2}$$

with  $\eta_i \approx O(1)$

$$E^2 = m^2 + p^2 + \eta_1 \frac{\mu^2}{M} p + \eta_2 \frac{\mu}{M} p^2 + \eta_3 p^3 / M + \eta_4 p^4 / M^2 + \dots$$

## Constraints at lowest orders

- In a such a framework the  $n=1,2$  terms will dominate at low energies  $p \ll \mu$ .
  - At high energies,  $p \gg \mu$ , the  $p^3$  term, if present, will dominate.
  - If  $p^3$  is absent then the  $p^4$  term will dominate if  $p^2 \gg \mu M$  and so on...

A large amount of both theoretical and experimental work has been carried out in the case  $n \leq 2$  which includes the “standard model extension proposal” and models like those proposed in VSL and by Coleman-Glashow

Compared to “Planck-suppressed” expectation  
(with  $\mu$ =relevant mass scale for observation/experiment)

- **Laboratory** ~ 1-2 orders weaker
- **High energy astrophysics** ~ 1-2 orders weaker
- **GZK (if confirmed)** ~ comparable
- **Vacuum birefringence** ~ few orders stronger

## An open problem: un-naturalness of small LV.

Renormalization group arguments might suggest that lower powers of momentum in

$$E^2 = p^2 + m^2 + \tilde{\eta}_1 p^1 + \tilde{\eta}_2 p^2 + \tilde{\eta}_3 p^3 + \tilde{\eta}_4 p^4 + \dots + \tilde{\eta}_n p^n$$

will be suppressed by lower powers of  $M$  so that  $n \geq 3$  terms will be further suppressed w.r.t.  $n \leq 2$  ones.

I.e. one could have something like

$$\tilde{\eta}_3 = \eta_3 \frac{1}{M} = \frac{\mu}{M} \frac{1}{M} \ll \tilde{\eta}_2$$

This need not be the case if a symmetry or other mechanism protects the lower dimensions operators from violations of Lorentz symmetry

Of course we do not know at the moment  
if this is indeed the case!

About how things can go wrong, see gr-qc/0403053 (Collis et al.)

However look also at gr-qc/0402028 (Myers-Pospelov) or

hep-ph/0404271 Nibblink-Pospelov (on SUSY possible role) for solutions in EFT framework

# Constraints on E/M terms

## Lab experiments:

- Sidereal variation of LV coupling as the Lab moves with respect to the preferred frame. Constraint needs assumptions on dynamics, ether-coupling

## Astrophysical observations:

- Cumulative effects: times of flight & birefringence: Purely kinematical effects (presume only modified dispersion relation and standard definition of group velocity).
- Anomalous threshold reactions (usually forbidden, e.g. gamma decay, Vacuum cherekov): Constraint needs assumptions on energy/momentum conservation (LIV vs DSR) reactions are too fast to be sensitive to suppressed changes in the matrix element.
- Shift of standard thresholds reactions: Constraint needs assumptions on energy/momentum conservation (LIV vs DSR) and dynamics (e.g. mean free path)
- Reactions affected by “speeds limits” (e.g. synchrotron radiation): Constraint needs assumptions on energy/momentum conservation (LIV vs DSR) and dynamics
- Dynamical effects of LV background fields (e.g. gravitational coupling): Constraint needs assumptions on dynamics, ether-coupling



# Theoretical Framework for LV?

EFT? Renormalizable, or higher dimension operators?

Stochastic spacetime foam?

Rotational invariant?

Lorentz Violation or Doubly Special Relativity?  
(i.e. preferred frame or possibly a relativity with two invariant scales?,  $c$  and  $l_p$ )

Universal, or species dependent?

# Framework choice:

EFT, all dimension ops, rotation inv., non-universal

- EFT
  - ✓ well-defined & simple
  - ✓ implies energy-momentum conservation (below the cutoff scale)
  - ✓ covers standard model, GR, condensed matter systems, string theory ...
- All dimension ops: who knows?
- Rot. invariance
  - ✓ simpler
  - ✓ cutoff idea only implies boosts are broken, rotations maybe not
  - ✓ boost violation constraints likely also boost + rotation violation constraints
- Non-universal
  - ✓ EFT implies it for different polarizations & spins
  - ✓ different particle interactions suggest different spacetime interactions
  - ✓ "equivalence principle" anyway not valid in presence of LV

# Dispersion relations from EFT

Let's consider all the Lorentz-violating dimension 5 terms ( $n=3$  LIV in dispersion relation) that are quadratic in fields, gauge & rotation invariant, not reducible to lower order terms (Myers-Pospelov, 2003).  
For  $\underline{E} \gg m$

All LIV terms also violate CPT

$$\frac{\xi}{M} u^m F_{ma} (u \cdot \partial) (u_n \tilde{F}^{na}) \implies \omega_{R,L}^2 = k^2 \pm \xi \frac{p^3}{M}$$

photon helicities have opposite LIV coefficients

$$\frac{1}{M} u^m \bar{\psi} \gamma_m (\eta_1 + \eta_2 \gamma_5) (u \cdot \partial)^2 \psi \implies E_{R,L}^2 = p^2 + m^2 + \eta_{R,L} \frac{p^3}{M}$$

$$\eta_R = 2(\eta_1 + \eta_2) \quad \eta_L = 2(\eta_1 - \eta_2)$$

electron helicities have independent LIV coefficients

Moreover electron and positron have inverted and opposite positive and negative helicities LIV coefficients (JLMS, 2003).

	Positive helicity	Negative helicity
Electron	$\eta_R$	$\eta_L$
Positron	$-\eta_L$	$-\eta_R$

Electron spin resonance in a Penning trap yields

$$|\eta_L - \eta_R| \leq 4$$

# Constraining $n=3$ LV in the QED sector

$$\omega_{\pm}^2 = k^2 \pm \frac{\xi}{M} k^3 \quad \text{photon}$$
$$E_{\pm}^2 = p^2 + m^2 + \frac{\eta_{\pm}}{M} p^3 \quad \text{electron}$$

## Times of flight

Constraint on the photon LIV coefficient  $\xi$  by using the fact that different colors will travel at different speeds. On long distances one expects different time of flight corresponding to different speed of propagations.

Using a purely phenomenological model (no opposite coefficients for photon helicities)

Best constraint up to date is Schaefer (1999) using GRB930131, a gamma ray burst at a distance of 260 Mpc that emitted gamma rays from 50 keV to 80 MeV on a time scale of milliseconds. The constraint is  $|\xi| < 122$ .

Very recently (Oct. 2003) Corburn et al. using GRB021206 obtained  $|\xi| < 77$

However, probably GRB are not "good" objects (different energies emission at different times), then best constraint is Biller (1998, Markarian 421)  $\xi < 252$ .

Using the above EFT disp.rel. the opposite coefficients for photon helicities imply larger dispersion  $2|\xi|p/M$  rather than that due to different energies  $\xi(p_2 - p_1)/M$ . Current best limits (using Biller, 1998, AGN)  $|\xi| < 63$  (or, using Boggs et al. 2003, GRB),  $|\xi| < 34$ .

# Birefringence

T. Jacobson, SL, D. Mattingly, F. Stecker: PRL (2004)  
Mitrofanov: Nature (2004)

Opposite  $\xi$  for the photon helicities imply different phase velocities: birefringence of vacuum  
Hence observation of polarized radiation from distant sources can hence be used to constraint  $\xi$

- There is a rotation of linear polarization direction through an angle. For a plane wave of wave-vector  $k$ :

$$\omega^2 = k^2 \pm \xi k^3 \rightarrow \omega = k \pm \frac{1}{2} \xi k^2$$

$$e^{-i\omega t + ikx} = e^{ik(x-t)} e^{\mp \frac{i}{2} \xi k^2 t} \Rightarrow \theta = \frac{1}{2} \xi \frac{k^2}{M} t \approx \frac{1}{2} \xi E^2 d \text{ rotation of linear polarization}$$

- The difference in rotation angle for two different energies is
- So for long distances the instantaneous polarization at the detector would fluctuate enough to suppress the net polarization well below the observed value.

$$\Delta\theta (E_2^2 - E_1^2) d/2M$$

Recently polarized gamma rays in the energy range 0.15–2 MeV were observed (Coburn-Boggs, 2003) in the prompt emission from the  $\gamma$ -ray burst GRB021206 using the RHESSI detector.

A linear polarization of  $80\% \pm 20\%$  was measured by analyzing the net asymmetry of their Compton scattering from a fixed target into different directions.

This then yields at least  $|\xi| < 5.0 \times 10^{-15} / d_{0.5}$  where  $d_{0.5}$  is the distance to the burst in units of 0.5 Gpc.  $|\xi| \leq 2 \times 10^{-4}$

***N.B. Criticized by Ritledge and Fox. Boggs-Coburn defended their analysis.***

***Otherwise best limit Gleiser and Kozameh (10% polarization from  $z=1.82$ , radio galaxy 3C 256)***

# Threshold reactions

Key point: the effect of the non  $\square$  dispersion relations can be important at energies well below the fundamental scale

$$E^2 = c^2 p^2 \left( 1 + \frac{m^2 c^2}{p^2} + \eta \frac{p^{n-2}}{M^{n-2}} \right)$$

Corrections start to be relevant when the last term is of the same order as the second.

If  $\eta$  is order unity, then

$$\frac{m^2}{p^2} \approx \frac{p^{n-2}}{M^{n-2}} \Rightarrow p_{crit} \approx \sqrt[n]{m^2 M^{n-2}}$$

$n$	$p_{crit}$ for $\nu_e$	$p_{crit}$ for $e^-$	$p_{crit}$ for $p^+$
2	$p \approx m_\nu \sim 1 \text{ eV}$	$p \approx m_e = 0.5 \text{ MeV}$	$p \approx m_p = 0.938 \text{ GeV}$
3	$\sim 1 \text{ GeV}$	$\sim 10 \text{ TeV}$	$\sim 1 \text{ PeV}$
4	$\sim 100 \text{ TeV}$	$\sim 100 \text{ PeV}$	$\sim 3 \text{ EeV}$

For  $n=3$

$$m^2 \approx \eta p^3 / M \leftrightarrow p \approx (m^2 M / \eta)^{1/3} \approx 10 \text{ TeV } \eta^{-1/3}$$

$$\eta \text{ constraint} \propto \frac{1}{p_{max}^3}$$

# Threshold reactions

Key point: the effect of the non  $\perp$  dispersion relations can be important at energies well below the fundamental scale because is the mass that does matter

$$E^2 = c^2 p^2 \left( 1 + \frac{m^2 c^2}{p^2} + \eta \frac{p^{n-2}}{M^{n-2}} \right)$$

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$p_{crit}$  for  $e^-$  and  $n=3$  is 10 TeV

## □ New threshold reactions

□ **Vacuum Cherenkov:**  $e^- \rightarrow e^- + \gamma$

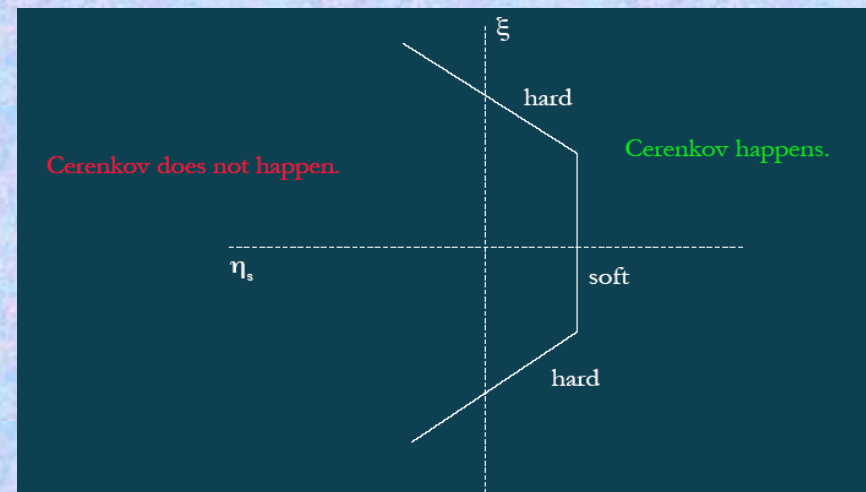
▪ Moreover now possible Cherenkov with emission of an hard photon

□ **Gamma decay:**  $\gamma \rightarrow e^+ + e^-$

▪ Moreover now possible asymmetric pair production of electron-positron pair

**These reactions are almost instantaneous (interaction with zero point modes)**

**If allowed the particle won't propagate.**



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## Anomalous thresholds (modification of standard threshold reactions)

**Shift of lower thresholds (Coleman-Glashow, JLM, Konopka-Major, etc...)**

**Emergence of upper thresholds (Klusniak, JLM)**

**Asymmetric pair production (JLM, Konopka-Major)**

**So far constraints from**

**Photon pair creation using AGN:**  $\gamma + \gamma_{CMB, FIRB} \rightarrow e^+ + e^-$

**Best limit so far from Mkr 501**

**For proton-pions GZK reaction:**  $p^+ + \gamma_{CMB} \rightarrow p^+ + \pi^0$  (if actually found)



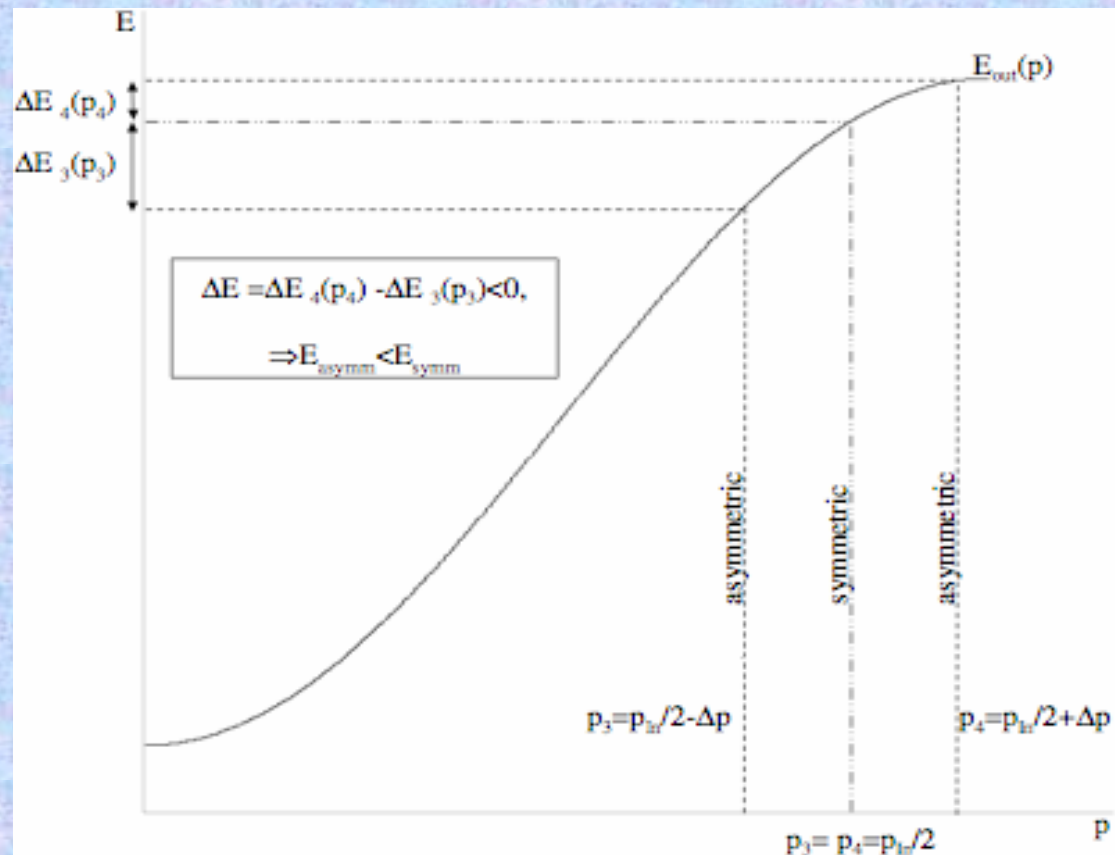
## Novelties in threshold reactions: why

- ❖ **Asymmetric configurations:**  
Pair production can happen with asymmetric distribution of the final momenta

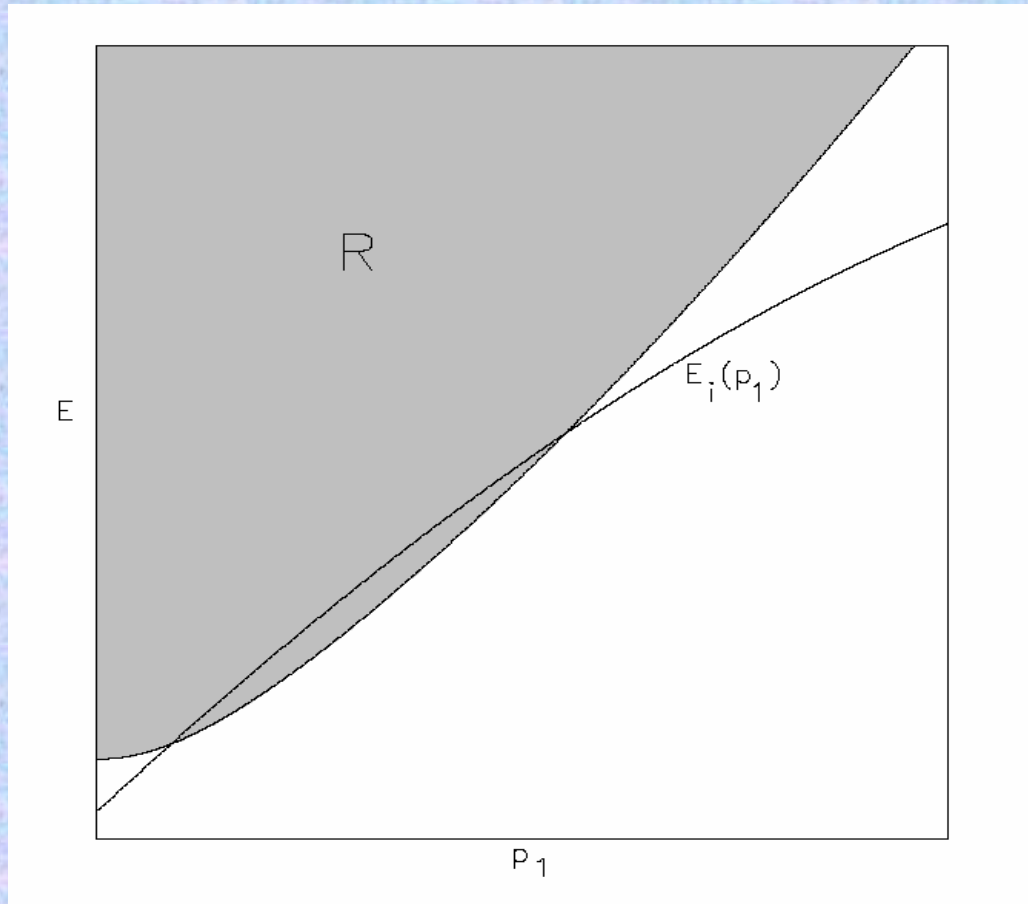
$$\Delta E_f = \left. \frac{\partial^2 E_0}{\partial p^2} \right|_{p=p_s} (\Delta p)^2$$

$$\text{if } \left. \frac{\partial^2 E_0}{\partial p^2} \right|_{p=p_s} < 0$$

**Sufficient condition for asymmetric Threshold.**



## Novelties in threshold reactions: why



❖ **Upper thresholds:**  
The range of available energies of the incoming particles for which the reactions happens is changed. Lower threshold can be shifted and upper thresholds can be introduced

If LI holds there is never an upper threshold

However the presence of different coefficients for different particles allows  $E_i$  to intersect two or more times  $E_f$  switching on and off the reaction!

## The synchrotron radiation

LI synchrotron critical frequency:  $\omega_c^{LI} = \frac{3 e B \gamma^2}{2 m}$        $e$  - electron charge,  
 $m$  - electron mass  
 $B$  - magnetic field

The key point is that for negative  $\eta$ ,  $\gamma$  is now a bounded function of  $E$ ! There is now a maximum achievable synchrotron frequency  $\omega^{\max}$  for ALL electrons!

$$\gamma = (1 - v^2)^{-1/2} \approx \left( \frac{m^2}{E^2} - 2\eta \frac{E}{M_{QG}} \right)^{-1/2}$$

So one gets a constraints from asking  $\omega^{\max} \geq (\omega^{\max})_{\text{observed}}$

Actually in order to get a real constraint one needs a detailed re-derivation of the synchrotron effect with LIV based on EFT.

Purely kinematical arguments (LI/LIV independent) can be used to derive  
 $R$ =radius of gyration,  $\theta$ =angular width synchrotron cone

$$\omega_c \propto 1/[R \theta (v_\gamma - v_e)]$$

Computing  $R$  and  $\theta$  in LIV theory imply adoption of a defined framework, for us EFT

Within this framework one can show that corrections to both quantities are negligible with respect to the LI values

# The synchrotron constraint

This leads to a modified formula for the peak frequency:

$$\omega_c^{LIV} = \frac{3 e B}{2 E} \gamma^3$$

We can now maximize the synchrotron frequency with respect to the electron energy ( $\eta < 0$ )

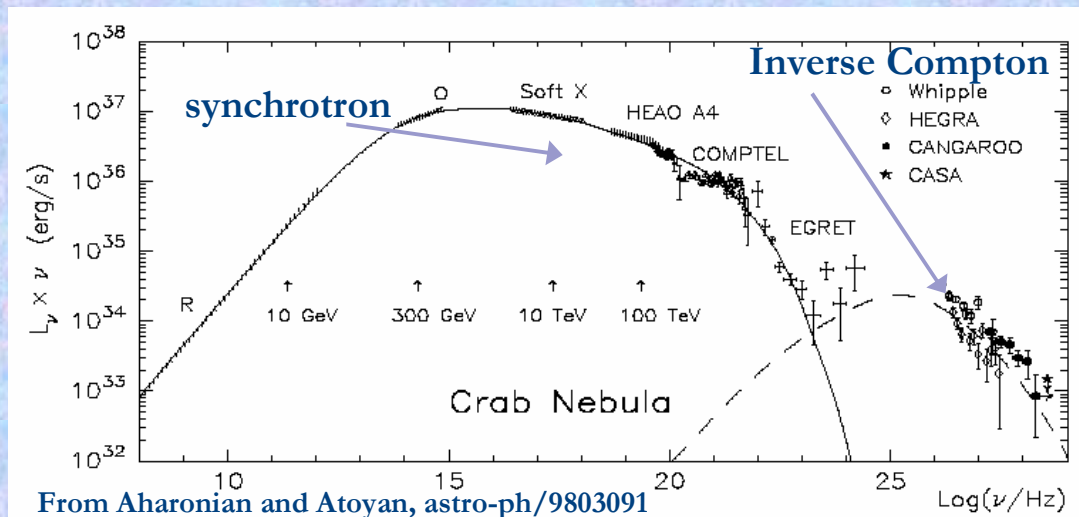
One gets that the maximal peak frequency achievable is

$$\omega_c^{\max} = 0.34 \frac{e B}{m} (-\eta m / M)^{-2/3}$$

Then if one observes some max frequency  $\omega_{\text{obs}}$  the LIV parameter must be such to allow it

$$\eta > -\frac{M}{m} \left( \frac{0.34 e B}{m \omega_{\text{obs}}} \right)^{3/2}$$

Stronger constraint for smaller  $B/\omega_{\text{obs}}$   
Best case is Crab nebula...



Crab nebula (and other SNR) well explained by synchrotron self-Compton model.

SSC Model:

1. Electrons are accelerated to very high energies at pulsar
2. High energy electrons emit synchrotron radiation
3. High energy electrons undergo inverse Compton with ambient photons

# The Crab nebula: a key object for QG phenomenology



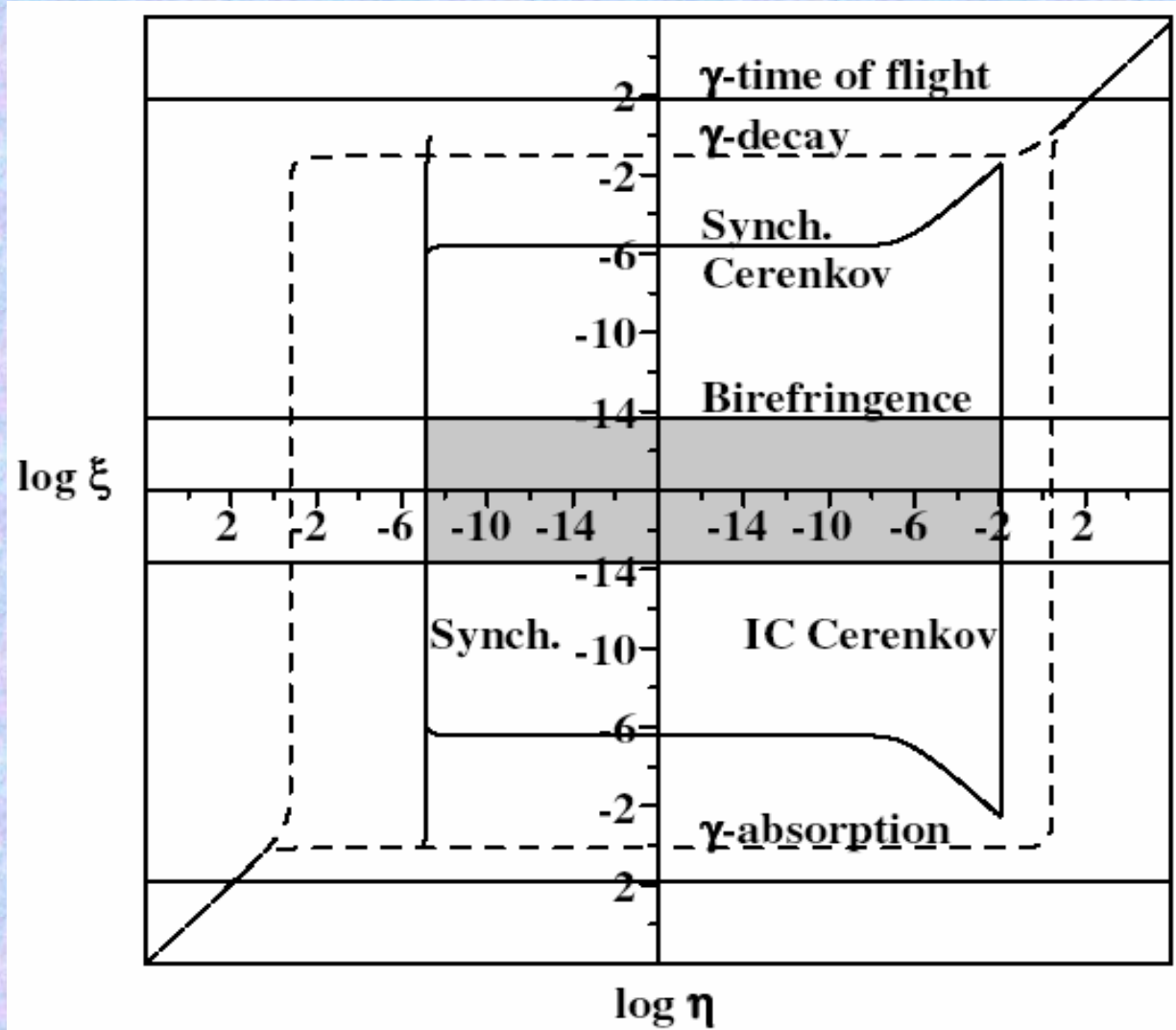
X-ray

- **IC vacuum Cherenkov:** By energy conservation during the IC process we can infer that electrons of at least 50 TeV propagate in the nebula: no vacuum Cherenkov up to 50 TeV. At least one of the  $\eta$  must satisfy this.
- **Synchrotron:** The synchrotron emission extends up to 100 MeV (corresponding to  $\sim 1500$  TeV electrons if LIV is preserved): LIV for electrons (with negative  $\eta$ ) should allow an  $\omega^{\max} \leq 100$  MeV.  $B$  at most 0.6 mG  $\Rightarrow \eta > -7 \times 10^{-8}$ . Moreover this  $\eta$  must be the same that satisfy the IC vacuum Cherenkov constraint because the Synch-IC spectrum requires a single population of emitters.
- **Improved vacuum Cherenkov:** The existence of electrons producing the synchrotron can be used to extend the vacuum Cherenkov constraint. For a given  $\eta$  satisfying the synchrotron bound, some definite electron energy  $E_{\text{synch}}(\eta)$  must be present to produce the observed synchrotron radiation. (This is higher for negative  $\eta$  and lower for positive  $\eta$  than the Lorentz invariant value)  
 Values of  $\xi$  for which the vacuum Cherenkov threshold is lower than  $E_{\text{synch}}(\eta)$  for either photon helicity can therefore be excluded. (use hard photon Cherenkov)

# The Crab nebula: a key object for QG phenomenology



X-ray



## Other constraints

- **Helicity decay:** a constraint on  $|\eta_+ - \eta_-|$  can be obtained from Crab. E.g. If negative helicity electrons do not satisfy the Synch-IC constraint then positive helicity one have to ( $\eta_- < \eta_+$ ). Then their energy must be at least above 50 TeV and they cannot decay to negative helicity one. So the transition energy for helicity decay must be greater than 50 TeV. If the reaction rate is fast enough then one gets  $|\eta_+ - \eta_-| < 10^{-2}$
- **Photon decay:** previous analysis was done before knowing different  $\eta$  for  $e^-/e^+$ . Analysis can be done in full EFT and constraint improved separately in  $\eta_+$  and  $\eta_-$  using 50 TeV gamma rays from Crab. However still won't be competitive with other constraints  $\eta \approx O(10^{-2})$
- **Photon absorption:** Constraint from Mkn 501 emission. Analysis complicated by uncertainty on original spectrum, IR background. Very complicated threshold shift. Needs framework to be sure that matrix element is not severely modified. However still won't be competitive with other constraints  $\eta \approx O(10^{-1})$ .
- **GZK reaction:** Uncertainty on the actual presence of the GZK cutoff. Possible evidence for new physics. LIV can shift the threshold and allow vacuum proton Cherenkov. If GZK particles are indeed protons strong Cherenkov constraint  $\eta \approx O(10^{-14})$  from  $5 \times 10^{19}$  eV protons. If GZK cutoff confirmed then  $\eta_{p,\pi} \approx O(10^{-11})$

# The future?

- Definitively rule out  $n=3$  LV,  $O(E/M)$ , EFT including chirality effects
  - Strengthen the positive  $\eta$  and  $|\eta_R - \eta_L|$  bounds e.g. via possible role of positrons in Crab nebula emission.
  - naturalness problem
- Constraint on  $n=4$  (favored if CPT fundamental also for QG):

$$m^2 \sim \eta p^4 / M^2 \Leftrightarrow p \sim \sqrt{mM} \eta^{-1/4}$$

$$p \sim 100 \text{ TeV (neutrino), } 3 \times 10^{18} \text{ eV (proton), } 100 \text{ PeV (electron)}$$

- No GZK protons Cherenkov:  $\eta \leq O(10^{-5})$
- If GZK cutoff seen:  $\eta \approx \geq O(10^{-2})$
- Neutrinos: 100 TeV neutrinos give order unity constraint by absence of vacuum Cherenkov but rate of energy loss too low. Recent calculations shows one need  $10^{20}$  eV UHE cosmological neutrinos. Possibly to be seen via EUSO and/or OWL satellites
- Better measures of energy, timing, polarization from distant  $\gamma$ -ray sources.  $O(1)$  constraint on  $|\xi|$  requires polarization detection of at 100 MeV
- A true messenger of QG phenomenology will arrive? Perhaps the missing GZK?

**We'll see soon...**