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CONFERENCE ON FUNDAMENTAL SYMMETRIES AND FUNDAMENTAL CONSTANTS

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GAUGE ANOMALIES AND EXTRA DIMENSIONS

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Gauge anomalies and extra dimensions

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Outline

- Chiral anomalies
- ”Anomalous" electrodynamics in 4D
- Gauge theories from extra dimensions
- QED from six-dimensional vortex
- Effective theory and gauge anomalies
- Conclusions

Chiral anomalies

General principles of model building in particle physics:

- Lorentz invariance
- Renormalizability
- Gauge symmetry
- Higgs mechanism

Particular situation: symmetry is exact on classical level but is broken on quantum level.

Famous example: Adler-Bell-Jackiw chiral anomaly

Chiral anomalies

Simplest case

Consider Quantum Electrodynamics of massless fermions:

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \gamma^{\nu} D_{\nu} \psi, \quad D_{\nu} = \partial_{\nu} - ieA_{\nu}$$

Two conserved (on classical level) currents:

gauge current

$$J_{\mu} = \bar{\psi} \gamma_{\mu} \psi$$

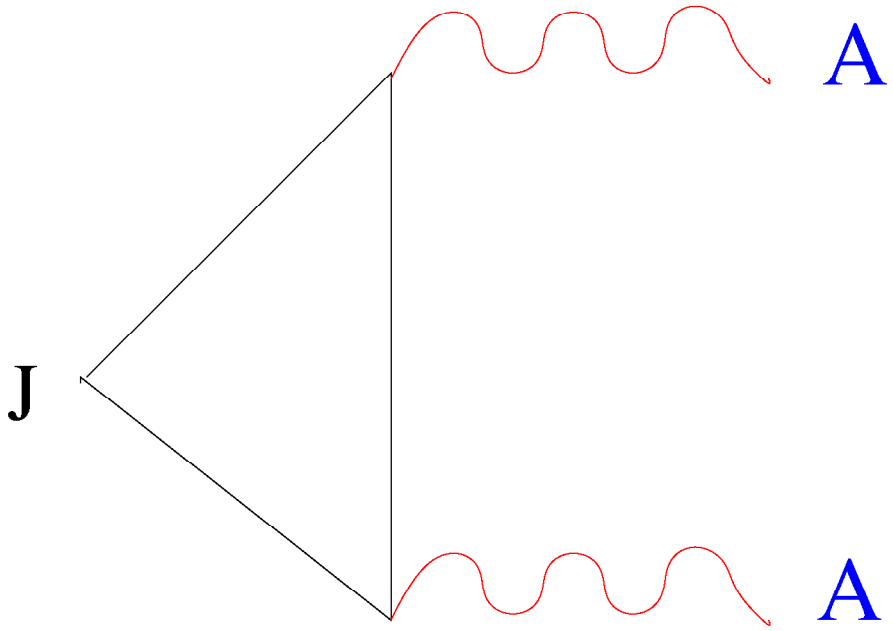
and global current

$$J^{5}_{\mu} = \bar{\psi} \gamma_{\mu} \gamma_{5} \psi$$

Chiral anomalies

Quantum corrections incorporated:

$$\partial^\mu J_\mu = 0, \quad \partial^\mu J^5_\mu = \frac{e^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$



Physical consequences

- QCD and electrodynamics:

  $\pi^0 \to \gamma\gamma$ decay

- Electroweak theory:

  Lepton and baryon number non-conservation

  Baryon asymmetry of the Universe (?)

Gauge anomalies

If J^5_μ is a gauge current: theory may be inconsistent!

Anomaly cancellation in Abelian U(1) theory:

$$\sum e_L^3 = \sum e_R^3$$

Electroweak theory:

$$\sum Y_L^3 = \sum Y_R^3$$

U(1) anomaly cancellation

$$\sum Y_L = \sum Y_R$$

mixed SU(2)$\times$U(1) anomaly cancellation

"Anomalous" electrodynamics in 4D

Is there any sense in anomalous theories?

What if $\sum e_L^3 - \sum e_R^3 = 10^{-100}$???

A way to approach this problem: Construct an anomaly free theory the low energy limit of which looks like anomalous theory.

Example of parity breaking electrodynamics:

Fermions:

$$\psi_1^L: \quad Q = 1 + \epsilon/2, \quad \psi_2^L: \quad Q = 1 - \epsilon/2,$$

$$\chi_1^R: \quad Q = 1 - \epsilon/2, \quad \chi_2^Q: \quad Q = 1 + \epsilon/2,$$

Scalar: $\phi: \quad Q = \epsilon.$

Anomalous QED

Lagrangian:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum\left(\bar{\psi}i\gamma^{\nu}D_{\nu}\psi + \bar{\chi}i\gamma^{\nu}D_{\nu}\chi\right)$$

$$+f_1\bar{\psi}_1\chi_1\phi + f_2\bar{\psi}_2\chi_2\phi^* + h.c.$$

$$+|D_{\mu}\phi|^2 - \frac{\lambda}{4}(|\phi|^2 - v^2)^2,$$

Let: $f_1 \ll f_2, \;\; \epsilon \ll 1, \;\; \lambda \sim f_2^2 \sim 1$

Light sector: fermion $\psi = \psi_1 + \chi_1$ and photon

Heavy sector: fermion $\chi = \psi_2 + \chi_2$ and scalar

Effective theory

Low energy Lagrangian:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\gamma^2 A_\mu A^\mu$$

$$+\bar{\psi}i\gamma^\mu\left(\partial_\mu - ie(1+\frac{\epsilon}{2}\gamma_5 A_\mu)\right)\psi + m\bar{\psi}\psi$$

Looks like a bad theory: anomaly + photon mass + explicit breaking of gauge invariance due to fermion mass.

But heavy sector makes it consistent!

Physical consequences

Parity breaking in QED:

- massive photon
- “new" physics at mass scales m_γ/ϵ

Numerology:

$m_\gamma < 3 \times 10^{-36}$ GeV (galactic magnetic fields)

$m_\gamma < 6 \times 10^{-26}$ GeV (MHD of solar wind)

$\Rightarrow \epsilon < 10^{-36}$!

Electron-proton mass difference:

$\frac{q_p+q_e}{e} < 10^{-21}$

Gauge theories from extra dimensions

- Odd number of dimensions - vector-like theories - no anomalies
- Chiral fermions as zero modes on branes (field theory - on topological defects like domain walls, strings, monopoles)

Extra challenge - localization of gauge fields : the theory must look like 4d gauge theory at low energies.

Possibility: localization due to gravity (a'la Randall-Sundrum for graviton)

Localization of gauge fields

Warped metric:

$$ds^2 = \sigma(y^a)\eta_{\mu\nu}dx^\mu dx^\nu + g(y^a)_{ab}dy^a dy^b$$

Relevant part of gauge Lagrangian

$$\int d^4x \int d^n y \sigma^2 \sigma^{-2} \sqrt{(g(y^a))} F_{\mu\nu}F^{\mu\nu}$$

Localization of gauge fields: $\int d^n y \sqrt{(g(y^a))} < \infty$

Localization of graviton: $\int d^n y \sigma \sqrt{(g(y^a))} < \infty$

$n = 1$, $D = 5$ - does not work

$n = 2$, $D = 6$ - may work

QED from six-dimensional vortex

Theory in 6D: Gravity + charged complex scalar + U(1) gauge field + fermions, anomaly free

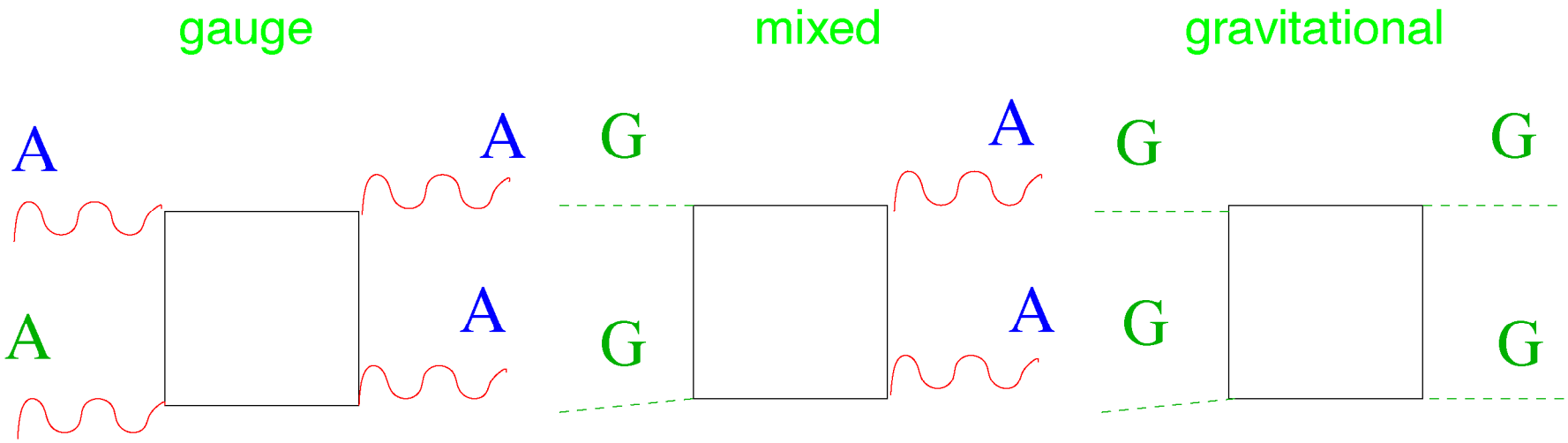
Effective theory in 4D: Gravity + U(1) gauge field + fermions, may be anomalous

Action:

$$S = \int \mathrm{d}^6x\sqrt{-G}\left\{\frac{1}{\kappa^2}R - \frac{1}{4}F_{MN}F^{MN} + (D_M\Phi)^\dagger D^M\Phi - U(\Phi)\right.$$

$$\left. + \sum_{i=1} \bar{\Psi}_i\Gamma^A E_A^M \nabla_M \Psi_i + \text{h.c.}\right\}$$

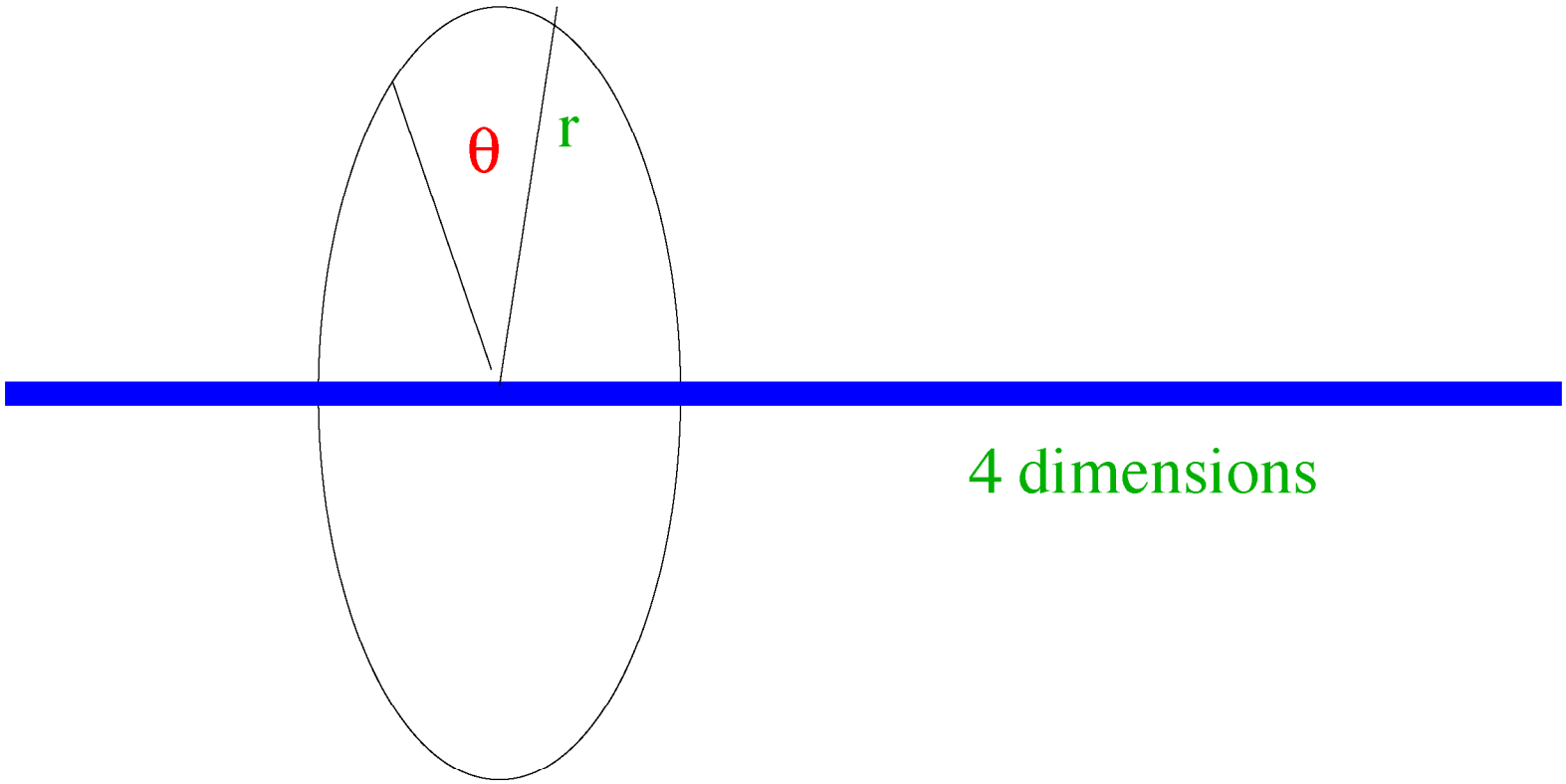
Anomalies in 6D



$$\sum e_L^4 = \sum e_R^4 \qquad \sum e_L^2 = \sum e_R^2 \qquad \sum e_L^0 = \sum e_R^0 = n_F$$

Vortex solution

Abrikosov line (Nielsen-Olesen string) residing in 6D



Vortex solution

$$
\begin{aligned}
\mathrm{d}s^2 &= \mathrm{e}^{A(r)}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}r^2 + \mathrm{e}^{B(r)}a^2\mathrm{d}\theta^2 \,, \\
\Phi &= f(r)\mathrm{e}^{\mathrm{i}n\theta}, \qquad aeA_\theta = (P(r)-n)\mathrm{d}\theta \,,
\end{aligned}
$$

boundary conditions

$$
\begin{aligned}
f(0) = 0 \,, \qquad & f(\infty) = f_0 \neq 0 \,, \\
P(0) = n \,, \qquad & P(\infty) = 0. \\
& A(0) = 1, \qquad B(r\to 0) = 2\ln\frac{r}{a} \,, \\
& A(r\to\infty) = B(r\to\infty) = -2cr, \quad c>0.
\end{aligned}
$$

U(1) gauge field

"Initial" symmetry U(1) (isometry)$\times$ U(1) (gauge) is broken down to "diagonal" U(1) by the Higgs and gauge backgrounds:

$$V_\mu = \frac{1}{ae} P(r) W_\mu(x,r), \quad h_{\mu\theta} = \mathrm{e}^{B(r)} W_\mu(x,r),$$

$W_\mu(x,r)$ is localized on a string (but - no mass gap for the bulk modes)

Fermion modes

- Fermion zero modes are localized on a string by gauge and gravitational interactions (no Yukawa coupling is needed !)
- Bulk fermions charged with respect to U(1) are massive (in the contrary to the case of domain wall residing in AdS)
- For $n_F = 1, 2$ 6D theory and effective 4D theories are vector-like
- For the vortex with topological number n the number of left-handed zero modes originating from a left 6D fermion with charge e_L is $N_L = [\frac{e_L}{e} n - \frac{1}{2}]$. They have 4D charges $\frac{1}{2} + m - n\frac{e_L}{e}$, where m is integer, $m = 0, \ldots N_L$

Effective theory and gauge anomalies

- For integer fermionic charges in 6D four-dimensional theory may be chiral but is anomaly free (i.e. $\sum e_L^3 = \sum e_R^3$)
- For fractional but anomaly free fermionic content in 6D the resulting 4D theory may be anomalous

Conclusions

Main question: physical consequences of anomalous theory coming from extra dimensions

- Dynamical photon mass generation?
- Electric charge non-conservation?

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But believe that have a good theory at hand in which this problem can be addressed!