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LORENTZ VIOLATION IN SUPERSYMMETRIC THEORIES

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Lorentz Violation in Supersymmetric Theories

Fundamental Symmetries and Fundamental Constants

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Symmetries in Particle Physics

- Spacetime symmetries and internal symmetries
- Local and global symmetries
- Exact and spontaneously broken symmetries

Experimental Status of Symmetries

	Exact	Broken
Local Internal	$SU(3)_{QCD}$ $U(1)_{EM}$	$SU(2) \times U(1)_Y$
Global Internal	Baryon number: B Individual lepton numbers: L_i	Isospin: $SU(2)_I$
Global Spacetime	Displacements: P Angular momentum: J Lorentz boosts: K	Supersymmetry: Q

The Lorentz symmetry and supersymmetry are both spacetime symmetries.

1) **Supersymmetry is experimentally determined to be a broken symmetry.**
2) **Could the Lorentz symmetry also be broken at some level?**

Evolution of the Knowledge of Spacetime Symmetries

- **Stern and Gerlach: Intrinsic spin, properties with respect to the rotation operator J doubles the number of electron states**

- **Dirac: particle/antiparticle, properties with respect to the Lorentz boost generator, K, doubling the number of electron states: electron-positron**

- **Supersymmetry: introduces a new generator Q doubling the number of states once again: electron and scalar electron (selectron)**

If we lived at the Planck scale, we might be surprised to learn from our experiments that supersymmetry is a broken spacetime symmetry. $M_{LV} << M_{SUSY} << M_{Pl}$

Physical Scales of Symmetry Breaking

- There is circumstantial evidence for electroweak-scale supersymmetry: dark matter, solution to the naturalness problem ($M_{EW} << M_{Pl}$), absence of substantial radiative corrections to precision electroweak measurements.

- Supersymmetry, if it is an approximate symmetry, must be broken at or above the electroweak scale.

- Compared to the fundamental Planck scale, this symmetry breaking is a small effect, and supersymmetry is an approximate symmetry. $M_{SUSY} << M_{Pl}$

- Supersymmetry breaking can be incorporated into a supersymmetric Standard Model in the context of field theory.

- The remaining spacetime symmetries such as the Lorentz symmetry could possibly have even smaller violations: $M_{LV} << M_{SUSY} << M_{Pl}$

Supersymmetry Algebra

- Poincare algebra (4 generators of translation and the 6 Lorentz generators):

$$\begin{aligned} [P_\mu, P_\nu] &= 0 \\ [P_\mu, M_{\rho\sigma}] &= i(\eta_{\mu\rho} P_\sigma - \eta_{\mu\sigma} P_\rho) \\ [M_{\mu\nu}, M_{\rho\sigma}] &= i(\eta_{\nu\rho} M_{\mu\sigma} - \eta_{\nu\sigma} M_{\mu\rho} - \eta_{\mu\rho} M_{\nu\sigma} + \eta_{\mu\sigma} M_{\nu\rho}) \end{aligned}$$

- 3 generators of the rotation group

$$J^i = \frac{1}{2}\epsilon^{ijk} M^{jk} \ , \qquad \left[J^i, J^j\right] = i\epsilon^{ijk} J^k$$

- 3 generators of Lorentz boosts

$$K^i = M^{0i}$$

- Supersymmetry extends the Poincare algebra to include a fermionic generator Q

$$[Q, P_\mu] = 0 \ , \qquad \left\{Q, \overline{Q}\right\} = 2\gamma^\mu P_\mu$$

Incorporation of symmetry breaking effects phenomenologically

1. Supersymmetry breaking

- Add fields to Supersymmetrize the Standard Model
- Add supersymmetry breaking terms to the supersymmetric Standard Model. These soft supersymmetry breaking terms can in principle be derived from a more fundamental model of supersymmetry breaking, but often they are treated in a purely phenomenological manner.
- The result is an effective Field Theory.
- Supersymmetry breaking is believed to be spontaneous by nonperturbative effects

2. Lorentz breaking

- 1) Lorentz violation can be incorporatted into the Standard Model in a general but phenomenological way: Standard Model Extension: Lorentz breaking terms.
- 2) Presumably the Lorentz symmetry breaking is spontaneous in some underlying fundamental theory.

Supersymmetric Model with Lorentz Violation

MB, Kostelecky, Phys. Rev. D65, 091701 (2002);

MB, Phys. Rev. D68, 085001 (2003).

Add Lorentz-violating terms to the Wess-Zumino supersymmetric model in the spirit of the Standard Model Extension.

Supersymmetry requires terms involving scalars and terms involving fermions, with related coefficients (similar to masses and couplings).

Modifications: supersymmetric transformations, supersymmetry algebra, kinetic terms of the supersymmetric Lagrangian.

The requirement of supersymmetry places strong constraints on the kinds of Lorentz-violating terms that can be added.

Wess-Zumino Model

Lagrangian

$$\mathcal{L} \quad = \frac{1}{2}\partial_\mu A \partial^\mu A + \frac{1}{2}\partial_\mu B \partial^\mu B + \frac{1}{2} i\overline{\psi} \not{\partial}\psi$$

Transforms under the supersymmetry transformations

$$\begin{aligned} \delta A &= \bar{\epsilon}\psi \\ \delta B &= i\bar{\epsilon}\gamma_5\psi \\ \delta\psi &= -i \not{\partial}(A + i\gamma_5 B)\epsilon \\ \delta\overline{\psi} &= i\bar{\epsilon} \not{\partial}(A - i\gamma_5 B) \end{aligned}$$

Into a total derivative

$$\delta\mathcal{L} = \frac{1}{2}\bar{\epsilon} \not{\partial}[\not{\partial}(A + i\gamma_5 B)\psi]$$

Superfield Formulation

Chiral superfield is a function of four spacetime coordinates and four anticommuting coordinates:

$$\begin{aligned}\Phi(x,\theta,\bar{\theta}) &= \phi(y) + \sqrt{2}\theta\psi(y) + (\theta\theta)\mathcal{F}(y)\,, \qquad y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta} \\ &= \phi(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\partial_\mu\partial^\mu\phi(x) \\ &\quad + \sqrt{2}\theta\psi(x) + i\sqrt{2}\theta\sigma^\mu\bar{\theta}\theta\partial_\mu\psi(x) + (\theta\theta)\mathcal{F}(x)\end{aligned}$$

The conjugate of the chiral superfield:

$$\begin{aligned}\Phi^*(x,\theta,\bar{\theta}) &= \phi^*(z) + \sqrt{2}\bar{\theta}\bar{\psi}(z) + (\bar{\theta}\bar{\theta})\mathcal{F}^*(z) \qquad z^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta} \\ &= \phi^*(x) - i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi^*(x) - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\partial_\mu\partial^\mu\phi^*(x) \\ &\quad + \sqrt{2}\bar{\theta}\bar{\psi}(x) + i\sqrt{2}\theta\sigma^\mu\bar{\theta}\bar{\theta}\partial_\mu\bar{\psi}(x) + (\bar{\theta}\bar{\theta})\mathcal{F}^*(x)\end{aligned}$$

The Wess-Zumino Lagrangian can be derived from the superspace integral:

$$\int d^4\theta\Phi^*\Phi + \int d^2\theta\left[\frac{1}{2}m\Phi^2 + \frac{1}{3}g\Phi^3 + h.c.\right]$$

The action of the supersymmetry generators on the superfield is

$$\begin{aligned}\delta_S\Phi(x,\theta,\bar\theta) &= -i(\epsilon Q+\bar\epsilon\bar Q)\Phi(x,\theta,\bar\theta)\\ &= \left[\epsilon^\alpha\partial_\alpha+\bar\epsilon_{\dot\alpha}\bar\partial^{\dot\alpha}+i\theta\sigma^\mu\bar\epsilon\partial_\mu-i\epsilon\sigma^\mu\bar\theta\partial_\mu\right]\Phi(x,\theta,\bar\theta)\end{aligned}$$

In components the Lagrangian is

$$\begin{aligned}\mathcal{L} &= \partial_\mu\phi^*\partial^\mu\phi+\frac{i}{2}[(\partial_\mu\psi)\sigma^\mu\bar\psi+(\partial_\mu\bar\psi)\bar\sigma^\mu\psi]+\mathcal{F}^*\mathcal{F}\\ &\quad +m\left[\phi\mathcal{F}+\phi^*\mathcal{F}^*-\tfrac{1}{2}\psi\psi-\tfrac{1}{2}\bar\psi\bar\psi\right]\\ &\quad +g\left[\phi^2\mathcal{F}+\phi^{*2}\mathcal{F}^*-\phi(\psi\psi)-\phi^*(\bar\psi\bar\psi)\right]\end{aligned}$$

As is well-known, the chiral superfield can be represented as

$$\begin{aligned}&X=(\theta\sigma^\mu\bar\theta)\partial_\mu\\ &U_x\equiv e^{iX}=1+i(\theta\sigma^\mu\bar\theta)\partial_\mu-\frac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\partial_\mu\partial^\mu\\ &\Phi(y,\theta)=U_x\Psi(x,\theta)\\ &\Phi^*(z,\theta)=U_x^*\Psi^*(x,\theta)\end{aligned}$$

Lorentz-violating superfield transformations

In an analogous way one can define two Lorentz-violating extensions of the Wess-Zumino model.

Define

$$X \equiv (\theta\sigma^\mu\bar{\theta})\partial_\mu$$
$$Y \equiv k_{\mu\nu}(\theta\sigma^\mu\bar{\theta})\partial^\nu$$
$$K \equiv k_\mu(\theta\sigma^\mu\bar{\theta})$$

so that

$$U_x \equiv e^{iX} = 1 + i(\theta\sigma^\mu\bar{\theta})\partial_\mu - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\partial^\mu\partial_\mu$$
$$U_y \equiv e^{iY} = 1 + ik_{\mu\nu}(\theta\sigma^\mu\bar{\theta})\partial^\nu - \frac{1}{4}k_{\mu\nu}k^{\mu\rho}(\theta\theta)(\bar{\theta}\bar{\theta})\partial^\nu\partial_\rho$$
$$T_k \equiv e^{-K} = 1 - k_\mu(\theta\sigma^\mu\bar{\theta}) + \frac{k^2}{4}(\theta\theta)(\bar{\theta}\bar{\theta})$$

CPT-Even Model with Lorentz Violation

Following the SM extension, consider adding Lorentz violating terms to the Wess-Zumino model.

Apply the derivative operator effects the substitution $\partial_\mu \longrightarrow \partial_\mu + k_{\mu\nu}\partial^\nu$

And give the superfields:

$$\begin{aligned} \Phi_y(x,\theta,\bar\theta) &= U_y U_x \Psi(x,\theta) \\ \Phi_y^*(x,\theta,\bar\theta) &= U_y^* U_x^* \Psi^*(x,\bar\theta) \\ &= U_y^{-1} U_x^{-1} \Psi^*(x,\bar\theta) \end{aligned}$$

The Lorentz-violating Lagrangian is then given by

$$\int d^4\theta \Phi_y^* \Phi_y + \int d^2\theta \left[\frac{1}{2} m \Phi_y^2 + \frac{1}{3} g \Phi_y^3 + h.c.\right]$$
$$= \int d^4\theta \left[U_y^* \Phi^*\right] \left[U_y \Phi\right] + \int d^2\theta \left[\frac{1}{2} m \Phi^2 + \frac{1}{3} g \Phi^3 + h.c.\right]$$

CPT-Even Model

The resulting Lagrangian in components is

$$
\begin{aligned}
\mathcal{L}_{\mathsf{CPT-even}} &= (\partial_\mu + k_{\mu\nu}\partial^\nu)\phi^*(\partial^\mu + k^{\mu\rho}\partial_\rho)\phi \\
&+\frac{i}{2}[((\partial_\mu + k_{\mu\nu}\partial^\nu)\psi)\sigma^\mu\bar{\psi} + ((\partial_\mu + k_{\mu\nu}\partial^\nu)\bar{\psi})\bar{\sigma}^\mu\psi] + \mathcal{F}^*\mathcal{F} \\
&+m\left[\phi\mathcal{F} + \phi^*\mathcal{F}^* - \tfrac{1}{2}\psi\psi - \tfrac{1}{2}\bar{\psi}\bar{\psi}\right] \\
&+g\left[\phi^2\mathcal{F} + \phi^{*2}\mathcal{F}^* - \phi(\psi\psi) - \phi^*(\bar{\psi}\bar{\psi})\right]
\end{aligned}
$$

The presence of the terms in the extension forces a relationship on the coefficients for Lorentz violation, analogous to the common mass and common couplings that are a standard consequence of supersymmetric theories.

The fermion propagator can be written as

$$
\begin{aligned}
iS_F(p) &= \frac{i}{p_\mu(\gamma^\mu + k_{\mu\nu}\gamma^\nu) - m} \\
&= i\frac{p_\mu(\gamma^\mu + k_{\mu\nu}\gamma^\nu) + m}{p^2 + 2p^\mu p^\nu k_{\mu\nu} + k_{\mu\rho}k^\rho{}_\nu p^\mu p^\nu}
\end{aligned}
$$

The scalar propagator has the same form for the denominator.

CPT-Even model (cont.)

The modified superfields can be understood by defining left-chiral and right-chiral coordinates as

$$x_{\pm}^{\mu} = x^{\mu} \pm i\theta\sigma^{\mu}\overline{\theta} \pm ik^{\mu\nu}\theta\sigma_{\nu}\overline{\theta}$$

The chiral superfield is then simply

$$\begin{aligned} \Phi_y(x,\theta,\overline{\theta}) &= \Phi(x,\theta,\overline{\theta};\partial_\mu \longrightarrow \partial_\mu + k_{\mu\nu}\partial^\nu) \\ &= \phi(x_+) + \sqrt{2}\theta\psi(x_+) + (\theta\theta)F(x_+)\ , \end{aligned}$$

A superfield covariant derivative can also be introduced in analogy with the usual case with the substitution

$$\partial_\mu \longrightarrow \partial_\mu + k_{\mu\nu}\partial^\nu$$

The modified superalgebra is

$$[P_\mu, Q] = 0, \qquad \left\{Q, \overline{Q}\right\} = 2\sigma^\mu P_\mu + 2k_{\mu\nu}\sigma^\mu P^\nu$$

CPT-Odd Model with Lorentz Violation

There exists a CP-odd extension of the Wess-Zumino model. It does not involve a derivative operator, but can be understood in terms of the transformation on superfields.The resulting superfields are

$$\begin{aligned}\Phi_k(x,\theta,\bar\theta) &= T_k U_x \Psi(x,\theta)\\ \Phi_k^*(x,\theta,\bar\theta) &= T_k^* U_x^* \Psi^*(x,\bar\theta)\\ &= T_k U_x^{-1} \Psi^*(x,\bar\theta)\end{aligned}$$

The Lorentz-violating Lagrangian is then given by

$$\int d^4\theta \Phi_k^* \Phi_k = \int d^4\theta \Phi^* e^{-2K} \Phi$$

No mass or couplings terms can be made supersymmetric since only vector superfields

$$\begin{aligned}\Phi_k &= \Phi(\partial_\mu \rightarrow \partial_\mu + ik_\mu)\\ \Phi_k^* &= \Phi(\partial_\mu \rightarrow \partial_\mu - ik_\mu)\end{aligned}$$

CPT-Odd Model (cont.)

The resulting Lagrangian is

$$\begin{aligned}\mathcal{L}_{\mathrm{CPT-odd}} &= [(\partial_\mu - ik_\mu)\phi^*]\,[(\partial^\mu + ik^\mu)\phi] \\ &\quad + \frac{i}{2}[((\partial_\mu + ik_\mu)\psi)\sigma^\mu\bar{\psi} + ((\partial_\mu - ik_\mu)\bar{\psi})\bar{\sigma}^\mu\psi] + \mathcal{F}^*\mathcal{F}\end{aligned}$$

The $(\theta\theta)(\overline{\theta}\overline{\theta})$ component of $\Phi^*\Phi$ no longer transforms into a total derivative. A certain combination of components of $\Phi^*\Phi$ does transform into a total derivative, and this combination can be understood as being the $(\theta\theta)(\overline{\theta}\overline{\theta})$ component of $\Phi_k^*\Phi_k$.

This model does not involve a derivative operator, but the combination of components of the superfield can be projected out using the vector superfield

$$e^{-2K} = e^{-2k_\mu(\theta\sigma^\mu\overline{\theta})}$$

Conclusion

- **Explicit Lorentz violation can be added to the Wess-Zumino model.**

- **By virtue of the Lorentz violation, manifest through the presence of the Lorentz-violating coefficient, the superalgebra**

$$[P_\mu, Q] = 0, \qquad \left\{Q, \overline{Q}\right\} = 2\sigma^\mu P_\mu + 2k_{\mu\nu}\sigma^\mu P^\nu.$$

  lies outside the usual list of possible supersymmetric extensions of the Poincare algebra.

- **Superfield description makes it clear that the vector supermultiplet (i.e. supersymmetric gauge theory) should have a Lorentz-violating extension.**

- **Realistic models should incorporate supersymmetry breaking as well as Lorentz violation. Since the source of spacetime symmetry breaking is not understood, there may be some relationship between the breaking of supersymmetry and possible breaking of the Lorentz symmetry.**