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CONFERENCE ON FUNDAMENTAL SYMMETRIES AND FUNDAMENTAL CONSTANTS

15 - 18 September 2004

NONTRIVIAL TOPOLOGY AND CPT VIOLATION

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Fundamental Symmetries and Fundamental Constants
ICTP Trieste, September 15–18, 2004

Nontrivial spacetime topology and CPT violation

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1. INTRODUCTION

Experiment has shown the violation of P, C, CP, and T, but not the combination CPT.

Indeed, there is the well-known CPT "theorem" [Lüders, 1954–57; Pauli, 1955; Bell, 1955; Jost, 1957]:

any local relativistic quantum field theory is invariant under the combined operation of charge conjugation (C), parity reflection (P), and time reversal (T).

The main inputs for this "theorem" are:

- flat spacetime $(M,\, g) = \big(\mathbb{R}^4,\, \eta_{\mu\nu}^{\text{Minkowski}}\big)$;
- invariance under proper orthochronous Lorentz transformations and spacetime translations;
- normal spin-statistics connection;
- locality and Hermiticity of the Hamiltonian.

BUT <u>CAN</u> CPT INVARIANCE BE VIOLATED AT ALL
IN A PHYSICAL THEORY
AND, IF SO, <u>IS</u> IT IN THE REAL WORLD?

It was widely believed that only quantum gravity effects or superstrings could give CPT violation.

A different result was, however, obtained recently [1,2]:

for certain spacetime topologies and certain classes of chiral gauge theories, CPT invariance is broken anomalously, that is, by quantum effects.

Crucial ingredients of the CPT anomaly are:

- chiral fermions and gauge interactions;
- nontrivial spacetime topology.

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[1] Klinkhamer, NPB 578 (2000) 277 [hep-th/9912169].

[2] K, in: *Proc. 7-th Int. Wigner Symp.* [hep-th/0110135].

A concrete example of an "anomalous" theory:

- chiral gauge theory $G = SO(10)$, $R_{\text{left}} = 3 \times (\mathbf{16})$,
- manifold $M = \mathbb{R}^3 \times S^1_{\text{PSS}}$, $e^a_\mu(x) = \delta^a_\mu$,

where PSS stands for periodic spin structure.

Note that this example incorporates the Standard Model with $N_{\text{fam}} = 3$ families of quarks and leptons.

Three possible applications:

• *optical activity of the vacuum*, e.g., for the CMB [2];

• *fundamental arrow-of-time*, e.g., for the Big Bang [3];

• *spacetime foam*, with CPT anomaly as diagnostic tool [4].

In this talk, we summarize the basic idea of the CPT anomaly and focus on the last application.

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[3] K, PRD 66 (2002) 047701 [gr-qc/0111090].

[4] K & Rupp, PRD 70 (2004) 045020 [hep-th/0312032].

2. CPT ANOMALY FOR $M = \mathbb{R}^3 \times S^1$

2.1 HEURISTICS

The main ingredients of the CPT anomaly for the 4D manifold $M = \mathbb{R}^3 \times S^1$ are:

- compact spacelike dimension ($x^3 \in [0, L]$) with periodic spin structure (i.e., the fermions can have momentum $p_3 = 0$);
- a single chiral fermion with $p_3 = 0$ corresponds to a single massless Dirac fermion in 3D;
- a single massless Dirac fermion in 3D has a "parity anomaly," provided gauge invariance is maintained exactly [5,6];
- this "parity" violation corresponds to T violation in 4D.

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$[5]$ Redlich, PRL 52 (1984) 18; PRD 29 (1984) 2366.

$[6]$ Alvarez-Gaumé & Witten, NPB 234 (1984) 269.

2.2 RIGOROUS RESULT ON THE LATTICE

Consider the 4D Abelian chiral gauge theory with

$$\begin{aligned} G &= U(1), \\ R_{\text{left}} &= 6 \times (1/3) + 3 \times (-4/3) + 3 \times (2/3) + \\ & \quad 2 \times (-1) + 1 \times (2) + 1 \times (0) \ , \end{aligned} \tag{1}$$

embedded in the $SU(2) \times U(1) \subset SO(10)$ theory $(Y \equiv 2\,Q - 2\,T_3)$. Chiral gauge anomalies cancel out.

Next, introduce a finite hypercubic lattice and define a chiral lattice gauge theory with

- periodic spin structure in one direction;
- Ginsparg-Wilson fermions;
- Neuberger's lattice Dirac operator;
- Lüscher's chiral constraints.

The goal is to establish that the Euclidean effective gauge field action changes under a CPT transformation,

$$\Gamma[U] \neq \Gamma[U^{\mathsf{CPT}}]\,, \tag{2}$$

where U denotes the set of link variables.

This has been shown [7] for an arbitrary odd number N of links in the periodic direction and for arbitrary values of the lattice spacing a (compact dimension has $L = Na$).

Moreover, the origin of the CPT anomaly has been identified as an ambiguity in the choice of basis vectors needed to define the fermion integration measure; cf. [8].

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[7] K & Schimmel, NPB 639 (2002) 241 [hep-th/0205038].

[8] Fujikawa, PRD 21 (1980) 2848.

2.3 PERTURBATIVE RESULT IN THE CONTINUUM

Return to the $SO(10)$ chiral gauge theory with $N_{\text{fam}} = 3$ and the spacetime manifold $\mathbb{R}^3 \times S^1_{\text{PSS}}$ with a Lorentzian signature of the metric and spacelike compact dimension.

The effective action $\Gamma[B]$, for $B \in \text{so}(10)$, is not known exactly (only in 2D are there exact results [9]).

But, the crucial term has been identified perturbatively [1]:

$$\Gamma^{\mathbb{R}^3 \times S^1}_{\text{anom}}[B\,] \;=\; \int_{\mathbb{R}^3} \mathrm{d}x^0 \mathrm{d}x^1 \mathrm{d}x^2 \int_0^L \mathrm{d}x^3 \;\; n\, x^3/L \\ \times\;(1/32\pi)\;\epsilon^{\kappa\lambda\mu\nu}\;\mathrm{tr}\;F_{\kappa\lambda}(x)\,F_{\mu\nu}(x)\;, \tag{3}$$

with an odd integer n and the Yang–Mills field strength $F_{\kappa\lambda}(x) \equiv \partial_\kappa B_\lambda(x) - \partial_\lambda B_\kappa(x) + [B_\kappa(x), B_\lambda(x)]$.

This local term is <u>Lorentz- and CPT-noninvariant</u>, because of the spacetime-dependent “coupling constant” x^3/L.

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[9] K & Nishimura, PRD 63 (2001) 097701 [hep-th/0006154].

3. STATIC SPACETIME FOAM

3.1 CPT ANOMALY FOR A SINGLE WORMHOLE

Consider an orientable three-space N_3 with a wormhole constructed by removing two identical open balls from $\mathbb{R}^3$ and identifying their surfaces. These balls have diameter b and are separated by a distance d in $\mathbb{R}^3$.

The length of the wormhole "throat" is then zero, whereas the long distance between the wormhole "mouths" is d.

As a particularly simple case, the "width" b of the wormhole mouths is put to zero. The three-space N_3 is then $\mathbb{R}^3$ with two points identified, which are denoted by $\vec{x} = \pm(d/2)\,\hat{e}$, for some unit vector $\hat{e}$.

Next, define

$$\Phi(\vec{x}) \equiv \arctan\left(\frac{d}{|\vec{x}-(d/2)\,\hat{e}|} - \frac{d}{|\vec{x}+(d/2)\,\hat{e}|}\right), \quad (4)$$

and introduce the coordinate $\eta \equiv \Phi(\vec{x})$.

For our purpose, it suffices to establish the CPT anomaly for one particular class of gauge fields. Take the 4D gauge fields over $N = \mathbb{R} \times N_3$ to be independent of η and without component in the direction of η. These fields will be indicated by a prime.

For the electromagnetic component A'_μ, the anomalous contribution to the effective action is then proportional to

$$\int_N \mathrm{d}^4x \;\; \frac{\alpha\,\Phi(x)}{\pi} \;\; \epsilon^{\kappa\lambda\mu\nu}\, F'_{\kappa\lambda}(x)\, F'_{\mu\nu}(x)\,, \tag{5}$$

with field strength $F'_{\mu\nu}(x) \equiv \partial_\mu A'_\nu(x) - \partial_\nu A'_\mu(x)$ and fine-structure constant $\alpha \approx 1/137$.

The structure of the term (5) is of the same form as (3):

$$\int_{\mathbb{R}^4} \mathrm{d}^4x \;\, f_N(x; A] \;\; \epsilon^{\kappa\lambda\mu\nu}\, F_{\kappa\lambda}(x)\, F_{\mu\nu}(x)\,, \tag{6}$$

with field strength $F_{\mu\nu}(x) \equiv \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$ and integration domain extended to $\mathbb{R}^4$

The factor $f_N(x; A]$ is both a function of the spacetime coordinates x^μ and a gauge-invariant functional of the gauge field $A_\mu(x)$.

3.2 PHOTON MODEL AND DISPERSION LAW

Introduce a random (time-independent) background field g to mimic the anomalous effects of a multiply connected (static) spacetime foam, generalizing the result (6) for a single wormhole.

The photon model is defined by the action

$$I_{\text{photon}} = -\frac{1}{4}\int_{\mathbb{R}^4} \mathrm{d}^4x \, \Big(F_{\mu\nu}(x) F^{\mu\nu}(x) + g(x)\, F_{\kappa\lambda}(x) \widetilde{F}^{\kappa\lambda}(x) \Big) \, , \tag{7}$$

where the Maxwell field strength tensor $F_{\mu\nu}$ and its dual $\widetilde{F}^{\kappa\lambda}$ are given by

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \, , \quad \widetilde{F}^{\kappa\lambda} \equiv \tfrac{1}{2}\, \epsilon^{\kappa\lambda\mu\nu}\, F_{\mu\nu} \, ,$$

with $\epsilon^{\kappa\lambda\mu\nu}$ the Levi–Civita symbol.

Models of the type (7) have been considered before, but only for coupling constants $g(x)$ varying smoothly over cosmological scales [10,11].

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[10] Carroll, Field, & Jackiw, PRD 41 (1990) 1231.

[11] Kostelecký, Lehnert, & Perry, PRD 68 (2003) 123511.

Assumed properties of the background field $g(x)$:

1. g is time-independent, $g = g(\vec{x})$,
2. g is weak, $|g(\vec{x})| = \mathrm{O}(\alpha) \ll 1$,
3. average of $g(\vec{x})$ vanishes in the large volume limit,
4. $g(\vec{x})$ varies over length scales which are small compared to the considered wavelengths of the photon field A_μ ,
5. autocorrelation function of $g(\vec{x})$ is finite and isotropic, and drops off "fast enough" at large distances.

The modified Maxwell equation (in Lorentz gauge) reads:

$$\Box\, A^\nu(x) = -\partial_\mu g(x)\, \widetilde{F}^{\mu\nu}(x)\,, \tag{8}$$

The dispersion law of the transverse modes is found by expanding the solution to second order in g, under the assumption that the power spectrum of g vanishes for momenta $|\vec{q}| < q_{\text{low}}$ and that the photons have $|\vec{k}| < q_{\text{low}}/2$.

The result is

$$\omega^2 = c_{\text{ren}}^2\, k^2 - c_{\text{ren}}^2\, \alpha^2\, l_{\text{foam}}^2\, k^4 + \text{O}(\alpha^4, k^6)\,, \qquad (9)$$

with the renormalized light velocity

$$c_{\text{ren}} \equiv\; c\,\sqrt{1-\alpha^2\,\gamma} \qquad (10)$$

and the simplified notation $k \equiv |\vec{k}|$.

The constants γ and l_{foam} are given by

$$\gamma \;\;=\;\; \frac{\pi}{18\,\alpha^2}\, C(0)\,, \qquad (11a)$$

$$l_{\text{foam}}^2 \;\;=\;\; \frac{2\pi}{15\,\alpha^2} \int_0^\infty \text{d}x\;\, x\, C(x)\,, \qquad (11b)$$

in terms of the isotropic autocorrelation function

$$C(x) = \widehat{C}(\vec{x})\,, \quad \text{for} \;\; x = |\vec{x}|\,,$$

with general definition

$$\widehat{C}(\vec{x}) \equiv \lim_{R\to\infty} \frac{1}{(4\pi/3)R^3} \int\limits_{|\vec{y}|<R} \text{d}^3\vec{y}\;\, g(\vec{y})\, g(\vec{y}+\vec{x})\,.$$

3.3 EXPERIMENTAL LIMIT ON l_{foam}

Calculate group velocity $v_{\text{g}}(k) \equiv \text{d}\omega/\text{d}k$ from (9).

Relative change between wave numbers k_1 and k_2 is

$$\left.\frac{\Delta c}{c}\right|_{k_1,k_2} \equiv \left|\frac{v_{\text{g}}(k_1) - v_{\text{g}}(k_2)}{v_{\text{g}}(k_1)}\right| \sim 2\left|k_1^2 - k_2^2\right| \alpha^2\, l_{\text{foam}}^2\,. \tag{12}$$

It has been suggested [12] to use the lack of time dispersion in GRBs to get an upper bound on $\Delta c/c$. For a particular TeV gamma-ray flare of the active galaxy Markarian 421 [13], this gives:

$$\left.\frac{\Delta c}{c}\right|^{\text{Mkn 421}}_{\substack{k_1=2.5\times10^{16}\ \text{cm}^{-1}\\ k_2=1.0\times10^{17}\ \text{cm}^{-1}}} < 2.5\times10^{-14}\,, \tag{13}$$

where the right-hand side is simply the ratio of the binning interval for the gamma-ray events ($\Delta t \approx 280$ s) over the inferred travel time ($D/c \approx 1.1\times10^{16}$ s).

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[12] Amelino-Camelia *et al.*, Nature 393 (1998) 763.

[13] Biller *et al.*, PRL 83(1999) 2108.

Combining our theoretical expression (12) and the astrophysical bound (13), we have the "experimental" limit:

$$l_{\text{foam}} < 1.6 \times 10^{-22}\ \text{cm}\,, \qquad (14)$$

for the length l_{foam} defined by (11b).

Two comments:

1. For a frozen ensemble of wormholes with average separation a between the different wormholes and transverse width l_h for the individual wormholes, one has: $l_{\text{foam}} \sim a\,(l_h/a)^{5/2}$.

2. The upper bound (14) is, of course, *very* far above the Planck length, $l_{\text{Planck}} \equiv \sqrt{G\hbar/c^3} \approx 1.6 \times 10^{-33}$ cm.

But it should be realized that we have no *real* understanding of the possible topologies of spacetime, be it at the very smallest scale or the very largest.

It is even possible that l_{foam} and l_{Planck} are unrelated.

Hence, the importance of the independent limit (14), which is already a factor 10^4 below the length scales to be probed by LHC ($\sqrt{s} \sim 10$ TeV).

4. CONCLUSIONS

The subtle role of topology on local properties of quantum field theory is well-known (e.g., the Casimir effect).

For certain chiral gauge theories, the interplay of UV and IR effects may also lead to:

Lorentz and CPT noninvariance,

even for flat spacetime manifolds, that is, without gravity.

Previously, we started from a given large-scale topology (e.g., $\mathbb{R} \times T^3_{\text{PSS}}$) and looked for new effects in the photon physics (e.g., birefringence [2,9] and photon decay [14]).

Here, we have gone the other way. Namely, the anomaly was used to probe the small-scale structure of spacetime.

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[14] Adam & K, NPB 657 (2003) 214 [hep-th/0212028].

Perhaps the simplest case would be a static multiply-connected spacetime foam consisting of many randomly-oriented, localized defects (e.g., wormholes, punctures).

Concentrating on the two simplest terms of the effective action for the photons, we have calculated the dispersion law in the long-wavelength limit.

Combined with experimental results from gamma-ray astronomy, we have obtained an upper bound on the typical length scale of the (static) spacetime foam.

More important than this particular bound is the general idea:

spacetime topology affects the second-quantized vacuum and fundamental symmetries of chiral gauge theory, which, in turn, allows us to probe certain properties of spacetime.